#### Reconstruction of GPDs in the small x region

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September 25, 2023 – 25th International Spin Symposium (SPIN2023) – hldutrieux@wm.edu



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# Generalized parton distributions

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Spin-1/2 hadron, parton-helicity averaged quark GPDs  $H^q$  and  $E^q$  in the lightcone gauge [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

$$\int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle p_{2} \left| \bar{\psi}^{q} \left( -\frac{z}{2} \right) \gamma^{+} \psi^{q} \left( \frac{z}{2} \right) \left| p_{1} \right\rangle \right|_{z_{\perp}=0, z^{+}=0}$$

$$= \frac{1}{2P^{+}} \left( H^{q}(x,\xi,t) \bar{u}(p_{2}) \gamma^{+} u(p_{1}) + E^{q}(x,\xi,t) \bar{u}(p_{2}) \frac{i\sigma^{+\mu}\Delta_{\mu}}{2M} u(p_{1}) \right) \qquad (1)$$

$$p_2 - p_1 = \Delta, \ t = \Delta^2, \ P = \frac{1}{2}(p_1 + p_2), \ \xi = -\frac{\Delta^+}{2P^+}.$$
 (2)



#### Forward limit

$$\begin{cases} H^{q}(x,\xi=0,t=0) &= q(x)\Theta(x) - \bar{q}(-x)\Theta(-x) \\ H^{g}(x,\xi=0,t=0) &= xg(x)\Theta(x) - xg(-x)\Theta(-x) \end{cases}$$

where  $\Theta(x)$  is the Heaviside step function.

#### Elastic form factors

$$\int_{-1}^{1} \mathrm{d}x \, H^{q}(x,\xi,t) = F_{1}^{q}(t), \qquad \int_{-1}^{1} \mathrm{d}x \, E^{q}(x,\xi,t) = F_{2}^{q}(t)$$

 $\rightarrow$  independent of  $\xi$  !

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#### Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_{a}(x,\mathbf{b}_{\perp}) = \int \frac{\mathrm{d}^{2}\Delta_{\perp}}{(2\pi)^{2}} e^{-i\mathbf{b}_{\perp}\cdot\Delta_{\perp}} F^{a}(x,0,t=-\Delta_{\perp}^{2})$$
(5)

is the density of partons with plus-momentum x and transverse position  $\mathbf{b}_{\perp}$  from the center of plus momentum in a hadron  $\rightarrow$  hadron tomography

#### Gravitational form factors [Lorcé et al, 2017]

$$\langle p', s' | T^{\mu\nu}_{a} | p, s \rangle = \bar{u}(p', s') \left\{ \frac{P^{\mu}P^{\nu}}{M} A_{a}(t) + \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^{2}}{M} C_{a}(t) + M\eta^{\mu\nu}\bar{C}_{a}(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\rho}\Delta_{\rho}}{4M} [A_{a}(t) + B_{a}(t)] + \frac{P^{[\mu}i\sigma^{\nu]\rho}\Delta_{\rho}}{4M} D^{GFF}_{a}(t) \right\} u(p, s)$$
(6)

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## Generalized parton distributions

• Link between GFFs and GPDs thanks to e.g. for quarks

$$\int_{-1}^{1} dx \, x \, H^q(x,\xi,t) = A_q(t) + 4\xi^2 C_q(t) \tag{7}$$

$$\int_{-1}^{1} dx \, x \, E^q(x,\xi,t) = B_q(t) - 4\xi^2 C_q(t) \tag{8}$$

• Ji's sum rule [Ji, 1997]

$$J^{q} = \frac{1}{2} \left( A_{q}(0) + B_{q}(0) \right) \tag{9}$$

• Radial distributions of hadron matter properties [Polyakov, 2003]: in the Breit frame  $(\vec{P} = 0, t = -\vec{\Delta}^2)$ , radial pressure anisotropy profile

$$s_{a}(r) = -\frac{4M}{r^{2}} \int \frac{\mathrm{d}^{3}\vec{\Delta}}{(2\pi)^{3}} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^{2}} \frac{\mathrm{d}^{2}}{\mathrm{d}t^{2}} \Big[ t^{5/2} C_{a}(t) \Big]$$
(10)

# **Evolution of GPDs**

GPD's dependence on scale is given by renormalization group equations.

