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# Collective dynamics of polarized spin-half fermions in relativistic heavy-ion collisions

IJMPA, 38 (20)  APPB 54  PRD 105

PRD 103  PRD 102  PRC 99

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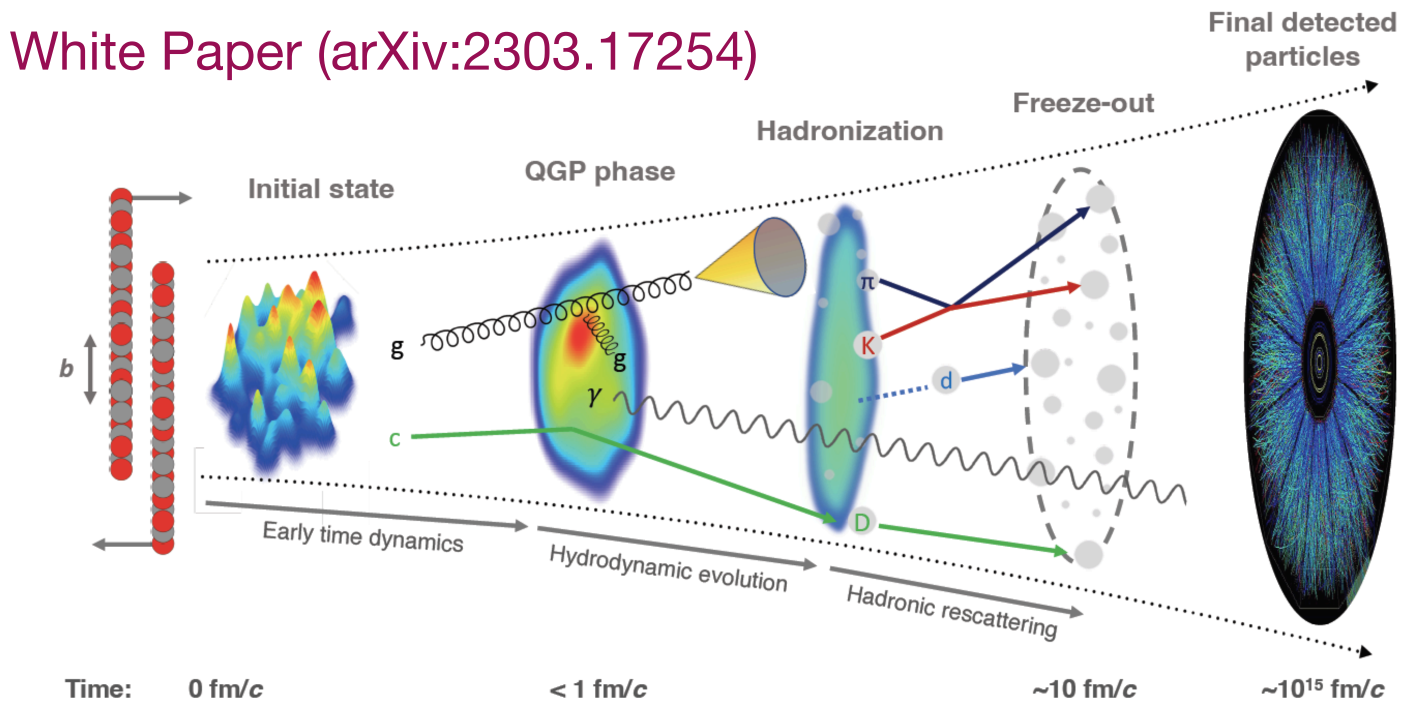
25 Sep 2023

SPIN 2023

# Basics

Hot QCD White Paper (arXiv:2303.17254)




Phys.Rev.Lett. 94 (2005) 102301, Phys. Rev. C 77, 024906

→ Non-central UR-HIC, due to spatial inhomogeneity, create large OAM, $\boldsymbol{L}_{\text{initial}} \approx 10^5\,\hbar$.

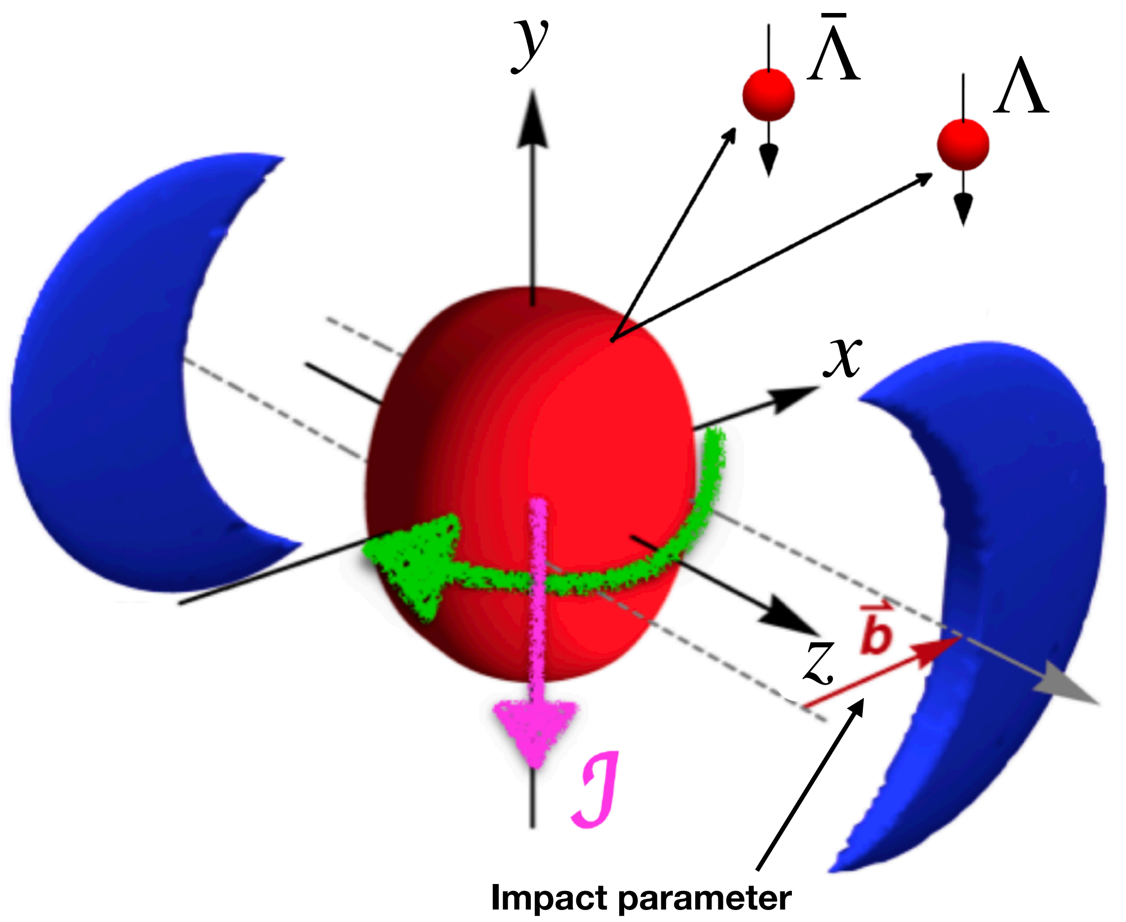
→ This OAM is along $y$-axis (orthogonal to reaction plane) & may polarize spin of the QGP constituents.

Phys. Rev. Lett. 94 (2005) 102301

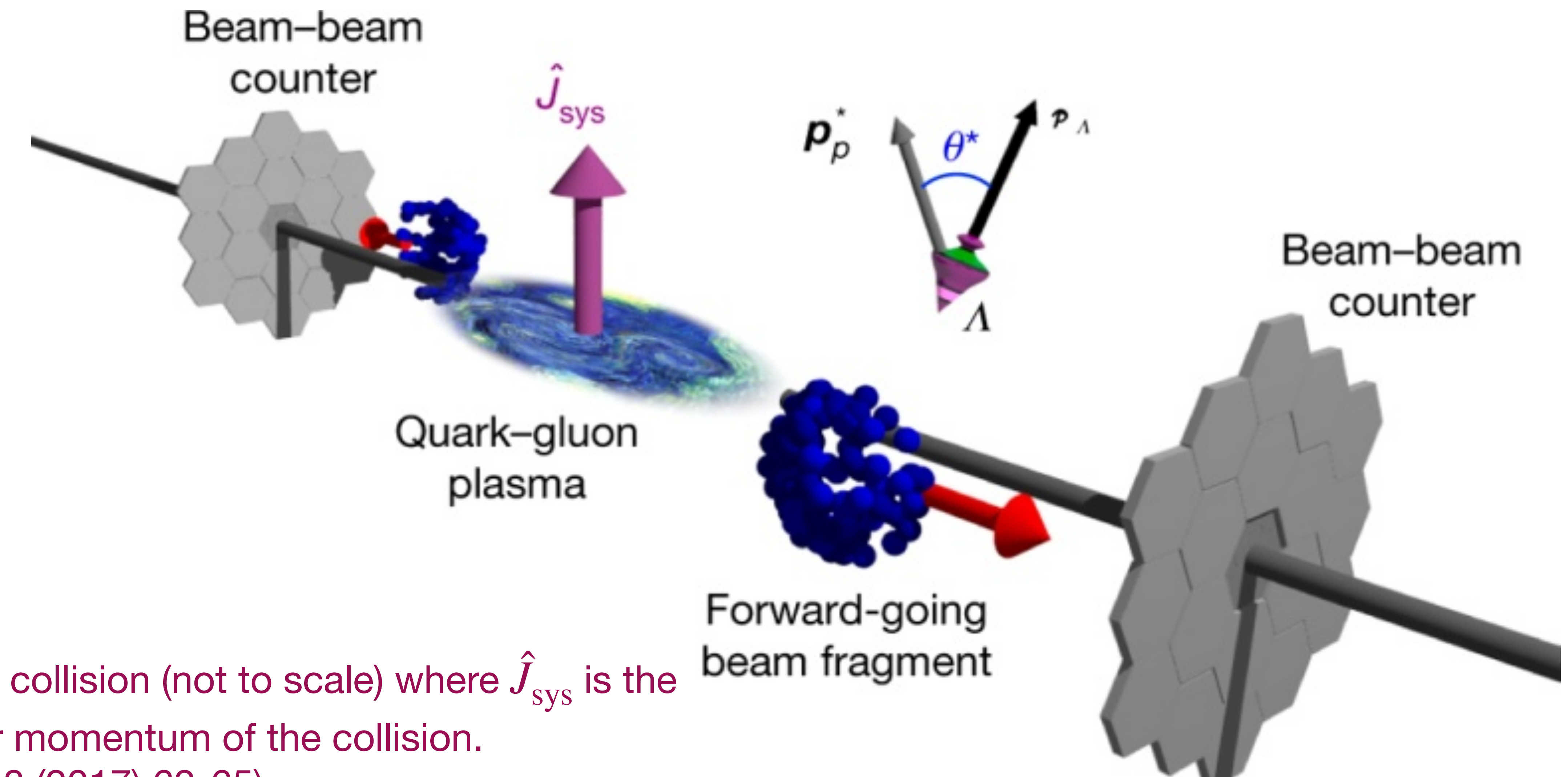
→ Spin polarization is expected to be transferred to the hadrons leading to their global spin polarization.

Predicted in 2005

→ Among various spin-polarizable hadrons, $\Lambda(\bar{\Lambda})$ hyperons are special as they are self-analyzing.




Schematic diagram of the initial angular momentum orientation in non-central heavy-ion collision.
(Prog.Part.Nucl.Phys. 108 (2019) 103709)



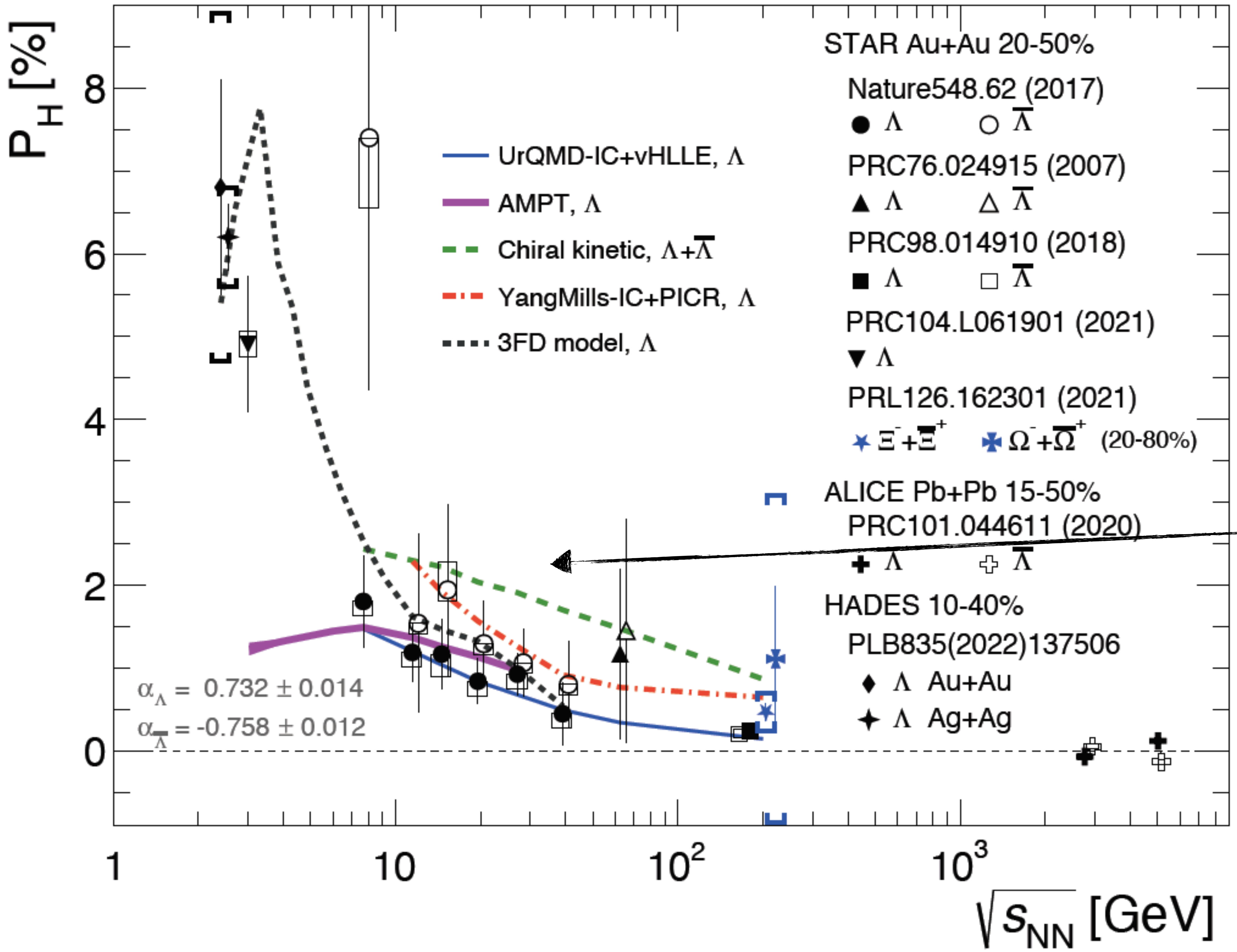

Schematic diagram of a $\mathrm{Au}+\mathrm{Au}$ collision (not to scale) where $\hat{J}_{\text{sys}}$ is the direction of the angular momentum of the collision.
(Nature 548 (2017) 62-65)

# Motivation

→ Observation of $\Lambda(\bar{\Lambda})$ global spin polarization provided evidence of QGP vortical structure.

