Reconstruction of GPDs in the small x region

Hervé Dutrieux
collaboration with K. Orginos (W&M), V. Bertone, M. Winn (Saclay) ...

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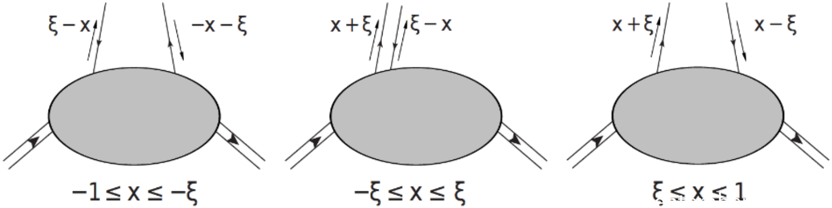
Generalized parton distributions

Spin-1=2 hadron, parton-helicity averaged quark GPDs H^q and E^q in the lightcone gauge [Müller et al, 1994], [Radyushkin, 1996], [Ji, 1997]

$$\frac{1}{2} Z \frac{\mathrm{d}z}{2} e^{ixP^+z} \quad p_2 \quad \bar{q} \quad \frac{z}{2} \quad + \quad q \quad \frac{z}{2} \quad p_1 \quad {}_{z_?=0;\ z^+=0}$$

$$= \frac{1}{2P^+} \quad H^q(x; \ ; t)\bar{u}(p_2) \ {}^+ u(p_1) + E^q(x; \ ; t)\bar{u}(p_2)\frac{i \ {}^+ \ \Delta}{2M} u(p_1) \tag{1}$$

$$p_2 \quad p_1 = \Delta; \ t = \Delta^2; \ P = \frac{1}{2}(p_1 + p_2); \quad = \frac{\Delta^+}{2P^+}: \tag{2}$$



Generalized parton distributions

Forward limit

$$\begin{cases} H^q(x; \quad = 0; t = 0) & = q(x)\Theta(x) \quad \bar{q}(\quad x)\Theta(\quad x) \\ H^g(x; \quad = 0; t = 0) & = xg(x)\Theta(x) \quad xg(\quad x)\Theta(\quad x) \end{cases} \quad (3)$$

where $\Theta(x)$ is the Heaviside step function.

Elastic form factors

$$\int_{1}^{1} dx\, H^q(x; \quad ; t) = F_1^q(t); \qquad \int_{1}^{1} dx\, E^q(x; \quad ; t) = F_2^q(t) \quad (4)$$

! **independent of** **!**

Generalized parton distributions

Impact parameter distribution (IPD) [Burkardt, 2000]

$$I_a(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} F^a(x, 0, t = -\Delta_\perp^2) \quad (5)$$

is the density of partons with plus-momentum x and transverse position $\mathbf{b}_\perp$ from the center of plus momentum in a hadron $\rightarrow$ **hadron tomography**

Gravitational form factors [Lorcé *et al*, 2017]

$$\langle p', s'|T_a^{\mu\nu}|p, s\rangle = \bar{u}(p', s')\left\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu} \bar{C}_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}[A_a(t) + B_a(t)] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t) \right\} u(p, s) \quad (6)$$

Link between **GFFs** and **GPDs** thanks to *e.g.* for quarks

$$\int_{1}^{1} \mathrm{d}x\, x\, H^q(x; \;; t) = A_q(t) + 4\; {}^2C_q(t) \tag{7}$$

$$\int_{1}^{1} \mathrm{d}x\, x\, E^q(x; \;; t) = B_q(t) \quad 4\; {}^2C_q(t) \tag{8}$$

Ji's sum rule **[Ji, 1997]**

$$J^q = \frac{1}{2}\left(A_q(0) + B_q(0)\right) \tag{9}$$

Radial distributions of hadron matter properties **[Polyakov, 2003]**: in the Breit frame ($\vec{P} = 0$, $t = \;\vec{\Delta}^2$), radial pressure anisotropy profile

$$s_a(r) = \frac{4M}{r^2} \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\;)^3} e^{\;i\vec{\Delta}\cdot\vec{r}}\, \frac{t^{\;1/2}}{M^2} \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[t^{5/2} C_a(t)\right] \tag{10}$$

Evolution of GPDs

GPD's dependence on scale is given by **renormalization group equations**.

