

# Azimuthal anisotropies at high- $p_T$ from transverse momentum dependent (TMD) parton distribution and fragmentation functions

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Ismail Soudi

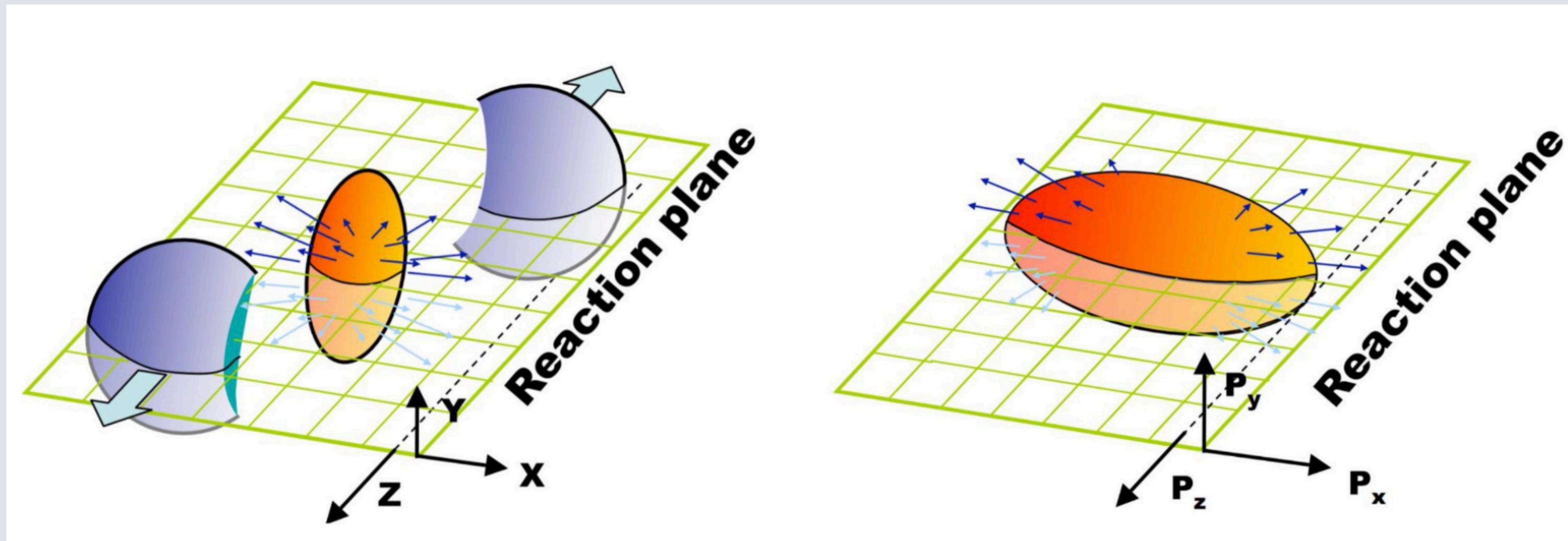
Wayne State University

Based on IS and Abhijit Majumder ArXiv:[2308.14702](https://arxiv.org/abs/2308.14702)

SPIN 2023, Durham



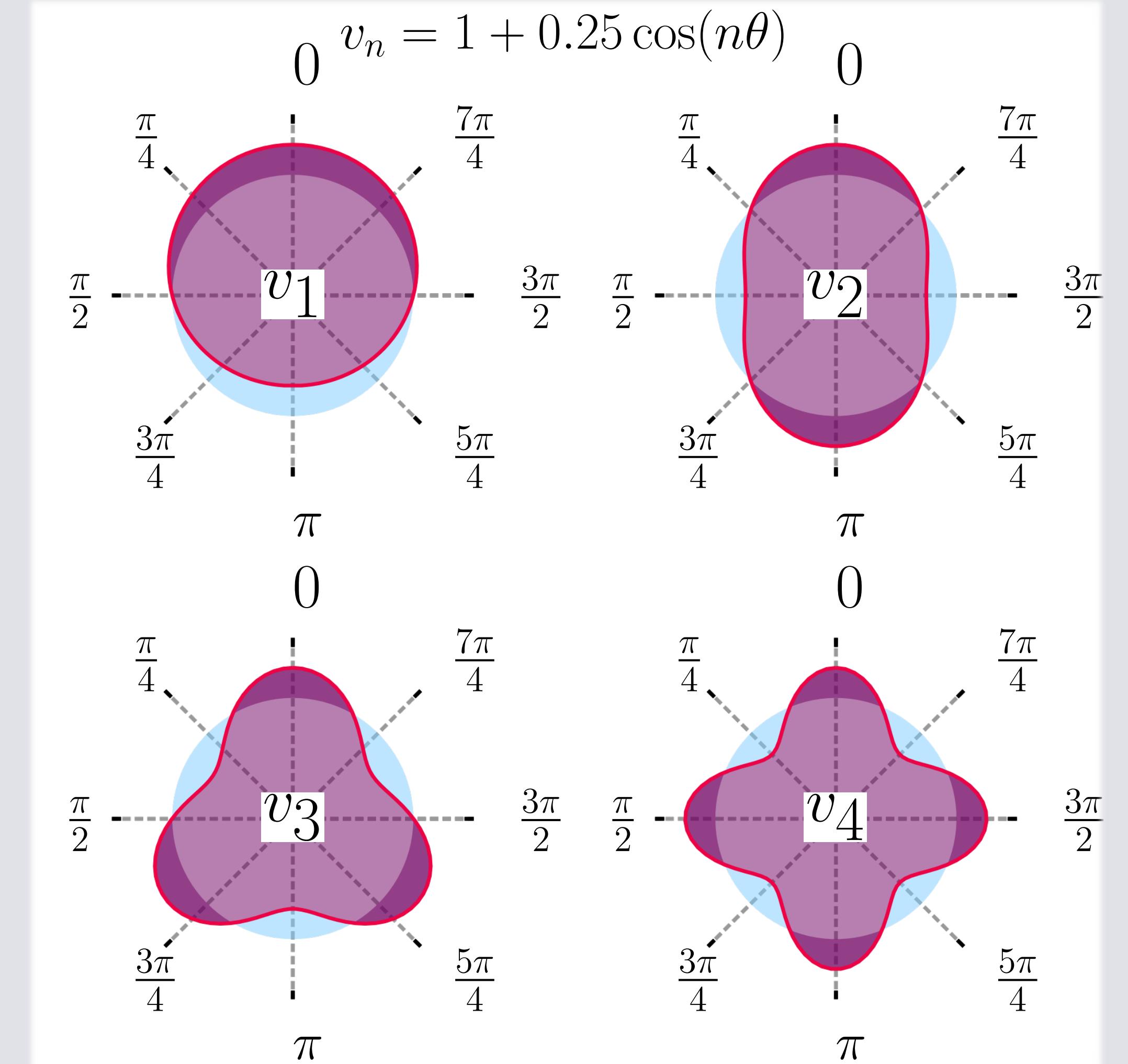
# Elliptic Flow in Relativistic Heavy-Ion Collisions



# $v_n$ Azimuthal Anisotropies

Azimuthal momentum correlated  
with soft bulk (event plane)

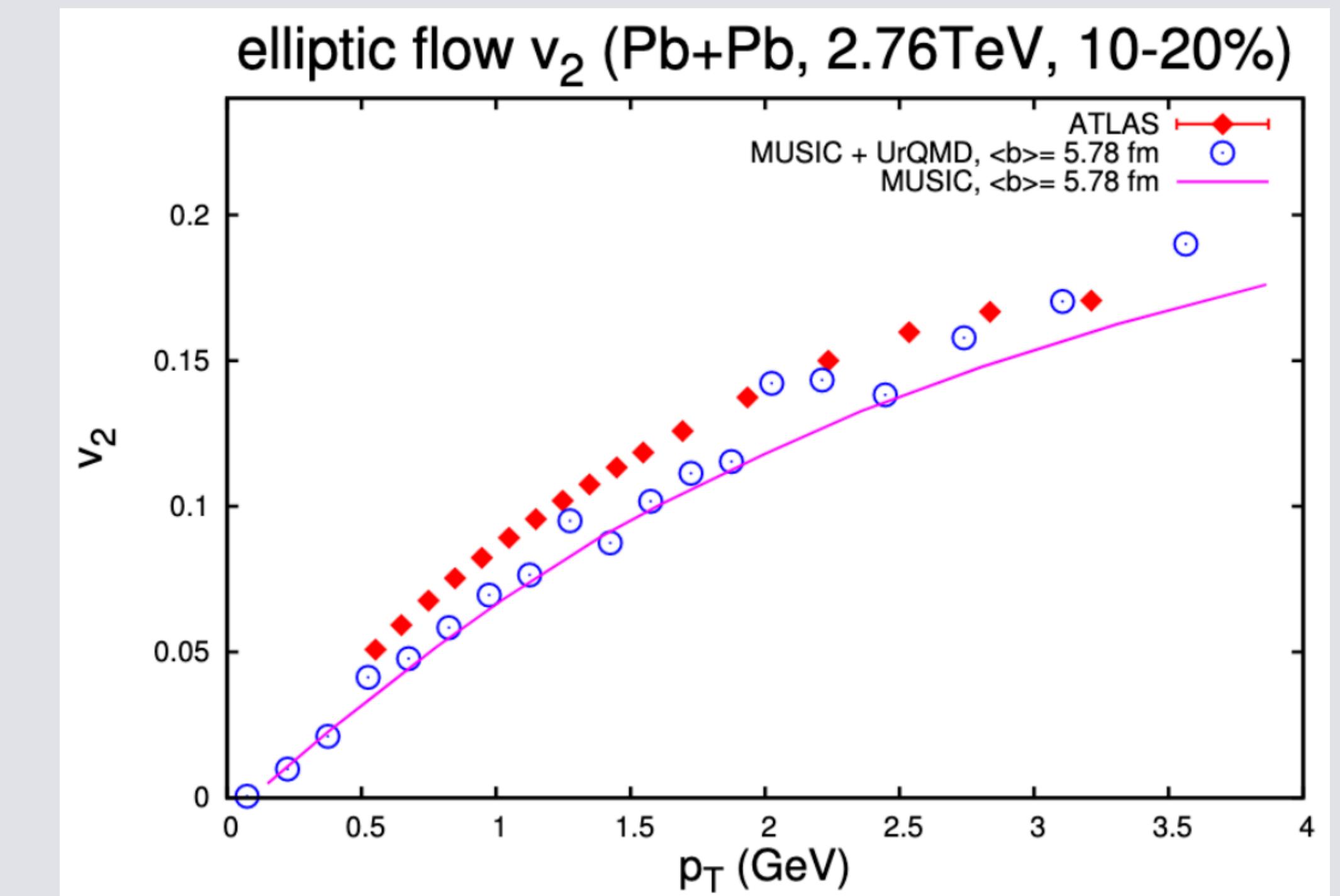
$$\frac{dN}{d\phi} \propto 1 + \sum_1^\infty 2v_n \cos[n(\phi - \Psi_n)]$$



# Signature of QGP formation

Measurement of elliptic flow  $v_2$   
one of the main signatures of  
the QGP

$$\frac{dN}{d\phi} \propto 1 + \sum_1^\infty 2v_n \cos[n(\phi - \Psi_n)]$$

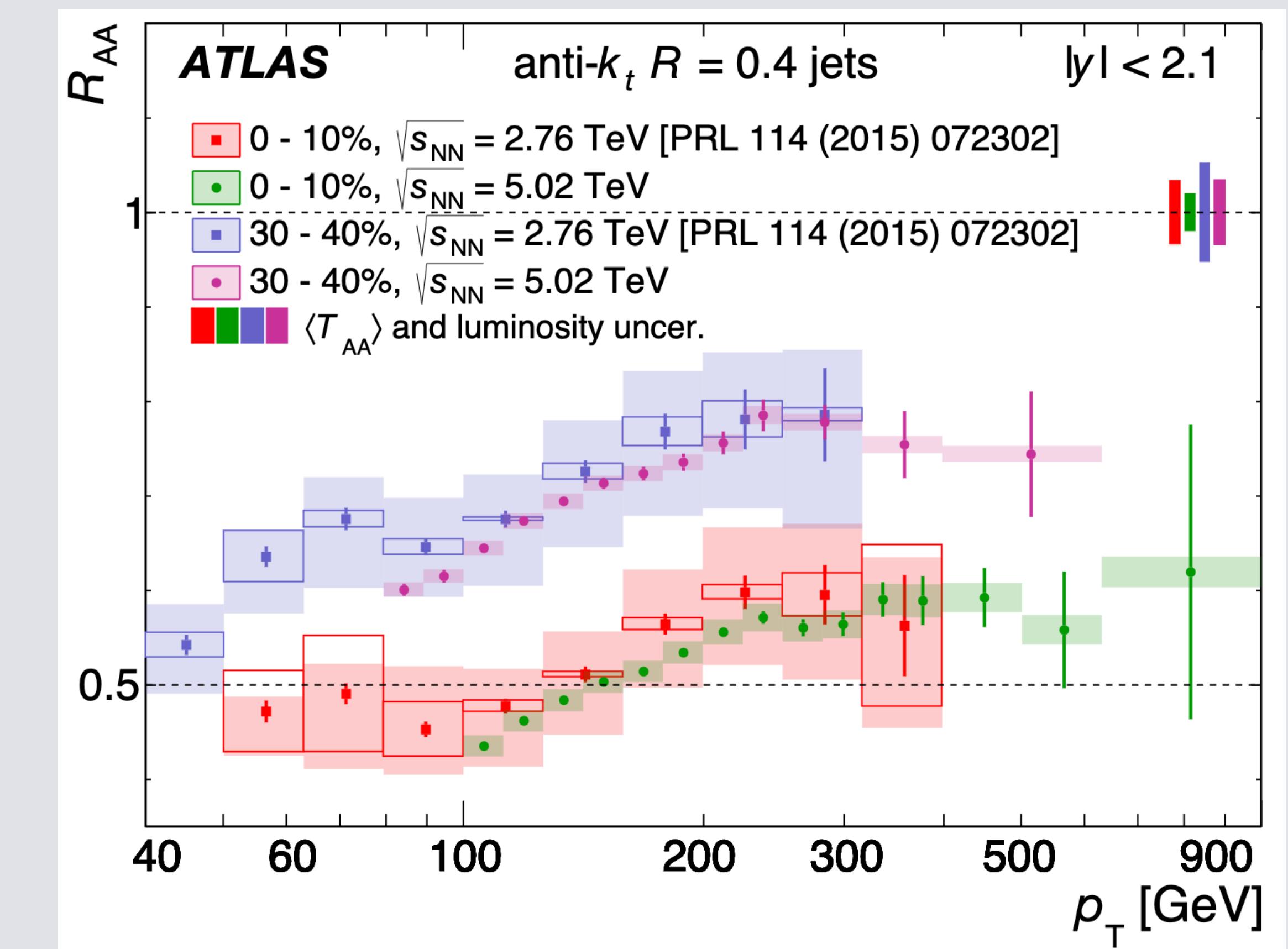


[Int. J. of Mod. Phys. A, Vol. 28, 1340011 (2013)]  
[ATLAS Phys.Lett. B707, 330 (2012)]

# Jet suppression: 2nd Signature of QGP formation

Energy loss leads to a suppression of jets

$$R_{AA} = \frac{\frac{d^2N_{jet}}{dp_T dy} \Big|_{PbPb}}{\langle T_{AA} \rangle \frac{d^2N_{jet}}{dp_T dy} \Big|_{pp}}$$

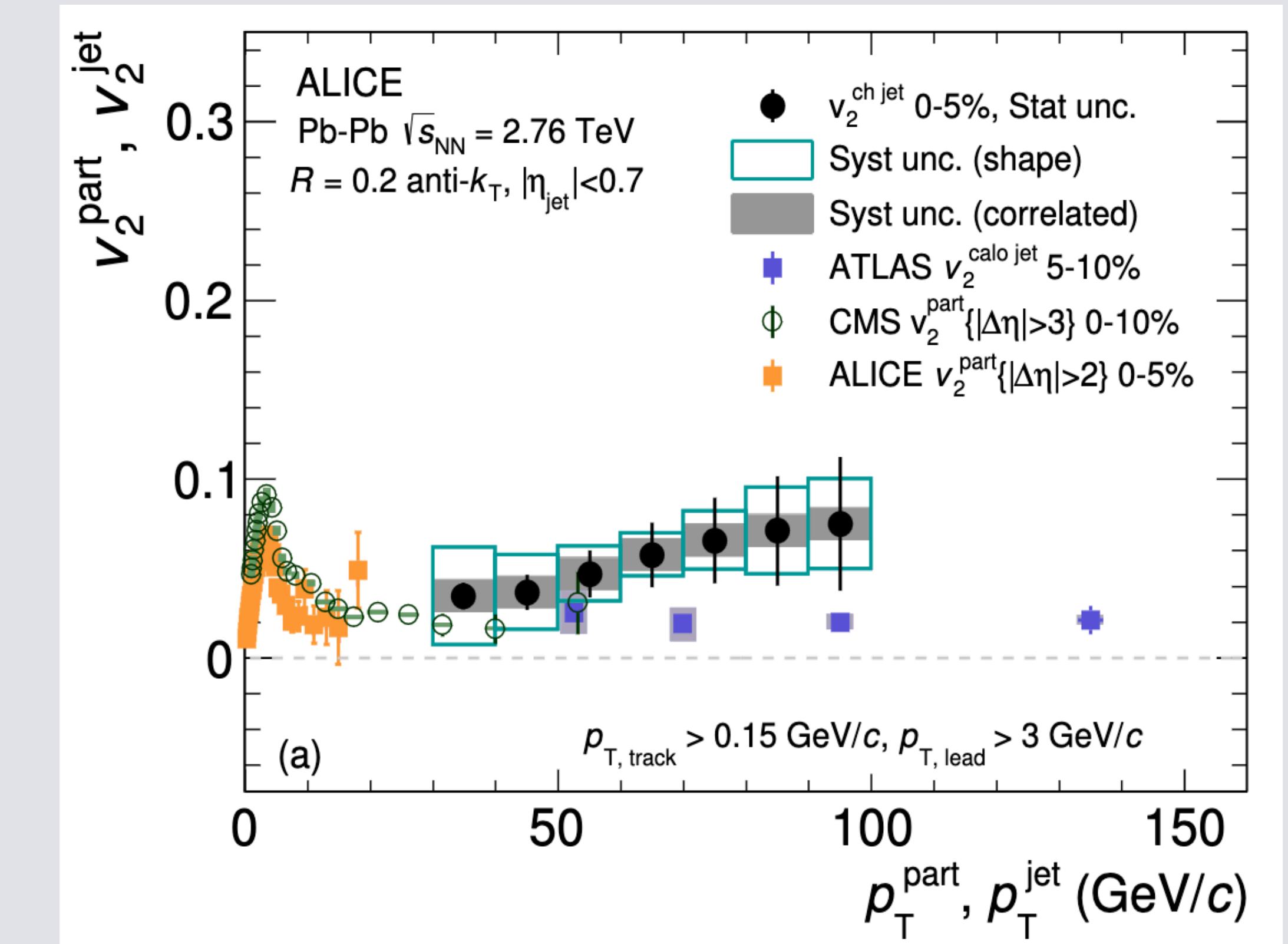
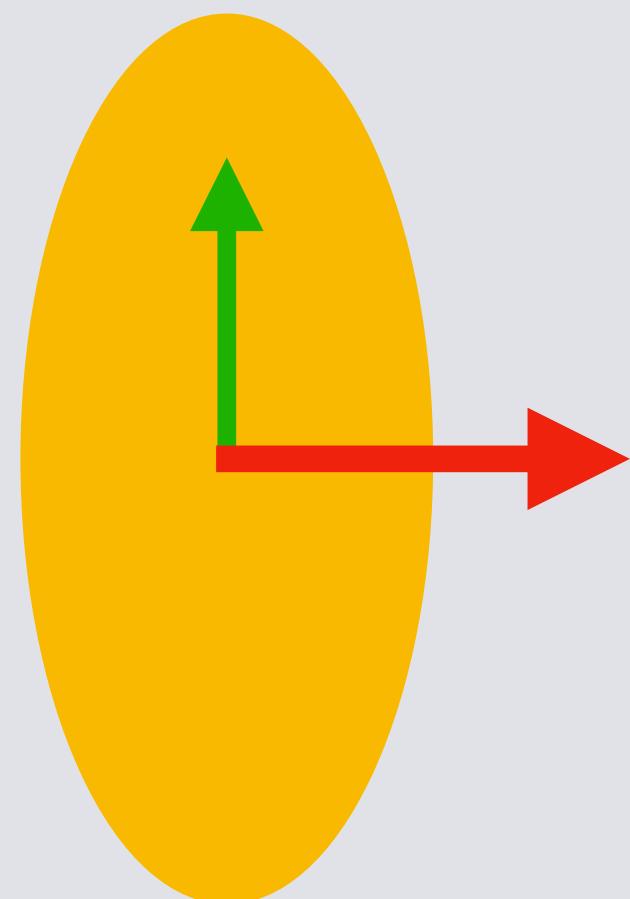


