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Gravitational form factors of nuclei in the Skyrme model

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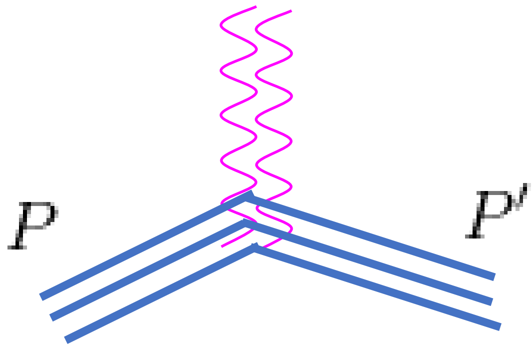
with Alberto Martin-Caro and Miguel Huidobro, arXiv:2304.05994 (PRD)

Nucleon gravitational form factors

Off-forward matrix element of the QCD energy momentum tensor



$$T^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{\eta^{\mu\nu}}{4}F^{\alpha\beta}F_{\alpha\beta} + \bar{\psi}i\gamma^{(\mu}D^{\nu)}\psi$$

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\left[A(t)\gamma^{(\mu}\bar{P}^{\nu)} + B(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_{\alpha}}{2M} + D(t)\frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^{2}}{4M}\right]u(P)$$

Form factors associated with scattering off a graviton

Energy momentum tensor carries information about mechanical properties (mass, spin, shear, pressure…)

Aligned with the core missions of the EIC. Already a lot of activities at Jlab.

D-term: the last global unknown

$D(0)$ is a fundamental constant of the proton!

The value, even the sign, is unknown at the moment.

Spatial components of the energy momentum tensor
→ May be interpreted as internal `force' exerted by quarks and gluons

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\, p(r) \qquad D = M\int d^3r r^2 p(r)$$

Conjecture: Stable systems must have a negative D-term $D(t=0) < 0$

D-term of atomic nuclei

Liquid drop model Polyakov (2003)

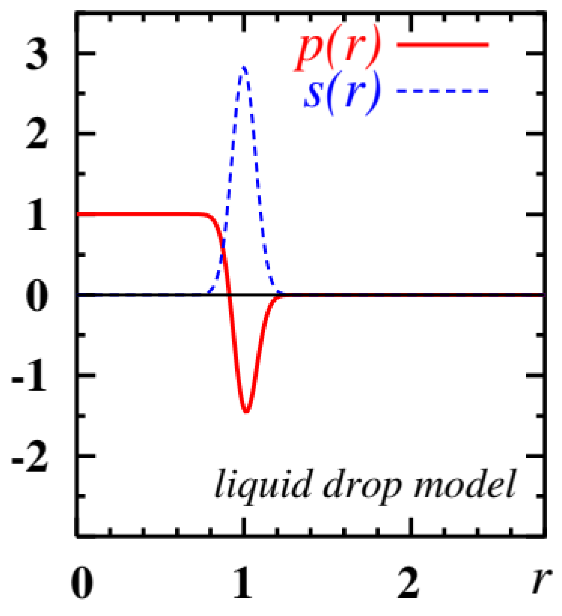
$$D = -\frac{4}{5}\frac{4\pi}{3} M\gamma R^4 \propto A^{7/3}$$

Microscopic approach Liuti, Taneja (2005)

$$D \propto A$$

Walecka model Guzey, Siddikov (2006)

$$D \propto A^{2.26}$$



Previous works limited to spin-0 nuclei with spherical symmetry
Nuclei come with various shapes and spins.

