Proton Polarizabilities

Recent Results from Compton Scattering at MAMI

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THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

Outline

1. Motivation
2. Recent MAMI Results
3. Global Extraction
4. Outlook and Plans

Non-Perturbative QCD

- Regime where the coupling is too strong and perturbative QCD (pQCD) is not appropriate.
- Very important for a thorough understanding of QCD.
- An understanding of the transition from non-pQCD (confinement) to pQCD (asymptotic freedom) is integral to the overall understanding of QCD.

Fundamental Question:

"Can the theory of quark and gluon confinement quantitatively describe the detailed properties of hadrons?"

Perspectives on Subatomic Physics in Canada 2006–2016.

- Theory: QCD describes the strong force in terms of quarks and gluons.
- Nobel Prize in 2004 for **Asymptotic Freedom** in the pQCD regime. . .
- However, in the non-perturbative region, QCD is still unsolved.

One of the top ten challenges for all of physics!

How do we test QCD in the non-perturbative regime?

High-precision measurements with polarization observables.

Hadron Polarizabilities

- Fundamental structure constants
- Response of internal structure to external fields
- Fertile meeting ground between theory and experiment
- Best measured via **Compton scattering**, both real and virtual

Theoretical Approaches

- Dispersion Relations (both subtracted and unsubtracted)
- Chiral Perturbation Theory
- Lattice QCD

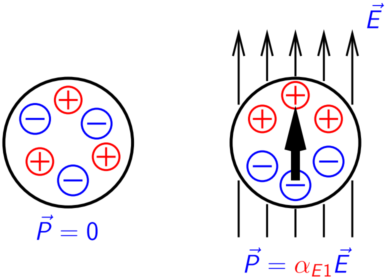
Why else do we care about the nucleon polarizabilities?

Limit precision in other areas of physics:

- Lamb shift and hyperfine structure (proton radius)
- EM contribution to $n - p$ mass difference
- Neutron star properties

Scalar Polarizabilities – Conceptual

Electric Dipole Polarizability

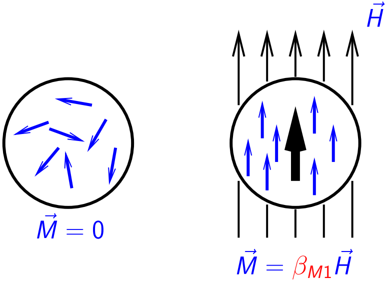


- Apply an electric field to a composite system
- Separation of Charge, or **"Stretchability"**
- Proportionality constant between electric dipole moment and electric field is the electric dipole polarizability, α_{E1}.

Provides information on force holding system together.

Scalar Polarizabilities – Conceptual

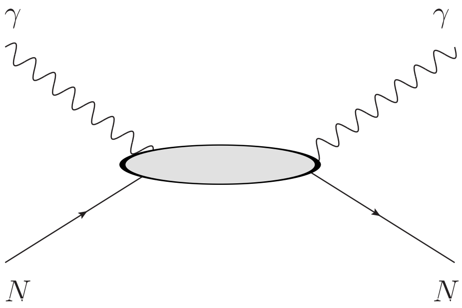
Magnetic Dipole Polarizability



- Apply a magnetic field to a composite system
- Alignment of dipoles or **"Alignability"**
- Proportionality constant between magnetic dipole moment and magnetic field is the magnetic dipole polarizability, β_{M1}.
- Two contributions, paramagnetic and diamagnetic, and they cancel partially, giving $\beta_{M1} < \alpha_{E1}$.

Provides information on force holding system together.

Real Compton Scattering from the Nucleon



Low-energy outgoing photon *plays the role of the applied EM field*.

$\Rightarrow$ Nucleon Response

$\Rightarrow$ **POLARIZABILTIES!**

Global response to internal degrees of freedom.

Real Compton Scattering – Hamiltonian

Expand the Hamiltonian in incident-photon energy.