[ATLAS Phys. Lett. B 790 (2019) 108]

# Jet $v_2$

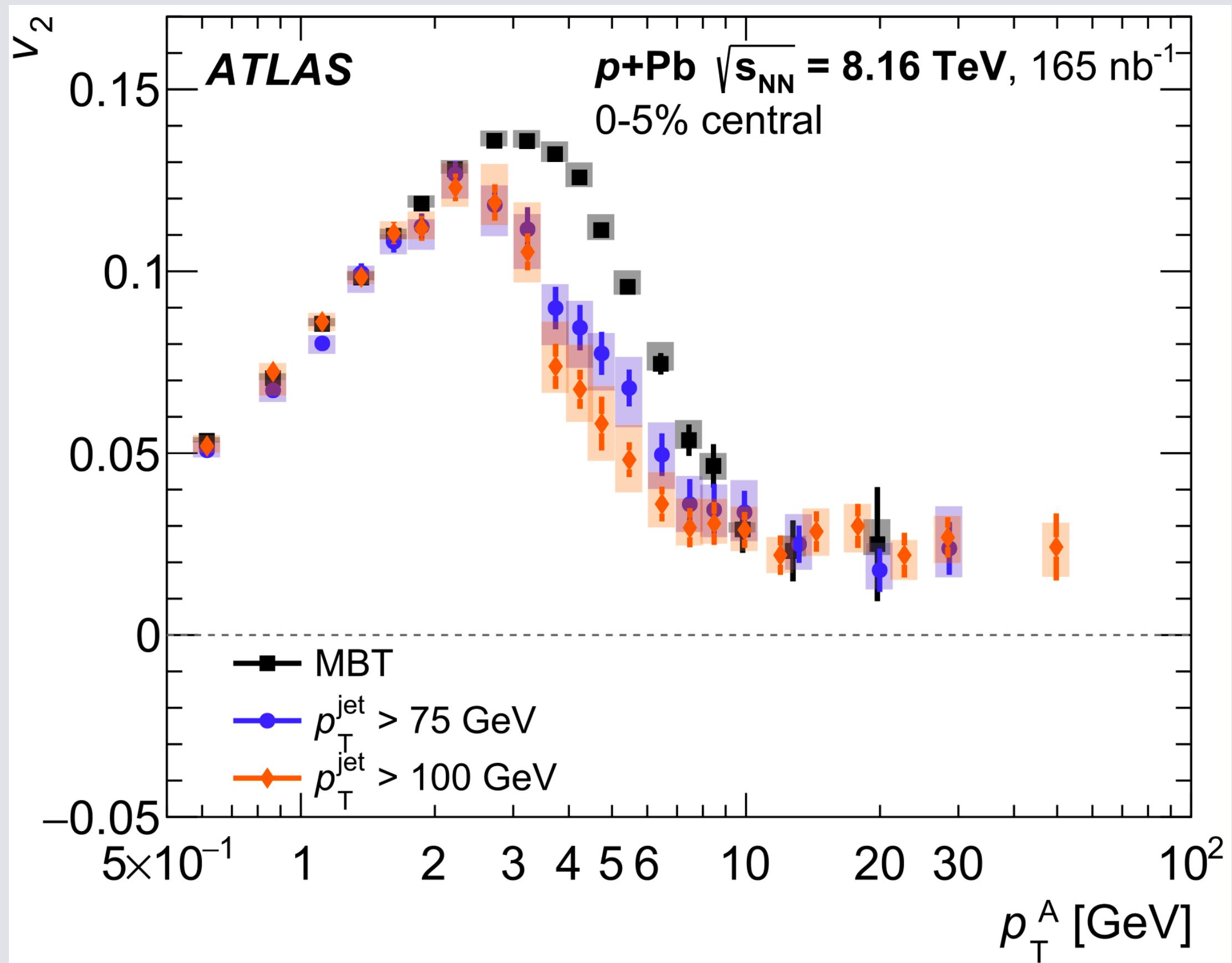
Path length dependence of energy loss

$$\frac{dN}{d\phi_{jet}} \propto 1 + \sum_1^8 2v_n^{jet} \cos[n(\phi_{jet} - \Psi_n)]$$



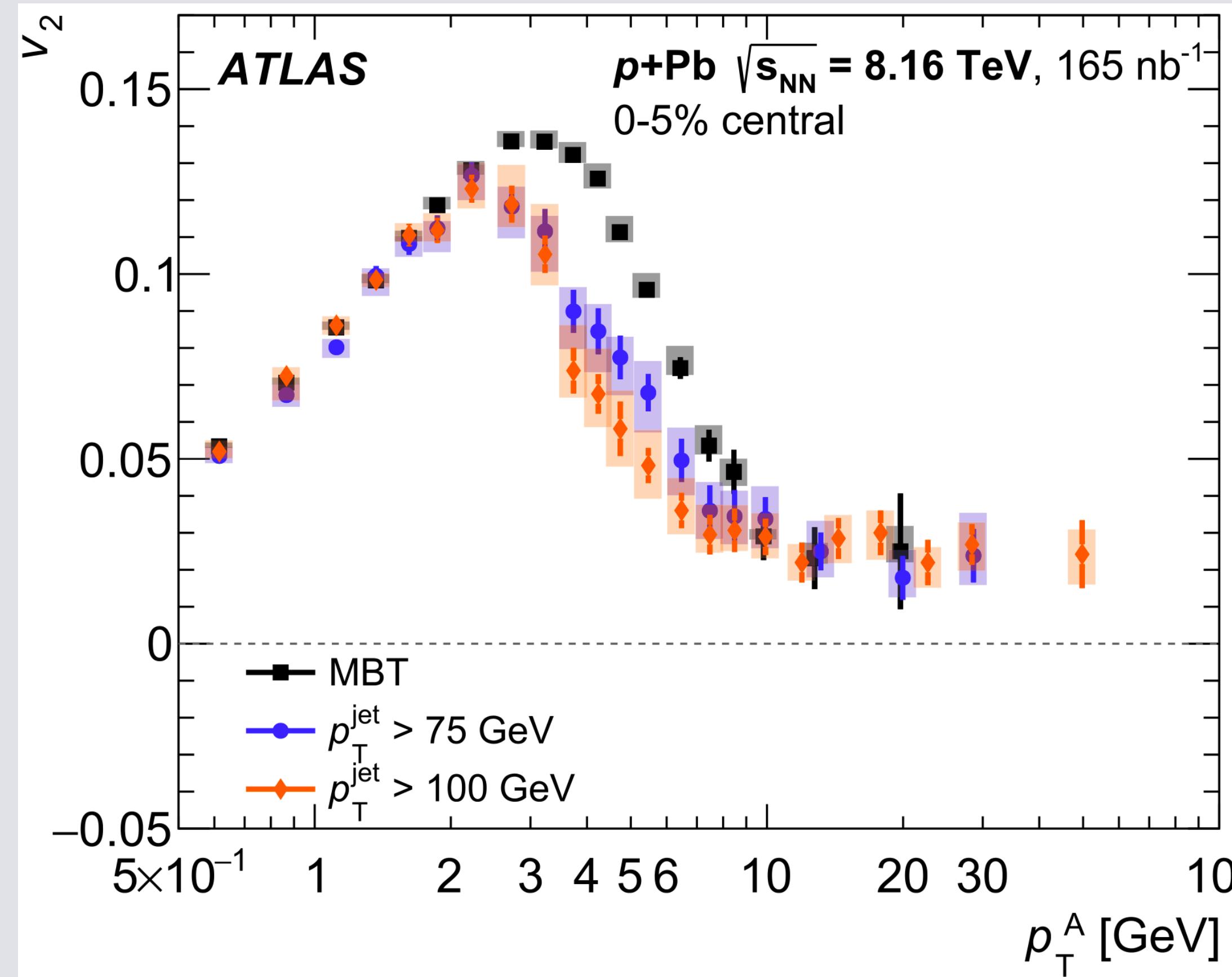
[ALICE Phys.Lett. B 753 (2016) 511-525]

# Small systems

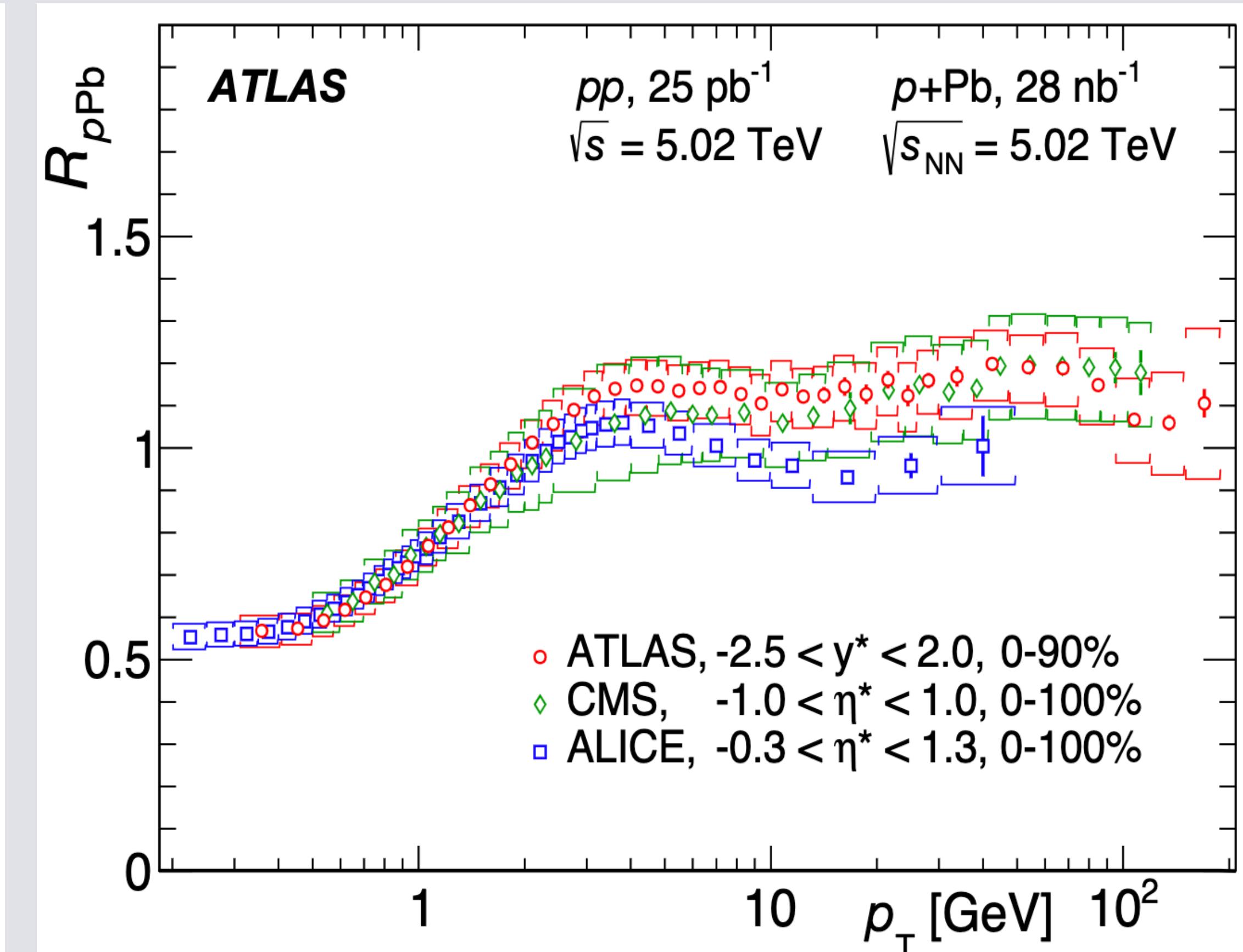


[ATLAS Eur. Phys. J. C 80 (2020) 73]

# Small systems



[ATLAS Eur. Phys. J. C 80 (2020) 73]

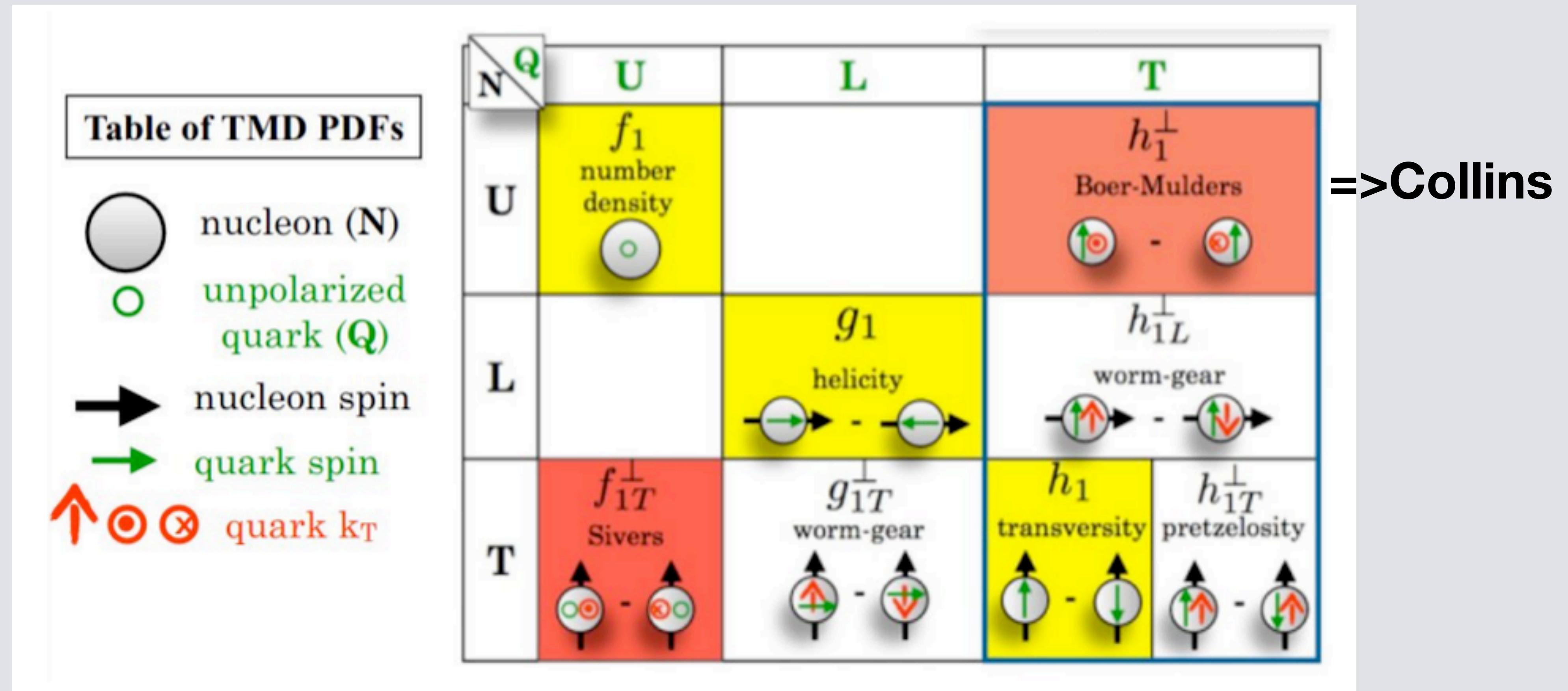


[ATLAS JHEP 07 (2023) 074]

[CMS JHEP 04 (2017) 039]

[ALICE JHEP 11 (2018) 013]

# Transverse Momentum Distributions



[Fig from PHENIX Spin Physics Overview]

[D. Boer, P. J. Mulders, J. C. Collins, J. Rodrigues, C. Pisano, S. J. Brodsky, M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, D.W. Sivers, F. G. Celiberto...]

# Transverse Momentum Distributions

Table of TMD PDFs			
N Q	U	L	T
U	$f_1$ number density 		$h_1^\perp$ Boer-Mulders 
L		$g_1$ helicity 	$h_{1L}^\perp$ worm-gear 
T	$f_{1T}^\perp$ Sivers 	$g_{1T}^\perp$ worm-gear 	$h_1$ transversity  $h_{1T}^\perp$ pretzelosity 

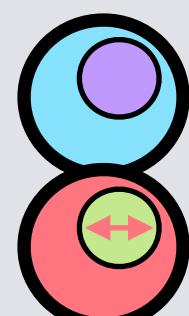
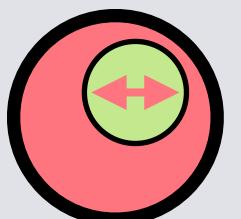
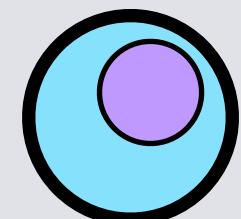
=>Collins

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# Transverse Momentum Distributions

$$\Phi^{\alpha\beta}\epsilon_{\alpha}^{\lambda_1}(k)\epsilon_{\beta}^{\lambda_2*}(k) = \frac{1}{2x} \left\{ \delta_{\lambda_1, \lambda_2} f^g(x, k_{\perp}) + \delta_{\lambda_1, -\lambda_2} \frac{k_{\perp}^2}{2M^2} h^{\perp g}(x, k_{\perp}) \right\}$$

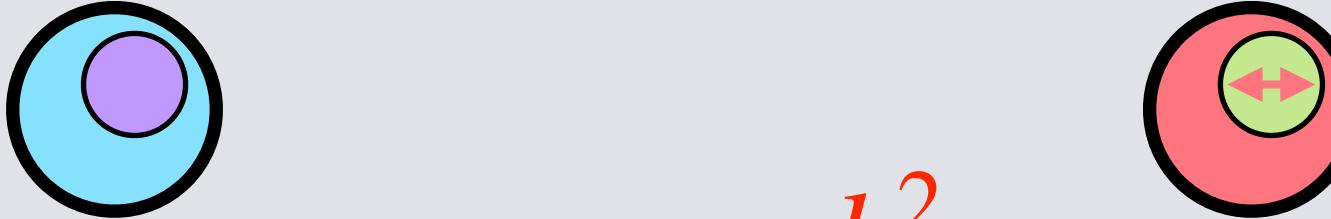


Unpolarized gluon inside unpolarized hadron



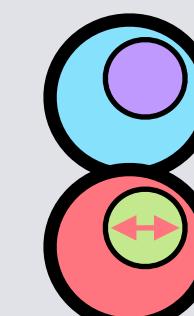
Linearly polarized gluon inside unpolarized hadron

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The Boer-Mulders/Collins flips helicity between amplitude and conjugate