The Skyrme model (1961)

One of the oldest, most well-known and phenomenologically successful models of the nucleon

$$\mathcal{L}_{\mathrm{SK}} = -\frac{f_\pi^2}{16}\mathrm{Tr}\{L_\mu L^\mu\} + \frac{1}{32e^2}\mathrm{Tr}\{[L_\mu, L_\nu]^2\} - \lambda^2\pi^4 B_\mu B^\mu + \frac{m_\pi^2 f_\pi^2}{8}\mathrm{Tr}\{U - \mathbf{1}\},$$

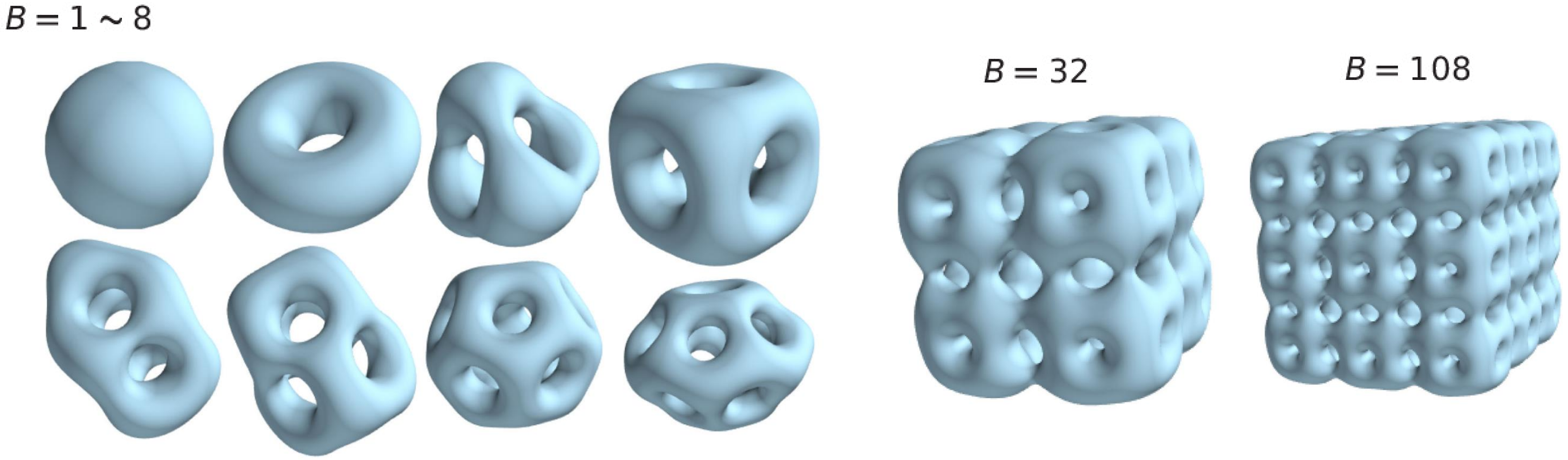
$$L_\mu = U^\dagger \partial_\mu U$$

$$B^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2}\mathrm{Tr}\{L_\nu L_\rho L_\sigma\}$$

Nucleon: Classical configuration (`hedgehog') with a definite baryon number B=1
Quantization of the collective coordinates → nucleon resonances

Gravitational form factors of the B=1 solution (nucleon) Cebulla, Goeke, Ossmann, Schweitzer (2007)

Nuclei in the Skyrme model



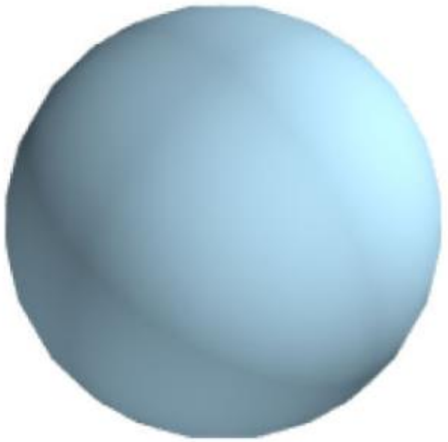
For more exotic solutions, see Gudnason, Halcrow (2022)

Electromagnetic form factors computed	$B = 2$	deuteron	Braaten, Carson (1989)
	$B = 3$	triton, helium-3	Carson (1991)

B=1, nucleon

$$D(t) = -6M_N \int d^3x \left(x^i x^j - \frac{1}{3}\delta^{ij}x^2 \right) \frac{j_2(qx)}{(qx)^2} T^{\mathrm{cl}}_{ij}(\boldsymbol{x})$$

$$T^{\mathrm{cl}}_{ij}(\boldsymbol{x}) = \left(\frac{x_i x_j}{x^2} - \frac{1}{3}\delta_{ij} \right) s(x) + p(x)\delta_{ij}$$



