Measurement of ^{3}He Spin Sum Rules at Low Q^2

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For the saGDH (JLab E97-110) Collaboration

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Outline

- **Spin Sum Rules**
- Experiment E97-110 at Jefferson Lab
- Experimental Results for neutron and ^3He

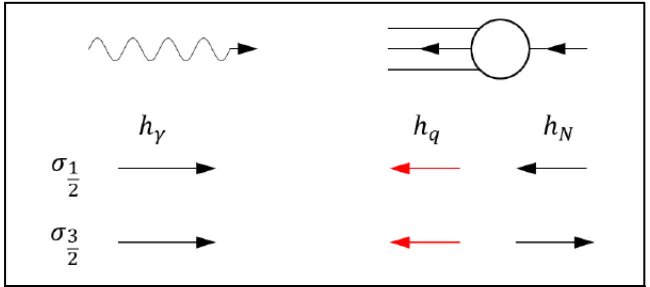
Gerasimov-Drell-Hearn (GDH) Sum Rule

- Relate photoabsorption cross sections to the target's static properties
  - Anomalous magnetic moment κ, spin S, mass M

$$I^{GDH} = \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} \left(\sigma_P(\nu) - \sigma_A(\nu)\right) = 4\pi^2\alpha \frac{\kappa^2}{M^2} S$$

- Prediction based on general principles
  - Dispersion relation
  - Unitarity
  - Lorentz and gauge invariance

- GDH sum rule can be tested experimentally



Circularly polarized photons incident on longitudinally polarized targets

Experimental Tests of the GDH Sum Rule

- Valid for nucleon and nuclear targets ($S = \frac{1}{2}, 1, \frac{3}{2}, \ldots$) with real photons ($Q^2 = 0$)

Target	I^{GDH} (μb)	Measured Range (MeV)	I^{GDH}_{exp} (μb)
Proton	204.8	200-2900	$254 \pm 5 \pm 12$ – 17.5 ± 5.6
Neutron	233.2		
Deuteron	0.65	200-1800	$452 \pm 9 \pm 24$
^{3}He	497.9	200-500	$135 \pm 20 \pm 12$

  - Proton: Verified in 10% - Mainz, Bonn, LEGS
  - Neutron (D and ^{3}He): Mainz, Bonn, LEGS, HIGS
  - Light nuclear target (D and ^{3}He) measurements are interesting
    - GDH integrands and sums

Generalized GDH Sum Rules

- Virtual Compton amplitudes are connected with the moments of spin dependent structure functions
  - Connect moments of spin-dependent structure functions with the Compton amplitudes

$$I_{TT}(Q^2) = \frac{M^2}{4\pi^2\alpha}\int_{\nu_{th}}^{\infty}\frac{K\sigma_{TT}(\upsilon, Q^2)}{\upsilon^2}d\upsilon$$

$$= \frac{2M^2}{Q^2}\int_0^{x_{th}}\left[g_1(x, Q^2) - \frac{4M^2x^2}{Q^2}g_2(x, Q^2)\right]dx$$

- g_1 and g_2 are experimentally accessible, $I_{TT}(Q^2)$ predicted by theories
  - Chiral Effective Field Theory (ChEFT)
  - Lattice QCD (not available yet)

Generalized Spin Polarizabilities

- Longitudinal-Transverse (LT) interference polarizability

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_{th}} [g_1(x,Q^2) + g_2(x,Q^2)]x^2 dx$$

  - Quantifies the spin precession from LT interference (analogous in classical view)
  - Arises because of virtual photon ($Q^2 \neq 0$) can be longitudinally polarized
  - "Gold-plated" observable for ChEFT because of suppression in $\Delta(1232)$ contributions

- Forward spin polarizability

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_{th}} \left[g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)\right] x^2 dx$$

Other Spin Sum Rules

- First Moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2)dx$$

  – Connects to the total spin carried by the quarks
  – Inelastic contribution: another generalized form of GDH sum

$$I_1(Q^2) = \frac{16\pi^2\alpha}{Q^2}\int_0^{x_{th}} g_1(x, Q^2)dx = 2\pi^2\alpha S_1(0, Q^2)$$

Ji and Osborne, J. Phys. G27, 127 (2001)

- Bjorken Sum Rule

$$\Gamma_1^P(Q^2) - \Gamma_1^N(Q^2) = \frac{g_A}{6} + O\left(\alpha_s(Q^2)\right) + O\left(\frac{1}{Q^2}\right)$$

  – g_A, nucleon axial charge
  – Consistent with experimental result within 10%

Importance of the Generalized GDH Sum Rule

- Constraints at two-ends
  - GDH sum for real photons $(Q^2 = 0)$
  - Bjorken sum rule $(Q^2 \rightarrow \infty)$
- Tests the theoretical calculations for the Compton amplitudes at very low Q^2
  - Baryon Chiral Perturbation Theory (HBChPT, IRBChPT, RBChPT)
  - Future Lattice QCD
- Study the transition from non-perturbative to perturbative QCD

Outline

- Spin Sum Rules
- **Experiment E97-110 at Jefferson Lab**
- Experimental Results for neutron and ^3He

Experiments Summary

Observable	H target	D target	^{3}He target
g_1, g_2, Γ_1 & Γ_2 at high Q^2	SLAC JLAB SANE	SLAC	SLAC JLAB E97-117 JLAB E01-012 JLAB E06-014
g_1 & Γ_1 at high Q^2	SMC HERMES JLAB EG1	SMC HERMES JLAB EG1	HERMES
Γ_1 & Γ_2 at low Q^2	JLab RSS	JLab RSS	JLab E94-010 JLab E97-103
Γ_1 at low Q^2	SLAC HERMES JLAB EG1	SLAC HERMES JLAB EG1	HERMES
Γ_1, $Q^2 << 1$ GeV^2	JLab EG4	JLab EG4	JLab E97-110
Γ_2, $Q^2 << 1$ GeV^2	JLab E08-027		JLab E97-110