0th order $\longrightarrow$ charge, mass

1st order $\longrightarrow$ magnetic moment

2nd order $\longrightarrow$ **scalar polarizabilities**:

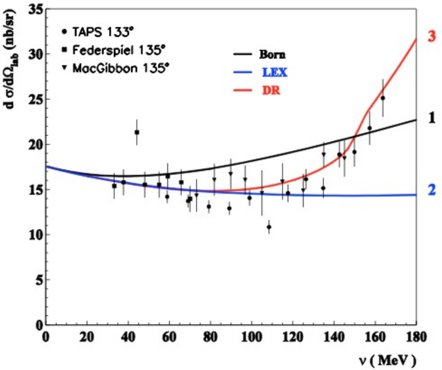
$$H_{\text{eff}}^{(2)} = -4\pi\left[\frac{1}{2}\alpha_{E1}\vec{E}^2 + \frac{1}{2}\beta_{M1}\vec{H}^2\right]$$

3rd order $\longrightarrow$ **spin (or vector) polarizabilities**:

$$H_{\text{eff}}^{(3)} = -4\pi\left[\tfrac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \tfrac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot(\vec{H}\times\dot{\vec{H}}) - \gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\right]$$

where $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$ and $H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$

Low-Energy Expansion in Proton Compton Scattering



1 Born (anomalous magnetic moment)

2 LEX (polarizabilities at leading order)

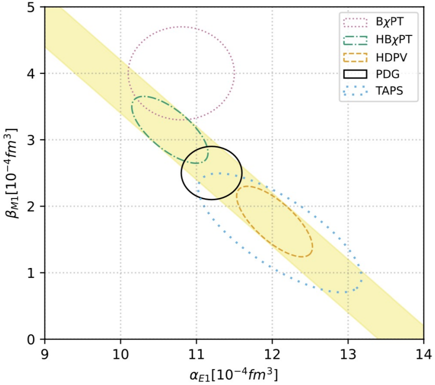
3 Dispersion relations (full calculation)

LEX:

$$\frac{d\sigma}{d\Omega}(\nu,\theta) = \frac{d\sigma}{d\Omega}^{Born}(\nu,\theta) - \nu\nu'\left(\frac{\nu'}{\nu}\right)\frac{e^2}{2m}\left[(\alpha_{E1}+\beta_{M1})(1+z)^2 + (\alpha_{E1}-\beta_{M1})(1-z)^2\right]$$

Measure low energies and precise cross sections/asymmetries!

Previous Results



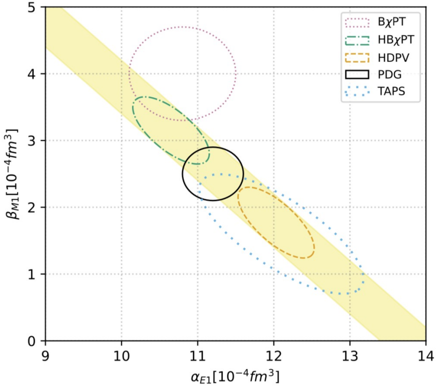
Baldin sum rule from optical theorem:

$$\alpha_{E1} + \beta_{M1} = \int_{\omega_0}^{\infty} d\omega \frac{\sigma_{tot}(\omega)}{\omega^2}$$

Various α_{E1} and β_{M1} extractions:

- Different experimental inputs
- Different theoretical inputs
- Different fitting strategies

Previous Results



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Various α_{E1} and β_{M1} extractions:

- Different experimental inputs
- Different theoretical inputs
- Different fitting strategies

New high-precision dataset needed!

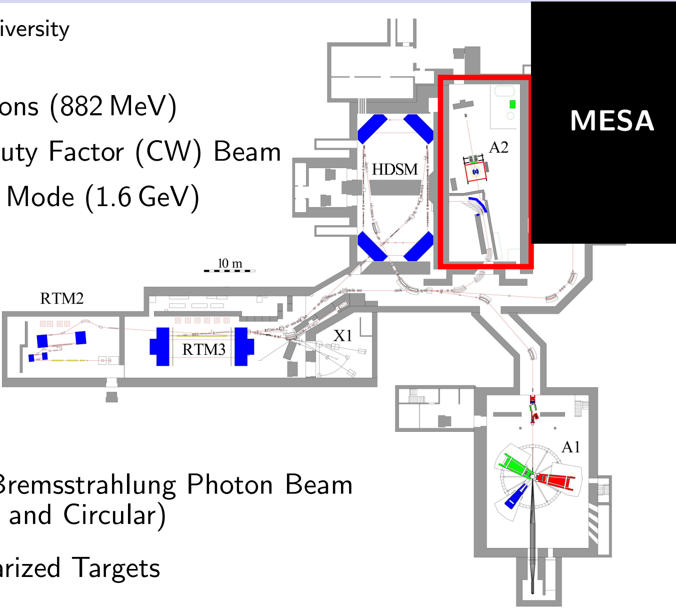
The Mainzer Mikrotron (MAMI)