Quantization

$$U(t,\boldsymbol{x}) = A(t) U_0(R_B(t)(\boldsymbol{x} - \boldsymbol{X}(t))) A^\dagger(t)$$

Rotation matrix which acts on external spin states

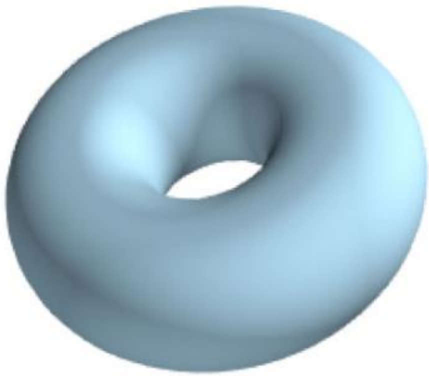
Drop out due to spherical symmetry.

$$\langle \boldsymbol{q}/2 | T_{ij}[U(R(B)(\boldsymbol{x} - \boldsymbol{X}))] | -\boldsymbol{q}/2 \rangle = e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} R^T_{ia}(B) R^T_{jb}(B) \int d^3x' e^{i\boldsymbol{q}\cdot R^T(B)\boldsymbol{x}'} T^{cl}_{ab}(\boldsymbol{x}') + \underline{\mathcal{O}(I^2, J^2)}$$

Neglected in this work

Cf. Kim, Sun (2021)

B=2, deuteron



There are three D-terms for a spin-1/2 hadron.

$$\begin{aligned}
&\langle p'\sigma'|T_{ij}|p\sigma\rangle \\
&= \frac{1}{2}(q_i q_j - \delta_{ij} q^2)\mathcal{D}_1(t)\epsilon^*_{\sigma'}\cdot\epsilon_\sigma \\
&\quad + (q_j q_k Q_{ik} + q_i q_k Q_{jk} - q^2 Q_{ij} - \delta_{ij} q_k q_l Q_{kl})_{\sigma'\sigma}\mathcal{D}_2(t) + \frac{1}{2M_D^2}(q_i q_j - \delta_{ij}q^2) q_k q_l Q_{kl,\sigma'\sigma}\mathcal{D}_3(t) + \cdots
\end{aligned}$$

Classical configurations with quadrupole deformation Polyakov, Sun (2019)

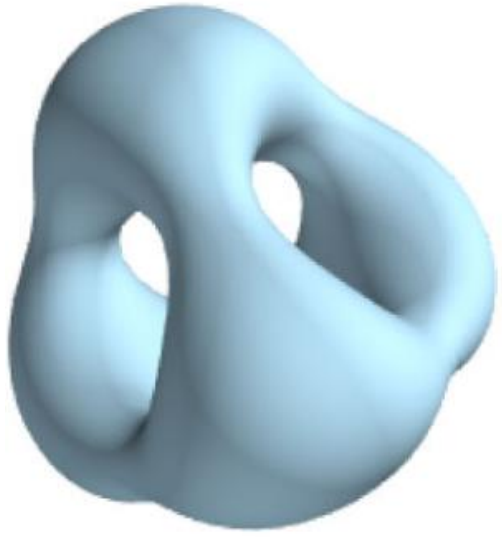
$$\begin{aligned}
T^{\rm cl}_{ij} \approx\ & Y_2^{ij} s(x) + p(x)\delta_{ij} + 2s'(x)(Q_{ik}Y_2^{kj} + Q_{jk}Y_2^{ki} - \delta_{ij}Q_{ab}Y_2^{ab}) \\
& + p'(x)Q_{ij} - \frac{1}{M_D^2} Q^{kl}\partial_k\partial_l(p''(x)\delta^{ij} + s''(x)Y_2^{ij})
\end{aligned}$$

$$Y_2^{ij} = \frac{x_i x_j}{x^2} - \frac{\delta_{ij}}{3}$$

$$D(t) = -6M_N \int d^3x \left(x^i x^j - \frac{1}{3}\delta^{ij}x^2\right)\frac{j_2(qx)}{(qx)^2} T^{\rm cl}_{ij}(\boldsymbol{x})$$

The monopole part given by the same formula

B=3, triton and helium-3



Spin ½, only one D-term, just like the nucleon

But the classical solution is not spherically symmetric.

