Can we measure Double DVCS at JLab and the EIC?

Víctor Martínez-Fernández

PhD student at the National Centre for Nuclear Research
(NCBJ, Warsaw, Poland)

Work in collaboration with:

Katarzyna Deja (NCBJ)
Bernard Pire (CPHT)
Paweł Sznajder (NCBJ)
Jakub Wagner (NCBJ)

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Outline

- Starting point: **GPD**
- Double deeply virtual Compton scattering **(DDVCS)**
  - Goal & motivation
  - Gauge invariance at LT
  - Formulation *à la* Kleiss & Stirling (KS)
  - Observables and MC simulations
- Summary and conclusions

Partonic distribution

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the fraction of the hadron's longitudinal momentum carried by a quark:

$$\mathrm{GPD}_f(x,\xi,t)=\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle N'|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|N\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2,\quad \xi=-\frac{\bar{q}\Delta}{2\bar{p}\bar{q}},\quad \rho=\frac{-\bar{q}^2}{2\bar{p}\bar{q}},\quad \bar{q}=\frac{q+q'}{2},\quad \bar{p}=\frac{p+p'}{2}$$

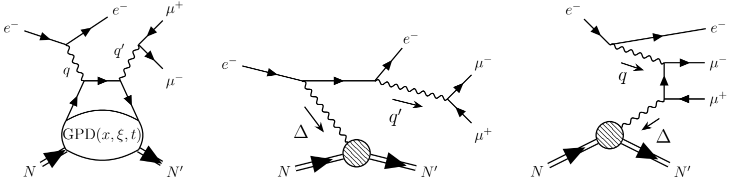
Importance

- Connected to QCD energy-momentum tensor, and so to spin. GPDs are a way to address the hadron's **spin puzzle**
- **Tomography**: distribution of quarks in terms of the longitudinal momentum and in the transverse plane

$$q(x,\mathbf{b}_\perp)=\int \frac{d^2\mathbf{\Delta}}{4\pi^2}\ e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}} H^q(x,0,t=-\mathbf{\Delta}^2)$$

Our goal

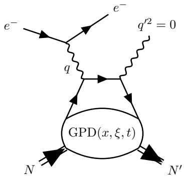
- **Goal:** phenomenology for JLab12, JLab20+ and EIC
- **What is DDVCS?** Subprocess in the electroproduction of a lepton pair



(from left to right) DDVCS, BH1, BH2. Complementary crossed-diagrams are not shown

Why DDVCS?

- **Problem:** currently, GPDs are accessible experimentally in processes such as deeply virtual (DVCS) and timelike Compton scattering (TCS), but the LO amplitudes are restricted to the line $x = \pm\xi$



DVCS

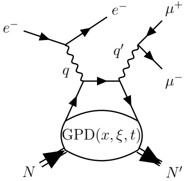
- GPDs enter amplitude at LO via CFF:

$$\mathrm{CFF}_{\mathrm{DVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\xi)\mathrm{GPD}(x,\xi,t) + \cdots$$

Similarly for TCS with $\xi \to -\xi$

Why DDVCS?

- **Solution by DDVCS:** the extra virtuality allows for the introduction of a new (generalized) Björken variable ρ so that we can access GPDs for $x = \rho \neq \xi$



Double DVCS (DDVCS)

- GPDs enter amplitude at LO via CFF:

$$\mathrm{CFF}_{\mathrm{DDVCS}} \sim \mathrm{PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho} \mathrm{GPD}(x,\xi,t)\right) - \int_{-1}^{1} dx \;\; i\pi\delta(x-\rho)\mathrm{GPD}(x,\xi,t) + \cdots$$

$$\rho = -\frac{\bar{q}^2}{2\bar{p}\bar{q}}\,, \quad \xi = \frac{-\bar{q}\Delta}{2\bar{p}\bar{q}}$$

Original papers in DDVCS: Belitsky & Muller, PRL 90, 022001 (2003); Guidal & Vanderhaeghen, PRL 90, 012001 (2003); Belitsky & Muller, PRD 68, 116005 (2003)

