Moments of GPDs from the OPE of nonlocal quark bilinears

Xiang Gao
Argonne National Laboratory

In collaboration with: S. Bhattacharya, K. Cichy, M. Constantinou, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, and Y. Zhao

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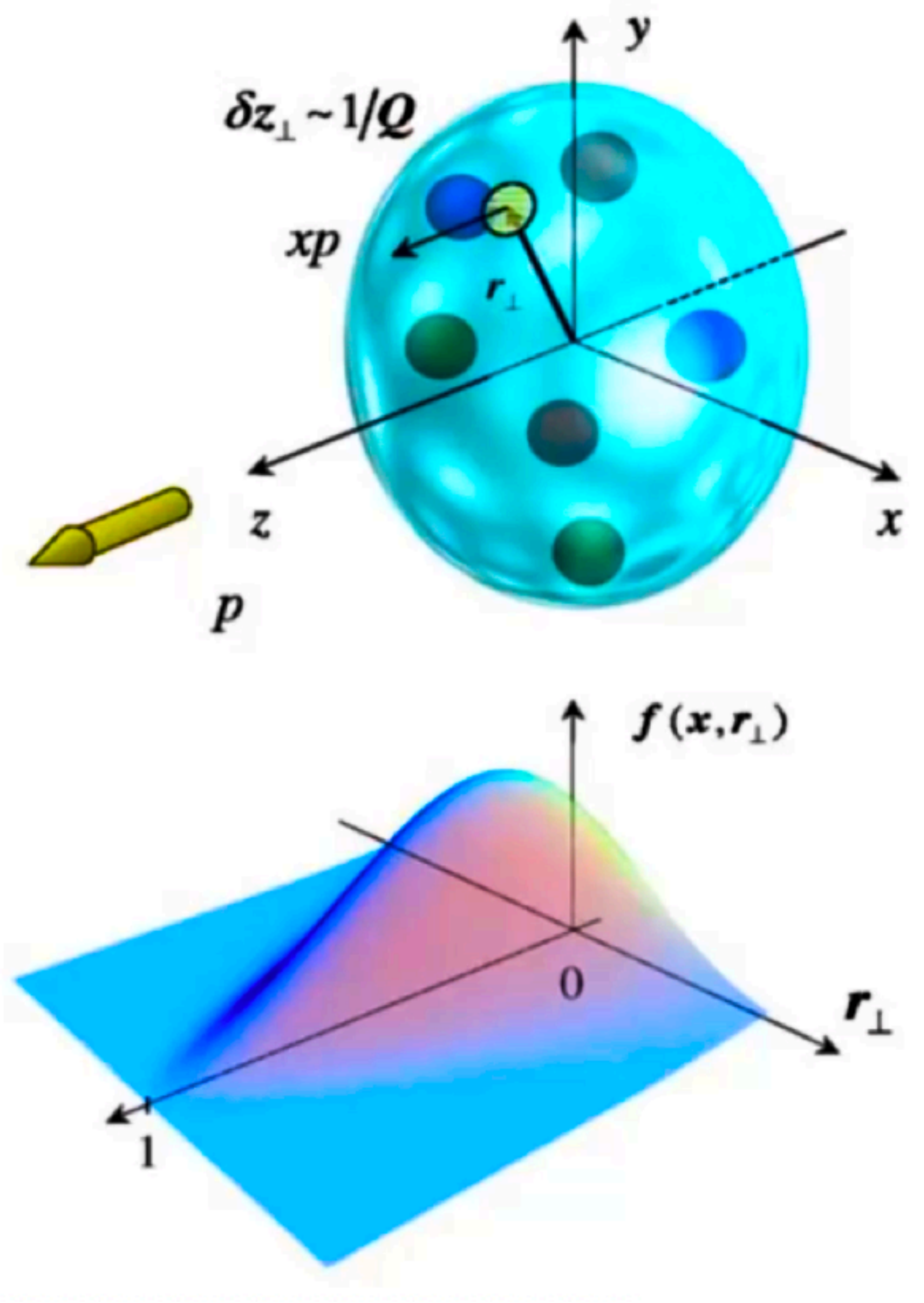
Generalized parton distributions

GPDs goes far beyond the 1D PDFs and the transverse structure encoded in the form factors.



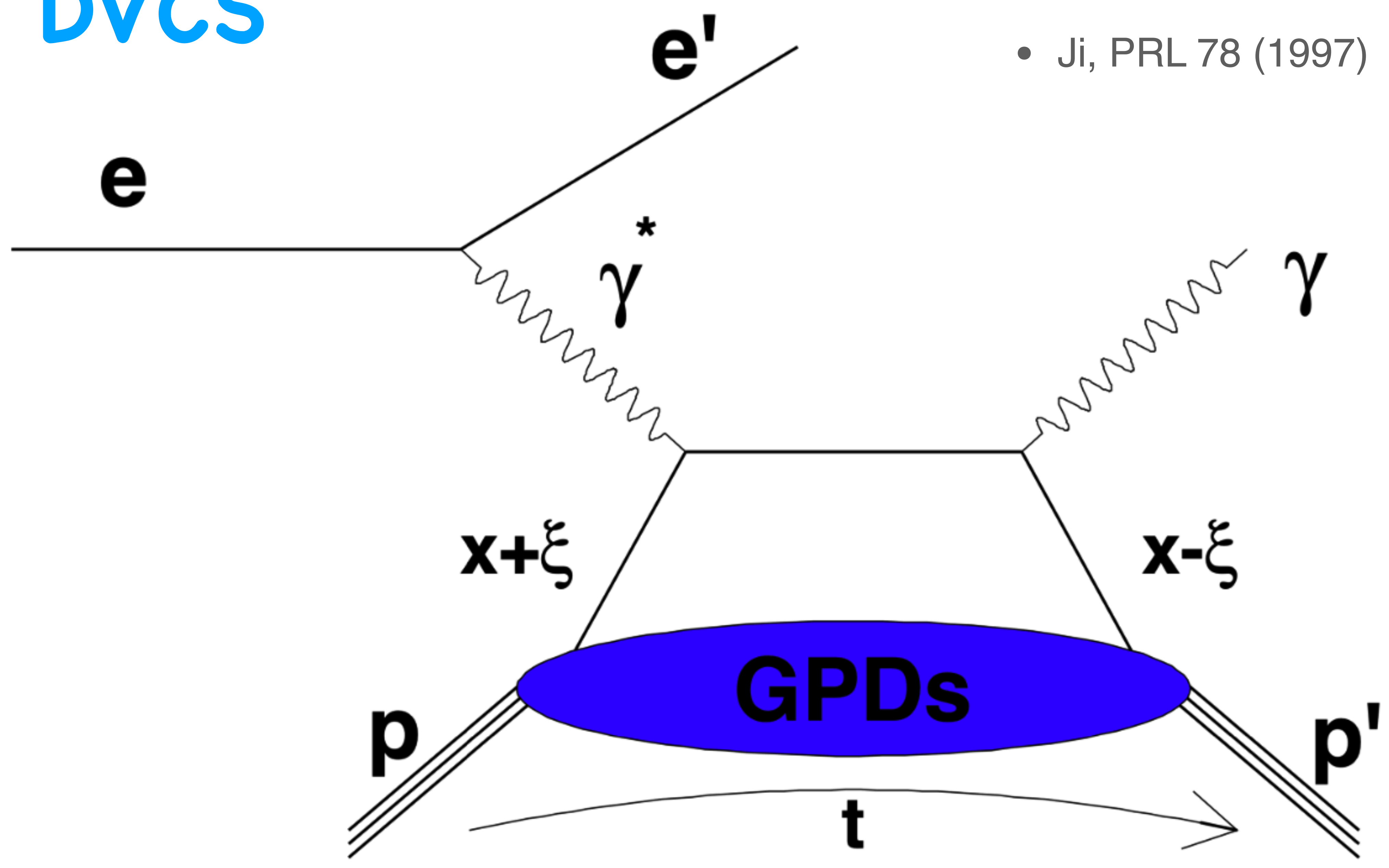
$$F_q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

- Offer insights into the 3D image of hadrons.
- Give access to the orbital motion and spin of partons.
- Have a relation to pressure and shear forces inside hadrons.

- Ji, PRL 78 (1997)
- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

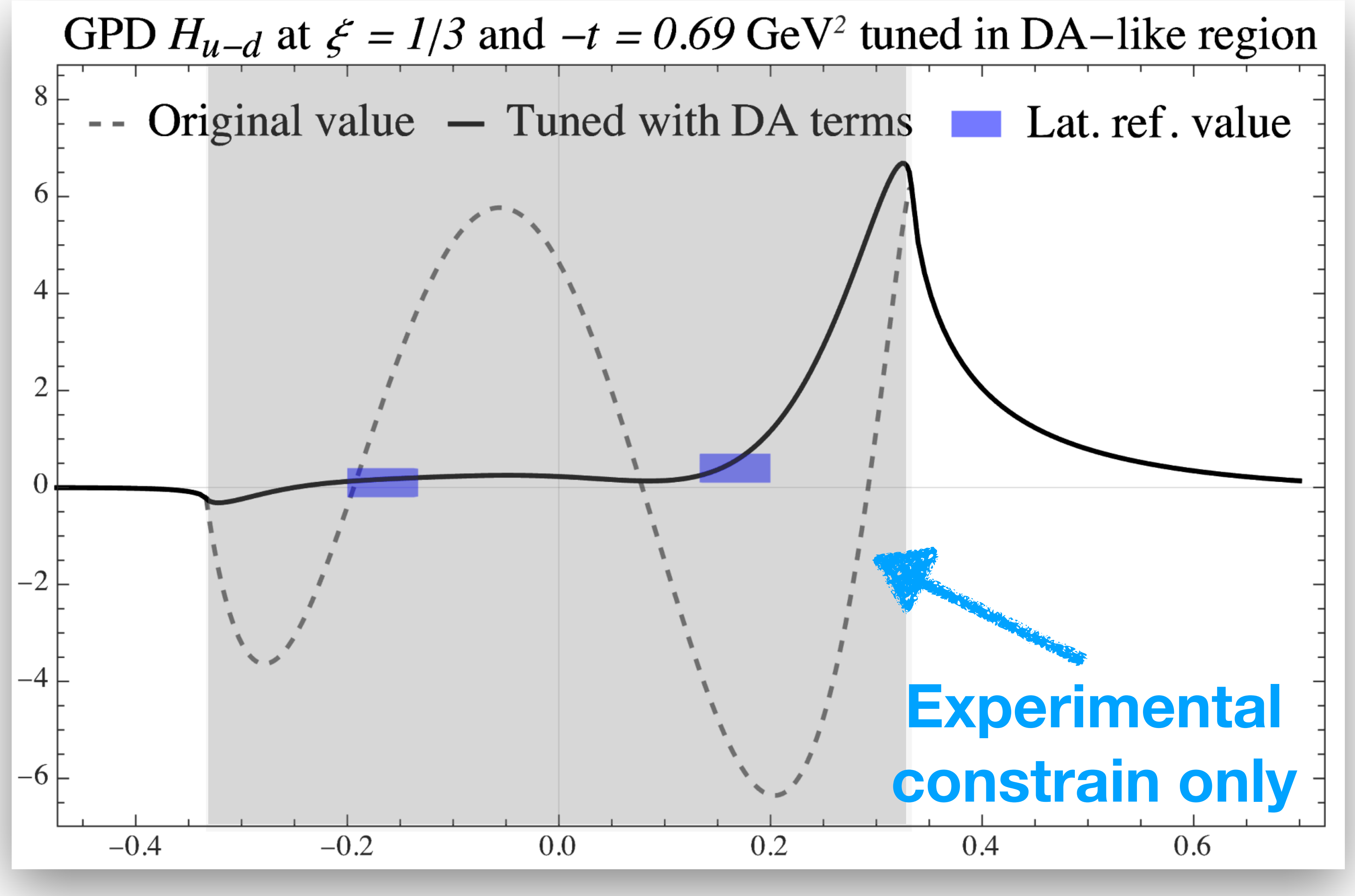
Generalized parton distributions

DVCS



- Ji, PRL 78 (1997)

- Y. Guo, et al., JHEP 05 (2023) 150



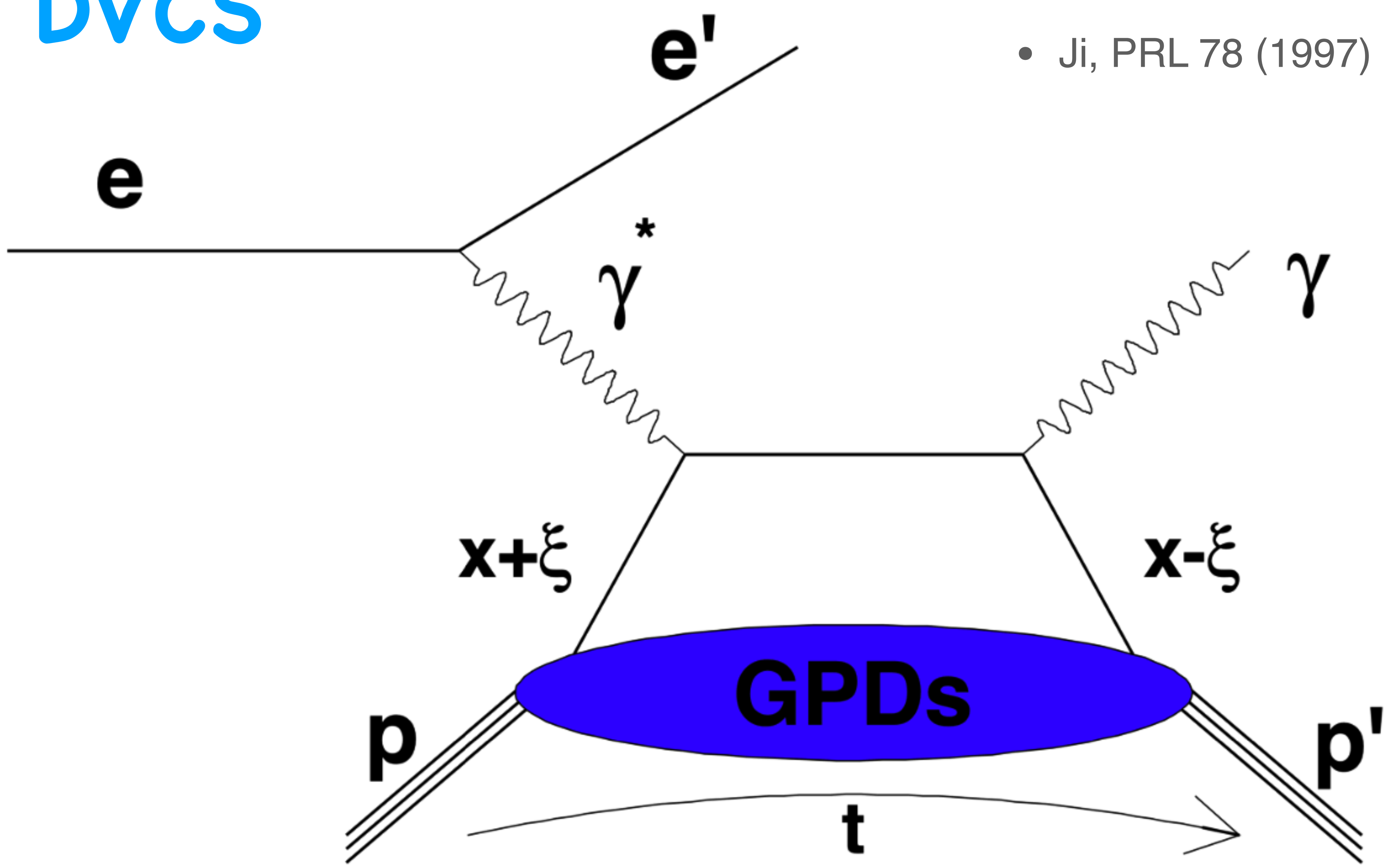
Challenging：

- Observables appear at the amplitude level.
- Multi-dimensionality $F(x, \xi, t)$.
- The momentum fraction x is integrated over (Compton Form Factors).