• In the limit  $\xi = 0$ , usual DGLAP equation:

$$\frac{\mathrm{d}f^{q_+}}{\mathrm{d}\log\mu}(x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi} \left\{ \int_x^1 \mathrm{d}y \, \frac{f^{q_+}(y,\mu) - f^{q_+}(x,\mu)}{y-x} \left[ 1 + \frac{x^2}{y^2} \right] + f^{q_+}(x,\mu) \left[ \frac{1}{2} + x + \log\left(\frac{(1-x)^2}{x}\right) \right] \right\}$$
(11)

• But in the limit  $x = \xi$ :

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$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\log\mu}(x,x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi} \bigg\{ \int_x^1 \mathrm{d}y \, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} \\ + H^{q+}(x,x,\mu) \left[\frac{3}{2} + \log\left(\frac{1-x}{2x}\right)\right] \bigg\}$$
(12)

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Even if one assumed that GPD = PDF at some scale, through the effect of evolution, an intrinsic  $\xi$  dependence would be generated! **SPIN2023** 

# Evolution of GPDs



Evolution of a PDF with GPD LO evolution for various values of  $\xi$ . From [Bertone et al, 2022]

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# Evolution of GPDs



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#### Vector meson production



LO depiction of  $J/\psi$  photoproduction.

The region  $x \sim \xi$  where significant perturbative  $\xi$  dependence occurs is crucial for the phenomenology of GPDs! Transfer of four-momentum to the hadron  $\rightarrow$  description in the framework of collinear

factorization by generalized parton distributions (GPDs) and non-relativistic QCD matrix element for moderate or small photon virtuality  $Q^2 = -q^2$ . Hard scale provided by  $m_V/2$  [Jones et al, 2015].

$$\xi = rac{p^+ - p'^+}{p^+ + p'^+} pprox rac{x_B}{2}, \ \ t = (p' - p)^2$$

#### Vector meson production

• Vector meson production amplitude up to NLO [Ivanov et al, 2004]:

$$\mathcal{F}(\xi,t) \propto \left(\frac{\langle O_1 \rangle_V}{m_V^3}\right)^{1/2} \sum_{a=q,g} \int_{-1}^1 \mathrm{d}x \ T^a(x,\xi) \ F^a(x,\xi,t) \tag{13}$$

where  $\langle O_1 \rangle_V^{1/2}$  is the NR QCD matrix element, T a hard-scattering kernel and  $F(x, \xi, t)$  is the GPD.

• The dominant region controlling the imaginary part of the amplitude is:

$$x \approx \xi \approx \frac{x_B}{2} \approx e^{-y} \frac{m_V}{2\sqrt{s}} \tag{14}$$

• At LHCb kinematics *e.g.*, typical values of  $x_B$  as low as  $\sim 10^{-5}$ .

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## GPDs at small skewness

 Significant asymmetry between incoming and outgoing (x + ξ ≫ x − ξ) parton momentum means very different dynamics, materialized *e.g.* by a very different behavior under evolution.



# GPDs at small skewness



- Evolution displaces the GPD from the large x to the small x region
- Significant ξ dependence arises perturbatively in the small x and ξ region
- But how does it compare to the unknown ξ dependence at initial scale?

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Obviously depends on the range of evolution, value of x and  $\xi$ , and profile of the known *t*-dependent PDF.