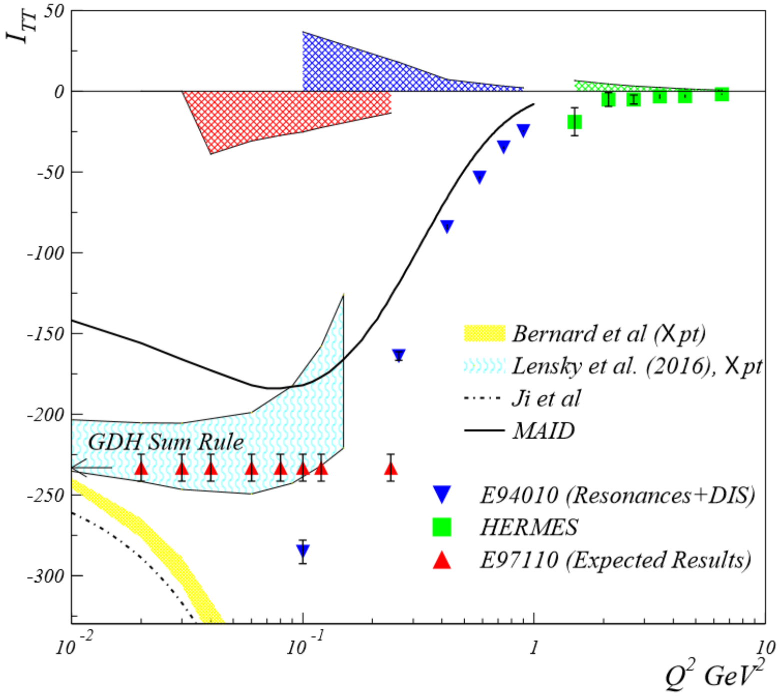
Recent Low-Q experiments at JLab:

1. EG4 (CLAS): NH_3 and ND_3 to measure g_1
2. E08-027 (Hall A): NH_3 to measure g_2
3. **E97-110 (Hall A): ^{3}He to measure g_1 and g_2**

All measurements down to at least **$Q^2 \sim 0.02$ GeV^2**

Motivation for E97-110

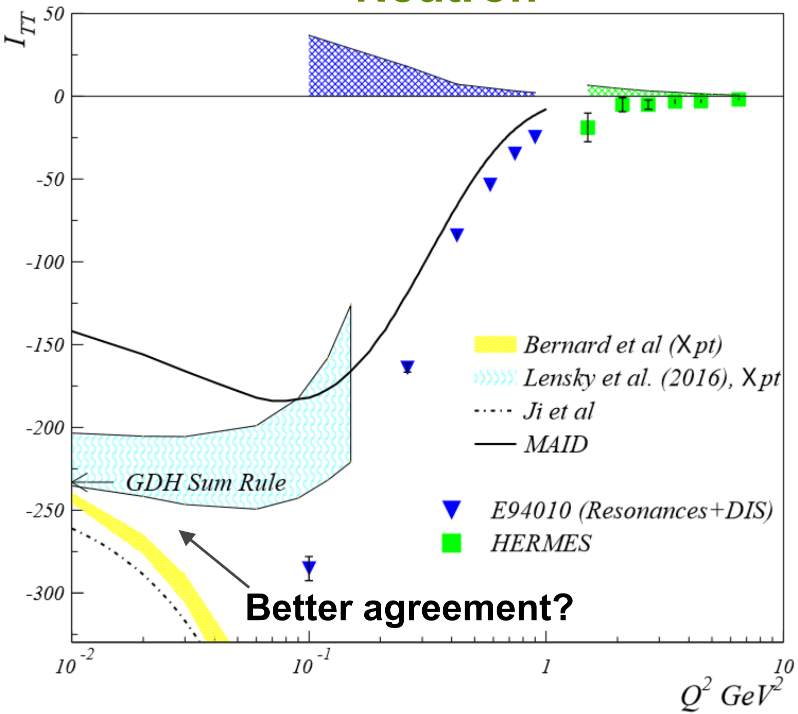
- Precision measurement of the moments of spin structure functions at low Q2, 0.02 to 0.24 GeV^2 for the neutron (^{3}He)
- Covered an unmeasured region of kinematics to test theoretical calculations (Chiral Perturbation theory)
- Complements data from experiment E94-010 covered region from 0.1 to 0.9 GeV^2



Spokespersons: J.-P. Chen, A. Deur, F. Garibaldi

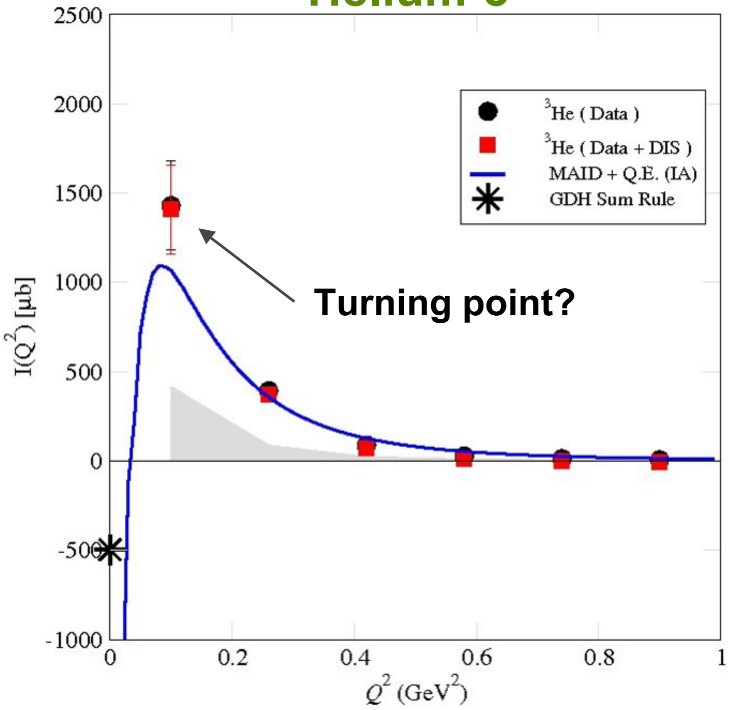
E94-010 Results

Neutron



M. Amarian et al., Phys. Rev. Lett., 89:242301, Nov 2002.

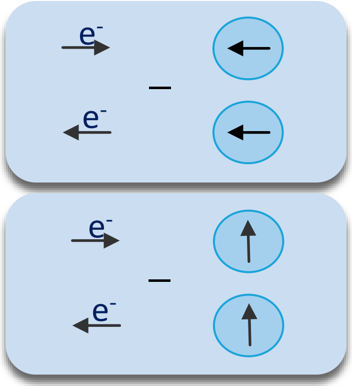
Helium-3



K. Slifer et al., Phys. Rev. Lett., 101:022303, Jul 2008.

Measurement of SSF

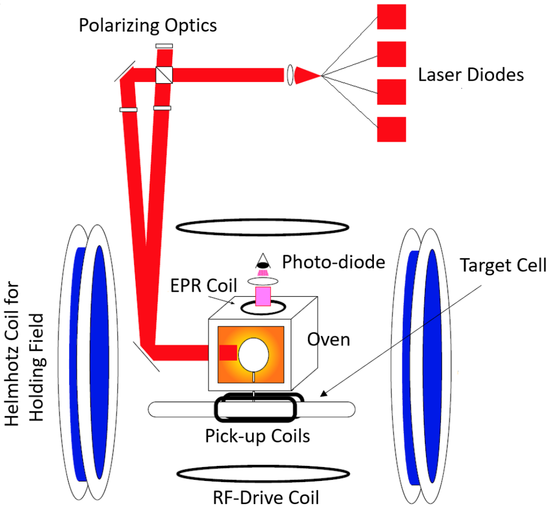
- Experimental access to the spin-dependent structure functions
  - Doubly polarized inclusive scattering
  - Different polarization configurations
  - Effective neutron target (^3He)



$$\Delta\sigma_{\parallel} = \frac{4\alpha^2}{M\nu Q^2}\frac{E'}{E}[(E + E'cos\theta)g_1 - 2Mxg_2]$$