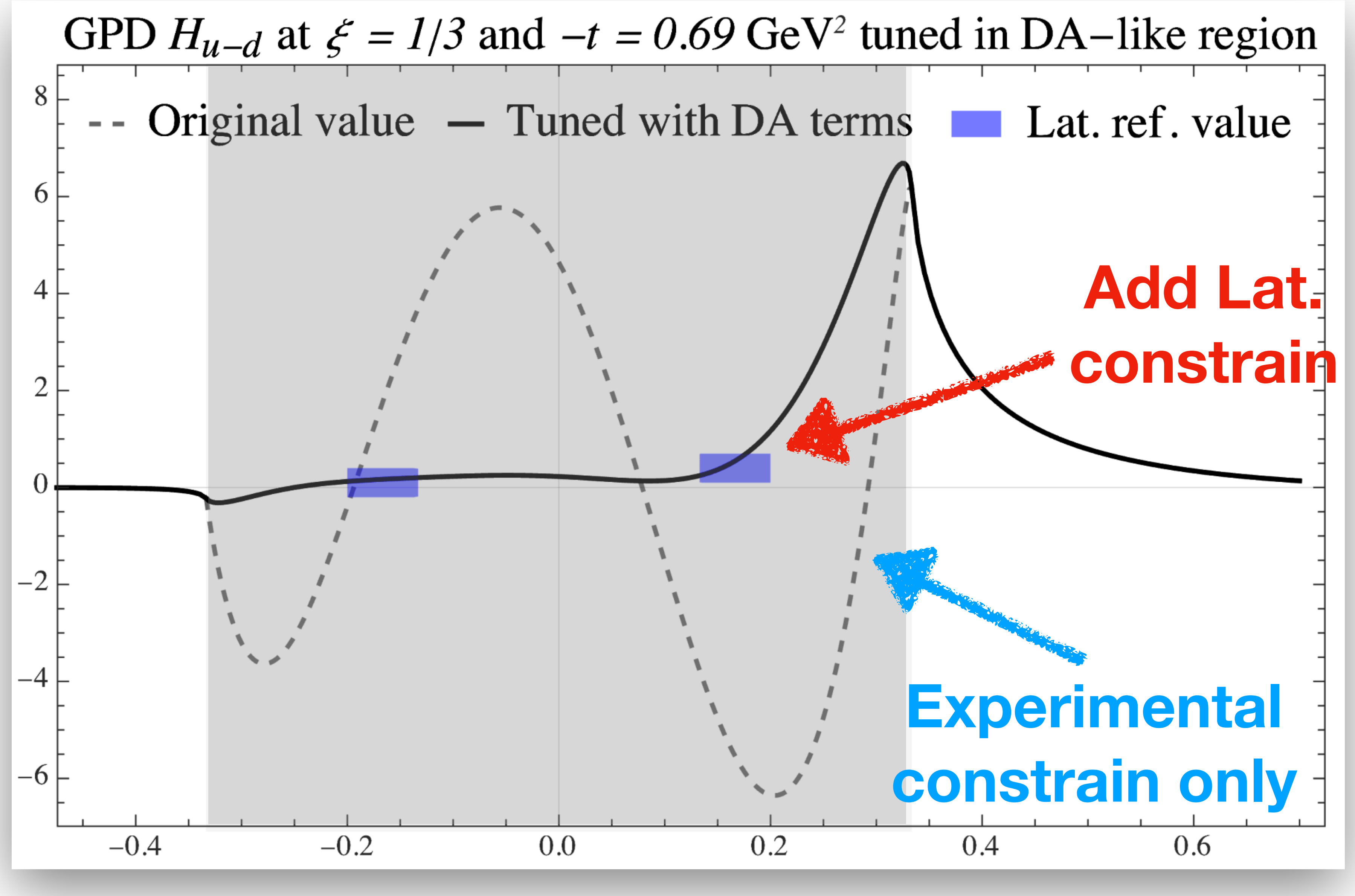
Generalized parton distributions

DVCS

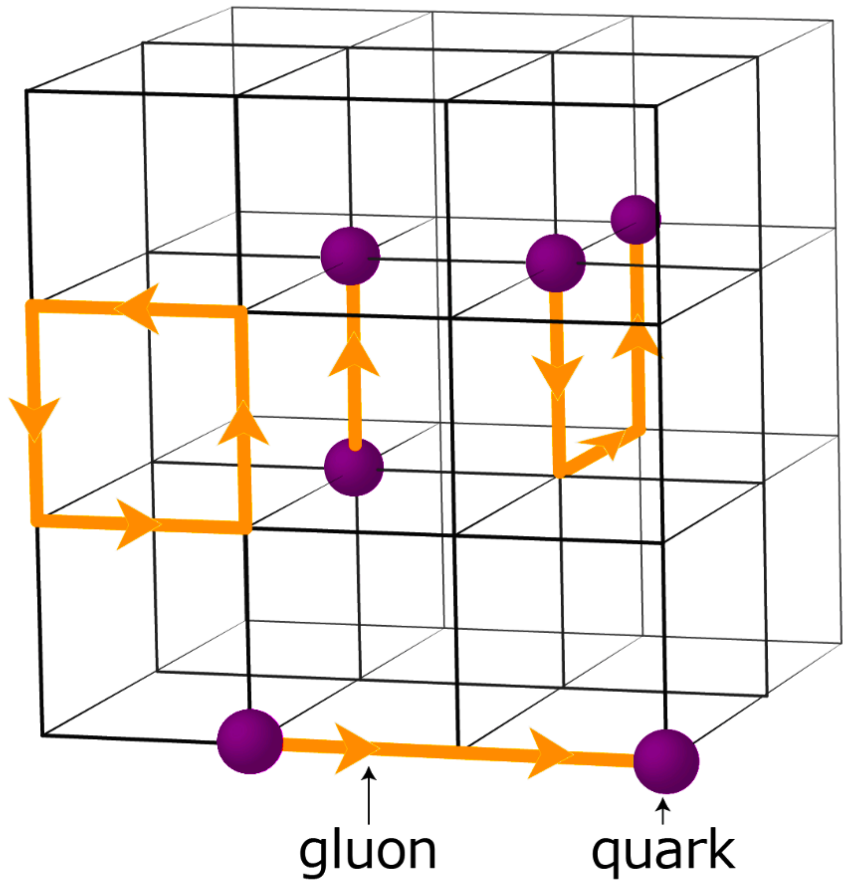
- Ji, PRL 78 (1997)



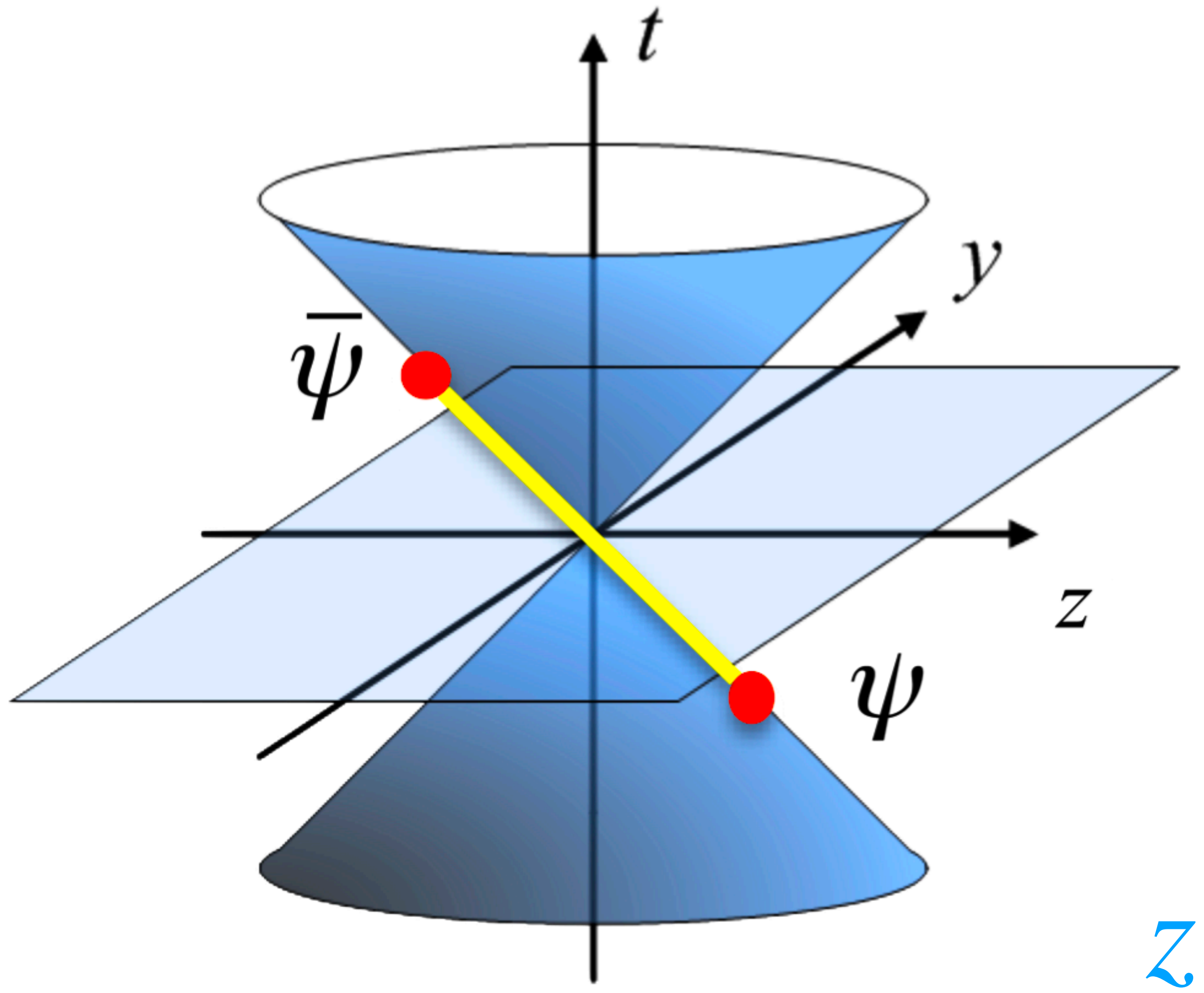
- Y. Guo, et al., JHEP 05 (2023) 150



Complementary knowledge from lattice QCD is essential.



GPDs from Lattice QCD: local operator



$z^2 = 0$

$$\langle p_f | \bar{q}(-\frac{z^-}{2})\gamma^\mu \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) | p_i \rangle$$

Light-cone correlation: Cannot be calculated on the lattice

OPE of the light-cone operator

$$\bar{q}(-\frac{z^-}{2})\gamma^+ \mathscr{W}(-\frac{z^-}{2}, \frac{z^-}{2}) q(\frac{z^-}{2}) = \sum_{n=0}^{\infty} \frac{(-iz^-)^n}{n!} O^{++\ldots+}(\mu)$$

- Moments from Local operator

$$\bar{q}\gamma^{\{\mu_0} iD^{\mu_1} \ldots iD^{\mu_n\}} q$$

GPDs from Lattice QCD: local operator

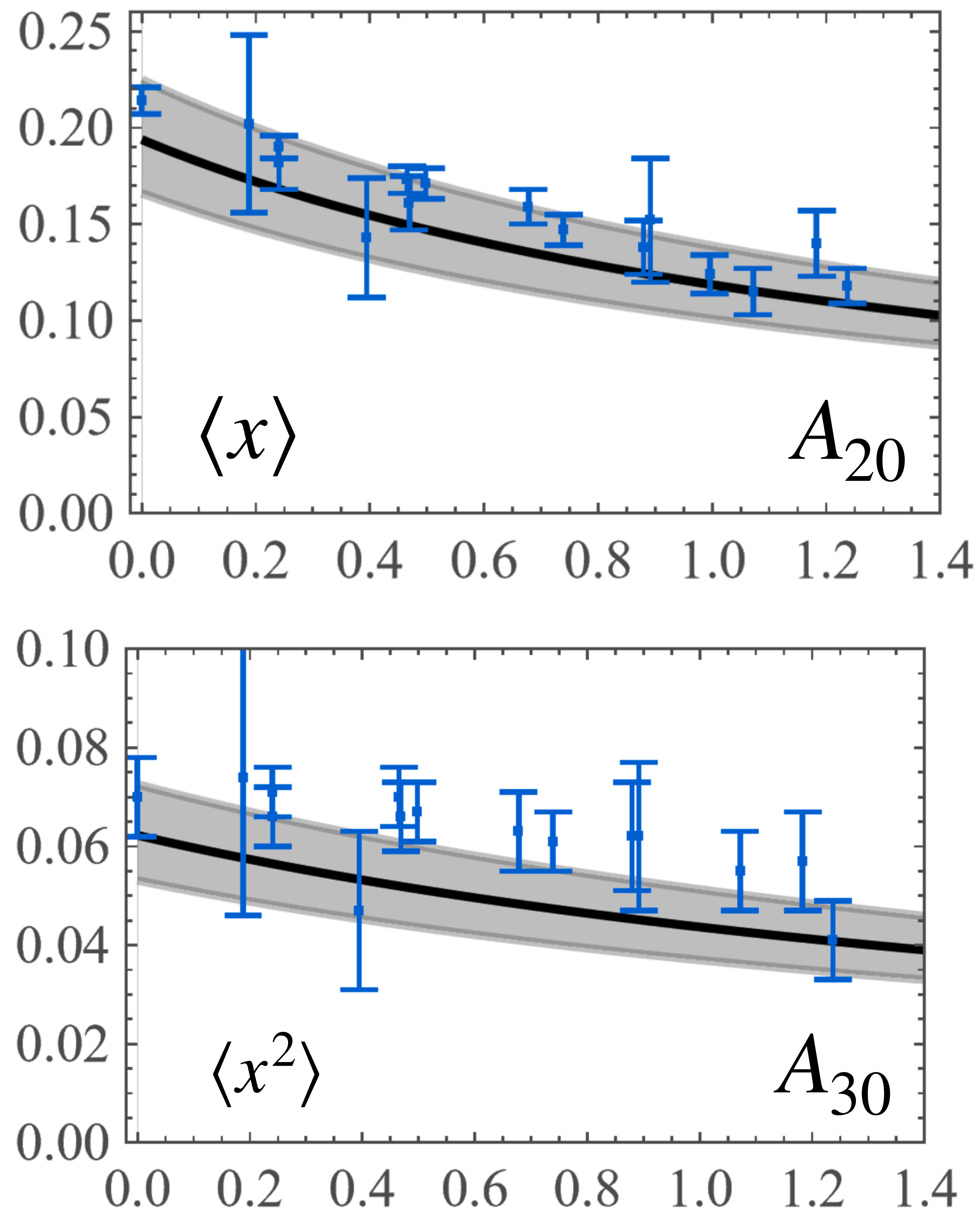
- Moments from Local operator

$$\bar{q}\gamma^{\{\mu_0}iD^{\mu_1}\dots iD^{\mu_n\}}q$$

High dimensional operator

- Limited up to $\langle x^3\rangle$ due to signal decay and power-divergent mixing under renormalization.

