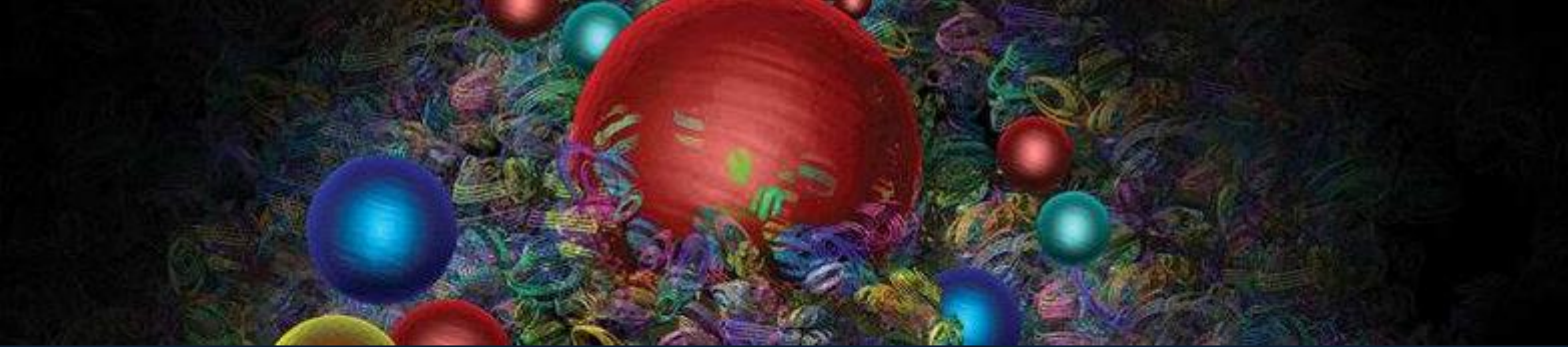

Threshold heavy quarkonium production and GPDs at large skewness

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In collaboration with Xiangdong Ji, Yizhuang Liu, Jinghong Yang, and Feng Yuan

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BERKELEY LAB

Outline

- Brief review of threshold heavy vector meson production

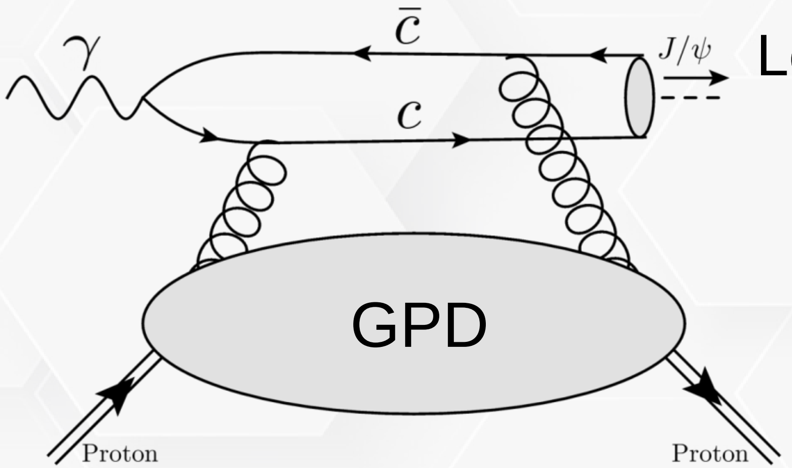
Z.-E. Meziani's Talk

L. Pentchev's Talk

- More about GPDs at large skewness
- Some phenomenology with J/psi production
- Summary and outlook

J/psi photoproduction near the threshold

Exclusive heavy vector meson, e.g., J/psi productions naturally probe the gluon GPD.



Leading order factorization with GPDs near the threshold

- **Same amplitude as the collinear case;**
- **Different (threshold) kinematics from the collinear case;**
- **Large momentum transfer/skewness in the heavy quark limit;**

Y. Guo et. al. Phys. Rev. D 103 9, 096010 (2021)

$$\frac{d\sigma}{dt} \propto \left[\left(1-\xi^2\right) \left|\mathcal{H}_{gC}\right|^2 - 2\xi^2 \mathrm{Re}\left[\mathcal{H}_{gC}^* \mathcal{E}_{gC}\right] - \left(\xi^2 + \frac{t}{4M_p^2}\right) \left|\mathcal{E}_{gC}\right|^2 \right] ,$$

In terms of the gluonic Compton-like amplitudes or gluonic Compton form factors (gCFFs)

Systematical corrections

Higher order corrections:

- Higher order in the strong coupling;
- Factorization at higher (all) order;
- Heavy meson (NRQCD) correction/Higher twist;
- …

Large skewness expansion:

$$\mathcal{H}_{gC}(\xi,t)=\frac{1}{2\xi}\int_{-1}^{1}\mathrm{d}x\left(\frac{1}{x+\xi-i\epsilon}-\frac{1}{x-\xi+i\epsilon}\right)H_g(x,\xi,t)$$

$$\Updownarrow$$

$$A^g(t)+(2\xi)^2C^g(t)=\int_0^1 dx H_g(x,\xi,t)$$

What could be the problem

The original arguments were based on Taylor expansion:

$$\mathrm{Re}\left[\frac{1}{2\xi}\left(\frac{1}{x+\xi-i\epsilon}-\frac{1}{x-\xi+i\epsilon}\right)\right]=\sum_{n=0}^{\infty}\frac{x^{2n}}{\xi^{2+2n}}$$

Take the leading term so that GFFs will be obtained with the x integral:

$$\mathrm{Re}\mathcal{H}_{gC}(\xi,t)\overset{?}{\approx}\frac{1}{\xi^2}\int_{-1}^{1}\mathrm{d}x H_g(x,\xi,t)$$

The Taylor series in principle diverge in the PDF region $x>\xi$

Instead of having higher-moment contamination,

the approximation might not hold!

We live in a (theoretically) divergent world

Don't panic, QFT is a theory about divergence, more or less.

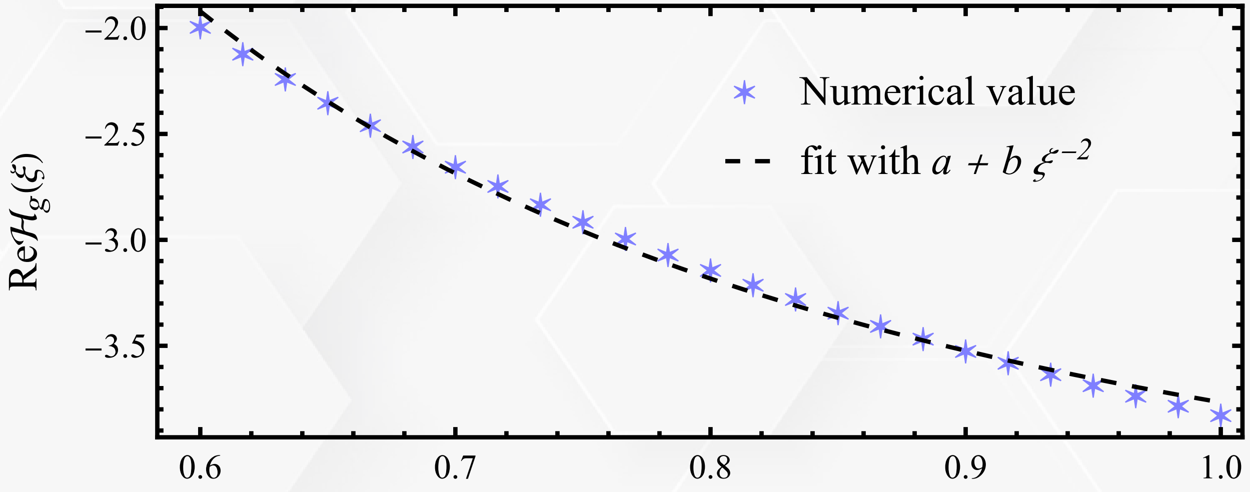
$$\mathrm{Re}\left[\frac{1}{2\xi}\left(\frac{1}{x+\xi-i\epsilon}-\frac{1}{x-\xi+i\epsilon}\right)\right]=\sum_{n=0}^{\infty}\frac{x^{2n}}{\xi^{2+2n}}$$