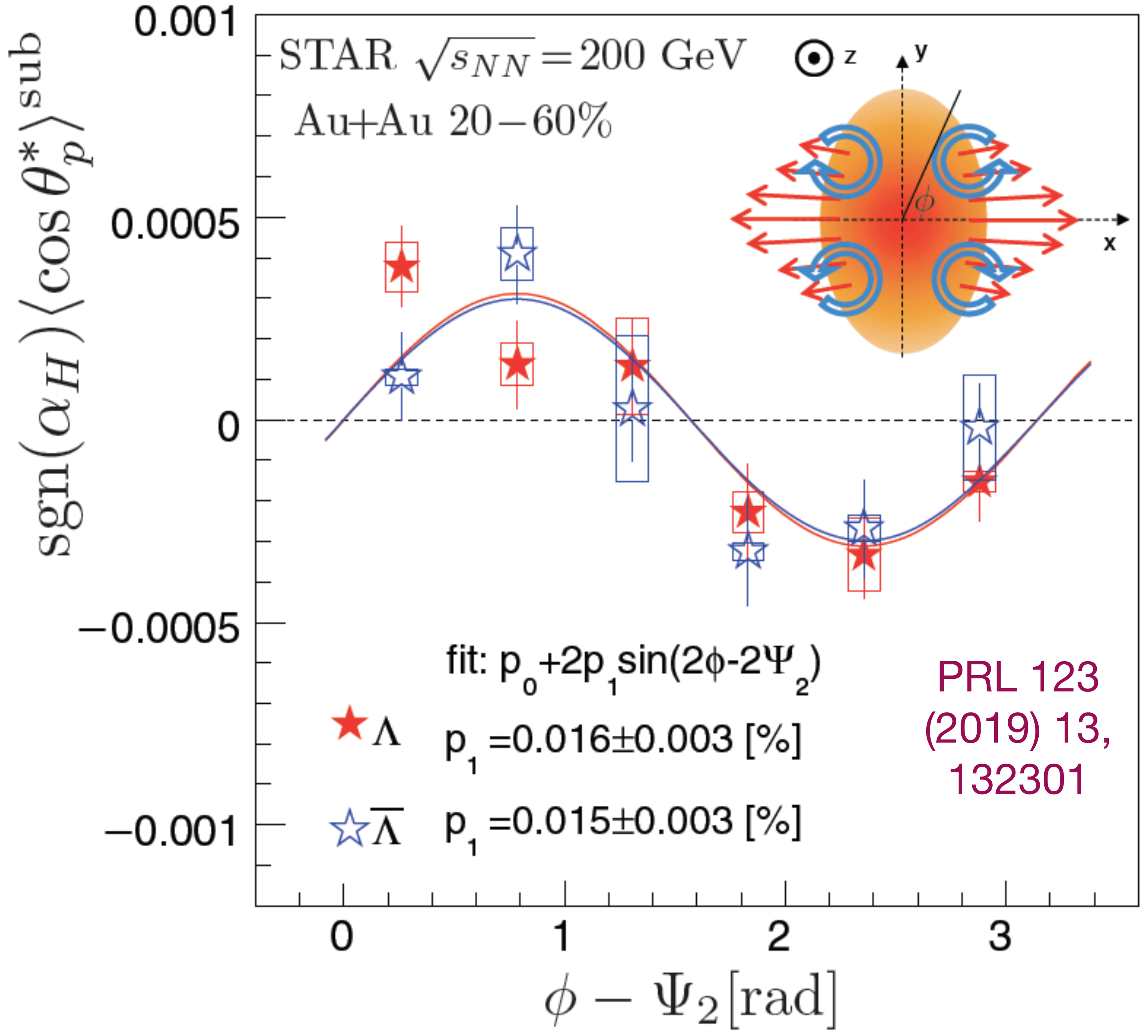
→ Shows decreasing behavior with increase in $\sqrt{s_{\mathrm{NN}}}$.

→ Differences between $\Lambda - \bar{\Lambda}$ polarization may be due to initial EM fields caused during the collisions.

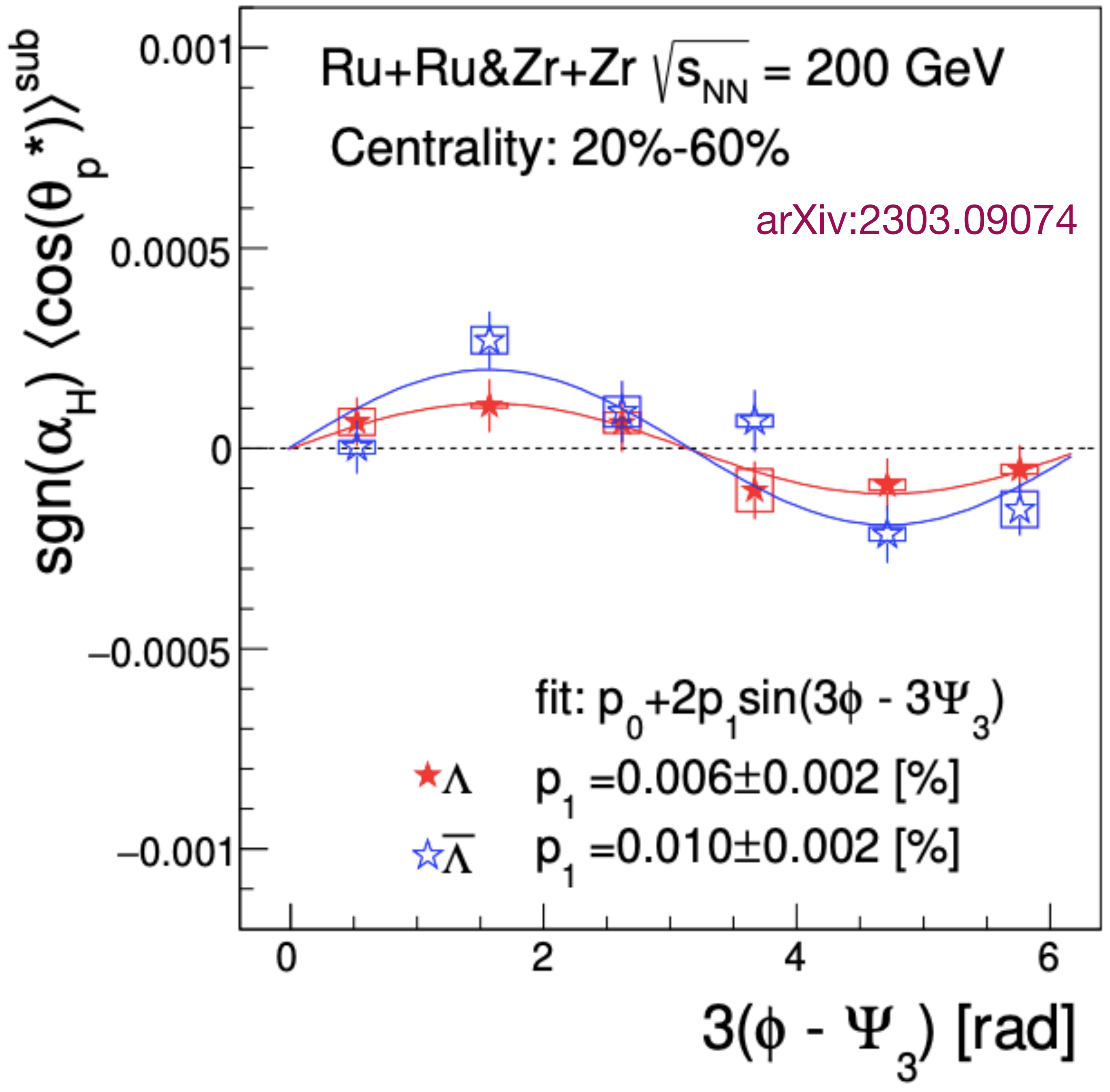



Finite differences (Open question!)

→ Also observed spin polarization along the beam dir. which may result from the transverse plane flow structure.




PRL 123 (2019) 13, 132301



# Outline

What do I mean?

→ Why we need spin hydrodynamics?

→ Pseudogauge transformations

→ GLW pseudogauge based spin hydro

# Developments towards hydrodynamics with spin

## Lagrangian effective field theory approach

D. Montenegro, G. Torrieri, Phys.Rev. D94 (2016) no.6, 065042
D. Montenegro, L. Tinti, G. Torrieri, Phys. Rev. D 96(5) (2017) 056012; Phys. Rev. D 96(7) (2017) 076016
D. Montenegro, G. Torrieri, Phys. Rev. D 100, 056011 (2019)

## Hydrodyna

## Hydrodyna

## Relativistic … on analysis

| | Citeable | Published |
|---|---|---|
| Papers | **446** | **365** |
| Citations | **10,862** | **10,289** |
| h-index | **58** | **58** |
| Citations/paper (avg) | **24.4** | **28.2** |

## Relativistic viscous spin hydrodynamics from chiral kinetic theory

S. Shi, C. Gale, and S. Jeon, Phys. Rev. C 103, 044906 (2021)

## Spin polarization generation from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-l. Sheng, Q. Wang, and D. H. Rischke, arXiv:2005.01506, arXiv:2103.04896

## Spin polarisation due to thermal shear

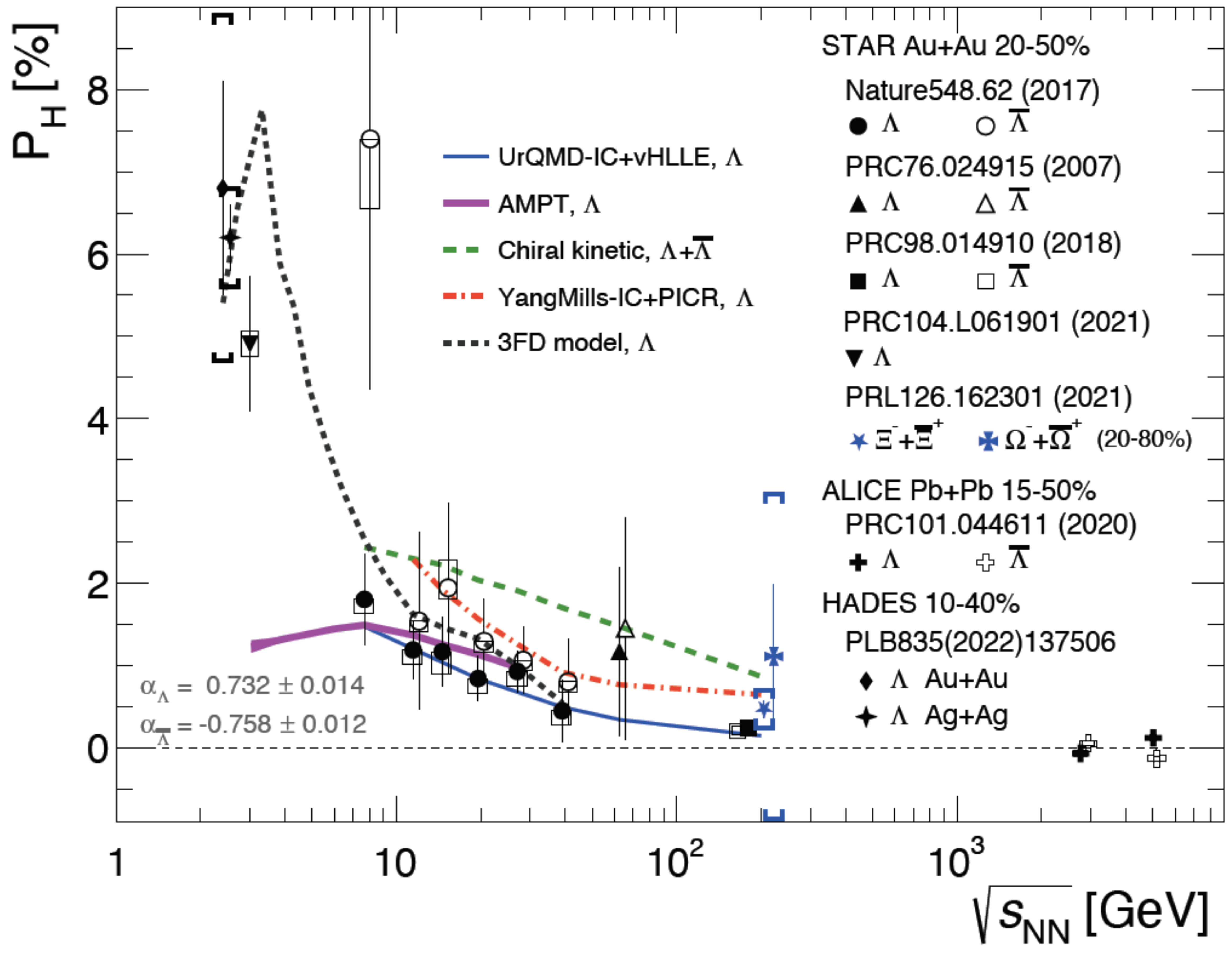
F. Becattini, M. Buzzegoli, and A. Palermo, arXiv:2103.10917
S. Y. F. Liu and Y. Yin, arXiv:2103.09200

Not enough space to include all papers. I apologize!

# Why we need spin hydro?

→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.

What does it mean?




Average global spin polarization for $\Lambda(\bar{\Lambda})$ hyperons in 20-50% centrality $\mathrm{Au}+\mathrm{Au}$ collisions as a function of collision energy. (Nature 548 (2017) 62-65)

# Why we need spin hydro?

**In local thermodynamic equilibrium, one can establish a link between spin and thermal vorticity**

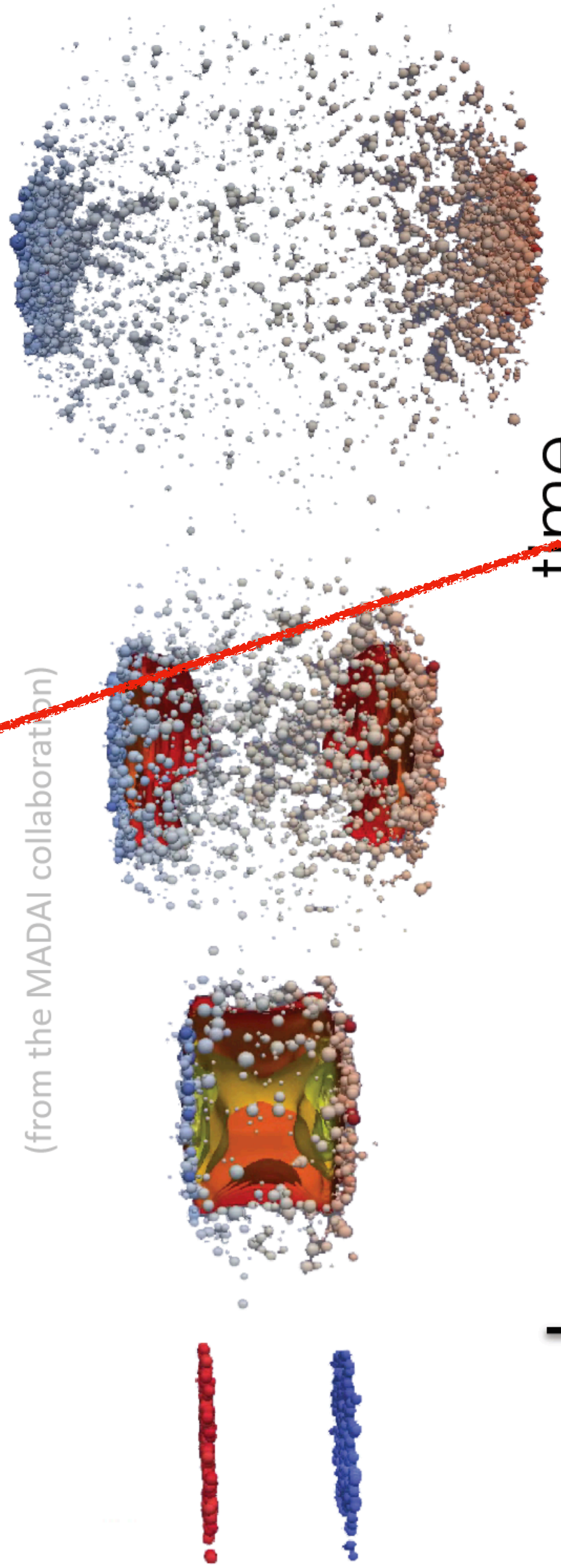
Ann. Phys. 323:2452 (2008), Ann. Phys. 338:32 (2013)
Phys. Rev. C 94:024904 (2016)

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau \frac{\int d\Sigma_\lambda p^\lambda n_F(1-n_F)\boxed{\varpi_{\rho\sigma}}}{\int d\Sigma_\lambda p^\lambda n_F}$$