Yet, the solution is highly symmetric.
Discrete transformations form the tetrahedral group T_d

$$\int d^3x T^{\mathrm{cl}}_{ab}(\boldsymbol{x}) j_0(qx) \propto \delta_{ab}$$

Group theory helps! cf. Carson (1991)
Decompose tensors into the irreducible representations of T_d

$$T_1 \times T_1 = A_1 + E + T_1 + T_2$$

Rank-4 tensor associated with the D-term

$$T_{abkl} = \int d^3\boldsymbol{x}\left(x_k x_l - \frac{1}{3}\delta_{kl}x^2\right)\frac{15 j_2(qx)}{(qx)^2}T^{\rm cl}_{ab}(\boldsymbol{x})$$

$$= \frac{1}{10}\left(\delta_{ak}\delta_{bl} + \delta_{al}\delta_{bk} - \frac{2}{3}\delta_{ab}\delta_{kl}\right)T_{cdcd} + \underline{C_{abkl}}$$

totally-symmetric, traceless

$$\langle \boldsymbol{q}/2|T_{ij}(-R\boldsymbol{X})|-\boldsymbol{q}/2\rangle|_{l=2}$$

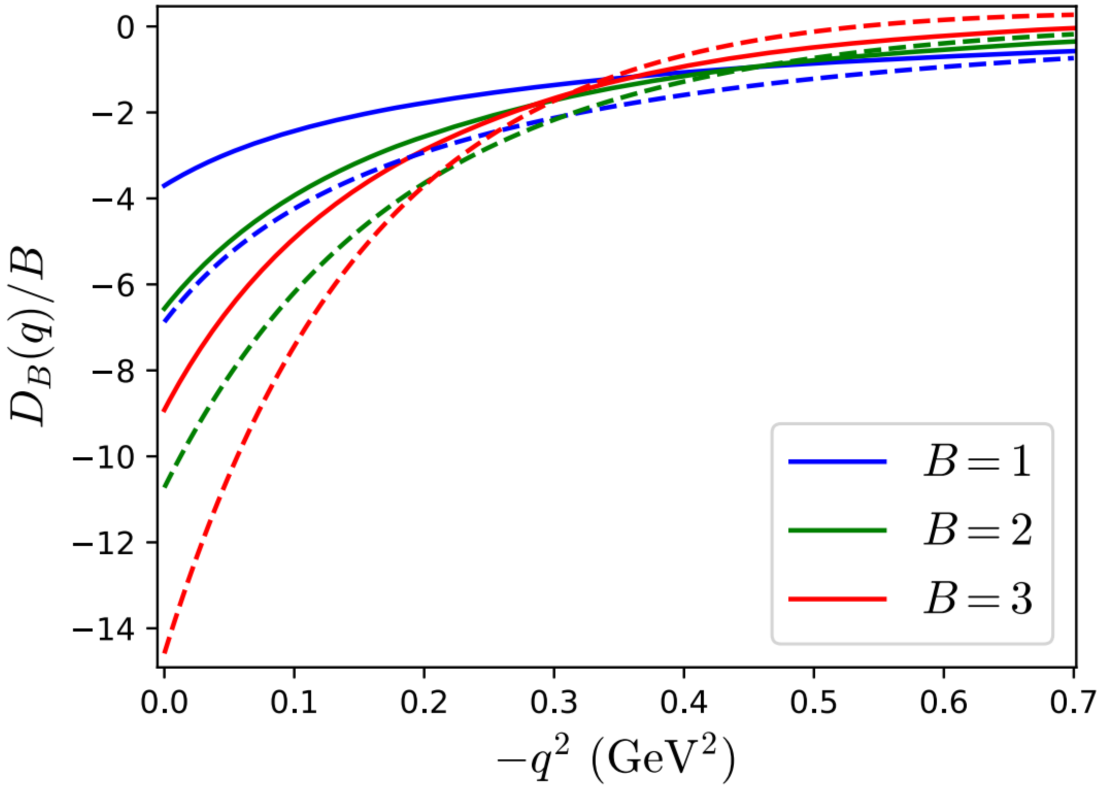
$$= -\left(q_i q_j - \frac{\delta_{ij}}{3}q^2\right)\frac{T_{cdcd}}{10} - \frac{1}{2}\left(q_c q_d - \frac{1}{3}\delta_{cd}q^2\right)\underline{R^T_{ia}(B)R^T_{jb}(B)R^T_{ck}(B)R^T_{dl}(B)C_{ab,kl}}$$