Plug all (not just the first) of them back, we get

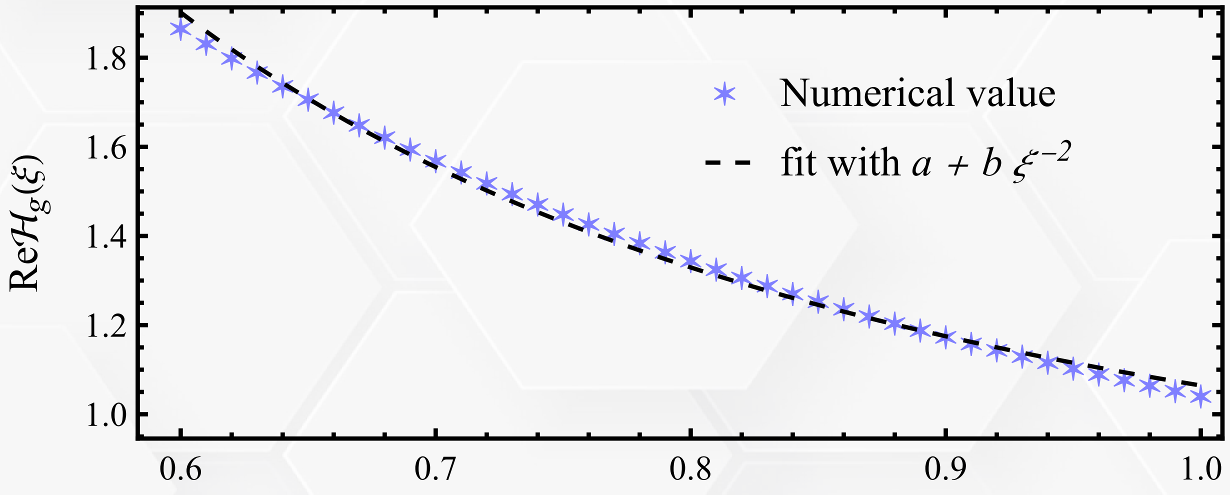
$$\begin{aligned}\mathrm{Re}\mathcal{H}_{g\mathrm{C}}(\xi,t)&=\mathcal{C}_g(t)+\sum_{n=1}^{\infty}\xi^{-2n}\mathcal{A}_g^{(2n)}(t)\\&\stackrel{?}{\approx}\mathcal{C}_g(t)+\xi^{-2}\mathcal{A}_g^{(2)}(t)\end{aligned}$$

$$\mathcal{A}_g^{(2n)}(t)\equiv\sum_{k=0}^{\infty}2^{2k+1}A_g^{(2k+2n,2k)}(t)\ ,\qquad \mathcal{C}_g(t)\equiv\sum_{k=0}^{\infty}2^{2k+3}C_g^{(2k+2)}(t)\ ,$$

