Orbital Angular Momentum at Small x

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SURGE COLLABORATION

Overview

* Motivation: proton spin puzzle
* Polarized observables / Light Cone Operator Treatment (LCOT) approach
* OAM distributions in terms of polarized S-matrices
* Polarized S-matrix (+moment) evolution
* Numerical results:
    * Small-x asymptotics of OAM distributions
    * OAM/helicity PDF ratios

Motivation: proton spin puzzle

* Jaffe-Manohar sum rule: Jaffe, Manohar '90

$$\int_0^1 dx \left[\frac{1}{2}\Delta\Sigma\left(x,Q^2\right) + L_{q+\bar{q}}\left(x,Q^2\right) + \Delta G\left(x,Q^2\right) + L_G\left(x,Q^2\right)\right] = \frac{1}{2}$$

* Current estimates of proton spin

$$\int_{0.001}^1 dx\, \frac{1}{2}\Delta\Sigma\left(x,Q^2=10\,\mathrm{GeV}^2\right) \approx [0.15, 0.20] \qquad \int_{0.05}^1 dx\, \Delta G\left(x,Q^2=10\,\mathrm{GeV}^2\right) \approx [0.13, 0.26]$$

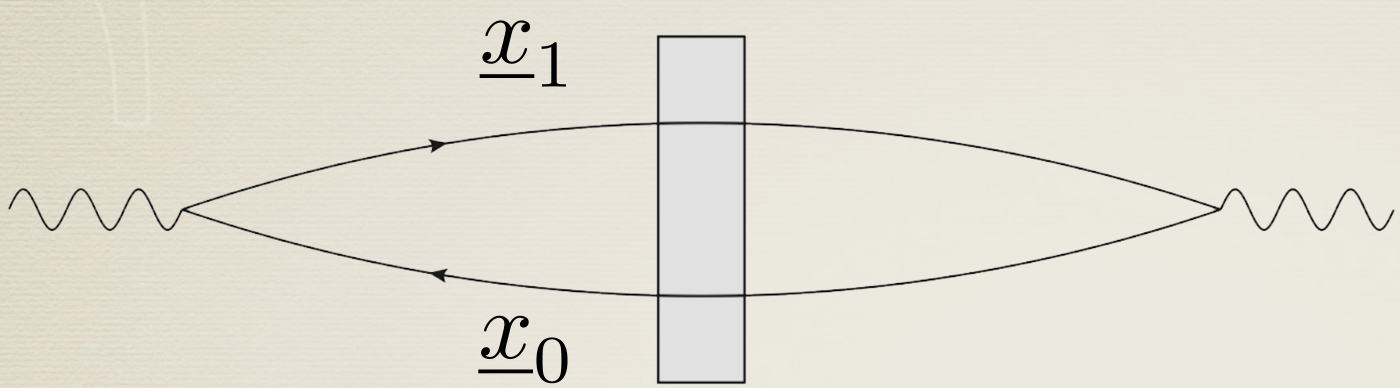
RHIC Spin Collaboration '15 (1501.01220)

  * Missing spin needs to be constrained at small-x

* Focus on OAM distributions at small-x

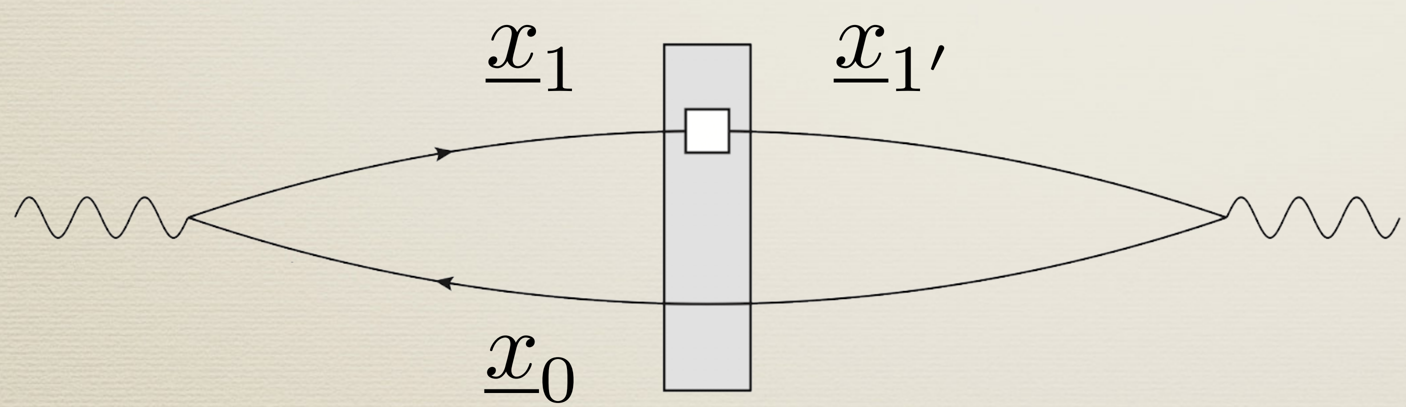
Polarized observables

* Unpolarized observables dominated by eikonal contributions



$$S \sim \mathrm{tr}\left[V_{\underline{x}_1} V^{\dagger}_{\underline{x}_0}\right]$$

* Polarized observables dominated by *sub*-eikonal contributions



$$S \sim \mathrm{tr}\left[V^{\mathrm{pol}}_{\underline{x}_1, \underline{x}_{1'}} V^{\dagger}_{\underline{x}_0}\right]$$

$$V_{\underline{x}}[x_f^-, x_i^-] \equiv \mathcal{P} \exp \left[ig \int\limits_{x_i^-}^{x_f^-} dx^- A^+(0^+, x^-, \underline{x}) \right]$$