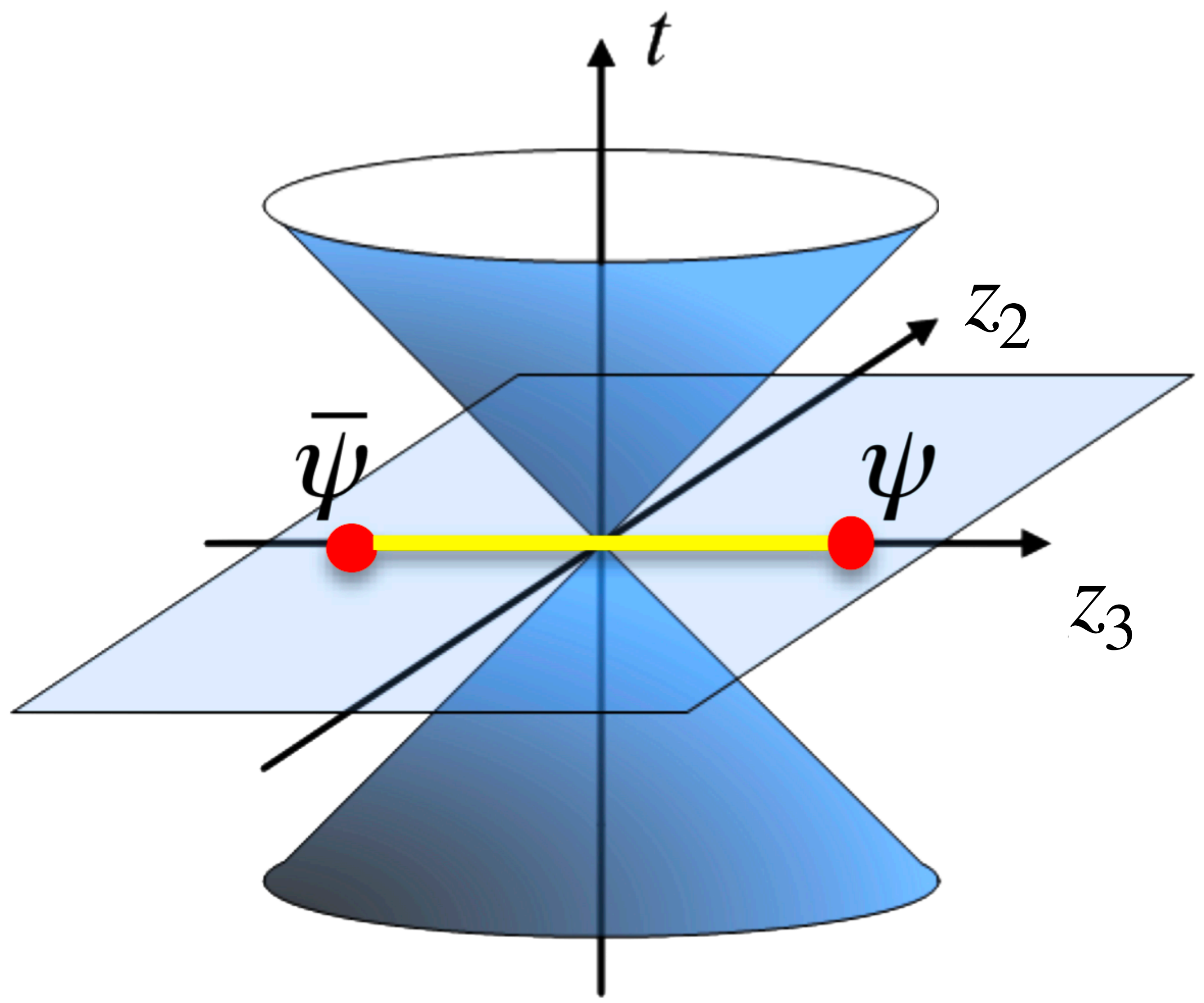
Moments of GPDs



• LHPC, Phys.Rev.D 77 (2008) 094502

GPDs from Lattice QCD: non-local operator

$t = 0, \quad z_3 \neq 0$



- X. Ji, PRL 2013

$$F^\mu(z, P, \Delta) = \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) \, | p_i \rangle$$

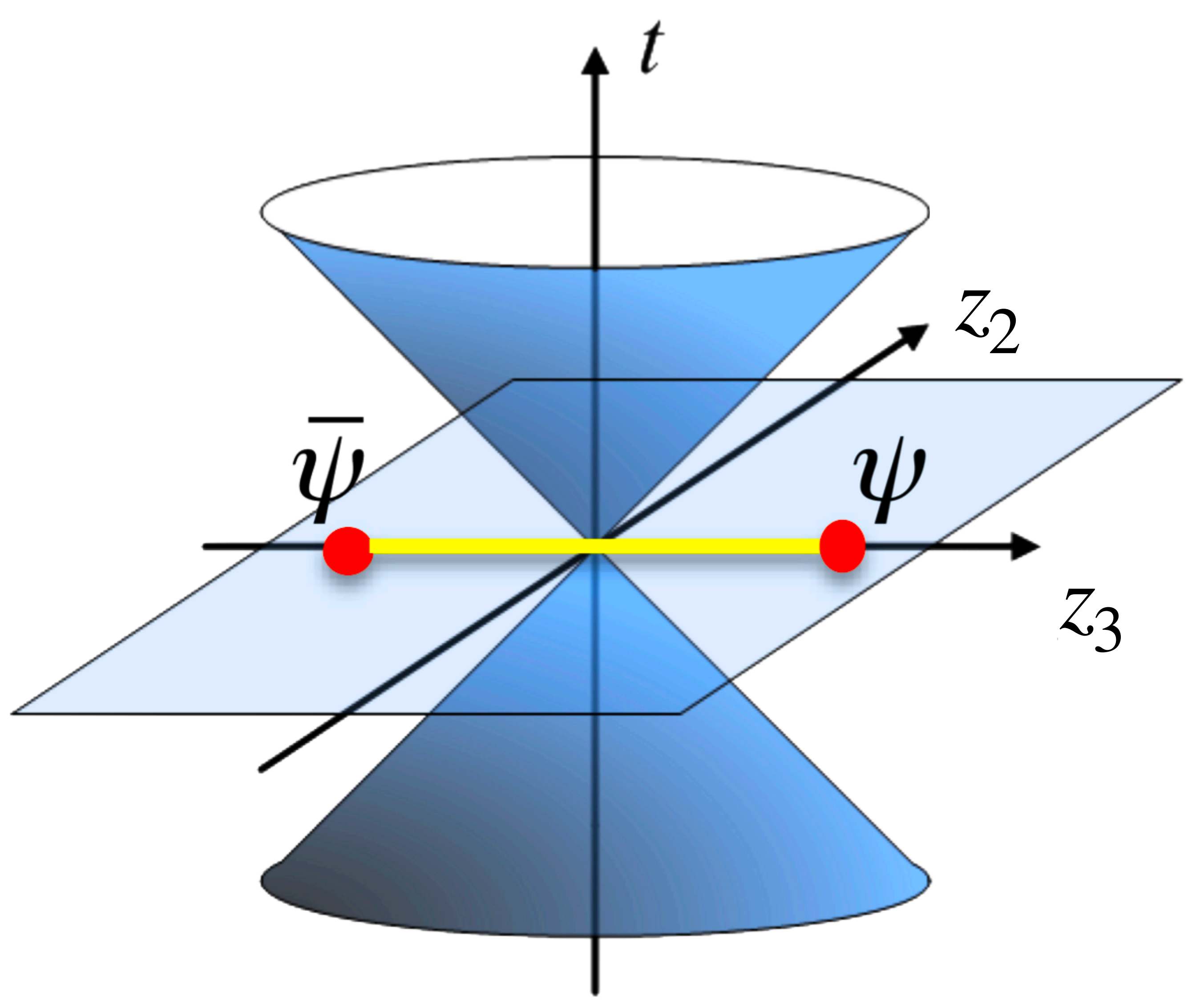
$$z = (0,0,0,z_3), \; z^2 = z_3^2$$

- **Large-momentum effective theory**: x-space matching of quasi-PDF.
  - X. Ji, PRL 2013
  - X. Ji, et al, RevModPhys 2021

- **Short distance factorization** of the quasi-PDF matrix elements or the pseudo-PDF approach.
  - A. Radyushkin, PRD 100 (2019)
  - A. Radyushkin, Int.J.Mod.Phys.A 2020

8 GPDs from short distance factorization

$t = 0, \quad z_3 \neq 0$



• X. Ji, PRL 2013

$$F^\mu(z, P, \Delta)$$
$$= \langle p_f | \bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | p_i \rangle$$

$$z = (0,0,0,z_3),\ z^2 = z_3^2$$

OPE of the equal-time operator

$$\bar{q}(-\tfrac{z}{2})\gamma^\mu \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2})$$
$$= \sum_{n=0}^{\infty} C_n(\mu^2 z^2) \frac{(-iz)^n}{n!} e_{\mu_1} \ldots e_{\mu_n} O^{\mu_0\mu_1\cdots\mu_n}(\mu)$$
+Higher twist operators

GPDs from short distance factorization

SDF/OPE of the quasi-GPD matrix elements:
zero skewness case

$$F^R(z, P, \Delta) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2)\langle x^n\rangle(t;\mu) + \mathcal{O}(z^2\Lambda^2_{\rm QCD})$$

Perturbative coefficients

E.g.

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$

- The perturbative matching is valid in short range of z^2.
- The information is limited to the first moments by the range of finite $\lambda = zP$.
- Free of power divergent mixing so that can be systematically improved.

quasi-GPD matrix elements

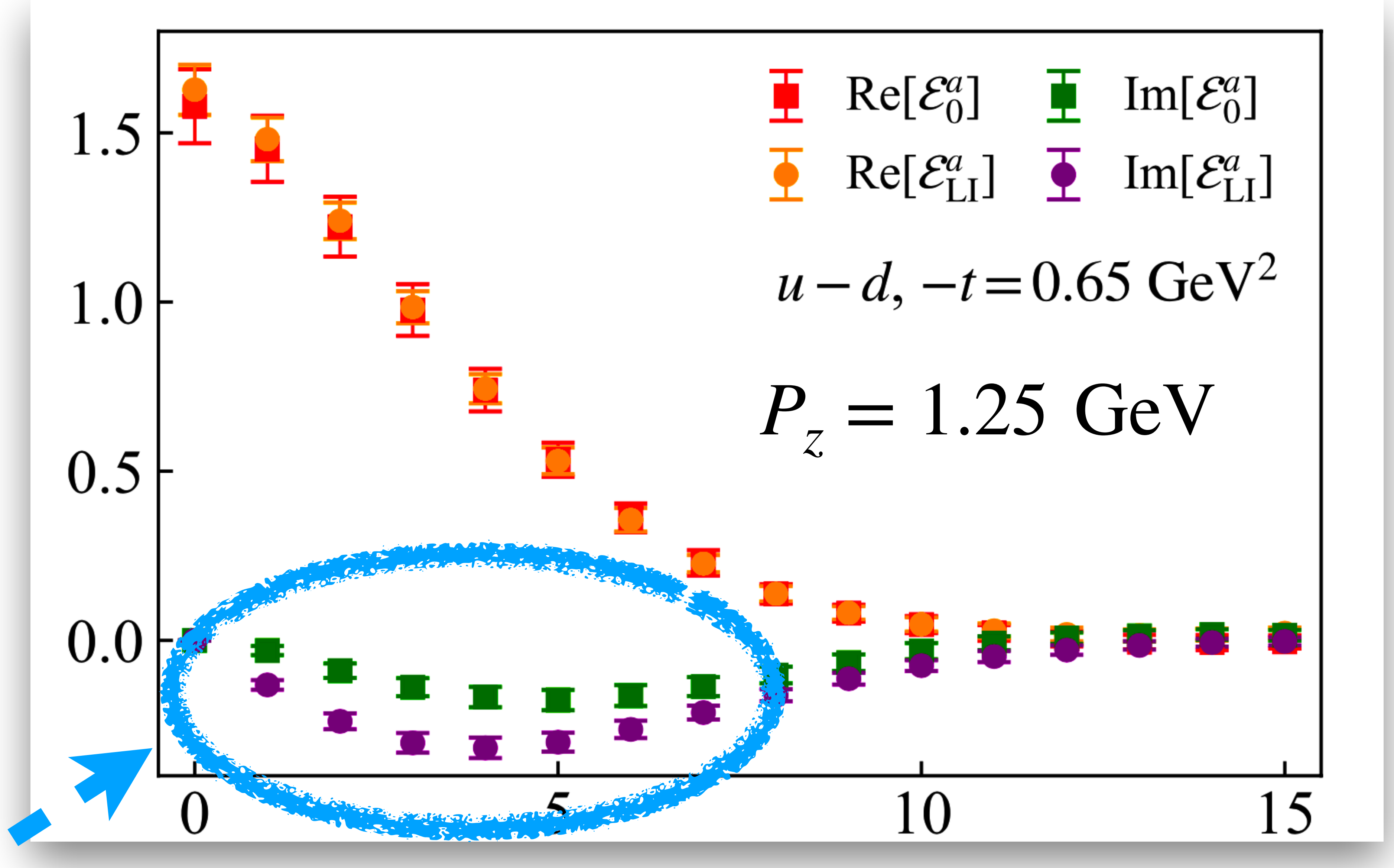
The unpolarized qGPD matrix elements in γ_0 definition:

$$F^0(z,P,\Delta) = \langle p_f|\,\bar{q}(-\tfrac{z}{2})\gamma^0\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})\,|p_i\rangle$$

$$= \bar{u}(p_f,\lambda')\left[\gamma^0\mathscr{H}_0(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathscr{E}_0(z,P,\Delta)\right]u(p_i,\lambda)$$

Problem:

- The qGPDs are frame dependent though light-cone GPDs are Lorentz invariant.
- Computationally expensive for multiple $-t = Q^2$ in symmetric frame.



frame-dependent power corrections

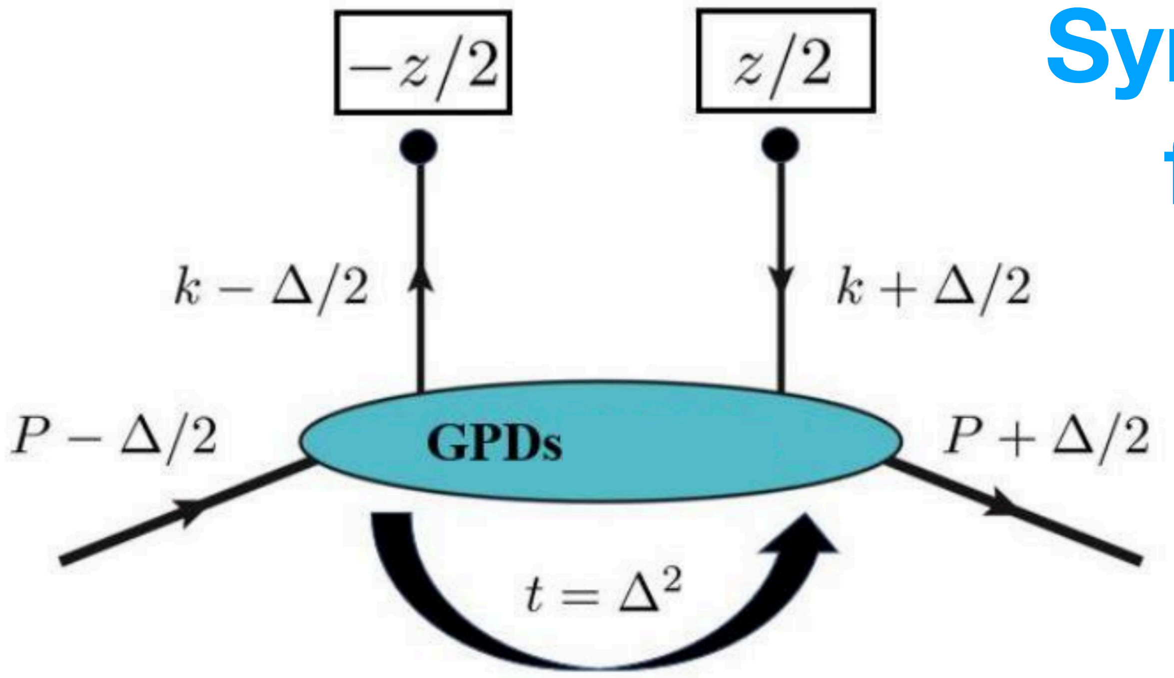
$\sim \Delta/P$ at the tree level

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of Lorentz invariant amplitudes $A_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$:

Shohini Bhattacharya's talk on Friday

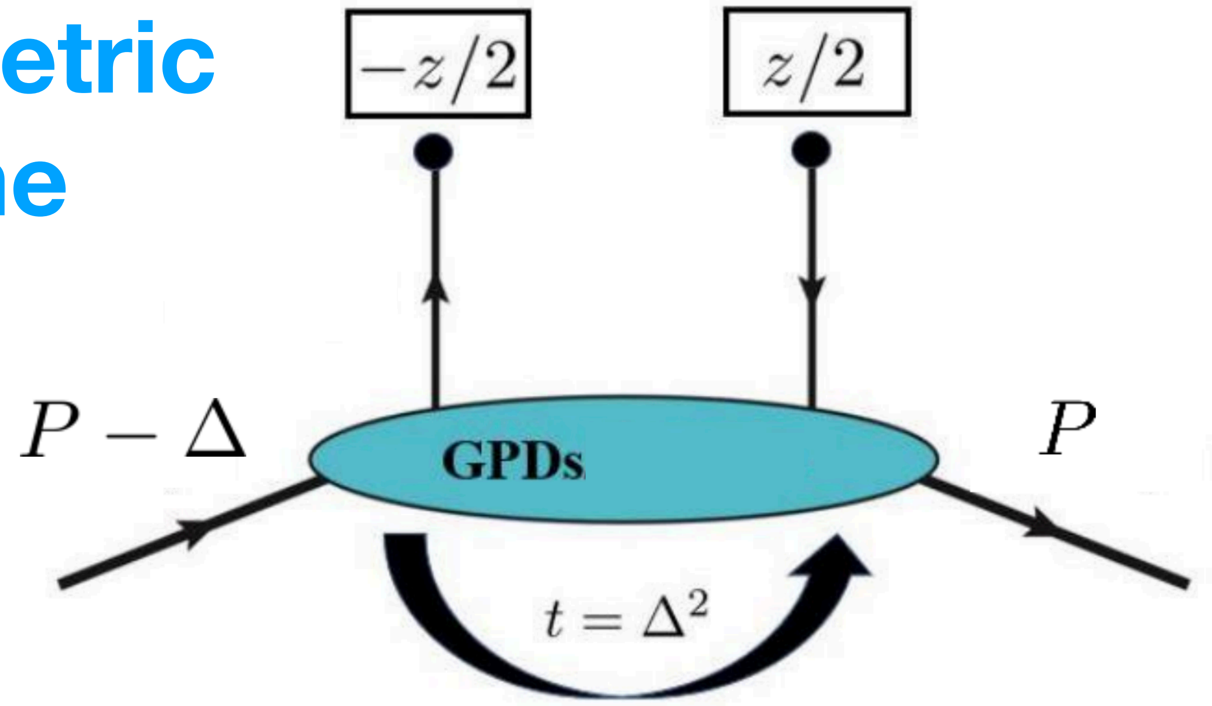
$$F^{\mu}(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + mz^{\mu}i\sigma^{z\Delta}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$



Symmetric frame

Lorentz transform

Asymmetric frame



New development:

- Construct qGPD from asymmetric-frame calculation.
- Computational much cheaper for multiple $-t$, and possibly reducing the power corrections with proper construction.

- S. Bhattacharya, XG, et al., Phys.Rev.D 106 (2022), 114512

quasi-GPD matrix elements

The matrix elements can be parametrized in terms of Lorentz invariant amplitudes $A_i\left(z\cdot P, z\cdot\Delta, \Delta^2, z^2\right)$**:**

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+mz^{\mu}i\sigma^{z\Delta}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\right]u(p_i,\lambda)$$

A Lorentz invariant (LI) choice analogous to the light-cone GPD:

$$\mathscr{H}(z,P,\Delta)=A_1+\frac{\Delta\cdot z}{P\cdot z}A_3$$

$$\mathscr{E}(z,P,\Delta)=-A_1-\frac{\Delta\cdot z}{P\cdot z}A_3+2A_5+2P\cdot zA_6+2\Delta\cdot zA_8$$

- Differ from Light-cone GPD only by $z^2\neq 0$

Renormalization

- The operator can be **multiplicatively renormalized**

- X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
- J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B = e^{-\delta m(a)|z|} Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

- Short distance factorization with **ratio scheme renormalization**

- A. V. Radyushkin et al., PRD 96 (2017)
- BNL, PRD 102 (2020)

$$\mathscr{M}(z^2, zP, \Delta^2) = \frac{\mathscr{H}^R(z, P, \Delta; \mu)}{\mathscr{H}^R(z, P=0, \Delta=0; \mu)} = \frac{\mathscr{H}^B(z, P, \Delta; a)}{\mathscr{H}^B(z, P=0, \Delta=0; a)}$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle (\Delta^2; \mu) + \mathscr{O}(z^2 \Lambda^2_{\text{QCD}})$$

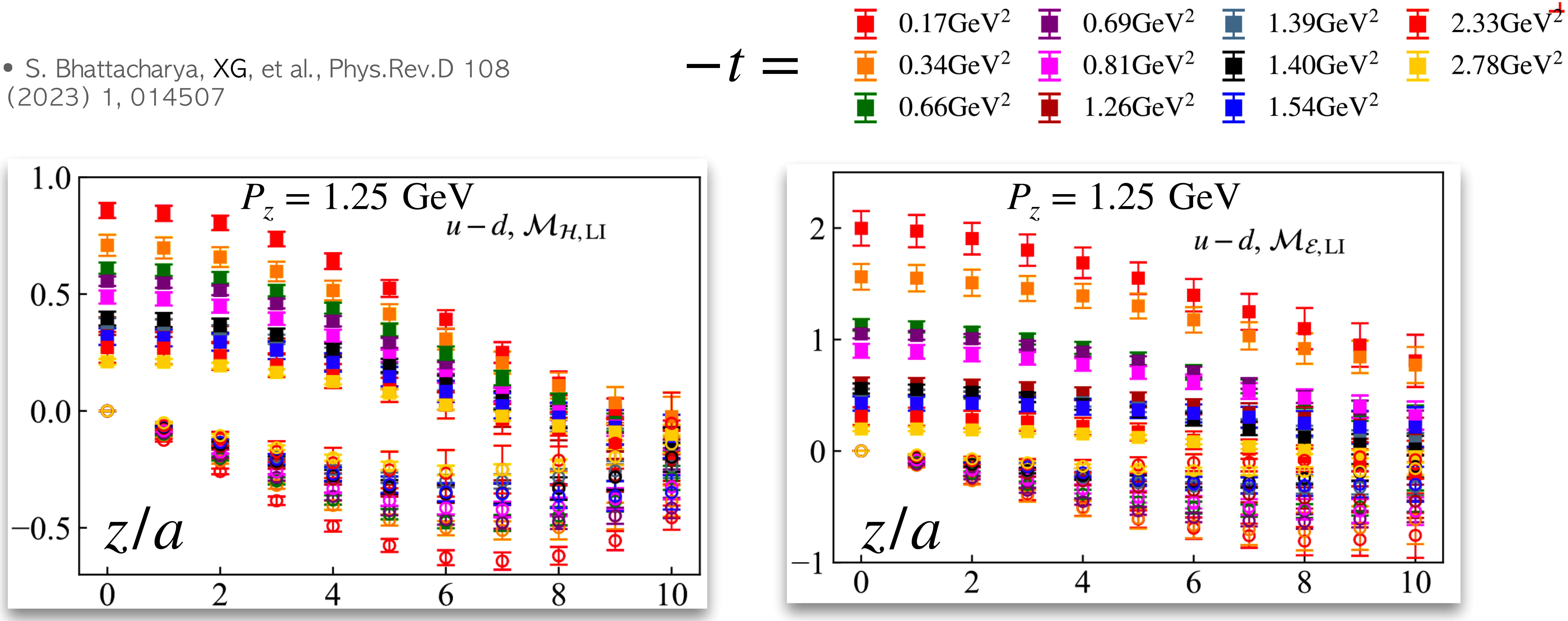
$C_n^{\overline{MS}}(\mu^2 z^2) = 1 + \alpha_s C^{(1)}(\mu^2 z^2) + \ldots$ up to NNLO

- **Lattice setup**

$m_\pi = 260$ MeV, $a = 0.093$ fm, $32^3 \times 64$, $N_f = 2+1+1$ twisted mass fermions

Renormalized matrix elements

- S. Bhattacharya, XG, et al., Phys.Rev.D 108 (2023) 1, 014507



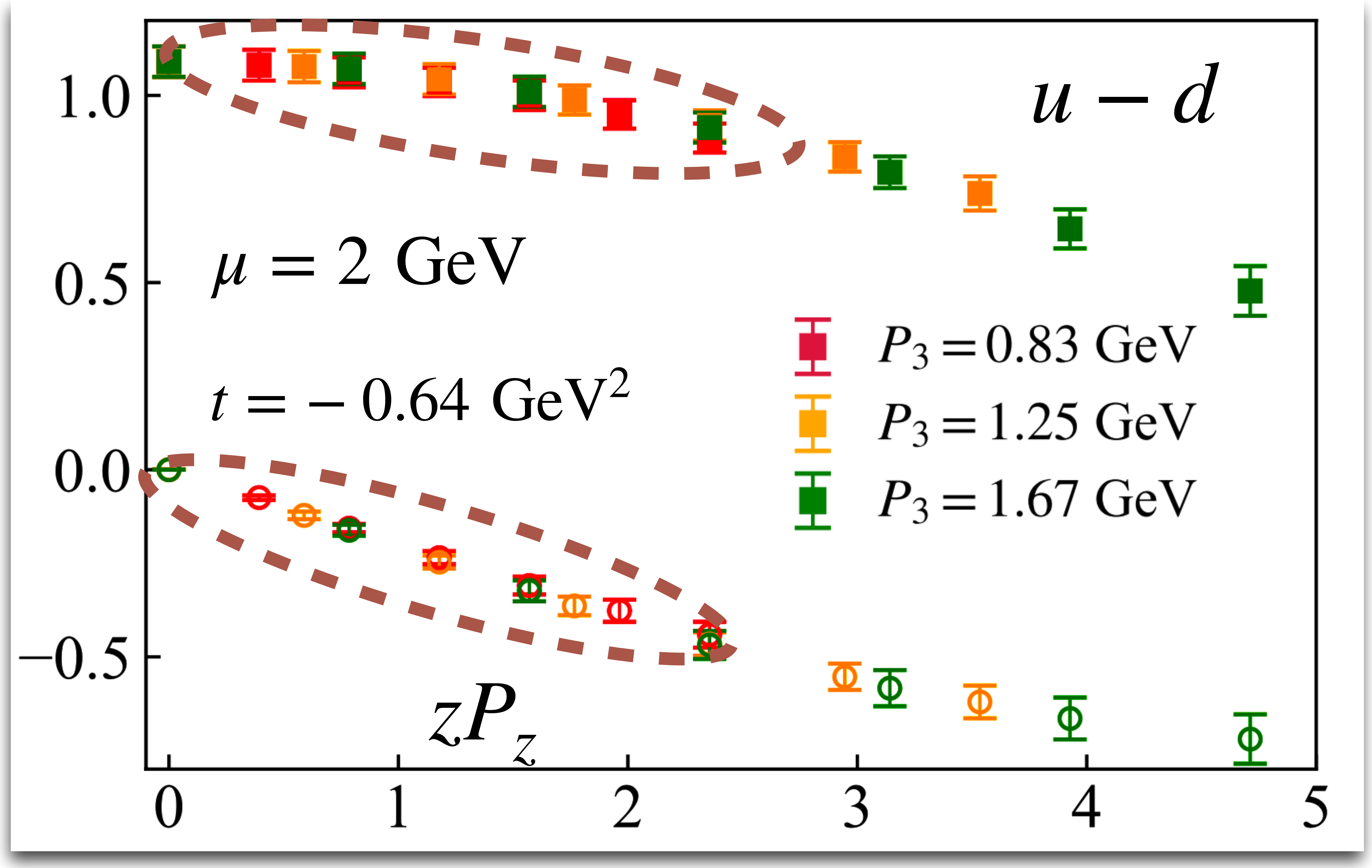
- filled symbols: real part, sensitive to even moments
- unfilled symbols: imaginary part, sensitive to odd moments

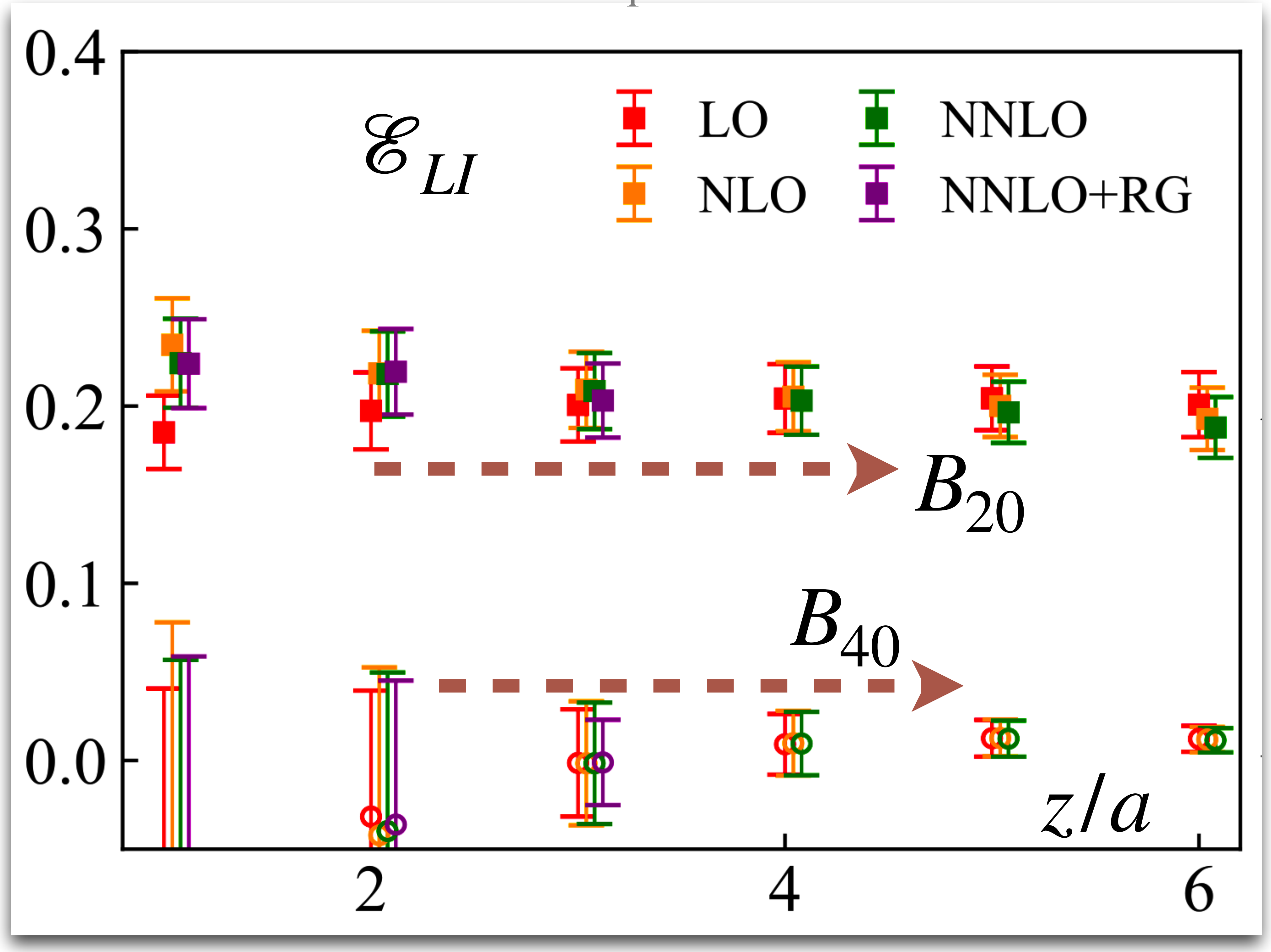
$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