Requires two flips for unpolarized pion production

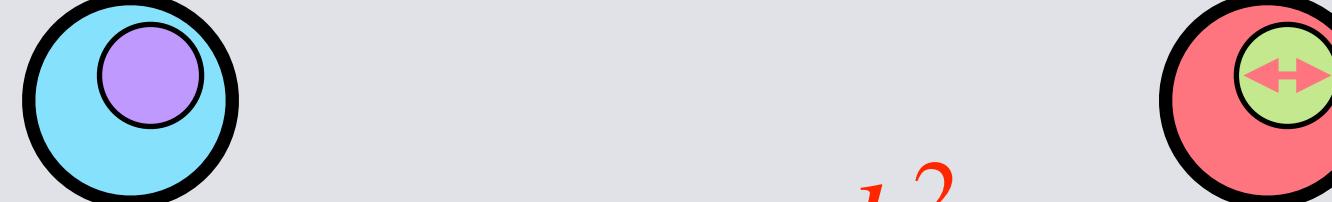


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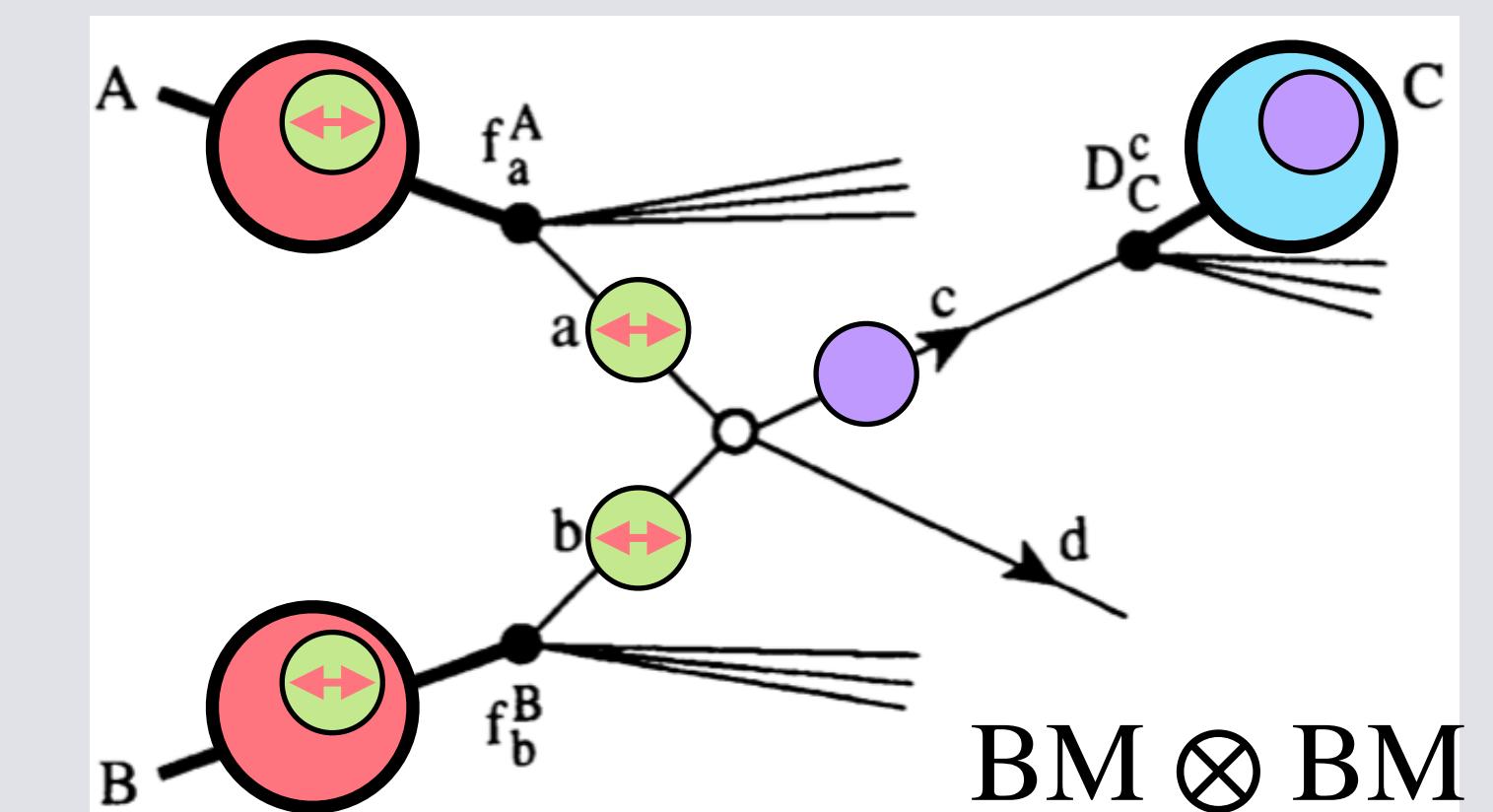
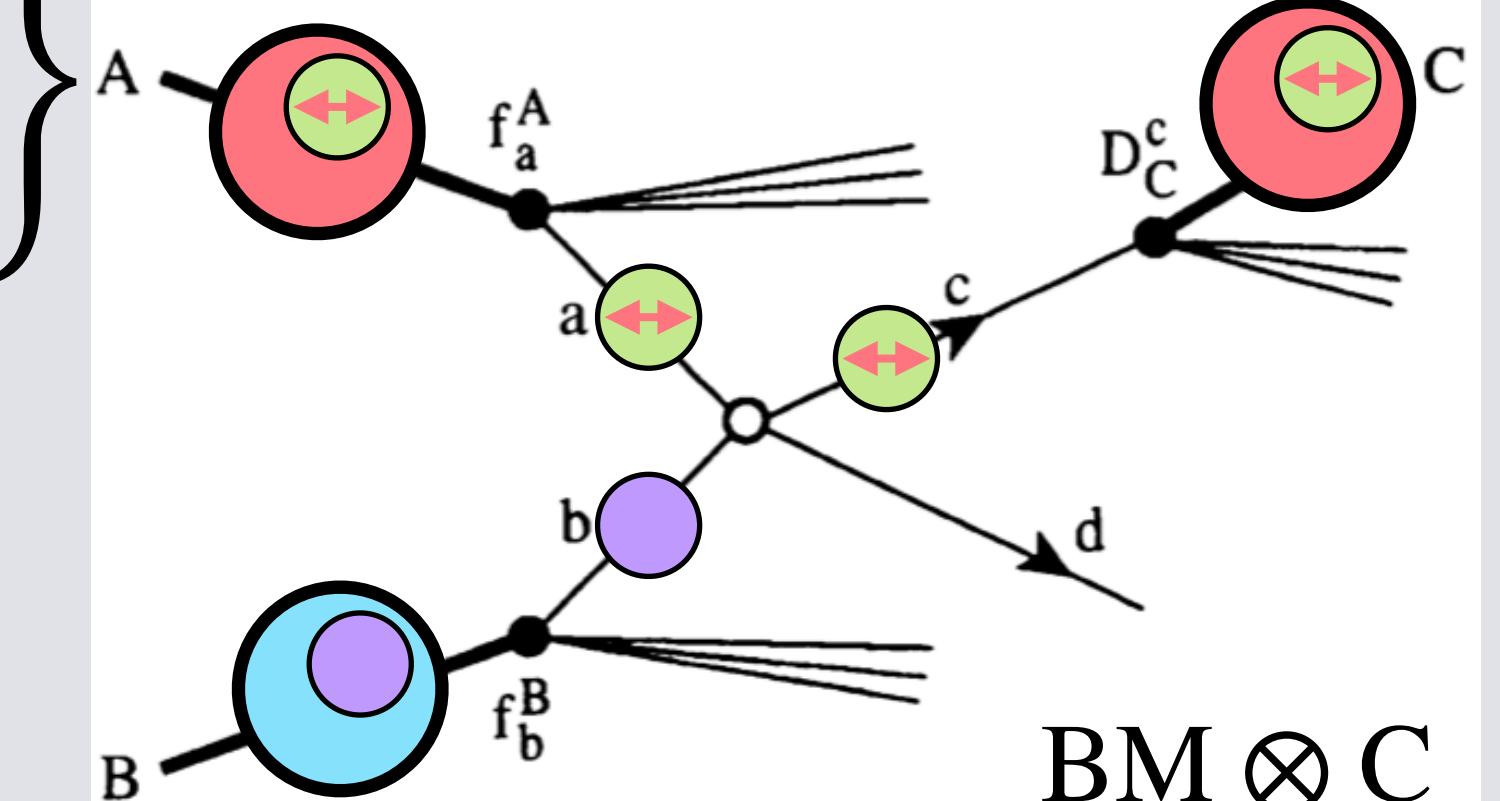
For  $gg \leftrightarrow gg$ :

- $BM \otimes C$

$$\Rightarrow M_{\lambda, \lambda}^{\lambda, \lambda} \left( M_{-\lambda, \lambda}^{-\lambda, \lambda} \right)^*$$

- $BM \otimes BM$

$$\Rightarrow M_{-\lambda, \lambda}^{\lambda, -\lambda} \left( M_{-\lambda, \lambda}^{-\lambda, \lambda} \right)^*$$



 Unpolarized gluon inside unpolarized hadron

 Linearly polarized gluon inside unpolarized hadron

# Spin-helicity formalism

Matrix element for  $\text{BM} \otimes \text{C}$  Naturally contains asymmetries

$$\Sigma^{\text{BM} \otimes \text{C}} = H^{\perp(1)}(z, k_{\perp C}) \left\{ h^{\perp(1)}(x_a, k_{\perp a}^2) f(x_b, k_{\perp b}^2) \hat{M}_1^0 \hat{M}_2^0 \cos(4(\phi_{ab} - \phi_{bc})) + f(x_a, k_{\perp a}^2) h^{\perp}(x_b, k_{\perp b}^2) \hat{M}_1^0 \hat{M}_3^0 \cos(4(\phi_{ab} - \phi_{ac})) \right\},$$

with

$$\hat{M}_1^0 \hat{M}_2^0 = \frac{9}{4} g_s^4 \frac{u^2 + tu + t^2}{t^2},$$

$$\hat{M}_1^0 \hat{M}_3^0 = \frac{9}{4} g_s^4 \frac{u^2 + tu + t^2}{u^2},$$

and

$$\tan \phi_{ij} = \tan \frac{\phi_j - \phi_i}{2} \frac{\sin \frac{\theta_j + \theta_i}{2}}{\sin \frac{\theta_j - \theta_i}{2}}.$$

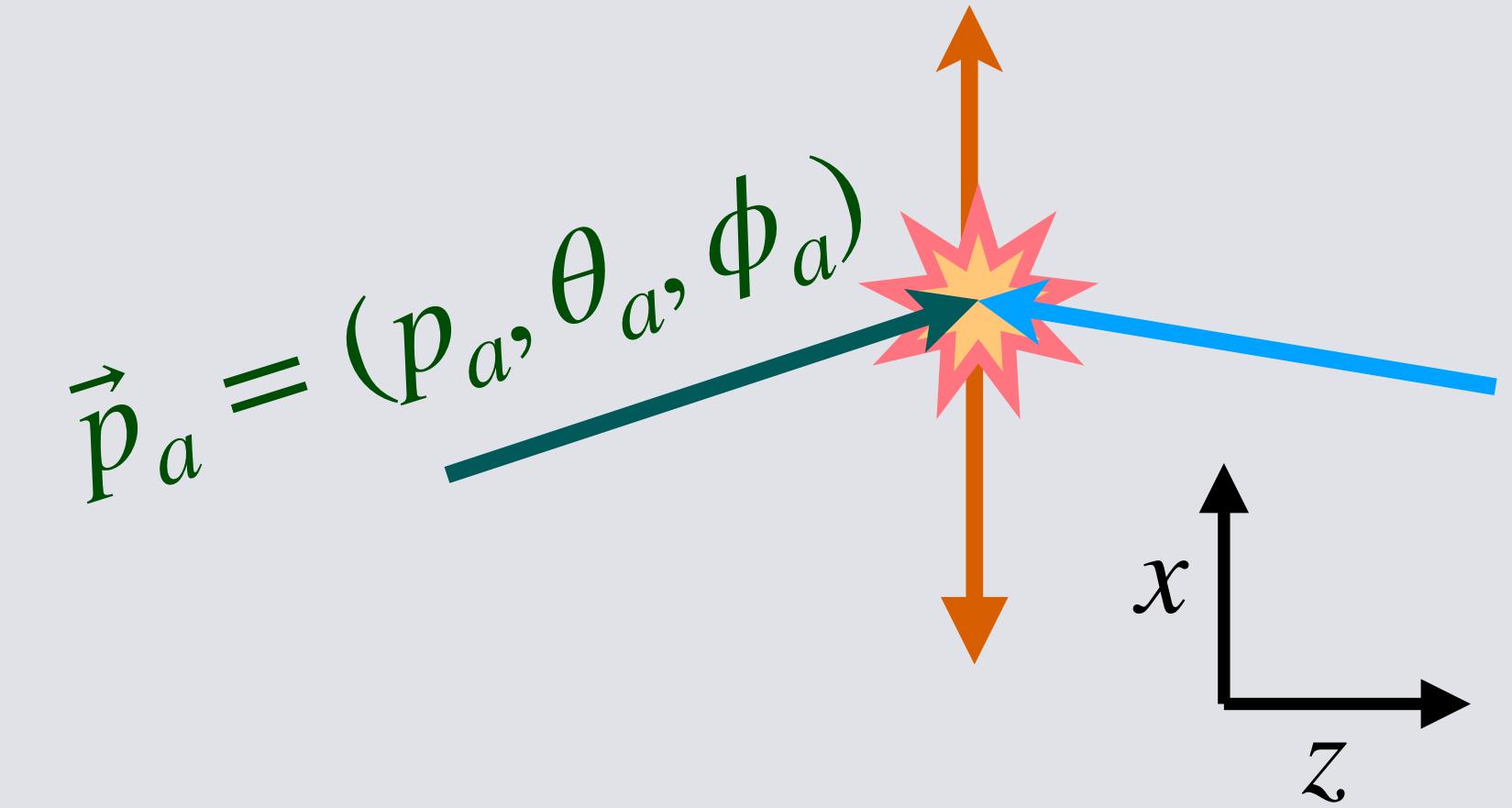
[I.S. A. Majumder ArXiv:2308.14702]

# Spin Independent

TMD matrix element contains angular correlations as well

$$\begin{aligned} \frac{\hat{s}}{\hat{t}} &= \frac{-\hat{s}}{2p_c \cdot p_a} = \frac{-\hat{s}}{2p_c p_a [1 - \theta_a \cos(\phi_a - \phi_c)]} \\ &= \frac{-\hat{s}}{2p_c p_a} \left[ 1 + \frac{\theta_a^2}{4} + \theta_a \cos(\phi_a - \phi_c) \right. \\ &\quad \left. + \frac{\theta_a^2 \cos(2(\phi_a - \phi_c))}{4} + \mathcal{O}(\theta_a^3) \right]. \end{aligned}$$

$$\vec{p}_c = (p_c, \pi/2, \phi_c)$$

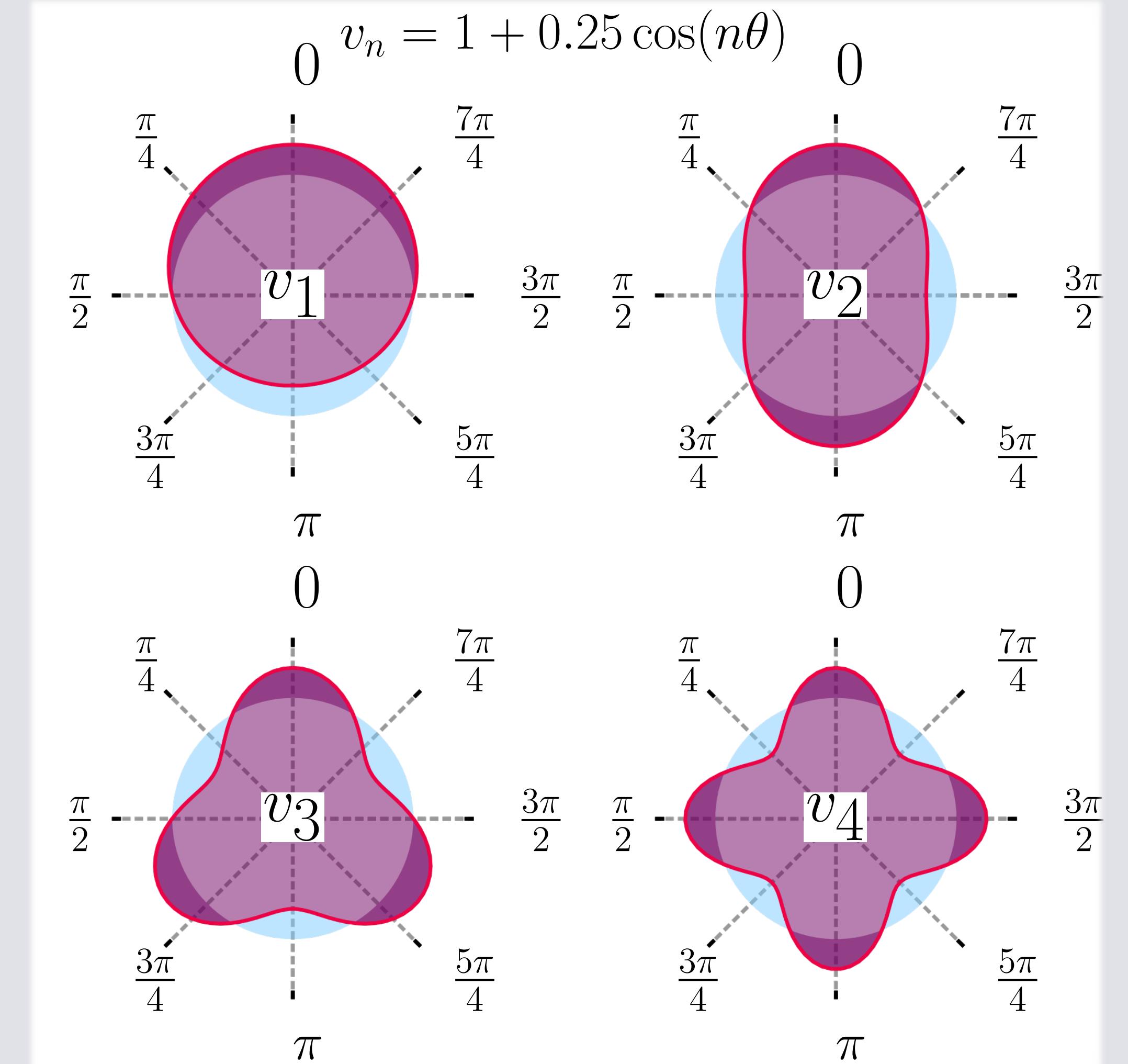


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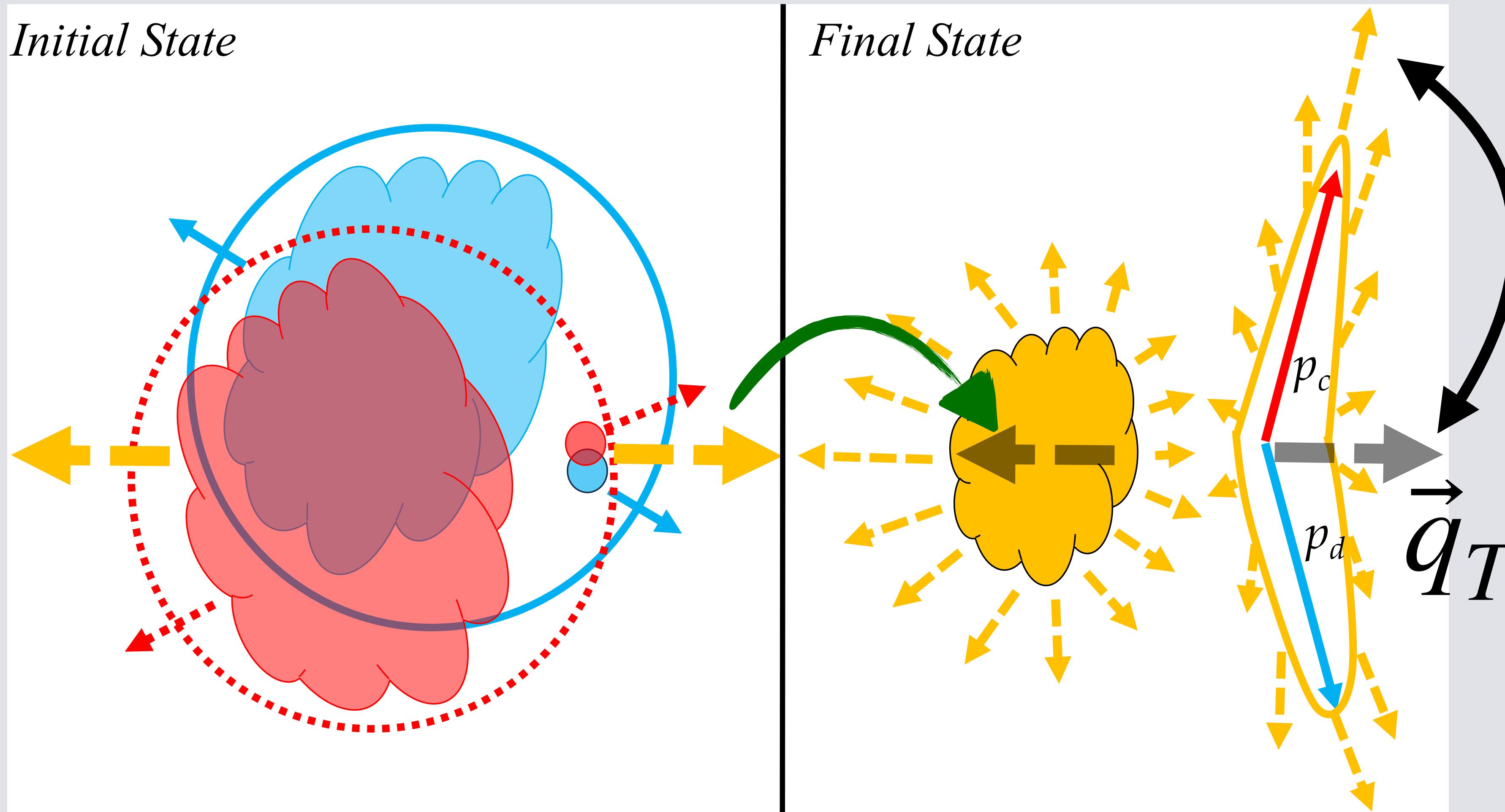
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Azimuthal momentum correlated  
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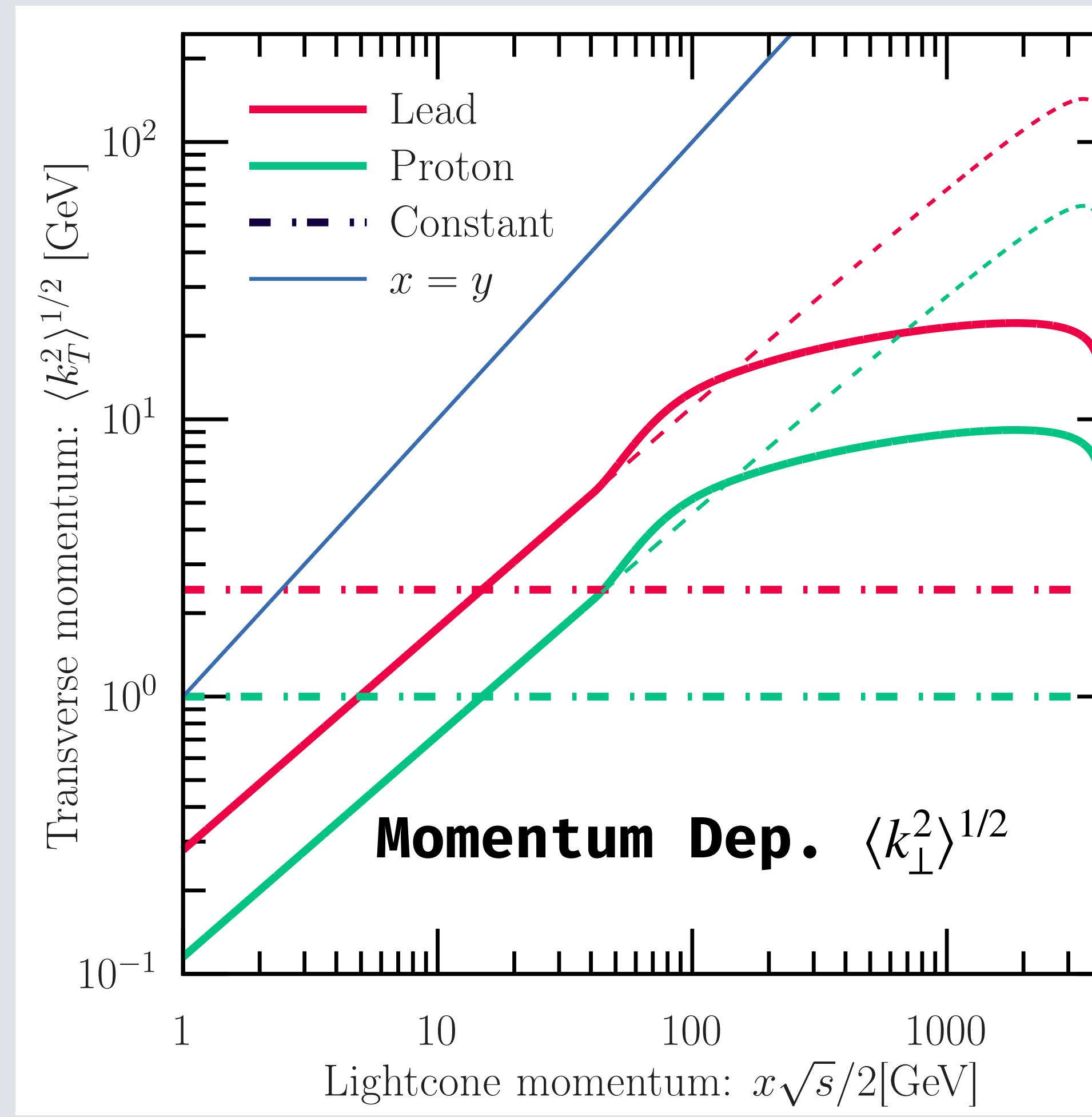
$$\frac{dN}{d\phi} \propto 1 + \sum_1^\infty 2v_n \cos[n(\phi - \Psi_n)]$$