Johannes Gutenberg University
Mainz, Germany

3 Race-Track Microtrons (882 MeV)

High-Quality 100% Duty Factor (CW) Beam

HDSM in Production Mode (1.6 GeV)



A2 Collaboration:

High-Flux, Tagged, Bremsstrahlung Photon Beam
(Unpolarized, Linear, and Circular)

Polarized and Unpolarized Targets

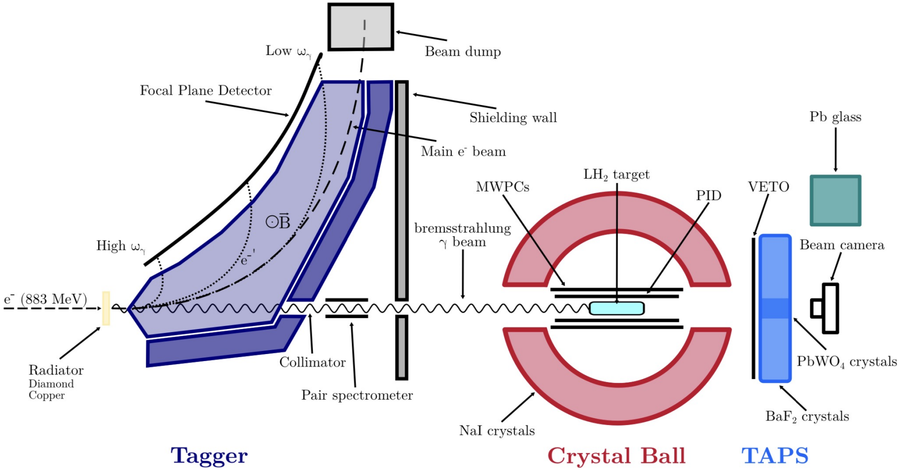
Run Conditions

Standard A2 Equipment was required:

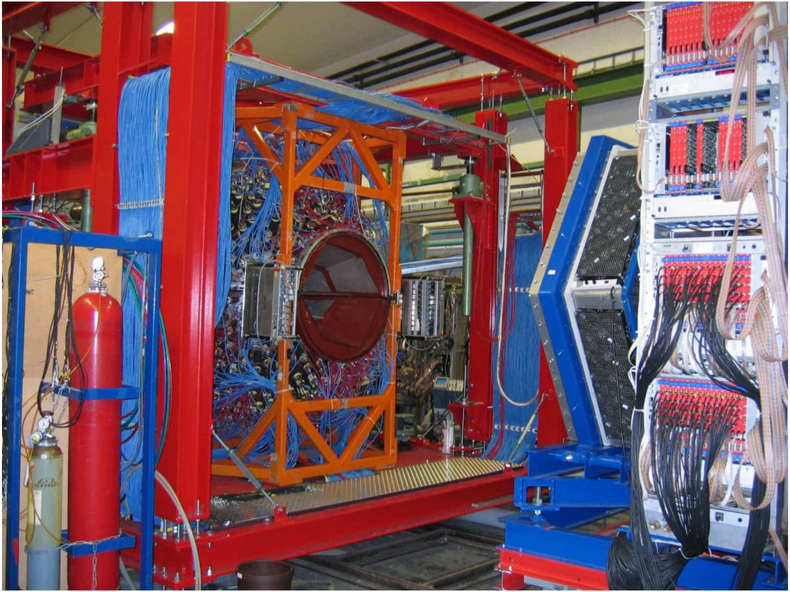
- MAMI electrons
- Glasgow-Mainz Tagger
- CB-TAPS detector system
- Cryogenic Target

Run Parameter	**Value**
Electron Beam Energy	883 MeV
Target	LH_2
Radiator	Diamond
Tagged Energy Range	100 – 400 MeV
Channel Energy Resolution	2 MeV
Beam Polarization	linear
Target Polarization	none