Skewness-scaling in the CFFs

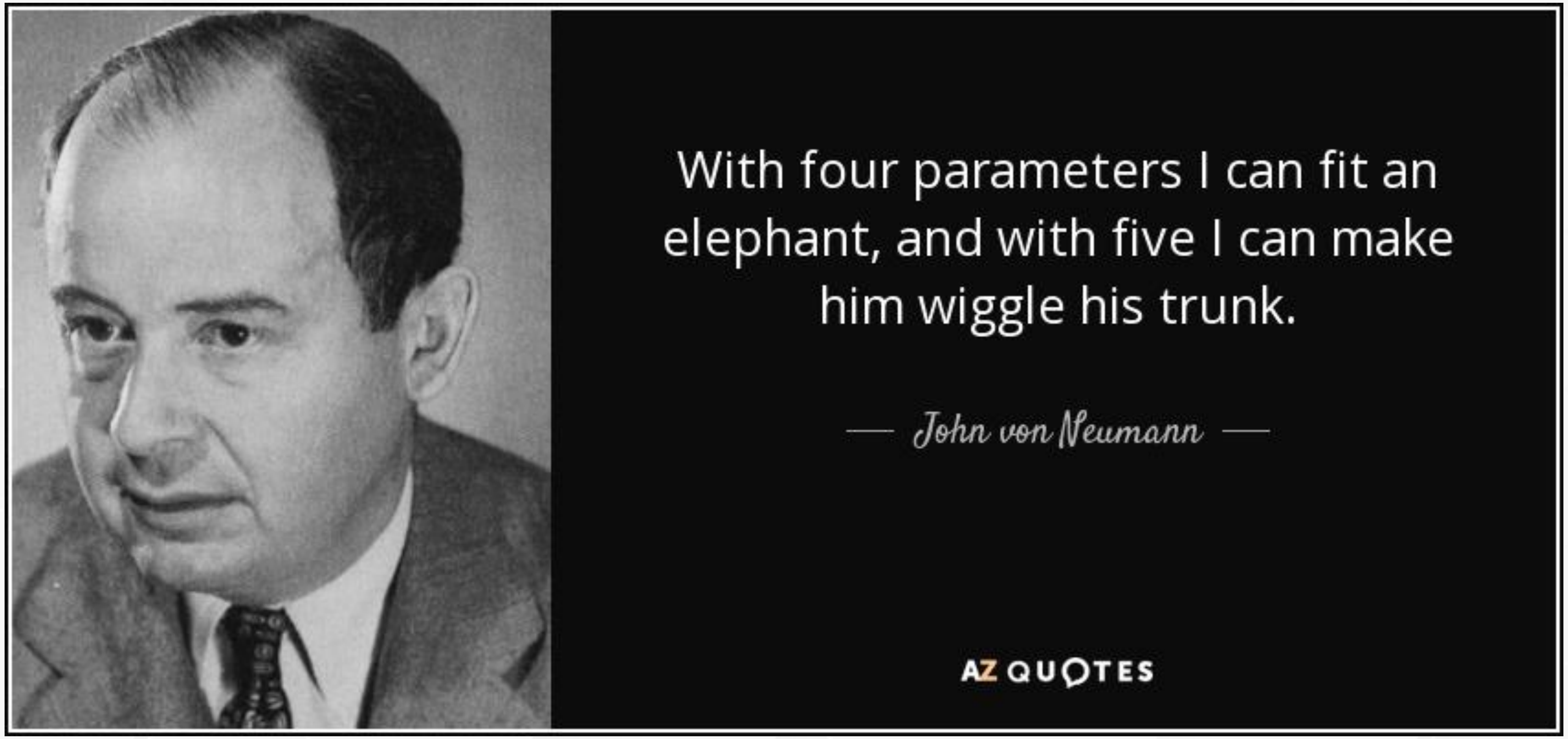


Double distribution model



Conformal moment model

Fit is meaningless if not physical



Extraction of leading moments

Those coefficients should be given by the input gluon GPD

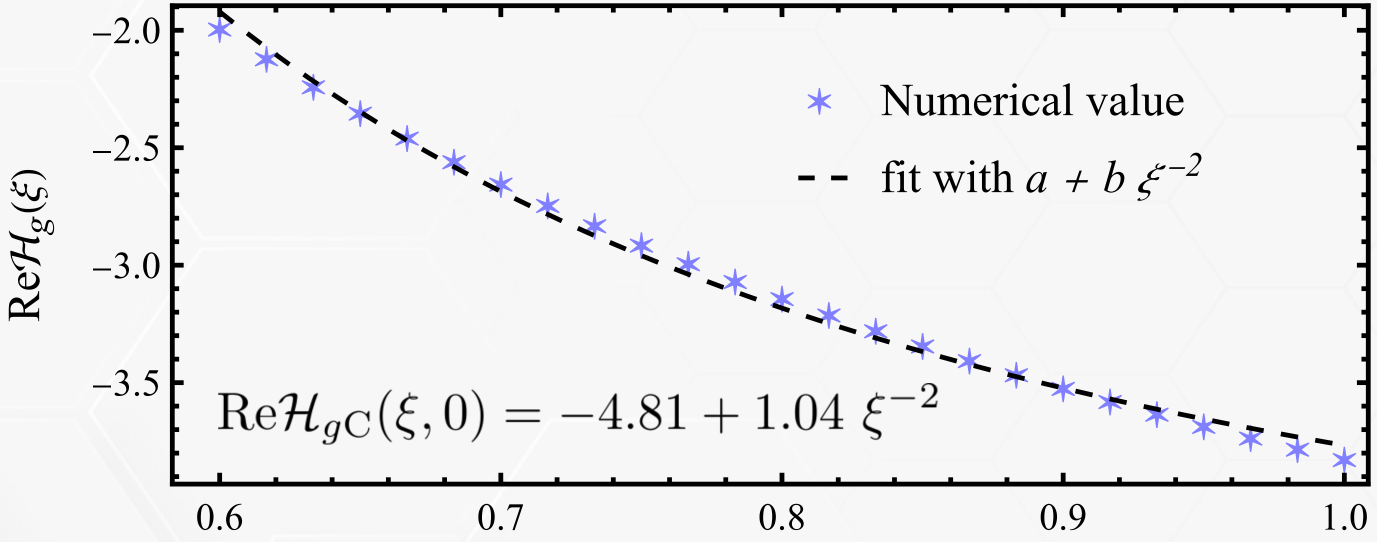
$$\mathrm{Re}\mathcal{H}_{g\mathrm{C}}(\xi,t) \approx \mathcal{C}_g(t) + \xi^{-2}\mathcal{A}_g^{(2)}(t) \qquad \mathcal{A}_g^{(2)}(t) \approx 2A_g(t)\ , \mathcal{C}_g(t) \approx 8C_g(t)\ ,$$

The input gluon GPD has

$$A_g(0) \approx 0.385\ , C_g(0) \approx -0.48$$

The extracted moments are about 25% larger than input:

$$\frac{1}{2}\mathcal{A}_g^{(2)}(0) \approx 0.52\ , \frac{1}{8}\mathcal{C}_g(0) \approx -0.60$$



The 25% excess in the extraction is from higher-moment contamination.