# TMD Scattering



# p-p Results



$$f(x, k_\perp) = \frac{4\pi}{\langle k_\perp^2 \rangle} e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}} f(x),$$

$$\langle k_\perp^2 \rangle_{p-Pb} = A^{1/3} \langle k_\perp^2 \rangle_{p-p}$$

Factorization of  $(x \otimes k_T)$   
 $\Rightarrow$  Allows to approximate to p-A  
 Soffer Bound:  $B \leq 1$     $b \leq 1$

$$\frac{k_\perp^2}{2M^2} |h^{\perp g}(x, k_\perp)| = b \cdot f^g(x, k_\perp),$$

$$\frac{k_\perp^2}{2M^2} |H^{\perp g}(x, k_\perp)| = B \cdot D^g(x, k_\perp),$$

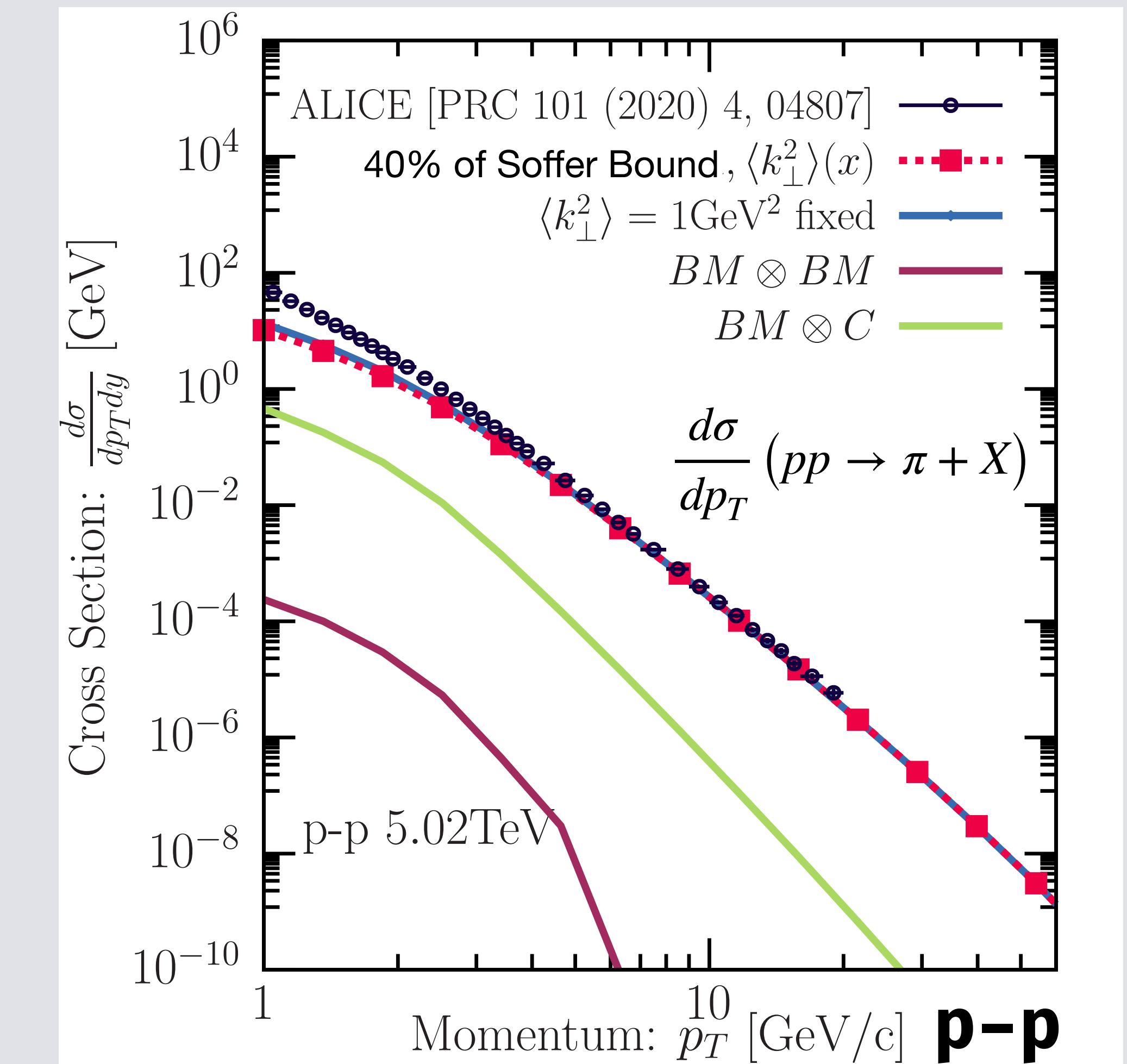
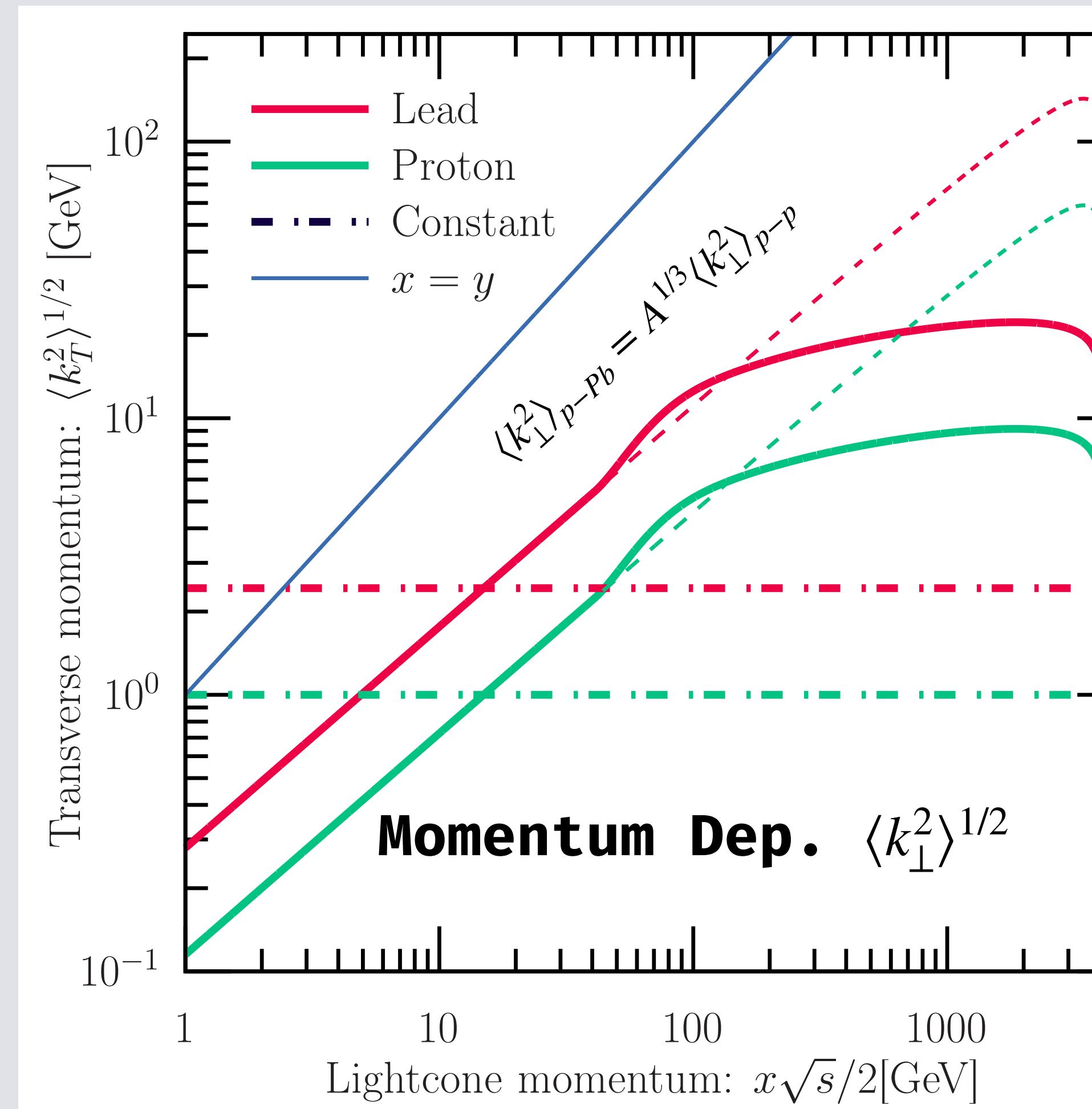
Results gg  $\rightarrow$  gg only  
 $\Rightarrow$  Most relevant for kinematics

[X. Guo, Phys. Rev. D 58, 036001(1998)]

[R. J. Fries, Phys. Rev. D 68, 074013 (2003)]

[A. Majumder and B. Müller, Phys. Rev. C 77, 054903 (2008)]

# p-p Results



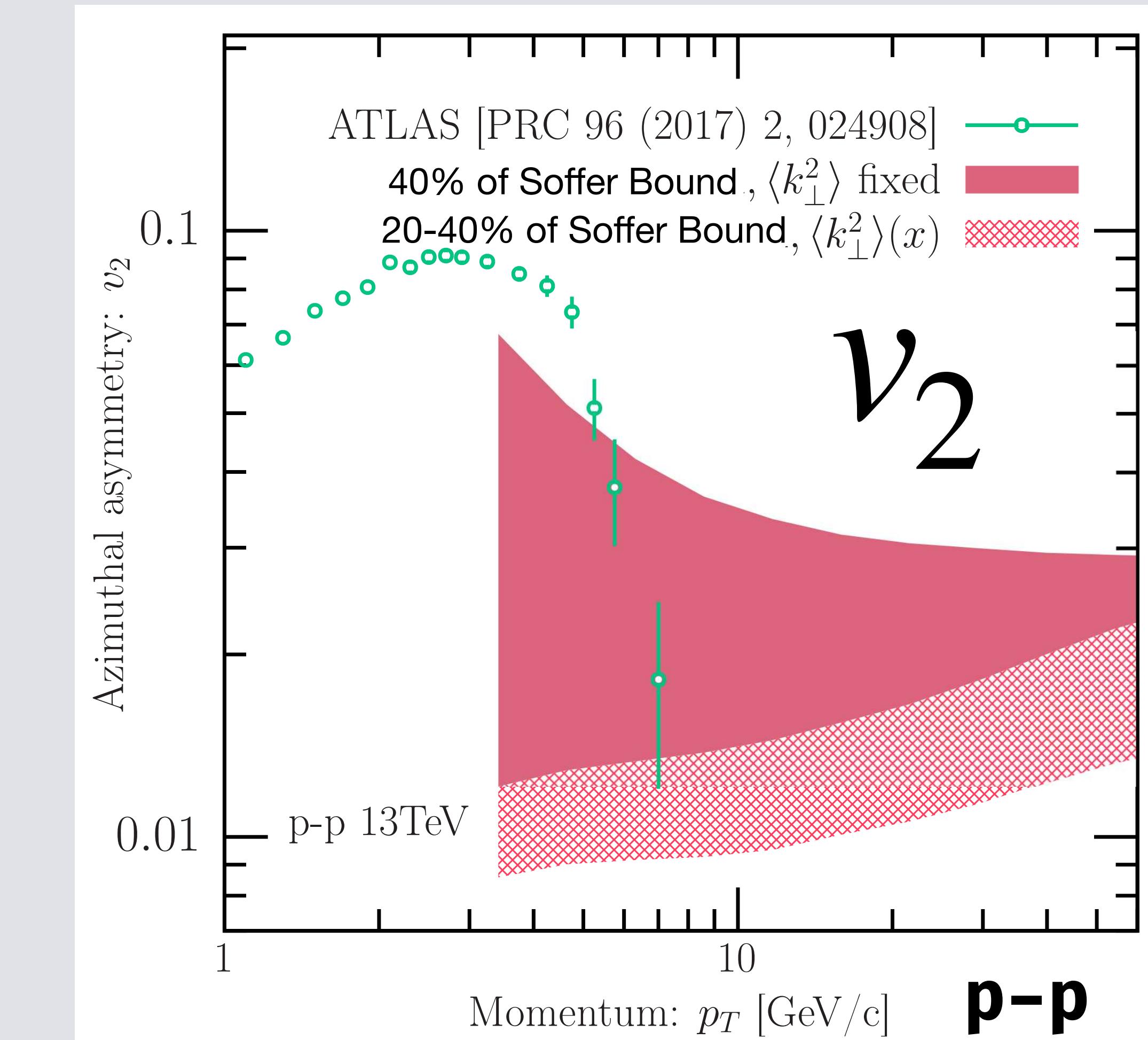
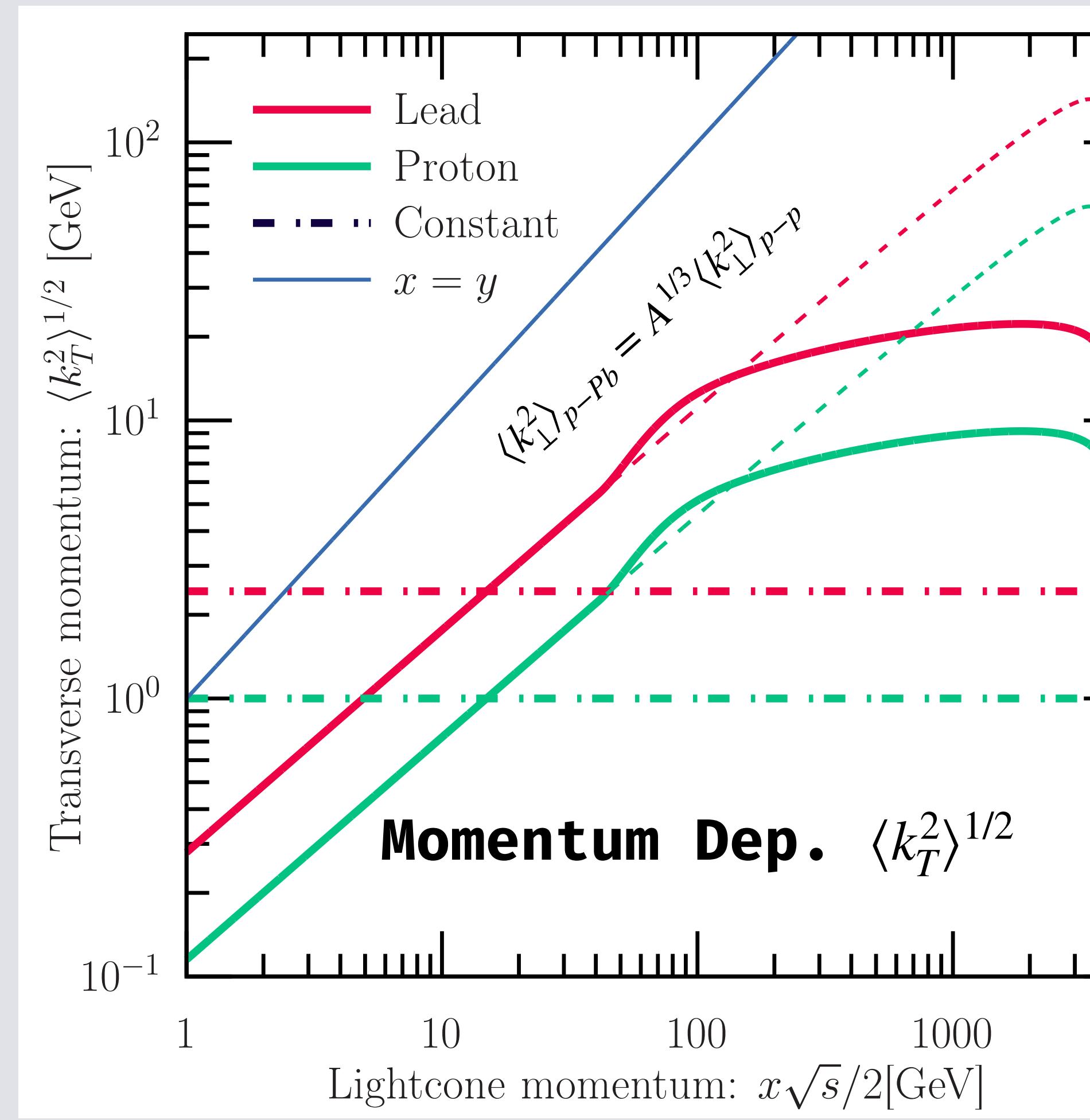
We use nCTEQ PDFs and KKP FFs

[K. Kovarik et al., Phys. Rev. D 93, 085037 (2016)]

[B. A. Kniehl et al., Nucl. Phys. B582, 514 (2000)]

