Results on proton GPDs from lattice QCD

Martha Constantinou

Temple University

25TH INTERNATIONAL SPIN PHYSICS SYMPOSIUM

September 24 – 29, 2023
Durham Convention Center
Durham, NC, USA

September 26, 2023

Recent results

PHYSICAL REVIEW LETTERS **125,** 262001 (2020)

Unpolarized and Helicity Generalized Parton Distributions of the Proton within Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1] Karl Jansen,[5] Aurora Scapellato,[3] and Fernanda Steffens[6]

PHYSICAL REVIEW D **105,** 034501 (2022)

Transversity GPDs of the proton from lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1,2] Karl Jansen,[5] Aurora Scapellato,[4] and Fernanda Steffens[6]

★ **Twist-2 GPDs: "traditional" calculations**

★ **Twist-3 GPDs**

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Chiral-even axial twist-3 GPDs of the proton from lattice QCD

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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

+ Joshua Miller

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K. Cichy , Wed @ 11 am

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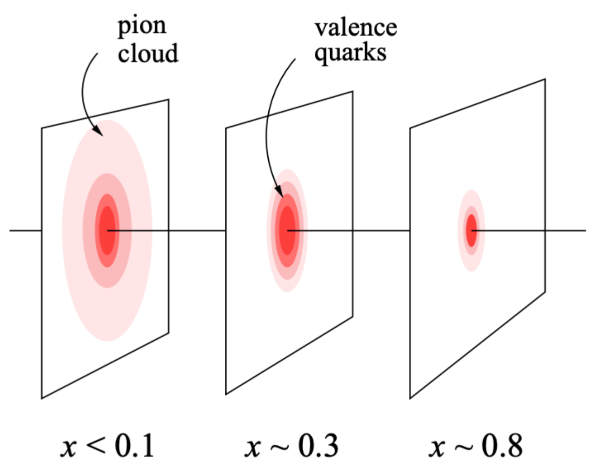
Motivation for GPDs studies



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{\mathrm{mom}} + 2_{\mathrm{coord}}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathscr{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

  (SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs

- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

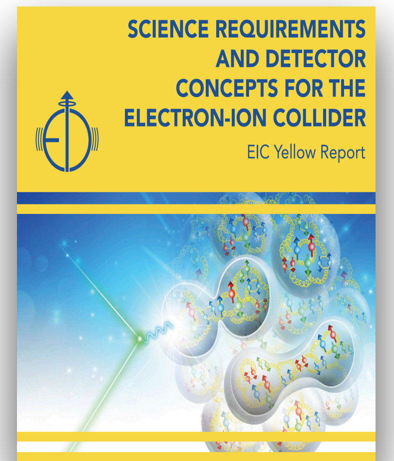
Hadron structure at core of nuclear physics

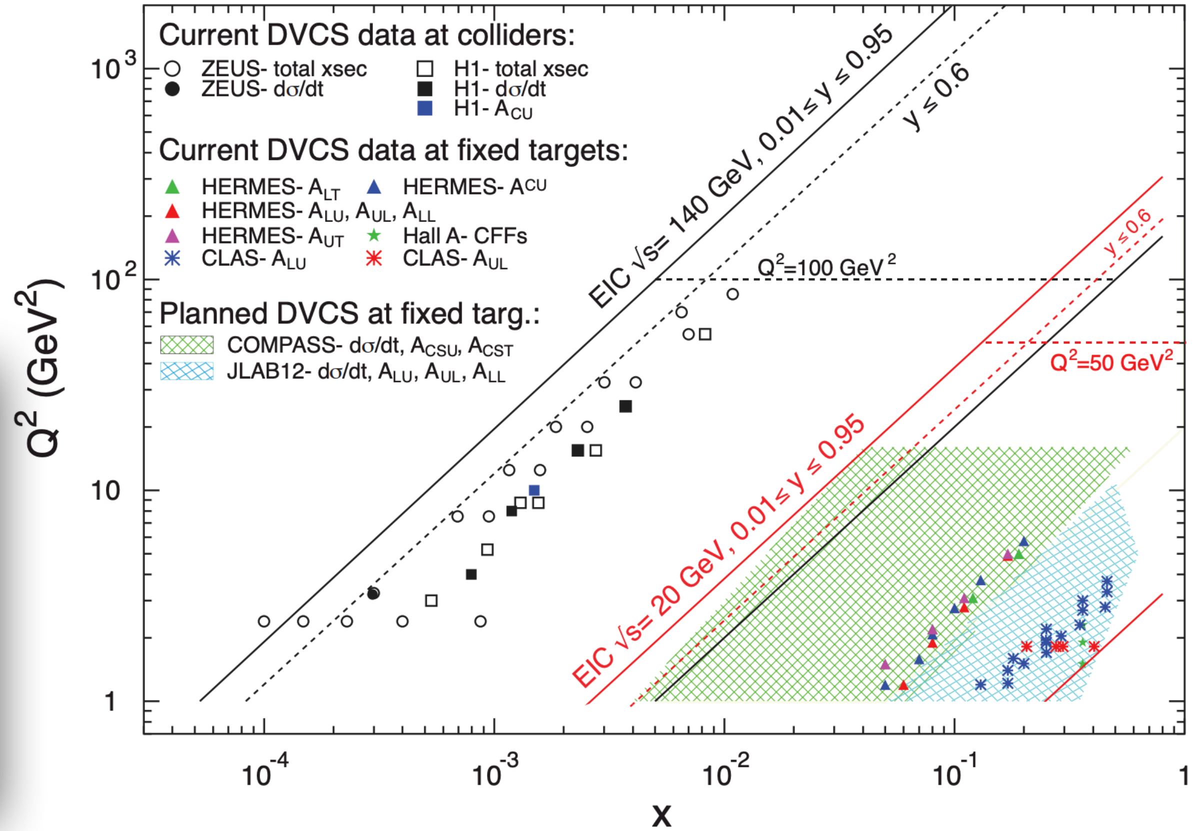
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





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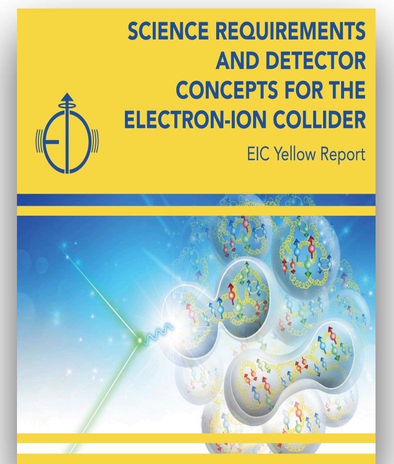

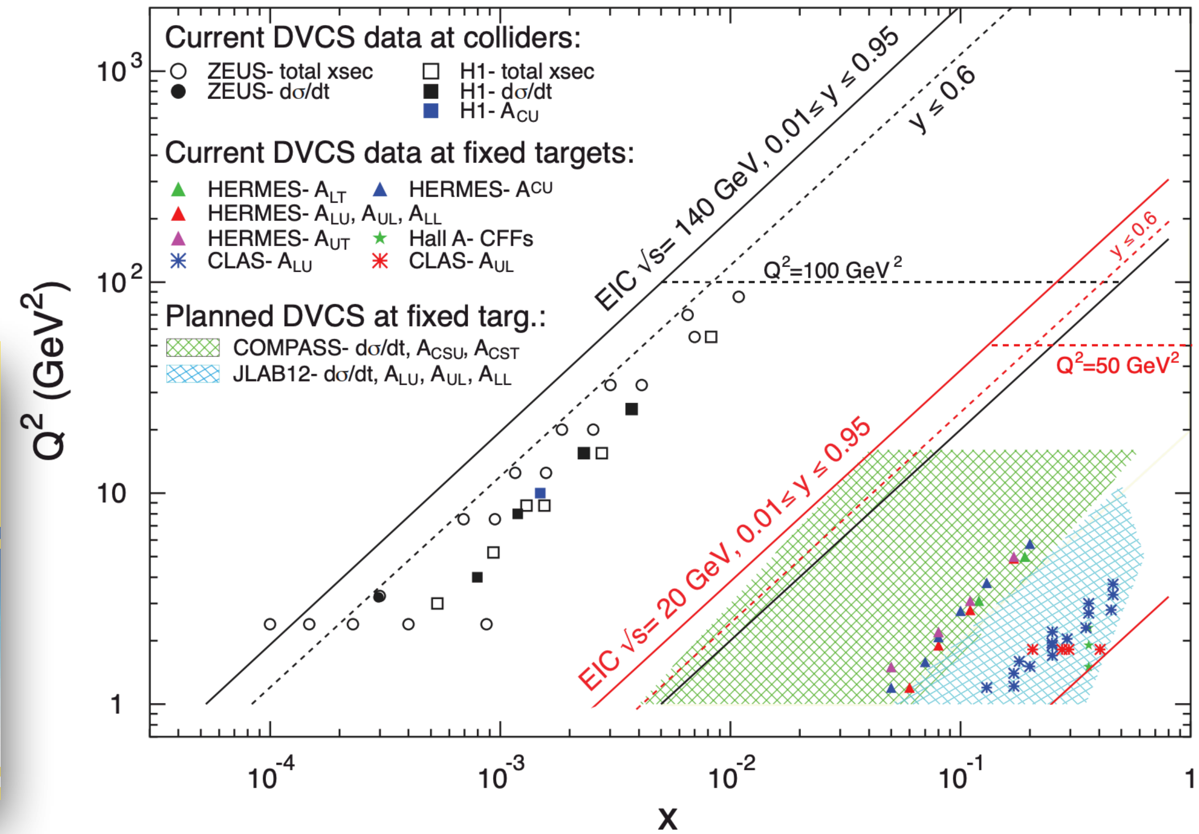

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QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number: DE-SC0023646

PHENOMENOLOGY · LATTICE QCD · THEORY

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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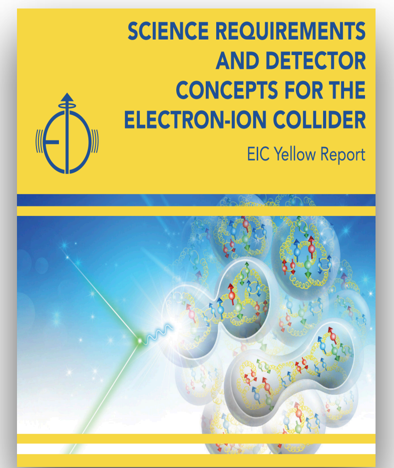

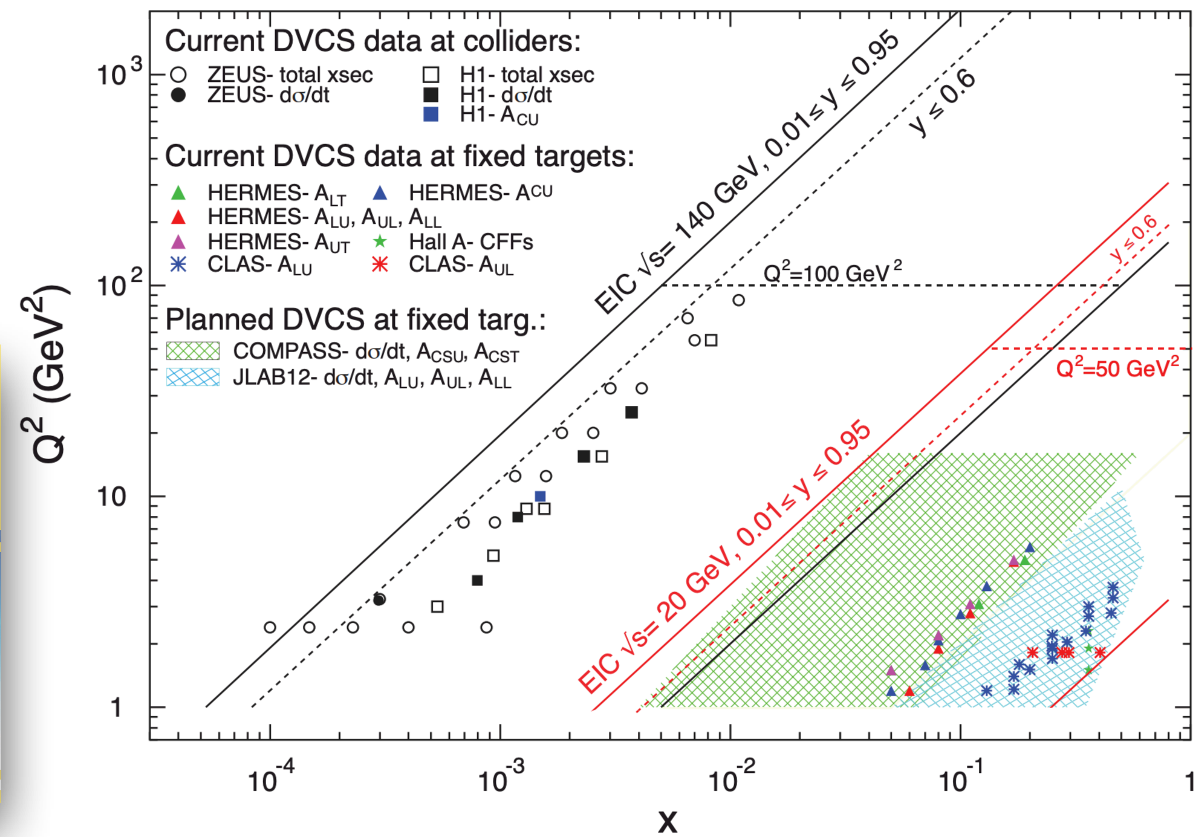

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Advances of lattice QCD are timely