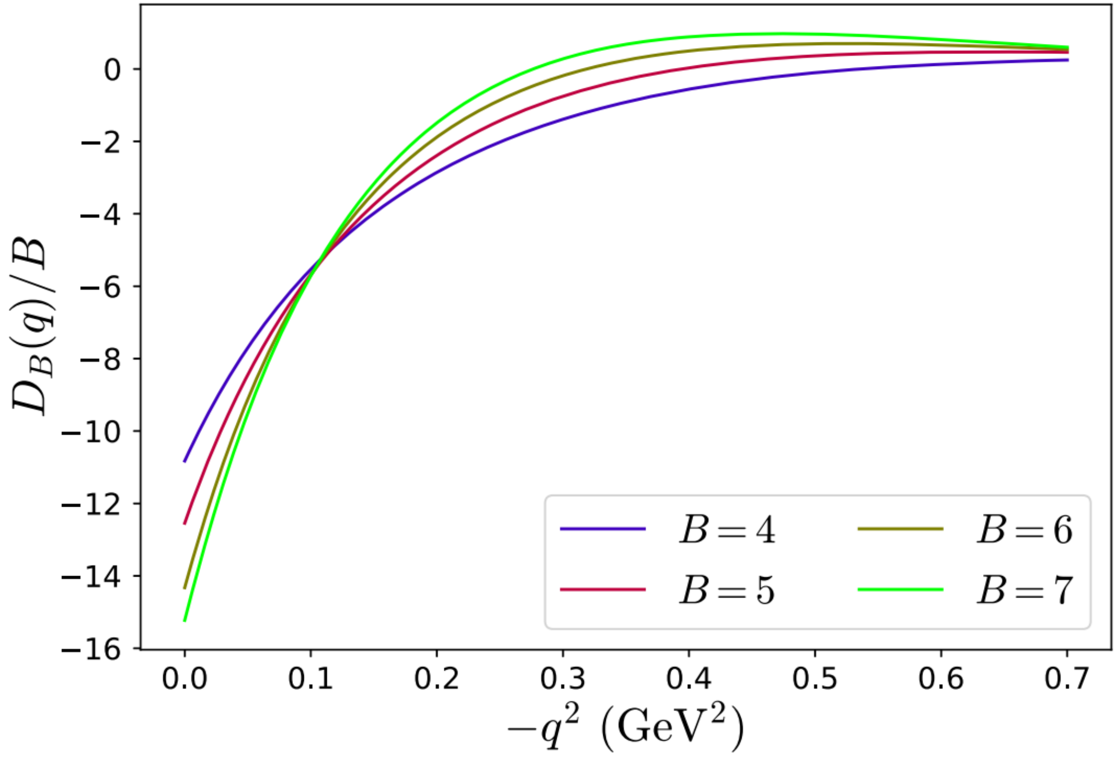
Spin-4 operator, vanishes when evaluated in spin-1/2 states

$$D(t) = -6M_N\int d^3x\left(x^i x^j - \frac{1}{3}\delta^{ij}x^2\right)\frac{j_2(qx)}{(qx)^2}T^{\rm cl}_{ij}(\boldsymbol{x})$$