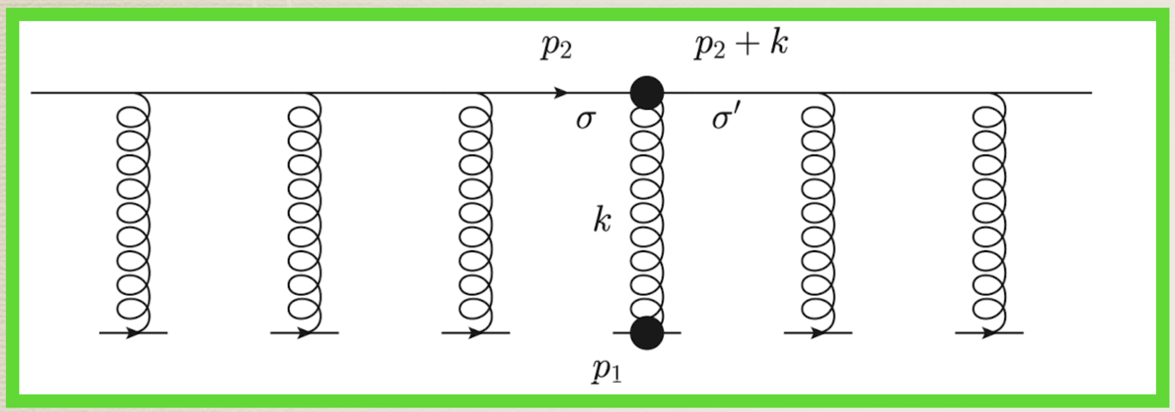
$$V_{\underline{x}} \equiv V_{\underline{x}}[\infty, -\infty]$$

Kovchegov, Pitonyak, Sievert '15 (1511.06737)

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Polarized propagators

- Sub-eikonal vertex sandwiched between semi-infinite Wilson lines (LCOT approach)



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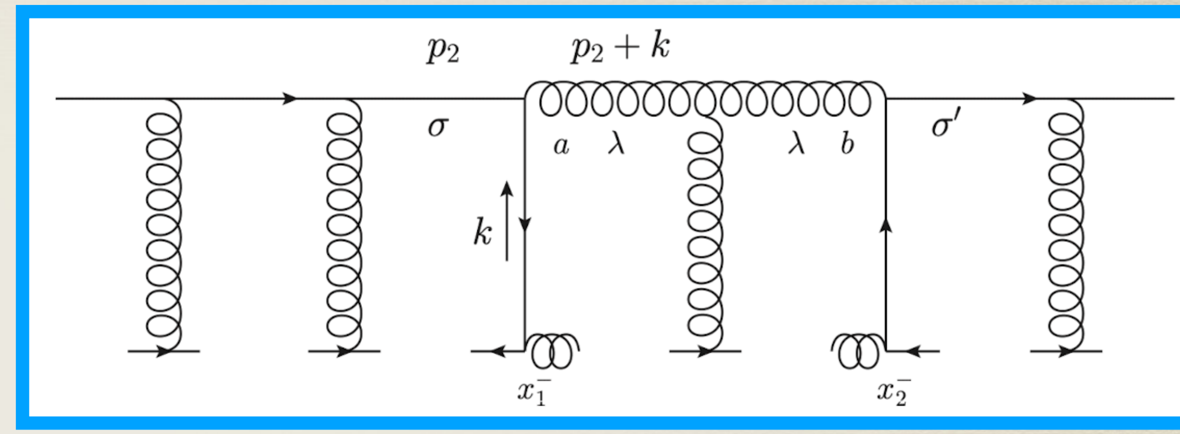


$$V^{\text{pol}}_{\underline{x},\underline{y};\sigma,\sigma'} = \sigma\delta_{\sigma,\sigma'}\left(V^{\text{q}[1]}_{\underline{x}} + V^{\text{G}[1]}_{\underline{x}}\right)_{\underline{y}=\underline{x}} + \delta_{\sigma,\sigma'}\left(V^{\text{q}[2]}_{\underline{x}}\Big|_{\underline{y}=\underline{x}} + V^{\text{G}[2]}_{\underline{x},\underline{y}}\right)$$

- *Polarized* dipole amplitudes

$$Q_{10}(zs) \propto \text{tr}\left[V_{\underline{x}_0}\left(V^{\text{pol}[1]}_{\underline{x}_1}\right)^{\dagger} + V^{\text{pol}[1]}_{\underline{x}_1}V^{\dagger}_{\underline{x}_0}\right] \qquad G^{i}_{10}(zs) \propto \text{tr}\left[V^{\dagger}_{\underline{x}_0}V^{i\,\text{G}[2]}_{\underline{x}_1} + \left(V^{i\,\text{G}[2]}_{\underline{x}_1}\right)^{\dagger}V_{\underline{x}_0}\right]$$

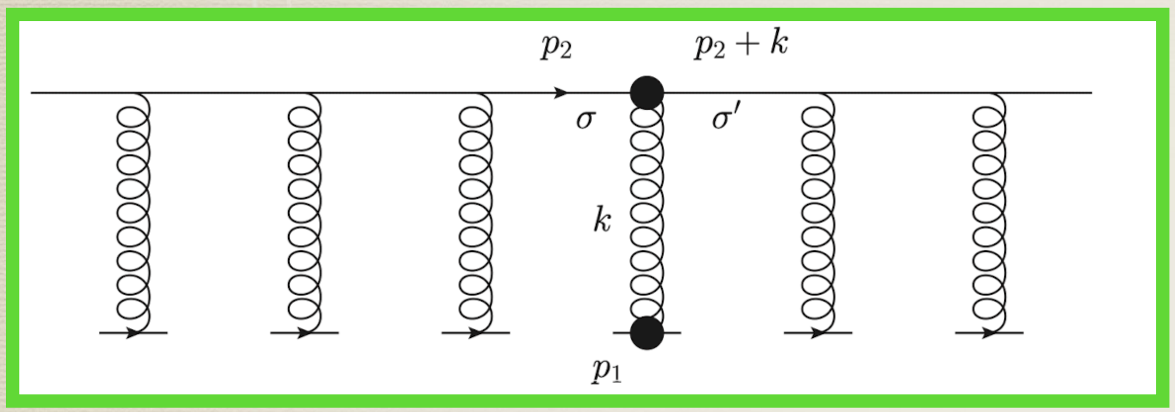
Kovchegov, Pitonyak, Sievert '15 (1511.06737)

$$V^i \propto \overleftarrow{D}^i - D^i$$

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Polarized propagators

- Sub-eikonal vertex sandwiched between semi-infinite Wilson lines (LCOT approach)