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# p-p Results



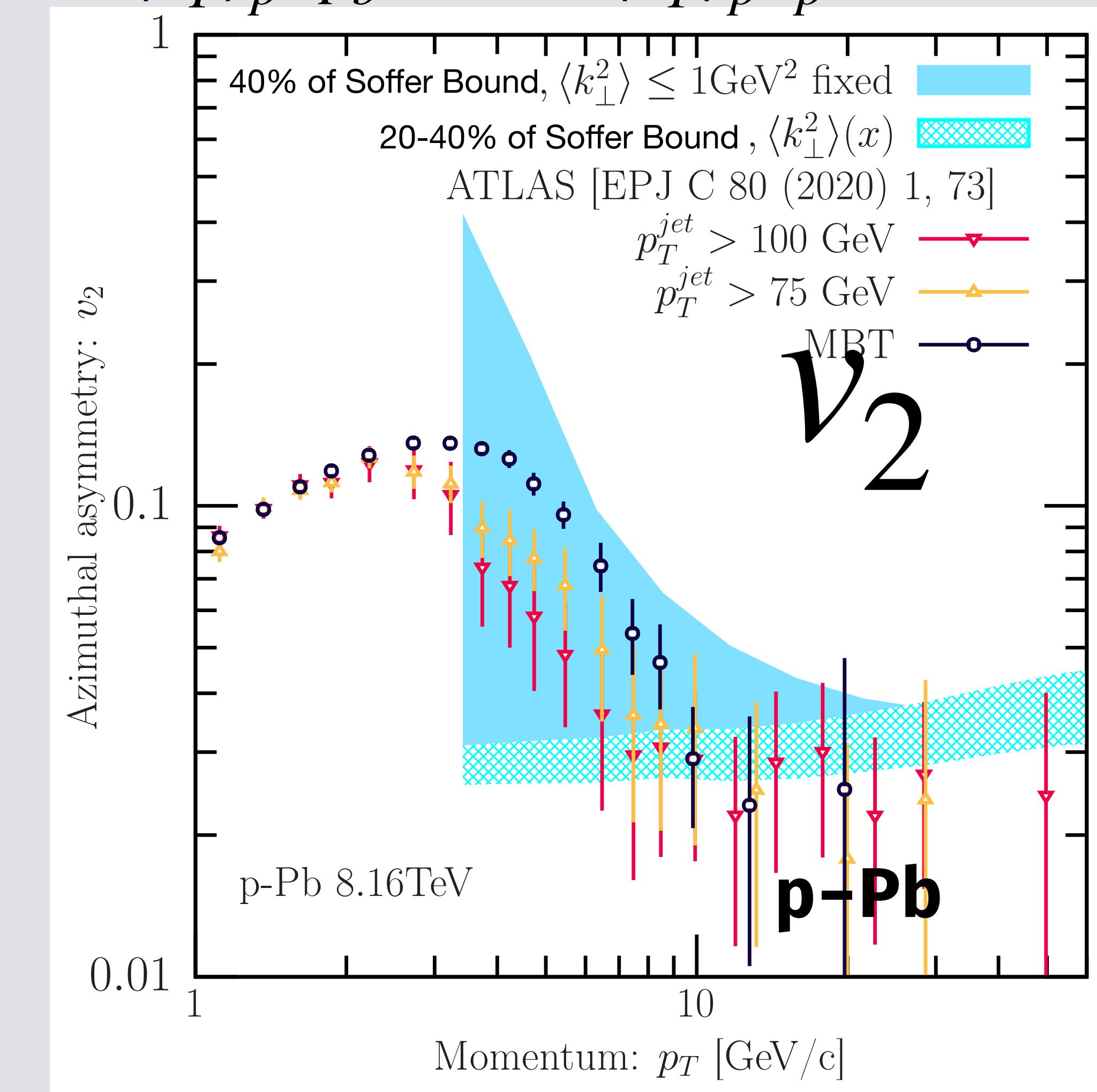
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# Results

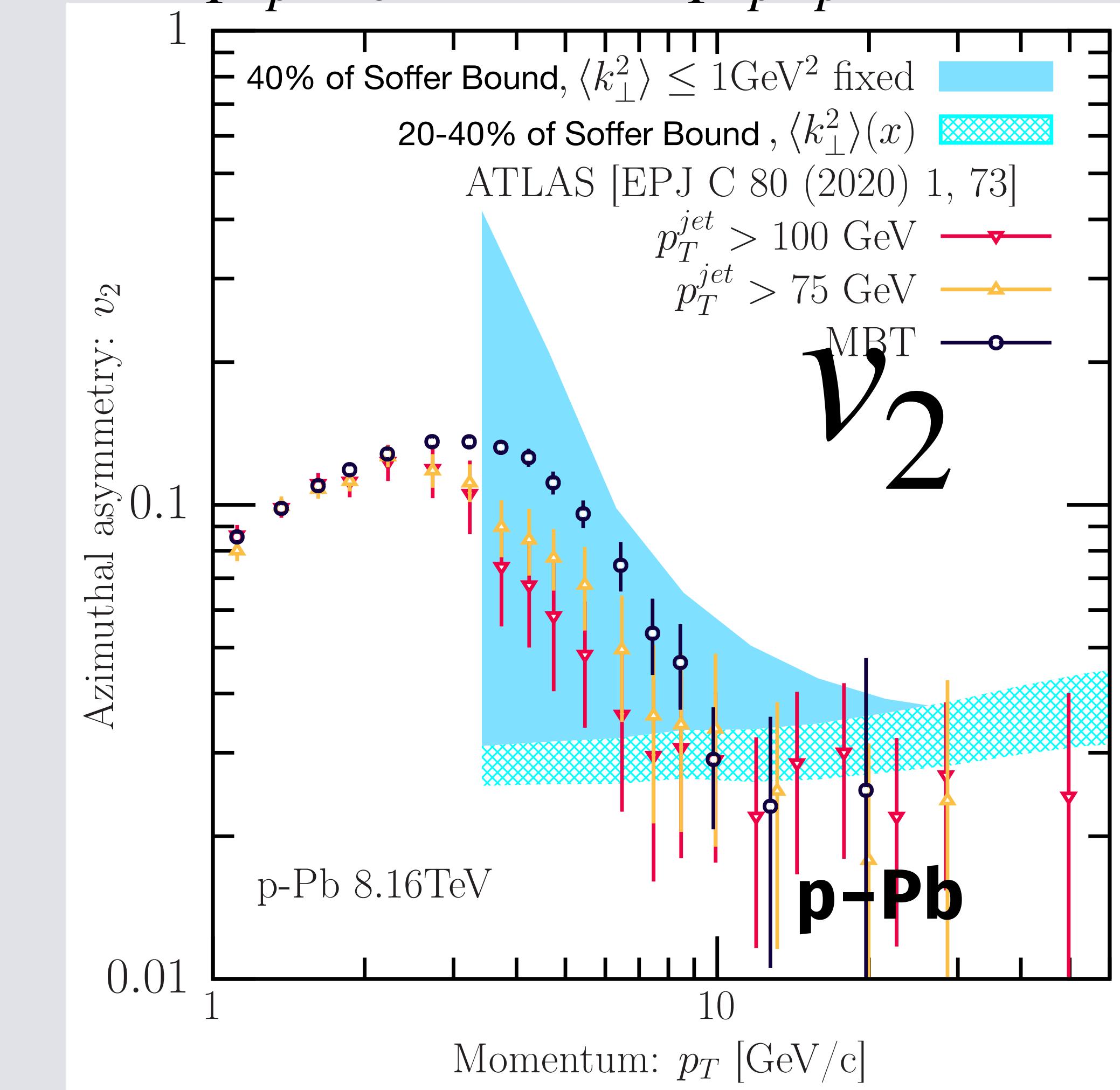
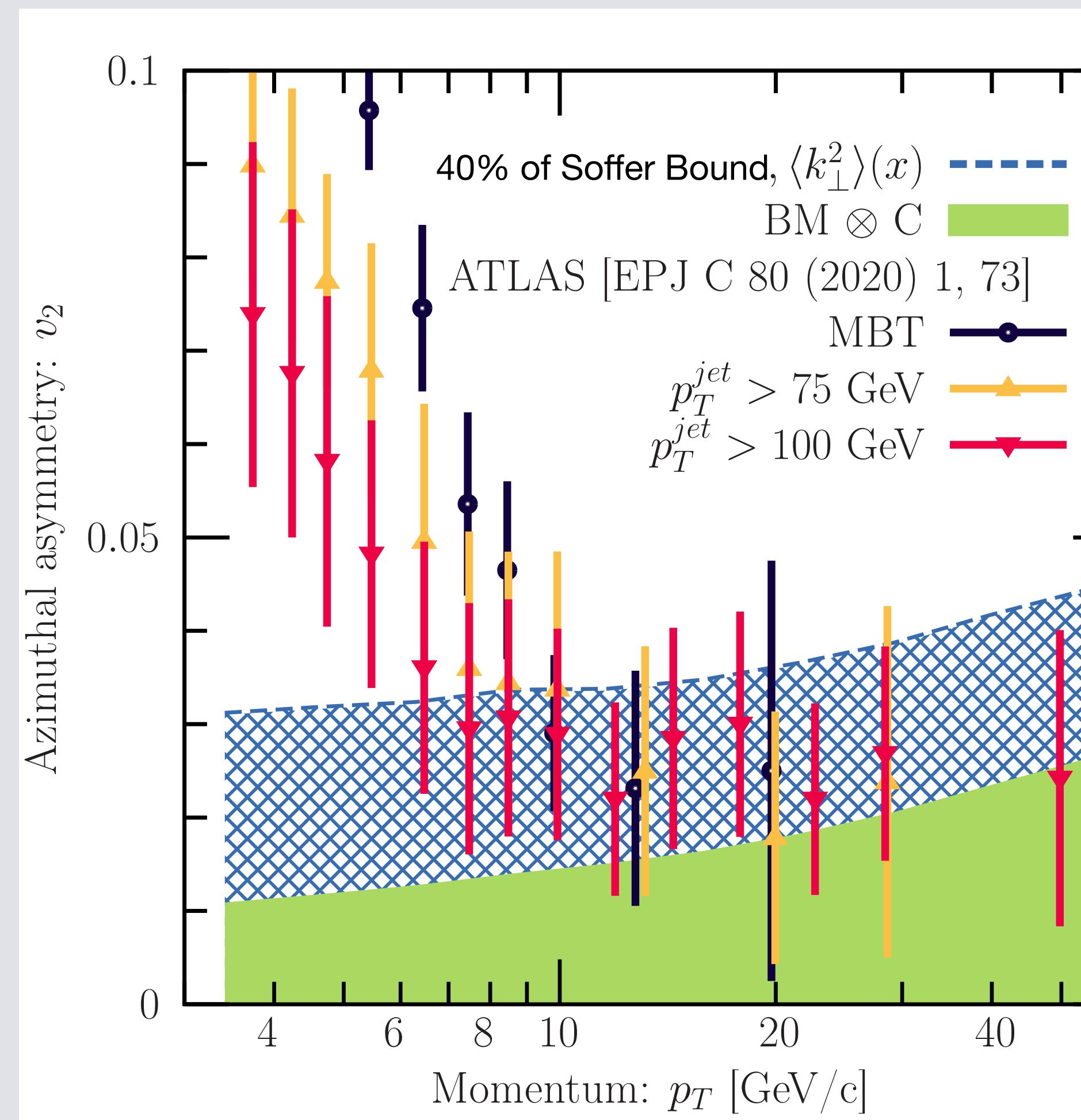
Approximation  $\Rightarrow \langle k_T^2 \rangle_{p-Pb} = A^{1/3} \langle k_T^2 \rangle_{p-p}$