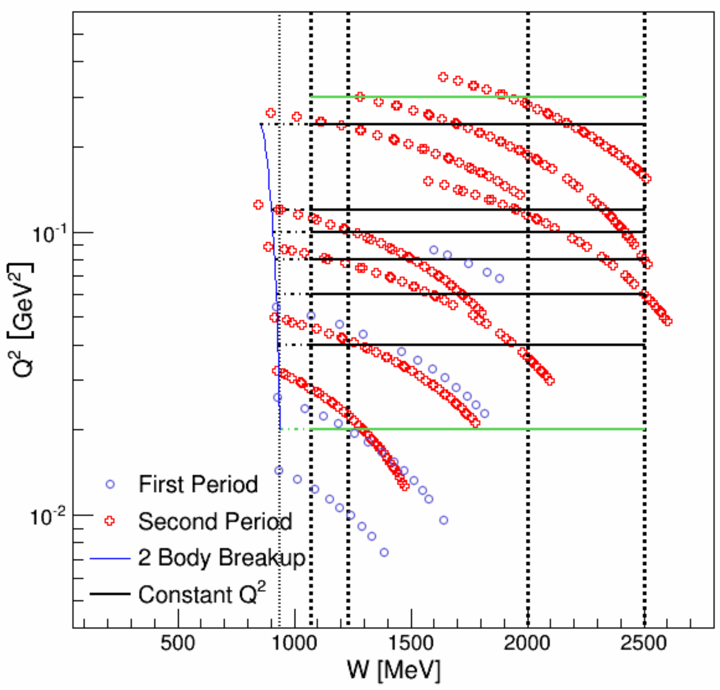
$$\Delta\sigma_{\perp} = \frac{4\alpha^2}{M\nu Q^2}\frac{E'^2}{E}sin\theta[g_1 + \frac{2E}{\nu}g_2]$$

E97-110: $P_t \approx 40\%$



JLab Polarized ^{3}He System

Kinematic Coverage



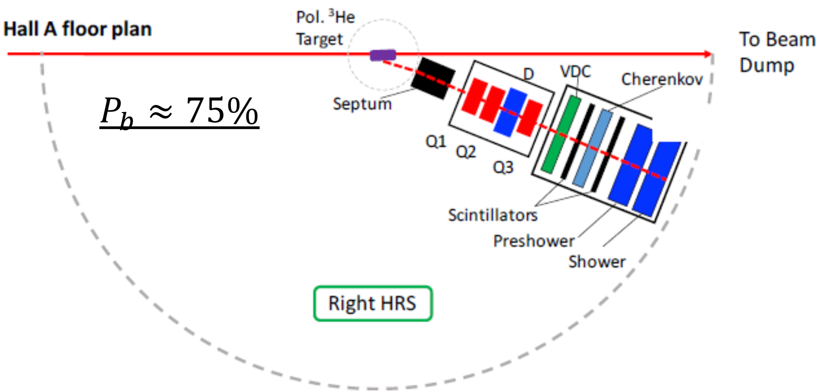
8 datasets covering 1.1 – 4.4 GeV @ 6° or 9°

- Quasi-elastic, resonances, and partially DIS
- Constant Q^2 interpolation based on the 5 low-Q datasets
- 3 high-Q datasets for constraining
- Q^2 values selected to minimize syst. from interpolation (close to real-data threshold region)

Target Cell	Angle	Beam Energy (MeV)
Penelope	6.10°	2134.2
Priapus	6.10°	2134.9
Priapus	6.10°	2844.8
Priapus	6.10°	4208.8
Priapus	9.03°	1147.3
Priapus	9.03°	2233.9
Priapus	9.03°	3318.8
Priapus	9.03°	3775.4
Priapus	9.03°	4404.2

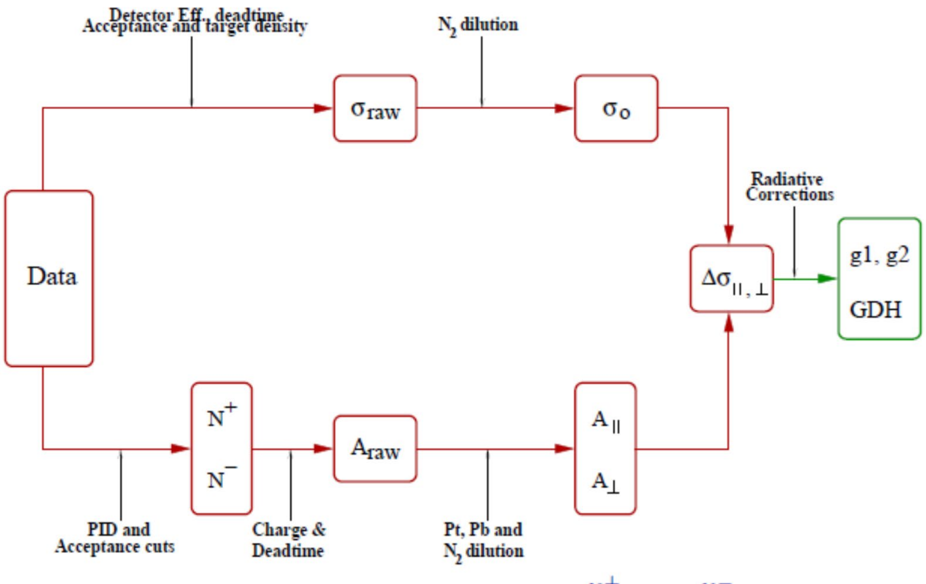
Setup and Analysis

- Right-arm HRS + septum magnet
  - Extend forward angle to below 12.5°
- Cross-section analysis and asymmetry analysis combined



$P_b \approx 75\%$

$$\sigma_o = \frac{N_{\text{cuts}}}{N_{\text{inc}}\rho\varepsilon_{\text{det}}LT} * Acc. - \frac{2\rho_{N_2}}{\rho + \rho_{N_2}}\sigma_{N_2}$$