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) = \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\underbrace{\left[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q\right]}_{\text{local operators}}$$

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\ldots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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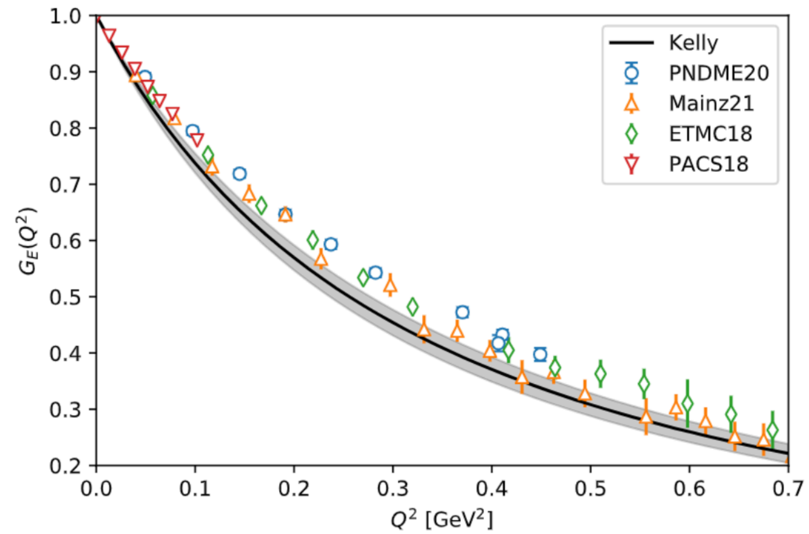

Wide -t range that comes at the cost of 1 (in the majority of cases)

Accessing information on GPDs

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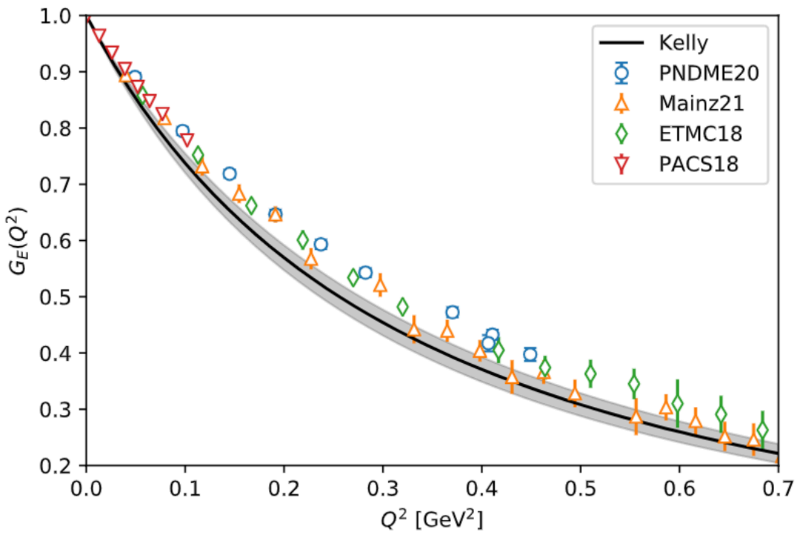

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★ **Matrix elements of non-local operators**
(quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\underbrace{\mathscr{W}(z,0)}_{\text{Wilson line}}\Psi(0)\,|\,N(P_i)\rangle_\mu$$

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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Accessing -t dependence:
Computationally intensive

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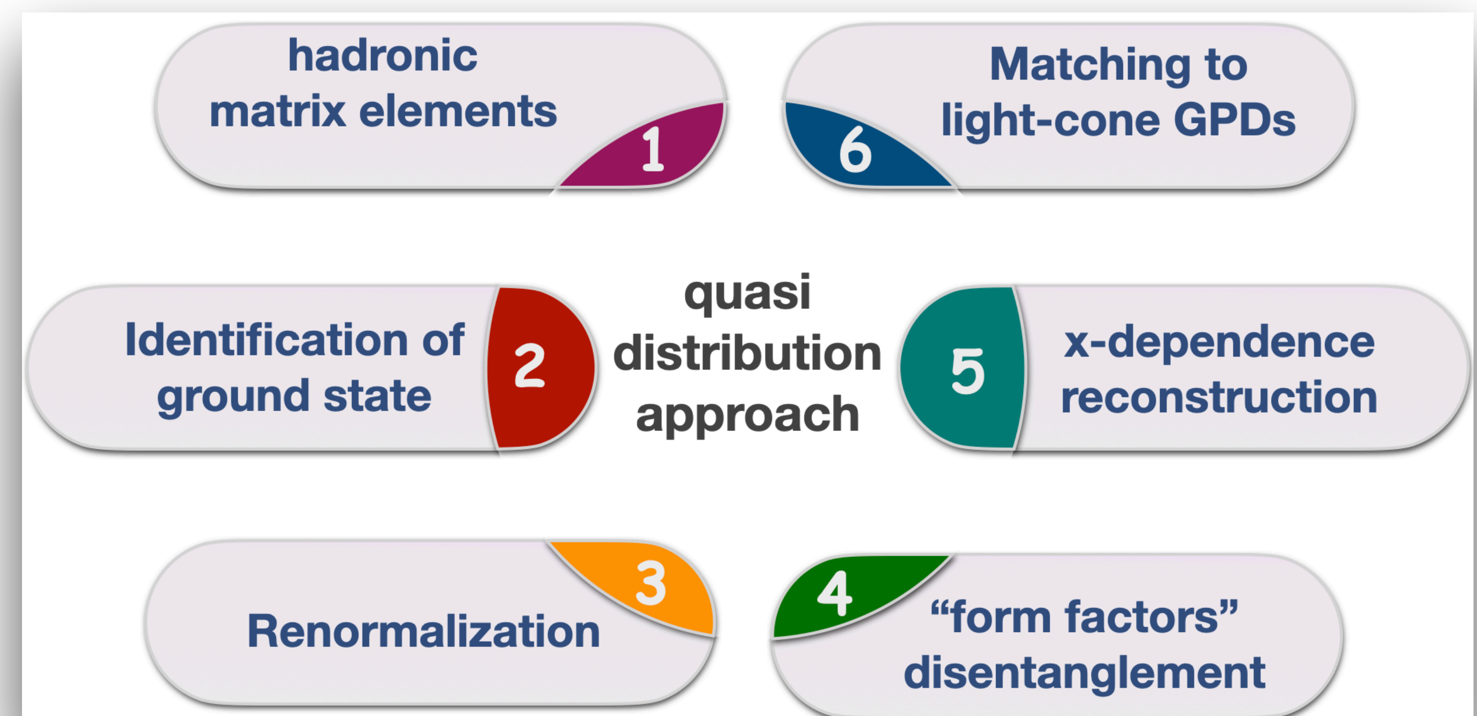

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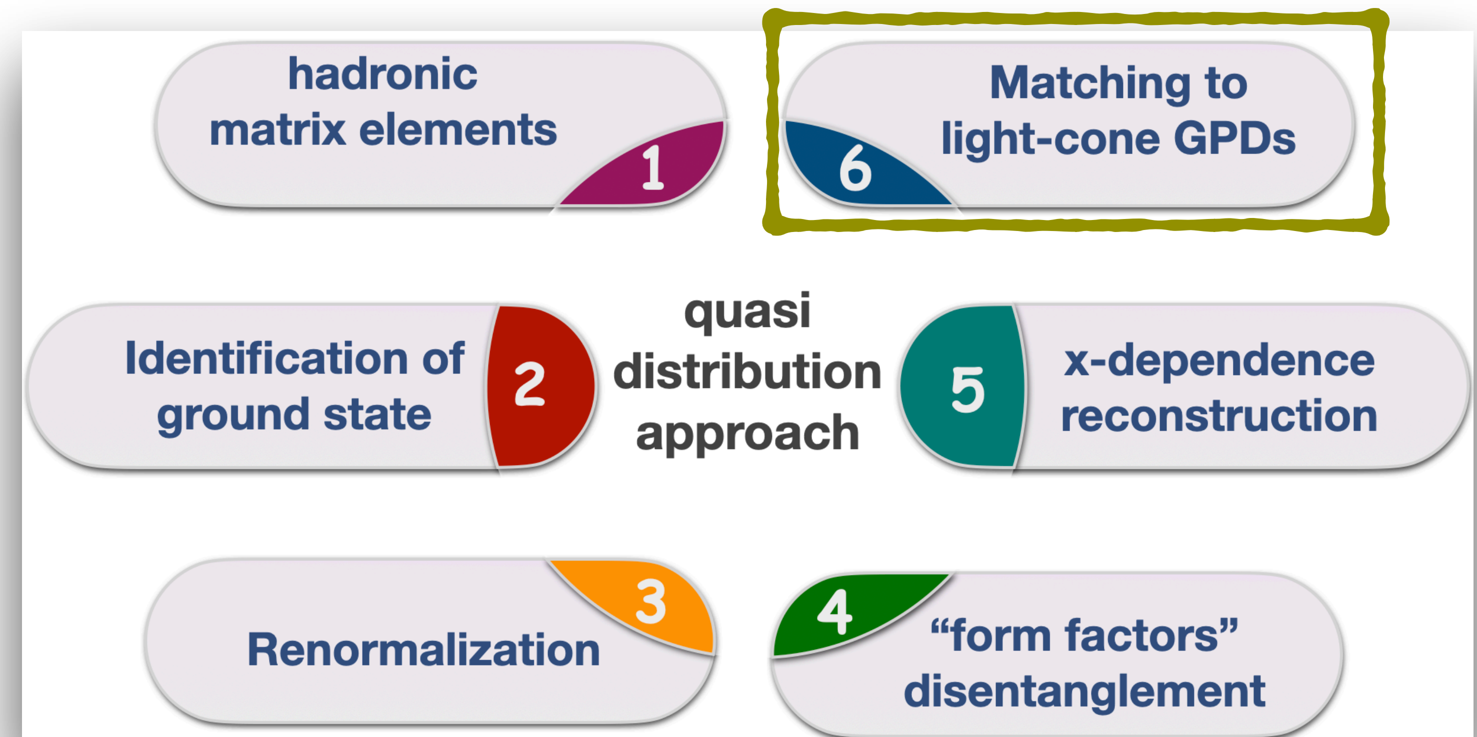

Traditional calculations of GPDs

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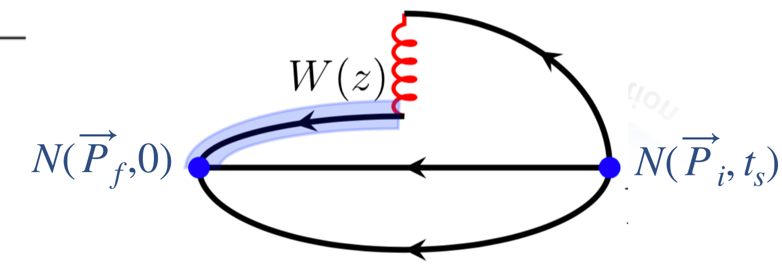
$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4



★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
±0.83	(0, 0, 0)	0	2	194	8	3104
±1.25	(0, 0, 0)	0	2	731	16	23392
±1.67	(0, 0, 0)	0	2	1644	64	210432
±0.83	(±2, 0, 0)	0.69	8	67	8	4288
±1.25	(±2, 0, 0)	0.69	8	249	8	15936
±1.67	(±2, 0, 0)	0.69	8	294	32	75264
±1.25	(±2, ±2, 0)	1.38	16	224	8	28672
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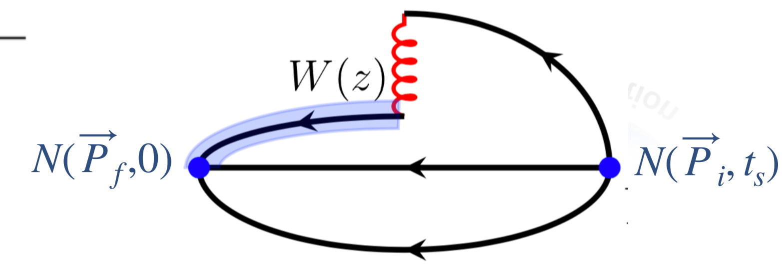

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Symmetric frame computationally expensive

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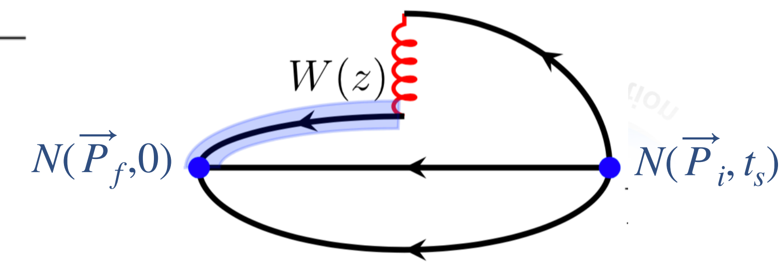

