Calculating parton distribution functions with NJL model

Bailing Ma

Argonne National Lab

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Outline

Overview of the Nambu-Jona-Lasinio model

- NJL model is a low energy effective theory of the strong interaction, that mimics many key features of QCD. Thus, it is a useful tool to help understand non-perturbative phenomena in low energy QCD.
- Only quarks as the explicit degrees of freedom, no gluons.
- Dynamics due to gluon-quark interaction and gluon self-couplings are absorbed into the four-fermion contact interaction.
- Local chiral symmetry is explicitly broken by non-vanishing current quark mass.
- Chiral symmetry is also dynamically broken, generating a mass gap.

Original NJL[1]

- Nucleons are the fundamental degrees of freedom.
- Is formulated in analogy to superconductivity.
- Nucleon-antinucleon form Cooper-like pair.
- Goldstone boson occurring in the theory is identified as pion.

[1] Nambu and Jona-Lasinio, Phys. Rev. 122, 345; Phys. Rev. 124, 246.

Subsequent NJL[2]

- Quarks as the fundamental degrees of freedom.
- Many successes in the study of meson and baryon properties.
- Did not have confinement.

[2] e.g., S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992)

Confining NJL[3]

- Confinement is simulated by the introduction of an infrared cutoff in the proper-time regularization scheme.
- Doing this eliminates free quark propagation (it gets rid of the imaginary part of hadron decaying into quarks). (In a way that maintains covariance.)
- We calculate PDF, FF, TMD, GPD, etc. with this model, as well as study nucleon in-medium modification, and the binding of atomic nuclei.

[3] H. Mineo et. al, Nucl. Phys. A 735, 482 (2004)

Proper-time regularization scheme

As an effective theory, NJL model is non-renormalizable, thus it needs a regularization prescription in order to be well-defined. We use the proper-time regularization scheme

$$\begin{aligned} \frac{1}{X} &= \frac{1}{(n-1)!} \int_0^\infty d\tau \; \tau^{n-1} e^{-\tau X} \\ &\longrightarrow \frac{1}{(n-1)!} \int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \; \tau^{n-1} e^{-\tau X}, \end{aligned}$$

where X represents a product of propagators that have been combined using Feynman parametrization. Only the ultraviolet cutoff Λ_{UV} is needed to render the theory finite, while Λ_{IR} is introduced to mimic confinement.

NJL Lagrangian

The SU(2) flavor NJL Lagrangian relevant to this study, in the $\bar{q}q$ interaction channel, reads[2]

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi \\ & + \tfrac{1}{2}\, G_\pi[(\bar{\psi}\psi)^2 - (\bar{\psi}\,\gamma_5\vec{\tau}\,\psi)^2] - \tfrac{1}{2}\, G_\omega(\bar{\psi}\,\gamma^\mu\,\psi)^2 \\ & - \tfrac{1}{2}\, G_\rho[(\bar{\psi}\,\gamma^\mu\vec{\tau}\,\psi)^2 + (\bar{\psi}\,\gamma^\mu\gamma_5\vec{\tau}\,\psi)^2], \qquad (1)\end{aligned}$$

where $\hat{m} \equiv \text{diag}[m_u, m_d]$ is the current quark mass matrix and the 4-fermion coupling constants in each chiral channel are labeled by G_π, G_ω, and G_ρ. Throughout this paper we take $m_u = m_d = m$. The interaction Lagrangian can be Fierz symmetrized, with the consequence that after a redefinition of the 4-fermion couplings one need only consider direct terms in the elementary interaction [28].