Schematic of the A2 Hall



CB-TAPS Detector System



CB-TAPS Detector System

Crystal Ball

Highly segmented EM calorimeter

$\Delta E/E = 0.020 \cdot E[\mathrm{GeV}]^{0.36}$

$\sigma_\phi = \sigma_\theta / \sin\theta$

$\sigma_\theta = 2 - 3^\circ$

Particle ID

Barrel of thin scintillators

$\Delta\phi = 15^\circ$

TARGET

Liquid Hydrogen

10-cm capton cell

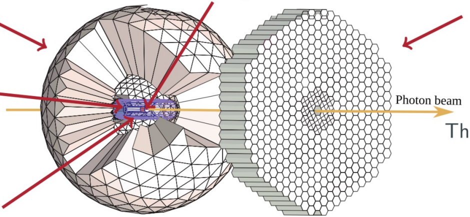
TAPS

Highly segmented EM calorimeter

$\Delta E/E = 0.018 + 0.008/E[\mathrm{GeV}]^{0.5}$

$\sigma_\phi = 14 \ldots 0.95^\circ$

$\sigma_\theta < 1^\circ$



TAPS-Veto

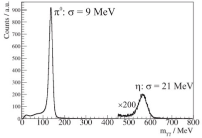
Thin scintillators before each TAPS crystal

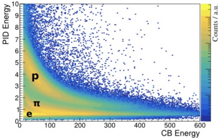
Multiwire Proportional Chambers

Precise charged tracking/positioning

$\sigma_\theta \sim 2^\circ$

$\sigma_\phi \sim 3^\circ$



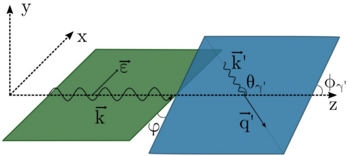


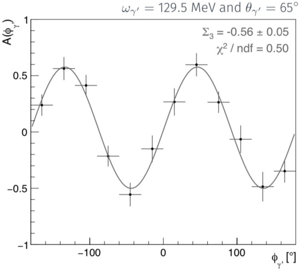
Run Summary

- **Ph.D. work of Edoardo Mornacchi**.
- Phys. Rev. Lett. **128**, 132503 (2022).
- Low-energy Compton scattering.
- Linearly polarized beam, (unpolarized) LH_2 target.
- High-statistics cross sections, $d\sigma/d\Omega$, and beam asymmetry, Σ_3. Most important data are below pion threshold.
- **Upgraded tagger, improved systematic errors:**
  - higher γ-flux with better flux monitoring
  - improved linpol peak stability
  - improved background subtraction
- 1.2×10^6 events, an improvement of $\times 6$ compared to the pilot measurement.
- Approximately $\times 10$ the statistics of the previous world best measurement with TAPS (also A2!) [OdL et al., EPJA **10** 207 (2001)], which make up of about 50% of the existing world data.

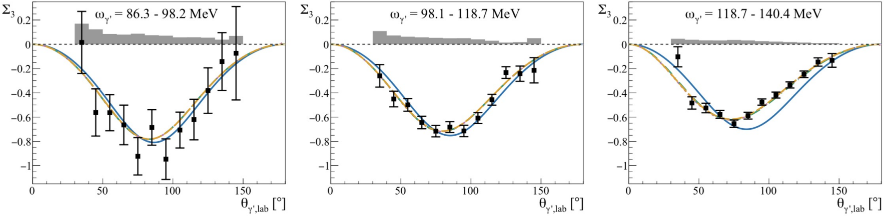
The beam asymmetry can be extracted by measuring the polarized cross-section with two orthogonal orientation of the polarization plane:

$$A(\varphi) = \Sigma_3 \cos(2\varphi) = \frac{N^{\parallel}(\omega_\gamma, \theta_{\gamma'}, \varphi) - N^{\perp}(\omega_\gamma, \theta_{\gamma'}, \varphi)}{p_\gamma^{\perp} N^{\parallel}(\omega_\gamma, \theta_{\gamma'}, \varphi) + p_\gamma^{\parallel} N^{\perp}(\omega_\gamma, \theta_{\gamma'}, \varphi)}$$





Beam Asymmetry



A2: Phys. Rev. Lett. **128** (2022)

Systematic errors

Born contribution

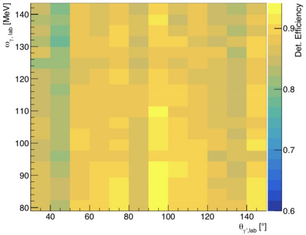
DR: Phys. Rev. C **76**, 015203 (2007)