DDVCS subprocess

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\mathrm{DDVCS}}=\frac{ie^4\bar{u}(\ell_-,s_\ell)\gamma_\mu v(\ell_+,s_\ell)\bar{u}(k',s)\gamma_\nu u(k,s)}{(q^2+i0)(q'^2+i0)}T^{\mu\nu}_{s_2s_1}$$

- Compton tensor decomposition at LT:

$$T^{\mu\nu}_{s_2s_1}=-\frac{1}{2}g^{\mu\nu}_\perp\bar{u}(p',s_2)\left[(\mathcal{H}+\mathcal{E})\not{n}-\frac{\mathcal{E}}{M}\bar{p}^+\right]u(p,s_1)-\frac{i}{2}\epsilon^{\mu\nu}_\perp\bar{u}(p',s_2)\left[\widetilde{\mathcal{H}}\not{n}+\frac{\widetilde{\mathcal{E}}}{2M}\Delta^+\right]\gamma^5u(p,s_1)$$

- Longitudinal plane is built with $\{\bar{q},\bar{p}\}$
- $q^\nu_\perp\sim\Delta^\nu_\perp\Rightarrow g^{\mu\nu}_\perp q_\nu\neq 0\Rightarrow$ EM gauge-violation. Solution:

$$q^\nu_\perp|_{t=t_0}\sim\Delta^\nu_\perp|_{t=t_0}=0$$

- This procedure is consistent with the longitudinal factorization which is at the core of the GPD description

- In the view of new experiments, revisiting DDVCS is timely: **PRD 107 (2023), no. 9, 094035 (our work)**
- Rederivation of DDVCS' formulae via Kleiss-Stirling's methods:
  - **Amplitudes as complex-numbers**
  - 2 scalars as building blocks, a and b as light-like vectors:

$$s(a,b) = \bar{u}(a,+)u(b,-) = -s(b,a)$$

$$t(a,b) = \bar{u}(a,-)u(b,+) = [s(b,a)]^*$$

$$s(a,b) = (a^2 + ia^3)\sqrt{\frac{b^0 - b^1}{a^0 - a^1}} - (a \leftrightarrow b)$$

Kleiss & Stirling, Nuclear Physics B262 (1985) 235-262

DDVCS subprocess *à la* Kleiss-Stirling

- DDVCS subprocess amplitude:

$$i\mathcal{M}_{\mathrm{DDVCS}}=\frac{-ie^4}{(Q^2-i0)(Q'^2+i0)}\left(i\mathcal{M}^{(V)}_{\mathrm{DDVCS}}+i\mathcal{M}^{(A)}_{\mathrm{DDVCS}}\right)$$

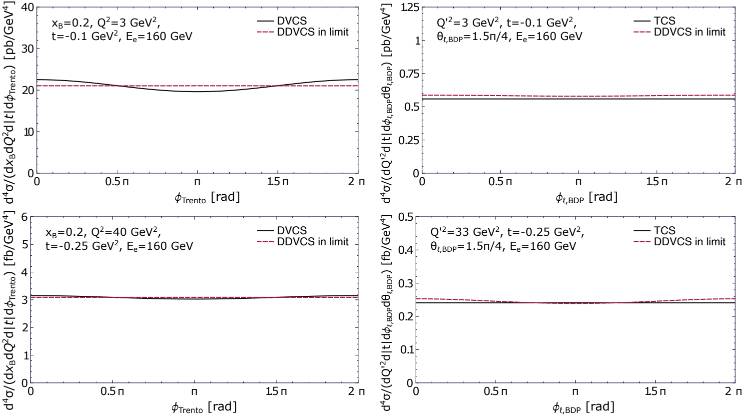
- Vector contribution:

$$i\mathcal{M}^{(V)}_{\mathrm{DDVCS}}=-\frac{1}{2}\left[f(s_\ell,\ell_-,\ell_+;s,k',k)-g(s_\ell,\ell_-,n^\star,\ell_+)g(s,k',n,k)-g(s_\ell,\ell_-,n,\ell_+)g(s,k',n^\star,k)\right]$$
$$\times\left[(\mathcal{H}+\mathcal{E})[Y_{s_2s_1}g(+,r'_{s_2},n,r_{s_1})+Z_{s_2s_1}g(-,r'_{-s_2},n,r_{-s_1})]-\frac{\mathcal{E}}{M}\mathcal{J}^{(2)}_{s_2s_1}\right]$$