Mass gap equation

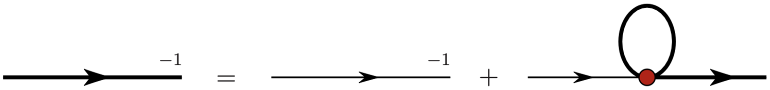


FIG. 1. (Color online) The NJL gap equation in the Hartree-Fock approximation, where the thin line represents the elementary quark propagator, $S_0^{-1}(k) = \not{k} - m + i\varepsilon$, and the shaded circle represents the $\bar{q}q$ interaction kernel given in Eq. (2). Higher-order terms, attributed to meson loops, for example, are not included in the gap equation kernel.

$$i S^{-1}(k) = i S_0^{-1}(k) - \sum_{\Omega} K_{\Omega}\, \Omega \int \frac{d^4\ell}{(2\pi)^4}\, \mathrm{Tr}[\bar{\Omega}\, i S(\ell)],$$

Mass gap equation

The interaction kernel in the gap equation of Fig. 1 is local and therefore the dressed quark mass, M, is a constant and satisfies

$$M = m + 12\, i\, G_\pi \int \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}_D[S(\ell)], \qquad (5)$$

where the remaining trace is over Dirac indices. For sufficiently strong coupling, $G_\pi > G_{\text{critical}}$, Eq. (5) supports a nontrivial solution with $M > m$, which survives even in the chiral limit ($m = 0$).[4] This solution is a consequence of dynamical chiral symmetry breaking (DCSB) in the Nambu-Goldstone mode and it is readily demonstrated, by calculating the total energy [39], that this phase corresponds to the ground state of the vacuum.

[4] In the proper-time regularization scheme defined in Eq. (6) the critical coupling in the chiral limit has the value $G_{\text{critial}} = \frac{\pi^2}{3}(\Lambda_{\text{UV}}^2 - \Lambda_{\text{IR}}^2)^{-1}$.

The dressed quark propagator thus has the solution

$$S(k) = \frac{1}{\not{k} - M + i\varepsilon}.$$

Dressed mass as a function of the coupling

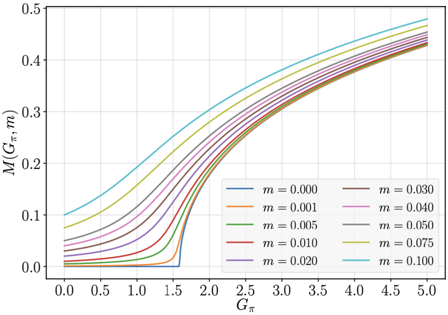


Figure 2.2. Dressed mass for the $1 + 1$ NJL model as a function of G_π and the bare mass.

Bethe-Salpeter equation

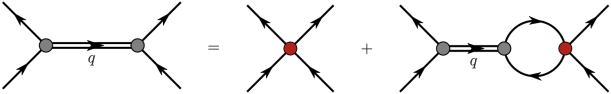


FIG. 2. (Color online) NJL Bethe-Salpeter equation for the quark-antiquark t matrix, represented as the double line with the vertices. The single line corresponds to the dressed quark propagator and the BSE $\bar{q}q$ interaction kernel, consistent with the gap equation kernel used in Eq. (5), is given by Eq. (2).

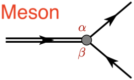
The NJL BSE, consistent with the gap equation of Fig. 1, is illustrated in Fig. 2 and reads

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K}\, S(k+q)\, S(k)\, \mathcal{T}(q), \tag{7}$$

This works for both meson and diquark very similarly.

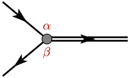
Bethe-Salpeter vertices

Meson



$$\Omega^{\sigma}_{\alpha\beta} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\alpha\beta}\,, \quad \Omega^{\eta}_{\alpha\beta} = \sqrt{Z_\eta}\,(\gamma_5)_{\alpha\beta}\,, \quad \Omega^{\omega}_{\alpha\beta} = \sqrt{-Z_\omega}\,(\gamma^\mu)_{\alpha\beta}\,, \quad \Omega^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