BχPT: Eur. Phys. J. C **65**, 195 (2010)

HBχPT: Eur. Phys. J. A **49**, 12 (2013)

The unpolarized cross-section can be determined by precisely measuring the detection, reconstruction and tagging efficiencies:

$$\frac{d\sigma}{d\Omega}(\omega_\gamma, \theta_{\gamma'}) = \frac{N_{\gamma'}(\omega_\gamma, \theta_{\gamma'})}{d\Omega} \frac{1}{N_p} \frac{1}{\epsilon_{rec}(\omega_\gamma, \theta_{\gamma'})} \frac{1}{N_{e^-}(\omega_\gamma)\epsilon_{tagg}(\omega_\gamma)}$$



Cross Sections

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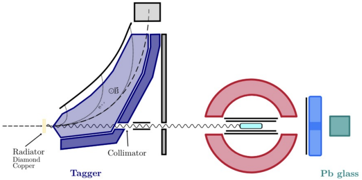

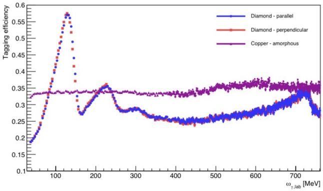

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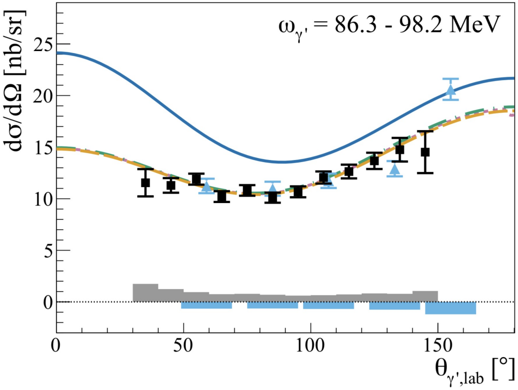

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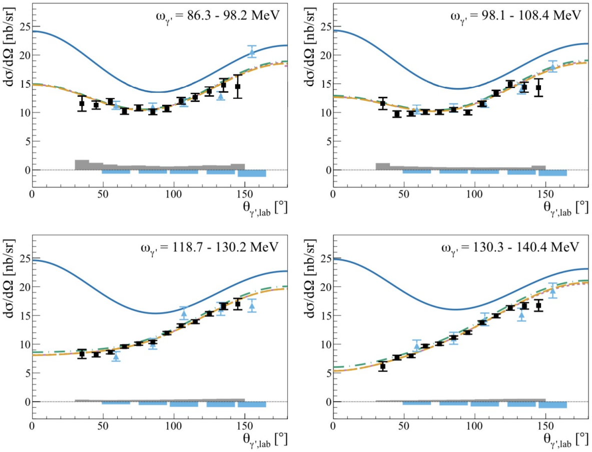

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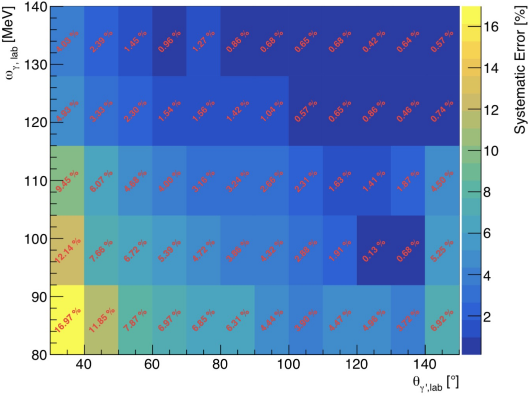
Systematic Errors

UCS	Event selection and MC correction	2%
	Target density	1%
	Flux normalization	2%
	Background	uncorr.
	TOTAL	**3%**
Σ_3	Linear polarization	5%
	Background	uncorr.
	TOTAL	**5%**

Systematic Errors



- Higher for low ω_γ and forward $\theta_{\gamma'}$ ($\sim$ 17%)
- Lower for high ω_γ and backward $\theta_{\gamma'}$ ($\sim$ 0.5%)
- Average $\sim$ 2%