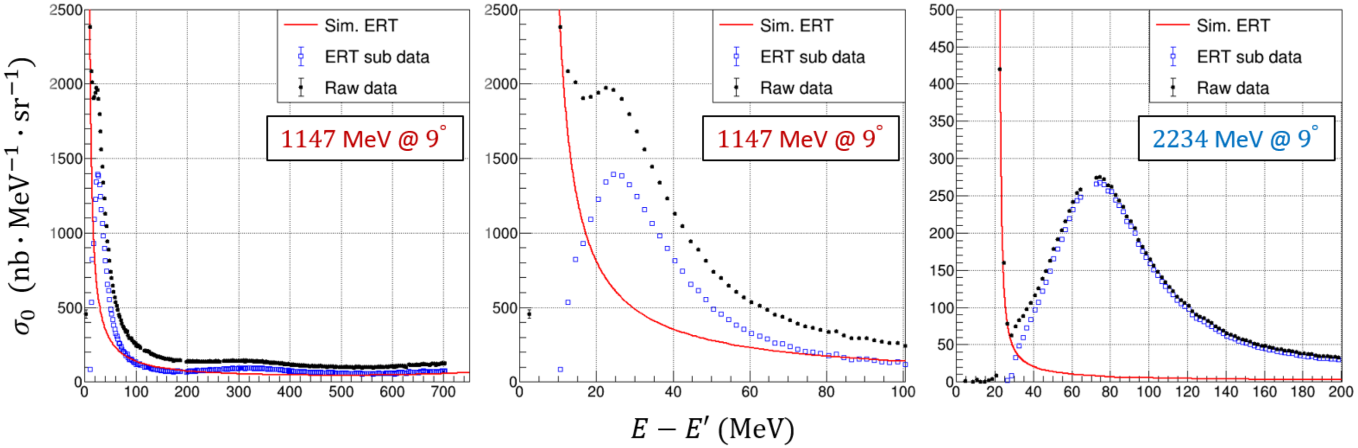


$$A_{\parallel,\perp} = \frac{1}{f_{N_2} P_{tg} P_{\text{beam}}} \frac{\frac{N^+}{Q^+ LT^+} - \frac{N^-}{Q^- LT^-}}{\frac{N^+}{Q^+ LT^+} + \frac{N^-}{Q^- LT^-}}$$

Elastic Radiative Tail Subtraction

- Significant contribution to syst. in Q.E. at the lowest Q^2
  - A dedicated simulation for elastic radiative tails
  - EM FF was based on Amroun et al. Nucl. Phys. A579, 596-626 (1994)
  - Radiative effects from different recipes evaluated

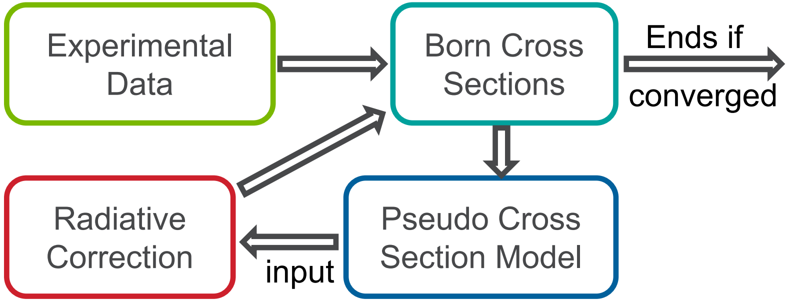
    Akushevich & Shumeiko (AS), Journal of Physics G, 20(4):513, 1994.
    Mo & Tsai (MT), Rev. Mod. Phys., 41:205–235, 1969.

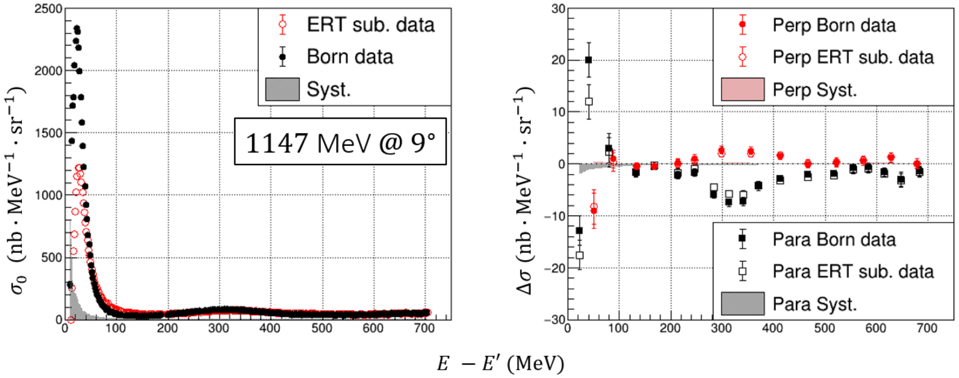


Inelastic Radiative Correction

- Iterative correction
  - Build a pseudo-model with the experimental data
  - Bilinear/bicubic interpolation for unmeasured points
  - A few of points extrapolated with models
  - Radiative effects calculated following POLRAD

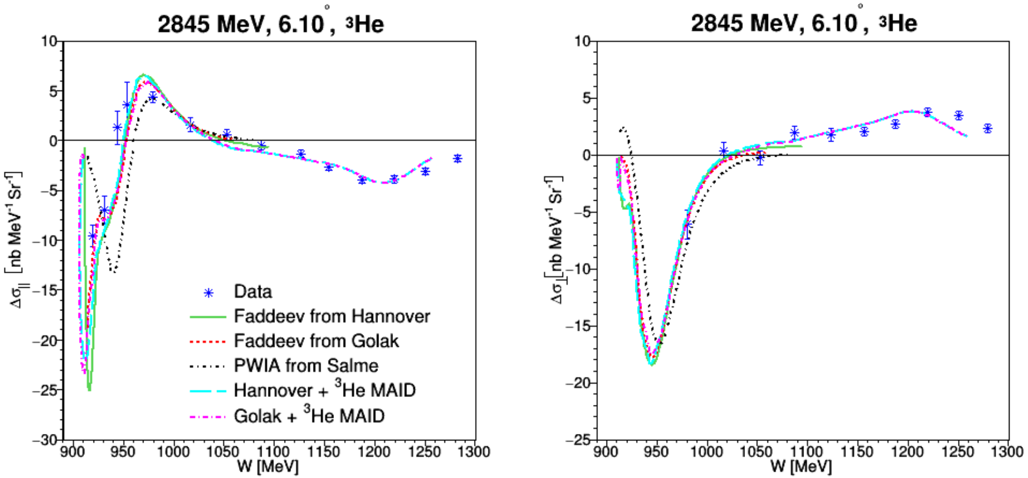
    I. Akushevich et al., Comput. Phys. Commun. 104, 201-244 (1997)





Quasi-elastic Calculations

- Quasi-elastic contribution subtracted for neutron results
  - Faddeev calculations from Hannover or Golak
  - MAID model with ENPA plus Smearing for ^{3}He
  - Separation of Q.E. and inelastic contributions with fits