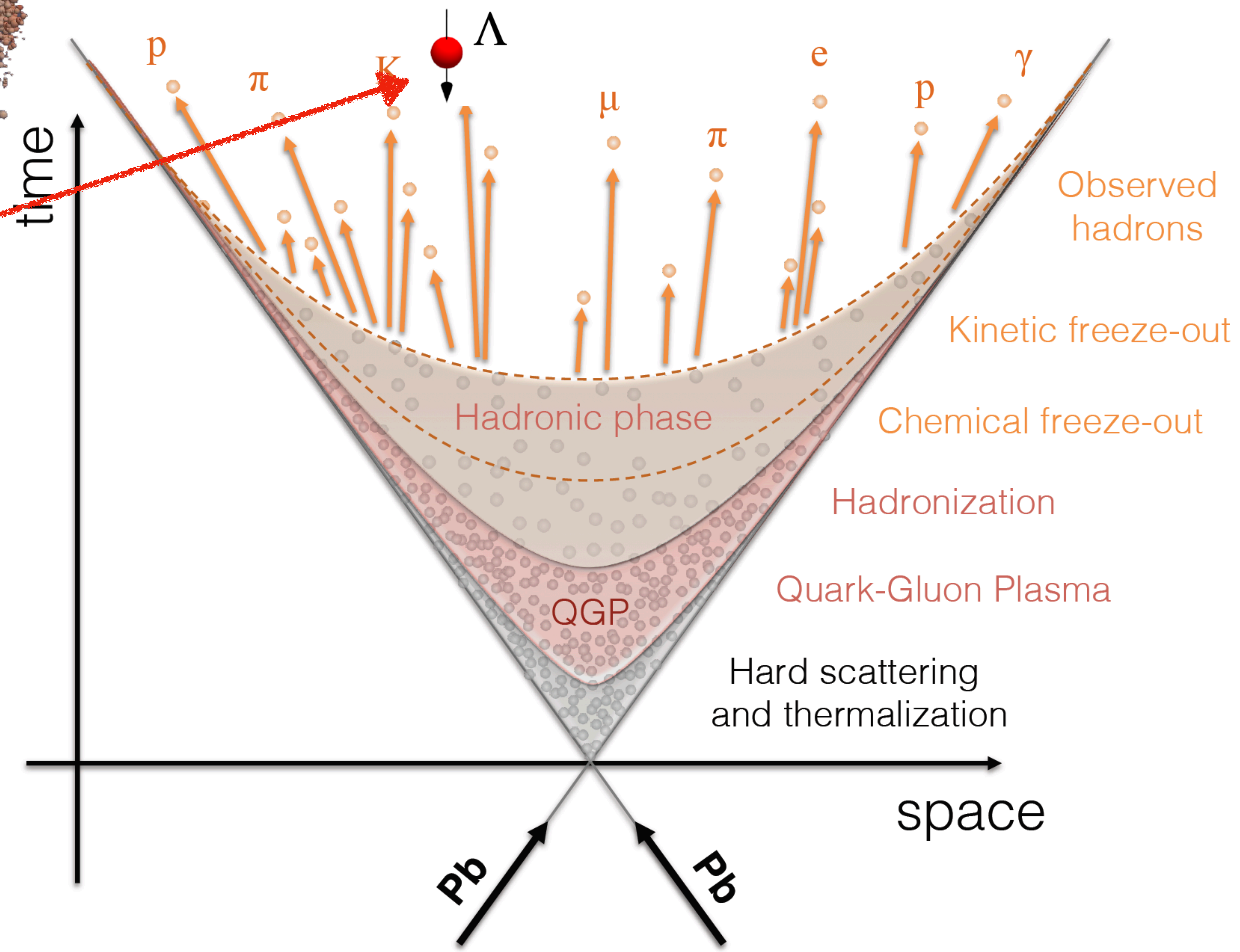
$$\boxed{\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) \qquad \beta^\mu = \frac{u^\mu}{T}}$$

$$n_F = (1+\exp[\beta\cdot p - \mu Q/T])^{-1}$$

**Allows to extract polarization at the freeze-out hypersurface in any model which provides $u^\mu$, $T$ and $\mu$**



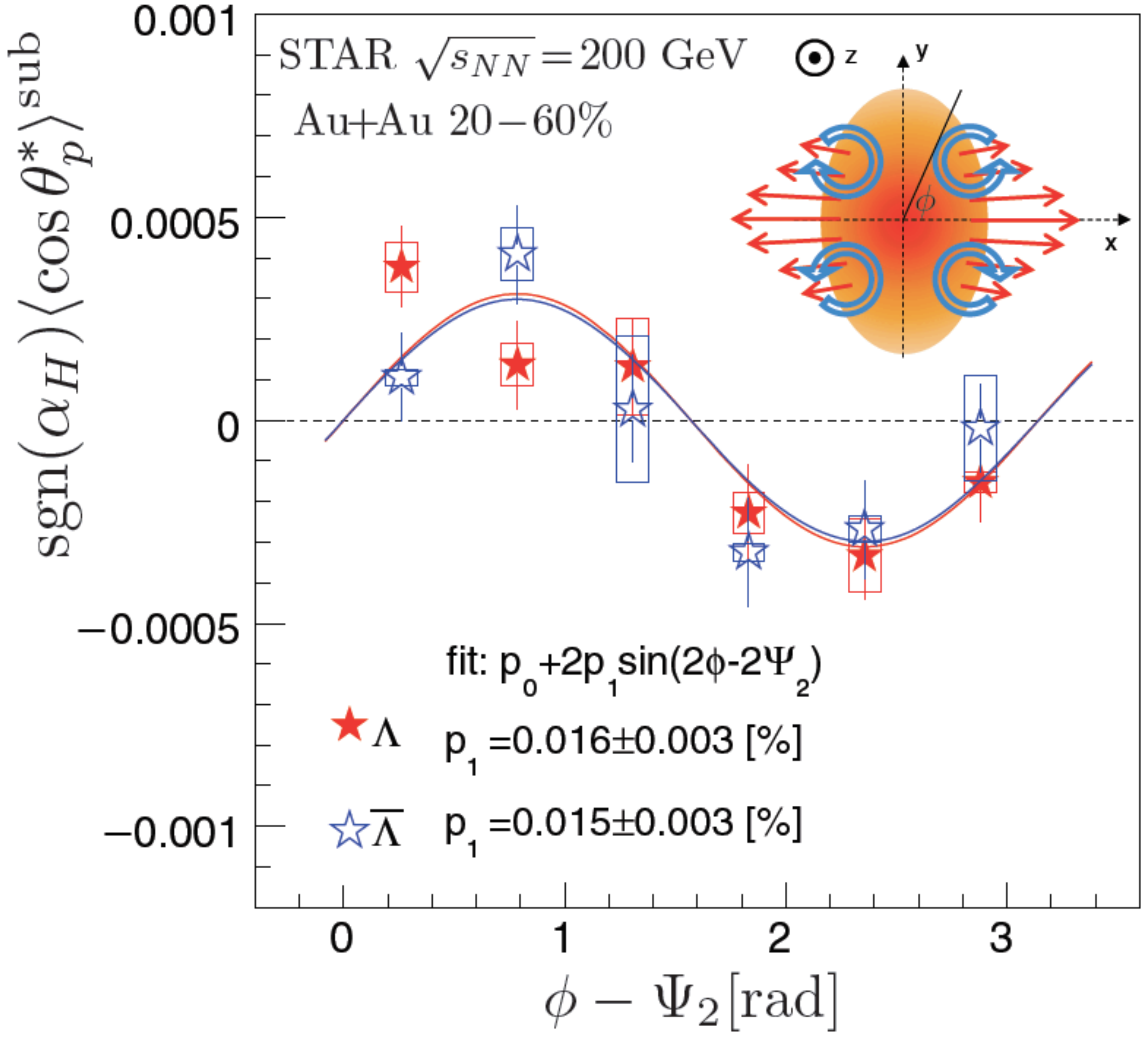

**relativistic heavy-ion collision**




D.D. Chinellato

# Why we need spin hydro?

→ Models that assume LTE of spin degrees of freedom are able to explain global spin polarization measurement.




→ But, unsuccessful to provide clear explanation for the azimuthal angle dependence of longitudinal polarization. Recent progress with thermal shear have some agreement.

Phys.Rev.Lett. 127 (2021) 27, 272302, Phys.Rev.Lett. 127 (2021) 14, 142301

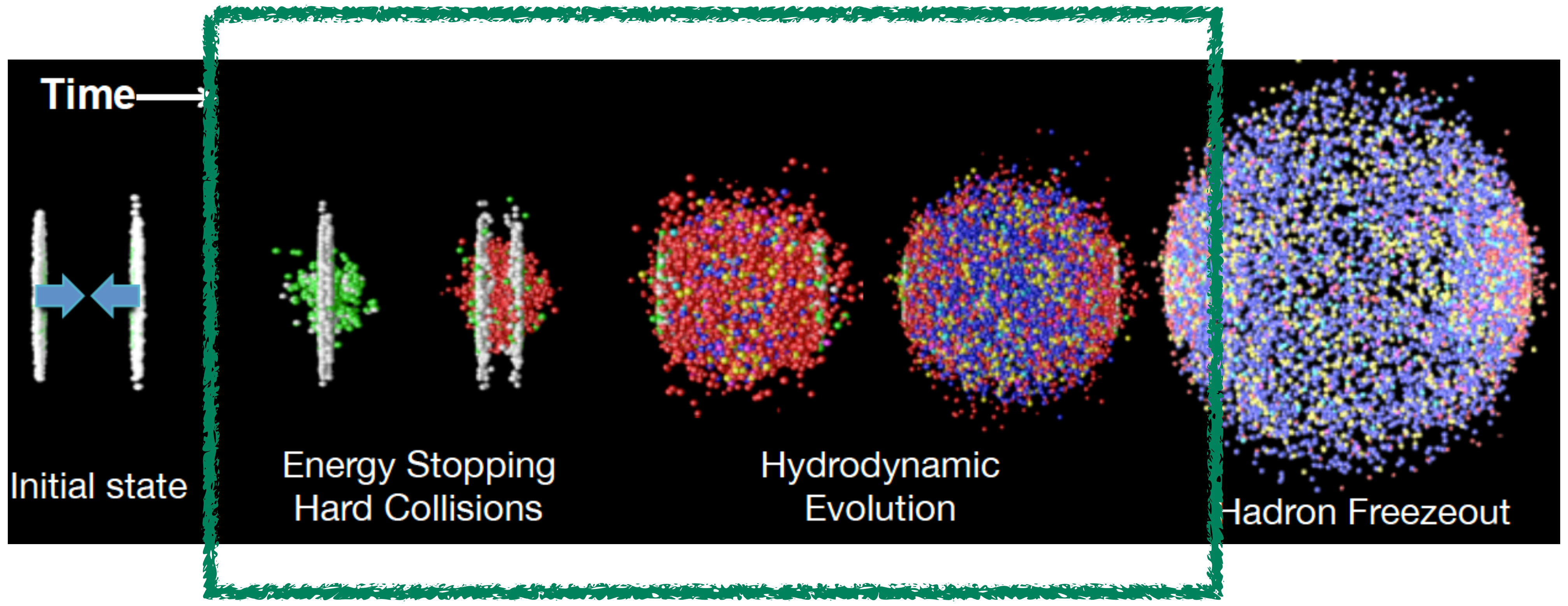
# Why we need spin hydro?

→ Why spin-thermal approach does not fully capture differential observables?

→ Is spin polarization always enslaved to thermal vorticity?

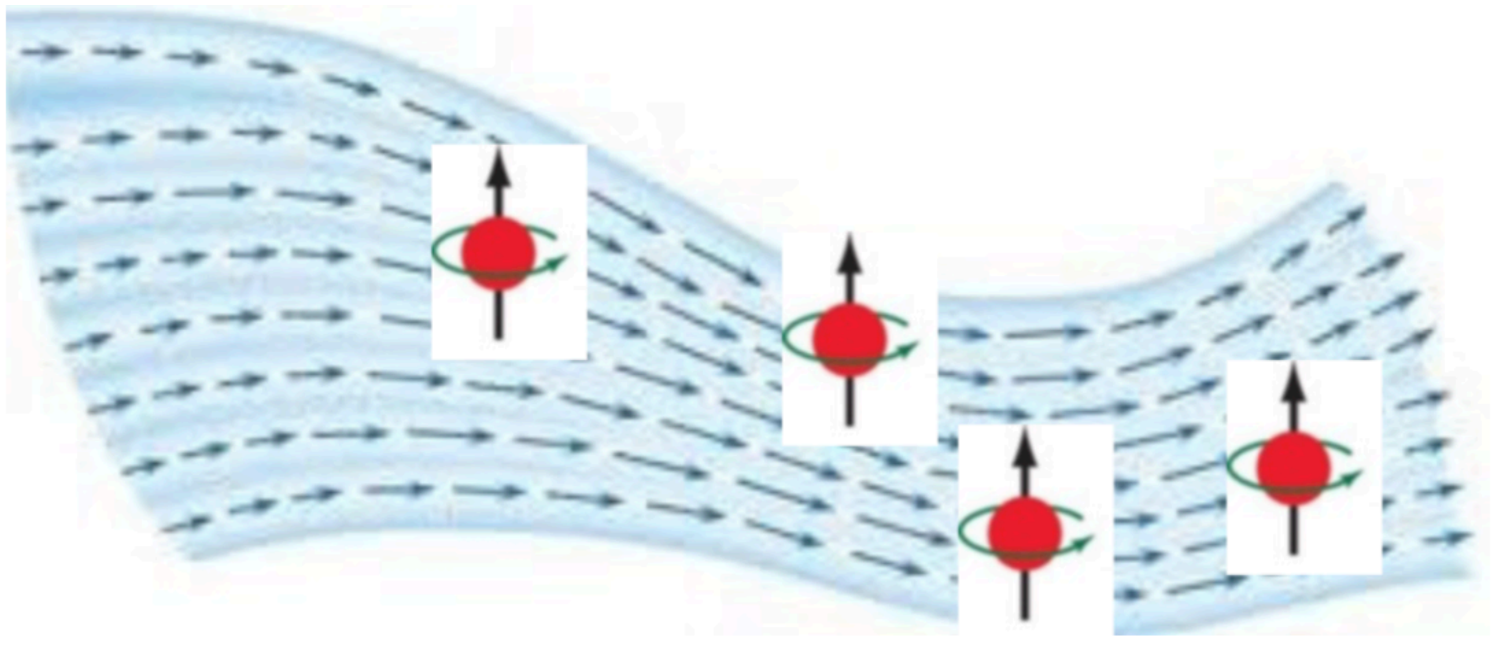
→ Is there non-trivial space-time dynamics of spin?

Relativistic fluid dynamics forms the basis of HIC models




Most of the time close to equilibrium but the dissipation is also important

**Spin Hydrodynamics ?**



# Boltzmann-like spin-kinetic equation

$$W_{\alpha\beta}(x,k) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}k\cdot y} \langle : \bar{\psi}_\beta(x_+)\,\psi_\alpha(x_-) : \rangle$$

Wigner function for massive Dirac particles

Dirac equation

$$(i\,\hbar\,\gamma^\mu\,\partial_\mu - m)\,\psi(x) = \hbar\,\rho(x) = -\frac{\partial\,\mathscr{L}_I}{\partial\,\bar{\psi}}$$

Using the total Lagrangian density

$$\mathscr{L}(x) = \mathscr{L}_D(x) + \mathscr{L}_I(x)$$

Does not contain gauge-field interactions

$$\mathscr{L}_D(x) = \frac{i\hbar}{2}\bar{\psi}(x)\,\gamma^\mu\,\overleftrightarrow{\partial}_\mu\,\psi(x) - m\,\bar{\psi}(x)\,\psi(x)$$

Transport equation

$$\left( i\hbar\frac{\gamma^\mu\partial_\mu}{2} + \gamma^\mu k_\mu - m \right) W(x,k) = \hbar\, C_{\alpha\beta}[W(x,k)]$$

Collisional kernel

$$C_{\alpha\beta}[W(x,k)] \equiv \int \frac{d^4y}{\left(2\pi\hbar\right)^4} e^{-\frac{i}{\hbar}k\cdot y} \langle : \rho_\alpha(x_-)\,\bar{\psi}_\beta\,(x_+) : \rangle$$

arXiv:2203.15562

Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

# Boltzmann-like spin-kinetic equation

$W(x,k) = W_{\rm eq}(x,k) + \delta W(x,k)$

- → Neglect initial correlations
- → Consider only binary collisions
- → Wigner function varies slowly in space and time on the microscopic scale corresponding to the interaction range

$$C_{\alpha\beta} = \frac{(2\pi\hbar)^6}{(2m)^4} \sum_{r_1,r_2,s_1,s_2} \int d^4k_1 d^4k_2 d^4q_1 d^4q_2 \;\; {}_{\rm in}\left\langle k_1 - \frac{q_1}{2}, k_2 - \frac{q_2}{2}; r_1, r_2 \middle| \Phi_{\alpha\beta}(k) \middle| k_1 + \frac{q_1}{2}, k_2 + \frac{q_2}{2}; s_1, s_2 \right\rangle_{\rm in}$$

$$\times \prod_{j=1}^{2} \bar{u}_{s_j}\left(k_j + \frac{q_j}{2}\right) \left\{ W(x,k_j)\delta^{(4)}(q_j) - i\hbar \left[\partial^{\mu}_{q_j}\delta^{(4)}(q_j)\right] \partial_\mu W(x,k_j) \right\} u_{r_j}\left(k_j - \frac{q_j}{2}\right)$$

Local

Non-Local

$$W(x,k) = \frac{1}{4}\left[\mathbf{1}_{4x4}\, F(x,k) + i\,\gamma^5\, P(x,k) + \gamma^\mu\, V_\mu(x,k) + \gamma^5\,\gamma^\mu\, A_\mu(x,k) + \Sigma^{\mu\nu}\, S_{\mu\nu}(x,k)\right]$$