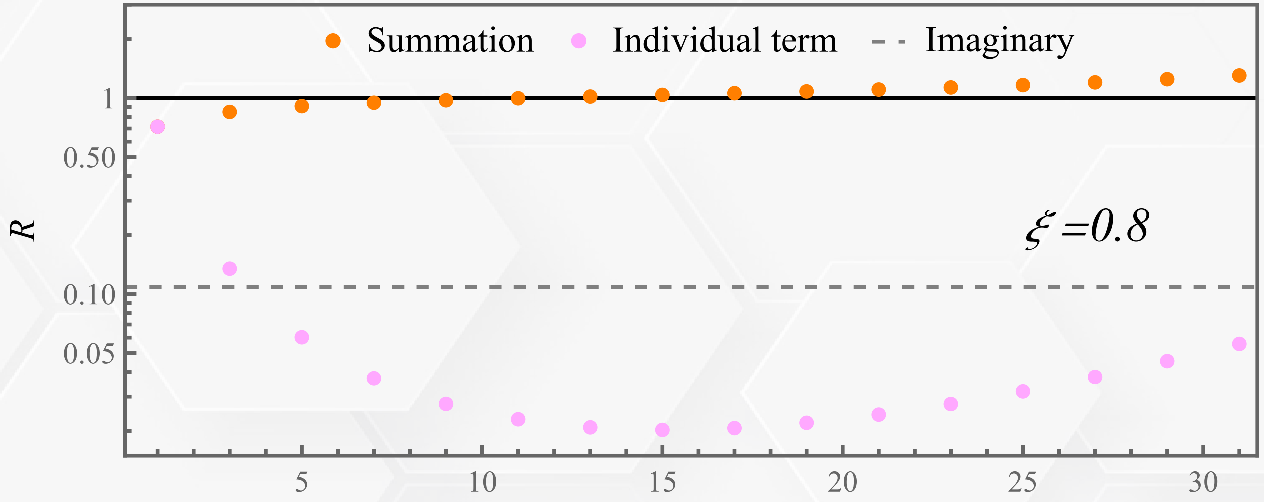
Two more comments

The same 25% excess is observed with the other GPD model

- ➢ Some magic of conformal moment ?

$$\mathcal{A}_j^{\text{conf}} = \frac{2^{j+2}\Gamma\left(5/2+j\right)}{\Gamma\left(3/2\right)\Gamma\left(4+j\right)} \qquad \mathcal{A}_1^{\text{conf}} = \frac{5}{4}$$

The series expansion corresponds to an asymptotic expansion.

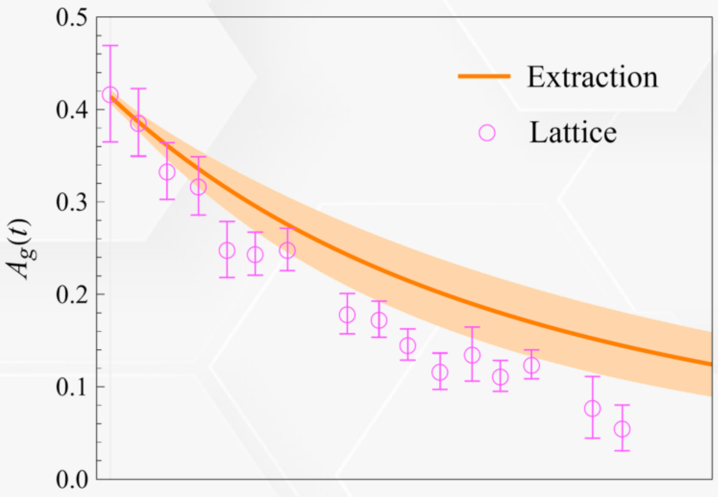


Gravitational form factor (GFF) extraction

In the large-skewness approximation we have

$$\mathcal{H}_{g\mathrm{C}}(\xi,t) \approx \mathcal{C}_g(t) + \xi^{-2}\mathcal{A}_g^{(2)}(t) \qquad \mathcal{E}_{g\mathrm{C}}(\xi,t) \approx -\mathcal{C}_g(t) + \xi^{-2}\mathcal{B}_g^{(2)}(t)$$

This implies the requirement of both large skewness AND higher-moment suppression.