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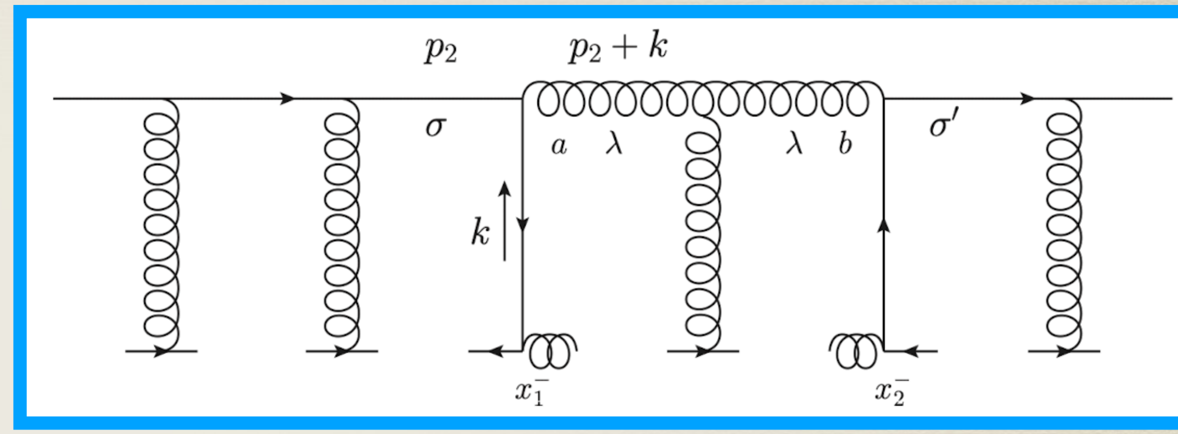


$$V^{\text{pol}}_{\underline{x},\underline{y};\sigma,\sigma'} = \sigma \delta_{\sigma,\sigma'} \left(V^{\text{q}[1]}_{\underline{x}} + V^{\text{G}[1]}_{\underline{x}} \right)_{\underline{y}=\underline{x}} + \delta_{\sigma,\sigma'} \left(V^{\text{q}[2]}_{\underline{x}} \Big|_{\underline{y}=\underline{x}} + V^{\text{G}[2]}_{\underline{x},\underline{y}} \right)$$

- *Polarized* dipole amplitudes

$$Q_{10}(zs) \propto \text{tr} \left[V_{\underline{x}_0} \left(V^{\text{pol}[1]}_{\underline{x}_1} \right)^{\dagger} + V^{\text{pol}[1]}_{\underline{x}_1} V^{\dagger}_{\underline{x}_0} \right]$$

$$G^i_{10}(zs) \propto \text{tr} \left[V^{\dagger}_{\underline{x}_0} V^{i\,\text{G}[2]}_{\underline{x}_1} + \left(V^{i\,\text{G}[2]}_{\underline{x}_1} \right)^{\dagger} V_{\underline{x}_0} \right]$$

Kovchegov, Pitonyak, Sievert '15 (1511.06737)

$$V^i \propto \overleftarrow{D}^i - D^i$$

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

OAM operator

Hatta '11 (1111.3547)

Ji, Xiong, Yuan '12 (1207.5221)

* Generic OAM operator

$$L_z\left(Q^2\right) = \int \frac{d^2b_\perp d^2k_\perp db^- dk^+}{(2\pi)^3} \left(\underline{b} \times \underline{k}\right) W(k,b)$$

where Wigner functions are

$$W^{q,\mathrm{SIDIS}}(k,b) = 2\sum_X \int d^2r dr^- e^{ik\cdot r} \left\langle \bar{\psi}\left(b-\frac{1}{2}r\right) V_{\underline{b}-\frac{1}{2}\underline{r}}\left[b^- - \frac{1}{2}r^-, \infty\right] |X\rangle \frac{1}{2}\gamma^+ \right.$$

$$\left. \times \langle X| V_{\underline{b}+\frac{1}{2}\underline{r}}\left[\infty, b^- + \frac{1}{2}r^-\right] \psi\left(b+\frac{1}{2}r\right)\right\rangle$$

$$W^{G,\mathrm{dipole}}(k,b) = \frac{4}{xP^+}\int d^2r dr^- e^{ixP^+r^- - i\underline{k}\cdot\underline{r}} \left\langle \mathrm{tr}\left[F^{+i}\left(b-\frac{1}{2}r\right)\mathcal{U}^{[+]}(b,r) F^{+i}\left(b-\frac{1}{2}r\right)\mathcal{U}^{[-]}(b,r)\right]\right\rangle$$

Kovchegov '19 (1901.07453)

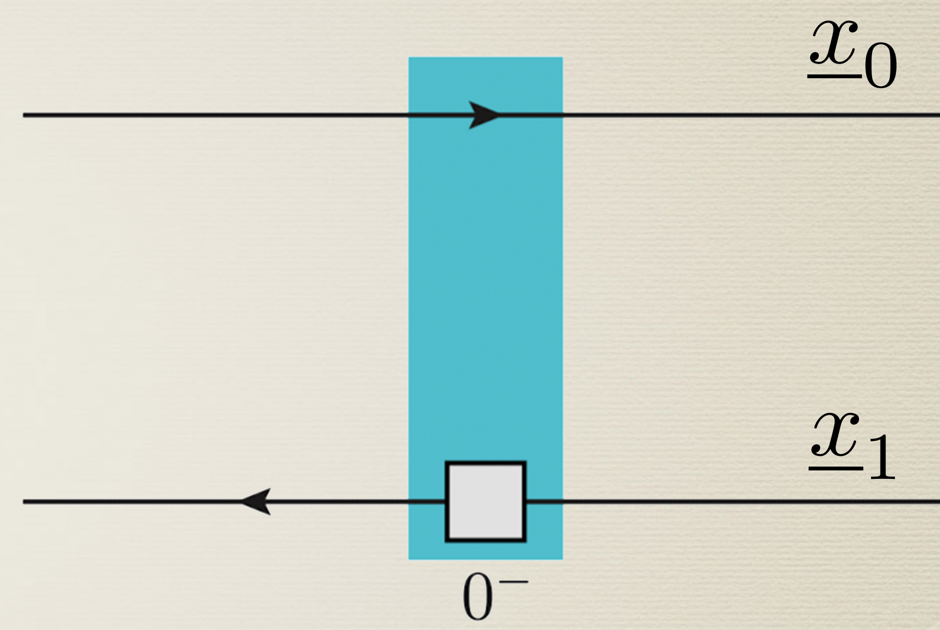
Future and past pointing staples

Polarized dipole amplitudes

* Impact-integrated polarized dipole amplitudes appear in calculations

$$Q(x_{10}^2, zs) \equiv \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) Q_{10}(zs)$$

$$G_2(x_{10}^2, zs) \equiv \frac{\epsilon^{ij} x_{10}^j}{x_{10}^2} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) G_{10}^i(zs)$$