In the limit $\ = 0$, usual DGLAP equation:

$$\frac{\mathrm{d} f^{q+}}{\mathrm{d} \log}(x;\ ) = \frac{C_F \ {}_s(\ )}{} \int_x^1 \mathrm{d}y \ \frac{f^{q+}(y;\ ) \quad f^{q+}(x;\ )}{y \quad x} \ 1 + \frac{x^2}{y^2}$$

$$+ f^{q+}(x;\ ) \ \frac{1}{2} + x + \log \ \frac{(1 \quad x)^2}{x} \qquad (11)$$

But in the limit $x = \ :$

$$\frac{\mathrm{d} H^{q+}}{\mathrm{d} \log}(x;x;\ ) = \frac{C_F \ {}_s(\ )}{} \int_x^1 \mathrm{d}y \ \frac{H^{q+}(y;x;\ ) \quad H^{q+}(x;x;\ )}{y \quad x}$$

$$+ H^{q+}(x;x;\ ) \ \frac{3}{2} + \log \ \frac{1 \quad x}{2x} \qquad (12)$$

Even if one assumed that GPD = PDF at some scale, through the effect of evolution, an intrinsic dependence would be generated!

Evolution of GPDs

Evolution of a PDF with GPD LO evolution for various values of From [Bertone et al, 2022]

Evolution of GPDs

Vector meson production

The region x where signi cant perturbative dependence occurs is crucial for the phenomenology of GPDs!

Transfer of four-momentum to the hadron description in the framework of collinear factorization by generalized parton distributions (GPDs) and non-relativistic QCD matrix element for moderate or small photon virtuality $Q^2 = q^2$. Hard scale provided by $m_V = 2$ [Jones et al, 2015].

LO depiction of J= photoproduction

$$= \frac{p^+ \quad p^{0+}}{p^+ + p^{0+}} \quad \frac{x_B}{2}; \quad t = (p^0 \quad p)^2$$

Vector meson production

Vector meson production amplitude up to NLO [Ivanov et al, 2004] :

$$\mathcal{F}(\xi;t) \propto \left(\frac{\langle O_1\rangle_V}{m_V^3}\right)^{1/2} \sum_{a=q,g} \int_{-1}^{1} dx\, T^a(x;\xi)\, F^a(x;\xi;t) \quad (13)$$

where $\langle O_1\rangle_V^{1/2}$ is the NR QCD matrix element, T^a a hard-scattering kernel and $F^a(x;\xi;t)$ is the GPD.

The dominant region controlling the imaginary part of the amplitude is:

$$x \sim \xi \approx \frac{x_B}{2} \approx e^{\pm y}\frac{m_V}{2\sqrt{s}} \quad (14)$$

At LHCb kinematics, e.g., typical values of x_B as low as $\sim 10^{-5}$.

GPDs at small skewness

Signi cant asymmetry between incoming and outgoing ($x +$ x) parton momentum means very di erent dynamics, materialized e.g. by a very di erent behavior under evolution.

No reason for the dependence to be negligible even at very small . Skewness ratios $\frac{H(x;x)}{H(x;0)}$ as large as 1.6 have been advocated at small x. [Frankfurt et al, 1998] [Shuvaev et al, 1999]

GPDs at small skewness

- Evolution displaces the GPD from the large x to the small x region
- Significant dependence arises perturbatively in the small x and region
- But how does it compare to the unknown dependence at initial scale?

Obviously depends on the range of evolution, value of x and , and profile of the known t-dependent PDF.