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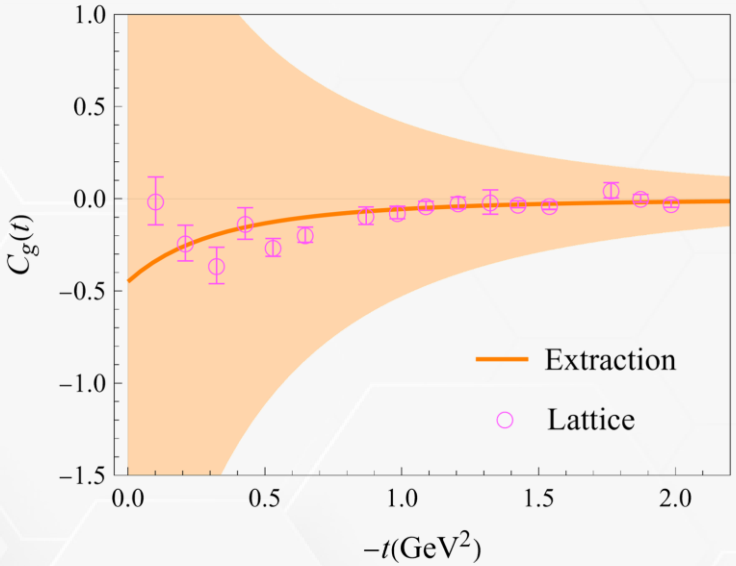

Could consider adding extra normalization factor, accounting for the conformal ratio of 5/4 or other higher-order effects.

Gravitational form factor (GFF) extraction

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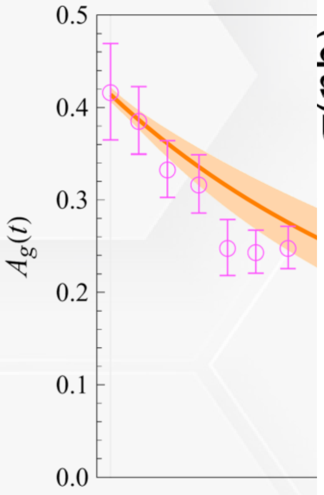

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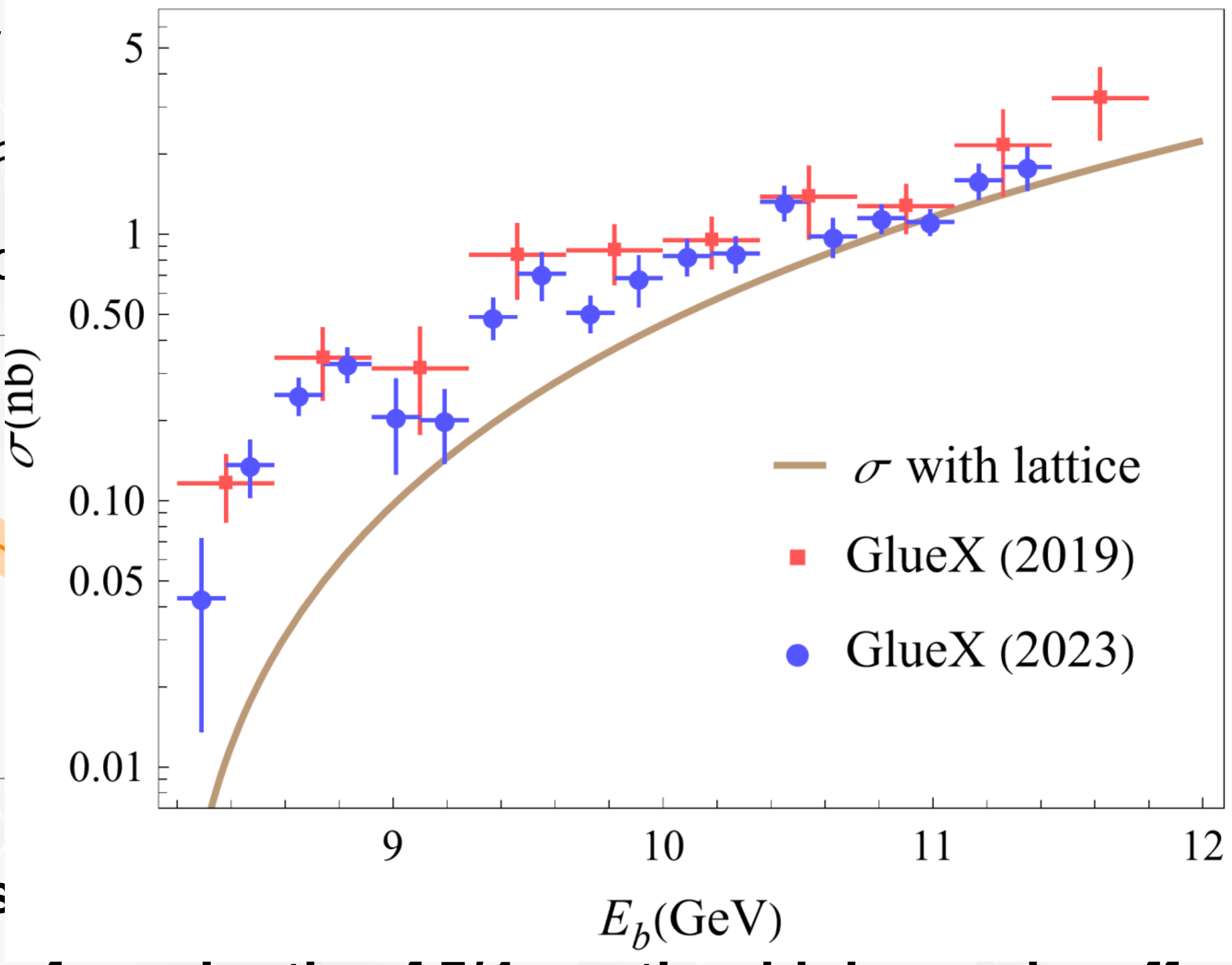
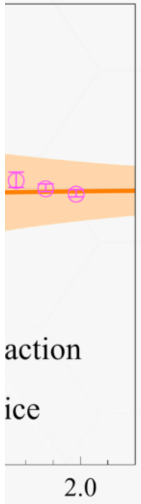