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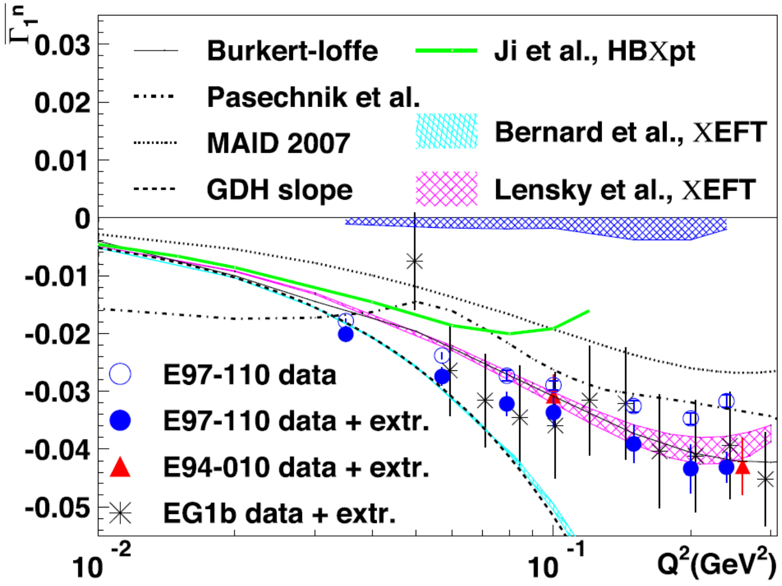
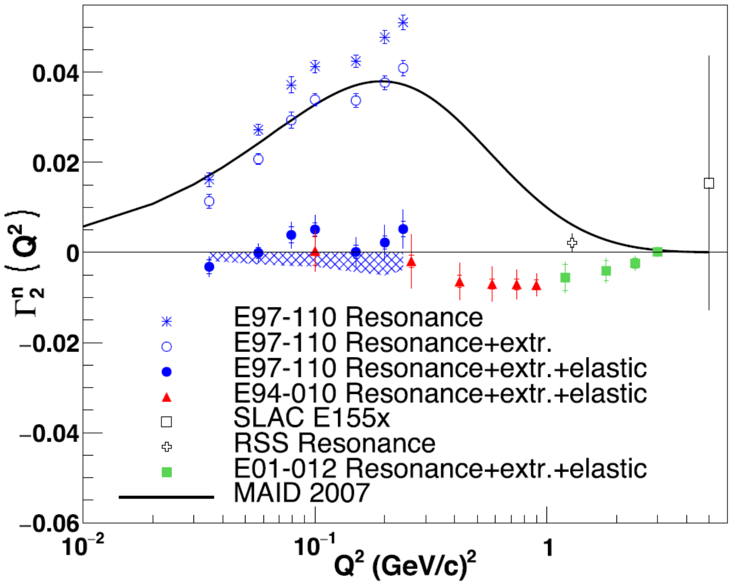
- **Experimental Results for neutron and ^{3}He**

Neutron Results – First Moments

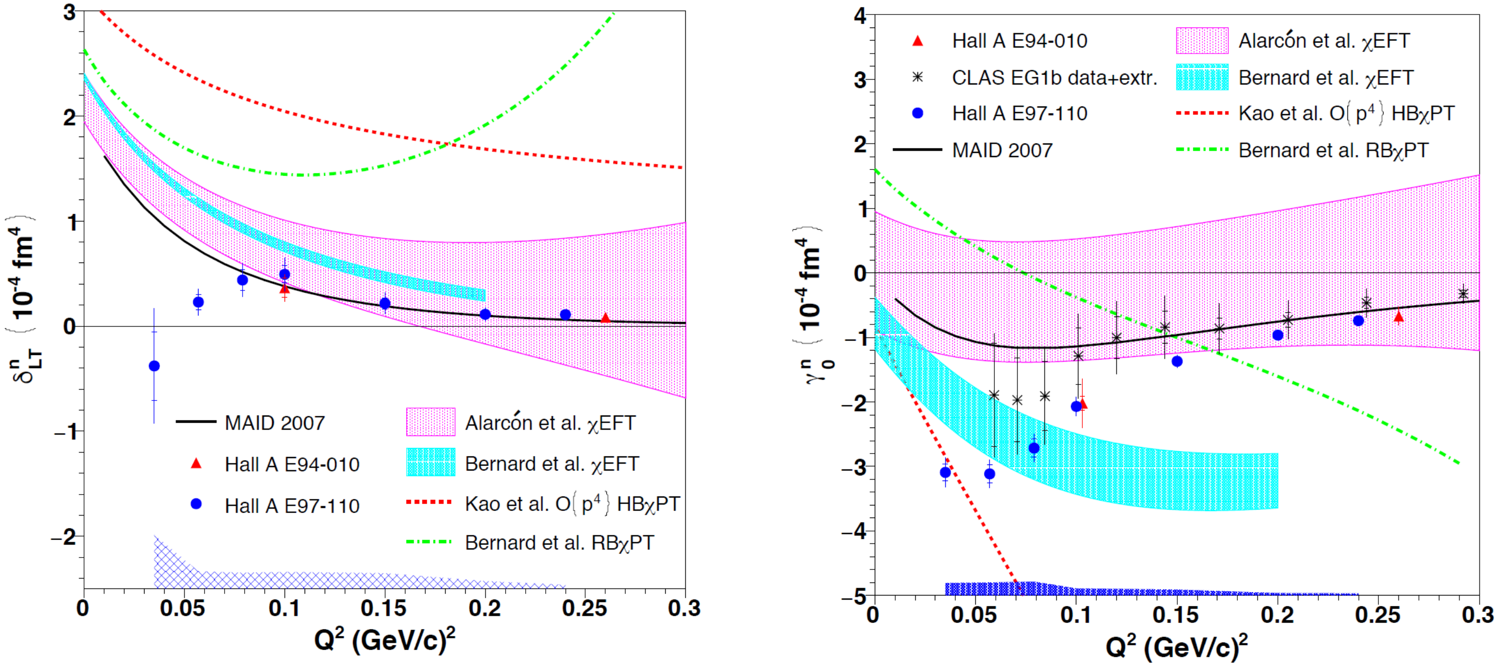
Nuclear corrections follow the recipe from C. Ciofi degli Atti and S. Scopetta (1997)