- Axial contribution:

$$i\mathcal{M}^{(A)}_{\mathrm{DDVCS}}=\frac{-i}{2}\epsilon^{\mu\nu}_{\perp}j_\mu(s_\ell,\ell_-,\ell_+)j_\nu(s,k',k)\left[\widetilde{\mathcal{H}}\mathcal{J}^{(1,5)+}_{s_2s_1}+\widetilde{\mathcal{E}}\frac{\Delta^+}{2M}\mathcal{J}^{(2,5)+}_{s_2s_1}\right]$$

DVCS & TCS limits of DDVCS



Comparison of DDVCS and (left) DVCS and (right) TCS cross-sections for pure VCS subprocess. **GK model for GPDs.**

Trento: PRD 70, 117504 (2004); **BDP:** EPJC23, 675 (2002)

Observables: cross-section

- For unpolarized beam and target:

$$\sigma_{UU}(\phi_{\ell,\mathrm{BDP}}) = \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\mathrm{BDP}} \;\; \sin\theta_{\ell,\mathrm{BDP}}$$

$$\times \left(\frac{d^7\sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}} + \frac{d^7\sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}} \right)$$

- Cosine components:

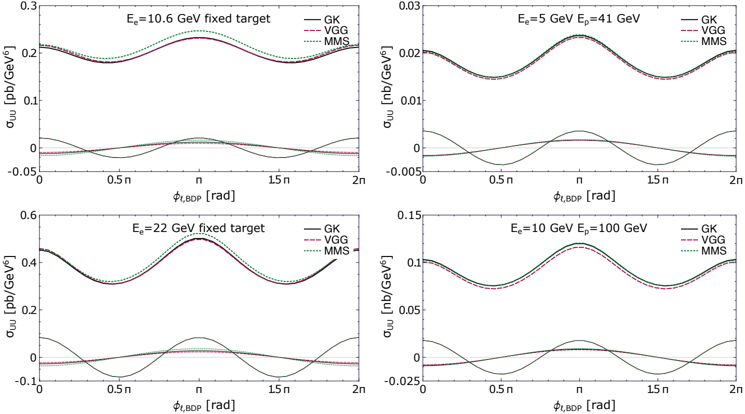
$$\sigma_{UU}^{\cos\left(n\phi_{\ell,\mathrm{BDP}}\right)}(\phi_{\ell,\mathrm{BDP}}) = M_{UU}^{\cos\left(n\phi_{\ell,\mathrm{BDP}}\right)} \cos\left(n\phi_{\ell,\mathrm{BDP}}\right)$$

- Cosine moments:

$$M_{UU}^{\cos\left(n\phi_{\ell,\mathrm{BDP}}\right)} = \frac{1}{N} \int_0^{2\pi} d\phi_{\ell,\mathrm{BDP}} \cos\left(n\phi_{\ell,\mathrm{BDP}}\right) \sigma_{UU}(\phi_{\ell,\mathrm{BDP}})$$

$N = 2\pi$ for $n = 0$, $N = \pi$ for $n > 0$.

Observables: cross-section



JLab12, JLab20+ EIC 5x41, EIC 10x100

Experiment	Beam energies [GeV]	y	$\|t\|$ [GeV^2]	Q^2 [GeV^2]	Q'^2 [GeV^2]
JLab12	$E_e = 10.6,\ E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22,\ E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5,\ E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10,\ E_p = 100$	0.15	0.1	0.6	2.5

Observables: beam-spin asymmetry

- Single beam-spin asymmetry for longitudinally polarized electrons:

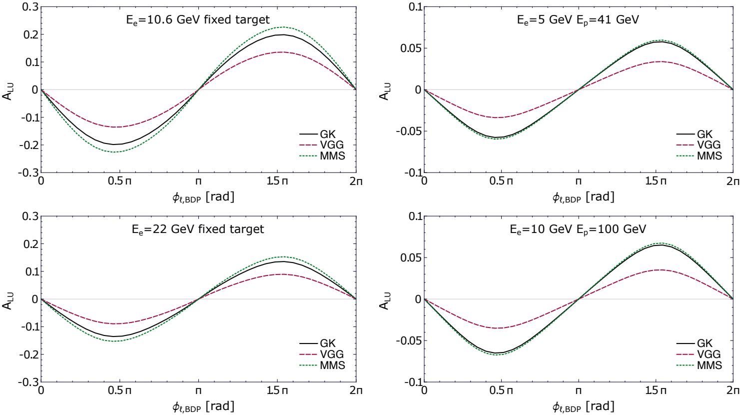
$$A_{LU}(\phi_{\ell,\mathrm{BDP}})=\frac{\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}})}{\sigma_{UU}(\phi_{\ell,\mathrm{BDP}})}$$