Evolution operators

$$\frac{1}{|x^{p_a}|} H^q(x;\ ;\ ) = \int_0^1 \frac{dz}{z} \ {}_{ab} \left(\frac{x}{z}; \frac{}{x};\ {}_0; \right) \frac{H^b(z;\ ; t;\ {}_0)}{|z^{p_a}|} \qquad (15)$$

The evolution operator can be interpreted as a parton-in-parton density probability of partonic splitting

Evolution operators

Since the evolution operator is a simple ordinary positive function, one can think the scale GPD as a true reweighting of the initial GPD ! gives a formal sense to the idea of \displacing" the distribution.

Evaluation of the dominance of the perturbative dependence over the initial unknown dependence:

- Start from a PDF at a low initial scale $_0$ = 1 GeV (need to be able to apply perturbation theory, so cannot go much below)
- Produce an arbitrary dependence at initial scale: pessimistic estimate { 60% of uncertain on the diagonal x = vs. PDF
- Evolve to higher scale and observe how the LO evolution of the true GPD and the LO evolution of the model GPD = PDF di er
- Both converge at very large scale as the dependence of evolution overwhelms the initial unknown dependence

Dominance of the perturbative dependence

Example: working at $t = 0$, with the MMHT2014 PDF [Harland-Lang et al, 2015] at 1 GeV (prior knowledge of t-dependent PDF)

Uncertainty on the diagonal of the light sea quarks (left) and gluons (right) depending on $x =$ and .
Stronger e ect for gluons, divergence of PDFs at small x visible.

[HD, Winn, Bertone, 2023]

Perspectives

- Generating perturbatively the dependence o ers a well de ned functional space for GPDs at small which veri es the main theoretical constraints (polynomiality of Mellin moments, positivity, limits, ...)
- By subtracting the degree of freedom of thedependence, we have regularized the deconvolution problem, and we have an evaluation of the uncertainty associated to thi regularization.
- Limitations: smallx resummation of the dependence is not available although this resummation is probably needed and could change the picture.
- Other schemes, notably those arising from lattice computations, see a lesser radiation small momentum fraction partons: stability of the perturbative expansion?
- What about the t-dependence?

Perspectives

[Shanahan, Detmold, 2018]

Conclusions

- We propose a procedure to evaluate the systematic uncertainty associated to the skewness dependence of GPDs at small x, within the context of the perturbative collinear evolution of GPDs.
- This procedure provides a regularization of the deconvolution problem at small, along with an estimate of the systematic uncertainty associated to the regularization.
- We point out that the uncertainty exhibits only a weak dependence on due to the steep increase of PDFs at small x, but a major dependence on the hard scale of the process. production provides a much safer channel to extract PDFs at small x with limited systematic uncertainty!

Thank you for your attention!

Perspectives

Other exclusive processes can be expressed in terms of GPDs. Close parent to DVCS
time-like Compton scattering (TCS) [Berger et al, 2002]. Although its measurement will
reduce the uncertainty, especially on Re H [Jlab proposal PR12-12-001], and produce a
valuable check of the universality of the GPD formalism, the similar nature of its
convolution (see [Müller et al, 2012]) makes it subject to the same shadow GPDs.

Deeply virtual meson production (DVMP) [Collins et al, 1997] is also an important
source of knowledge on GPDs, with currently a larger lever arm in Q^2. The process
involves form factors of the general form

$$F(\ ;t) = \int_0^1 du \int_1^1 \frac{dx}{\ } \ (u) T \ \frac{x}{\ }; u \ F(x;\ ;t) \qquad (16)$$

where (u) is the leading-twist meson distribution amplitude (DA).

At LO, the GPD and DA parts of the integral factorize and shadow GPDs cancel the fo
factor.

Situation at NLO remains to be clari ed, it is foreseeable new shadow GPDs (depende
on the DA) could be generated also for this process.