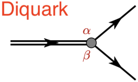
$$\Omega^{a_0}_{\alpha\beta} = \sqrt{-Z_{a_0}}\,(\tau_i)_{\alpha\beta}\,, \quad \Omega^{\pi}_{\alpha\beta} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i)_{\alpha\beta}\,, \quad \Omega^{\rho}_{\alpha\beta} = \sqrt{-Z_\rho}\,(\gamma^\mu\,\tau_i)_{\alpha\beta}\,, \quad \Omega^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i)_{\alpha\beta}$$



$$\overline{\Omega}^{\sigma}_{\beta\alpha} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\beta\alpha}\,, \quad \overline{\Omega}^{\eta}_{\beta\alpha} = \sqrt{Z_\eta}\,(\gamma_5)_{\beta\alpha}\,, \quad \overline{\Omega}^{\omega}_{\beta\alpha} = \sqrt{-Z_\omega}\,(\gamma^\nu)_{\beta\alpha}\,, \quad \overline{\Omega}^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

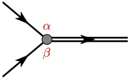
$$\overline{\Omega}^{a_0}_{\beta\alpha} = \sqrt{-Z_{a_0}}\,(\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{\pi}_{\beta\alpha} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{\rho}_{\beta\alpha} = \sqrt{-Z_\rho}\,(\gamma^\nu\,\tau_i^\dagger)_{\beta\alpha}, \quad \overline{\Omega}^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i^\dagger)_{\alpha\beta}$$

Diquark



$$\Omega^{s}_{\alpha\beta} = \sqrt{Z_s}\,(\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}\,, \qquad \Omega^{p}_{\alpha\beta} = \sqrt{-Z_p}\,(C\,\tau_2\,\beta_A)_{\alpha\beta}\,,$$

$$\Omega^{a}_{\alpha\beta} = \sqrt{-Z_a}\,(\gamma^\mu\,C\,\tau_i\tau_2\,\beta_A)_{\alpha\beta}\,, \qquad \Omega^{v}_{\alpha\beta} = \sqrt{-Z_v}\,(\gamma^\mu\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}$$



$$\overline{\Omega}^{s}_{\alpha\beta} = \sqrt{Z_s}\left(C^{-1}\,\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta}, \qquad \overline{\Omega}^{p}_{\alpha\beta} = \sqrt{-Z_p}\left(C^{-1}\,\tau_2\,\beta_A\right)_{\alpha\beta},$$

$$\overline{\Omega}^{a}_{\alpha\beta} = \sqrt{-Z_a}\left(C^{-1}\,\gamma^\nu\,\tau_2\tau_i^\dagger\,\beta_A\right)_{\alpha\beta}, \qquad \overline{\Omega}^{v}_{\alpha\beta} = \sqrt{-Z_v}\left(C^{-1}\,\gamma^\nu\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta}$$

Approximated diquark propagator

In the following I use scalar diquark as an example.

$$\tau_s(q) = 4i\,G_s - 4i\,N_c\,G_s \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5\,S(k)\,\gamma_5\,S(q+k)\right]\tau_s(q) = 4i\,G_s - 2\,G_s\,\Pi_{PP}(q^2)\,\tau_s(q).$$

Therefore

$$\tau_s(q) = \frac{4i\,G_s}{1 + 2\,G_s\,\Pi_{PP}(q^2)} \qquad \text{where} \qquad \Pi_{PP}(q^2) = 2i\,N_c \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma_5\,S(k)\,\gamma_5\,S(q+k)\right].$$

Expanding the bubble diagrams about the pole mass, where

$$\Pi(q^2) = \Pi(m^2) + (q^2 - m^2)\,\frac{\partial}{\partial q^2}\Pi(q^2)\bigg|_{q^2=m^2} + \ldots,$$

and using the pole condition $1 + 2\,G_s\,\Pi_{PP}(m_s^2) = 0$, gives

$$\tau_s(q) = \frac{4i\,G_s}{1 + 2\,G_s\,\Pi_{PP}(M_s^2) + (q^2 - M_s^2)\,2\,G_s\,\frac{\partial}{\partial q^2}\Pi_{PP}(q^2)\Big|_{q^2=M_s^2}}.$$

Therefore the pole form of the propagator reads

$$\tau_s(q) = -\frac{i\,Z_s}{q^2 - m_s^2 + i\varepsilon} \qquad \text{where} \qquad Z_s^{-1} = -\frac{1}{2}\,\frac{\partial}{\partial q^2}\Pi_{PP}(q^2)\bigg|_{q^2=m_s^2},$$

where Z_s is defined to be positive.