* For OAM, need first x_1-moments (moment amplitudes)

$$I_3(x_{10}^2, zs) \equiv \frac{x_{10}^i}{x_{10}^2} \int d^2 x_1 \, x_1^i \, Q_{10}(zs)$$

$$\epsilon^{ij} \, I_4(x_{10}^2, zs) + \epsilon^{ik} x_{10}^k x_{10}^j \, I_5(x_{10}^2, zs) + \epsilon^{jk} x_{10}^k x_{10}^i \, I_6(x_{10}^2, zs) \equiv \int d^2 x_1 \, x_1^i \, G_{10}^j(zs)$$

OAM distributions at small x

* OAM distributions can be written in terms of the regular and moment dipole amplitudes

$$L_{q+\bar{q}}(x, Q^2) = \frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{1/zs}^{\min[1/zQ^2, 1/\Lambda^2]} \frac{dx_{10}^2}{x_{10}^2} \Big[Q - 3\, G_2 - I_3 - 2\, I_4 + I_5 + 3\, I_6 \Big] (x_{10}^2, zs)$$

$$L_G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} [2\, I_4 + 3\, I_5 + I_6] \left(zs = \frac{Q^2}{x}, x_{10}^2 = \frac{1}{Q^2} \right)$$

* Compare to helicity PDFs at small x

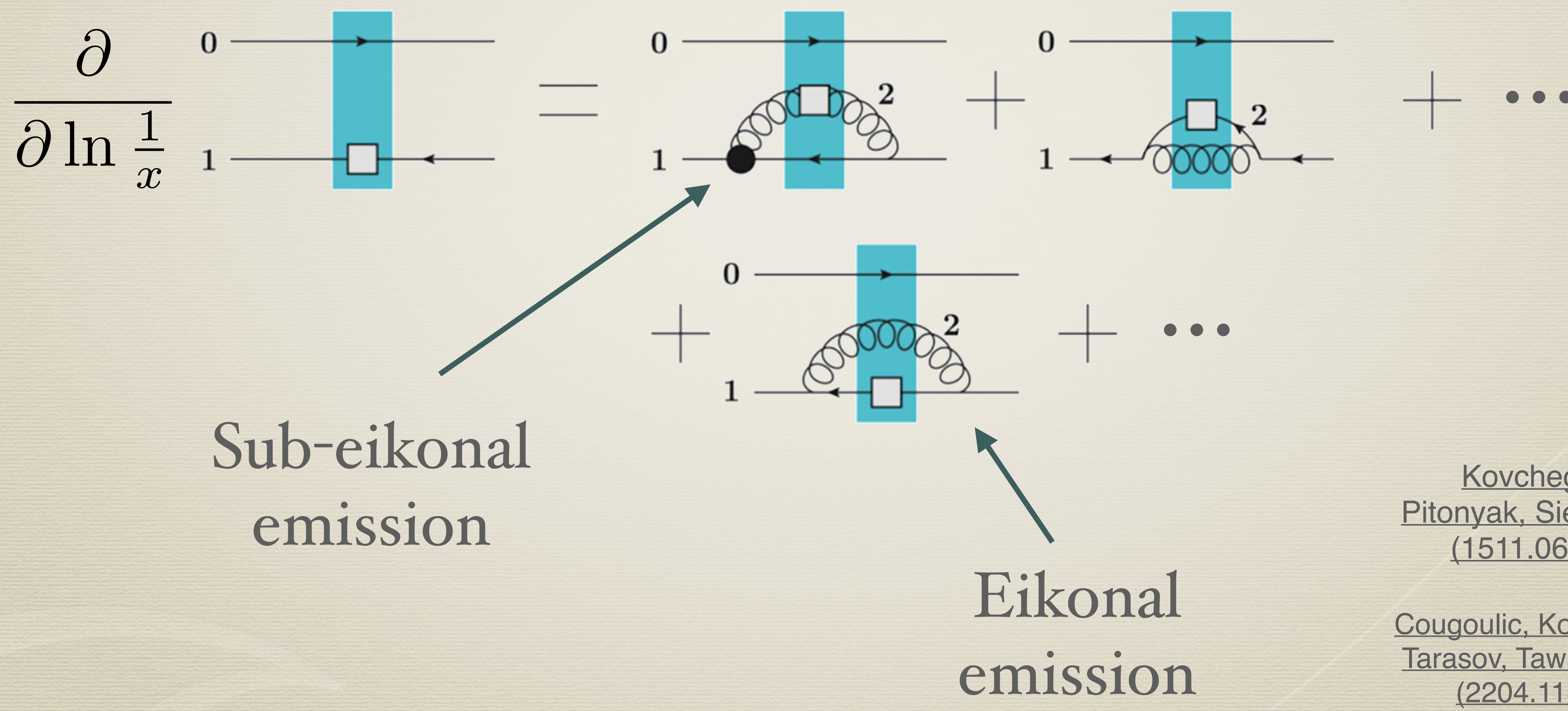
$$\Delta\Sigma(x, Q^2) = -\frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{1/zs}^{\min\left[1/zQ^2, 1/\Lambda^2\right]} \frac{dx_{10}^2}{x_{10}^2} [Q + 2\, G_2] (x_{10}^2, zs)$$

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s \pi^2} G_2 \left(zs = \frac{Q^2}{x}, x_{10}^2 = \frac{1}{Q^2} \right)$$

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Polarized S-matrix evolution

* Helicity evolution similar to BK/JIMWLK emission resummation parameter is $\alpha_s \ln^2(1/x)$



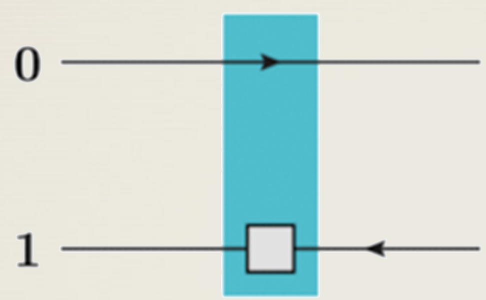
Kovchegov, Pitonyak, Sievert '15 (1511.06737)