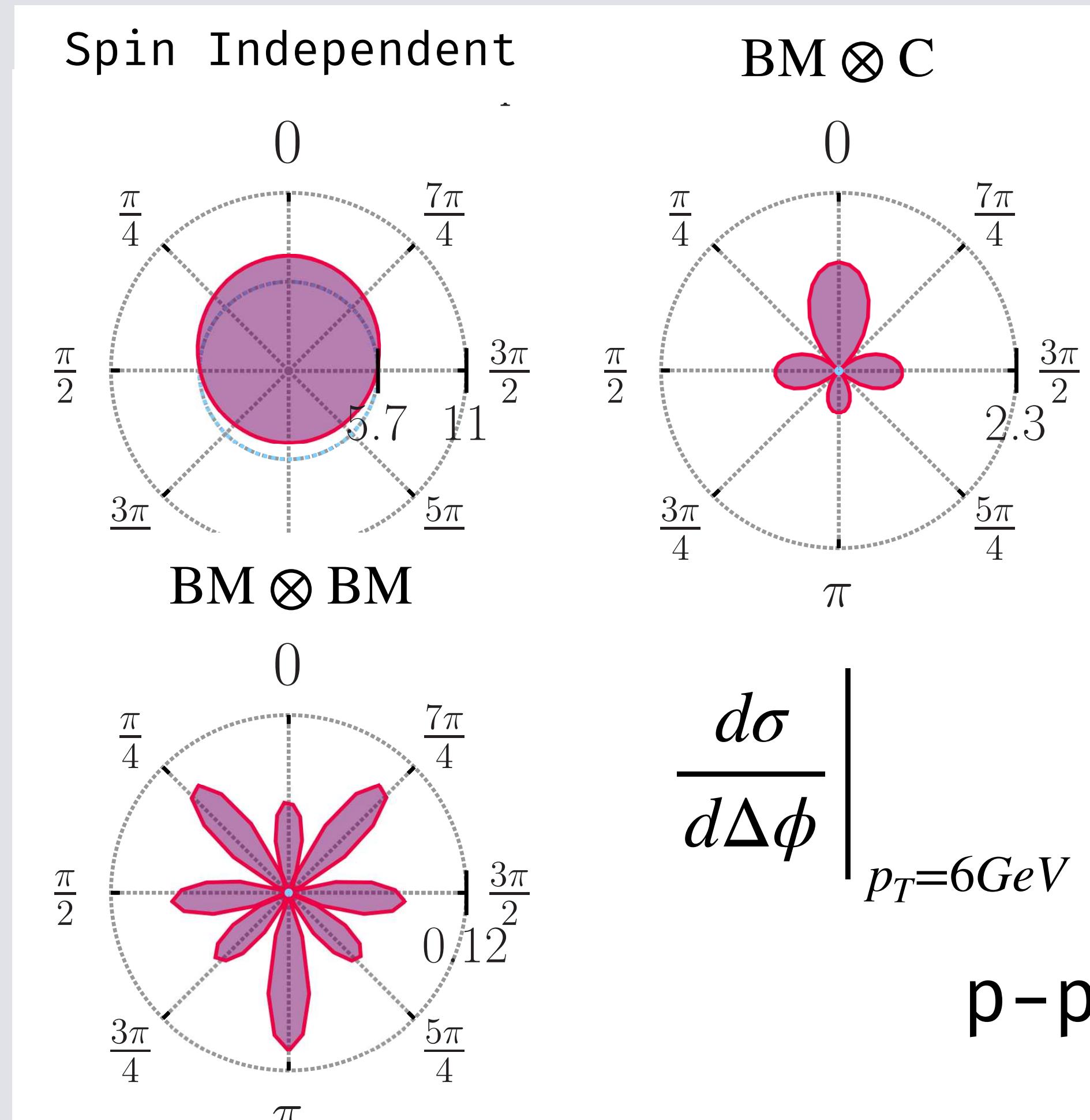
# Results

Approximation

$$\langle k_T^2 \rangle_{p-Pb} = A^{1/3} \langle k_T^2 \rangle_{p-p}$$



# Angular Distribution $\Delta\phi \equiv \phi_{q_T} - \phi_\pi$

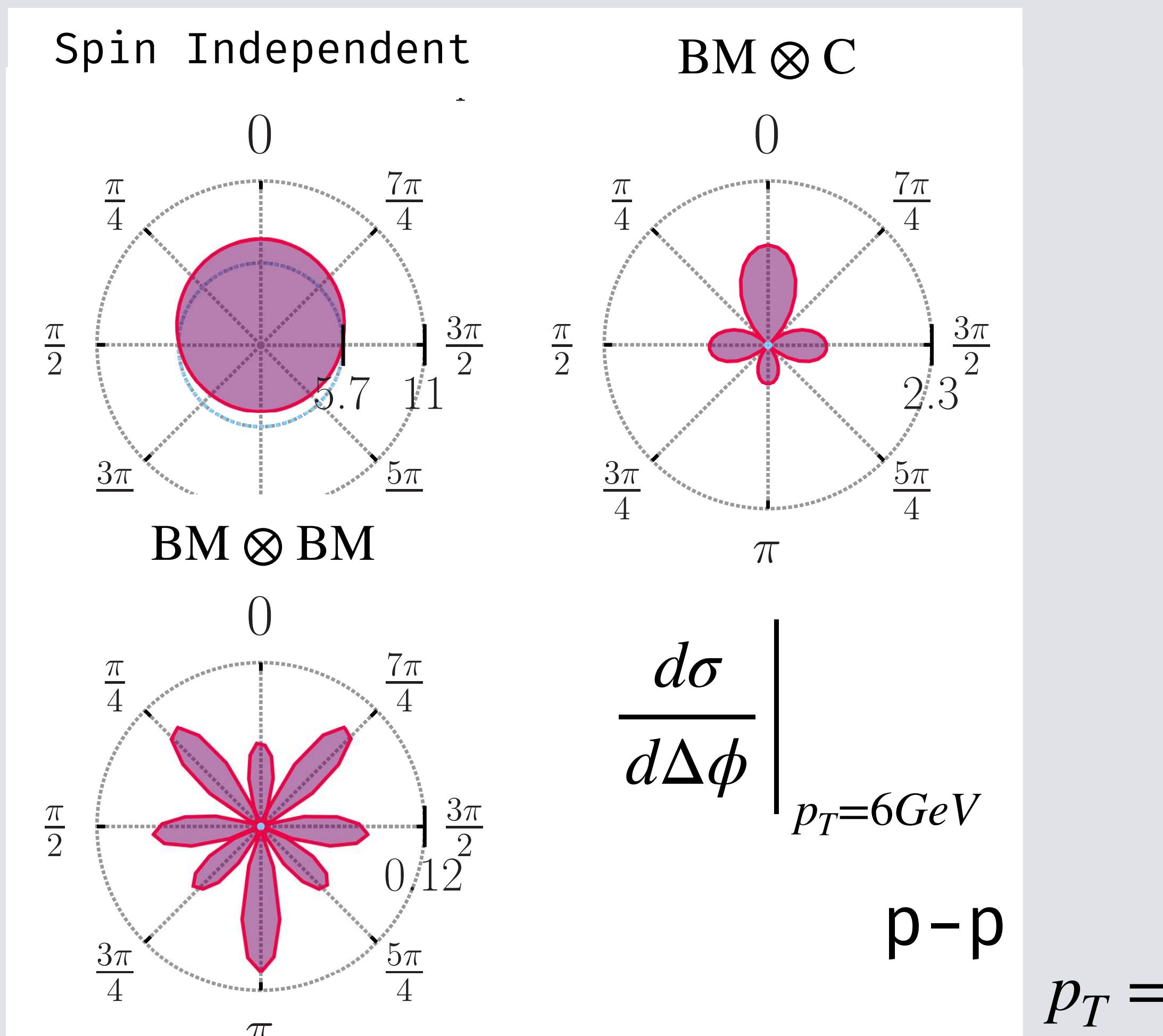


$$\left| \frac{d\sigma}{d\Delta\phi} \right|_{p_T=6GeV}$$

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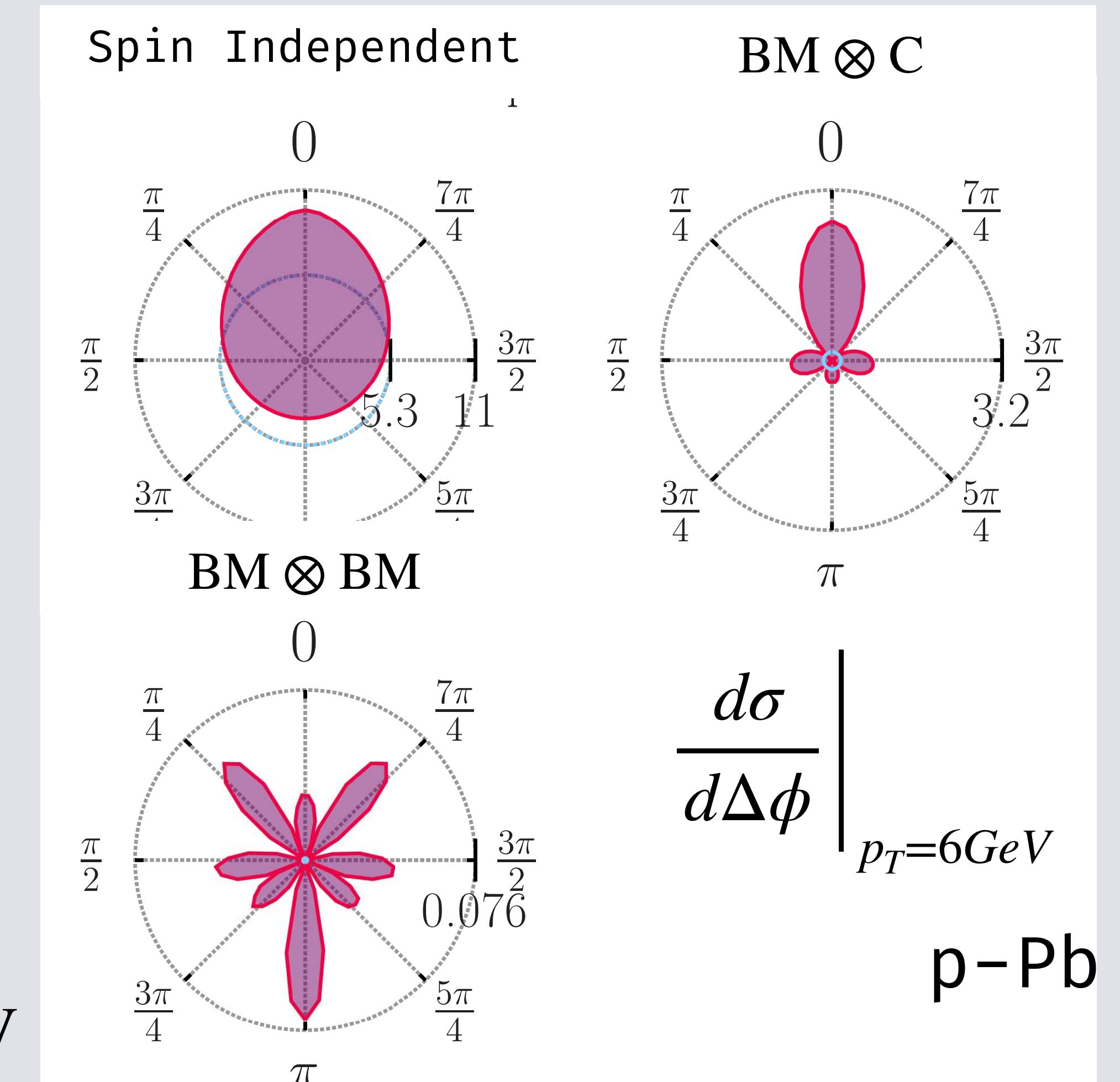
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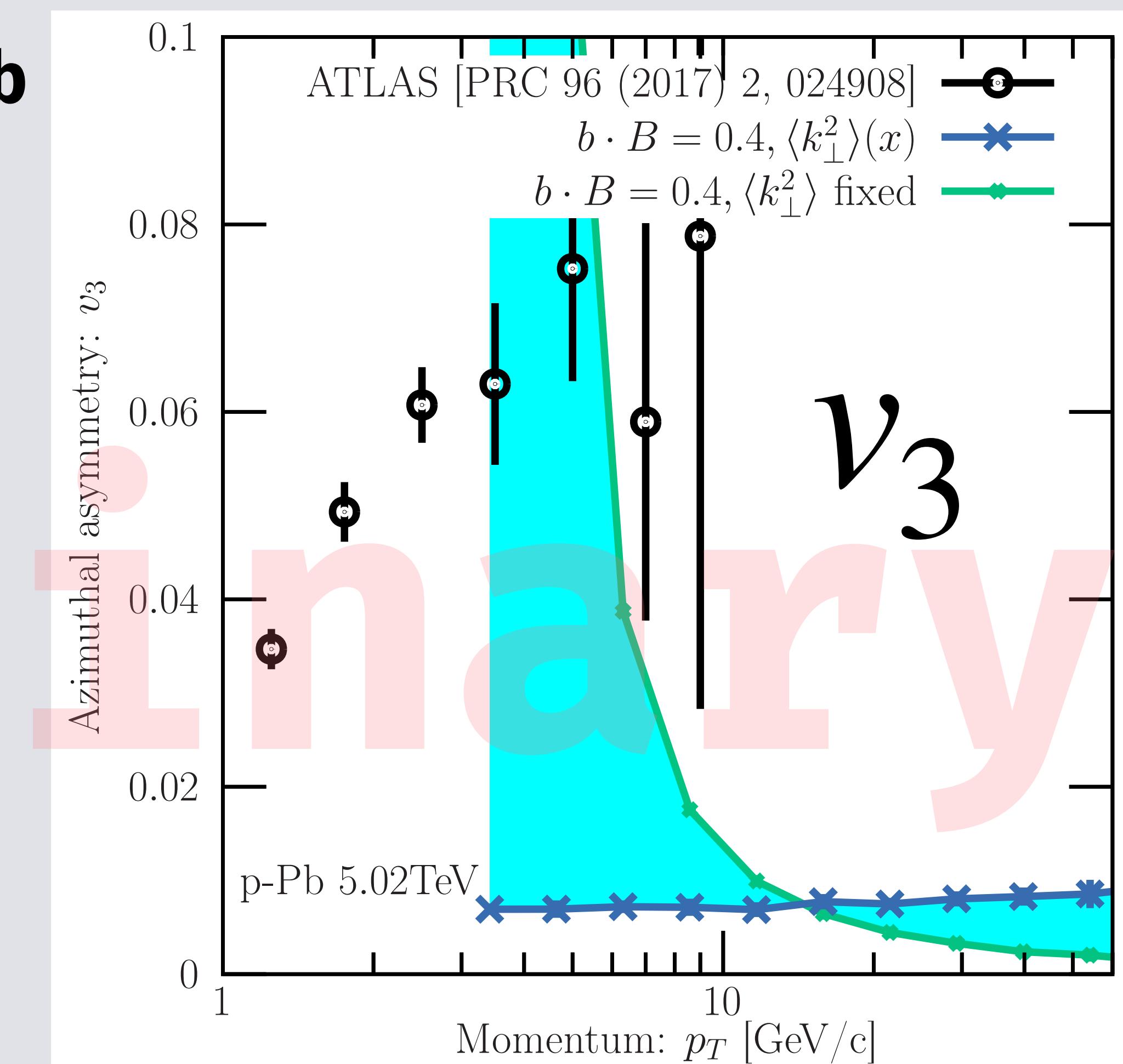
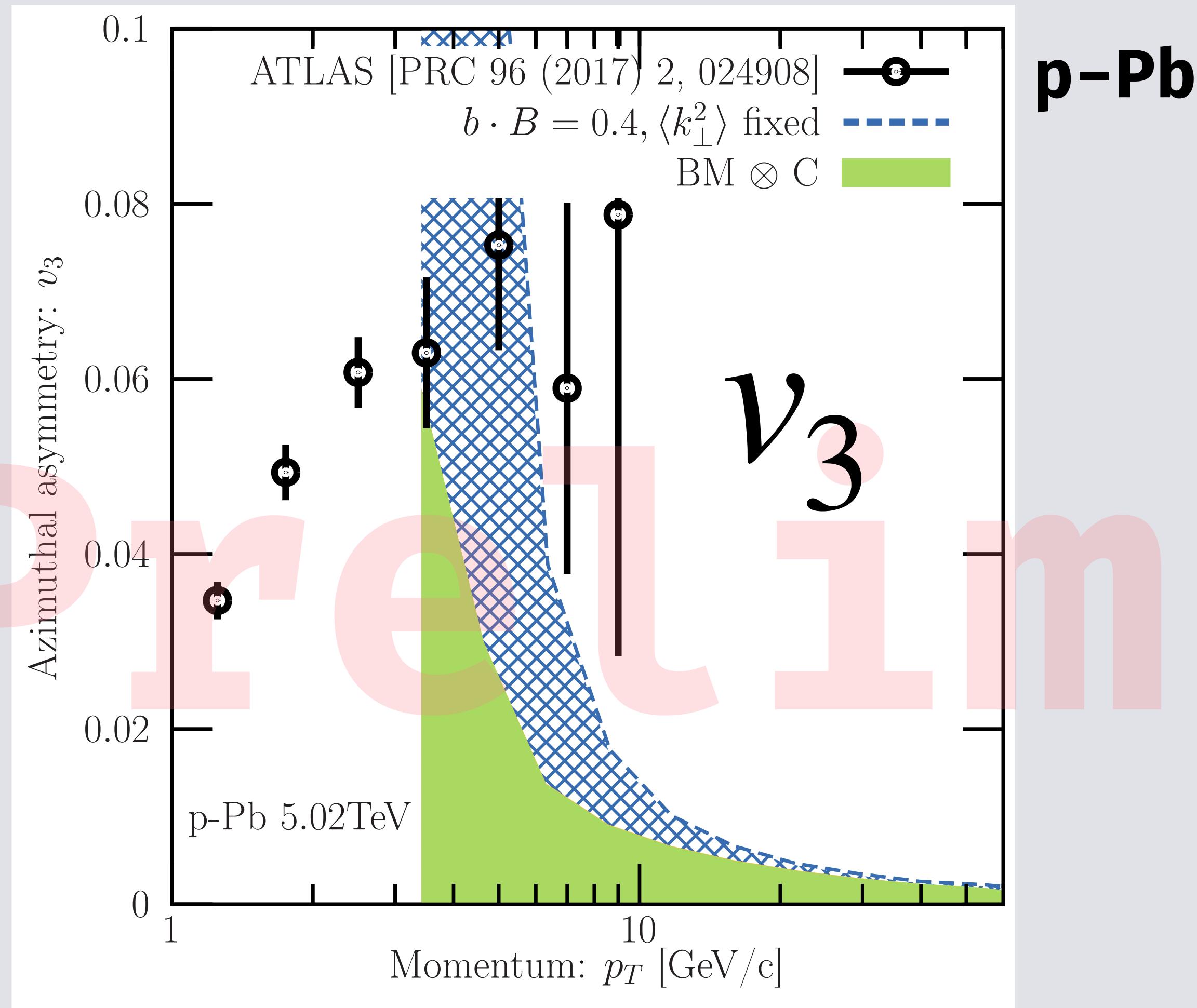


Ismail Soudi

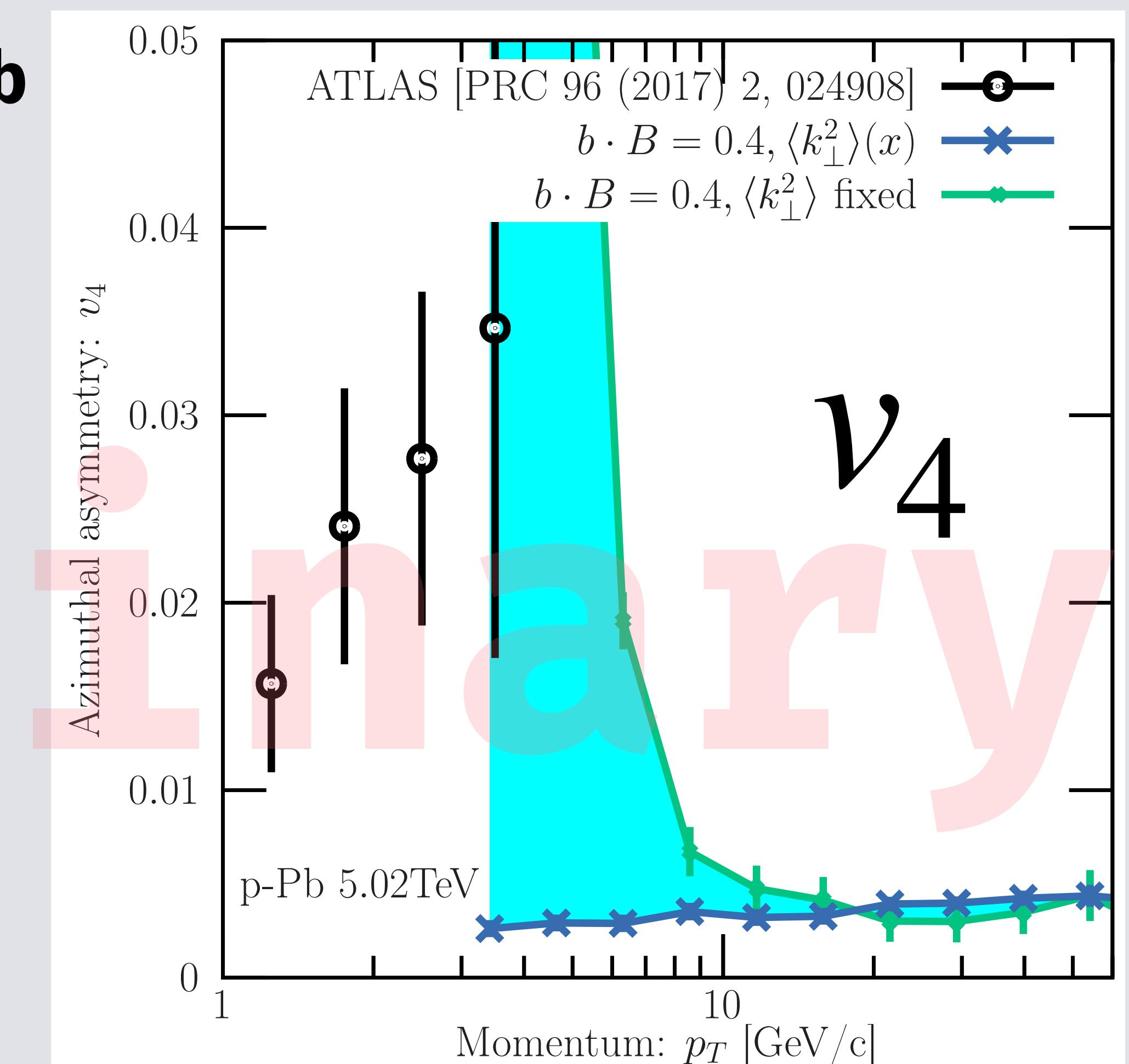
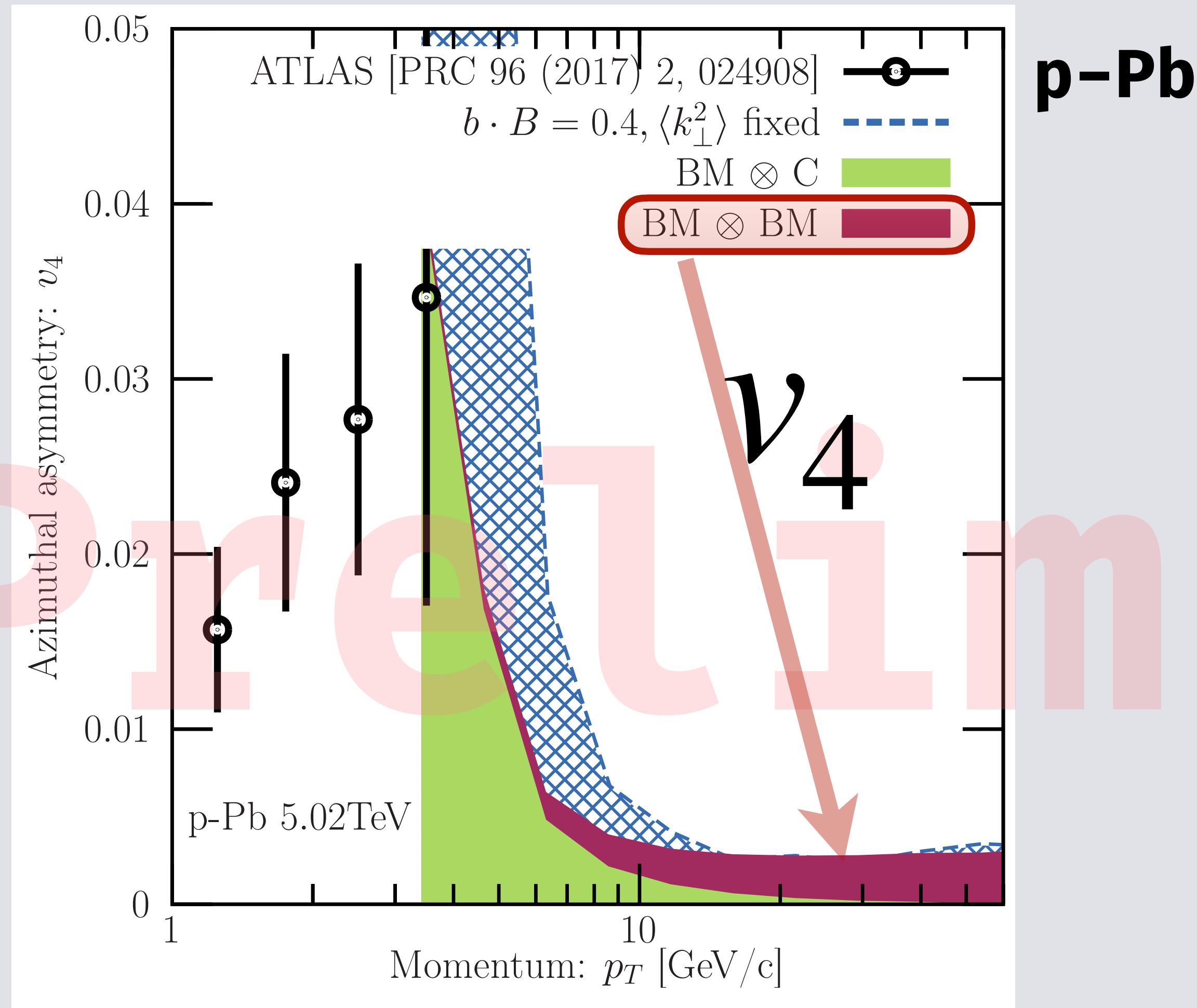
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# Preliminary results

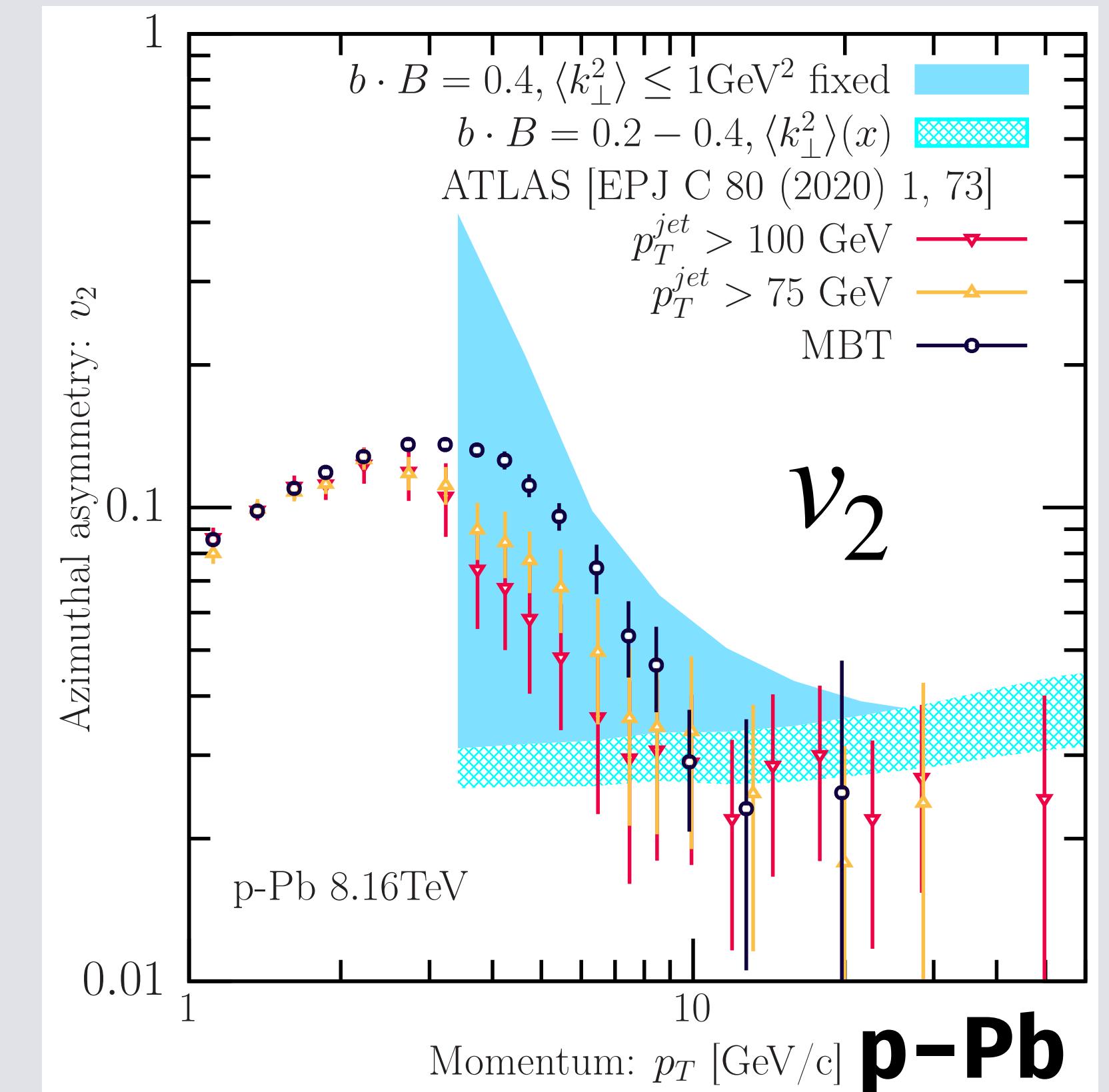


# Preliminary results



# Conclusion

- High- $p_T$  azimuthal correlations can be explained using TMD PDF/FF
- Heavy-ion studies provide a new approach to understand ( $T$ -even/ $T$ -odd) TMDs
- Include  $qg \leftrightarrow qg$  /  $qq \leftrightarrow qq$ : important at higher  $p_T$
- Beyond the  $A^{1/3} \Rightarrow$  Compute Spin-dependent rescattering in p-A