Approximated diquark propagator

We use "contact+pole" approximation for the meson/diquark propagator, namely

For the diquarks we have

$$\tau_s(q) = \frac{4i\,G_s}{1+2\,G_s\,\Pi_{PP}(q^2)} \longrightarrow 4i\,G_s - \frac{i\,Z_s}{q^2 - M_s^2 + i\,\varepsilon},$$

$$\tau_p(q) = \frac{-4i\,G_p}{1-2\,G_p\,\Pi_{SS}(q^2)} \longrightarrow -4i\,G_p + \frac{i\,Z_p}{q^2 - M_p^2 + i\,\varepsilon},$$

$$\tau_a^{\mu\nu}(q) = \frac{4i\,G_a}{1+2\,G_a\,\Pi_{VV}(q^2)}\left[g^{\mu\nu} + 2\,G_a\,\Pi_{VV}(q^2)\,\frac{q^\mu q^\nu}{q^2}\right] \longrightarrow 4i\,G_a\,g^{\mu\nu} - \frac{i\,Z_a}{q^2 - M_a^2 + i\,\varepsilon}\left(g^{\mu\nu} - \frac{q^\mu\,q^\nu}{M_a^2}\right),$$

$$\tau_v^{\mu\nu}(q) = \frac{4i\,G_a}{1+2\,G_a\,\Pi_{AA}^{(T)}(q^2)}\left[g^{\mu\nu} + \frac{2\,G_a\,[\Pi_{AA}^{(T)}(q^2) - \Pi_{AA}^{(L)}(q^2)]}{1+2\,G_a\,\Pi_{AA}^{(L)}(q^2)}\,\frac{q^\mu q^\nu}{q^2}\right] \longrightarrow 4i\,G_a\,g^{\mu\nu} - \frac{i\,Z_v}{q^2 - M_v^2 + i\,\varepsilon}\left(g^{\mu\nu} - \frac{q^\mu\,q^\nu}{M_v^2}\right),$$

$$Z_s^{-1} = -\frac{1}{2}\,\hat{\Pi}_{PP}(m_s^2), \qquad Z_p^{-1} = \frac{1}{2}\,\hat{\Pi}_{SS}(m_p^2), \qquad Z_a^{-1} = -\frac{1}{2}\,\hat{\Pi}_{VV}(m_a^2), \qquad Z_v^{-1} = -\frac{1}{2}\,\hat{\Pi}_{AA}^{(T)}(m_v^2),$$

where $\hat{\Pi}_i(m_j^2) = \frac{\partial}{\partial p^2}\,\Pi_i(p^2)\big|_{p^2=m_j^2}$.

Faddeev equation

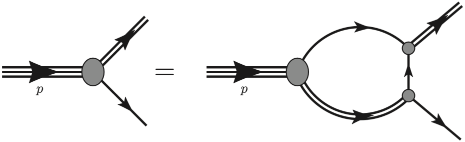


FIG. 3. Homogeneous Faddeev equation for the nucleon in the NJL model. The single lines represent the quark propagator and the double lines the diquark propagators.

$$X^a_{\alpha,i}(P,p) = \int \frac{d^4\ell}{(2\pi)^4}\, Z^{ab,ij}_{\alpha\beta}(p,\ell)\, S_j(\tfrac{1}{2}P+\ell)_{\beta\gamma}\, \tau^{ki}_{bc}(\tfrac{1}{2}P-\ell)\, X^c_{\gamma,j}(P,p).$$

where

$$Z^{ab,ij}_{\alpha\beta}(p,\ell) = \left[\Omega^{b,ik}\, S^T_k(-\ell-p)\, \overline{\Omega}^{a,kj}\right]_{\alpha\beta}$$