V. Sulkosky, J. Singh, C. Peng, J.-P. Chen, A. Deur *et al.*, Phys. Lett. B 805 (2020) 135428

Neutron Results – Spin Polarizabilities

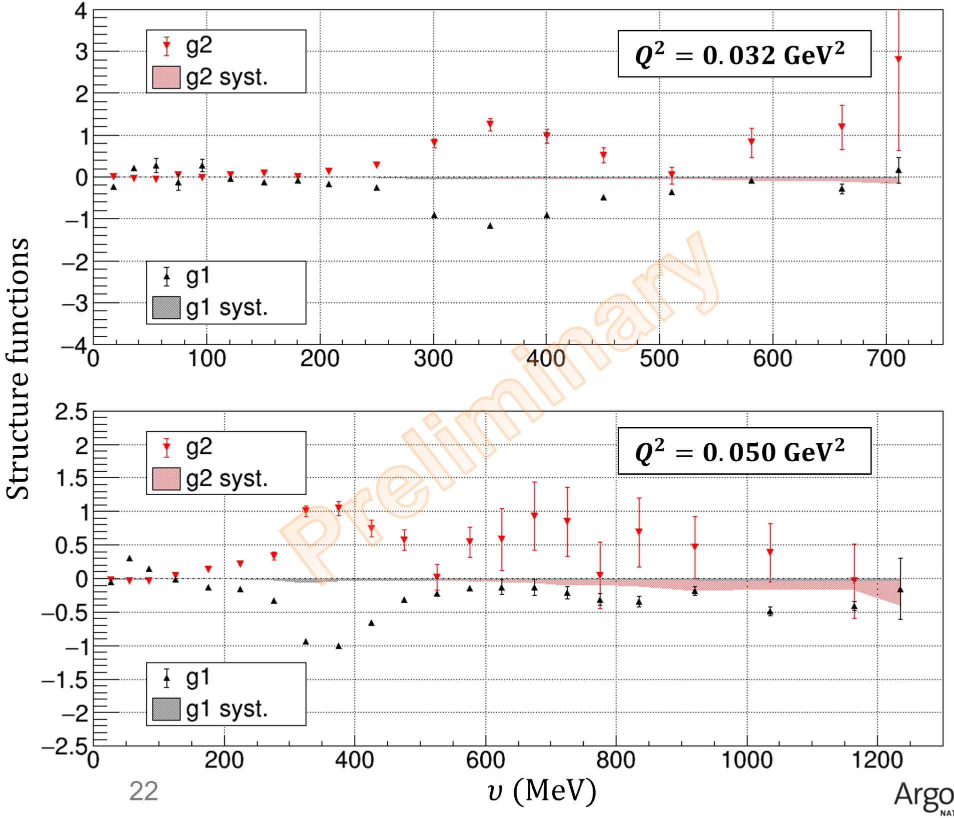


V. Sulkosky, C. Peng, J.-P. Chen, A. Deur *et al.*, *Nature Physics* 17, p687–692 (2021)

^{3}He Spin-dependent Structure Functions

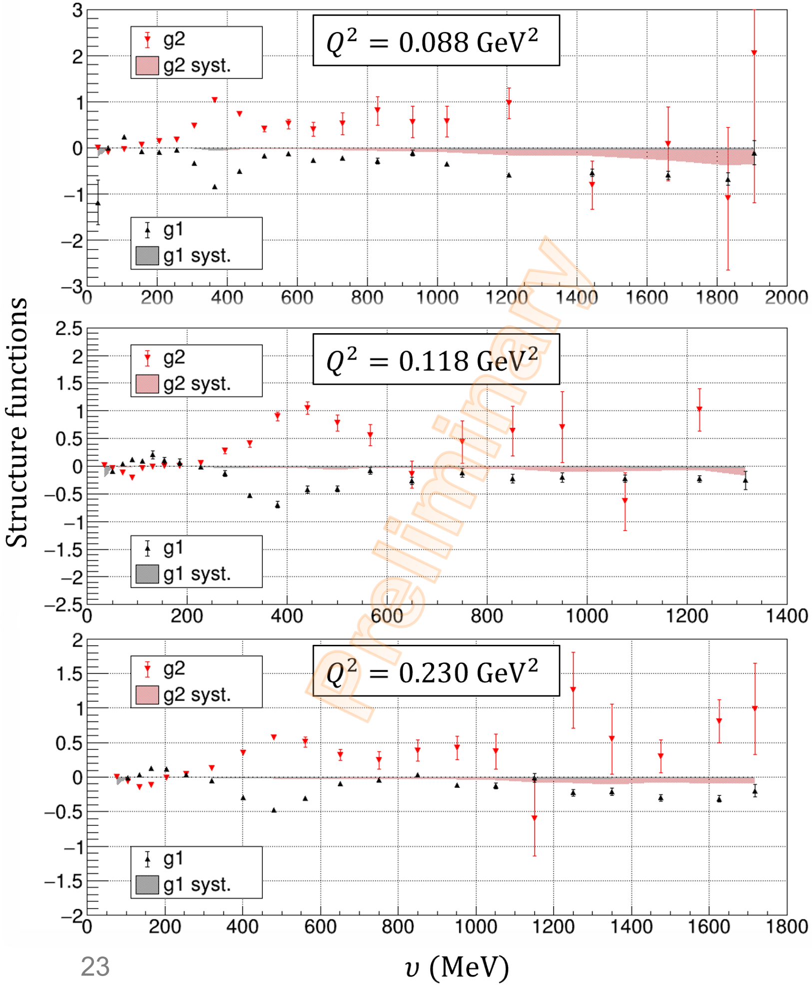
- Two lowest Q^2 point
  - Interpolated from
    1.1 GeV @ 9°
    2.1 GeV @ 6°
    2.8 GeV @ 6°

- $g_1 + g_2 \approx 0$ for $\Delta(1232)$



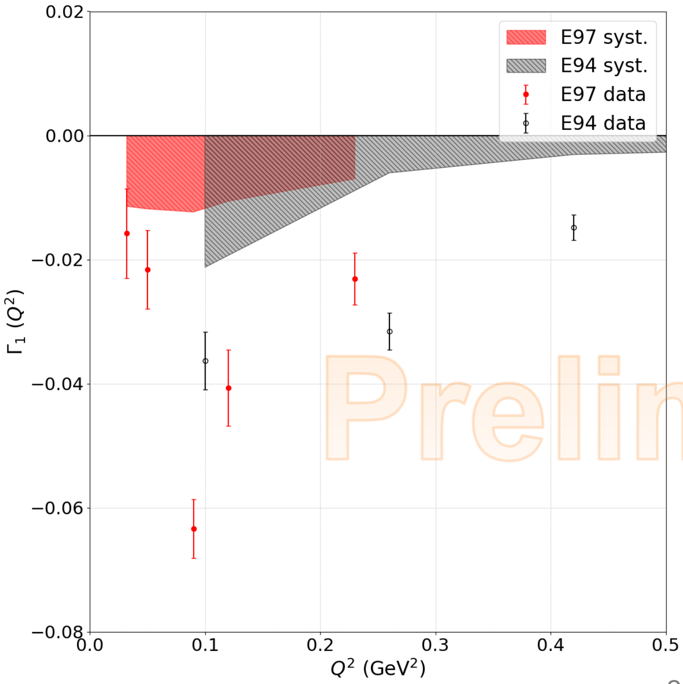
^{3}He Spin-dependent Structure Functions