Clifford algebra decomposition

arXiv:2203.15562

Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

# Boltzmann-like spin-kinetic equation

$A^{\star\mu\nu} = (1/2)\,\epsilon^{\mu\nu\alpha\beta}A_{\alpha\beta}$

$X = \Sigma_n \hbar^n X^{(n)}\,,\, C_X = \Sigma_n \hbar^n C_X^{(n)}$

Real parts

$$k\cdot V - m\,F = \hbar\,D_F$$
$$-\frac{\hbar}{2}\partial\cdot A - m\,P = \hbar\,D_P$$
$$k_\mu F - \frac{\hbar}{2}\partial^\nu S_{\nu\mu} - m\,V_\mu = \hbar\,D_{V,\mu}$$
$$\frac{\hbar}{2}\partial_\mu P - k^\beta S^\star{}_{\mu\beta} - m\,A_\mu = \hbar\,D_{A,\mu}$$
$$\hbar\,\partial_{[\mu}V_{\nu]} - \epsilon_{\mu\nu\alpha\beta}\,k^\alpha A^\beta - m\,S_{\mu\nu} = \hbar\,D_{S,\mu\nu}$$

Imaginary parts

$$\frac{\hbar}{2}\partial\cdot V = \hbar\,C_F$$
$$k\cdot A = \hbar\,C_P$$
$$\frac{\hbar}{2}\partial_\mu F + k^\nu S_{\nu\mu} = \hbar\,C_{V,\mu}$$
$$-k_\mu P - \frac{\hbar}{2}\partial^\beta S^\star{}_{\mu\beta} = \hbar\,C_{A,\mu}$$
$$-2k_{[\mu}V_{\nu]} - \frac{\hbar}{2}\epsilon_{\mu\nu\alpha\beta}\,\partial^\alpha A^\beta = \hbar\,C_{S,\mu\nu}$$

Kinetic equations using semi-classical expansion

For scalar component

$$k\cdot\partial\,F^{(0)} = 2\,m\,C_F^{(0)} \qquad k\cdot\partial\,F^{(1)} = 2\,m\,C_F^{(1)} + \partial\cdot D_V^{(0)}$$

For axial-vector component

$$k\cdot\partial\,A_\mu^{(0)} = 2\,m\,C_{A,\mu}^{(0)} - 2\,k_\mu\,D_P^{(0)} \qquad k\cdot\partial\,A_\mu^{(1)} = 2\,m\,C_{A,\mu}^{(1)} - 2\,k_\mu\,D_P^{(1)} - \frac{1}{2}\epsilon_{\mu\beta\gamma\delta}\,\partial^\beta D_{S(0)}^{\gamma\delta}$$

arXiv:2203.15562

Phys.Rev.D 104 (2021) 1, 016022

GLW, Relativistic Kinetic Theory and its Applications

Spin effects ?

# Boltzmann-like spin-kinetic equation

$$\int d\Gamma = \int d^4k\,\delta\left(k^2-m^2\right)\int dS(k)$$

which can then be combined to have

$$k\cdot\partial f(x,k,\mathfrak{s}) = m\,\mathfrak{C}(f) = m\left(\tilde{C}_F - \mathfrak{s}\cdot\tilde{C}_A\right)$$

where

$$f(x,k,\mathfrak{s}) = \frac{1}{2}\left(\tilde{F}(x,k) - \mathfrak{s}\cdot\tilde{A}(x,k)\right)$$

$$\mathfrak{C}[f] = \mathfrak{C}_l[f] + \hbar\mathfrak{C}_{nl}^{(1)}[f] = \mathfrak{C}_l^{(0)}[f] + \hbar\mathfrak{C}_l^{(1)}[f] + \hbar\mathfrak{C}_{nl}^{(1)}[f]$$

arXiv:2203.15562
Phys.Rev.D 104 (2021) 1, 016022
GLW, Relativistic Kinetic Theory and its Applications

# Canonical currents

→ Hydrodynamics is defined at a length scale larger than the mean free path of microscopic particles but smaller than the system size.

→ For formulating spin hydrodynamics, we need to define $T^{\mu\nu}$ & $S^{\lambda,\mu\nu}$ currents as ensemble averages of their respective normal-ordered QFT operators.

$$\boxed{T^{\mu\nu} = \langle : \hat{T}^{\mu\nu} : \rangle\,, \quad S^{\lambda,\mu\nu} = \langle : \hat{S}^{\lambda,\mu\nu} : \rangle}$$

→ For a system with spin we have

$$\hat{J}^{\lambda,\mu\nu} = \hat{L}^{\lambda,\mu\nu} + \hat{S}^{\lambda,\mu\nu} = x^\mu \hat{T}^{\lambda\nu} - x^\nu \hat{T}^{\lambda\mu} + \hat{S}^{\lambda,\mu\nu}$$

Conservation of TAM

$$\partial_\lambda \hat{J}^{\lambda,\mu\nu} = \partial_\lambda \hat{L}^{\lambda,\mu\nu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}^{\nu\mu} + \partial_\lambda \hat{S}^{\lambda,\mu\nu} = 0$$

gives

$$\partial_\lambda \hat{S}^{\lambda,\mu\nu} = \hat{T}^{\nu\mu} - \hat{T}^{\mu\nu}$$

Antisymmetric parts of $T^{\mu\nu}$

We also have $\partial_\mu \hat{T}^{\mu\nu} = 0$

For massive free Dirac particles:

$$\hat{T}^{\mu\nu}_{\rm Can} = \frac{i}{2}\,\bar{\psi}\,\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\rm Can} = \frac{i}{8}\,\bar{\psi}\left\{\gamma^\lambda, \left[\gamma^\mu, \gamma^\nu\right]\right\}\psi$$

$\psi$ & $\bar{\psi}$ are Dirac field operators

$\mathscr{L}_D$ is Dirac Lagrangian

$g^{\mu\nu} = \{1, -1, -1, -1\}$

$\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$

$\hat{S}^{\mu\nu}_{\rm Can} \equiv \int_\Sigma d\Sigma_\lambda \hat{S}^{\lambda,\mu\nu}_{\rm Can}$

QFT, Itzykson and Zuber (Saclay 1980)

# de Groot–van Leeuwen–van Weert pseudo-gauge

→ One obtains new pair of $\hat{T}^{\mu\nu}$ and $\hat{S}^{\lambda,\mu\nu}$ using $\hat{T}^{\mu\nu}_{\rm Can}$ and $\hat{S}^{\lambda,\mu\nu}_{\rm Can}$ through pseudo-gauge transformation.

Rept.Math.Phys. 9 (1976) 55-82,

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\rm Can} + \frac{1}{2}\partial_\lambda(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda})$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\rm Can} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho \hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{\Pi}^{\lambda,\mu\nu} = -\hat{\Pi}^{\lambda,\nu\mu}$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = -\hat{\Upsilon}^{\nu\mu,\lambda\rho} = -\hat{\Upsilon}^{\mu\nu,\rho\lambda}$$

$$\sigma^{\mu\nu} = (i/2)\left[\gamma^\mu, \gamma^\nu\right]$$

→ There are several choices of $\hat{\Pi}^{\lambda,\mu\nu}$ & $\hat{\Upsilon}^{\mu\nu,\lambda\rho}$, however, we choose

$$\hat{\Pi}^{\lambda,\mu\nu} = \frac{i}{4m}\bar{\psi}(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu})\psi$$

$$\hat{\Upsilon}^{\mu\nu,\lambda\rho} = 0$$

$$\boxed{\begin{aligned}\hat{T}^{\mu\nu}_{\rm GLW} &= -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu}\psi \\ \hat{S}^{\lambda,\mu\nu}_{\rm GLW} &= \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu\overleftrightarrow{\partial}^{\nu} - \gamma^\nu\overleftrightarrow{\partial}^{\mu}\right)\right]\gamma^\lambda\psi + \mathrm{h.c}\end{aligned}}$$

S. De Groot, W. Van Leeuwen, and C. Van Weert, Relativistic Kinetic Theory. Principles and Applications. North Holland, 1, 1980

Prog.Part.Nucl.Phys. 108 (2019) 103709

# Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

$$X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]\left[1 \pm \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$$

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right]$, $\xi(x) = \mu_B/T$, $\beta_\mu(x) = U^\mu/T$

$z = m/T$

"+" means particle contribution

"−" means antiparticle contribution

→ Using Wigner function (in equilibrium) $W_{\rm eq}(x,k) = W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)$

$$\left(i\hbar\frac{\gamma^\mu\partial_\mu}{2} + \gamma^\mu k_\mu - m\right)W_{\rm eq}(x,k) = \hbar\, C[W_{\rm eq}(x,k)]$$

Transport equation

$$W^+_{\rm eq}(x,k) = \frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k-p)\,\mathcal{U}^r(p)\,\bar{\mathcal{U}}^s(p)\,f^+_{rs}(x,p)$$

$$W^-_{\rm eq}(x,k) = -\frac{1}{2}\sum_{r,s}\int {\rm dP}\,\delta^{(4)}(k+p)\,\mathcal{V}^s(p)\,\bar{\mathcal{V}}^r(p)\,f^-_{rs}(x,p)$$

and ansatz for local equilibrium distribution functions

$$f^+_{rs}(x,p) = \frac{1}{2\,m}\bar{\mathcal{U}}_r(p)\,X^+\,\mathcal{U}_s(p) = \frac{1}{2\,m}\bar{\mathcal{U}}_r(p)\,\exp\left[-\beta_\mu(x)p^\mu + \xi(x)\right]\left[1 + \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{U}_s(p)$$

$$f^-_{rs}(x,p) = -\frac{1}{2\,m}\bar{\mathcal{V}}_s(p)\,X^-\,\mathcal{V}_r(p) = -\frac{1}{2\,m}\bar{\mathcal{V}}_s(p)\,\exp\left[-\beta_\mu(x)p^\mu - \xi(x)\right]\left[1 - \frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]\mathcal{V}_r(p)$$

Dirac spinors

We obtain

Spin polarization tensor
(Spin chemical potential)

$$W^{\pm}_{\rm eq}(x,k) = \frac{1}{4m}\int {\rm dP}\, e^{-\beta\cdot p \pm \xi}\,\delta^{(4)}(k \mp p)\left[2\,m(m \pm \gamma^\mu p_\mu) \pm \frac{1}{2}\,\omega_{\mu\nu}(\gamma^\mu p_\mu \pm m)\Sigma^{\mu\nu}(\gamma^\mu p_\mu \pm m)\right]$$

# Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

→ Decomposing Wigner function using Clifford algebra expansion

$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu, \gamma^\nu\right], \quad \xi(x) = \mu_B/T, \quad \beta_\mu(x) = U^\mu/T$

$\mathscr{C} = \cosh(\xi), \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$

$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)}+\mathscr{P}_{(0)}}{T}, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$

$z = m/T$

One can derive the <u>constitutive relations</u> for

- Net baryon current

$$N^\alpha(x) = \langle : \bar{\psi}\gamma^\alpha\psi : \rangle = \mathrm{tr}\int d^4k\,\gamma^\alpha\left(W^+_{\rm eq}(x,k) - W^-_{\rm eq}(x,k)\right)$$

$$N^\alpha(x) = \mathscr{N}\,U^\alpha$$

with

$$\mathscr{N} = 4\sinh(\xi)\,\mathscr{N}_{(0)}(T)$$

$$\mathscr{N}_{(0)}(T) = \frac{T^3}{2\pi^2}z^2K_2(z)$$

- Energy-momentum tensor

$$T^{\mu\nu}_{\rm GLW}(x) = \langle : \hat{T}^{\mu\nu}_{\rm GLW} : \rangle = \frac{1}{m}\mathrm{tr}\int d^4k\,k^\mu k^\nu\left(W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)\right)$$

$$T^{\mu\nu}_{\rm GLW}(x) = (\mathscr{E}+\mathscr{P})\,U^\mu U^\nu - \mathscr{P}\,g^{\mu\nu}$$

with

$$\mathscr{E} = 4\cosh(\xi)\,\mathscr{E}_{(0)}(T)$$

$$\mathscr{P} = 4\cosh(\xi)\,\mathscr{P}_{(0)}(T)$$

$$\mathscr{E}_{(0)}(T) = \frac{T^4}{2\pi^2}z^2\left[zK_1(z) + 3K_2(z)\right]$$

$$\mathscr{P}_{(0)}(T) = T\mathscr{N}_{(0)}(T)$$

# Relativistic hydrodynamics with spin

Prog.Part.Nucl.Phys. 108 (2019) 103709

- Spin tensor

$$S^{\alpha,\beta\gamma}_{\rm GLW} = \langle : \hat{S}^{\alpha,\beta\gamma}_{\rm GLW} : \rangle = \frac{\hbar}{4}\int d^4k \,{\rm tr}\left[\left(\left\{\sigma^{\beta\gamma},\gamma^\alpha\right\} + \frac{2i}{m}\left(\gamma^{[\beta}k^{\gamma]}\gamma^\alpha - \gamma^\alpha\gamma^{[\beta}k^{\gamma]}\right)\right)\left(W^+_{\rm eq}(x,k) + W^-_{\rm eq}(x,k)\right)\right]$$

$$S^{\alpha,\beta\gamma}_{\rm GLW} = U^\alpha\left(\mathscr{A}_1\,\omega^{\beta\gamma} + \mathscr{A}_2\,U^{[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right) + \mathscr{A}_3\left(U^{[\beta}\,\omega^{\gamma]\alpha} + g^{\alpha[\beta}\,\omega^{\gamma]}{}_\delta\,U^\delta\right)$$

Fluid-flow four-velocity

Spin polarization tensor

with

$$\left.\begin{aligned} \mathscr{A}_1 &= \mathscr{C}\left(\mathscr{N}_{(0)} - \mathscr{B}_{(0)}\right) \\ \mathscr{A}_2 &= \mathscr{C}\left(\mathscr{A}_{(0)} - 3\mathscr{B}_{(0)}\right) \\ \mathscr{A}_3 &= \mathscr{C}\,\mathscr{B}_{(0)} \end{aligned}\right\}$$ Thermodynamic coefficients

$$\longrightarrow \boxed{\partial_\alpha N^\alpha = 0\,,\ \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0\,,\ \partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW} = 0.}$$

$$\Sigma^{\mu\nu} = (i/4)\left[\gamma^\mu,\gamma^\nu\right], \quad \xi(x) = \mu_B/T\,, \quad \beta_\mu(x) = U^\mu/T$$

$$\mathscr{C} = \cosh(\xi)\,, \quad \Delta^{\mu\nu} = g^{\mu\nu} - (U^\mu U^\nu)/(U\cdot U)$$

$$\mathscr{B}_{(0)} = -\frac{2}{z^2}\frac{\mathscr{E}_{(0)} + \mathscr{P}_{(0)}}{T}\,, \quad \mathscr{A}_{(0)} = 2\mathscr{N}_{(0)} - 3\mathscr{B}_{(0)}$$

$$z = m/T$$

# Modeling of the spin polarization dynamics

→ Mean spin polarization per particle (momentum-dependent)

$$\langle \pi_\mu \rangle_p = \frac{E_p \dfrac{d\Pi_\mu(p)}{d^3p}^{*}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}} = \frac{-\dfrac{1}{(2\pi)^3 m}\int \cosh(\xi)\,\Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}\,(\omega^{\star}_{\mu\beta}p^\beta)^*}{\dfrac{4}{(2\pi)^3}\int \cosh(\xi)\,\Delta\Sigma_\lambda\, p^\lambda\, e^{-\beta\cdot p}}$$

Pauli Lubanski four-vector

Momentum density of all particles

Freeze-out hyper-surface element

→ Mean spin polarization per particle (momentum-independent)

$$\langle \pi_\mu \rangle = \frac{\int dP\,\langle \pi_\mu \rangle_p\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}{\int dP\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