Could cons … nting for the conformal ratio of 5/4 or other higher-order effects.

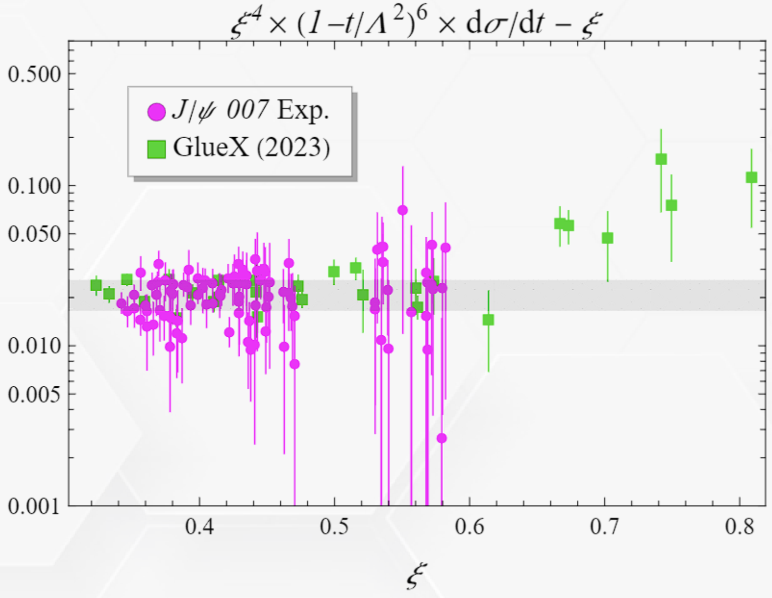
Utilizing the skewness dependence

How to test the large-skewness framework and quality of the extraction?

$$|G(\xi,t)|^2 = \left[(1-\xi^2)\,|\mathcal{H}_{gC}|^2 - 2\xi^2\mathrm{Re}\left[\mathcal{H}_{gC}^*\mathcal{E}_{gC}\right] - \left(\xi^2+\frac{t}{4M_p^2}\right)|\mathcal{E}_{gC}|^2\right]$$
$$= \xi^{-4}\left[G_0(t)+\xi^2G_2(t)+\xi^4G_4(t)\right]+\cdots$$

The xi-scaling of the cross-section has non-trivial behaviors at large xi.