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Moment amplitude evolution

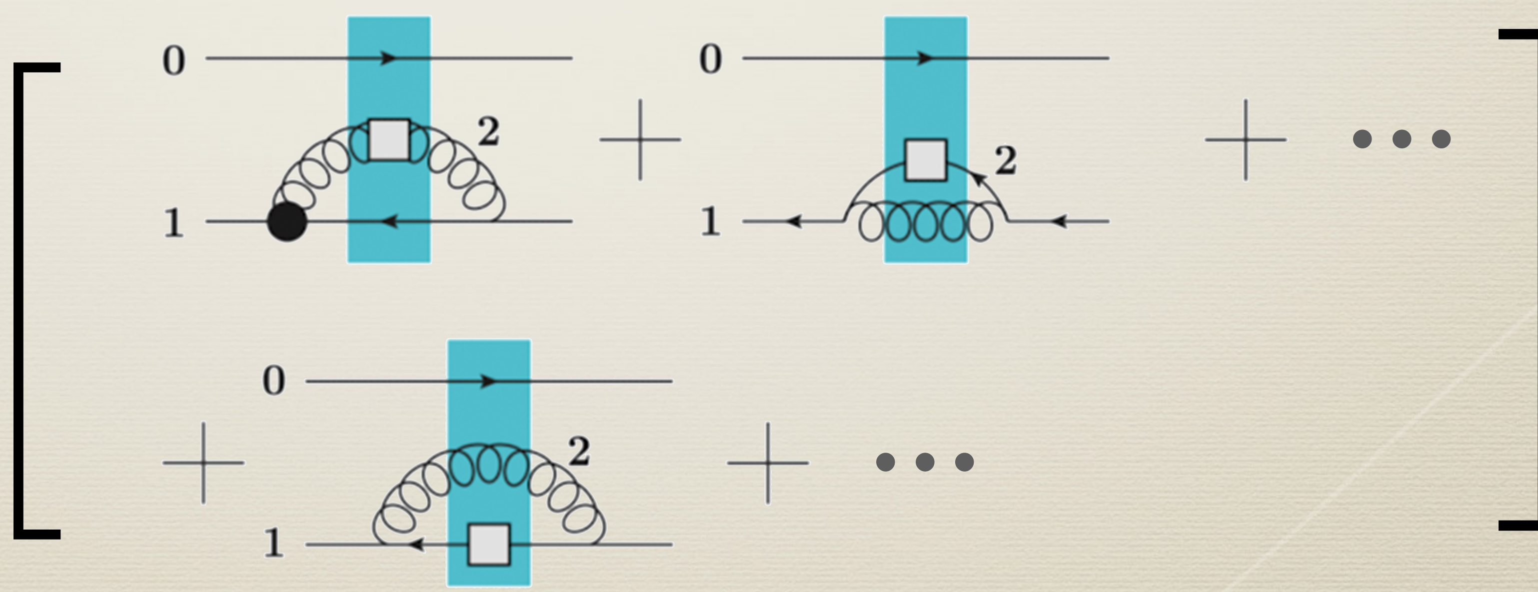
* Moment evolution: integrate over x_1 and match tensor structures

$$\frac{\partial}{\partial \ln \frac{1}{x}} \int d^2x_1\, x_1^i$$



$$=$$

$$\int d^2x_1\, x_1^i$$



Moment amplitude evolution

* Large-N_c evolution equations

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}(x_{10}^2, zs) = \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}_0 (x_{10}^2, zs)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 + 6\,\Gamma_6 - 2\,\Gamma_2 \\ 0 \\ 0 \\ 0 \end{pmatrix}(x_{10}^2, x_{21}^2, z's)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 4 & -4 & 2 & 6 & -4 & -6 \\ 0 & 4 & 2 & -2 & 0 & 1 \\ -2 & 2 & -1 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \\ G \\ G_2 \end{pmatrix}(x_{21}^2, z's).$$

Large N_c

$Q = G$

Moment amplitude evolution

* Large-N_c evolution equations

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} (x_{10}^2, zs) = \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}_0 (x_{10}^2, zs)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 + 6\,\Gamma_6 - 2\,\Gamma_2 \\ 0 \\ 0 \\ 0 \end{pmatrix} (x_{10}^2, x_{21}^2, z's)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z' s}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 4 & -4 & 2 & 6 & -4 & -6 \\ 0 & 4 & 2 & -2 & 0 & 1 \\ -2 & 2 & -1 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \\ G \\ G_2 \end{pmatrix} (x_{21}^2, z's).$$

Mixing with "regular" polarized amplitudes!

(cf. polarized DGLAP)

Numerical results: small-x asymptotics

* OAM distributions have same asymptotics as helicity counterparts (large N_c)

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

$$L_{q+\bar{q}}\left(x,Q^2\right) \sim L_G\left(x,Q^2\right) \sim \Delta\Sigma\left(x,Q^2\right) \sim \Delta G\left(x,Q^2\right) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

* Previously found

$$L_{q+\bar{q}}\left(x,Q^2\right) \sim \Delta\Sigma\left(x,Q^2\right) \sim \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

$$L_G\left(x,Q^2\right) \sim \Delta G\left(x,Q^2\right) \sim \left(\frac{1}{x}\right)^{1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Kovchegov '19 (1901.07453)

Numerical results: OAM/hPDF ratios

* Prediction for OAM/hPDF ratios

Boussarie, Hatta, Yuan '19 (1904.02693)

$$\Delta\Sigma\left(x,Q^2\right),\ \Delta G\left(x,Q^2\right) \sim \left(\frac{1}{x}\right)^{\alpha} \longrightarrow \begin{aligned} L_G\left(x,Q^2\right) &\approx -\frac{2}{1+\alpha}\Delta G\left(x,Q^2\right) \\ L_{q+\bar{q}}\left(x,Q^2\right) &\approx -\frac{1}{1+\alpha}\Delta\Sigma\left(x,Q^2\right) \end{aligned}$$