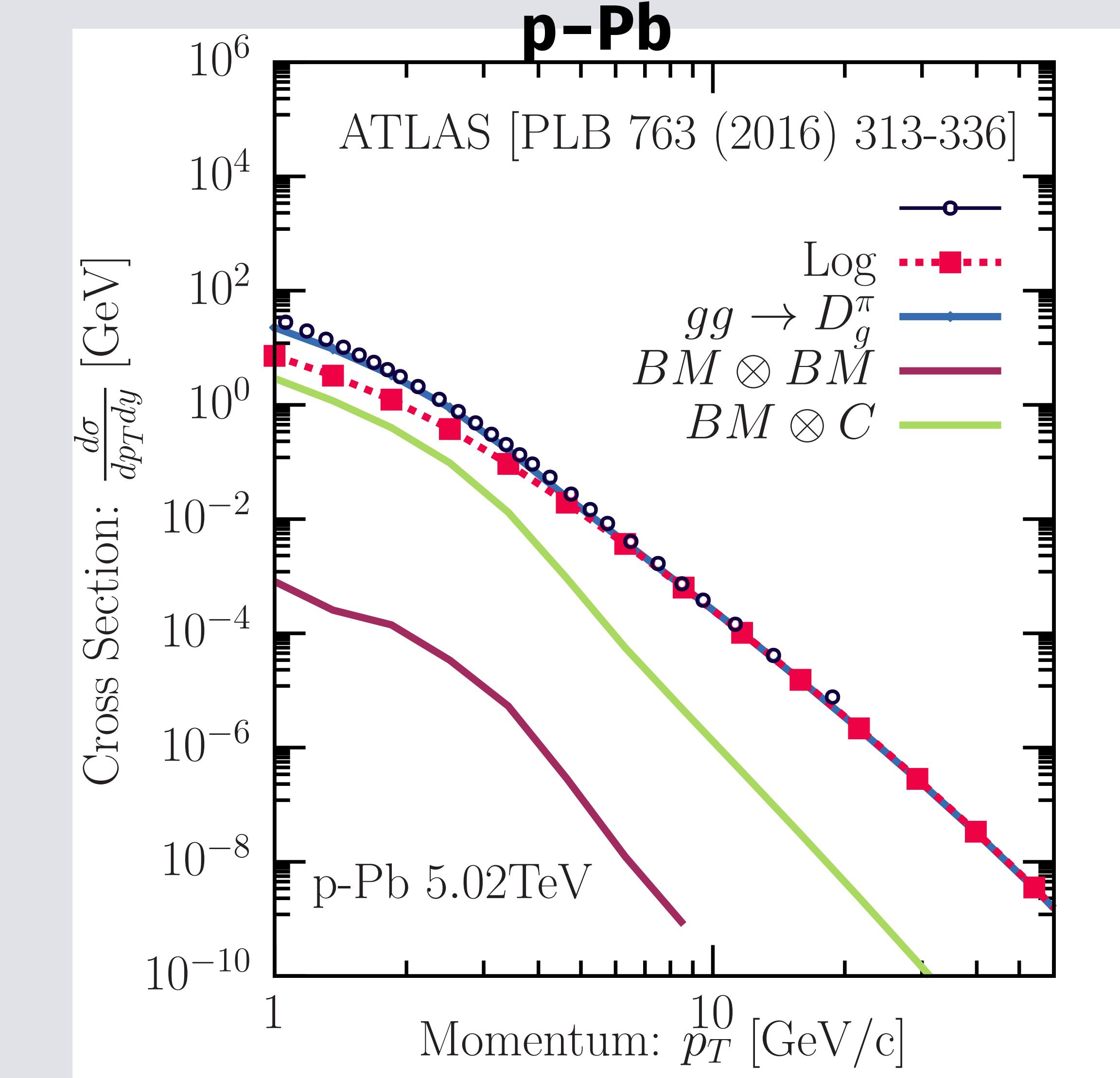
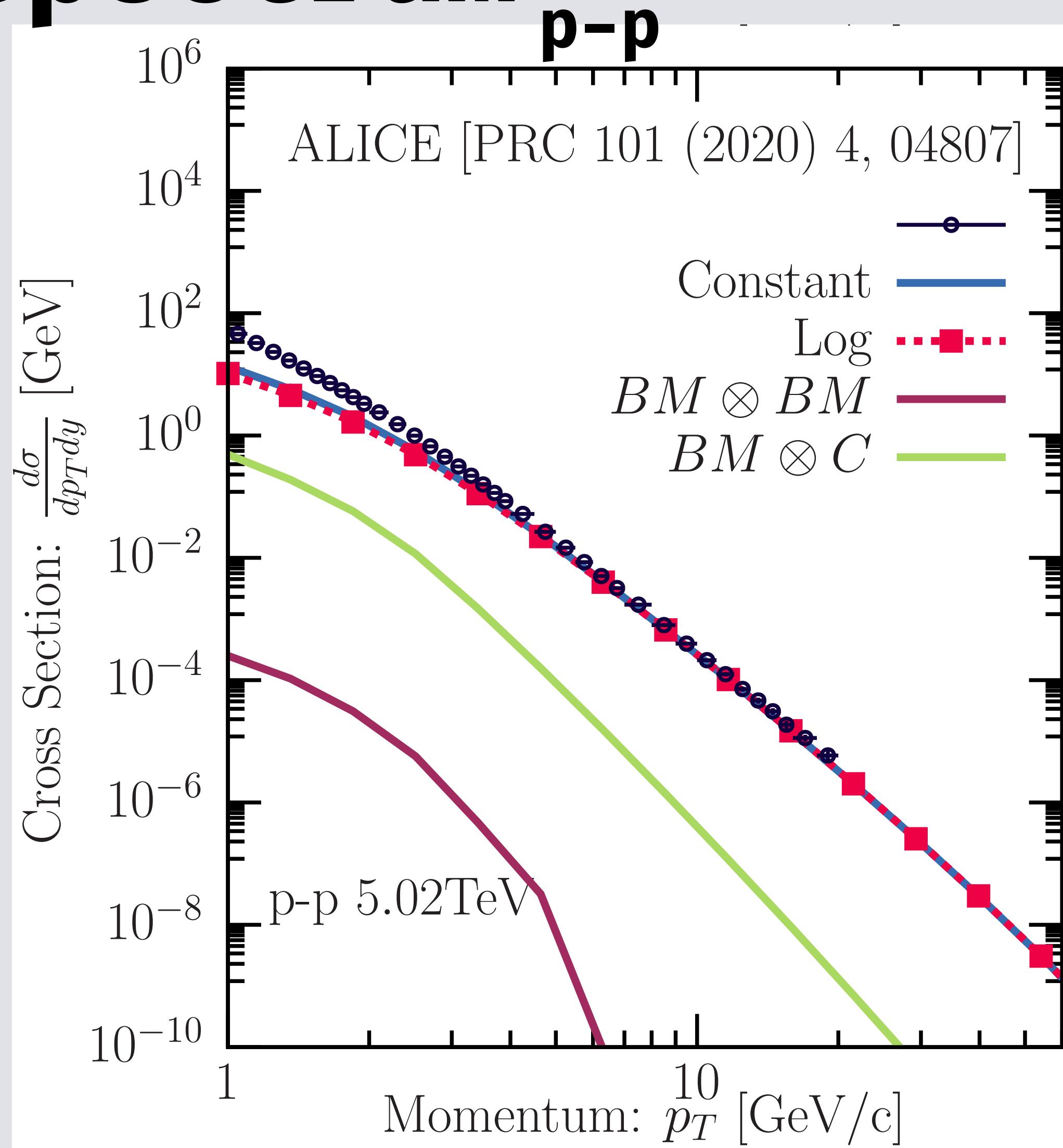


[I.S. A. Majumder ArXiv:[2308.14702](https://arxiv.org/abs/2308.14702)]

\o/ Thank you for listening!

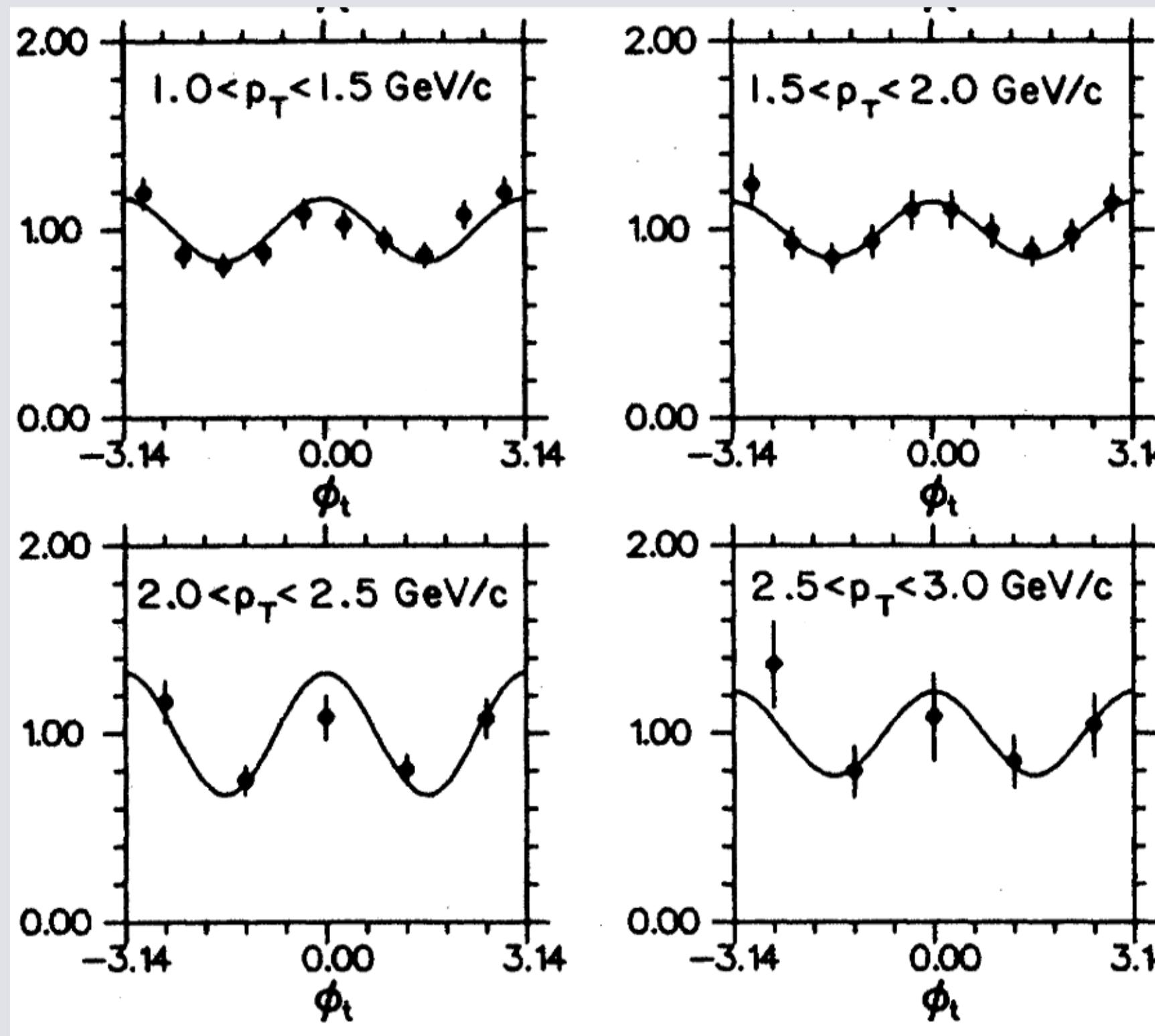
# Backup

# Spectrum

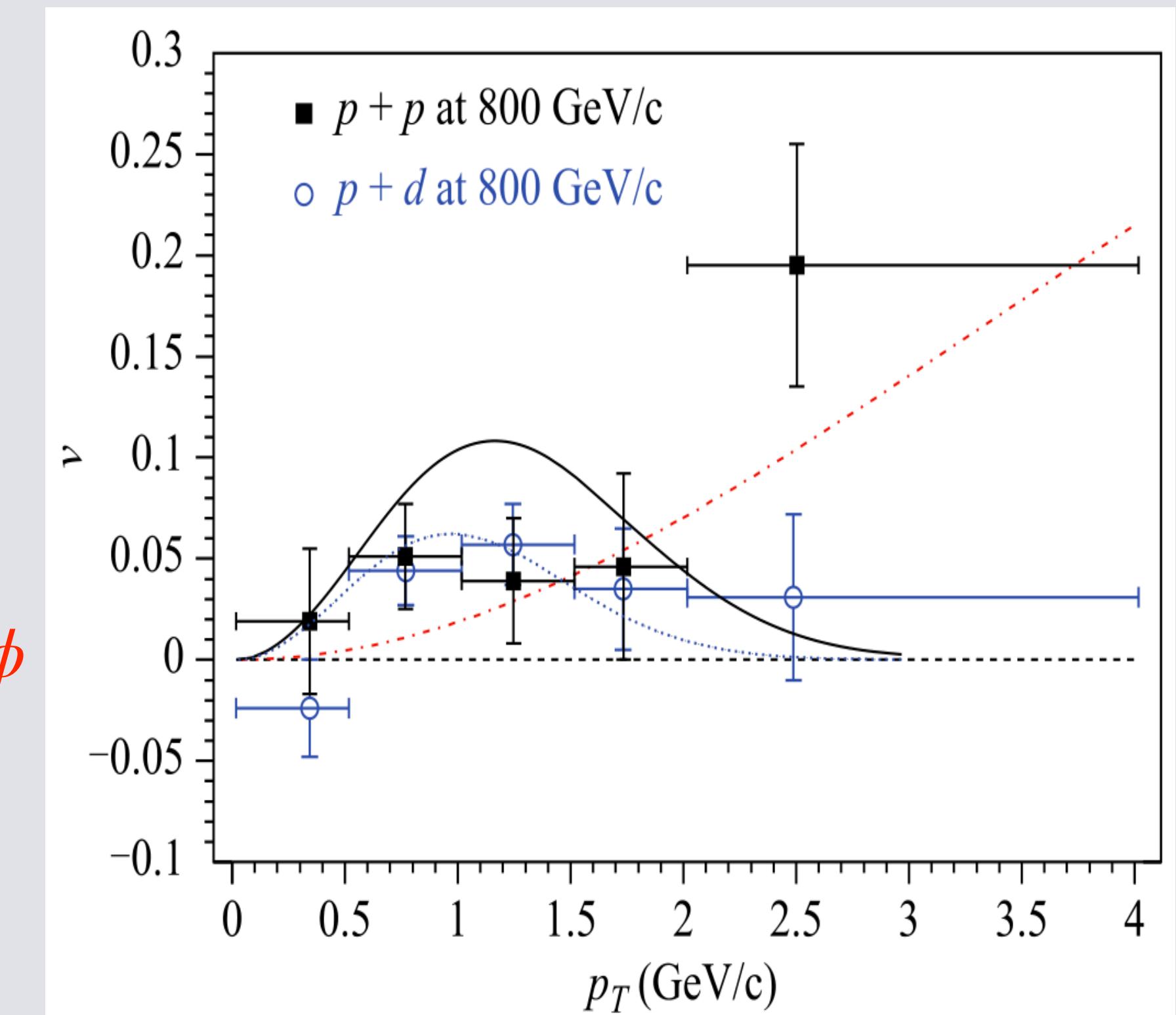


# Anisotropies in Drell-Yann

## Distribution



## Average



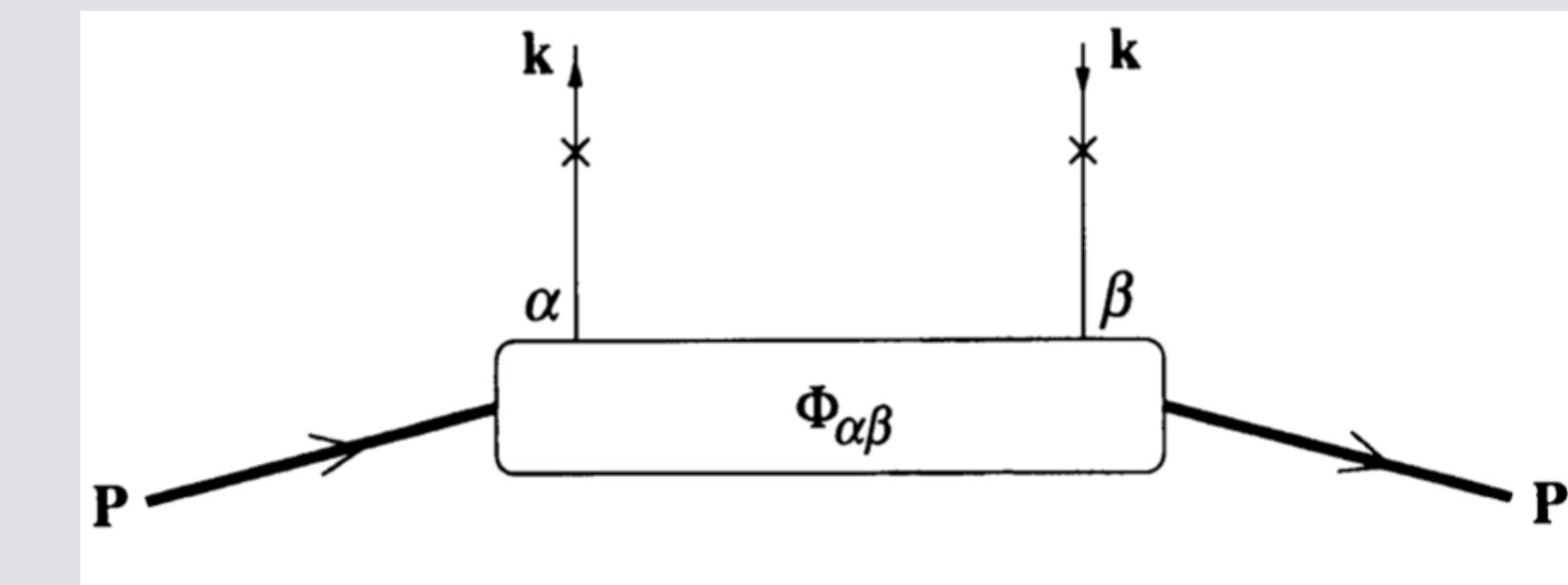
[J. S. Conway et al., PRD 39, 92 (1989).]

Ismail Soudi

[B. Zhang et al., PRD 77, 054011]  
[Z. Lu, FP 11 (2016) 1, 111204]

# Collinear Factorization

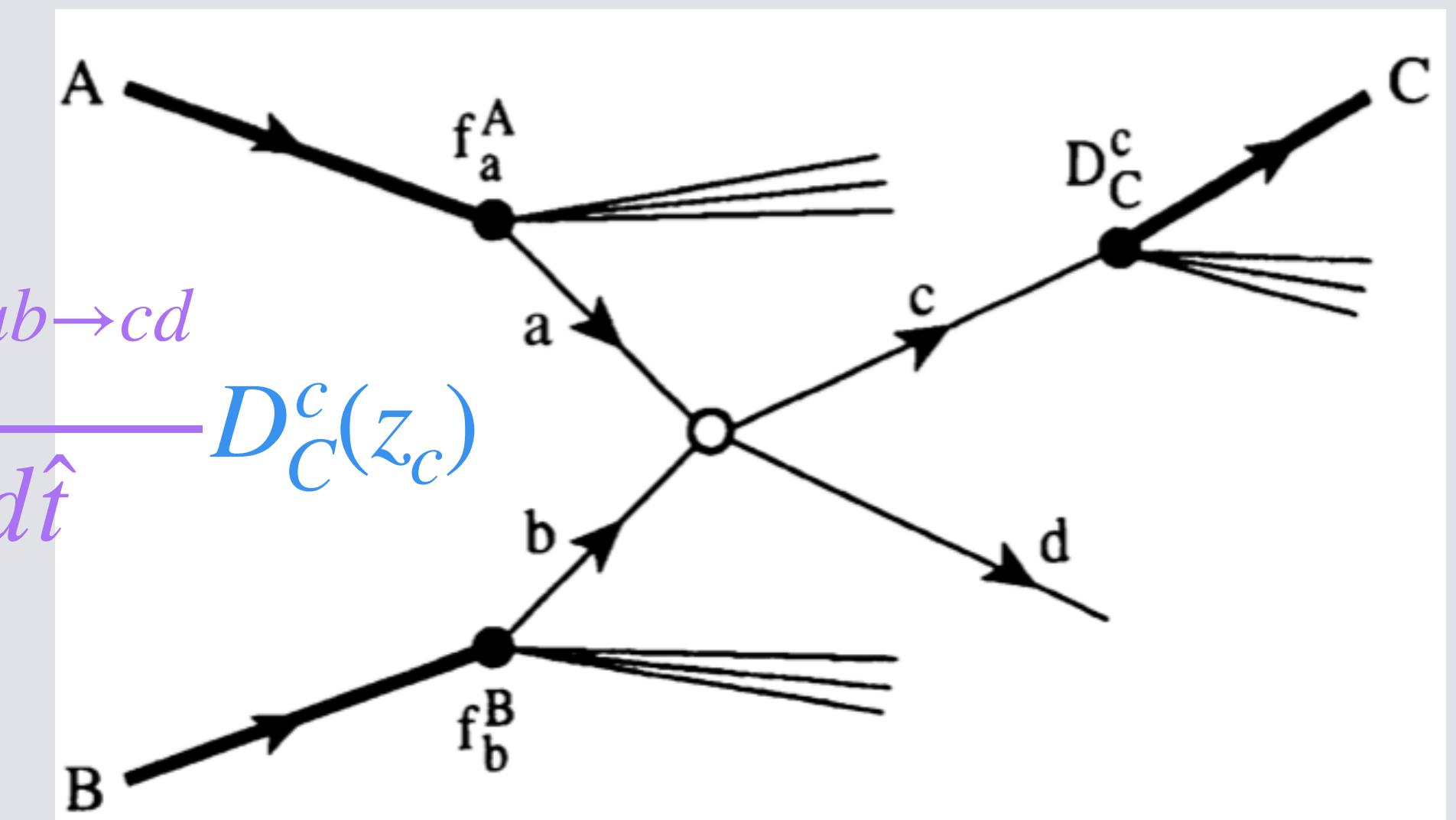
Partonic scattering



$$A^\alpha A^\beta \rightarrow \Phi_{\alpha\beta} \equiv \int d^3\xi \frac{e^{-ip\cdot\xi}}{(p^+)^2} \langle P, S | F^{-\mu}(\xi) F^{-\nu}(0) | P, S \rangle$$