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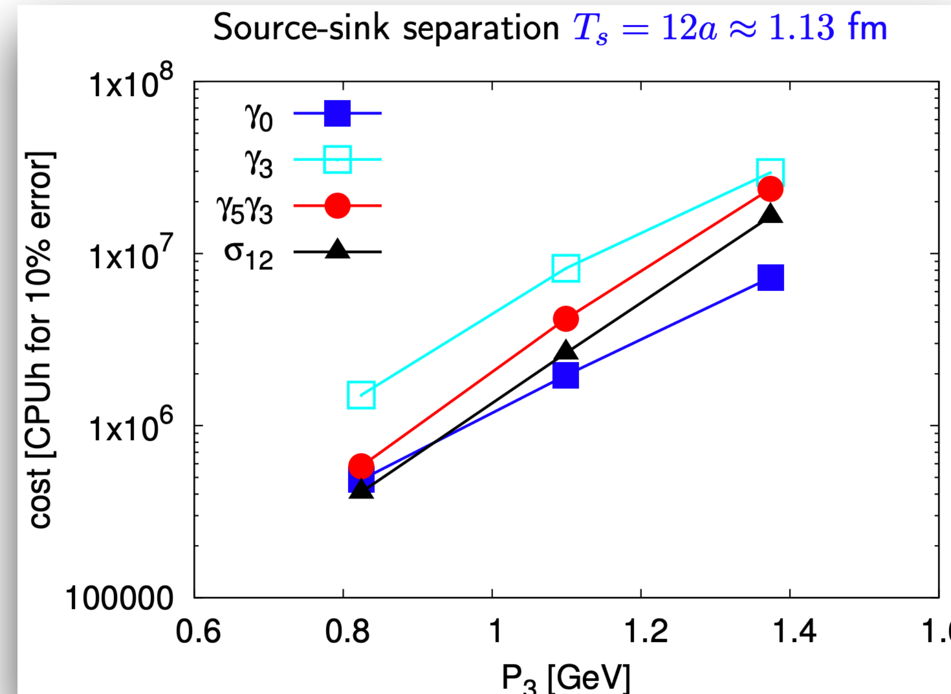

Symmetric frame computationally expensive

Suppressing gauge noise and reliably extracting the ground state comes at a significant computational cost

First lattice calculation of x-dependent GPDs

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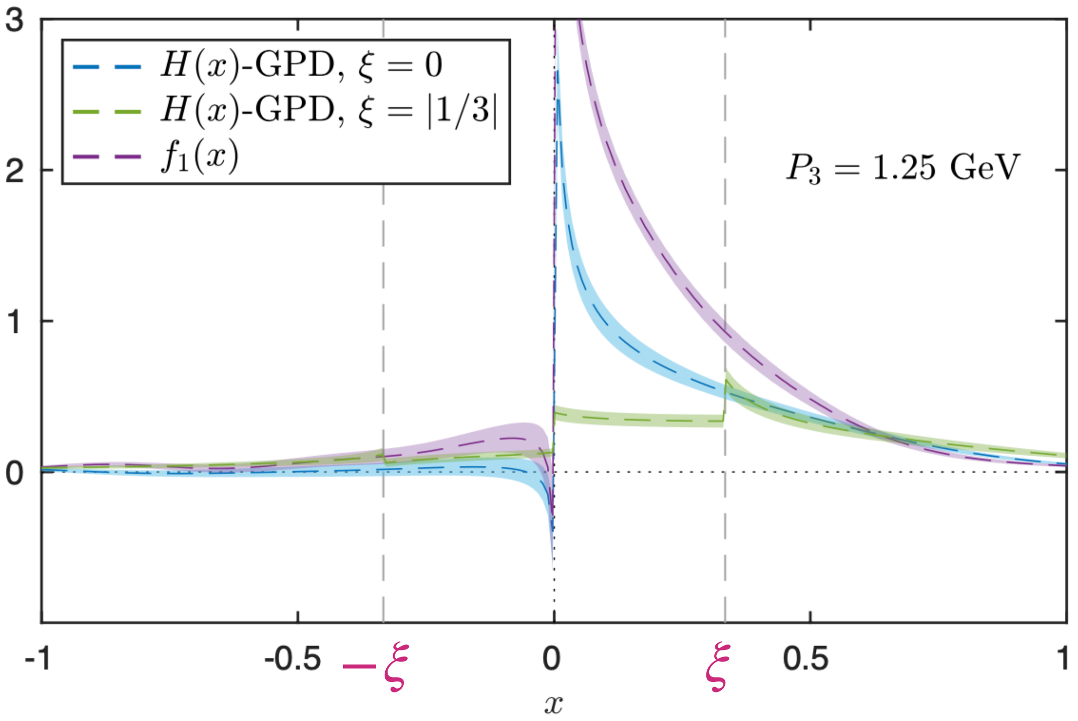

[C. Alexandrou et al., PRL 125, 262001 (2020)]

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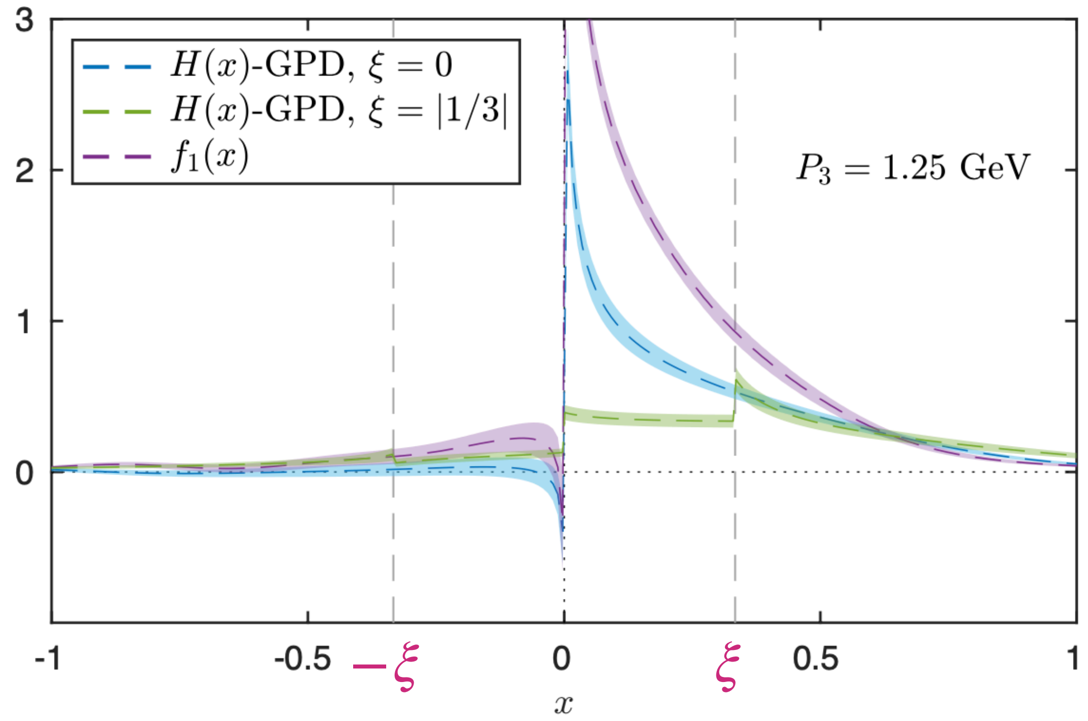

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★ ERBL/DGLAP: Qualitative differences

★ $\xi = \pm x$ inaccessible (formalism breaks down)

★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]

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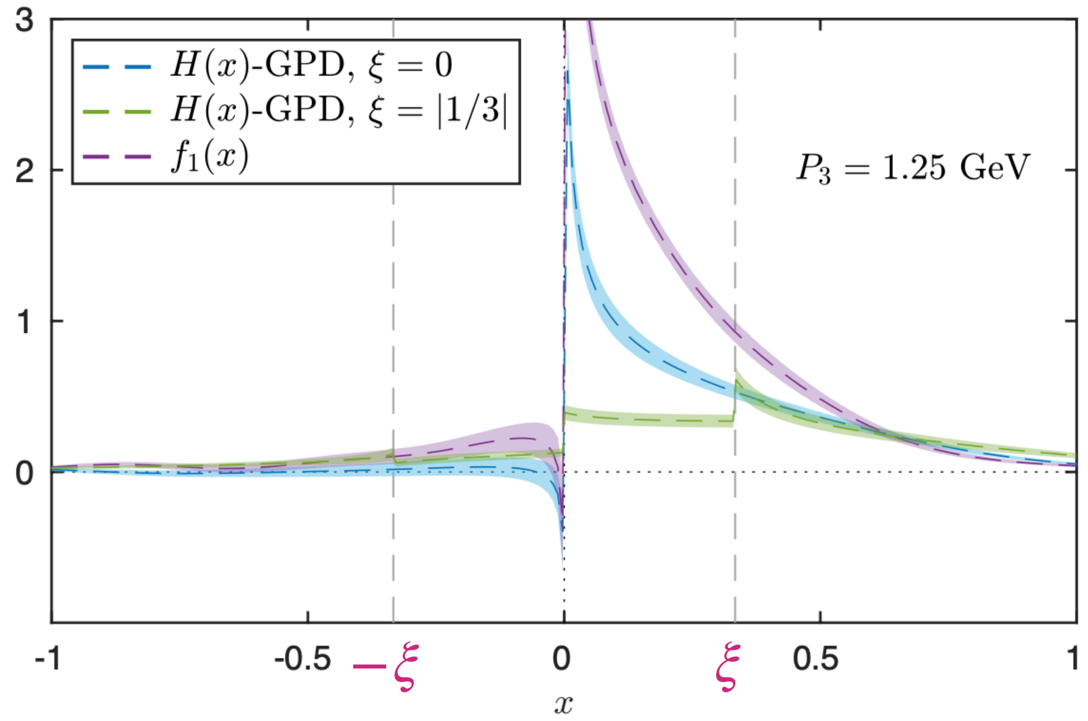

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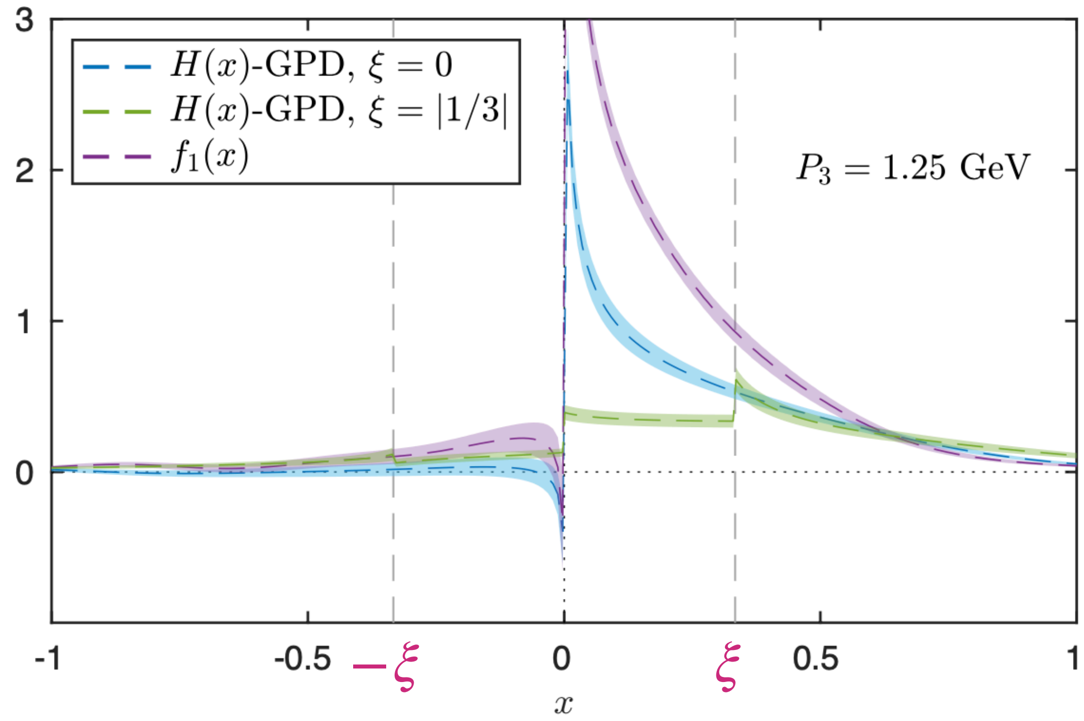