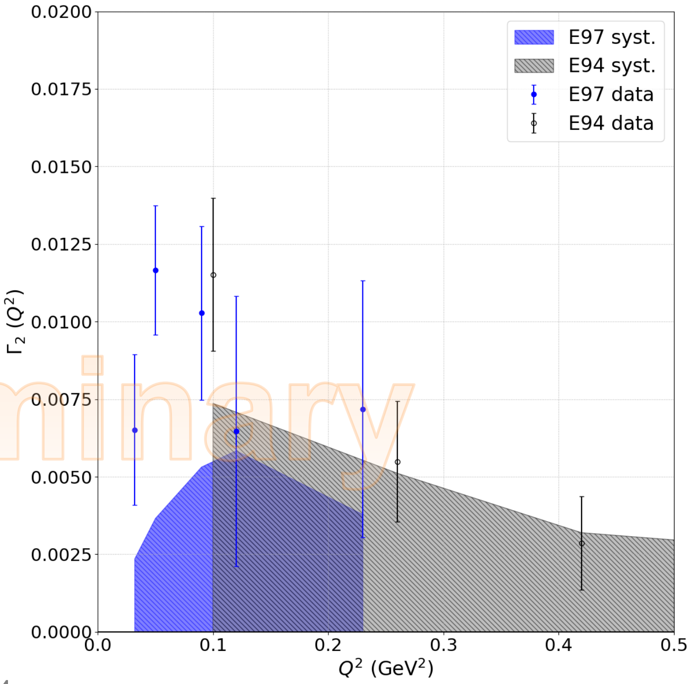
- Three higher Q^2 point
  - Interpolated from
    2.8 GeV @ 6°
    2.2 GeV @ 9°
    3.3 GeV @ 9°
  - Additional constraints from
    4.2 GeV @ 6°
    3.8 GeV @ 9°
    4.4 GeV @ 9°



^{3}He First Moments Γ_1, Γ_2

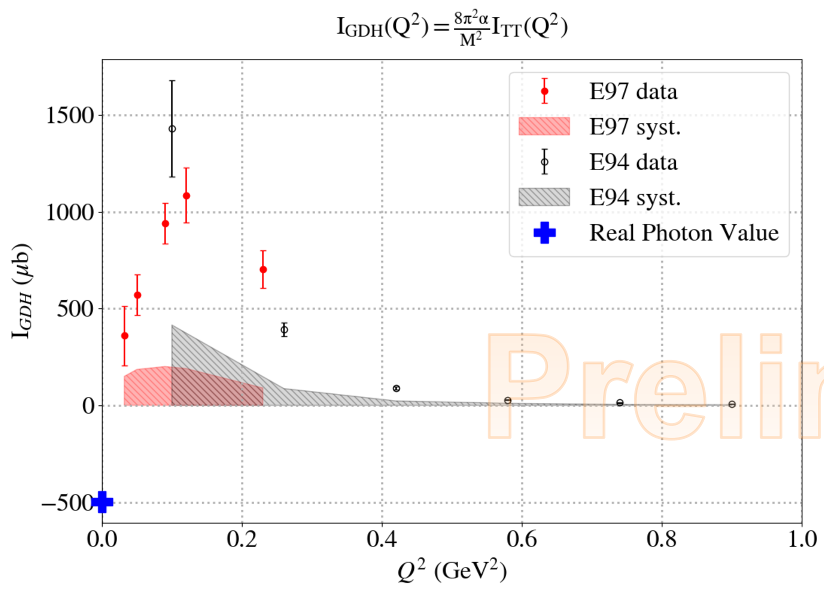
Consistent with E94 results at overlapping Q^2, trend is turning at < 0.1 GeV^2

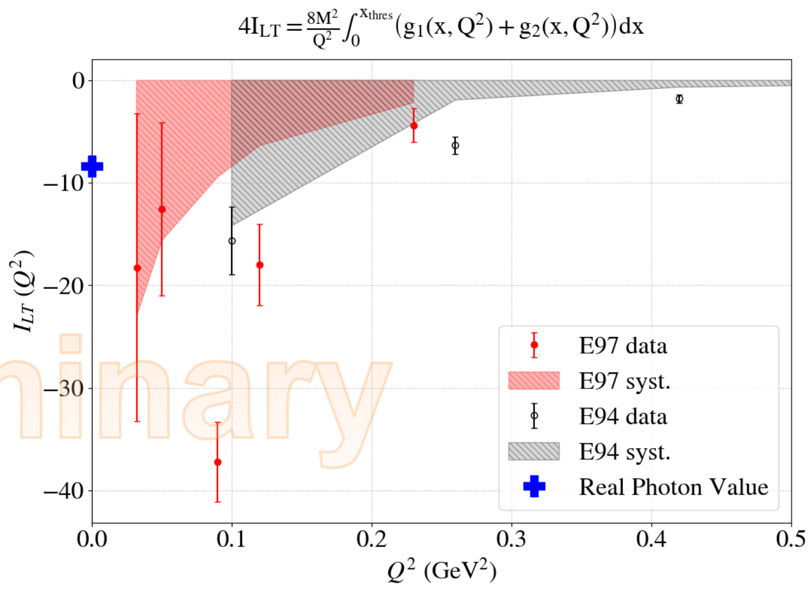




^{3}He Results - Sum Rules I_{TT}, I_{LT}

A turning point observed at < 0.1 GeV², data curve is approaching the real photon point.





Summary

- JLab E97-110 has measured the spin sum rules for neutron and ^{3}He at low Q^2
- Generalized spin polarizabilities of the neutron do not agree well with ChEFT predictions at $Q^2 < 0.1$ GeV^2
  - Lattice QCD inputs will be important to understand the discrepancy on neutron result
- The turning point of ^{3}He GDH sum rule is observed at around $Q^2 = 0.1$ GeV^2, correct trending to the real photon point

Thank you

This work is supported in part by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Contract No. DE-AC02-06CH11357.