Extracted Proton Polarizabilities

E. Mornacchi (A2), Phis. Rev. Lett. ***128****, 132503 (2022)*

	HDPV	BChPT	HBChPT
α_{E1}	11.23 ± 0.49	10.65 ± 0.50	11.10 ± 0.52
β_{M1}	2.79 ± 0.32	3.28 ± 0.33	3.36 ± 0.38
s_σ	1.011 ± 0.015	1.013 ± 0.015	1.043 ± 0.016
s_Σ	0.994 ± 0.015	0.996 ± 0.015	1.001 ± 0.015
χ^2/DOF	$82.10/93 = 0.89$	$82.96/93 = 0.89$	$83.16/93 = 0.89$

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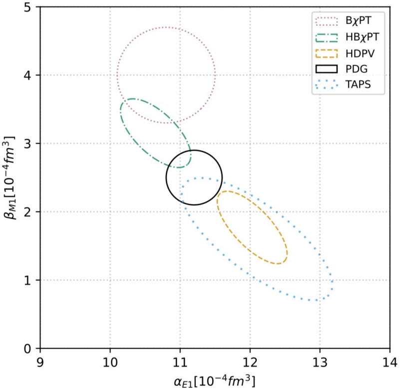

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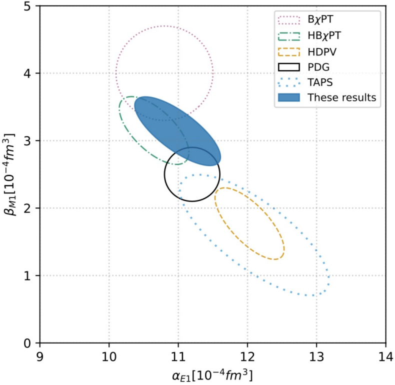
$$\alpha_{E1} = 10.99 \pm 0.16 \pm 0.47 \pm 0.17 \pm 0.34$$

$$\beta_{M1} = 3.14 \pm 0.21 \pm 0.24 \pm 0.20 \pm 0.35$$



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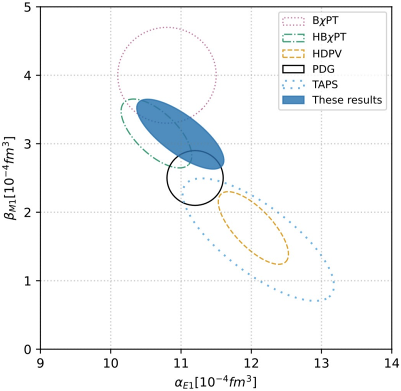
- Highest precision Compton scattering dataset below π-photoproduction threshold!
- Precise extraction of the scalar polarizabilities from one single dataset



Bootstrap Technique

- fixed-t Dispersion relation model
- Three different PWA solution used: MAID-2021, SAID-MA19, BnGA-2019
- All six polarizabilities are treated as free parameters
- Parametric bootstrap technique needed to include all possible sources of systematic uncertainties

$$e_{i,j}^{(0)} \to e_{i,j}^{(b)} = (1+\delta_{j,b})(e_{i,j}^{(0)} + r_{i,j,b}\sigma_{i,j}^{(0)})$$

  - inclusion of common systematic uncertainties without any *a priori* distribution assumption
  - probability distribution of the fit parameters obtained by the procedure
  - uncertainties on nuisance model parameters are taken into account in the sampling procedure
  - fit p-value is provided if goodness-of-fit distribution is not given by the χ^2

Compton Scattering Datasets

As many data points as possible were initially included in the fit!

- All existing unpolarized low-energy data ($E_\gamma < 150$ MeV)
  - 14 datasets, 218 points[1]
- New-generation (a.k.a. photon-tagged) unpolarized high-energy data ($E_\gamma = [150 - 300]$ MeV)
  - 6 datasets, 156 points
- Polarized ($\sigma_{||}$, $\sigma_\perp$, Σ_{2x}, Σ_{2z}, and Σ_3) data
  - 7 datasets, 137 points[2]

[1] including 10 above-thr points from TAPS
[2] 65 below- and 72 above-thr

Compton Scattering Datasets

First author	# of points	$\theta_{\gamma'}$ [°]	E_γ [MeV]
Unpolarized low-energy data			
Baranov	7	90, 150	82 – 111
Bernardini	2	135	120, 139
de Leon	55	59 – 155	59 – 150
Federspiel	16	60, 135	30 – 70
Goldansky	5	75 – 150	55
Hallin	13	45 – 135	130 – 150
Hyman	12	50, 90	60 – 130
Li	8	55, 90, 125	81
MacGibbon	8	90, 135	70 – 100
MacGibbon	10	90, 135	100 – 140
Mornacchi	60	35 – 145	85 – 140
Oxley	4	70 – 150	60
Pugh	16	45, 90, 135	55 – 125
Zieger	2	180	98, 132