Transverse momentum

Azimuthal angle

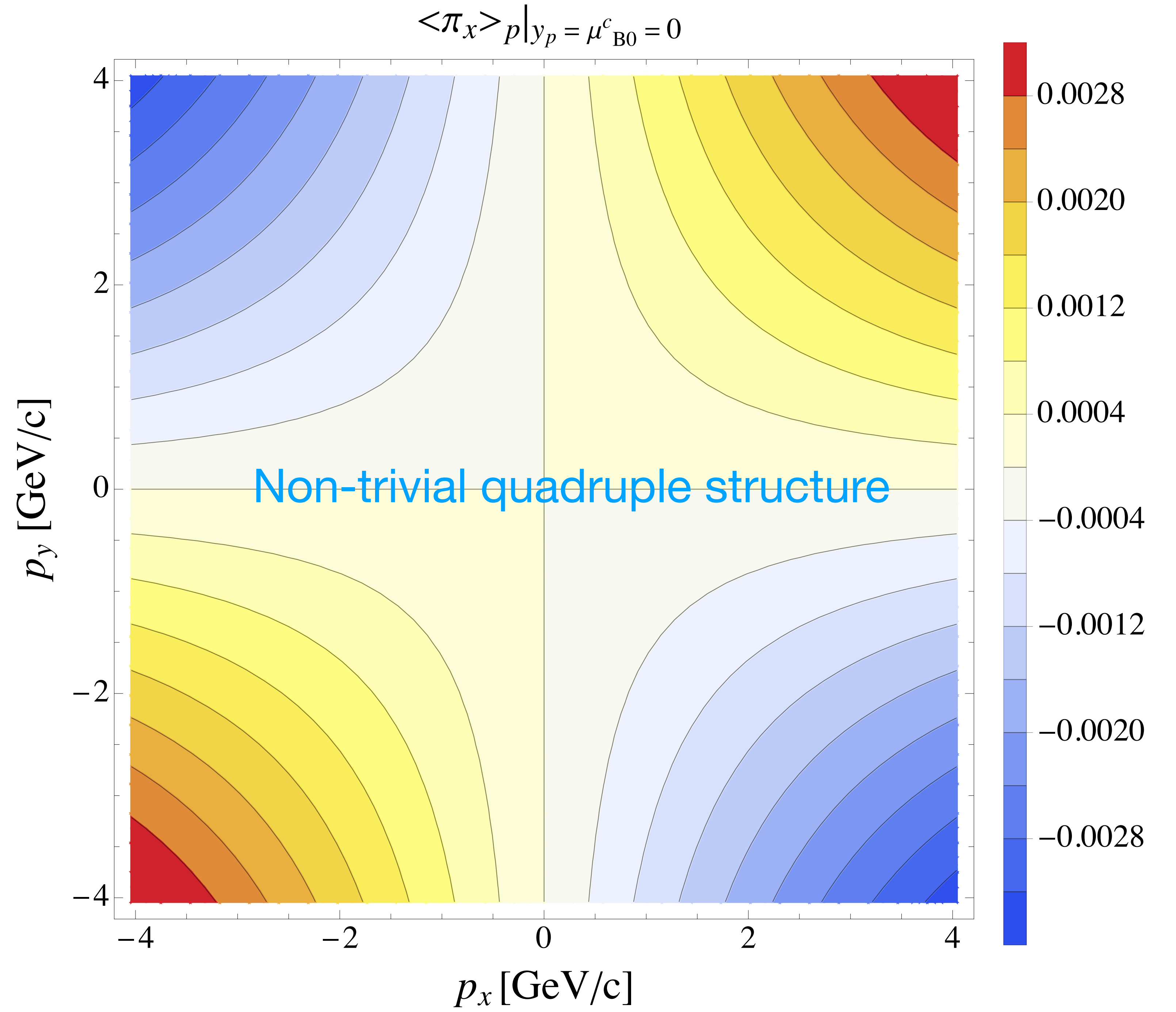
$$\langle \pi_\mu(p_T) \rangle = \frac{\frac{1}{2\pi}\int d\phi_p\, \sin(2\phi_p)\, E_p \dfrac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

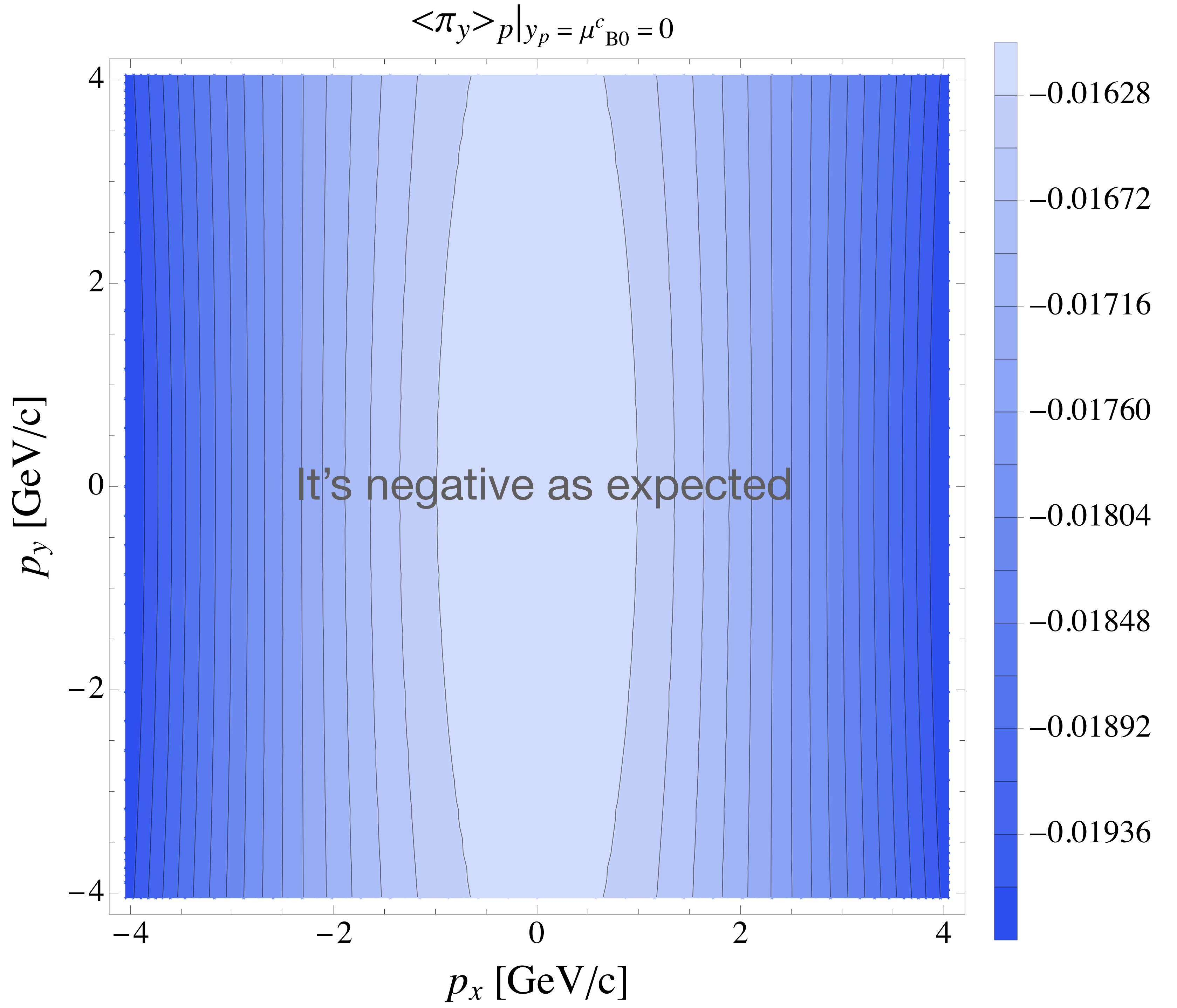
$$\langle \pi_\mu(\phi_p) \rangle = \frac{\int p_T\, dp_T\, E_p \dfrac{d\Pi^*_\mu(p)}{d^3p}}{\int d\phi_p\, p_T\, dp_T\, E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$

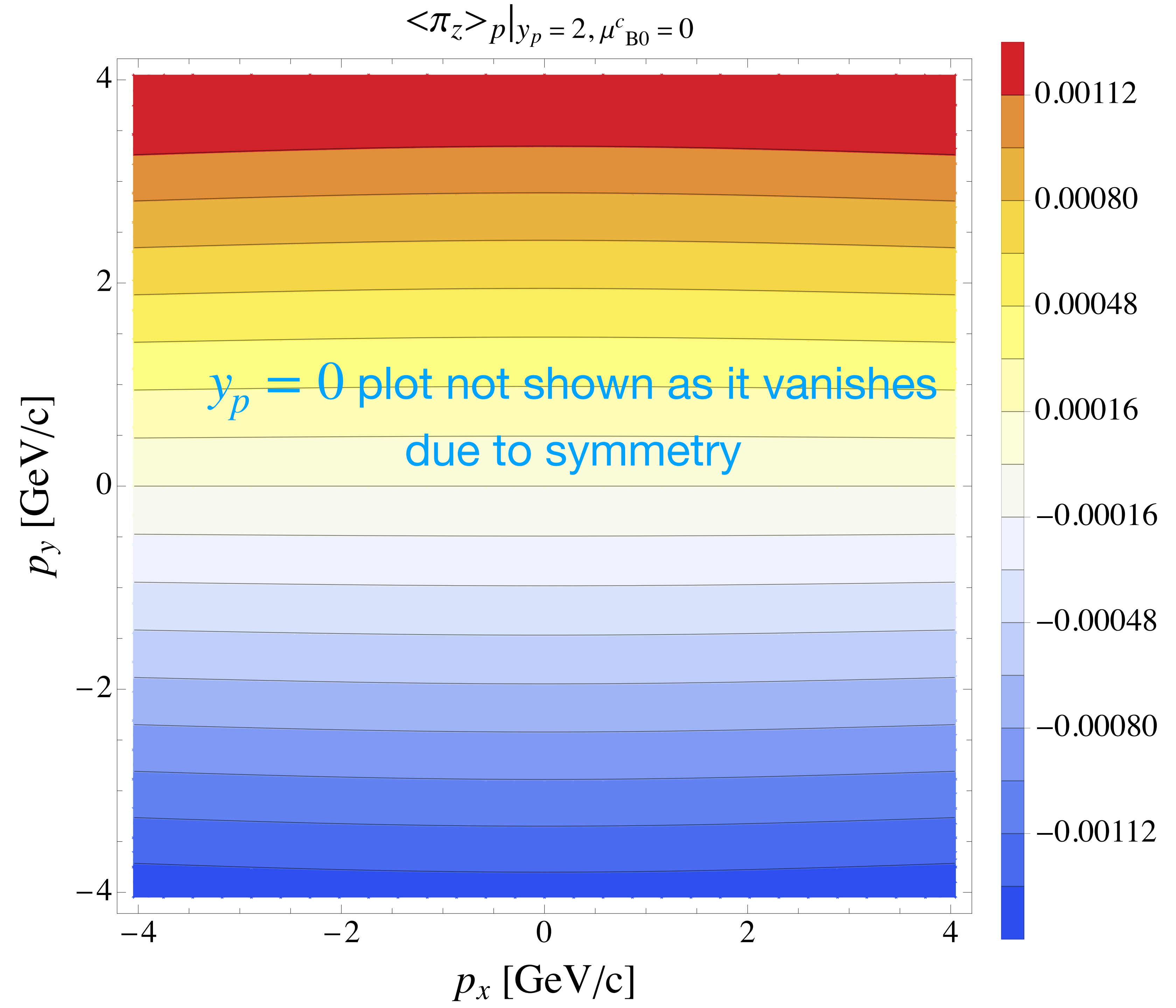
# Modeling of the spin polarization dynamics

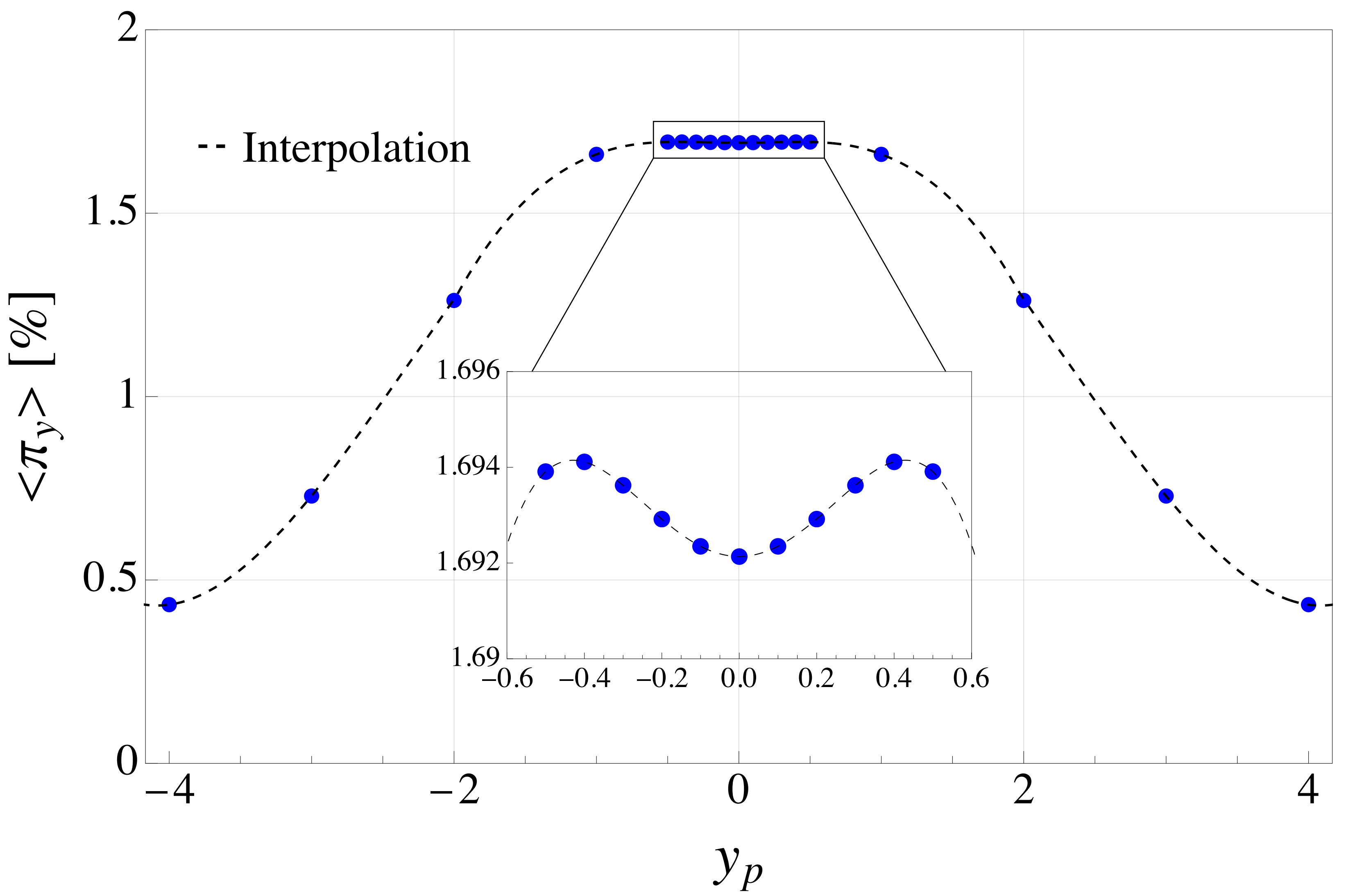
$\mu_B \neq 0$ plots not shown as they are qualitatively similar

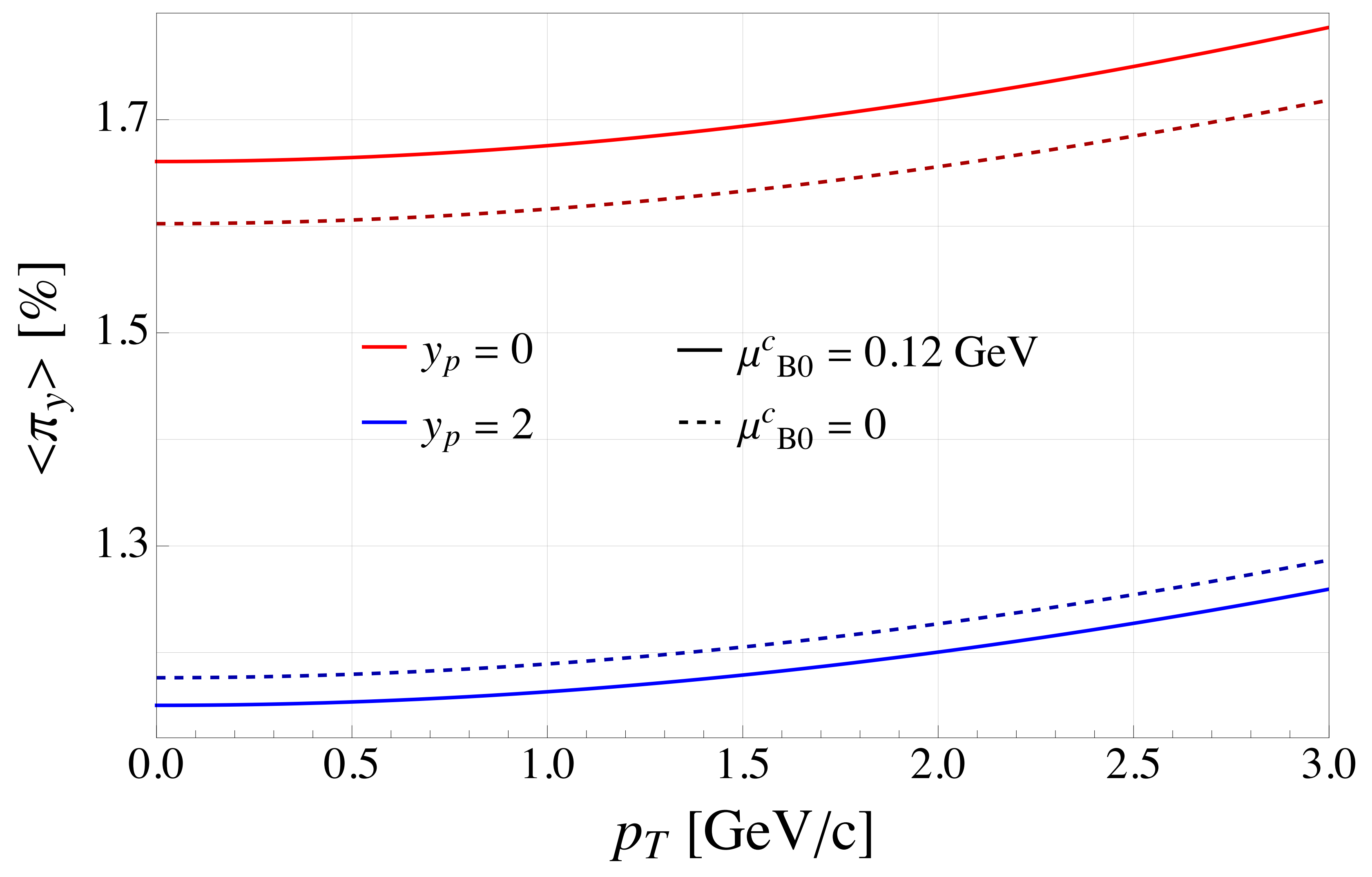
→ Non-boost-invariant and transversely homogeneous