$$\Delta\sigma_{LU}(\phi_{\ell,\mathrm{BDP}})=\int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} d\theta_{\ell,\mathrm{BDP}} \; \sin\theta_{\ell,\mathrm{BDP}}$$

$$\times\left(\frac{d^7\sigma^{\rightarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}}-\frac{d^7\sigma^{\leftarrow}}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_{\ell,\mathrm{BDP}}}\right)$$

- We consider $Q'^2 > Q^2$: our DDVCS is “more” timelike than spacelike

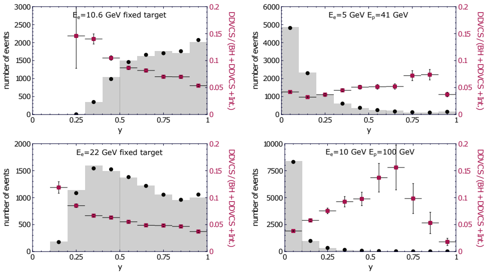
Observables: beam-spin asymmetry



JLab12, JLab20+: up to **15-20%**

EIC 5x41, EIC 10x100: **3-7%**

Experiment	Beam energies [GeV]	y	$\|t\|$ [GeV^2]	Q^2 [GeV^2]	Q'^2 [GeV^2]
JLab12	$E_e = 10.6,\ E_p = M$	0.5	0.2	0.6	2.5
JLab20+	$E_e = 22,\ E_p = M$	0.3	0.2	0.6	2.5
EIC	$E_e = 5,\ E_p = 41$	0.15	0.1	0.6	2.5
EIC	$E_e = 10,\ E_p = 100$	0.15	0.1	0.6	2.5



JLab12, JLab20+ EIC 5x41, EIC 10x100

EpIC MC
● integrated cross-section
pure DDVCS fraction

Kinematic cuts:
$Q^2 \in (0.15, 5)$ GeV^2
$Q'^2 \in (2.25, 9)$ GeV^2
JLab: $-t \in (0.1, 0.8)$ GeV^2
EIC: $-t \in (0.01, 1)$ GeV^2
$\phi, \phi_\ell \in (0.1, 2\pi - 0.1)$ rad
$\theta_\ell \in (\pi/4, 3\pi/4)$ rad
JLab: $y \in (0.1, 1)$
EIC: $y \in (0.05, 1)$

10000 events/distribution. Neither acceptance nor detectors response are taken into account in this study

Experiment	Beam energies [GeV]	Range of $\|t\|$ [GeV^2]	$\sigma\|_{0<y<1}$ [pb]	$\mathcal{L}^{10k}\|_{0<y<1}$ [fb^{-1}]	$y_{\min}$	$\sigma\|_{y_{\min}<y<1}/\sigma\|_{0<y<1}$
JLab12	$E_e = 10.6,\ E_p = M$	$(0.1, 0.8)$	0.14	70	0.1	1
JLab20+	$E_e = 22,\ E_p = M$	$(0.1, 0.8)$	0.46	22	0.1	1
EIC	$E_e = 5,\ E_p = 41$	$(0.05, 1)$	3.9	2.6	0.05	0.73
EIC	$E_e = 10,\ E_p = 100$	$(0.05, 1)$	4.7	2.1	0.05	0.32

Summary and conclusions

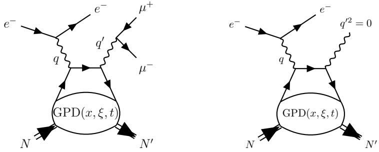
- New analytical formulae for the electroproduction of a lepton pair have been derived.
- It is already implemented in PARTONS and EpIC MC (LO + LT).
- Asymmetries are large enough for DDVCS to be measurable at both current (JLab12) and future (JLab20+, EIC) experiments.
- Addressing GPD model dependence with cross-sections and asymmetries is possible.