Faddeev equation

Therefore, the quark exchange kernel is given by

$$Z_{\alpha\beta}(p,\ell) = -3 \begin{pmatrix} \gamma_5\, S(p+\ell)\,\gamma_5 & \gamma^\sigma\, S(p+\ell)\,\gamma_5\,\tau_n & S(p+\ell)\,\gamma_5 & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma_5 \\ \gamma_5\, S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\, S(p+\ell)\,\gamma^\mu\,\tau_n\tau_m^\dagger & S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma^\mu\,\tau_m^\dagger \\ \gamma_5\, S(p+\ell) & \gamma^\sigma\, S(p+\ell)\,\tau_n & S(p+\ell) & \gamma^\sigma\gamma_5\, S(p+\ell) \\ \gamma_5\, S(p+\ell)\,\gamma^\mu\gamma_5 & \gamma^\sigma\, S(p+\ell)\,\gamma^\mu\gamma_5\,\tau_n & S(p+\ell)\,\gamma^\mu\gamma_5 & \gamma^\sigma\gamma_5\, S(p+\ell)\,\gamma^\mu\gamma_5 \end{pmatrix}_{\alpha\beta}, \tag{223}$$

where we have used $(\tau_i\tau_2)(\tau_2\tau_i) = (\tau_m\tau_2)(\tau_2\tau_m^\dagger)$. In the static approximation, $S(p+\ell) \to -\frac{1}{M}$, the quark exchange kernel becomes

$$Z_{\alpha\beta} = \frac{3}{M} \begin{pmatrix} 1 & \gamma^\sigma\,\gamma_5\,\tau_n & \gamma_5 & \gamma^\sigma \\ \gamma_5\,\gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\,\gamma^\mu\,\tau_n\tau_m^\dagger & \gamma^\mu\,\tau_m^\dagger & \gamma^\sigma\gamma_5\,\gamma^\mu\,\tau_m^\dagger \\ \gamma_5 & \gamma^\sigma\,\tau_n & 1 & \gamma^\sigma\gamma_5 \\ \gamma_5\,\gamma^\mu\gamma_5 & \gamma^\sigma\,\gamma^\mu\gamma_5\,\tau_n & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}_{\alpha\beta}. \tag{224}$$

Projecting the kernel onto isospin one-half gives

$$Z_{\alpha\beta} = \frac{3}{M} \begin{pmatrix} 1 & \sqrt{3}\,\gamma^\sigma\,\gamma_5 & \gamma_5 & \gamma^\sigma \\ \sqrt{3}\,\gamma_5\,\gamma^\mu & -\gamma^\sigma\,\gamma^\mu & \sqrt{3}\,\gamma^\mu & -\sqrt{3}\,\gamma^\sigma\gamma^\mu\gamma_5 \\ \gamma_5 & \sqrt{3}\,\gamma^\sigma & 1 & \gamma^\sigma\gamma_5 \\ -\gamma^\mu & \sqrt{3}\,\gamma^\sigma\,\gamma^\mu\gamma_5 & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}_{\alpha\beta}. \tag{225}$$

The Faddeev equation reads

$$X^a_\alpha(P) = \int \frac{d^4\ell}{(2\pi)^4}\, Z_{\alpha\beta}\, S(\tfrac{1}{2}P+\ell)_{\beta\gamma}\,\tau_{bc}(\tfrac{1}{2}P-\ell)\, X^\gamma_c(P) \equiv Z^{ab,ij}_{\alpha\beta}\,\Pi^{bc,ki}_{\beta\gamma,j}(P)\, X^c_{\gamma,j}(P). \tag{226}$$

Faddeev equation

4.2 Quark–Scalar-Diquark Bubble Diagram

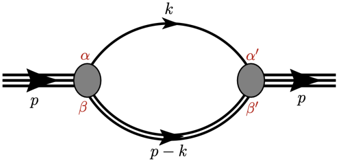


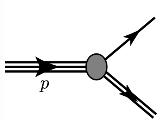
Figure 2: Quark–scalar-diquark diagram contribution to a baryon Faddeev vertex.