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$$\frac{1}{|x^{p_a}|}H^q(x,\xi,\mu) = \int_0^1 \frac{\mathrm{d}z}{z} \,\Gamma^{ab}\left(\frac{x}{z},\frac{\xi}{x};\mu_0,\mu\right) \frac{H^b(z,\xi,t,\mu_0)}{|z^{p_a}|} \tag{15}$$

The evolution operator can be interpreted as a parton-in-parton density  $\rightarrow$  probability of partonic splitting



- Since the evolution operator is a simple ordinary positive function, one can the **final scale GPD** as a true reweighting of the initial GPD  $\rightarrow$  gives a formal sense to the idea of "displacing" the distribution.
- Evaluation of the dominance of the perturbative  $\xi$  dependence over the initial unknown  $\xi$  dependence:
  - Start from a PDF at a low initial scale  $\mu_0 = 1$  GeV (need to be able to apply perturbation theory, so cannot go much below)
  - Produce an arbitrary  $\xi$  dependence at initial scale: pessimistic estimate 60% of uncertainty on the diagonal  $x = \xi$  vs. PDF
  - $\bullet\,$  Evolve to higher scale and observe how the LO evolution of the true GPD and the LO evolution of the model GPD = PDF differ
  - Both converge at very large scale as the  $\xi$  dependence of evolution overwhelms the initial unknown  $\xi$  dependence

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Example: working at t = 0, with the MMHT2014 PDF [Harland-Lang et al, 2015] at 1 GeV (prior knowledge of *t*-dependent PDF)



Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on  $x = \xi$  and  $\mu$ . Stronger  $\mu$  effect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, 2023]

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- Generating perturbatively the ξ dependence offers a well defined functional space for GPDs at small ξ which verifies the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)
- By subtracting the degree of freedom of the  $\xi$  dependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to this regularization.
- Limitations: small x resummation of the  $\xi$  dependence is not available although this resummation is probably needed and could change the picture.
- Other schemes, notably those arising from lattice computations, see a lesser radiation of small momentum fraction partons: stability of the perturbative expansion?
- What about the *t*-dependence?

#### Perspectives



[Shanahan, Detmold, 2018]

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- We propose a procedure to evaluate the systematic uncertainty associated to the skewness dependence of GPDs at small *x*, within the context of the perturbative collinear evolution of GPDs.
- This procedure provides a regularization of the deconvolution problem at small  $\xi$ , along with an estimate of the systematic uncertainty associated to the regularization.
- We point out that the uncertainty exhibits only a weak dependence on ξ due to the steep increase of PDFs at small x, but a major dependence on the hard scale of the process. Υ production provides a much safer channel to extract PDFs at small x with limited systematic uncertainty!

# Thank you for your attention!

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# Perspectives

- Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS is **time-like Compton scattering** (TCS) [Berger et al, 2002]. Although its measurement will reduce the uncertainty, especially on  $\operatorname{Re} \mathcal{H}$  [Jlab proposal PR12-12-001], and produce a valuable check of the universality of the GPD formalism, the similar nature of its convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.
- Deeply virtual meson production (DVMP) [Collins et al, 1997] is also an important source of knowledge on GPDs, with currently a larger lever arm in  $Q^2$ . The process involves form factors of the general form

$$\mathcal{F}(\xi,t) = \int_0^1 \mathrm{d}u \int_{-1}^1 \frac{\mathrm{d}x}{\xi} \,\phi(u) \,T\left(\frac{x}{\xi},u\right) \,F(x,\xi,t) \tag{16}$$

where  $\phi(u)$  is the leading-twist meson distribution amplitude (DA).

- At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the form factor.
- Situation at NLO remains to be clarified, it is foreseeable new shadow GPDs (dependent on the DA) could be generated also for this process.

# Shadow GPDs at next-to-leading order



Color plot of an NLO shadow GPD at initial scale 1 GeV<sup>2</sup>, and its evolution for  $\xi = 0.5$  up to 10<sup>6</sup> GeV<sup>2</sup> via APFEL++ and PARTONS [Bertone].

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