SDF of qGPDs: LI definition

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$





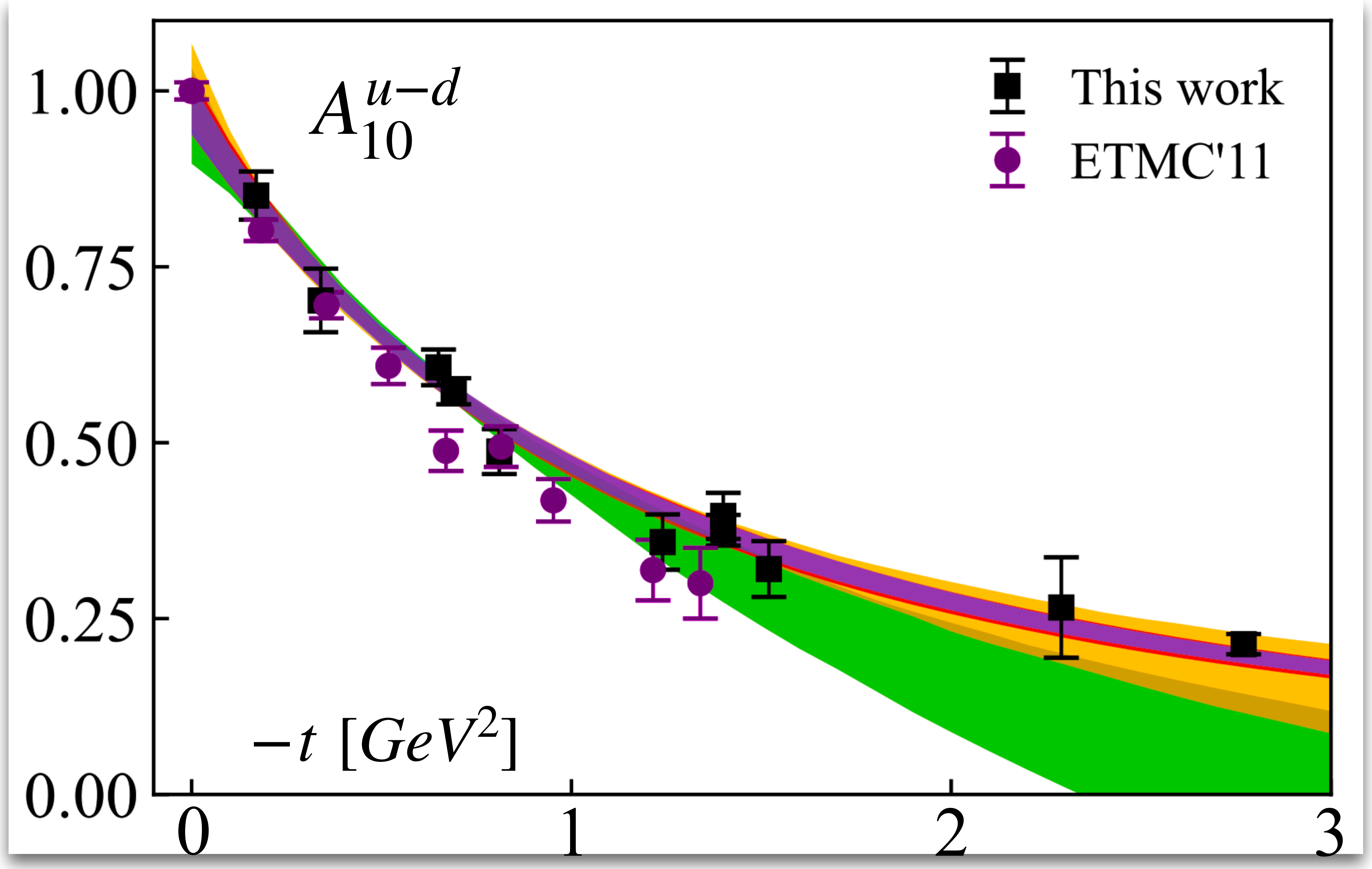
- Perturbative corrections $C_n(z^2\mu^2) = 1 + \mathcal{O}(\alpha_s)$
- Stable moments $\langle x^n \rangle(\mu)$

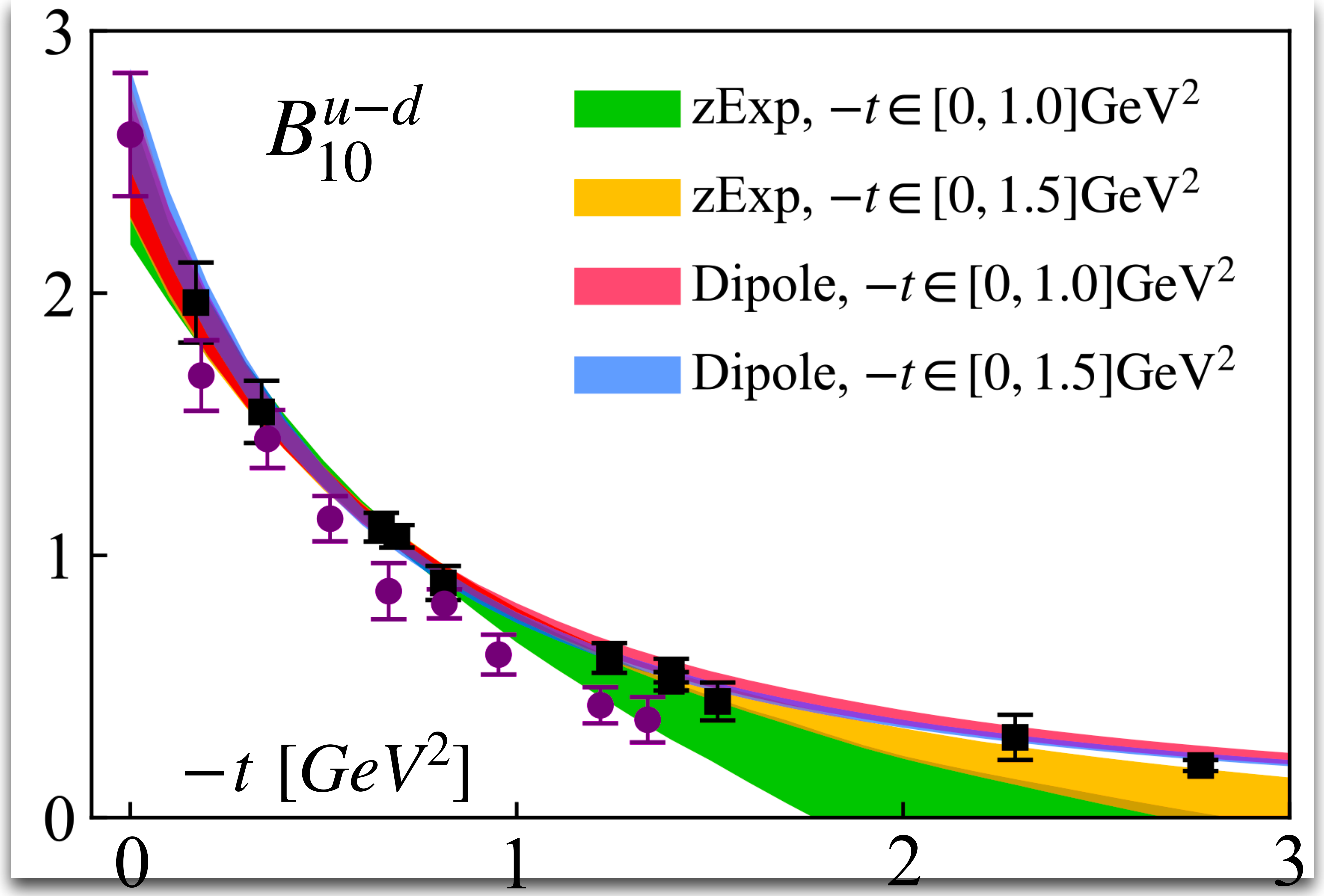
$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda^2_{\mathrm{QCD}})$$

Mellin moments of GPDs

$$\int_{-1}^{1} dx\, x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx\, x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$





- Good agreement with available traditional lattice QCD calculations of GPD moments using local operators (ETMC’11)