First author	# of points	$\theta_{\gamma'}$ [°]	E_γ [MeV]
Unpolarized high-energy data			
Blanpied	57	51 – 126	213 – 298
Camen	5	136	210 – 293
Molinari	4	90 (cms)	250 – 289
Peise	8	75 (cms)	200 – 291
Wissmann	6	131	199 – 295
Wolf	76	48 – 148	264 – 294

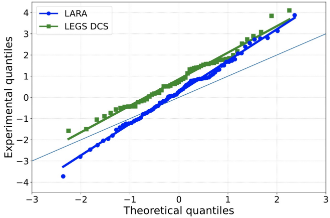
First author	Observable	# of points	$\theta_{\gamma'}$ [°]	E_γ [MeV]
Polarized low-energy data				
Li	$\sigma_{\parallel}$	5	55, 90, 125	83
Li	$\sigma_{\perp}$	3	55, 90, 125	83
Mornacchi	Σ_3	36	35 – 145	92, 108, 129
Sokhoyan	Σ_3	21	60 – 150	87, 109, 129
Polarized high-energy data				
Blanpied	Σ_3	58	65 – 135	213 – 298
Martel	Σ_{2x}	4	90 – 150	285
Paudyal	Σ_{2z}	10	85 – 150	275, 295

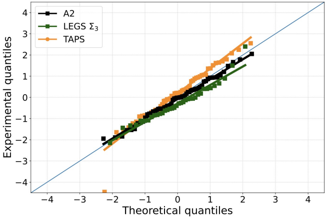
Excluded Datasets

Inconsistencies among unpolarized high-energy data are known to exist, especially between the LARA (Wolf) and LEGS (Blanpied) datasets!
A consistency check of the database was performed:

- Fit all 6 polarizabilities using MAID-2021 alternatively including LARA or LEGS.
- Using the polarizability best-values, the residuals were calculated.
- For every big dataset, the residual normal distribution was assessed using a probability plot.

All datasets had normally distributed residual, except both LARA and LEGS:





Final Dataset

LARA and LEGS DCS datasets were excluded from the fit!

The final database included

- All existing unpolarized low-energy data ($E_\gamma < 150$ MeV)
  - 14 datasets, 218 points[3]
- New-generation (a.k.a. photon-tagged) unpolarized high-energy data ($E_\gamma = [150 - 300]$ MeV)
  - 4 datasets, 23 points
- Polarized ($\sigma_{||}$, $\sigma_\perp$, Σ_{2x}, Σ_{2z}, and Σ_3) data
  - 7 datasets, 137 points[4]

For a total of 388 data points divided in 25 datasets!

---

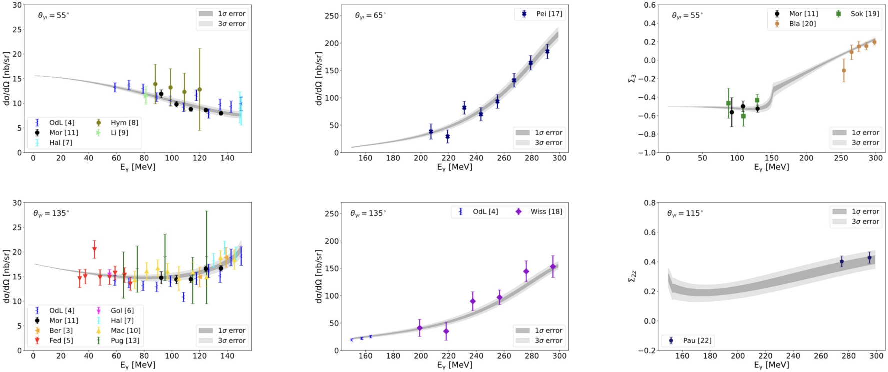
[3] including 10 above-thr points from TAPS