The baryon quark–scalar-diquark bubble diagram has the form

$$\Pi_N^s(p) = -i \int \frac{d^4k}{(2\pi)^4}\, \tau_s(p-k)\, iS(k) = \int \frac{d^4k}{(2\pi)^4} \left[4i\,G_s - \frac{i\,Z_s}{(p-k)^2 - m_s^2 + i\varepsilon}\right] \frac{\not{k} + M}{k^2 - M^2 + i\varepsilon}.$$

Faddeev equation

The nucleon vertex function is parametrized by



$$\Gamma_s = \sqrt{-Z_N}\, \alpha_1\, \chi_t\, u(p,s), \qquad \Gamma_a = \sqrt{-Z_N}\left[\alpha_2 \frac{p^\mu}{M_N}\gamma_5 + \alpha_3\, \gamma^\mu \gamma_5\right]\frac{\tau_i}{\sqrt{3}}\,\chi_t\, u(p,s),$$

$$\Gamma_p = \sqrt{-Z_N}\, \alpha_4\, \gamma_5\, \chi_t\, u(p,s), \qquad \Gamma_v = \sqrt{-Z_N}\left[\alpha_5 \frac{p^\mu}{M_N} + \alpha_6\, \gamma^\mu\right]\chi_t\, u(p,s),$$

Therefore the Faddeev equation reads

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \frac{p^\mu}{M_N}\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5 \frac{p^\mu}{M_N} + \alpha_6\,\gamma^\mu \end{bmatrix} u(p,\lambda)$$

$$= \frac{3}{M}\begin{pmatrix} 1 & \sqrt{3}\,\gamma^\sigma\,\gamma_5 & \gamma_5 & \gamma^\sigma \\ \sqrt{3}\,\gamma_5\,\gamma^\mu & -\gamma^\sigma\,\gamma^\mu & \sqrt{3}\,\gamma^\mu & -\sqrt{3}\,\gamma^\sigma\gamma^\mu\gamma_5 \\ \gamma_5 & \sqrt{3}\,\gamma^\sigma & 1 & \gamma^\sigma\gamma_5 \\ -\gamma^\mu & \sqrt{3}\,\gamma^\sigma\,\gamma^\mu\gamma_5 & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}\begin{pmatrix} \Pi_{Ns} & 0 & 0 & 0 \\ 0 & \Pi^{Na}_{\sigma\nu} & 0 & 0 \\ 0 & 0 & \Pi_{Np} & 0 \\ 0 & 0 & 0 & \Pi^{Nv}_{\sigma\nu} \end{pmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \frac{p^\nu}{M_N}\gamma_5 + \alpha_3\,\gamma^\nu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5 \frac{p^\nu}{M_N} + \alpha_6\,\gamma^\nu \end{bmatrix} u(p,\lambda)$$

Model parameters

The two-flavor NJL has the following parameters:

$\bar{q}q$ couplings:	G_π, G_ρ, G_ω
qq couplings:	$G_s(=G_p)$, $G_a(=G_v)$
masses:	$m_u = m_d$
regularization:	Λ_{IR}, Λ_{UV}

We assign values *a priori* to the following parameters:

$$\Lambda_{IR} = 240\ MeV \text{ and } M = 400\ MeV$$

The remaining parameters can then be fixed by

$$\Lambda_{UV} \leftrightarrow f_\pi$$
$$G_{\pi,\rho,\omega} \leftrightarrow m_{\pi,\rho,\omega}$$
$$G_s,\, G_a \leftrightarrow M_N,\, M_\Delta$$

Model parameters

The Λ_{UV} and G_π together is determined by the pion decay constant and pion mass.

$$< 0|\bar{\psi}\gamma_\mu\gamma_5\psi|\pi(q) >= 2if_\pi q_\mu$$

From this we can obtain

$$f_\pi = -12i\sqrt{Z_\pi}M\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2)\left((k+q)^2-M^2\right)}|_{q^2=m_\pi^2}$$

By solving the Bethe-Salpeter equation for the mesons, m_π and Z_π can be related to the parameter G_π. Thus, this equation, together with the value of m_π, determines the parameters of Λ_{UV} and G_π.