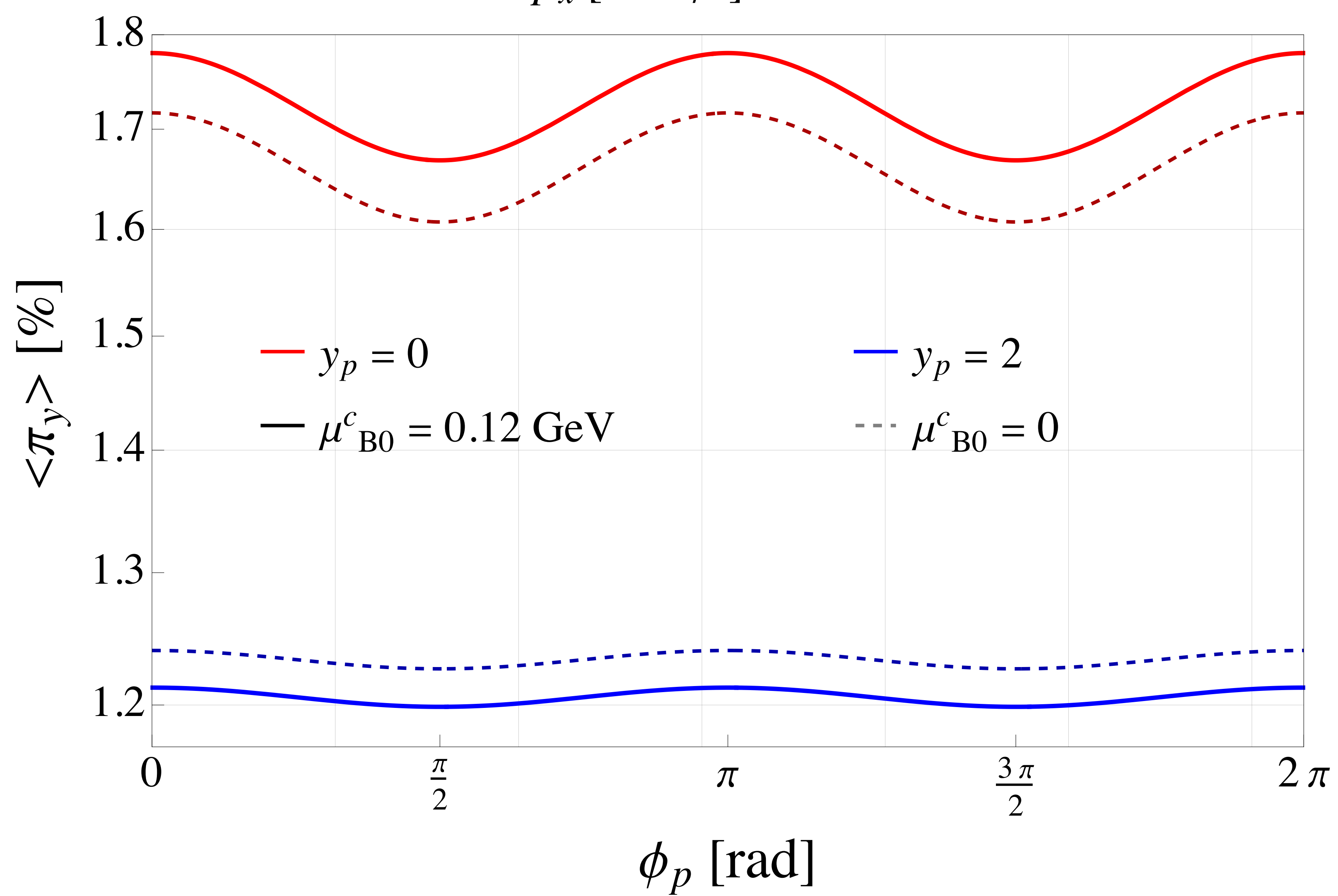


# Summary

The spin polarization provides a sensitive new probe of the QGP properties

The disagreement motivates developments of dynamical models

Spin hydrodynamics depends on pseudo gauge

The fluid dynamics with spin seems to be a natural framework


*Thank you for listening!*

Probably, yes!

**Some questions to my mind!**

Can we remove the pseudo-gauge ambiguity?

Can we derive pseudo-gauge independent spin hydro?

Rel. viscous spin hydro for boosted fluid?

Stochastic hydro with spin?

**Works in progress!**

# Pseudogauge ambiguity

Can: Canonical
BR: Belinfante–Rosenfeld
GLW: de Groot-van Leeuwen-van Weert
HW: Hilgevoord-Wouthuysen

→ Various pseudogauges

**Which one is physical?**

**Which one describes data?**

**Is there any general pseudogauge?**

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{1}{2}\partial_\lambda(\hat{\Pi}^{\lambda,\mu\nu} + \hat{\Pi}^{\nu,\mu\lambda} + \hat{\Pi}^{\mu,\nu\lambda})$$

$$\hat{S}^{\lambda,\mu\nu} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \hat{\Pi}^{\lambda,\mu\nu} + \partial_\rho \hat{\Upsilon}^{\mu\nu,\lambda\rho}$$

$$\hat{T}^{\mu\nu}_{\text{Can}} = \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{Can}} = \frac{i}{8}\bar{\psi}\left\{\gamma^\lambda, \left[\gamma^\mu, \gamma^\nu\right]\right\}\psi$$

$$\hat{T}^{\mu\nu}_{\text{BR}} = \frac{i}{4}\bar{\psi}\left(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu\right)\psi - g^{\mu\nu}\mathscr{L}_D$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{BR}} = 0$$

$$\hat{T}^{\mu\nu}_{\text{GLW}} = -\frac{1}{4m}\bar{\psi}\overleftrightarrow{\partial}^\mu \overleftrightarrow{\partial}^\nu \psi$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{GLW}} = \bar{\psi}\left[\frac{\sigma^{\mu\nu}}{4} - \frac{1}{8m}\left(\gamma^\mu \overleftrightarrow{\partial}^\nu - \gamma^\nu \overleftrightarrow{\partial}^\mu\right)\right]\gamma^\lambda \psi + \text{h.c}$$

$$\hat{T}^{\mu\nu}_{\text{HW}} = \hat{T}^{\mu\nu}_{\text{Can}} + \frac{i}{2m}\left(\partial^\nu \bar{\psi}\sigma^{\mu\beta}\partial_\beta \psi + \partial_\alpha \bar{\psi}\sigma^{\alpha\mu}\partial^\nu \psi\right) - \frac{i}{4m}g^{\mu\nu}\partial_\lambda\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_\alpha \psi\right)$$

$$\hat{S}^{\lambda,\mu\nu}_{\text{HW}} = \hat{S}^{\lambda,\mu\nu}_{\text{Can}} - \frac{1}{4m}\left(\bar{\psi}\sigma^{\mu\nu}\sigma^{\lambda\rho}\partial_\rho \psi + \partial_\rho \bar{\psi}\sigma^{\lambda\rho}\sigma^{\mu\nu}\psi\right)$$

Rept.Math.Phys. 9 (1976) 55-82
Eur.Phys.J.A 57 (2021) 5, 155

# Pseudogauge dependence of quantum fluctuations

$$\hat{T}_a^{00} = \frac{1}{(a\sqrt{\pi})^3}\int d^3\boldsymbol{x}\ \hat{T}^{00}(x)\ \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right)$$

→ Quantum fluctuations of energy in subsystems of a hot relativistic gas of spin-1/2 particles

$$\sigma^2(a,m,T) = \langle : \hat{T}_a^{00} :: \hat{T}_a^{00} : \rangle - \langle : \hat{T}_a^{00} : \rangle^2$$

Variance

$$\sigma_n(a,m,T) = \frac{\left(\langle : \hat{T}_a^{00} :: \hat{T}_a^{00} : \rangle - \langle : \hat{T}_a^{00} : \rangle^2\right)^{1/2}}{\langle : \hat{T}_a^{00} : \rangle}$$

Normalized standard deviation

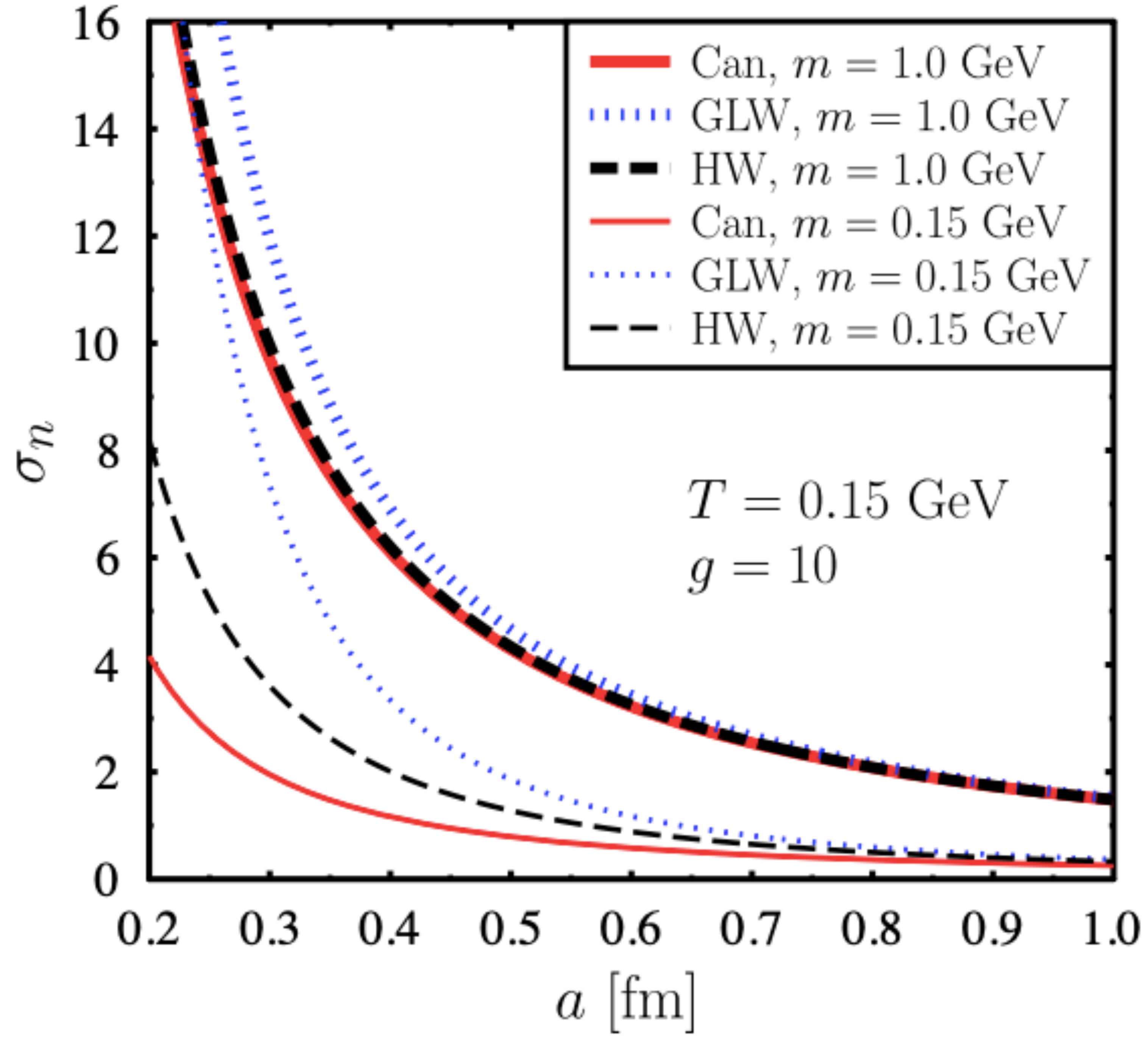
$$\sigma^2_{\rm Can}(a,m,T) = 2\int dKdK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'+m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

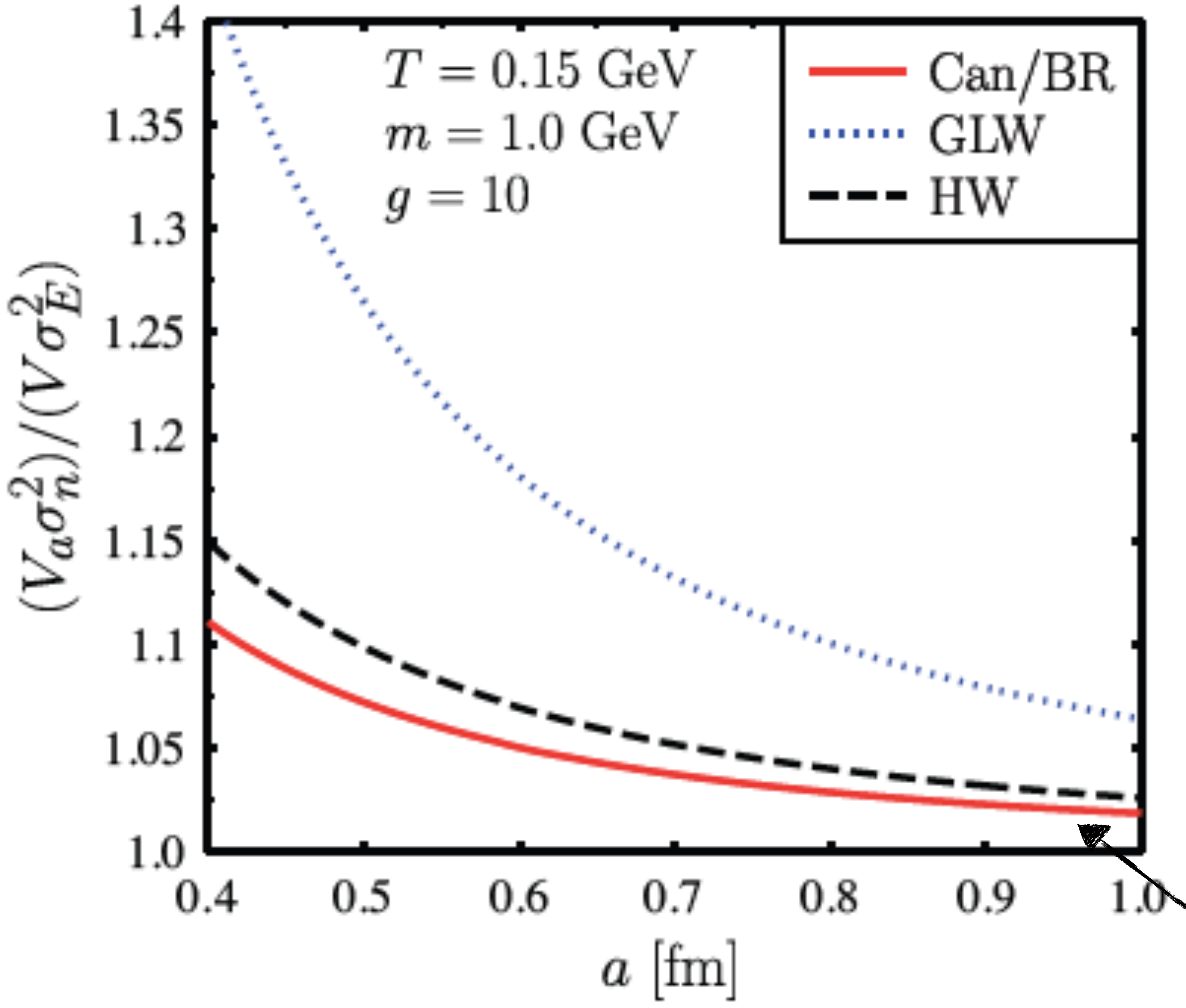
$$\sigma^2_{\rm GLW}(a,m,T) = \frac{1}{2m^2}\int dKdK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'-m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

$$\sigma^2_{\rm HW}(a,m,T) = \frac{2}{m^2}\int dKdK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\left[\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)^2\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'+m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}+\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'}-\boldsymbol{k}\cdot\boldsymbol{k}'-m^2)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\right]$$