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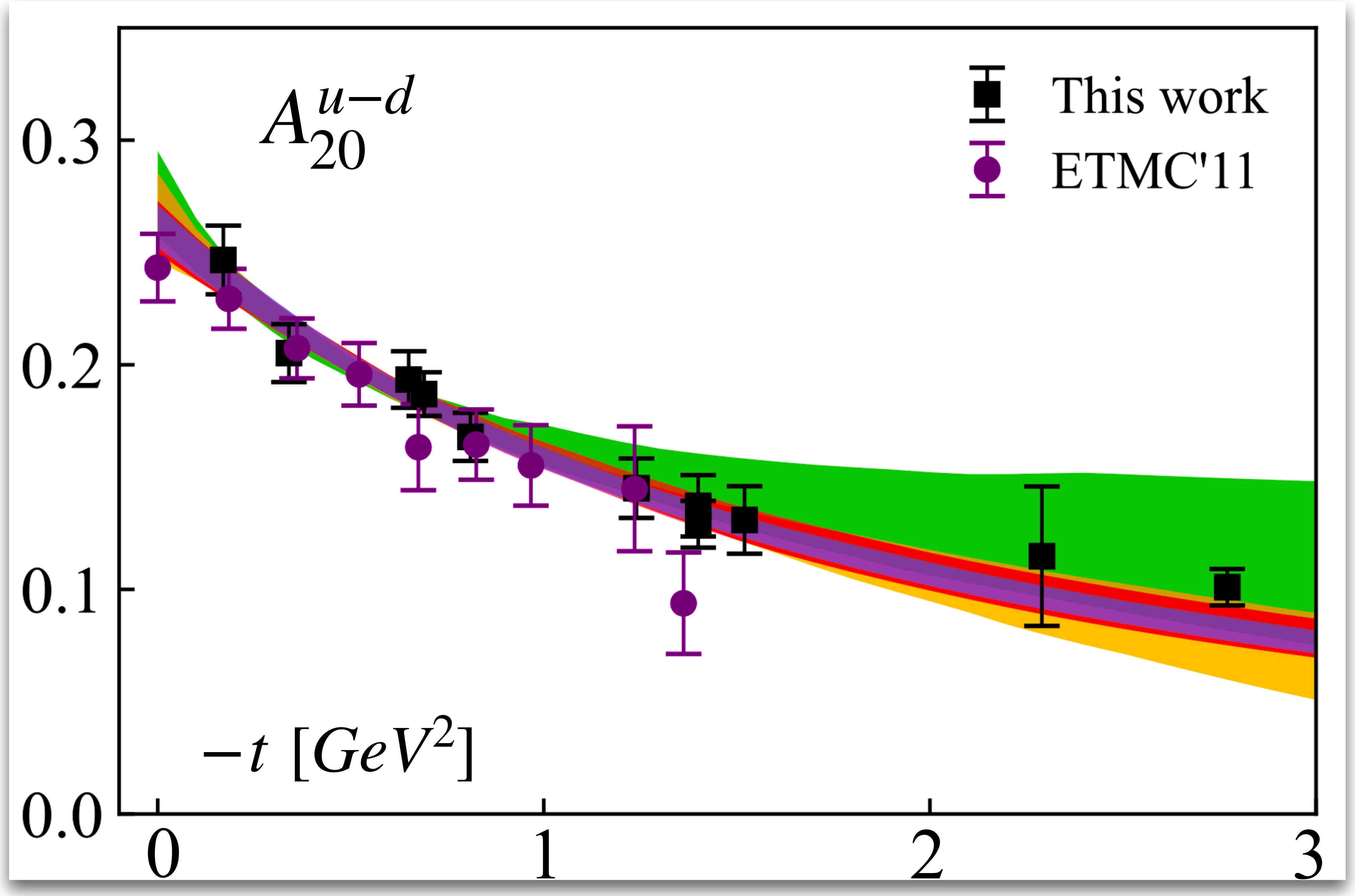
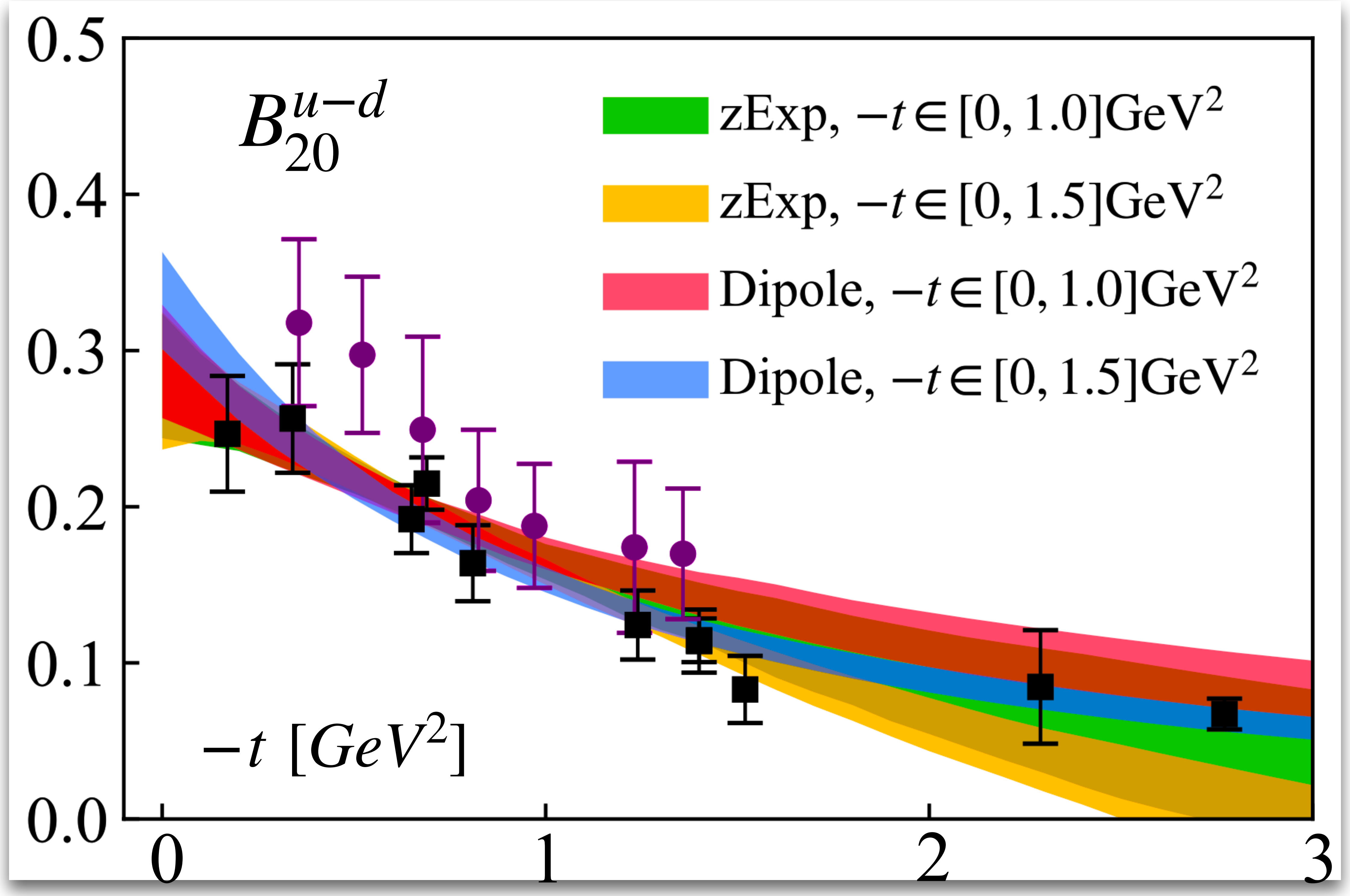
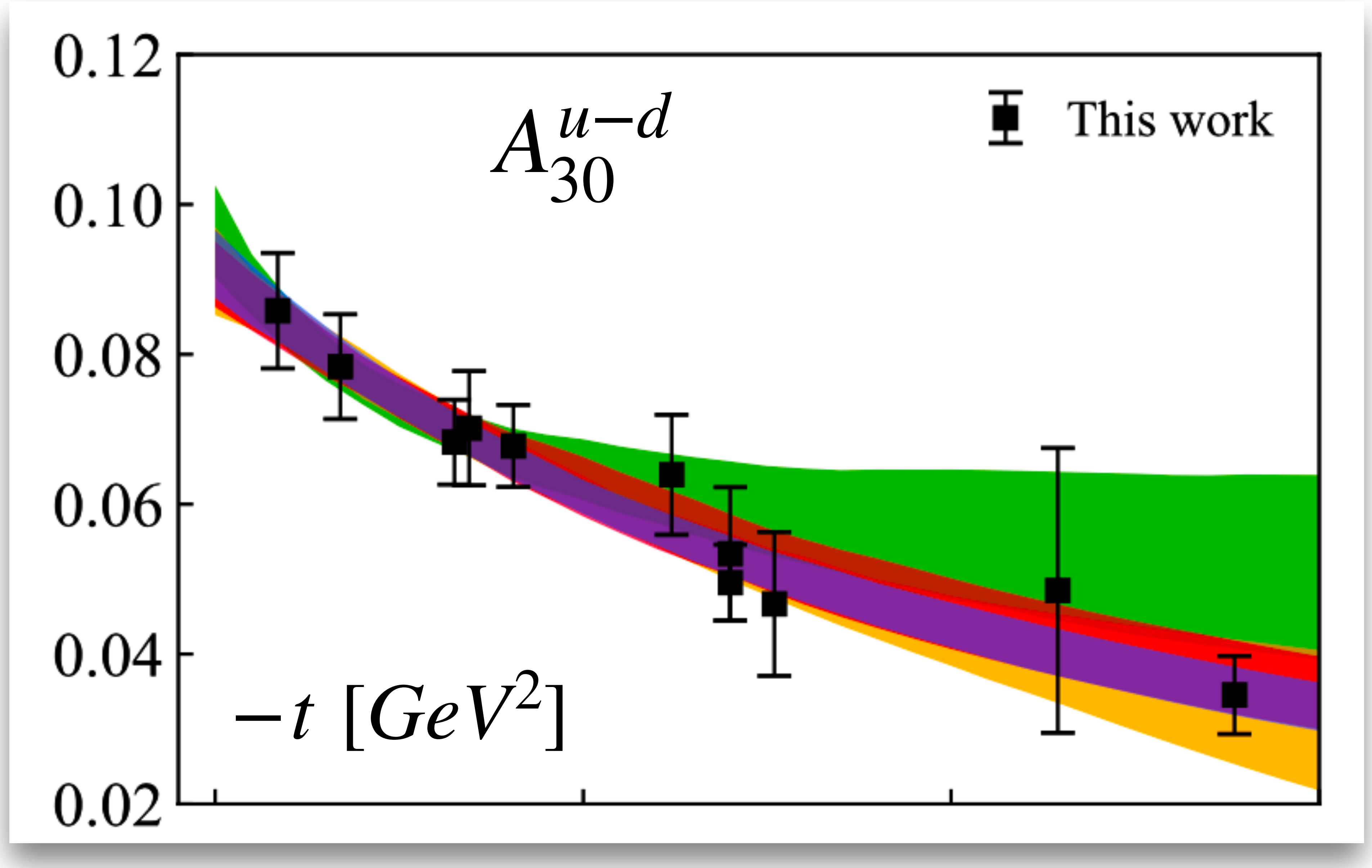
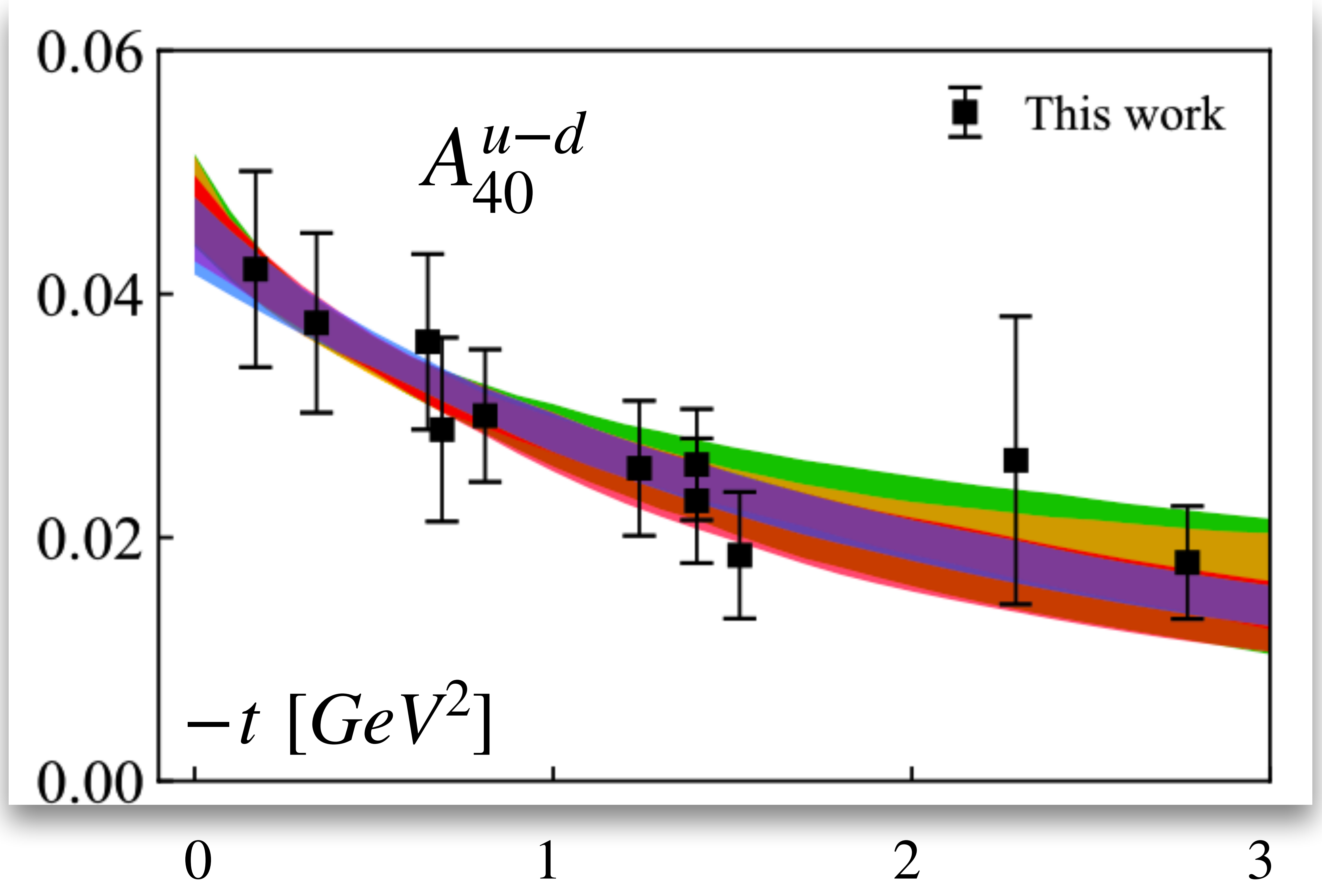


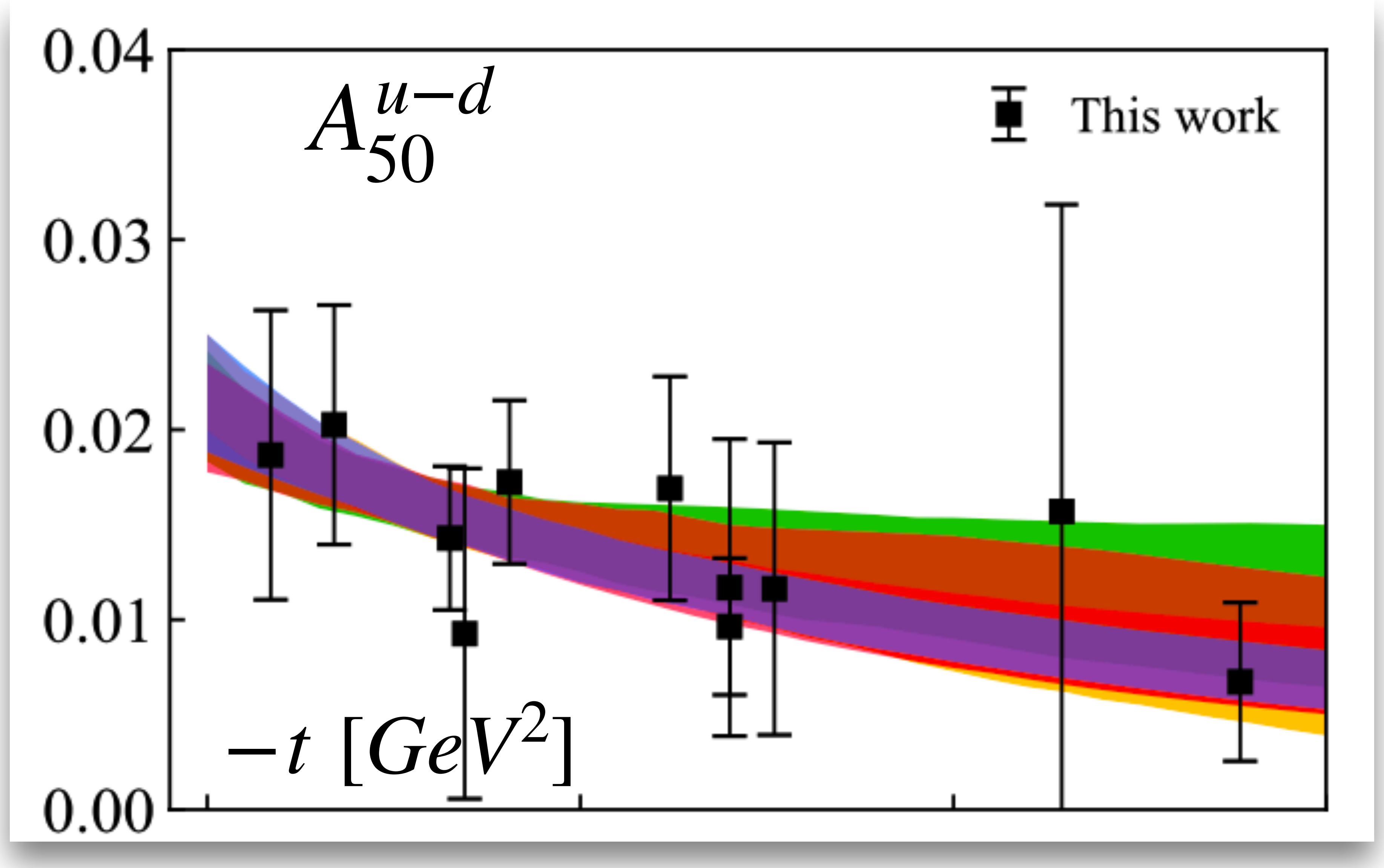


- Good agreement with available traditional lattice QCD calculations of GPD moments using local operators (ETMC'11)

Mellin moments of GPDs





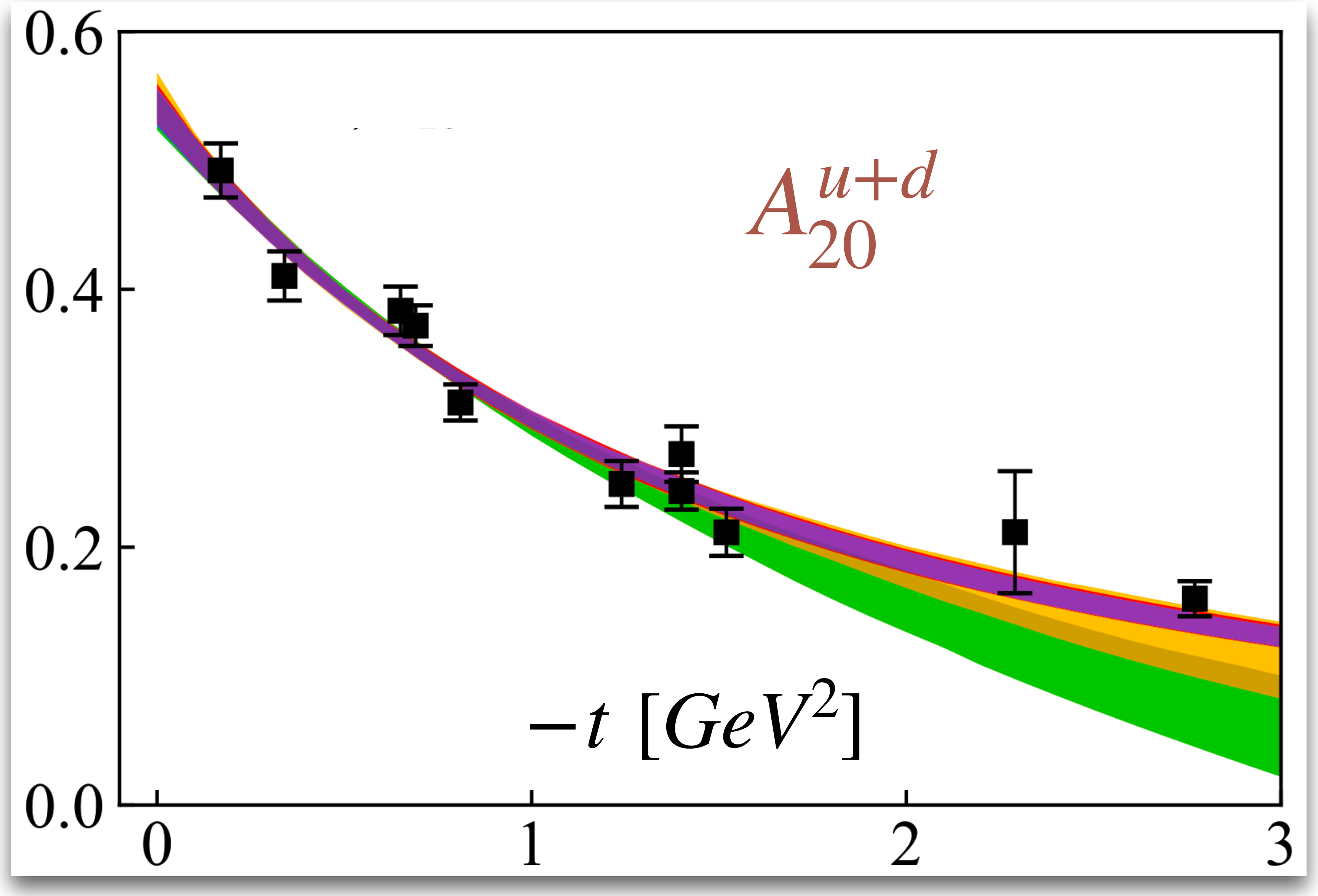


- Up to 5th moments of GPDs show reasonable signals and smooth $-t$ dependence.
- Higher moments can be constrained by increasing the hadron momentum.