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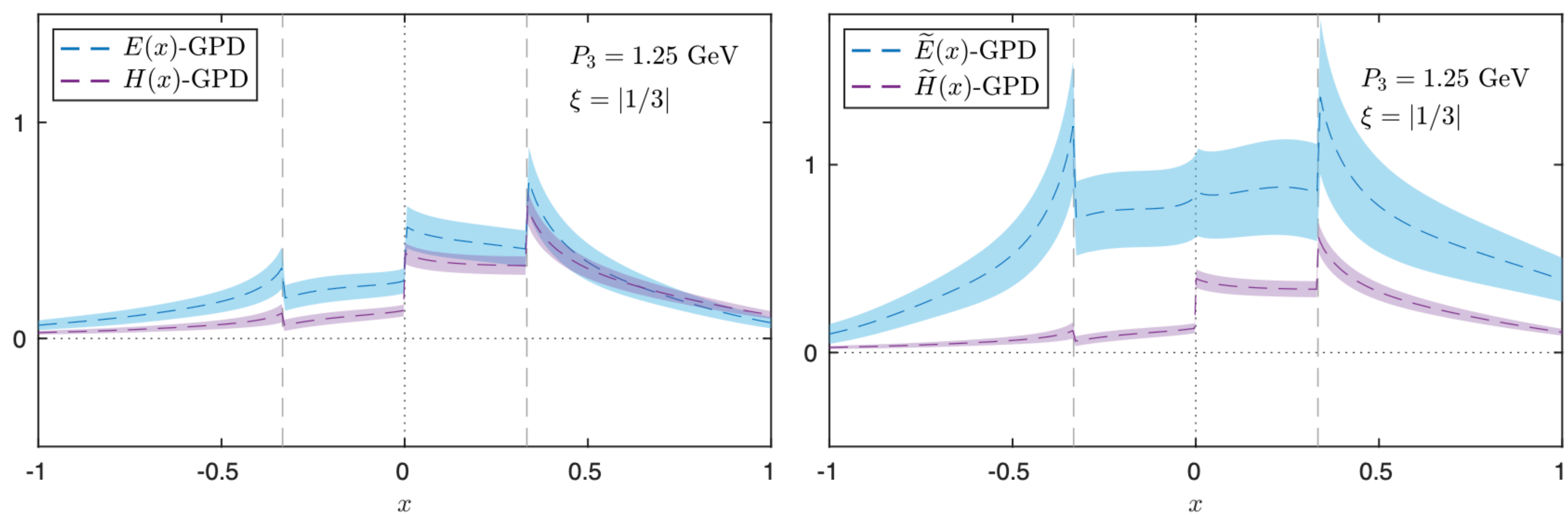
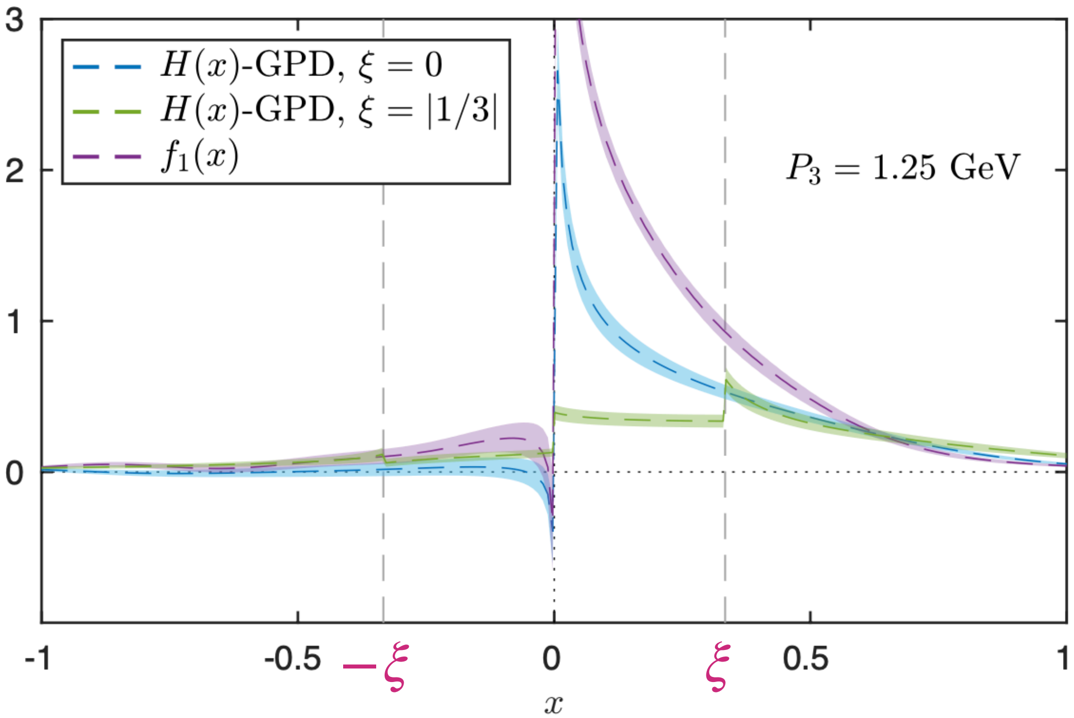
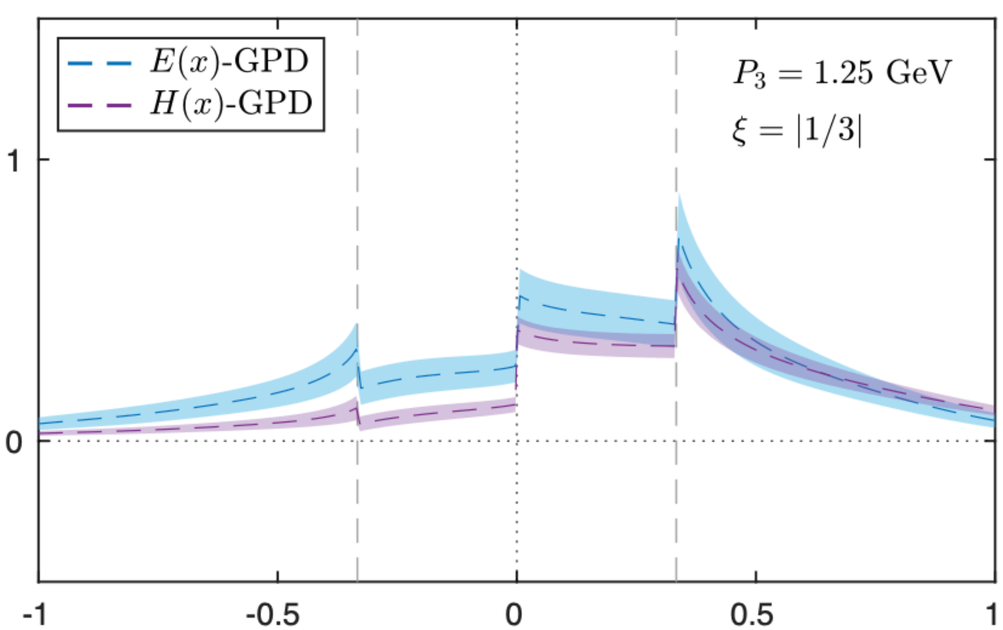


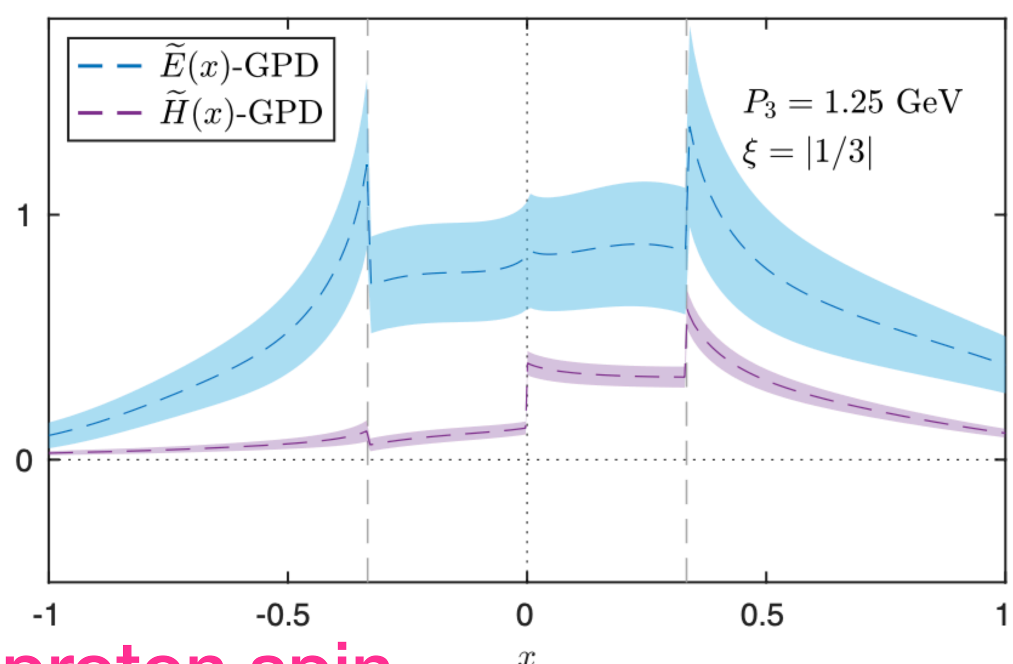
First lattice calculation of x-dependent GPDs



[C. Alexandrou et al., PRL 125, 262001 (2020)]

- ★ ERBL/DGLAP: Qualitative differences
- ★ $\xi = \pm x$ inaccessible (formalism breaks down)
- ★ $x \to 1$ region: qualitatively comparison with power counting analysis [F. Yuan, PRD69 (2004) 051501, hep-ph/0311288]
  - ✦ t-dependence vanishes at large-x
  - ✦ $H(x,0)$ asymptotically equal to $f_1(x)$





★ important contribution in the proton spin

$$\int_{-1}^{+1} dx\, x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx\, x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

Exploration of twist-3 GPDs

PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

K. Cichy , Wed @ 11 am

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$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

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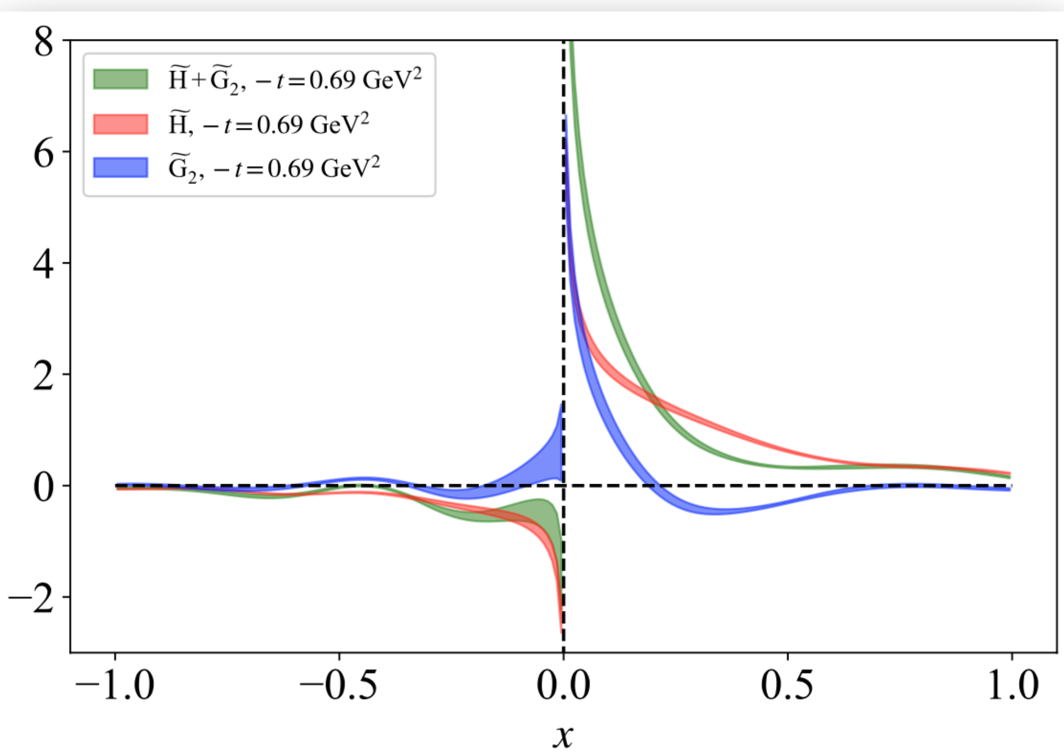

New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Recent addition:
Josh Miller (Temple University)

For latest updates see Miller's Lattice Conference 2023 talk [here]

Theoretical setup

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{Q(}\ \cdots\ ) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{Q(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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$$F^{[\gamma^0]}(x, \Delta; \lambda, \lambda'; P^3) = \frac{1}{2P^0}\bar{u}(p', \lambda')\left[\gamma^0 H_{\mathrm{Q(}}\right.$$

$$\left.) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M} E_{\mathrm{Q(0)}}(x, \xi, t; P^3)\right] u(p, \lambda)$$

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p', \lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right] u(p, \lambda)$$

A_i : - Lorentz invariant amplitudes
- have definite symmetries

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★ Two decompositions can be related

$$\mathcal{H}_0^s(A_i^s;z) = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3}A_6\,,$$

$$\mathcal{E}_0^s(A_i^s;z) = -A_1 - \frac{m^2 z}{P_3}A_4 + 2A_5 - \frac{z\left(4E^2+\Delta_1^2+\Delta_2^2\right)}{2P_3}A_6$$

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Light-cone GPDs using lattice correlators in non-symmetric frames

Theoretical setup

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} MA_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu}i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

(A) A_i are to the standard H, E GPDs $\quad \mathcal{H}_0^s(A_i^s;z) = A_1 + \frac{z(\Delta_1^2+\Delta_2^2)}{2P_3}A_6$

(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

Theoretical setup

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} M A_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

(A) A_i are to the standard H, E GPDs $\quad \mathcal{H}^s_0(A^s_i; z) = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3}A_6$

(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$E(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = -A_1 - \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3 + 2A_5 + 2P_{avg,s/a}\cdot z A_6 + 2\Delta_{s/a}\cdot z A_8$$

Theoretical setup

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(A) Proof-of-concept calculation ($\xi = 0$):

- symmetric frame: $\vec{p}^{\,s}_f = \overrightarrow{P} + \frac{\overrightarrow{Q}}{2}, \quad \vec{p}^{\,s}_i = \overrightarrow{P} - \frac{\overrightarrow{Q}}{2} \qquad -t^s = \overrightarrow{Q}^2 = 0.69\,GeV^2$

- asymmetric frame: $\vec{p}^{\,a}_f = \overrightarrow{P}, \qquad \vec{p}^{\,a}_i = \overrightarrow{P} - \overrightarrow{Q} \qquad t^a = -\overrightarrow{Q}^2 + (E_f - E_i)^2 = 0.65\,GeV^2$