Phys.Rev.D 103 (2021) 9, L091502

# Pseudogauge dependence of quantum fluctuations



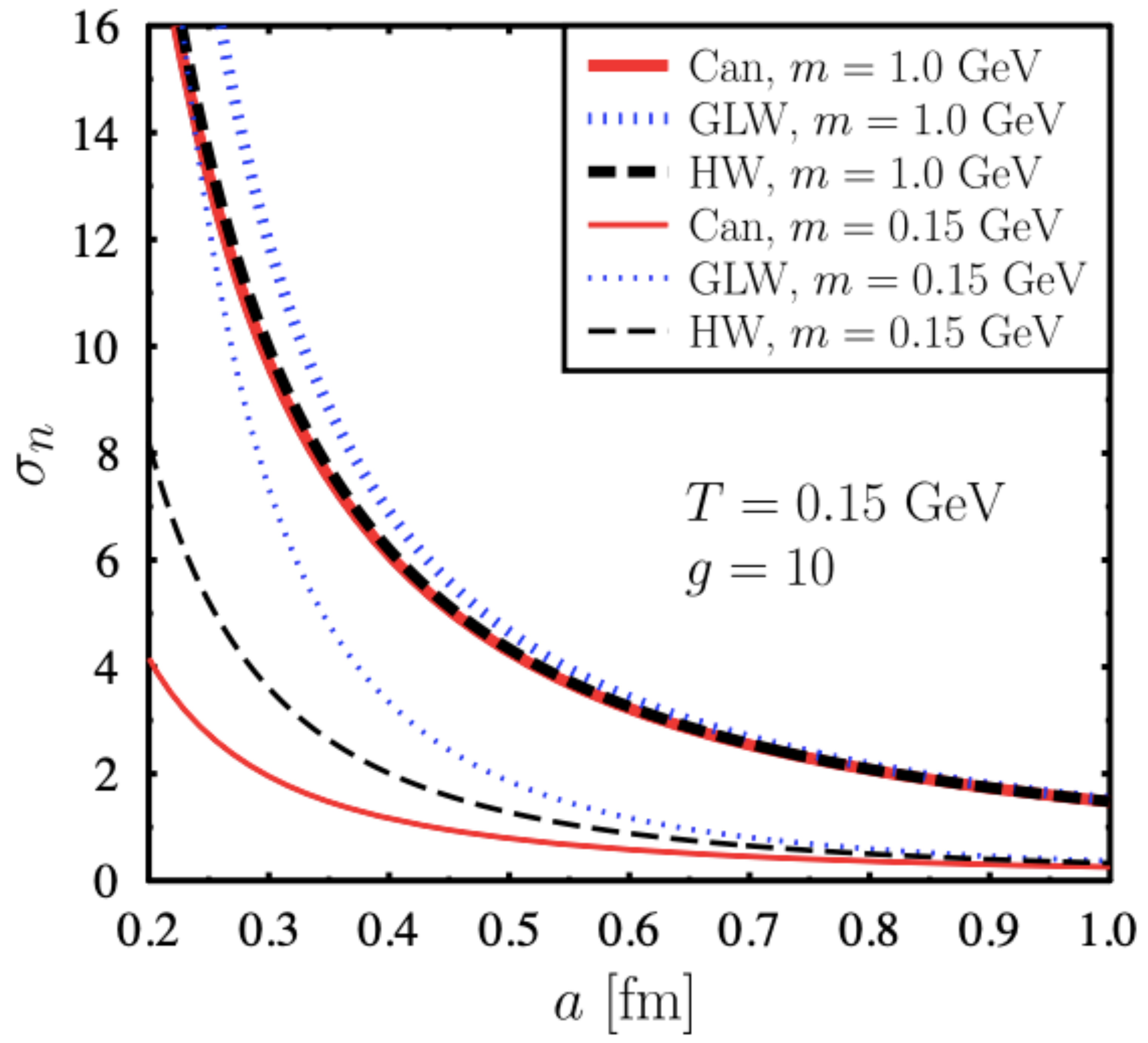



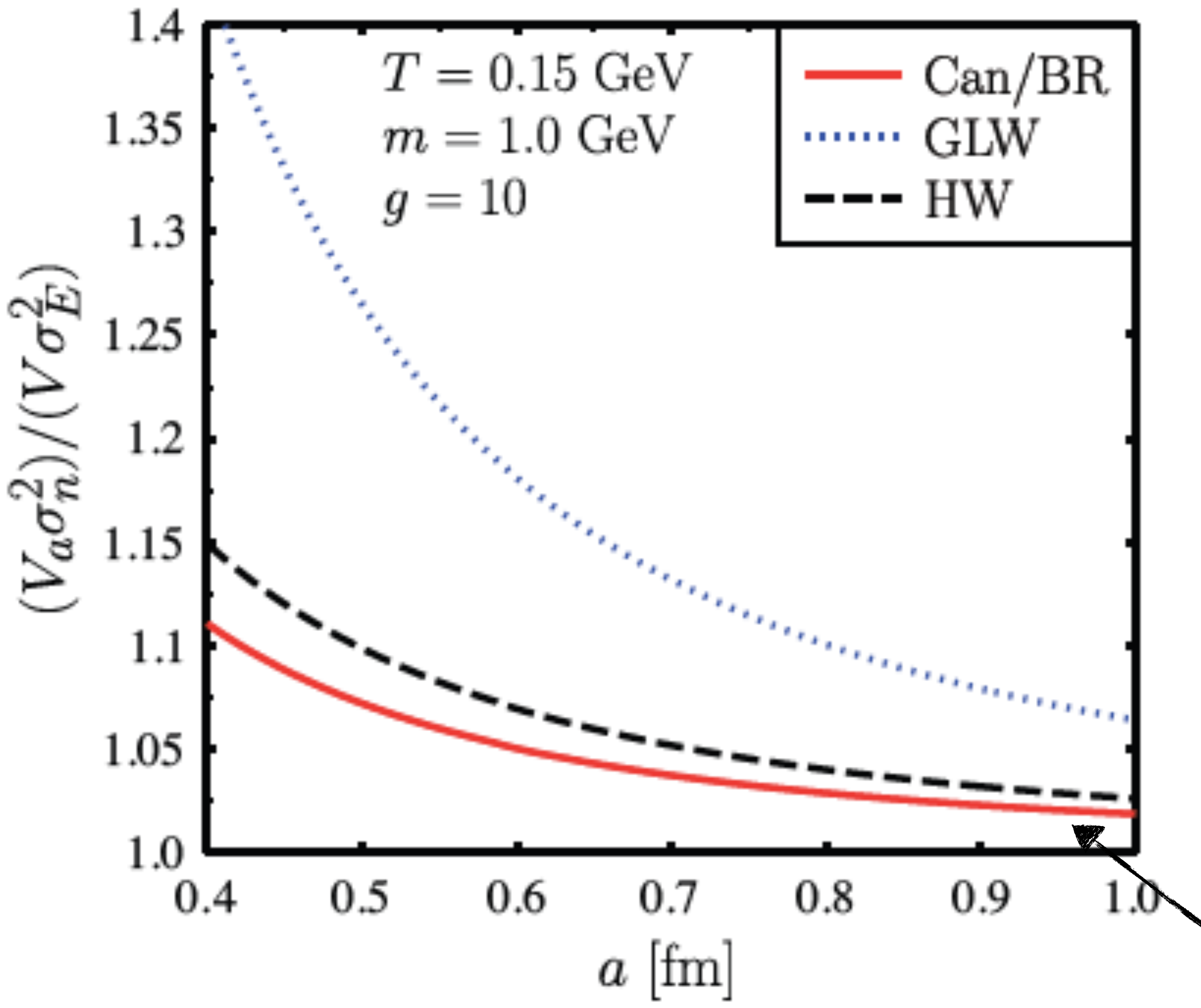

Agree with the known canonical ensemble formula

**Quantum fluctuations of energy in very small thermodynamic systems is pseudogauge dependent**

Phys.Rev.D 103 (2021) 9, L091502

# Pseudogauge dependence of quantum fluctuations




Agree with the known canonical ensemble formula

**Not all spin operators follow SO(3) angular momentum algebra except canonical spin**

Physics Letters B 843 (2023) 137994

Phys.Rev.D 103 (2021) 9, L091502

# Can we remove the ambiguity?

→ Canonical $\hat{T}^{\mu\nu}$

- Not gauge invariant in theories with gauge fields
- Asymmetric in the presence of spin even there are no interactions
- Not unique in linearized gravity
- Fails to be traceless in theories which are manifestly scale invariant

→ Use pseudo gauge, but?

Emmy Noether, “Invariante Variationsprobleme”, Gottinger Nachrichten (1918), 235-257

Annals Phys. 309 (2004) 306-389

Physics Letters B 843 (2023) 137994

Coming soon (arXiv:2310.xxxxx)

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$\boxed{\textbf{Noether's second theorem}}$

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Coming soon (arXiv:2310.xxxxx)

# Can we remove the ambiguity?

**Noether's second theorem**

Emmy Noether, "Invariante Variationsprobleme",
Gottinger Nachrichten (1918), 235-257

$$\to \hat{\mathcal{T}}^{\mu\nu} = \frac{i}{4}\bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{\partial}^{\nu} + \gamma^{\nu}\overleftrightarrow{\partial}^{\mu}\right)\psi - g^{\mu\nu}\mathcal{L}_D$$

Gauge invariant in theories with gauge fields

Symmetric in the presence of spin even there are no interactions

Traceless in theories which are manifestly scale invariant

$$\to \hat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{i}{8}\bar{\psi}\left\{\gamma^{\lambda}, \left[\gamma^{\mu}, \gamma^{\nu}\right]\right\}\psi$$

Conserved when there are no interactions

Satisfy SO(3) angular momentum algebra

Can be applied to massless case

Lead to covariant description of spin for free fields

PRD 106, 125012 (2022)

Coming soon (arXiv:2310.xxxxx)

# Classical phase-space distribution function

$s^{\mu\nu} = (1/m)\,\epsilon^{\mu\nu\alpha\beta} p_\alpha s_\beta$

Prog.Part.Nucl.Phys. 108 (2019) 103709

→ Using classical single-particle distribution function in a phase space extended to spin

→ Identifying the collisional invariants of the local Boltzmann equation

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-\beta(x)\cdot p \pm \xi(x)\right)\left[1 + \frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right]$$

→ And using relations

$$N^\mu(x) = \int \mathrm{dP}\ \mathrm{dS}\ p^\mu \left[f^+_{\rm eq}(x,p,s) - f^-_{\rm eq}(x,p,s)\right]$$

$$T^{\mu\nu}_{\rm GLW}(x) = \int \mathrm{dP}\ \mathrm{dS}\ p^\mu p^\nu \left[f^+_{\rm eq}(x,p,s) + f^-_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu}_{\rm GLW} = \int \mathrm{dP}\ \mathrm{dS}\ p^\lambda s^{\mu\nu} \left[f^+_{\rm eq}(x,p,s) + f^-_{\rm eq}(x,p,s)\right]$$

Considering spin polarization is small

→ $\boxed{\partial_\alpha N^\alpha = 0\,,\ \partial_\alpha T^{\alpha\beta}_{\rm GLW} = 0\,,\ \partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW} = 0.}$

Not true if we include non-local collisions

# Coupling between spin and electromagnetic fields - I

$\alpha_M = \mu_M / T$

→ Modified single particle distribution function

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-\beta(x)\cdot p \pm \xi(x)\right)\left(1+\frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right)\left(1 \mp \alpha_M F_{\mu\nu}(x)s^{\mu\nu}\right)$$

- Net baryon current

$$N^{\alpha}(x) = \int {\rm dP}\ {\rm dS}\ p^{\alpha}\left[f^{+}_{\rm eq}(x,p,s) - f^{-}_{\rm eq}(x,p,s)\right]$$

$$N^{\alpha}(x) = \left(\mathcal{N}_{\rm PF} + \mathcal{N}_{\rm EM}\right)U^{\alpha} + N^{\alpha}_{\perp}$$

with

$$\mathcal{N}_{\rm EM} = \alpha_M \cosh(\xi)\,\mathcal{N}_{(0)}\,\epsilon^{\beta\gamma\nu\mu}\,\omega_{\beta\gamma}U_{\mu}B_{\nu}$$

$$N^{\lambda}_{\perp} = \alpha_M \cosh(\xi)\mathcal{A}_3\left(U^{\lambda}F^{\beta\gamma} + 6U^{\lambda}U^{[\beta}E^{\gamma]} - U^{[\beta}F^{\gamma]\lambda} - g^{\lambda[\beta}E^{\gamma]}\right)\omega_{\beta\gamma}$$

# Coupling between spin and electromagnetic fields - II

→ Modified single particle distribution function

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-\beta(x)\cdot p \pm \xi(x)\right)\left(1+\frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right)\left(1 \mp \alpha_M F_{\mu\nu}(x)s^{\mu\nu}\right)$$

$\alpha_M = \mu_M/T$

$A:B \equiv A_{\mu\nu}B^{\mu\nu}$

- Energy-momentum tensor

$$T^{\mu\nu}_{\rm GLW}(x) = \int {\rm dP}\ {\rm dS}\ p^\mu p^\nu \left[f^+_{\rm eq}(x,p,s) + f^-_{\rm eq}(x,p,s)\right]$$

$$T^{\mu\nu}_{\rm GLW}(x) = (\mathscr{E}+\mathscr{P})\,U^\mu U^\nu - \mathscr{P}\,g^{\mu\nu} + \mathscr{Q}^\mu\mathscr{U}^\nu + \mathscr{Q}^\nu\mathscr{U}^\mu + \mathscr{T}^{\mu\nu}$$

with

$$\mathscr{E} \equiv \mathscr{U}_\mu\mathscr{U}_\nu\mathscr{T}^{\mu\nu} = \mathscr{E}_{\rm PF} + \mathscr{E}_{\rm EM}$$

$$\mathscr{P} \equiv -\frac{1}{3}\Delta : T = \mathscr{P}_{\rm PF} + \mathscr{P}_{\rm EM}$$

$$\mathscr{Q}^\mu \equiv \Delta^\mu{}_\alpha\, U_\beta T^{\alpha\beta} = 2\,\alpha_M \sinh(\xi) I^{(0)}_{41}\epsilon^{\mu\nu\alpha\beta}\,U_\nu\left(E_\alpha\omega_\beta - B_\alpha\kappa_\beta\right)$$

$$\mathscr{T}^{\mu\nu} \equiv \Delta^{\mu\nu}_{\alpha\beta}\,T^{\alpha\beta} = 4\,\alpha_M \sinh(\xi) I^{(0)}_{42}\left(E^{(\mu}\kappa^{\nu)} + B^{(\mu}\omega^{\nu)} - \frac{1}{3}\Delta^{\mu\nu}\left(\kappa\cdot E + \omega\cdot B\right)\right)$$

$$\mathscr{E}_{\rm EM} = \alpha_M \sinh(\xi)\left\{\mathscr{E}_{(0)}\,\omega : F + 2\left[\left(I^{(0)}_{40} + I^{(0)}_{41}\right)\kappa\cdot E - 2I^{(0)}_{41}\omega\cdot B\right]\right\}$$

$$\mathscr{P}_{\rm EM} = \alpha_M \sinh(\xi)\left\{\mathscr{P}_{(0)}\,\omega : F - 2\left[\left(I^{(0)}_{41} + \frac{5}{3}I^{(0)}_{42}\right)\kappa\cdot E - \frac{10}{3}I^{(0)}_{42}\omega\cdot B\right]\right\}$$

# Coupling between spin and electromagnetic fields - III

→ Modified single particle distribution function

$\alpha_M = \mu_M / T$

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-\beta(x)\cdot p \pm \xi(x)\right)\left(1+\frac{1}{2}\omega_{\mu\nu}(x)s^{\mu\nu}\right)\left(1 \mp \alpha_M F_{\mu\nu}(x)s^{\mu\nu}\right)$$

- Spin tensor

$$S^{\lambda,\mu\nu}_{\rm GLW} = \int {\rm dP\ dS}\ p^{\lambda}\, s^{\mu\nu}\left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}_{\rm PF} - 2\alpha_M \tanh(\xi)\, S^{\lambda,\mu\nu}_{\rm EM}$$

with

$$S^{\alpha,\beta\gamma}_{\rm EM} = \cosh(\xi)\left[U^{\alpha}\mathscr{A}_1\, F^{\beta\gamma} + \mathscr{A}_2\, U^{\alpha}U^{[\beta}\, F^{\gamma]}{}_{\delta}\, U^{\delta} + \mathscr{A}_3\left(U^{[\beta}\, F^{\gamma]\alpha} + g^{\alpha[\beta}\, F^{\gamma]}{}_{\delta}\, U^{\delta}\right)\right]$$

# Dispersion relation of spin-wave velocity - I

$$\kappa \cdot U = \omega \cdot U = 0$$

$$\kappa_\mu = \omega_{\mu\alpha} U^\alpha , \quad \omega_\mu = (1/2)\, \epsilon_{\mu\alpha\beta\gamma}\, \omega^{\alpha\beta} U^\gamma$$

- $\omega_{\mu\nu}$ is an antisymmetric tensor of rank 2: $\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta}\, U^\alpha \omega^\beta$
- We introduce a basis formed by a set of mutually orthogonal four-vectors: $U, X, Y, \& \ Z$

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha$$

Thus

$$\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$$

where

$$C_{\boldsymbol{\kappa}} = (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z})$$

$$C_{\boldsymbol{\omega}} = (C_{\omega X}, C_{\omega Y}, C_{\omega Z})$$

are (scalar and dimensionless) spin components

- In an unpolarized fluid at rest, $U^\mu = (1,0,0,0) \ \& \ \omega^{\mu\nu} = 0$. Considering small perturbations along z, we look for oscillations in $\omega_{\mu\nu}$.
- Perfect fluid background leads to well-known sound speed

$$c_s^2 = \left(\frac{\partial \mathscr{P}}{\partial \mathscr{E}}\right)_{\mathscr{N}} + \frac{\mathscr{N}}{\mathscr{E} + \mathscr{P}} \left(\frac{\partial \mathscr{P}}{\partial \mathscr{N}}\right)_{\mathscr{E}}$$

- We find $\partial_t C_{\kappa Z} = \partial_t C_{\omega Z} = 0$ and remaining components propagate as transverse waves.