Model parameters

- Similarly, m_ρ and m_ω determine the parameters G_ρ and G_ω, respectively.
- G_s and G_a are determined by solving the two Faddeev equations for the nucleon and the delta baryon.
- We obtain $G_s = 7.65\ GeV^{-2}$ and $G_a = 4.91\ GeV^{-2}$.
- The corresponding diquark masses are $M_s = 0.679\ GeV$, $M_p = 0.945\ GeV$, $M_a = 0.929\ GeV$, and $M_v = 1.099\ GeV$.
- Compared to the previous values obtained without the pseudoscalar and vector diquark channels, $M_s = 0.768\ GeV$ and $M_a = 0.929\ GeV$, the scalar diquark mass got smaller, while the axial vector diquark is exactly the same. The axial vector diquark mass does not change because the delta baryon Faddeev equation only concerns the axial vector diquark, and is thus unchanged from the previous work.

Quark light-cone momentum distributions

The leading twist spin-independent and spin-dependent quark light-cone momentum distributions in the nucleon are defined by the following equations:

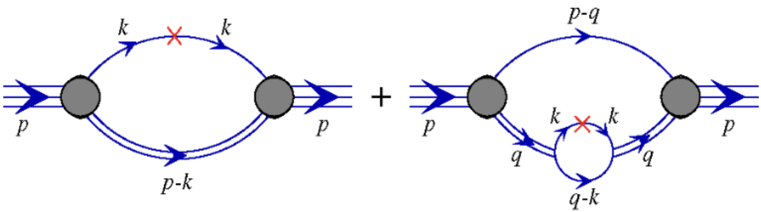
$$f_q(x) = p_- \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} < p, s|\bar{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s >_c,$$

$$\Delta f_q(x) = p_- \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} < p, s|\bar{\psi}_q(0)\gamma^+\gamma_5\psi_q(\xi^-)|p, s >_c,$$

where ψ_q is the quark field of flavor q, x is the Bjorken scaling variable and the subscript c means that only connected matrix elements are included.

Feynman diagrams for the PDF

At our model scale, there is no sea quark and no gluons. The Feynman diagram for calculating the valence quark PDFs are shown below.



The single line represents the quark propagator and the double line the diquark propagator. The shaded oval denotes the quark-diquark vertex function and the red cross represents the operator insertion which has the form of $\gamma^+\delta(x - k^+/p^+)\frac{1}{2}(1 \pm \tau_z)$ for the spin-independent distribution and $\gamma^+ \to \gamma^+\gamma_5$ for the spin-dependent one.

Isospin factors

By separating the isospin factors, the spin-independent u and d distributions in the proton can be expressed as

$$
\begin{aligned}
u_v(x) = {} & f^s_{q/N}(x) + f^p_{q/N}(x) + \frac{1}{3} f^a_{q/N}(x) + f^v_{q/N}(x) \\
& + f^{ss}_{q(D)/N}(x) + f^{pp}_{q(D)/N}(x) + \frac{5}{3} f^{aa}_{q(D)/N}(x) + f^{vv}_{q(D)/N}(x) \\
& + f^{sp}_{q(D)/N}(x) + f^{ps}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{sa}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{as}_{q(D)/N}(x) \\
& + f^{sv}_{q(D)/N}(x) + f^{vs}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{pa}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{ap}_{q(D)/N}(x) \\
& + f^{pv}_{q(D)/N}(x) + f^{vp}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{av}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{va}_{q(D)/N}(x).
\end{aligned}
$$

Isospin factors

And

$$d_v(x) = \frac{2}{3} f^{a}_{q/N}(x)$$
$$+ f^{ss}_{q(D)/N}(x) + f^{pp}_{q(D)/N}(x) + \frac{1}{3} f^{aa}_{q(D)/N}(x) + f^{vv}_{q(D)/N}(x)$$
$$+ f^{sp}_{q(D)/N}(x) + f^{ps}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{sa}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{as}_{q(D)/N}(x)$$
$$+ f^{sv}_{q(D)/N}(x) + f^{vs}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{pa}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{ap}_{q(D)/N}(x)$$
$$+ f^{pv}_{q(D)/N}(x) + f^{vp}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{av}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{va}_{q(D)/N}(x).$$