Thank you!

Complementary slides

DVCS limit of DDVCS

- **DDVCS to DVCS:**

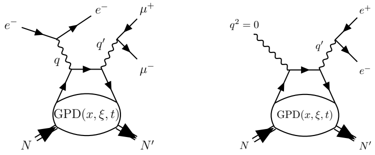


DDVCS (left), DVCS (right)

$$\int d\Omega_\ell \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q'^2\to 0} \underbrace{\left(\frac{d^4\sigma}{dx_B dQ^2 d|t| d\phi}\right)}_{\text{DVCS}} \frac{\mathcal{N}}{Q'^2}$$

$$\mathcal{N}=\alpha_{\rm em}/(3\pi)$$

- **DDVCS to TCS:**



DDVCS (left), TCS (right)

$$\int d\phi \underbrace{\frac{d^7\sigma}{dx_B dQ^2 dQ'^2 d|t| d\phi d\Omega_\ell}}_{\text{DDVCS}} \xrightarrow{Q^2\to 0} \underbrace{\left(\frac{d^4\sigma}{dQ'^2 d|t| d\Omega_\ell}\right)}_{\text{TCS}} \underbrace{\frac{d^2\Gamma}{dx_B dQ^2}}_{\text{EPA photon flux}}$$

$$\frac{d^2\Gamma}{dx_B dQ^2} = \frac{\alpha_{\text{em}}}{2\pi Q^2}\left(1+\frac{(1-y)^2}{y}-\frac{2(1-y)Q^2_{\text{min}}}{yQ^2}\right)\frac{\nu}{Ex_B} \leftarrow \text{EPA photon}-\text{flux}$$

DDVCS subprocess *à la* Kleiss-Stirling

- **f = contraction of 2 currents**

$$f(\lambda,k_0,k_1;\lambda',k_2,k_3)=\bar{u}(k_0,\lambda)\gamma^{\mu}u(k_1,\lambda)\bar{u}(k_2,\lambda')\gamma_{\mu}u(k_3,\lambda')=2[s(k_2,k_1)t(k_0,k_3)\delta_{\lambda-}\delta_{\lambda'+}$$

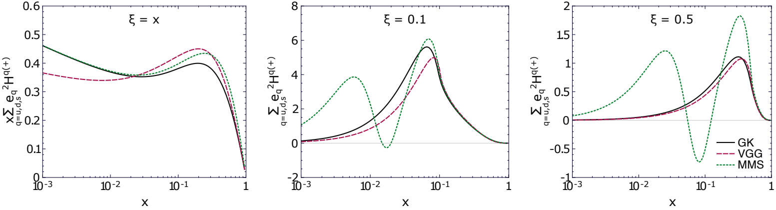
$$+t(k_2,k_1)s(k_0,k_3)\delta_{\lambda+}\delta_{\lambda'-}+s(k_2,k_0)t(k_1,k_3)\delta_{\lambda+}\delta_{\lambda'+}+t(k_2,k_0)s(k_1,k_3)\delta_{\lambda-}\delta_{\lambda'-}]$$

- **g = contraction of a current with a lightlike vector a**

$$g(s,\ell,a,k)=\bar{u}(\ell,s)\not{a}u(k,s)=\delta_{s+}s(\ell,a)t(a,k)+\delta_{s-}t(\ell,a)s(a,k)$$

- **BH diagrams can be treated in a similar manner**

Models for the C-even part of GPD H^q



Distributions of $\sum_q e_q^2 H^{q(+)}(x, \xi, t)$ at $t = -0.1\ \mathrm{GeV}^2$, where $q = u, d, s$ flavours for (left) $\xi = x$, (middle) $\xi = 0.1$ and (right) $\xi = 0.5$. The solid black, dashed red and dotted green curves describe the GK, VGG and MMS GPD models, respectively. The C-even part of a given vector GPD is defined as:
$H^{q(+)}(x, \xi, t) = H^q(x, \xi, t) - H^q(-x, \xi, t)$. The scale is chosen as $\mu_F^2 = 4\ \mathrm{GeV}^2$.