* Previously found

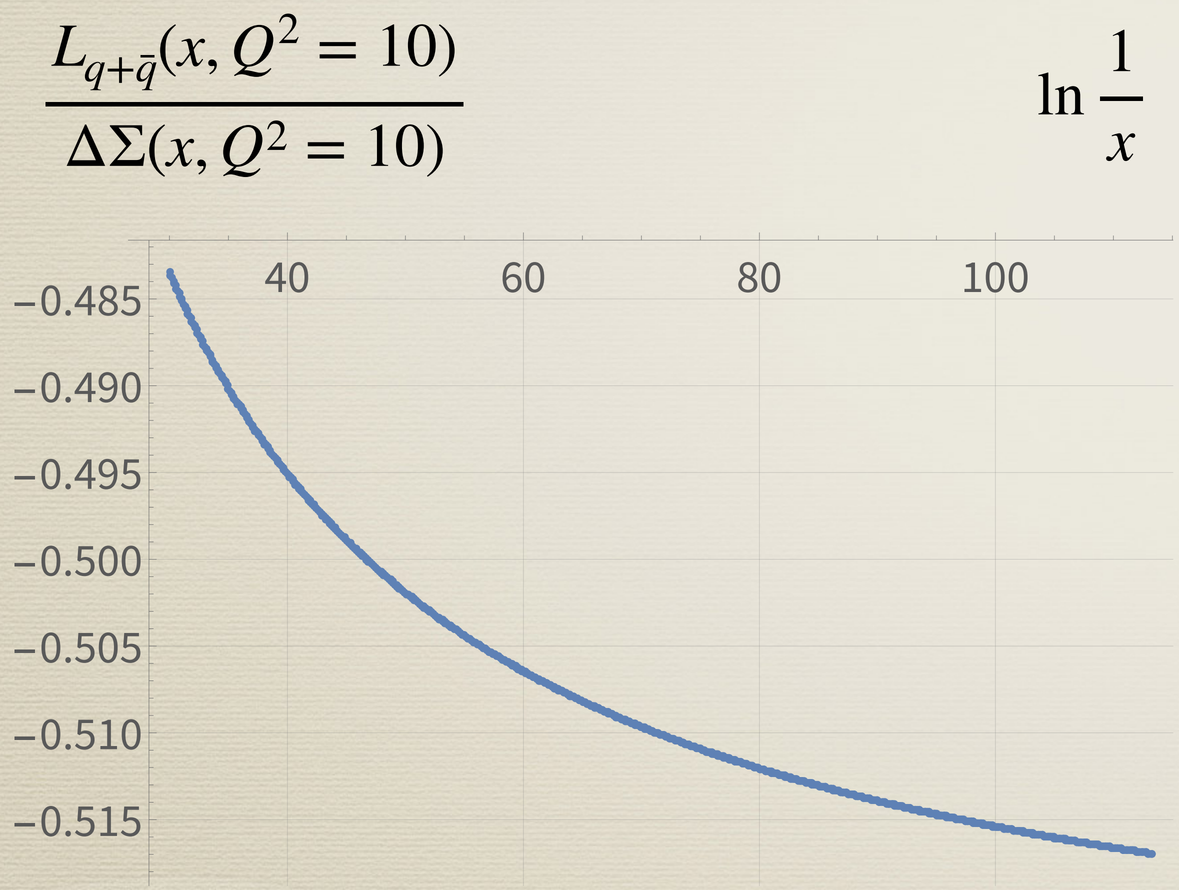
Kovchegov '19 (1901.07453)

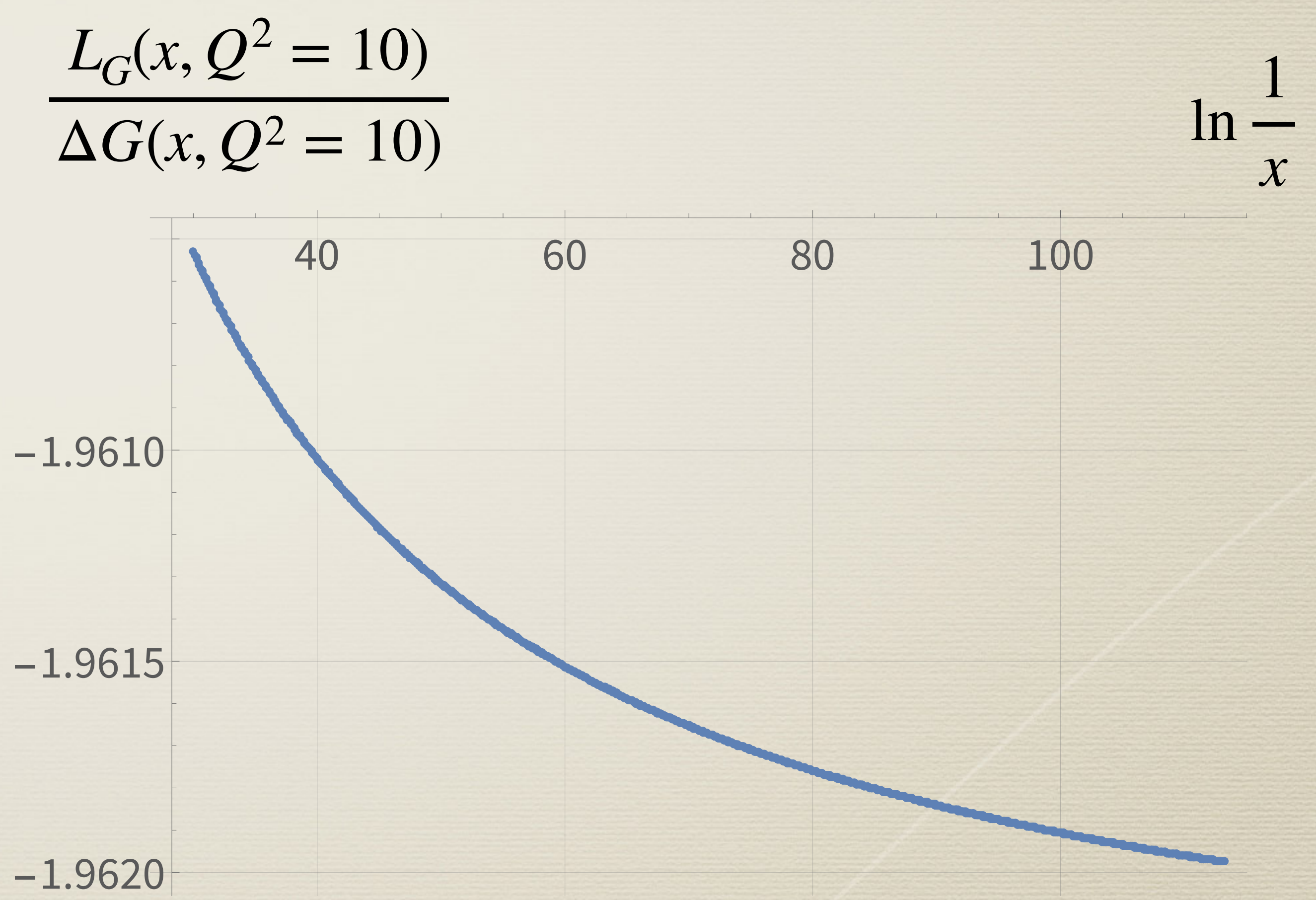
$$L_{q+\bar{q}}\left(x,Q^2\right) = -\Delta\Sigma\left(x,Q^2\right)$$

$$L_G\left(x,Q^2\right) = \left[\#\left(\sqrt{\alpha_s}\right)\ln\frac{Q^2}{\Lambda^2}\right]\Delta G\left(x,Q^2\right)$$