[4] 65 below- and 72 above-thr

Bootstrap Technique

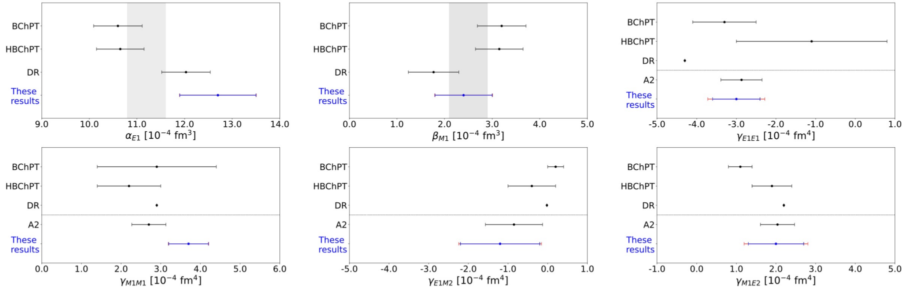
- Six free parameters
  - $\alpha_{E1} + \beta_{M1}$, $\alpha_{E1} - \beta_{M1}$, γ_{E1E1}, γ_{M1M1}, γ_0, and $\gamma_\pi^{\mathrm{disp}}$
- N = 10^4 bootstrap cycles
- Point-to-point systematic errors added in quadrature to statistical ones
- Common systematic errors are assumed to be uniform distributed (unless otherwise specified)
- Polarizability best-values are the mathematical average of the three results using the three different PWAs

Global Fitting Results



E.M., S. Rodini, B. Pasquini, P. Pedroni, Phys. Rev. Lett. **129**, *102501 (2022).*

Global Fitting Results



$$\alpha_{E1} = 12.7 \pm 0.8 \pm 0.1; \quad \beta_{M1} = 2.4 \pm 0.6 \pm 0.1; \quad \gamma_{E1E1} = -3.0 \pm 0.6 \pm 0.4$$

$$\gamma_{M1M1} = 3.7 \pm 0.5 \pm 0.1; \quad \gamma_{E1M2} = -1.2 \pm 1.0 \pm 0.3; \quad \gamma_{M1E2} = 2.0 \pm 0.7 \pm 0.4$$

Global Fitting Results

- E.M. *et al.* (A2), Phys. Rev. Lett. **128**, 132503 (2022)

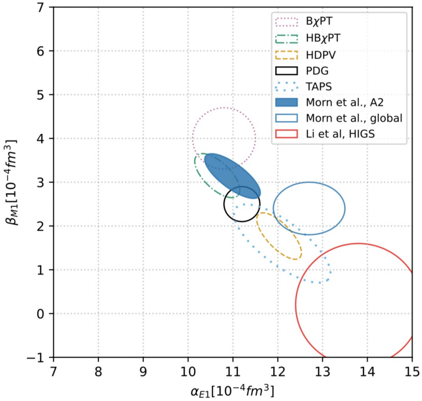
  $$\alpha_{E1} = 10.99 \pm 0.16 \pm 0.47 \pm 0.17 \pm 0.34$$
  $$\beta_{M1} = 3.14 \pm 0.21 \pm 0.24 \pm 0.20 \pm 0.35$$

- Li *et al.* (HIGS), Phys. Rev. Lett. **128**, 132502 (2022)

  $$\alpha_{E1} = 13.8 \pm 1.2 \pm 0.1 \pm 0.3$$
  $$\beta_{M1} = 0.2 \mp 1.2 \pm 0.1 \mp 0.3$$

- E.M., S. Rodini, B. Pasquini, and P. Pedroni, Phys. Rev. Lett. **129**, 102501 (2022)

  $$\alpha_{E1} = 12.7 \pm 0.8 \pm 0.1$$
  $$\beta_{M1} = 2.44 \pm 0.6 \pm 0.1$$



Polarizabilities – Outlook and Plans

It's been a good couple of years for proton Compton scattering:

1. The highest statistics Compton scattering dataset below pion threshold was finally published by the A2 Collaboration.
2. Compton@HIGS published a complemetary dataset at lower energy.
3. Fixed-t DR Bootstrap technique extraction of 6 leading-order proton polarizabilities has been performed.

Also in other news:

1. A high-pressure TPC target/detector has been for approved neutron polarizability (and threshold pion) experiments at MAMI.
2. The Primakoff effect has been measured with the GlueX at JLab with the goal of extracting $\alpha_\pi - \beta_\pi$ for both the charged and neutral pion to high precision.