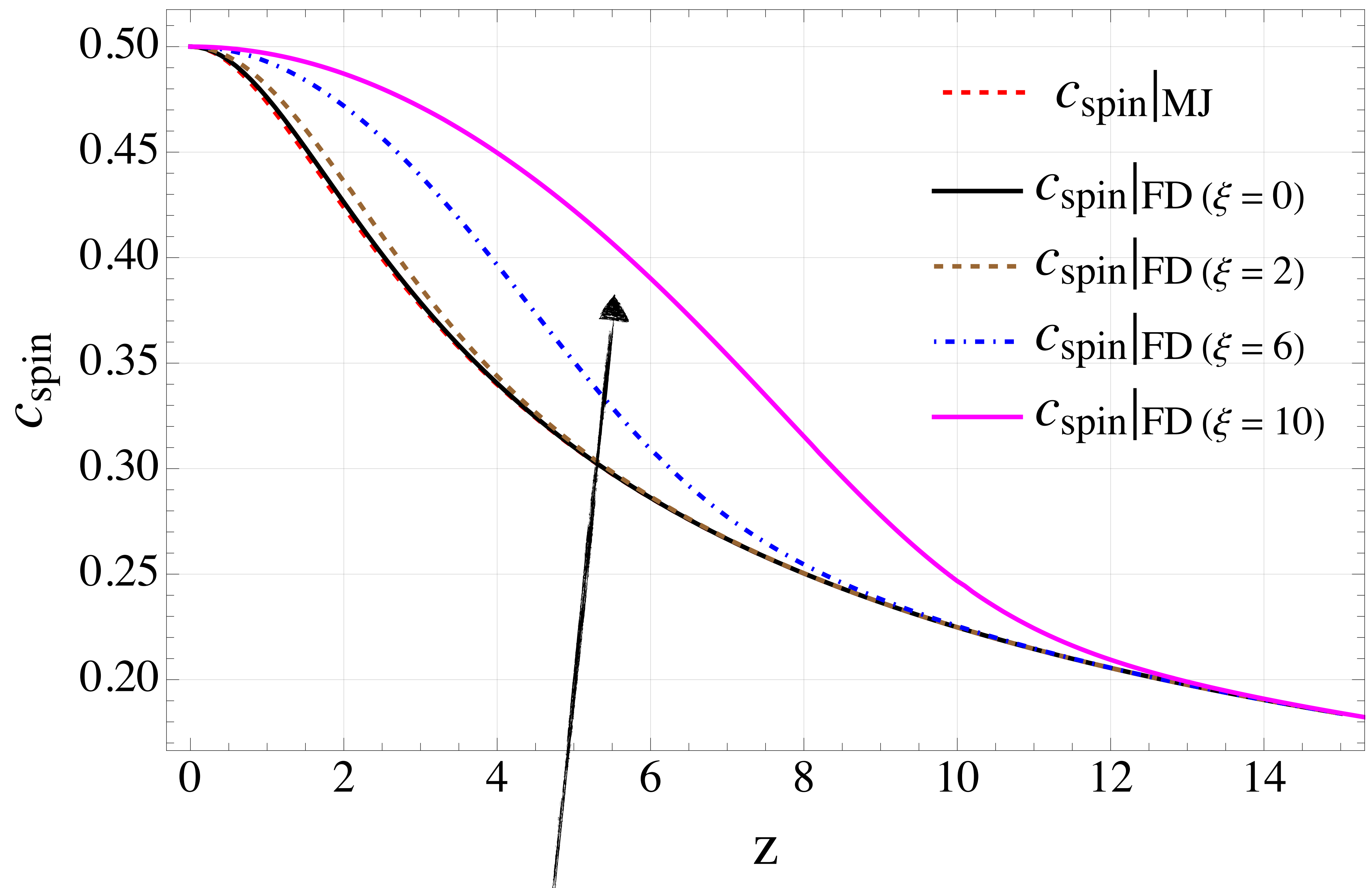
$$\boxed{c_{\rm spin}^2 = \frac{1}{4}\, \frac{\left(\partial \mathscr{E}/\partial T\right)_\xi - z^2 \left(\partial \mathscr{N}/\partial \xi\right)_T}{\left(\partial \mathscr{E}/\partial T\right)_\xi + \frac{z^2}{2} \left(\partial \mathscr{N}/\partial \xi\right)_T}}$$

# Dispersion relation of spin-wave velocity - II

→ Ideal-gas limit: $$c_{\rm spin}^2\Big|_{\rm MJ} = \frac{1}{4}\left[\frac{K_3(z)}{K_3(z)+\frac{z}{2}K_2(z)}\right]$$

→ Fermi-Dirac gas limit:

$$c_{\rm spin}^2\Big|_{\rm FD} = \frac{1}{4}\,\frac{\sum_{\ell=1}^{\infty}\frac{(-1)^{\ell+1}}{\ell}\cosh(\ell\xi)K_3(\ell z)}{\sum_{\ell=1}^{\infty}\frac{(-1)^{\ell+1}}{\ell}\cosh(\ell\xi)\left[K_3(\ell z)+\frac{\ell z}{2}K_2(\ell z)\right]}$$




$\mu$ enhances the $c_{\rm spin}$

→ Linearly polarized solutions:

$$C_{\boldsymbol\kappa} = C_0\,{\rm Re}\left[e^{-ik\left(c_{\rm spin}t - z\right)}\right]\left(\hat{\boldsymbol e}_1\cos(\theta)+\hat{\boldsymbol e}_2\sin(\theta)\right)$$

$$C_{\boldsymbol\omega} = 2\,c_{\rm spin}\,C_0\,{\rm Re}\left[e^{-ik\left(c_{\rm spin}t - z\right)}\right]\left(\hat{\boldsymbol e}_1\sin(\theta)-\hat{\boldsymbol e}_2\cos(\theta)\right)$$

where analogy to the EM waves is evident since $C_{\boldsymbol\omega} = 2c_{\rm spin}\hat{\boldsymbol n}\times C_{\boldsymbol\kappa}$

$kc_{\rm spin}$ is the angular frequency

$\xi = \mu_B/T\,,\quad z = m/T$

$\theta$ is the inclination angle with respect to x-axis

$\hat{\boldsymbol n} = \hat{\boldsymbol e}_3$ being the direction of the wave propagation

$C_0$ is real amplitude of the wave

# Pseudogauge dependence of quantum fluctuations-III

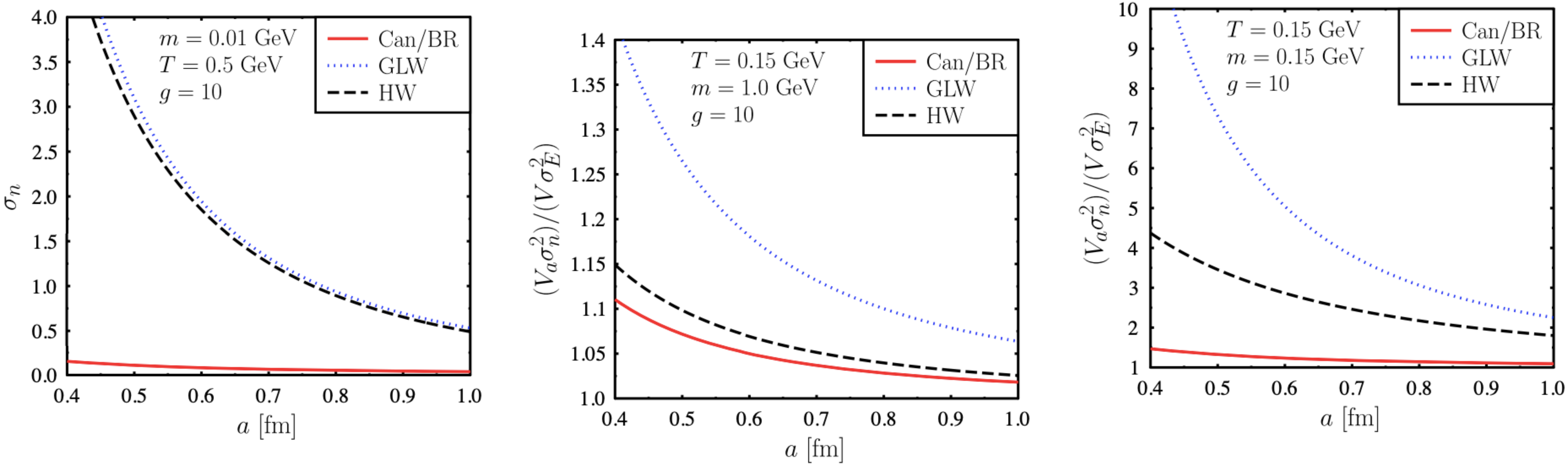
→ Quantum fluctuations of energy in subsystems of a hot relativistic gas of spin-1/2 particles

$$\sigma^2(a,m,T) = \langle :\hat{T}^{00}_a :: \hat{T}^{00}_a :\rangle - \langle :\hat{T}^{00}_a :\rangle^2$$

Variance

$$\sigma_n(a,m,T) = \frac{\left(\langle :\hat{T}^{00}_a :: \hat{T}^{00}_a :\rangle - \langle :\hat{T}^{00}_a :\rangle^2\right)^{1/2}}{\langle :\hat{T}^{00}_a :\rangle}$$

Normalized standard deviation




Phys.Rev.D 103 (2021) 9, L091502

# Spin polarization tensor

$$\kappa \cdot U = \omega \cdot U = 0$$

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- $\omega_{\mu\nu}$ is an antisymmetric tensor of rank 2: $\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$
- We introduce a basis formed by a set of mutually orthogonal four-vectors: $U, X, Y, \& Z$

Thus

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha$$

$$\omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$$

where

$$\boldsymbol{C}_{\boldsymbol{\kappa}} = (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z})$$

$$\boldsymbol{C}_{\boldsymbol{\omega}} = (C_{\omega X}, C_{\omega Y}, C_{\omega Z})$$

are (scalar and dimensionless) spin components

# Modeling of the spin polarization dynamics - I

→ Non-boost-invariant and transversely homogeneous

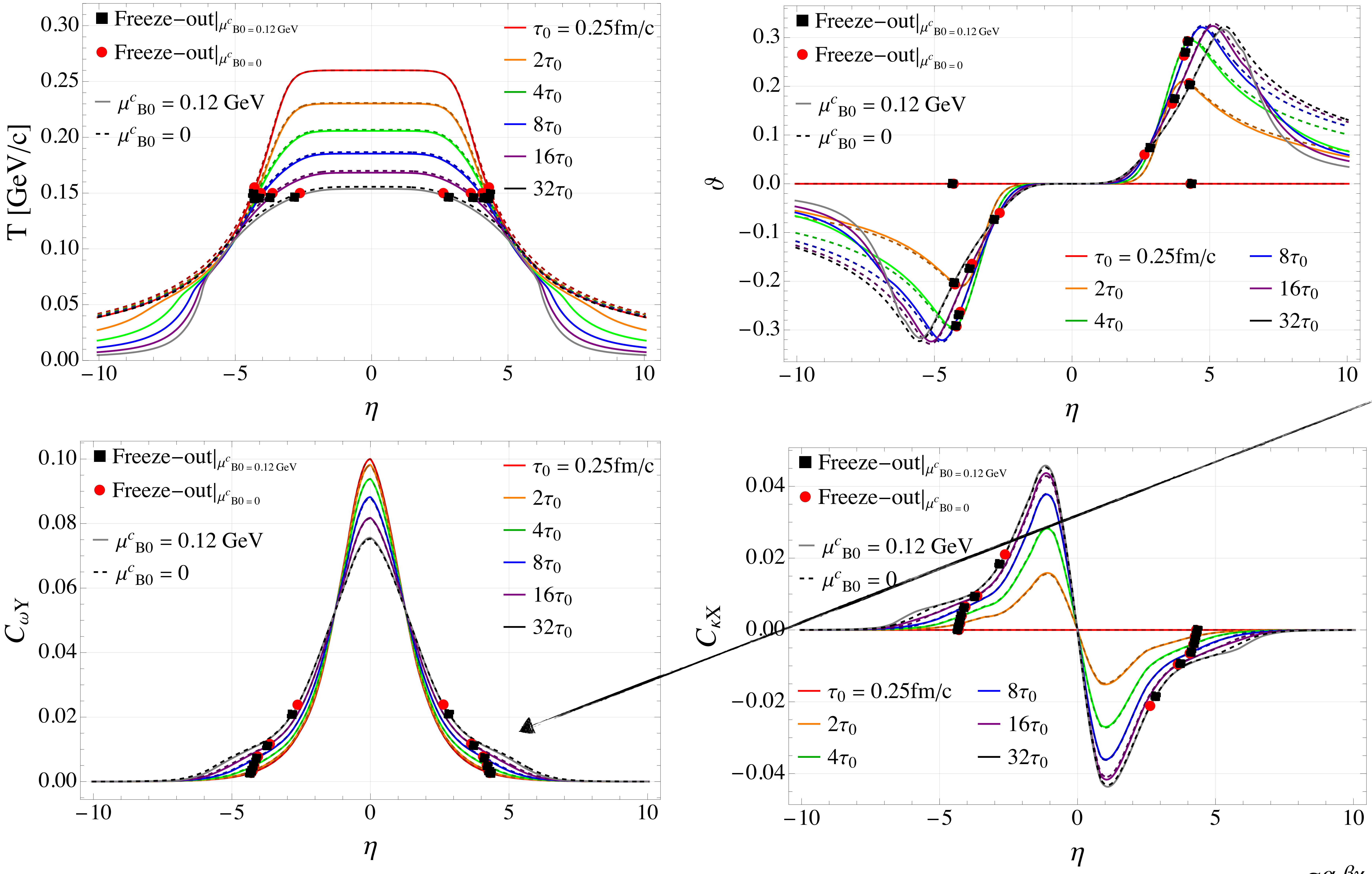
$$\mathscr{E}_0(\eta) = \frac{\mathscr{E}_0^c}{2}\left[\Theta(\eta)\left(\tanh(a-\eta\, b)+1\right) + \Theta(-\eta)\left(\tanh(a+\eta\, b)+1\right)\right]$$

$a = 6.2\,, b = 1.9$

$T_0^c = 0.26$ GeV

$\mathscr{E}_0^c = \mathscr{E}(T_0^c, \mu_{B0}^c)$

$C_{\omega Y_0} = 0.1\ \mathrm{sech}(\eta)$




Pattern is opposite in comparison to $\eta = 0$

$$S^{\alpha,\beta\gamma}_{z\gg 1} = \cosh(\xi)\,\mathscr{N}_{(0)}\,U^\alpha\left[\omega^{\beta\gamma} + 2\,U^\delta\,U^{[\beta}\,\omega^{\gamma]}{}_\delta\right]$$