Numerical results: OAM/hPDF ratios

* Compute OAM/hPDF ratios numerically (e.g. for rapidity step size of 0.05, $Q^2 = 10\,\text{GeV}^2$)





PRELIMINARY

Numerical results: OAM/hPDF ratios

* Take numerical continuum limit (e.g. for $Q^2 = 10\,\mathrm{GeV}^2$)

PRELIMINARY

$$\frac{L_{q+\bar{q}}\,(x)}{\Delta\Sigma(x)} = -0.57 + \frac{1.3}{\ln(1/x)}$$

$$\frac{L_G\,(x)}{\Delta G(x)} = -1.96 + \frac{0.061}{\ln(1/x)}$$

* OAM and helicity of **equal importance** at small-*x*

* Rapidity dependence suppressed but likely relevant for current/near-future kinematics

Summary

* LCOT evolution technique works for OAM at small-x
* Requires introduction of new ***moment*** polarized dipole amplitudes
    * Derived **novel** large N_c evolution equations for moment amplitudes
* Determined **small-x asymptotics** of OAM distributions

$$L_{q+\bar{q}}(x, Q^2) \sim L_G(x, Q^2) \sim \Delta\Sigma(x, Q^2) \sim \Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

* Numerically calculated OAM/hPDF ratio and found OAM just as important as helicity at small-x and **rapidity dependence** of ratios

* Backup

Polarized Wilson lines

$$V_{\underline{x},\underline{y};\sigma,\sigma'} = \sigma\delta_{\sigma,\sigma'}V_{\underline{x}}^{\text{pol}\,[1]}\delta^2(\underline{x}-\underline{y}) + \delta_{\sigma,\sigma'}V_{\underline{x},\underline{y}}^{\text{pol}\,[2]}$$

$$V_{\underline{x}}^{\text{pol}\,[1]} = V_{\underline{x}}^{\text{G}\,[1]} + V_{\underline{x}}^{\text{q}\,[1]}$$

$$V_{\underline{x},\underline{y}}^{\text{pol}\,[2]} = \delta^2(\underline{x}-\underline{y})V_{\underline{x}}^{\text{q}\,[2]} + V_{\underline{x},\underline{y}}^{\text{G}\,[2]}$$

$$V_{\underline{x}}^{\text{G}[1]} = \frac{i\,g\,P^+}{s}\int\limits_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-]\, F^{12}(x^-, \underline{x})\; V_{\underline{x}}[x^-, -\infty],$$

$$V_{\underline{x}}^{\text{q}[1]} = \frac{g^2 P^+}{2\,s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\; \left[\gamma^+\gamma^5\right]_{\alpha\beta}\; \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty],$$

$$V_{\underline{x},\underline{y}}^{\text{G}[2]} = -\frac{i\,P^+}{s}\int\limits_{-\infty}^{\infty} dz^- d^2z\; V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x}-\underline{z})\, \overleftarrow{D}^i(z^-, \underline{z})\, D^i(z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y}-\underline{z}),$$

$$V_{\underline{x}}^{\text{q}[2]} = -\frac{g^2 P^+}{2\,s}\int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\; \left[\gamma^+\right]_{\alpha\beta}\; \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty].$$

$$V_{\underline{z}}^{i\,\text{G}[2]} \equiv \frac{P^+}{2s}\int\limits_{-\infty}^{\infty} dz^-\; V_{\underline{z}}[\infty, z^-]\; \left[D^i(z^-, \underline{z}) - \overleftarrow{D}^i(z^-, \underline{z})\right]\; V_{\underline{z}}[z^-, -\infty].$$

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

CGC Averaging

* Standard *polarization-dependent* CGC averaging

$$\left\langle \hat{\mathcal{O}}(b, r) \right\rangle = \frac{1}{2P^+} \int \frac{d^2\Delta \, d\Delta^+}{(2\pi)^3} \, e^{ib\cdot\Delta} \left\langle P + \frac{\Delta}{2} \right| \hat{\mathcal{O}}(0, r) \left| P - \frac{\Delta}{2} \right\rangle .$$

$$\left\langle \ldots \right\rangle = \frac{1}{2} \sum_{S_L} S_L \frac{1}{2P^+V^-} \langle P, S_L | \ldots | P, S_L \rangle$$

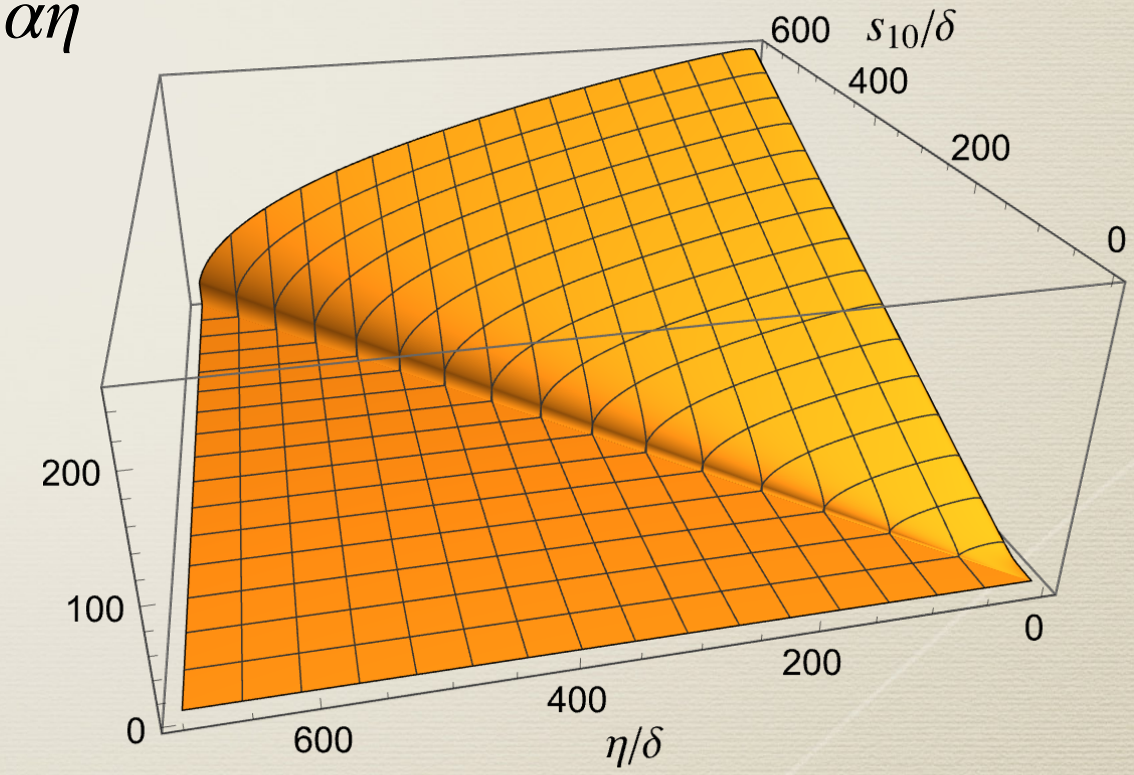
Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Moment amplitude asymptotics