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Results: matrix elements

★ Eight independent matrix elements needed to disentangle the A_i

asymmetric frame

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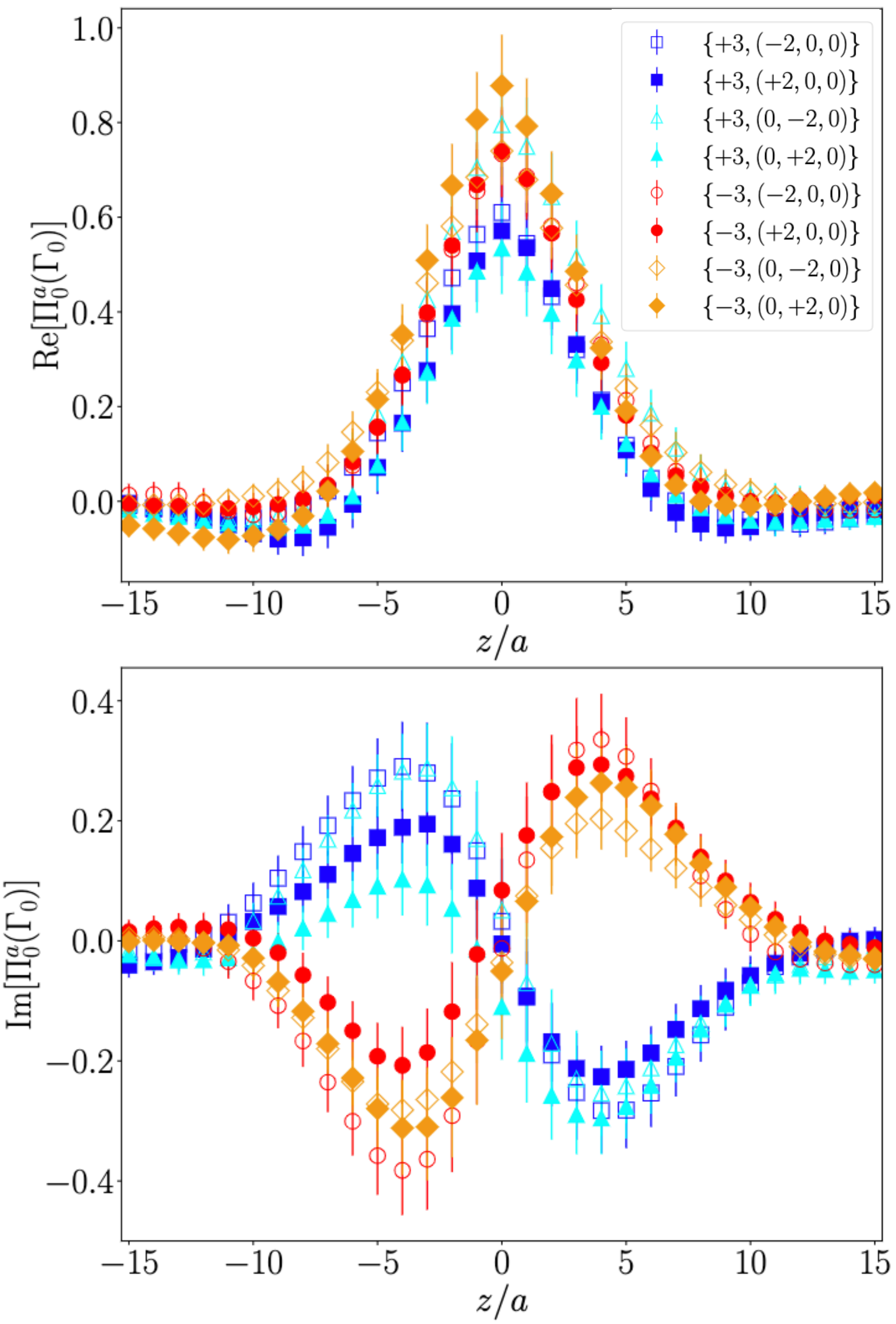

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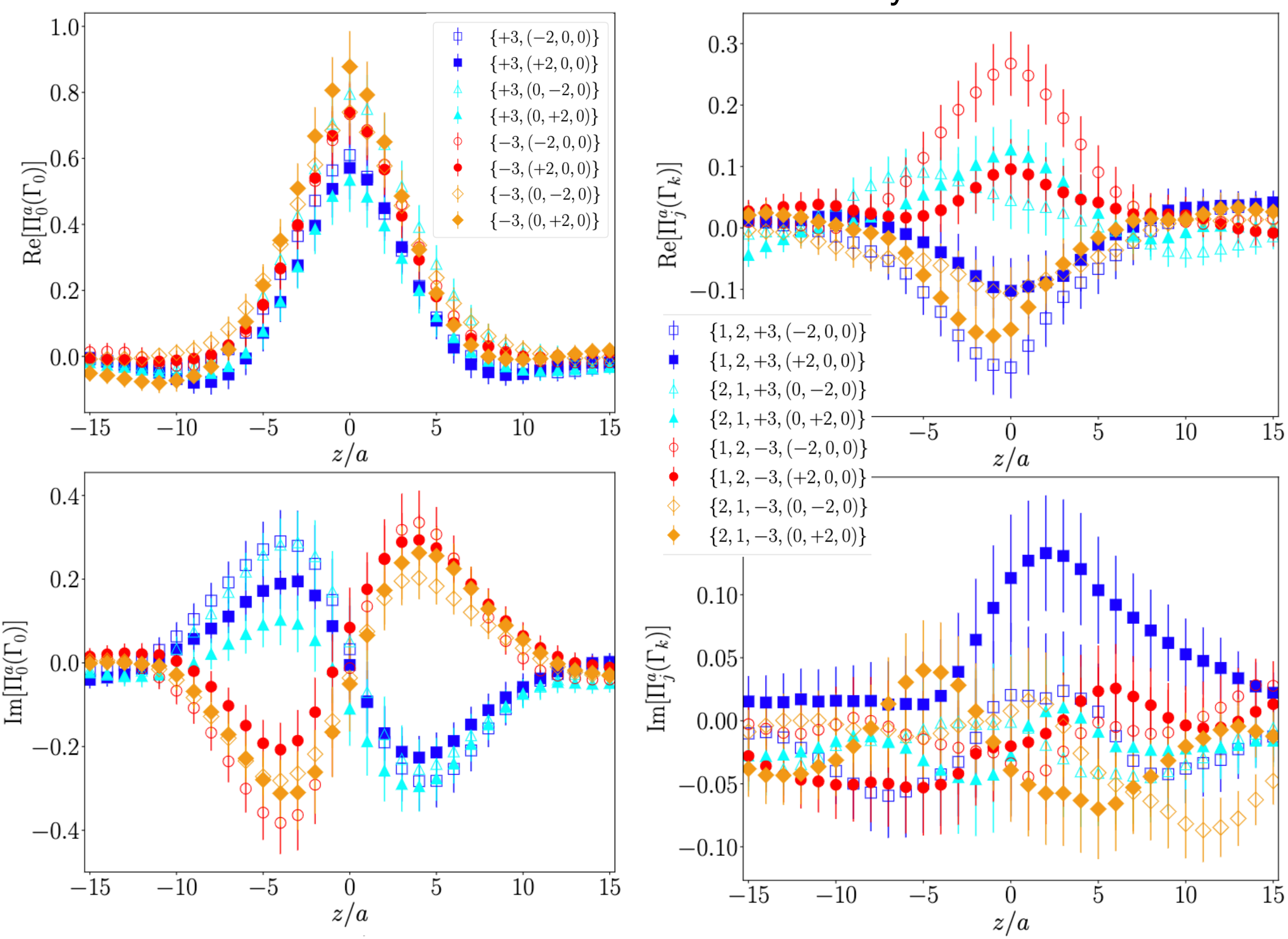

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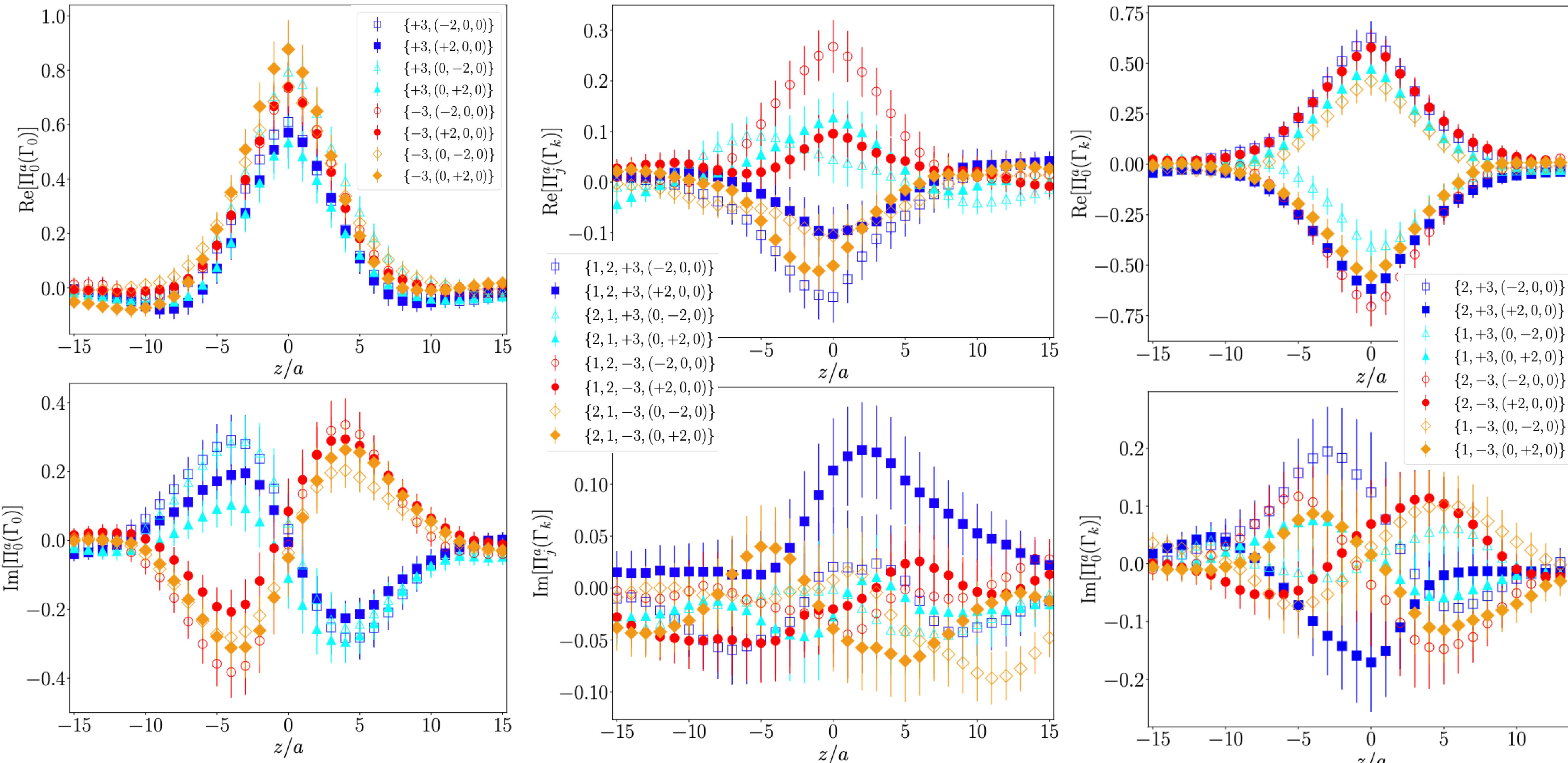