$\Rightarrow$  PDFs  $\otimes$  FFs

$$E_C \frac{d^3\sigma}{d^3\mathbf{p}_C} = \frac{1}{\pi} \sum_{a,b,c,d} \int dx_a dx_b \hat{f}_a^A(x_a, Q^2) \hat{f}_b^B(x_b, Q^2) \frac{d\sigma^{ab \rightarrow cd}}{d\hat{t}} D_C^c(z_c)$$



# Transverse Momentum Distributions

Fierz decomposition of quark-quark correlator contains additional terms than the leading twist distributions

$\Gamma_S$	$\Gamma_V^\mu$	$\Gamma_T^{\mu\nu}$	$\Gamma_A^\mu$	$\Gamma_P$
$I$	$\gamma^\mu$	$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$	$\gamma^5 \gamma^\mu$	$i \gamma^5$

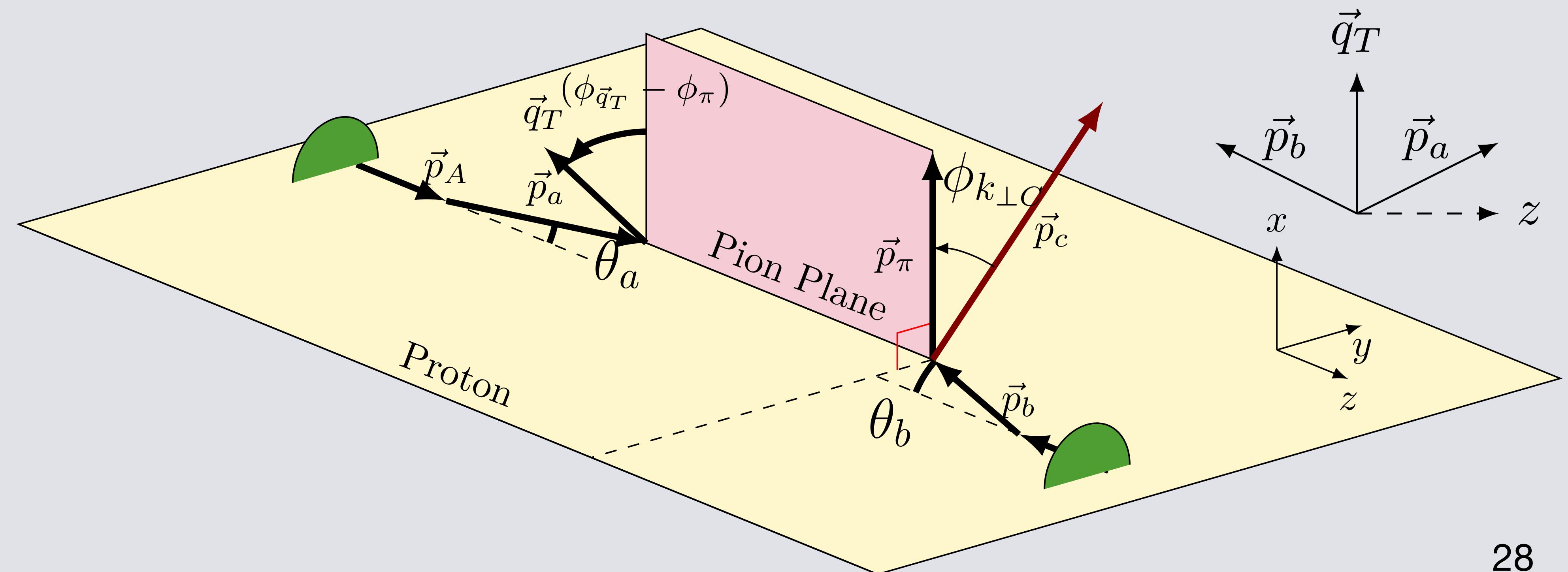
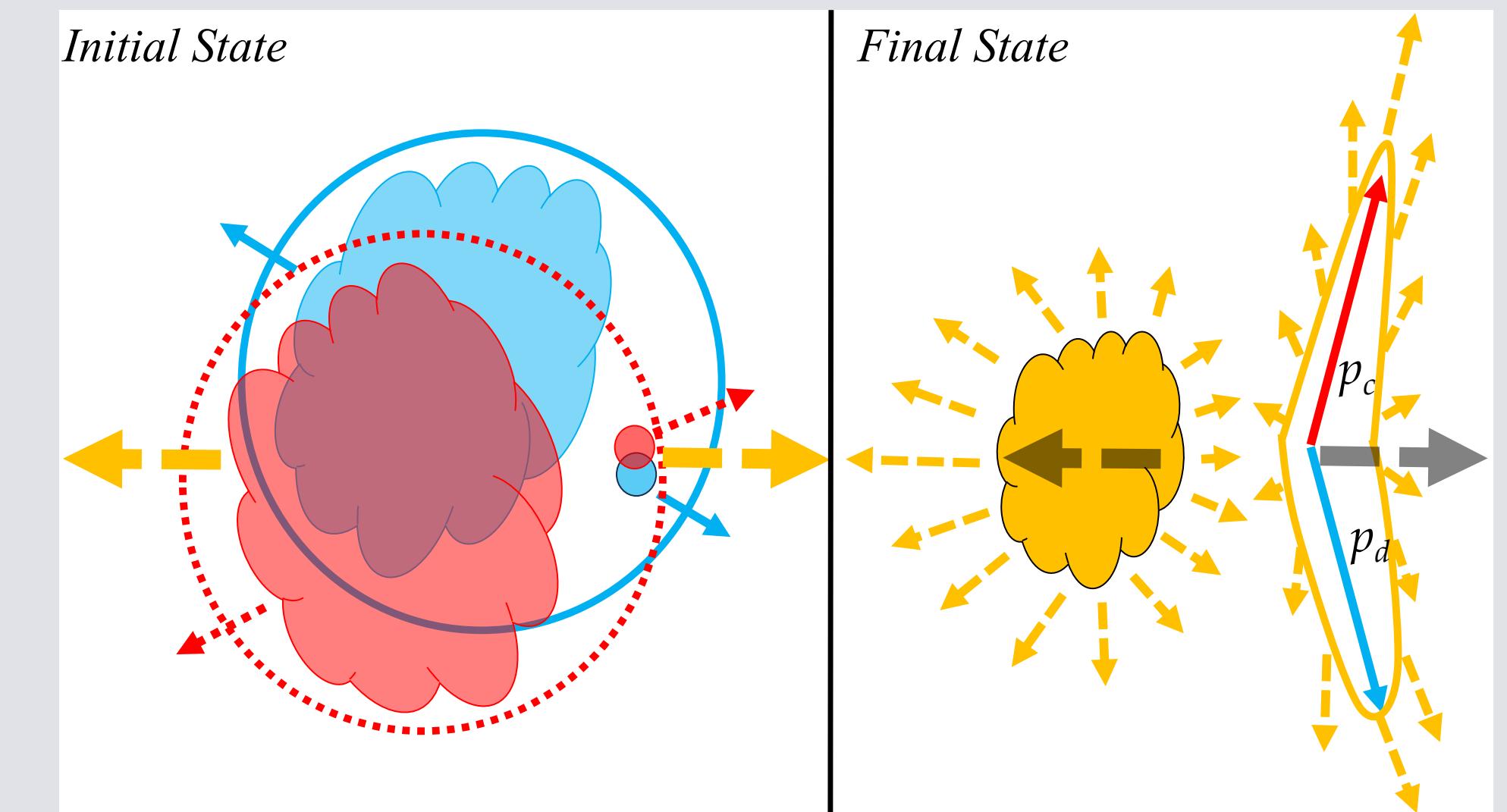
$$X = x_\alpha \Gamma^\alpha = \frac{1}{4} \Gamma^\alpha \text{Tr}(X \Gamma_\alpha)$$

# TMD Scattering

In hadronic C.M.:

Due to momentum conservation

⇒ Direction of bulk given by  $\vec{q}_T$



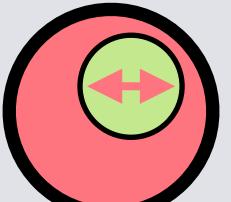
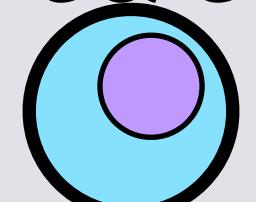
# Transverse Momentum Distributions

[F. G. Celiberto Universe 8 (2022) 12]

For unpolarized hadrons

⇒ Gluon correlator

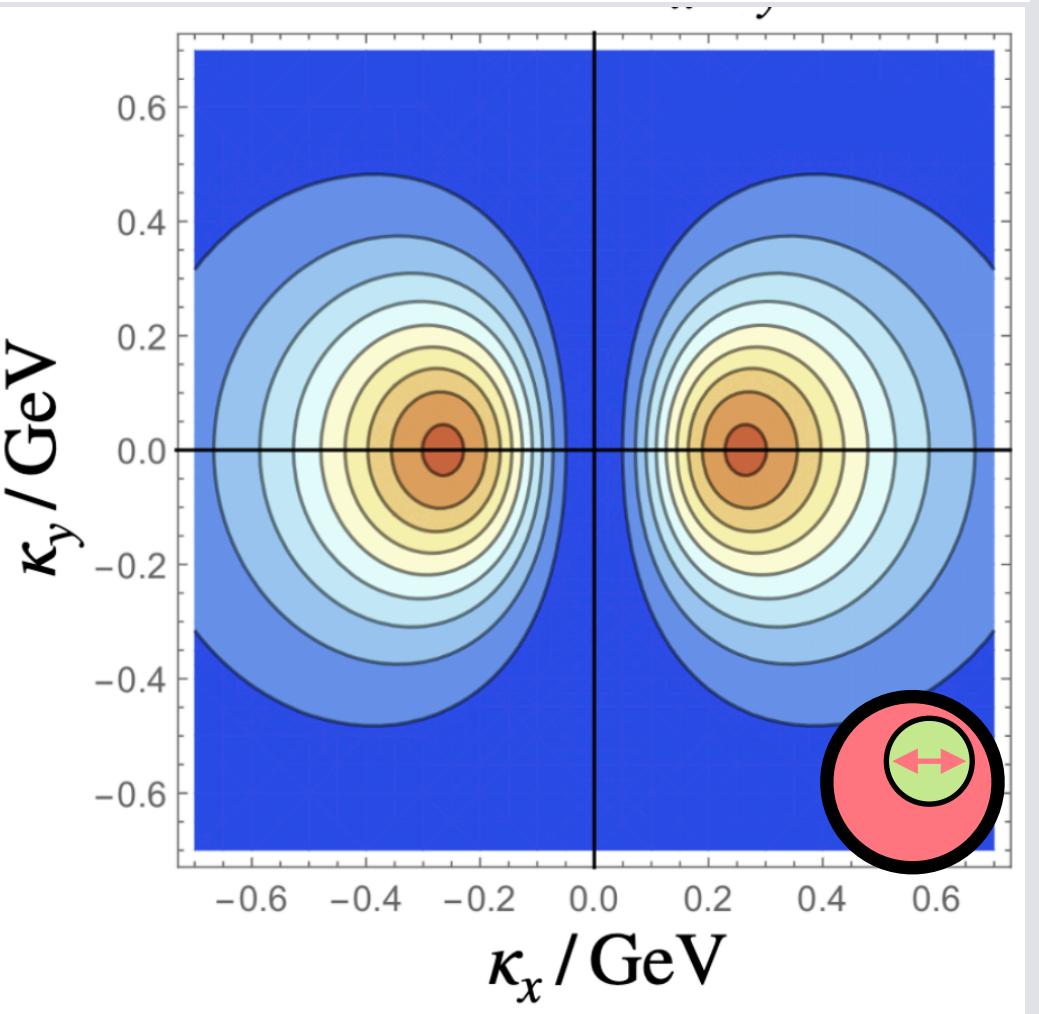
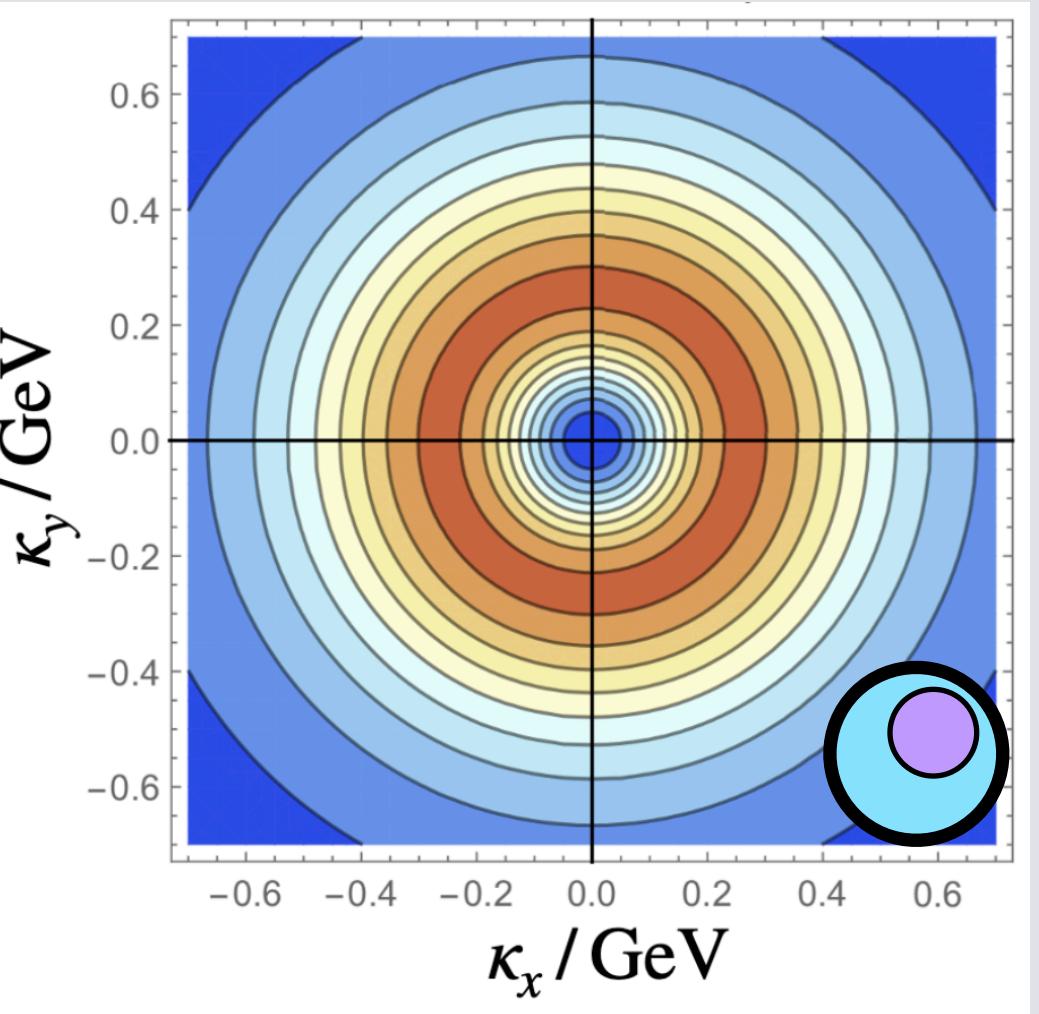
$$\Phi^{\alpha\beta} = \frac{1}{2x} \left\{ -g_T^{\alpha\beta} f^g(x, k_\perp) + \left( \frac{k_\perp^\alpha k_\perp^\beta}{M^2} + g_T^{\alpha\beta} \frac{k_\perp^2}{2M^2} \right) h^\perp g(x, k_\perp) \right\}$$



Unpolarized + Boer-Mulders/Collins

Soffer Bound:

$$\frac{k_\perp^2}{2M^2} |h^\perp g(x, k_\perp)| \leq f^g(x, k_\perp)$$



# kT Dist

$$\langle k_{\perp}^2 \rangle^{1/2}(x) = \langle k_{\perp}^2 \rangle_0^{1/2} \left( (1-x) \sqrt{\frac{s}{s_0}} \right)^{0.15} A^{1/3}$$
$$\times \begin{cases} \left( \frac{x}{x_0} \sqrt{\frac{s}{s_0}} \right)^{0.8}, & \text{if } x \leq x_0 \\ \left[ 1 + b_0 \log \left( \frac{x}{x_0} \sqrt{\frac{s}{s_0}} \right) \right]^{1/2}, & \text{if } x > x_0 \end{cases}$$

# Transverse Momentum Distributions

For unpolarized hadrons

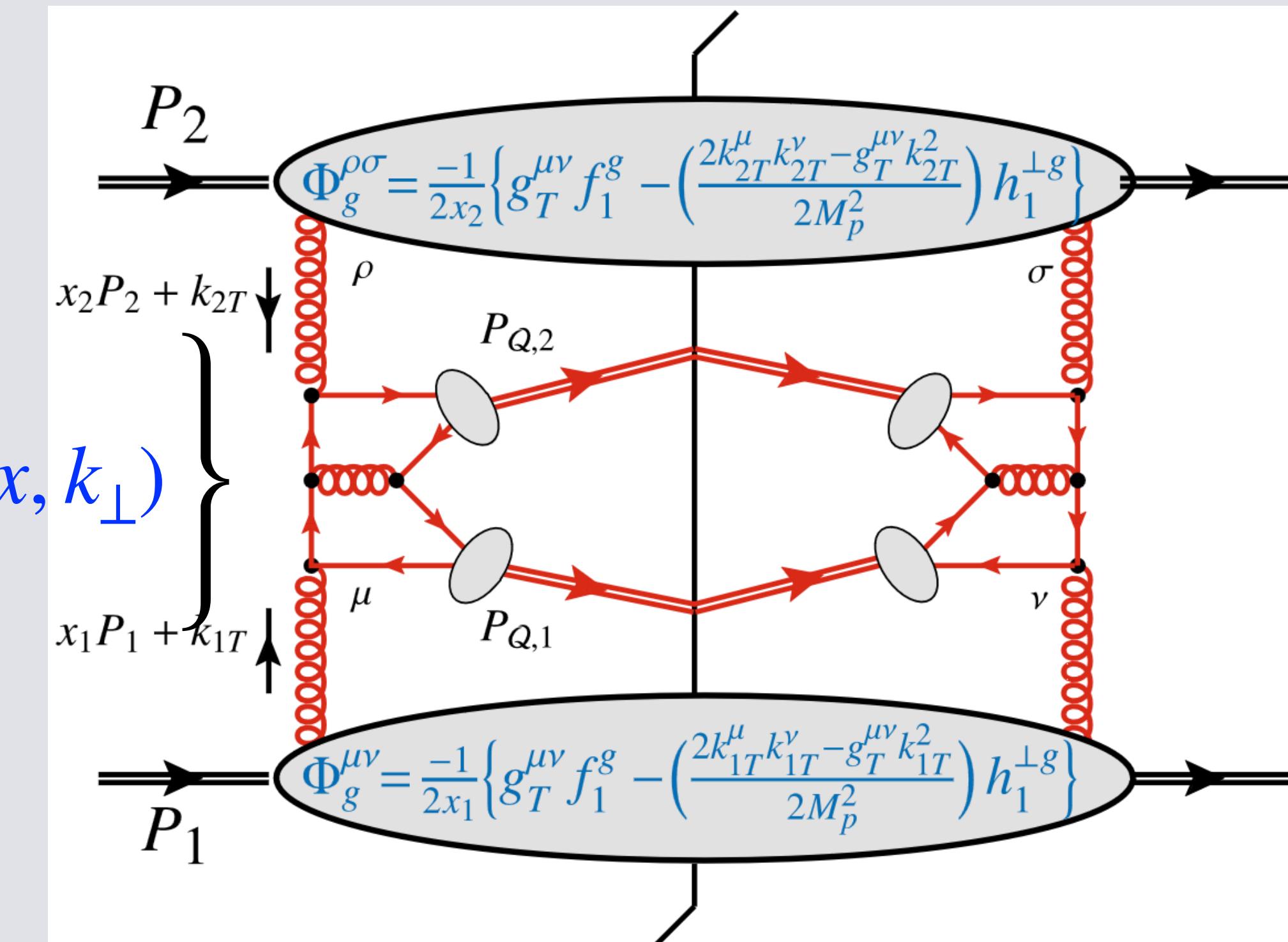
$\Rightarrow$  Gluon correlator

$$\Phi^{\alpha\beta} = \frac{1}{2x} \left\{ -g_T^{\alpha\beta} f^g(x, k_\perp) + \left( \frac{k_\perp^\alpha k_\perp^\beta}{M^2} + g_T^{\alpha\beta} \frac{k_\perp^2}{2M^2} \right) h^{\perp g}(x, k_\perp) \right\}$$

Unpolarized + Boer-Mulders/Collins

Soffer Bound:

$$\frac{k_\perp^2}{2M^2} |h^{\perp g}(x, k_\perp)| \leq f^g(x, k_\perp)$$



[F. Scarpa et al., Eur.Phys.J.C 80 (2020) 2, 87]

# Centrality