* Define logarithmic coordinates $\eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{zs}{\Lambda^2}$ $\quad s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2}$
* Moment amplitudes are exponential in η at large η
  * Enables ansatz: $I(0,\eta) \propto e^{\alpha\eta}$
* Can extract α numerically

  find it agrees w/ helicity

  $\alpha_h = 3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}$

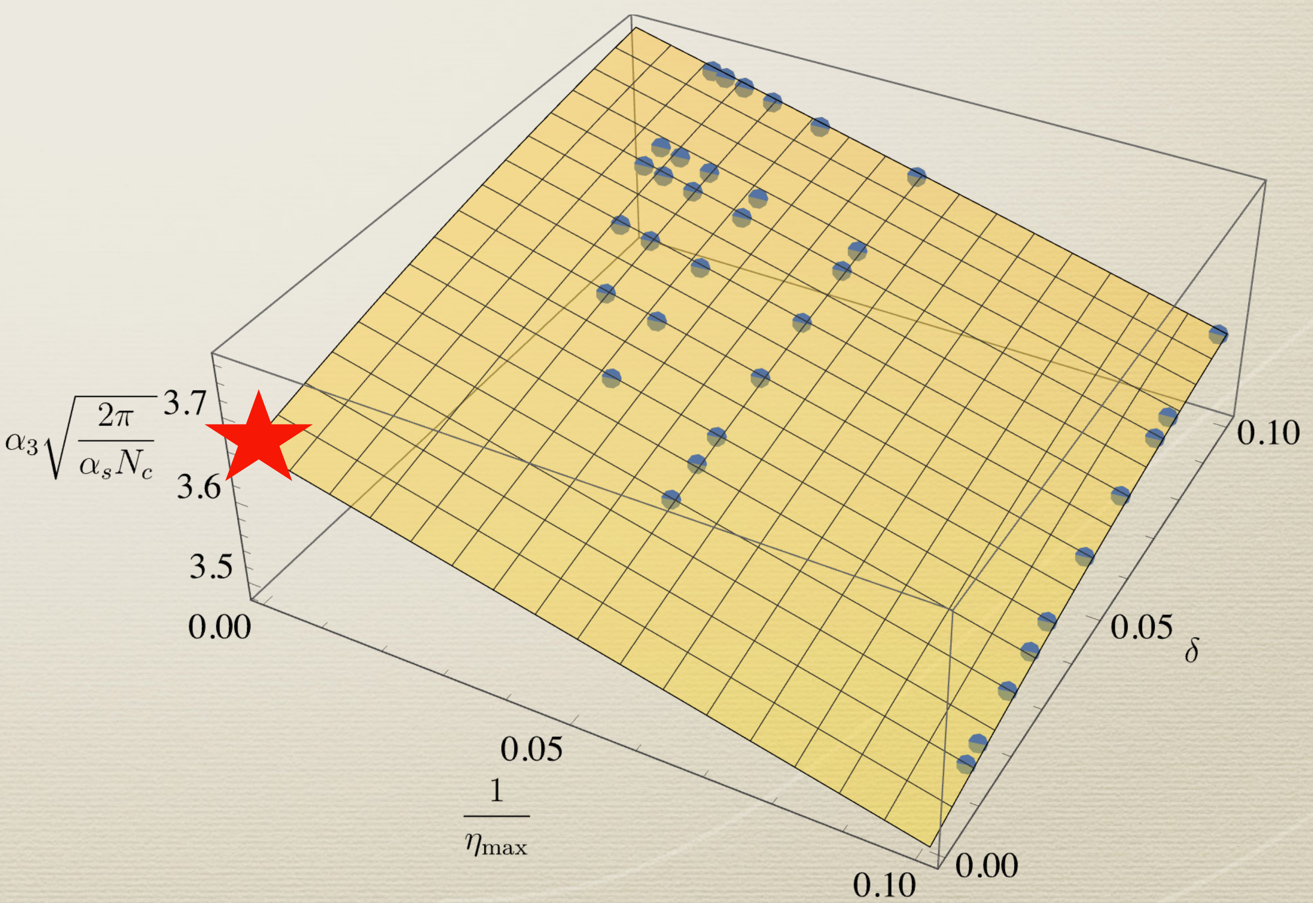


Numerical continuum limit

* For a numerical value, can create surface in step size and maximum rapidity/energy evolved

* Using polynomials, can fit the surface to extrapolate to the continuum (no step size, infinite energy)

* e.g. for I_3 intercept



Numerical results: OAM/hPDF ratios

* Can numerically investigate OAM/hPDF ratios another way
* Ansätze for distribution functions (motivated by helicity solution)

$$\left| L_{q+\bar{q}}\left(y = \ln\frac{1}{x}, Q^2\right) \right| = \exp\left(\bar{\alpha} y + \bar{\beta} + \bar{\gamma}\ln y + \frac{\bar{\delta}}{y}\right)$$

$$\left| \Delta\Sigma\left(y = \ln\frac{1}{x}, Q^2\right) \right| = \exp\left(\alpha y + \beta + \gamma\ln y + \frac{\delta}{y}\right)$$

* We see $\alpha = \bar{\alpha},\ \gamma = \bar{\gamma}$

$$\Rightarrow \quad \left| \frac{L_{q+\bar{q}}(x, Q^2)}{\Delta\Sigma(x, Q^2)} \right| = \exp(\bar{\beta} - \beta)\left(1 + \frac{\bar{\delta} - \delta}{y}\right)$$