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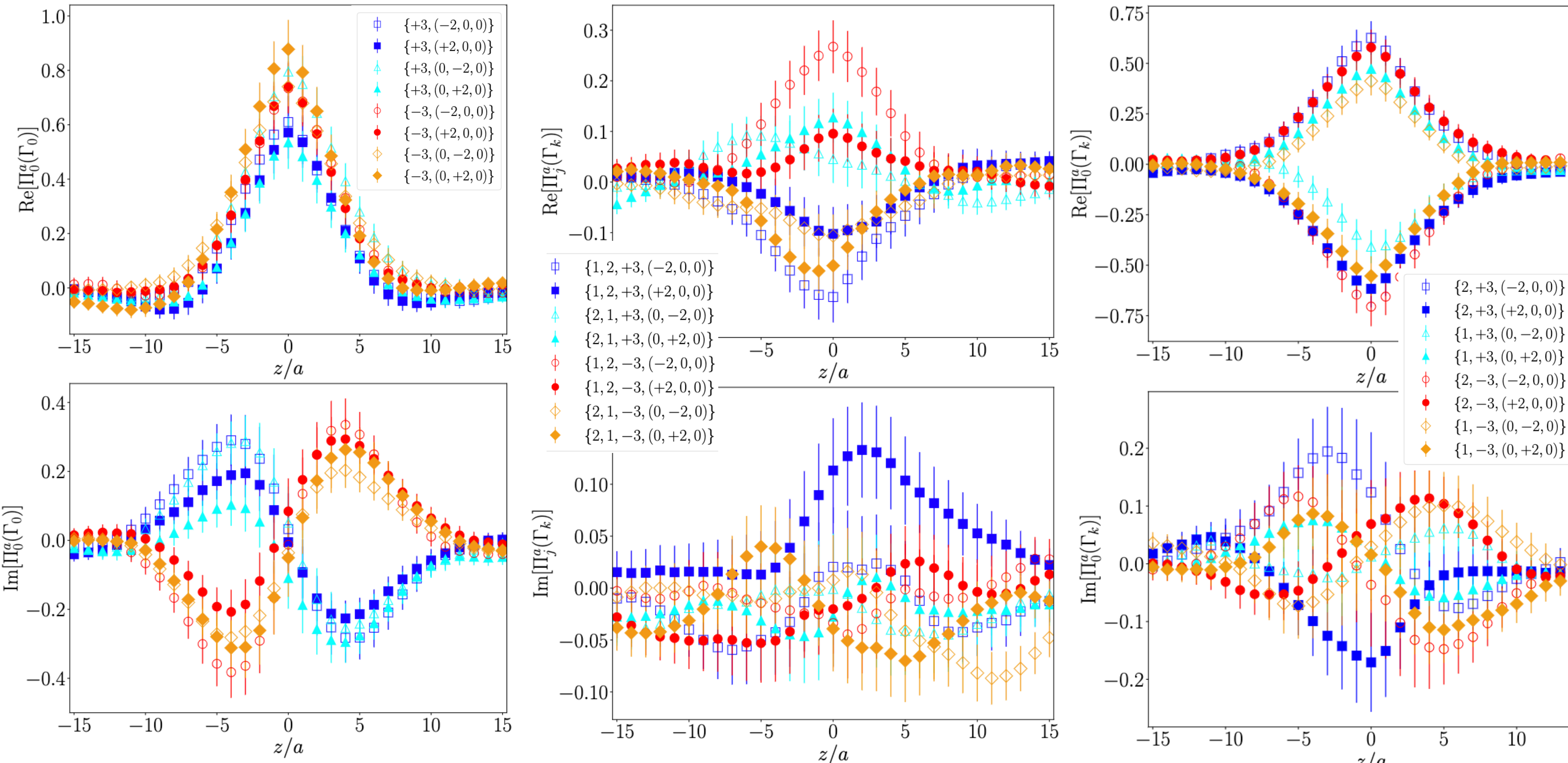
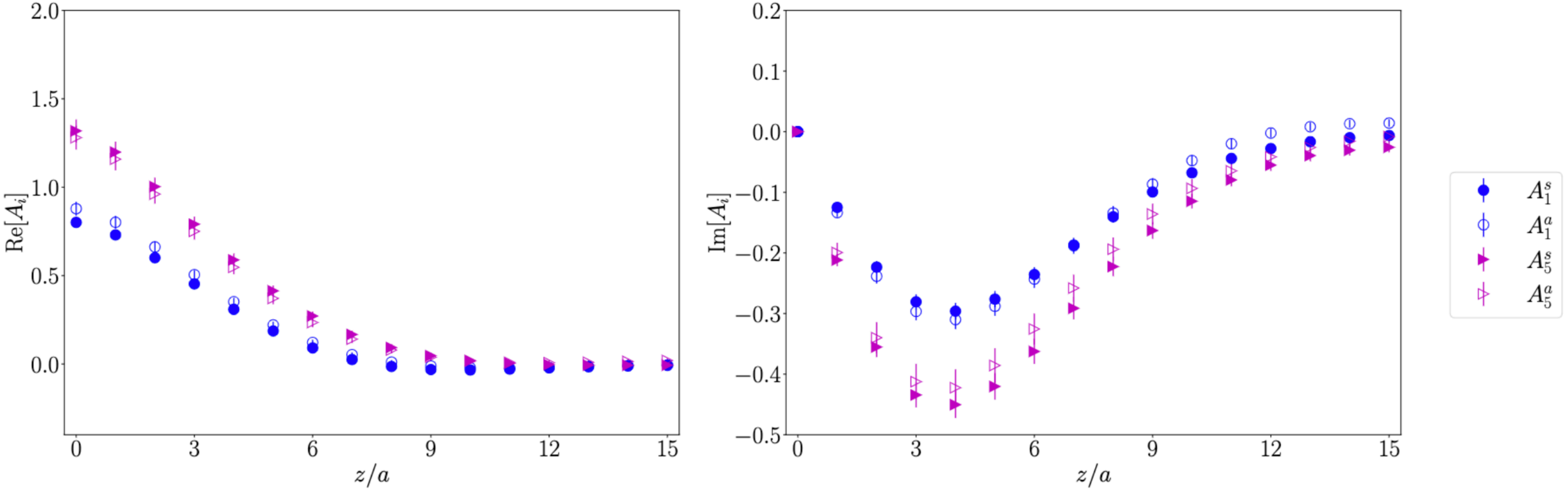


★ Asymmetric frame: ME do not have definite symmetries in $\pm P_3, \pm Q, \pm z$

★ Noisy ME lead to challenges in extracting A_i of sub-leading magnitude

Comparison of A_i in two frames

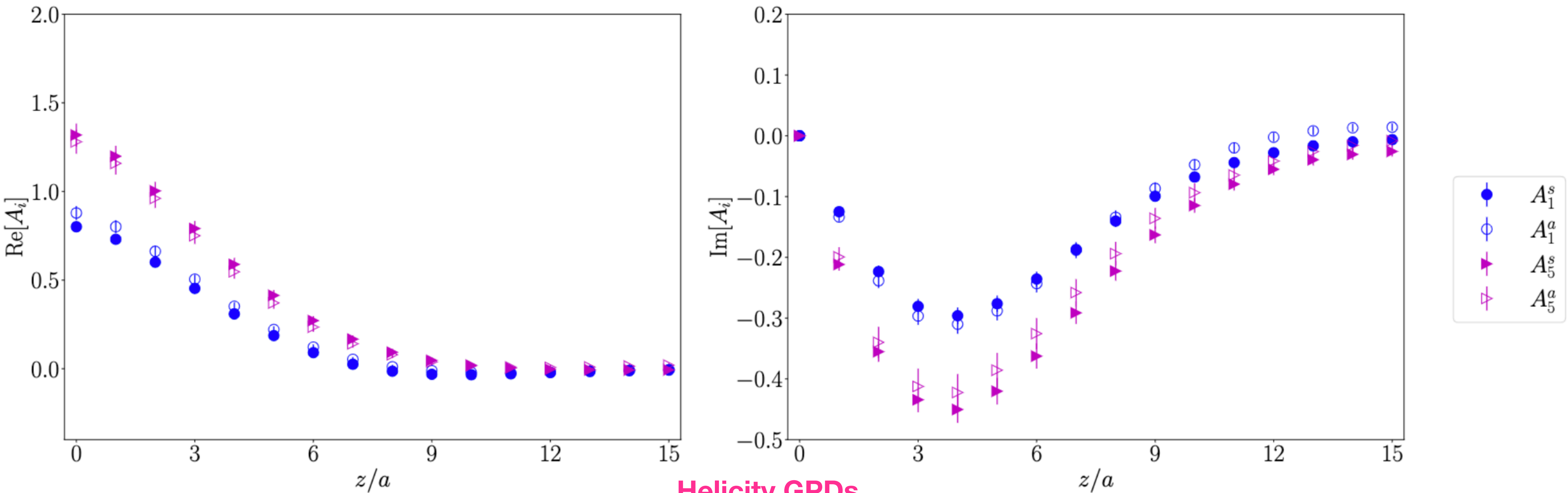
Unpolarized GPDs



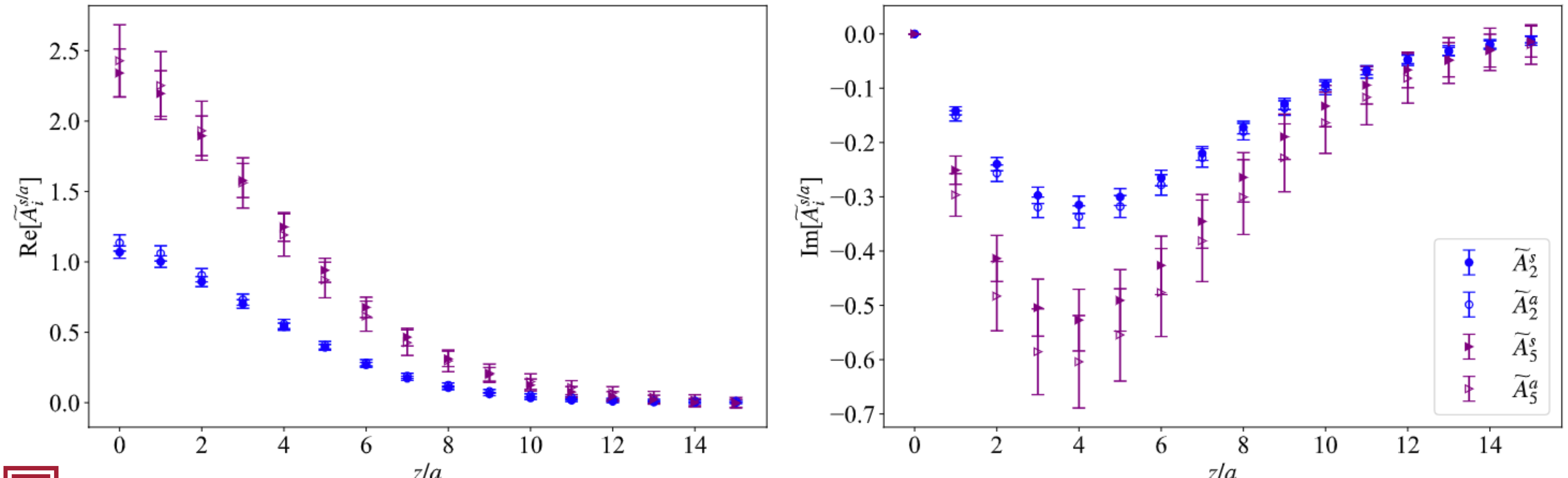
- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_3, A_4, A_8 zero at $\xi = 0$
- ★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