Mellin moments of GPDs

$$\int_{-1}^{1} dx x^n H^q(x, \xi = 0, t) = A^q_{n+1,0}(t)$$

$$\int_{-1}^{1} dx x^n E^q(x, \xi = 0, t) = B^q_{n+1,0}(t)$$

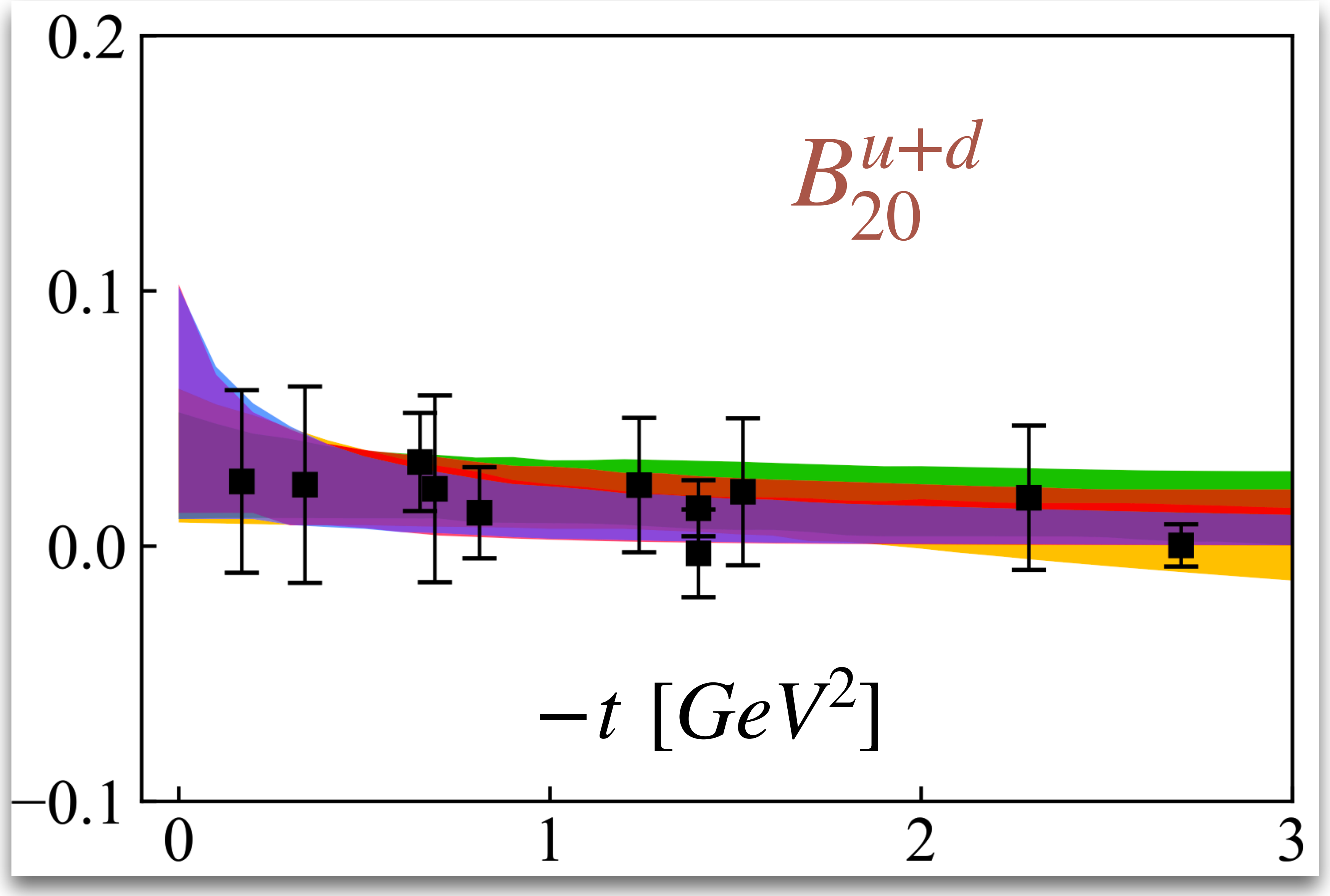


- 2nd moments: Gravitational form factors

Ji sum rule: $$J^q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

$$J^{u-d} = 0.281(21)(11)$$

$$J^{u+d} = 0.296(22)(33)$$



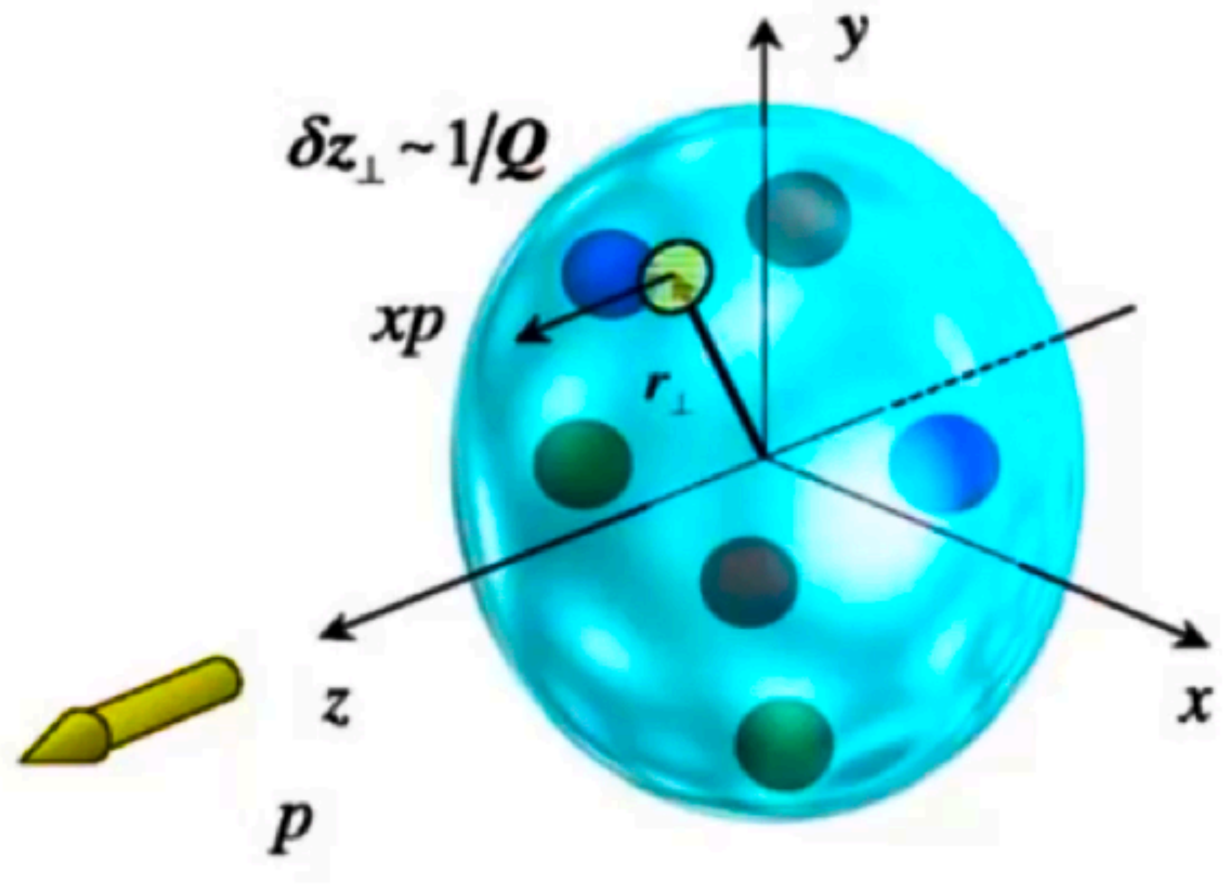
- $m_\pi = 260$ MeV, $a = 0.093$ fm
- Disconnected diagrams neglected

Impact parameter space interpretation

- Unpolarized quark inside unpolarized nucleon

$$q(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} H(x,-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

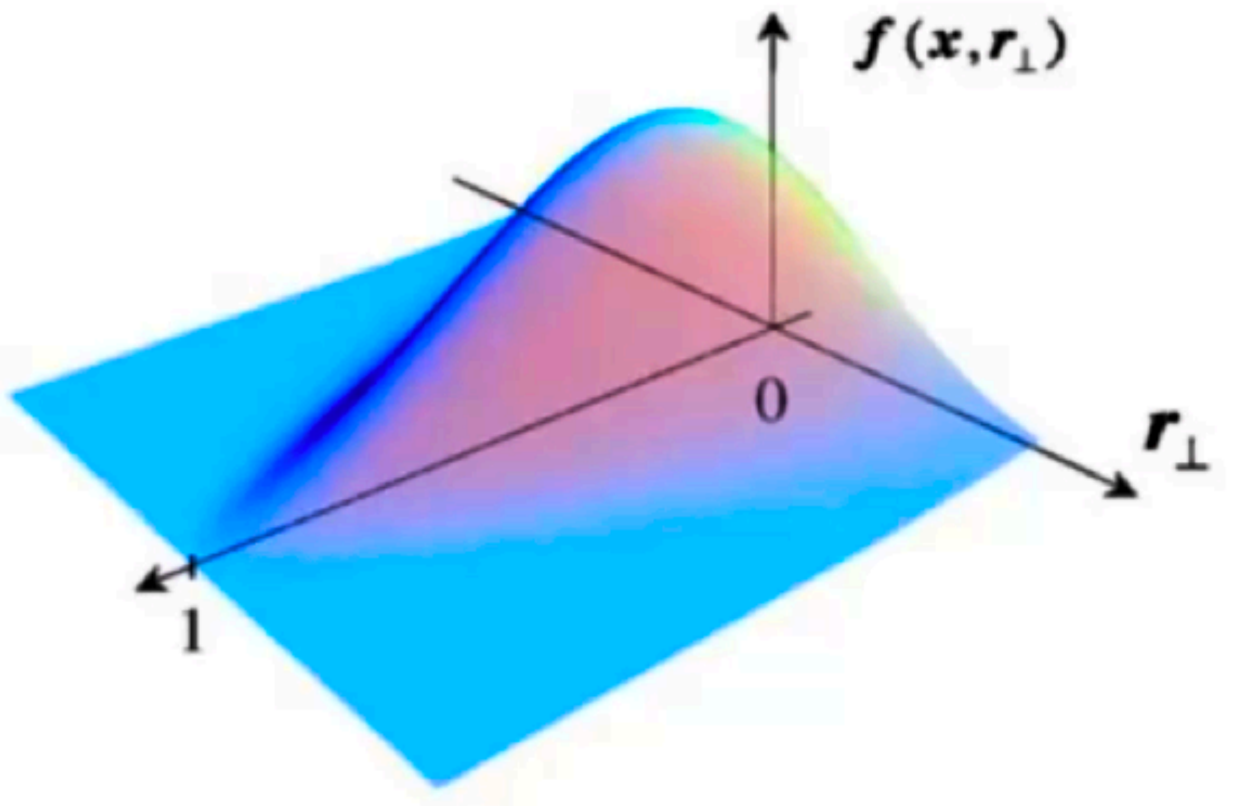
$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$



- Unpolarized quark inside transversely polarized nucleon

$$q^T(x,\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} \left[H(x,-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} E(x,-\overrightarrow{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

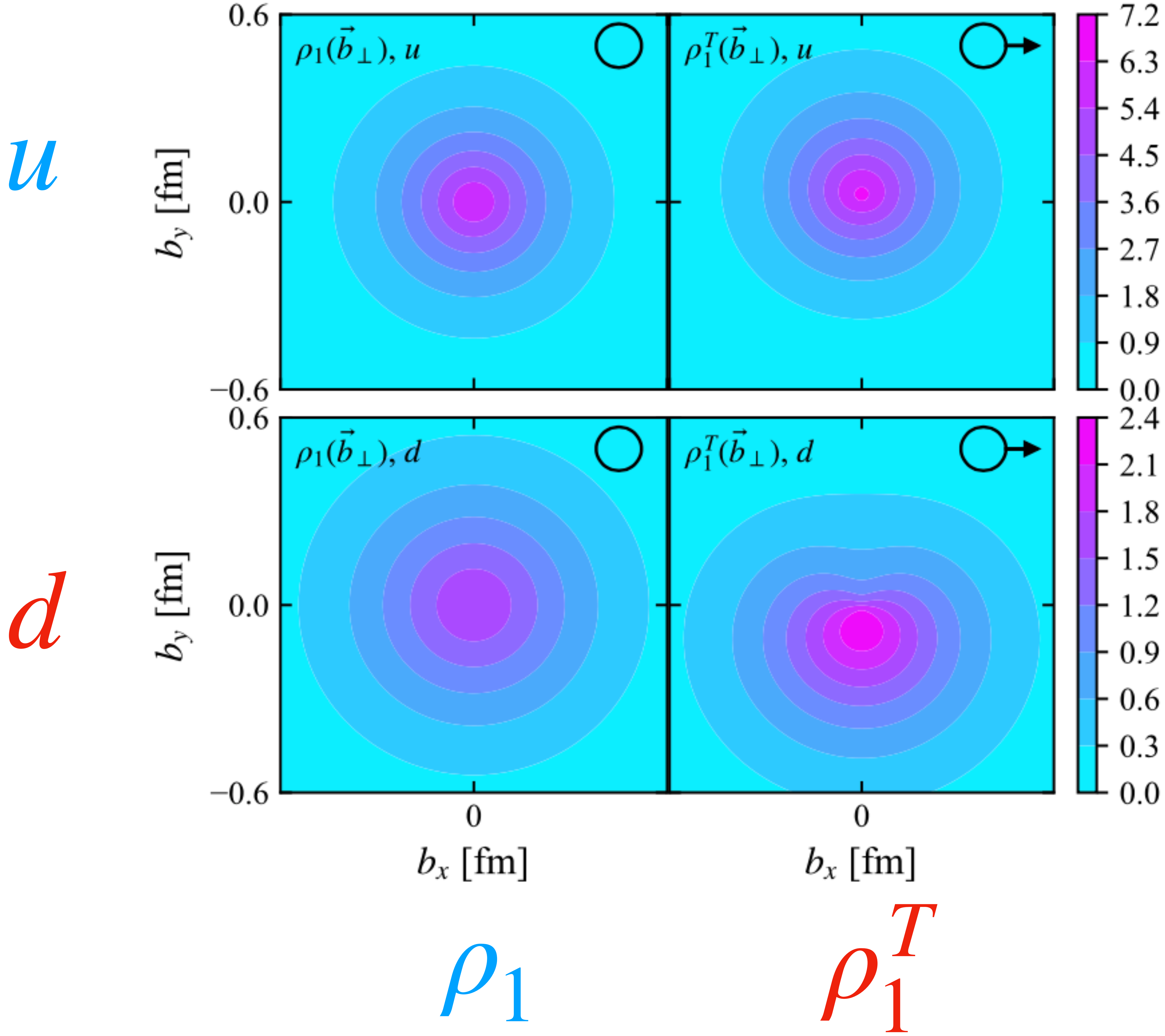
$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} \left[A_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2M} B_{n+1,0}(-\overrightarrow{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$