- **The overall xi-scaling are consistent with** ξ^{-4}
- **Extra xi-scaling crucial for the separation of A and C form factors**



What can we do with CFF

Compton-like amplitudes basically measure GPDs at x=xi

- Imaginary parts given by GPDs at x=xi

$$\mathrm{Im}\mathcal{H}_{g\mathrm{C}}(\xi,t)=\frac{\pi}{\xi}H_g(\xi,\xi,t)$$

- Real parts given by dispersion relation

$$\mathrm{Re}\mathcal{H}_{g\mathrm{C}}(\xi,t)=\frac{1}{\pi}\int_0^{\xi_{\mathrm{th}}}\mathrm{d}\xi'\frac{2\xi'\mathrm{Im}\mathcal{H}_{g\mathrm{C}}(\xi,t)}{(\xi-\xi')(\xi+\xi')}+\mathcal{C}_g(t)$$

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- Real parts can be formally written as

$$\mathrm{Re}\mathcal{H}_{g\mathrm{C}}(\xi,t)=\mathcal{C}_g(t)+\sum_{n=1}^{\infty}\xi^{-2n}\mathcal{A}_g^{(2n)}(t)$$

- Imaginary parts --- inverse Mellin transform:

$$\mathrm{Im}\mathcal{H}_{g\mathrm{C}}(\xi,t)\propto\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\xi^{-s}\mathcal{A}_g^{(s+2)}(t)\mathrm{d}s$$

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The two statements are mathematically equivalent!

It's the applicable region (near forward/ large skewness) that distinguishes them

What can we do with CFF

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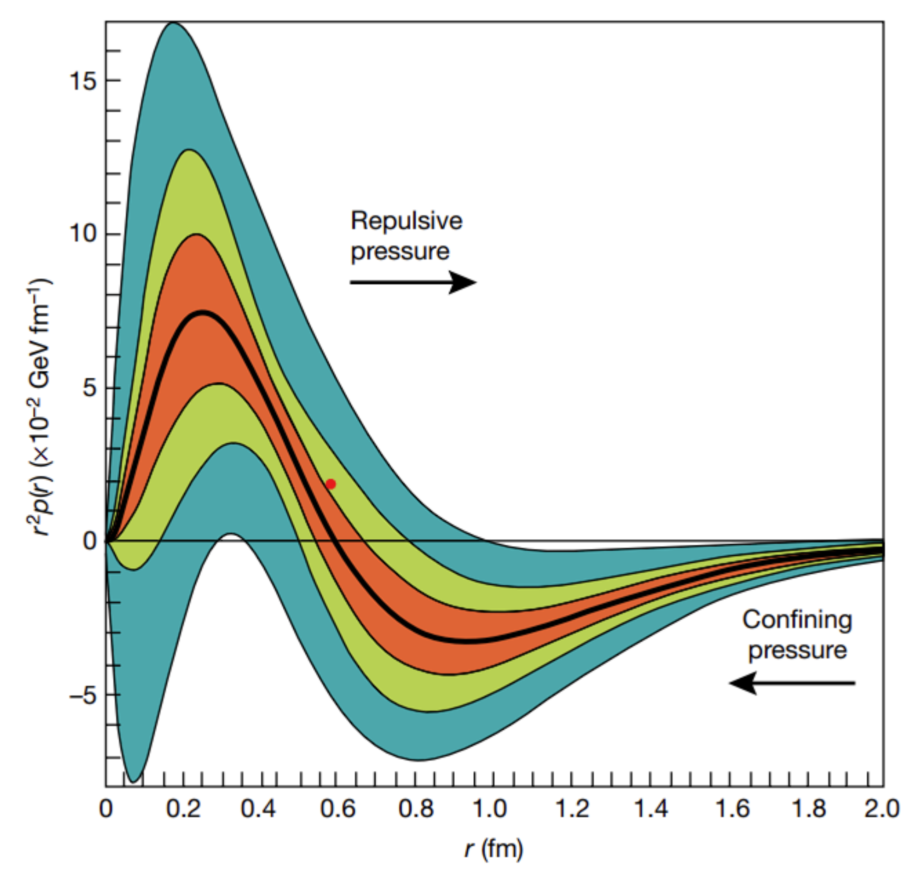

V. D. Burkert et. al., Nature 557 7705, 396-399 (2018)

Summary and outlook

Summary

- Large skewness behavior of gluonic GPD/Compton-like amplitude
- Application to heavy vector meson production
- Potential extraction of gluonic GFFs

Outlook

- Next-to-leading order effects
- Relations to the other frameworks
- Extension to quark GPDs

Thank you!