Comparison of A_i in two frames

Unpolarized GPDs

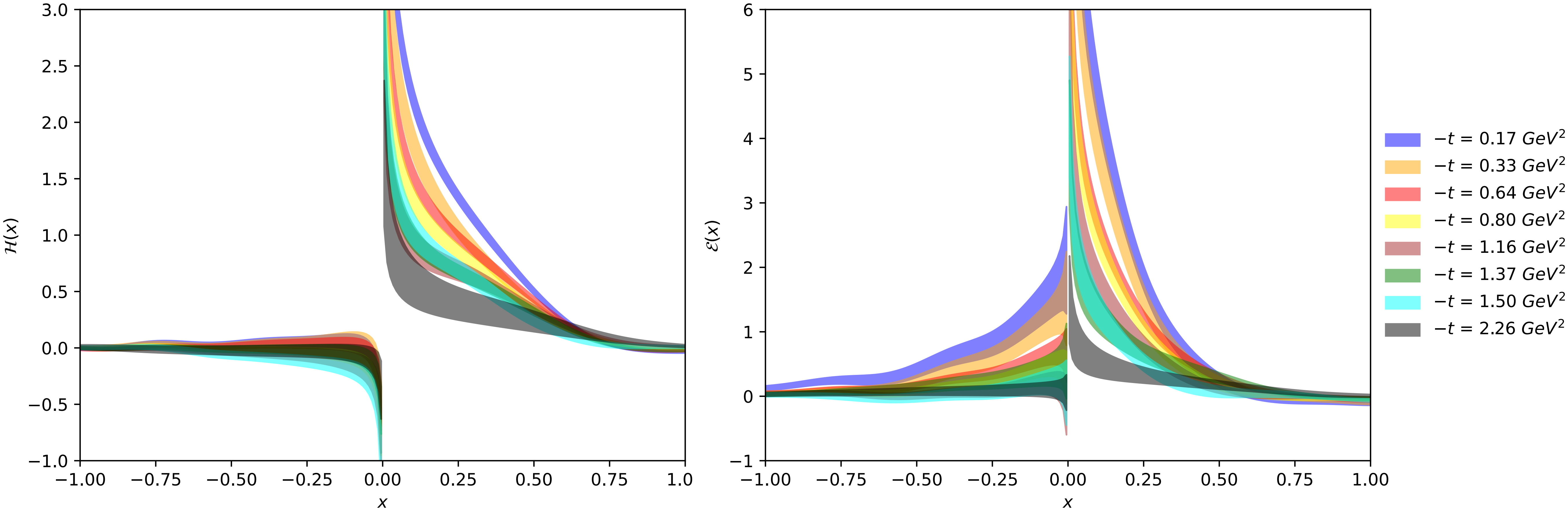


Helicity GPDs



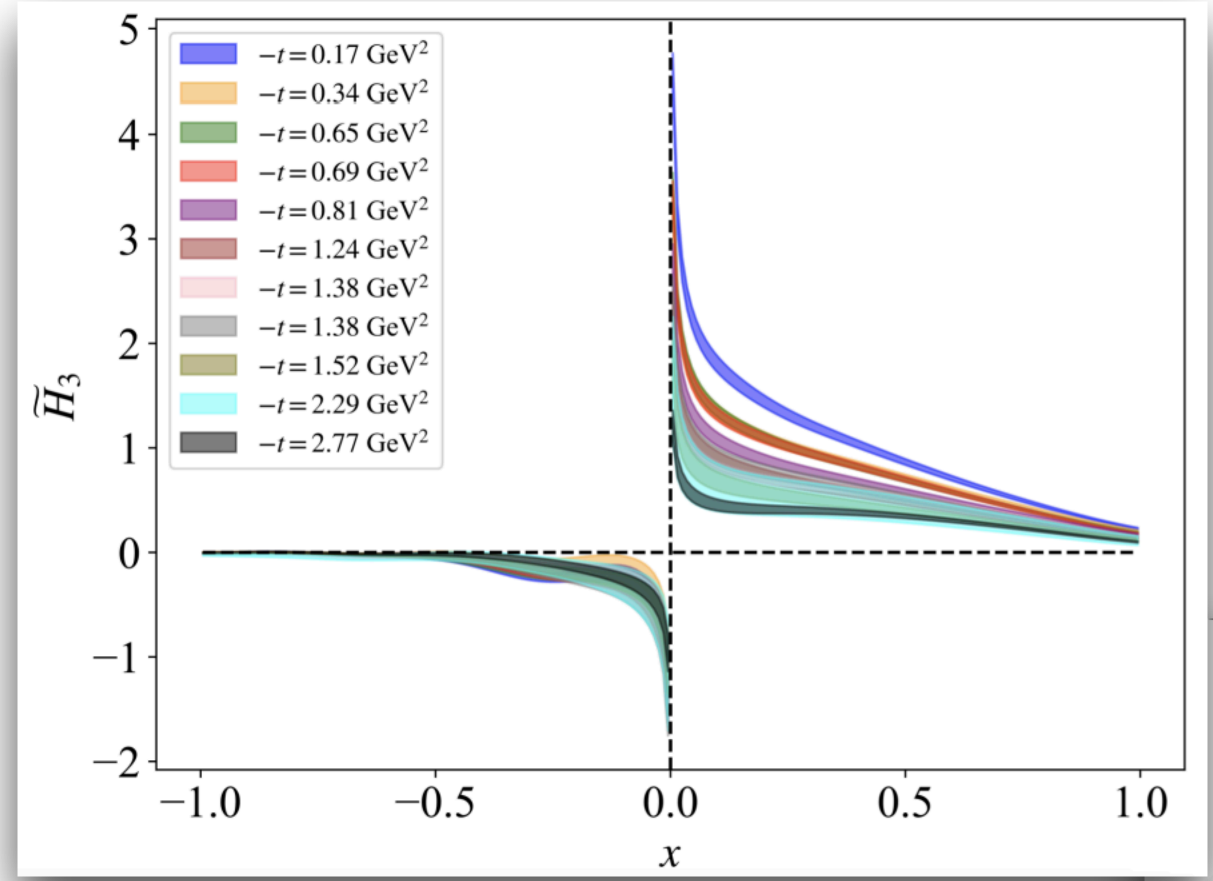
H, E light-cone GPDs

- ★ quasi-GPDs transformed to momentum space
- ★ Matching formalism to 1 loop accuracy level
- ★ +/-x correspond to quark and anti-quark region
- ★ Anti-quark region susceptible to systematic uncertainties.

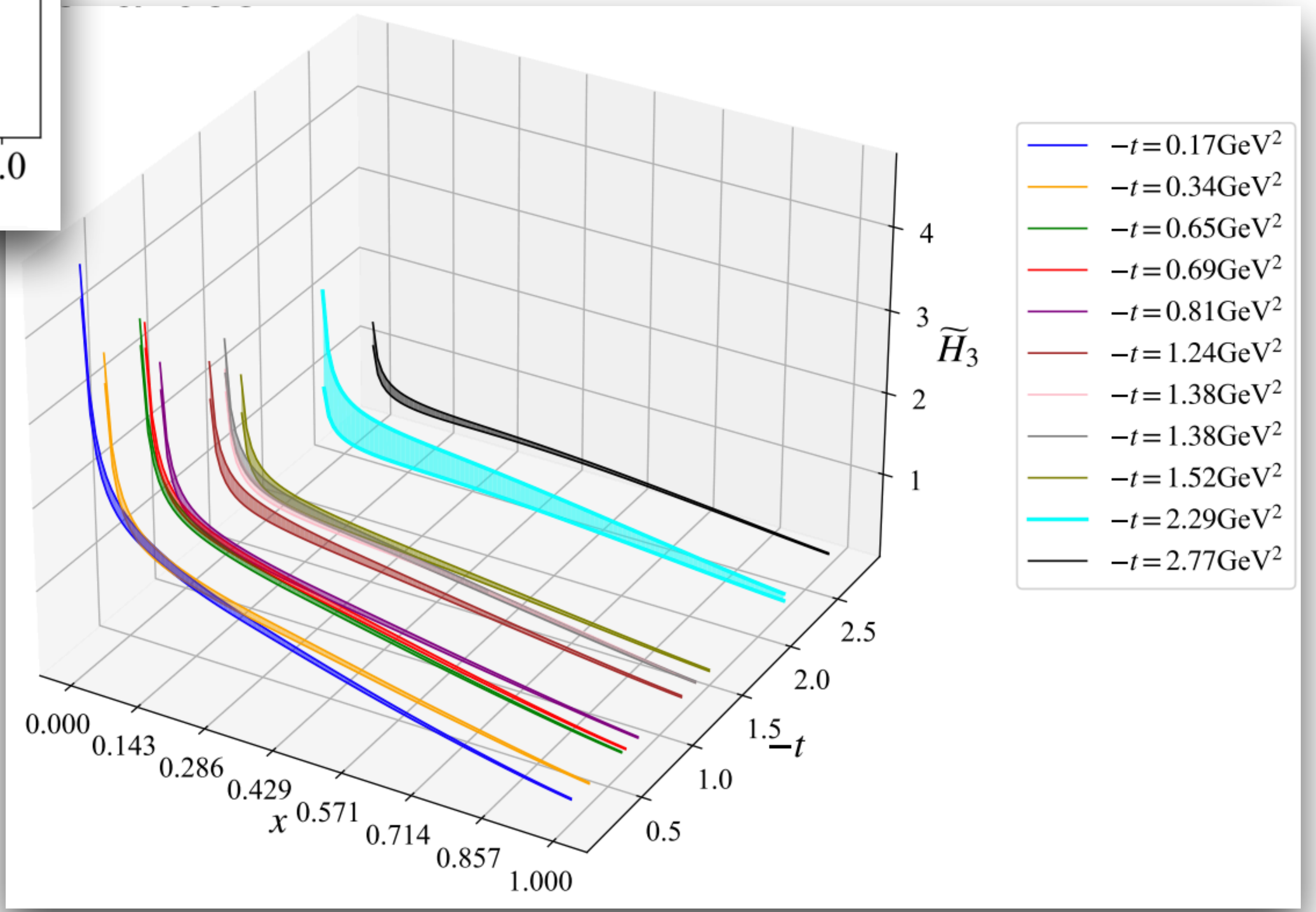


- ★ Similar analysis for helicity GPDs

$\widetilde{H}$ light-cone GPD



$$\widetilde{F}^\mu(z,P,\Delta) \equiv \langle p_f;\lambda'|\bar\psi(-\tfrac{z}{2})\,\gamma^\mu\gamma_5\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle$$
$$= \bar u(p_f,\lambda')\left[\frac{i\epsilon^{\mu P z\Delta}}{m}\widetilde{A}_1 + \gamma^\mu\gamma_5\widetilde{A}_2 + \gamma_5\left(\frac{P^\mu}{m}\widetilde{A}_3 + mz^\mu\widetilde{A}_4 + \frac{\Delta^\mu}{m}\widetilde{A}_5\right)\right.$$
$$\left. + m\not{z}\gamma_5\left(\frac{P^\mu}{m}\widetilde{A}_6 + mz^\mu\widetilde{A}_7 + \frac{\Delta^\mu}{m}\widetilde{A}_8\right)\right]u(p_i,\lambda)\,,$$



How to lattice QCD data fit into the overall effort for hadron tomography

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★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

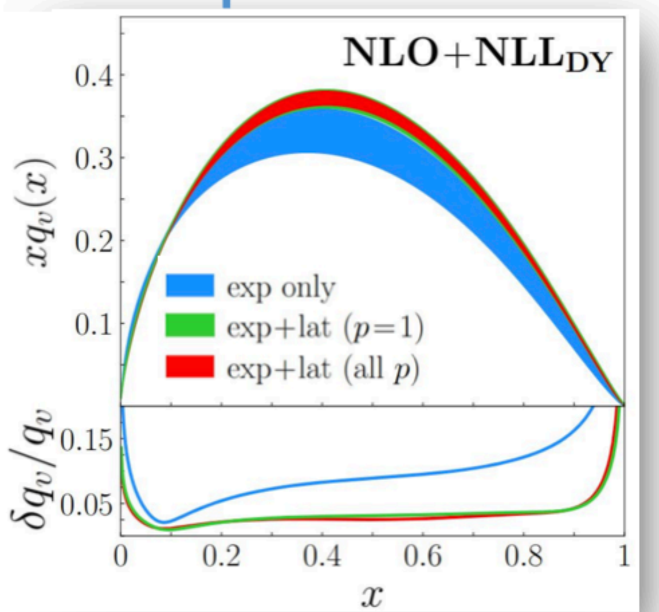
1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

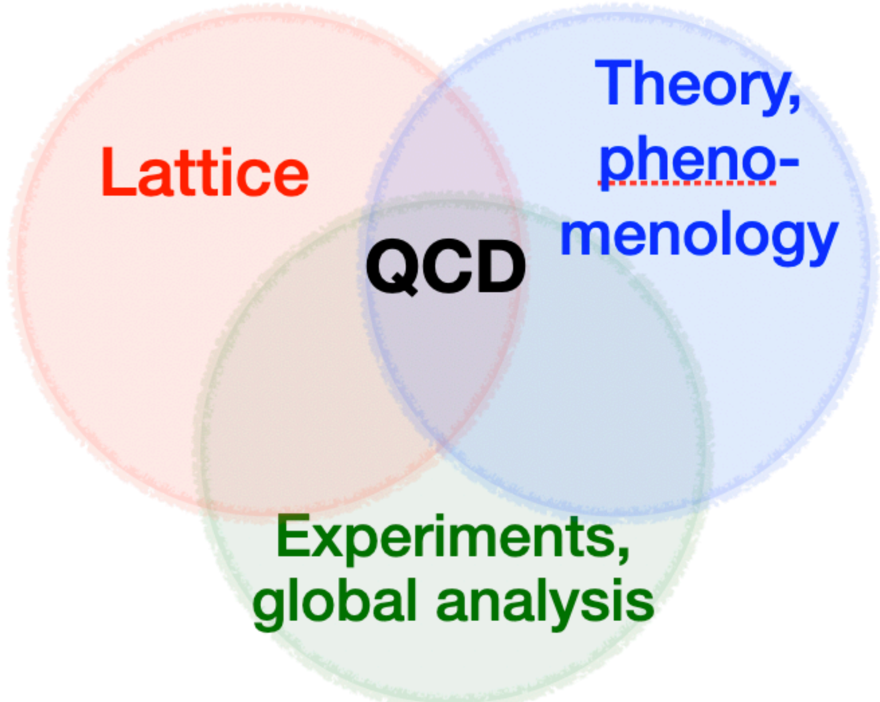
3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergies: constraints & predictive power of lattice QCD

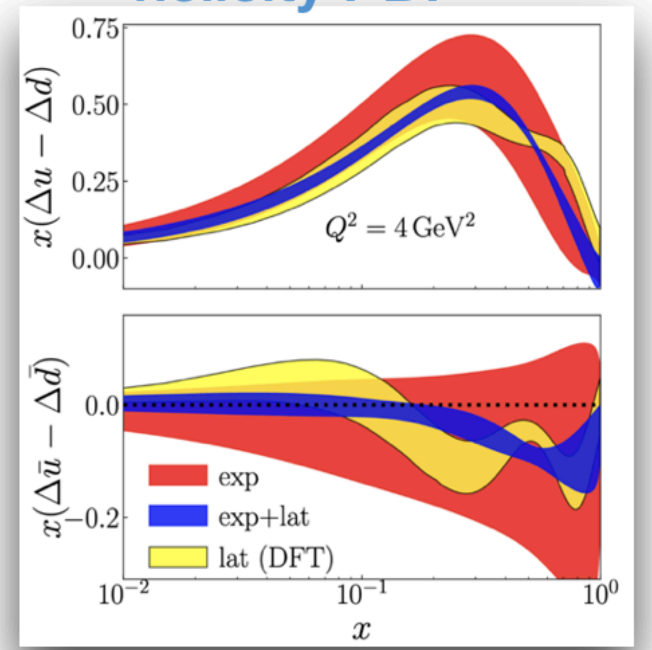
pion PDF



[JAM/HadStruc, PRD105 (2022) 114051]