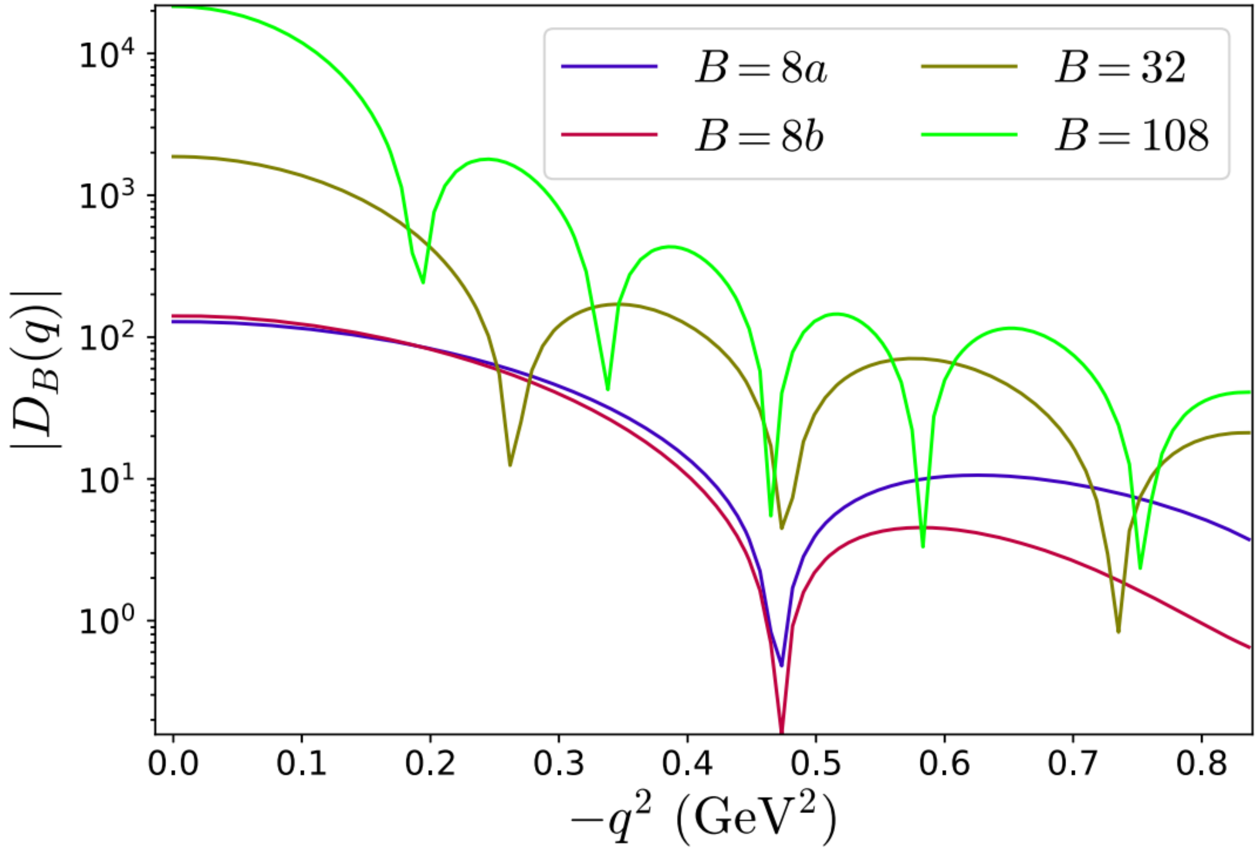
The same formula as for the B=1 solution.

Results





The dashed lines include the sextic term $\Delta\mathcal{L} \sim B_\mu B^\mu$ $\qquad B^\mu = \dfrac{\epsilon^{\mu\nu\rho\sigma}}{24\pi^2}\mathrm{Tr}\{L_\nu L_\rho L_\sigma\}$



The form factor changes signs, oscillates around zero for large-B nuclei.

Similar to the diffractive pattern in elastic scattering off nuclei.