Helicity distributions

For the calculation of the spin-dependent PDFs, we use the result

$$u(p,s)\bar{u}(p,s) = (\not{p} + M_N)\,\frac{1+\gamma_5\not{s}}{2},$$

where s^μ is the spin vector of the particle satisfying $s^2 = -1$ and $s \cdot p = 0$. In general, s^μ can be written as

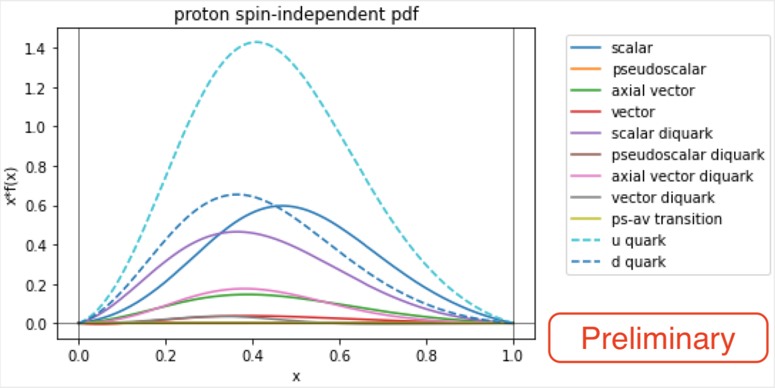
$$s^\mu = \left(\frac{\vec{p}\cdot\vec{n}}{M_N},\ \vec{n} + \frac{(\vec{p}\cdot\vec{n})\vec{p}}{M_N\,(M_N + p^0)}\right)$$

where $\vec{n} = \frac{\vec{p}}{|\vec{p}|}$ if the particle is longitudinally polarised, $\vec{n}\cdot\vec{p} = 0$ if transversely polarized.
For the helicity distribution, the proton is longitudinally polarized, and the helicity distribution is defined as

$$\Delta f(x) = f_+(x) - f_-(x),$$

i.e., the difference in the distributions of the quark's spin aligned with the proton's versus the quark's spin anti-aligned with the proton's.

Results for the spin-independent PDFs



Results for the spin-independent PDFs

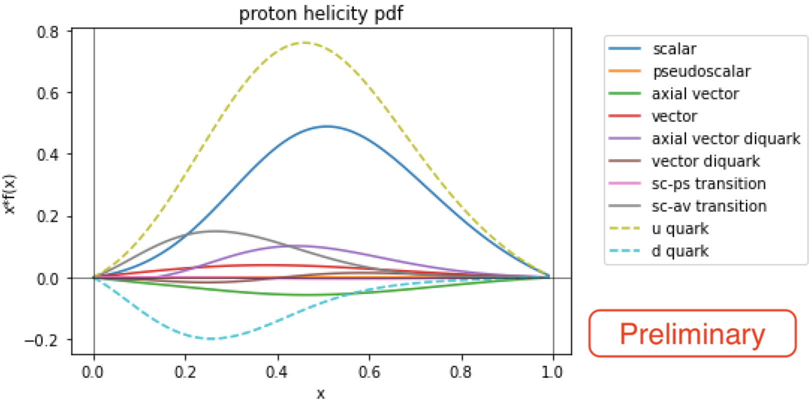
- The model scale is found to be 0.19 GeV^2, which is slightly higher than the previous 0.16 GeV^2. This is understandable because we've added more complexity to the model.
- The number and momentum sum rules are satisfied

$$\int_0^1 dx f_{q/P}(x) = N_{q/P},$$

$$\int_0^1 dx x \left[f_{u/P}(x) + f_{d/P}(x) \right] = 1.$$

- To compare our results to the experimental data, need to evolve to a higher energy scale where empirical PDFs are available.

Results for the spin-dependent PDFs



Preliminary

Summary and future work

- We used the framework of the relativistic Faddeev equation in the NJL model to calculate the quark LC momentum distributions in the nucleon based on a straightforward Feynman diagram evaluation.
- The work can be extended to calculate GPDs and TMDs.
- Or to a finite density calculation.

Thank you for your attention!