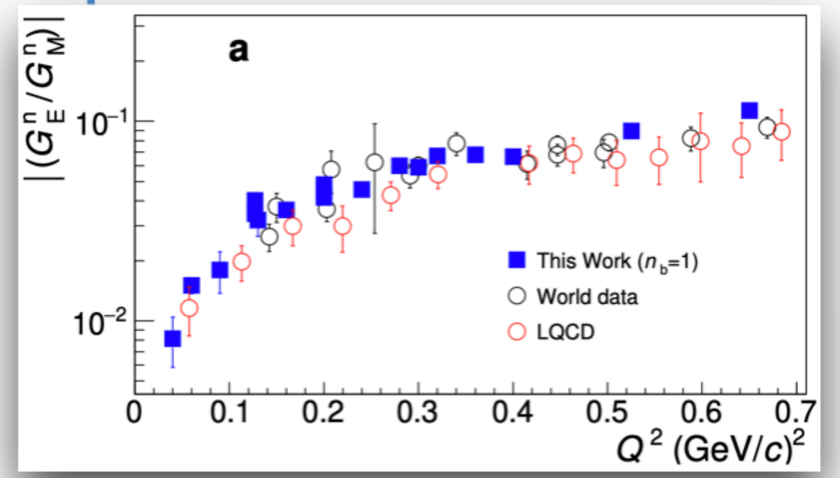


helicity PDF



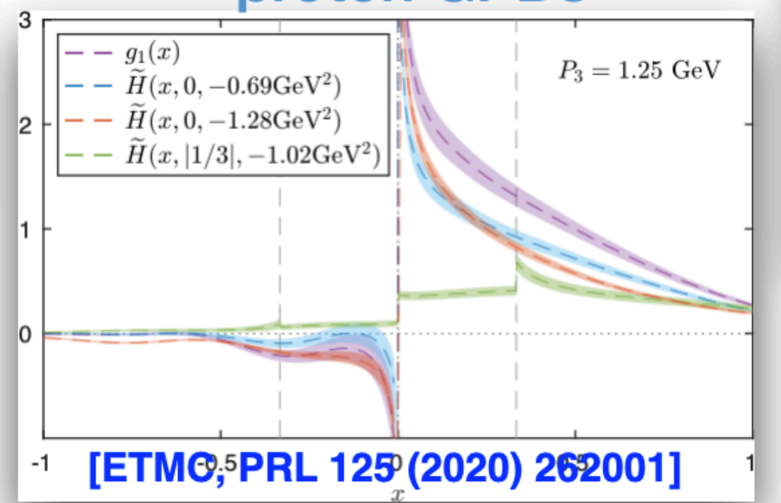
[JAM & ETMC, PRD 103 (2021) 016003]

proton & neutron radius



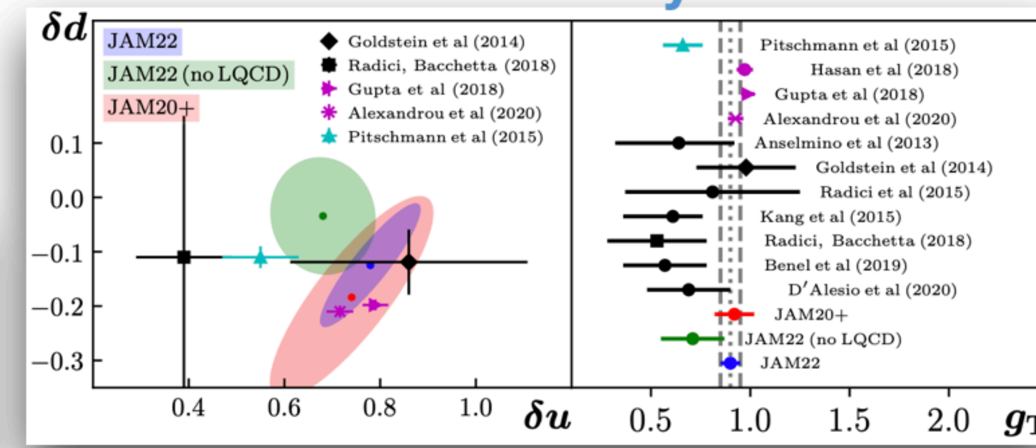
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

And many more!

Summary

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Future calculations have the potential to transform the field of GPDs

★ Synergy with phenomenology is an exciting prospect!
QGT Collaboration will be instrumental in such effort

Summary

★ Lattice QCD data on GPDs will play an important role in the pre-EIC era and can complement experimental efforts of JLab@12GeV

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ Future calculations have the potential to transform the field of GPDs

★ Synergy with phenomenology is an exciting prospect!
QGT Collaboration will be instrumental in such effort

Thank you

U.S. DEPARTMENT OF ENERGY | Office of Science
QUARK-GLUON TOMOGRAPHY COLLABORATION
Award Number: DE-SC0023646

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

U.S. DEPARTMENT OF ENERGY | Office of Science
DOE Early Career Award (NP)
Grant No. DE-SC0020405

Miscellaneous

Consistency checks

★ Norms satisfied   encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

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★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2$$

$$F_{\widetilde{G}_3} = \frac{1}{2m^2}\Big(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\Big) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5$$

$$F_{\widetilde{G}_3} = \frac{1}{m^2}\Big(z_3 P_0 P_3^2 - z_3 P_0^3\Big) A_1$$

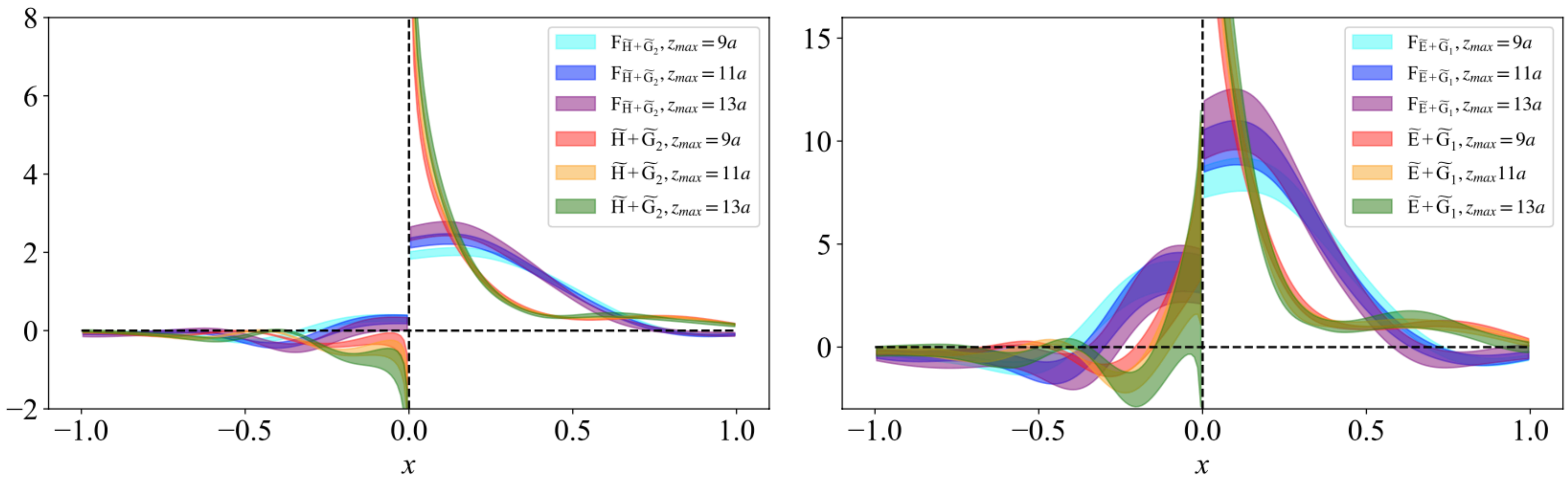

FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

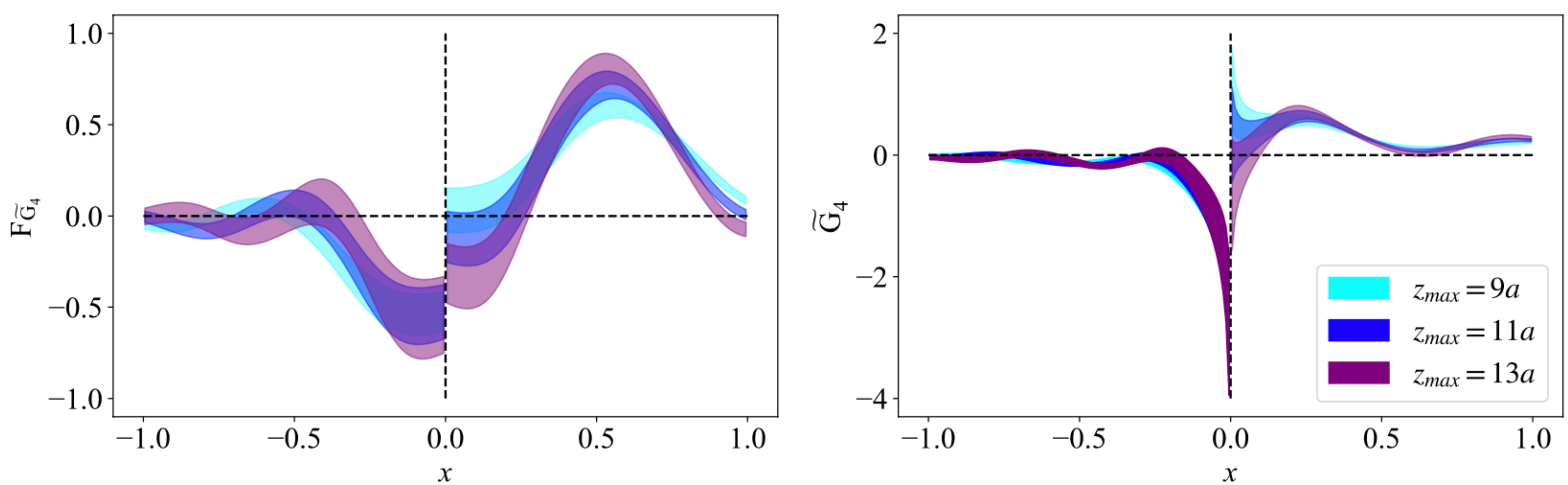

FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

Extension to twist-3 tensor GPDs

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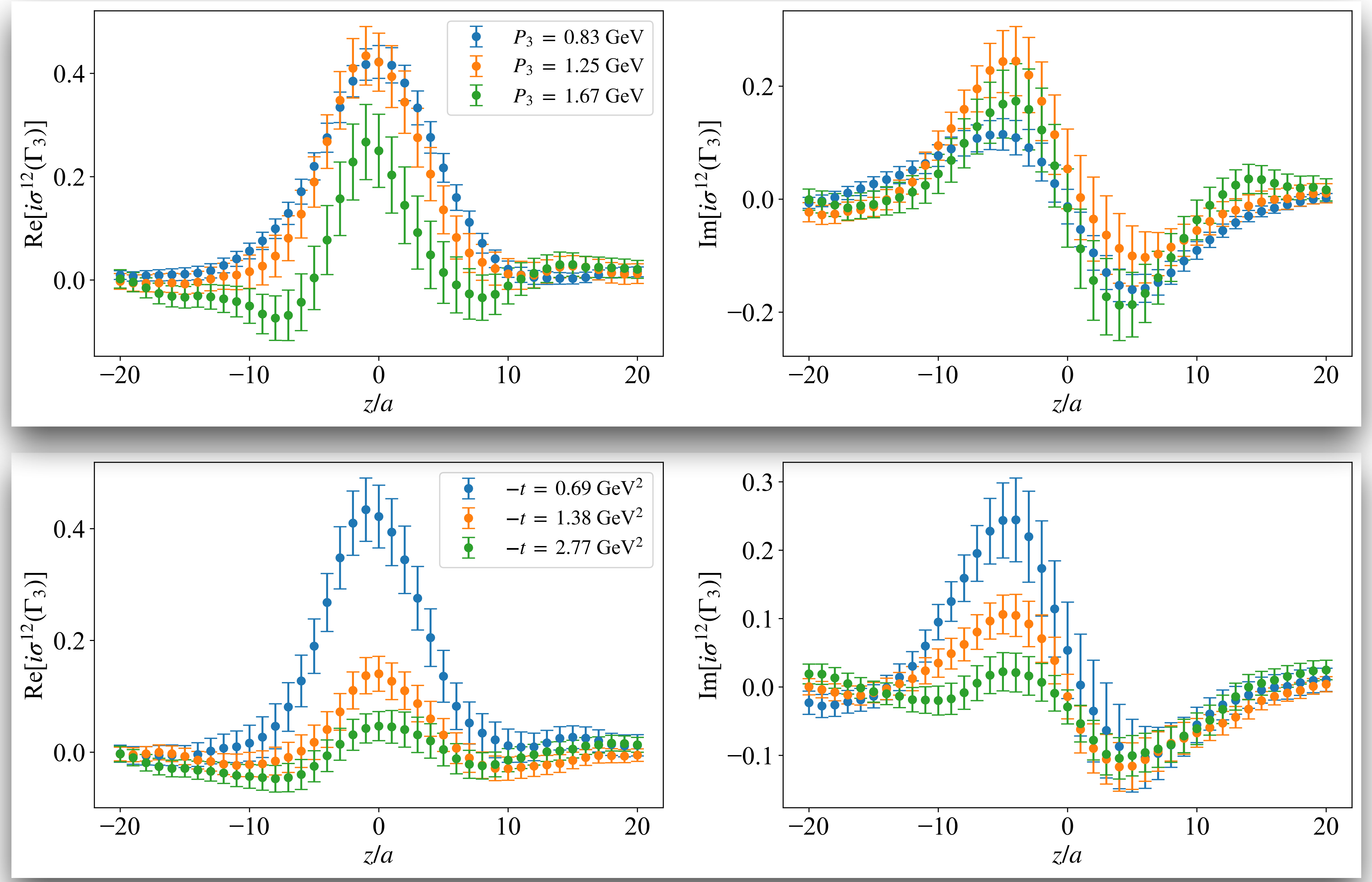

Extension to twist-3 tensor GPDs

★ **Parametrization** [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+\gamma_5\, \widetilde{H}'_2 + \frac{P^+\gamma_5}{M}\, \widetilde{E}'_2 \right) u(p)$$