- Belitsky and Radyushkin: Phys.Rept. 418 (2005) 1-387

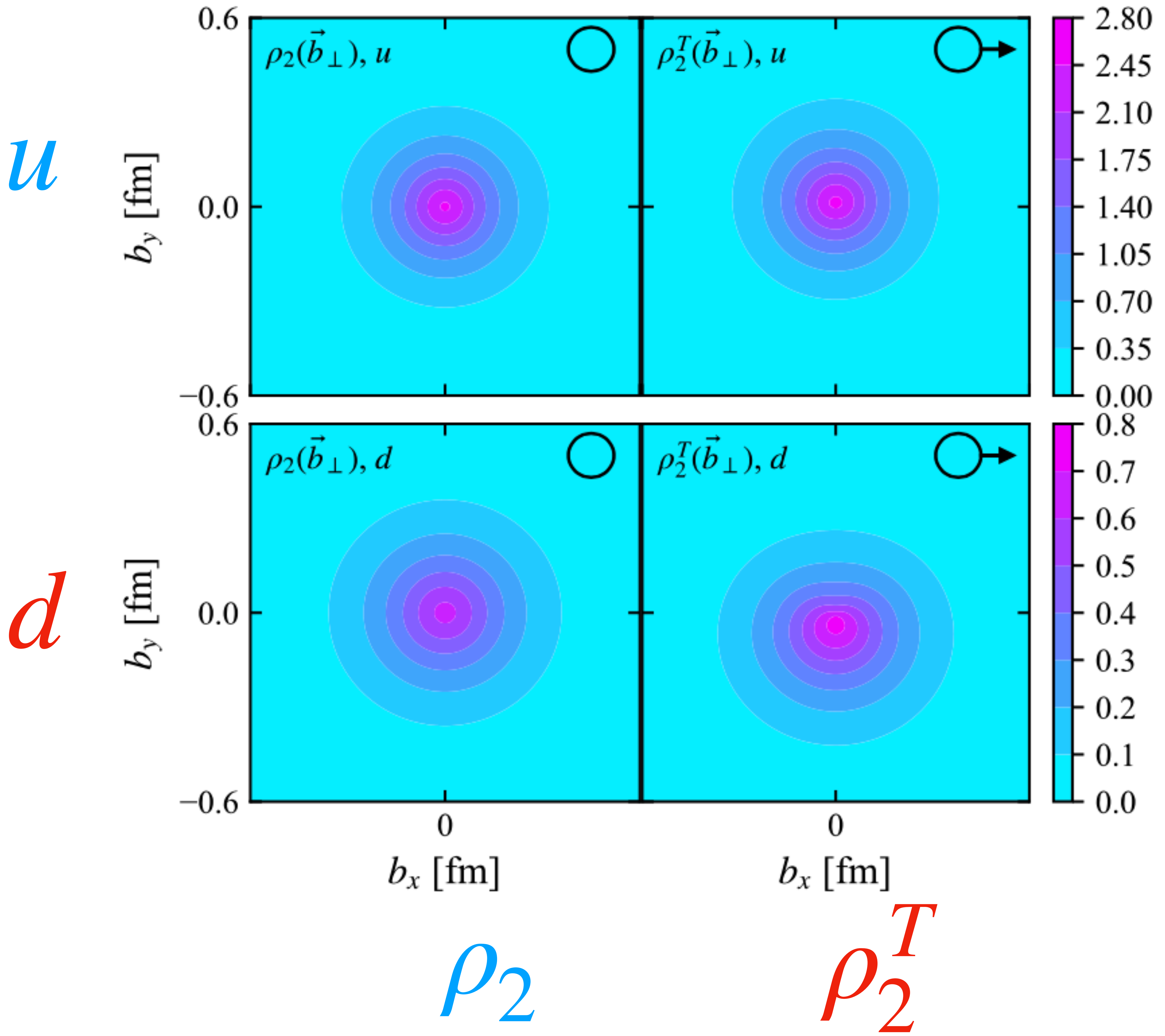
- Matthias Burkardt, Int.J.Mod.Phys.A 18 (2003) 173-208

Impact parameter space interpretation



- The **1st** moment: charge distribution
- **d** quark exhibits a broader distribution and smaller amplitudes
- When transversely polarized, the **u** and **d** quarks shift in different directions, with the **d** quarks showing larger distortion.

Impact parameter space interpretation

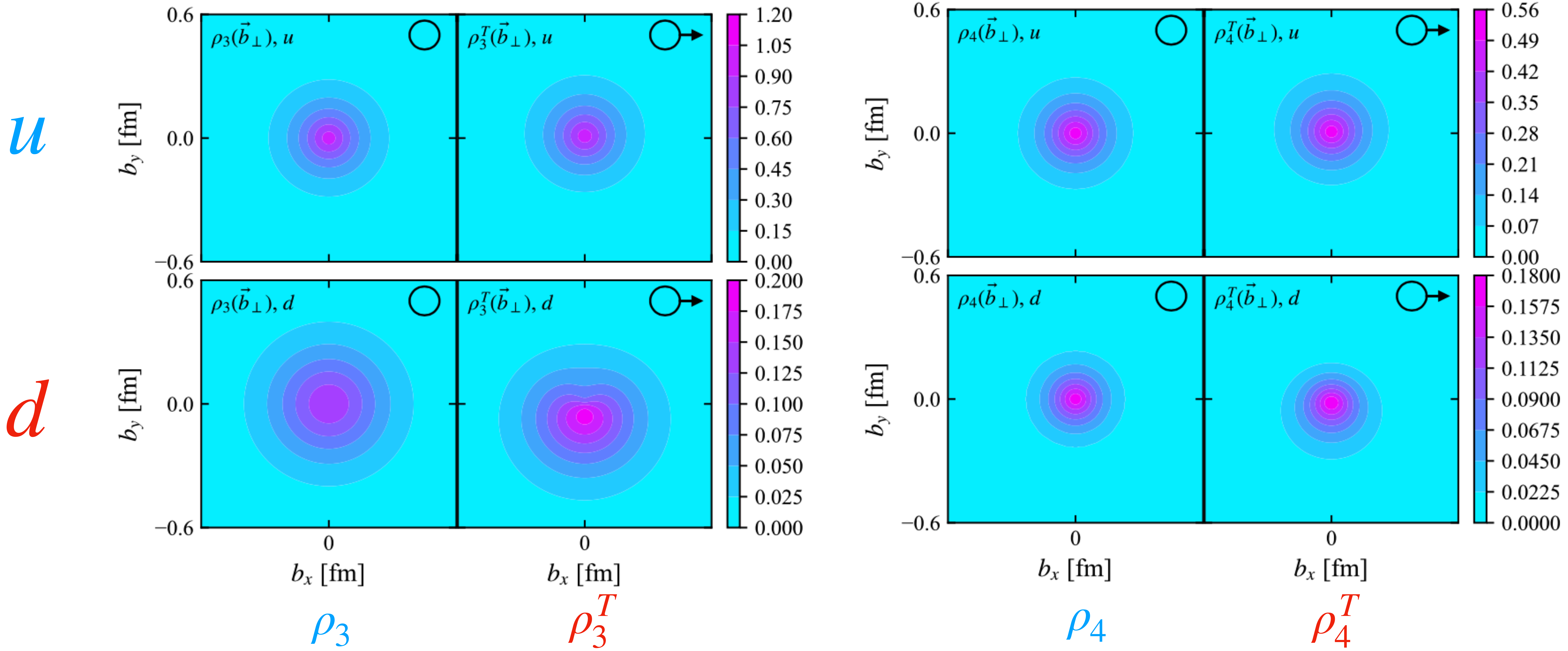


- The **2nd** moment: momentum distribution.
- The total contribution of **u** and **d** quarks to the transverse center of energy is small.

$$\sum_{u,d} \int d^2\vec{b}_\perp \, \vec{b}_\perp \rho_2^T(\vec{b}_\perp) = 1/(2M) B_{20}^{u+d}(0)$$

$$B_{20}^{u+d}(0) = 0.047(33)(65)$$

Impact parameter space interpretation



- Higher moments are weighted by x^n, exhibiting a sharper drop in the transverse distance.

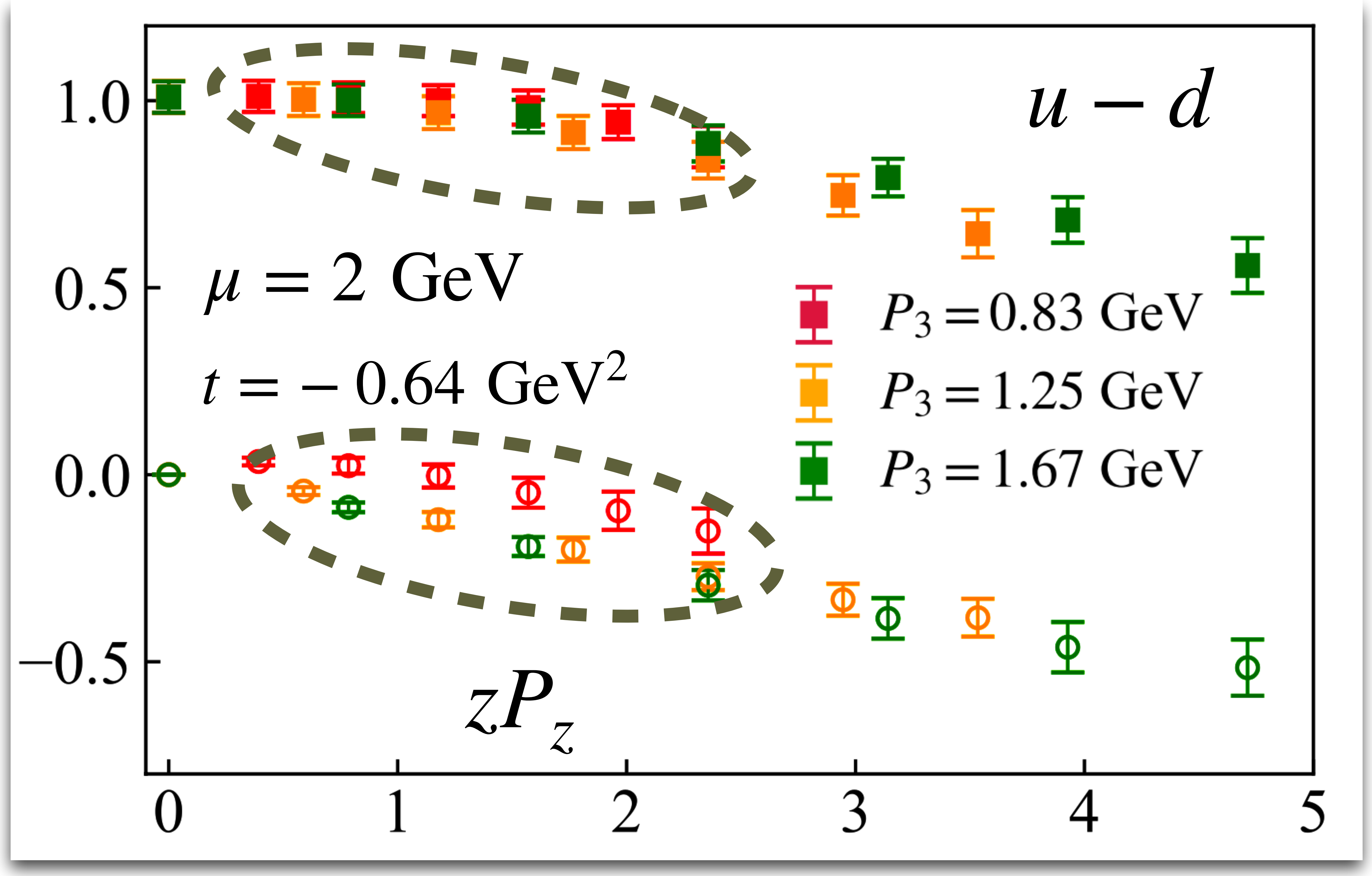
Summary and outlook

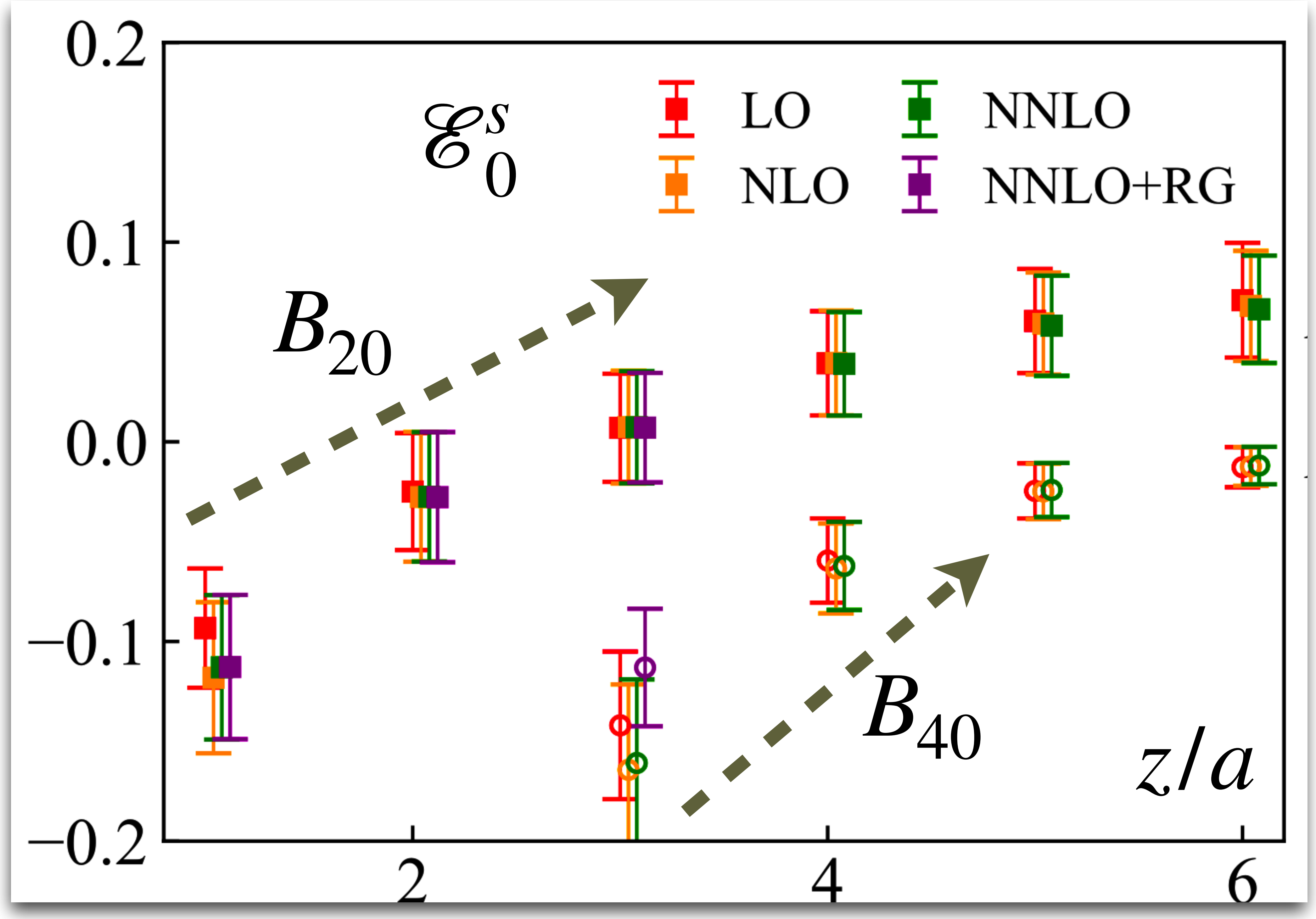
- We carried out lattice calculation of the quasi-GPD matrix elements of proton using the Lorentz invariant amplitudes.
- The matrix elements are renormalized in ratio scheme and the Mellin moments up to the 5th ones were extracted using the leading-twist short distance factorization.

  - The methods can be extended to other kind of GPDs and non-zero skewness.
  - Using hybrid renormalization and LaMET matching for x dependence.

Thanks for your attention!

SDF of qGPDs: γ_0 definition





- no scaling with zP_z
- not constant in z

$$\mathcal{M}(z^2, zP, \Delta^2) = \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} \frac{C_n(z^2\mu^2)}{C_n(z^2\mu^2)} \langle x^n \rangle(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$