B	1	2	3	4	5	6	7	$8a$	$8b$	32	108
$D(0)$	-3.701	-13.126	-26.757	-43.304	-62.72	-85.95	-106.596	-128.368	-140.816	-1.874×10^3	-2.152×10^4

The value D(0) grows quickly with increasing B

Baryon number scaling $D(0) \propto B^\beta$

We find $\beta \approx 1.7$

to be compared with $\beta = \frac{7}{3}$ $\beta \approx 1$ $\beta \approx 2.26$

Polyakov Liuti, Taneja Guzey, Siddikov

Implication for the scalar and mass radii

$$\langle r^2 \rangle_s = \langle r^2 \rangle_m - \frac{3D(0)}{M^2}$$

scalar radius mass radius

B^2

$\beta > 2$ $\langle r^2 \rangle_s \gg \langle r^2 \rangle_m$

$\beta < 2$ $\langle r^2 \rangle_s \to \langle r^2 \rangle_m$

as $B \to \infty$

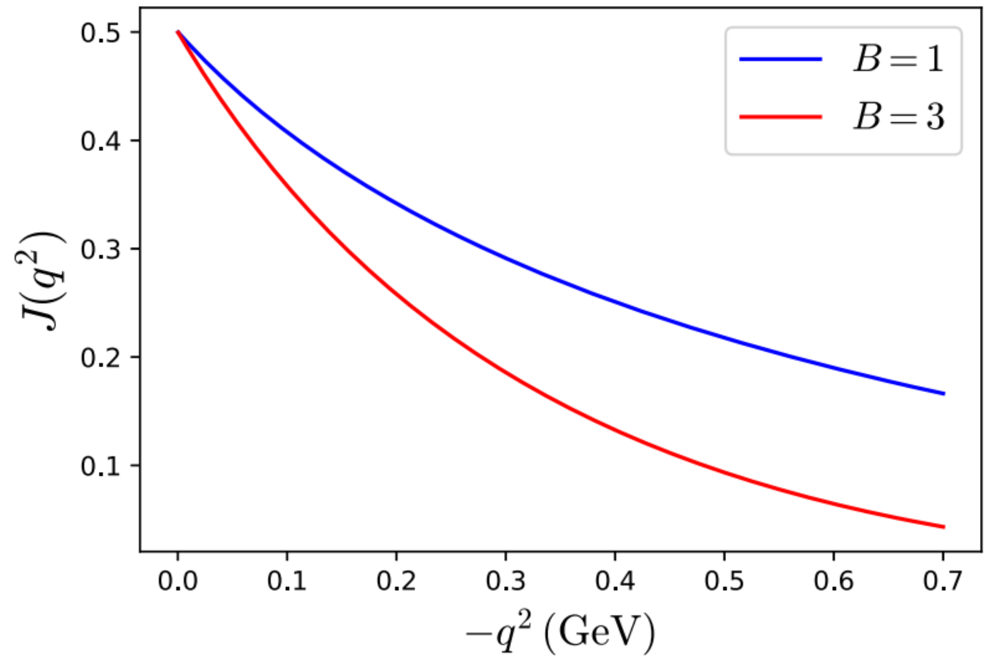
Angular-momentum form factor $J(t) = \frac{1}{2}(A(t) + B(t))$

$$\frac{\langle p', s'|T_{0i}(0)|p, s\rangle}{2P^0} = -J(t) i\epsilon_{ijk} \frac{\tau^j_{s's}}{2} q^k$$

Off-diagonal components vanish for classical configuration → quantization crucial

Straightrforward for B=1   Cebulla, et al.
Nontrivial for B=3

Again, group theory helps!



$$\langle \boldsymbol{q}/2|T_{0i}(-R\boldsymbol{X})| -\boldsymbol{q}/2\rangle|_{l=1} = iR^T_{ij}(B)R^T_{kl}(B)q^k \int d^3x \frac{3j_1(qx)}{qx}[I_{jm}a^m - J_{jm}b^m]x^l$$

$$A_1 \in T_2 \times T_1 \times T_1$$

Outlook

- Computation of mass/scalar radii (work in progress)
- Full quantization including the $\mathcal{O}(J^2, I^2)$ corrections.
- Excited states, nuclear vibration
- Solutions with B>8
- Multipole D-terms for spin>1/2 nuclei
- More realistic calculations from low energy nuclear theory techniques
- How to measure in experiments?