Data Table - δ_{LT}

Q^2 [GeV^2]	ν_{max} (W_{max}) [GeV]	$\delta_{\mathrm{LT}}^{^3He,\ res}(Q^2)\pm$(stat)$\pm$(syst) [$10^{-4}$ fm^4]	$\delta_{\mathrm{LT}}^{n,\ res}(Q^2)\pm$(stat)$\pm$(syst) [$10^{-4}$ fm^4]	$\delta_{\mathrm{LT}}^{n}(Q^2)\pm$(stat)$\pm$(syst) [$10^{-4}$ fm^4]
0.035	1.690 (2.00)	$-0.356 \pm 0.280 \pm 0.583$	$-0.379 \pm 0.326 \pm 0.677$	$-0.383 \pm 0.326 \pm 0.677$
0.057	1.700 (2.00)	$0.174 \pm 0.061 \pm 0.169$	$0.229 \pm 0.071 \pm 0.197$	$0.225 \pm 0.071 \pm 0.197$
0.079	1.710 (2.00)	$0.360 \pm 0.084 \pm 0.168$	$0.439 \pm 0.098 \pm 0.195$	$0.435 \pm 0.098 \pm 0.195$
0.100	2.885 (2.49)	$0.410 \pm 0.072 \pm 0.180$	$0.493 \pm 0.083 \pm 0.209$	$0.491 \pm 0.083 \pm 0.209$
0.150	2.910 (2.48)	$0.178 \pm 0.045 \pm 0.149$	$0.216 \pm 0.053 \pm 0.173$	$0.215 \pm 0.053 \pm 0.173$
0.200	2.655 (2.38)	$0.091 \pm 0.024 \pm 0.078$	$0.112 \pm 0.028 \pm 0.091$	$0.111 \pm 0.028 \pm 0.091$
0.240	2.320 (2.23)	$0.090 \pm 0.017 \pm 0.041$	$0.110 \pm 0.020 \pm 0.043$	$0.108 \pm 0.020 \pm 0.043$

Data Table - γ_0

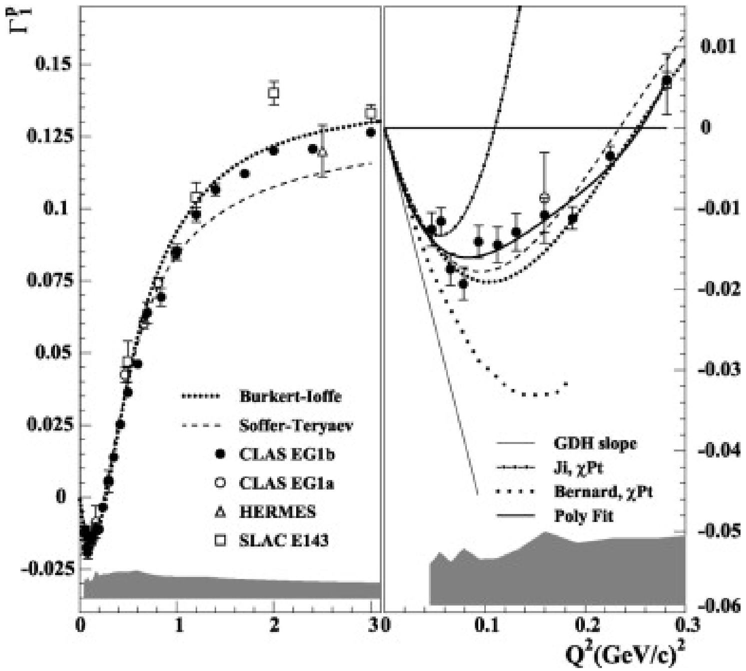
Q^2 [GeV^2]	ν_{max} (W_{max}) [GeV]	$\gamma_0^{^3He,\ res}(Q^2)\pm$(stat)$\pm$(syst) [$10^{-4}$ fm^4]	$\gamma_0^{n,\ res}(Q^2)\pm$(stat)$\pm$(syst) [$10^{-4}$ fm^4]	$\gamma_0^{n}(Q^2)\pm$(stat)$\pm$(syst) [10^{-4} fm^4]
0.035	1.690 (2.00)	$-2.590 \pm 0.111 \pm 0.225$	$-3.092 \pm 0.129 \pm 0.270$	$-3.094 \pm 0.129 \pm 0.270$
0.057	1.700 (2.00)	$-2.613 \pm 0.121 \pm 0.215$	$-3.115 \pm 0.141 \pm 0.259$	$-3.117 \pm 0.141 \pm 0.259$
0.079	1.710 (2.00)	$-2.274 \pm 0.121 \pm 0.226$	$-2.715 \pm 0.140 \pm 0.270$	$-2.717 \pm 0.140 \pm 0.270$
0.100	2.885 (2.49)	$-1.725 \pm 0.063 \pm 0.143$	$-2.070 \pm 0.074 \pm 0.170$	$-2.070 \pm 0.074 \pm 0.170$
0.150	2.910 (2.48)	$-1.135 \pm 0.044 \pm 0.105$	$-1.370 \pm 0.051 \pm 0.125$	$-1.370 \pm 0.051 \pm 0.125$
0.200	2.655 (2.38)	$-0.798 \pm 0.027 \pm 0.056$	$-0.964 \pm 0.032 \pm 0.065$	$-0.965 \pm 0.032 \pm 0.065$
0.240	2.320 (2.23)	$-0.612 \pm 0.022 \pm 0.043$	$-0.740 \pm 0.026 \pm 0.050$	$-0.742 \pm 0.026 \pm 0.050$

Data Table - I_{LT}

Q^2 [GeV^2]	ν_{max} (W_{max}) [GeV]	$I_{\rm LT}^{^3He,\, res}(Q^2)\pm$(stat)$\pm$(syst)	$I_{\rm LT}^{n,\, res}(Q^2)\pm$(stat)$\pm$(syst)	$I_{\rm LT}(Q^2)^n\pm$(stat)$\pm$(syst)
0.035	1.690 (2.00)	$-0.326 \pm 0.272 \pm 0.520$	$-0.294 \pm 0.316 \pm 0.604$	$-1.112 \pm 0.316 \pm 0.606$
0.057	1.700 (2.00)	$0.285 \pm 0.117 \pm 0.333$	$0.413 \pm 0.136 \pm 0.387$	$-0.862 \pm 0.136 \pm 0.389$
0.079	1.710 (2.00)	$0.610 \pm 0.128 \pm 0.268$	$0.786 \pm 0.149 \pm 0.312$	$-0.721 \pm 0.149 \pm 0.314$
0.100	2.885 (2.49)	$0.879 \pm 0.098 \pm 0.280$	$1.092 \pm 0.114 \pm 0.327$	$-0.126 \pm 0.114 \pm 0.329$
0.150	2.910 (2.48)	$0.339 \pm 0.049 \pm 0.198$	$0.458 \pm 0.057 \pm 0.231$	$-0.266 \pm 0.057 \pm 0.233$
0.200	2.655 (2.38)	$0.289 \pm 0.047 \pm 0.228$	$0.389 \pm 0.055 \pm 0.265$	$-0.345 \pm 0.055 \pm 0.267$
0.240	2.320 (2.23)	$0.391 \pm 0.057 \pm 0.163$	$0.511 \pm 0.067 \pm 0.190$	$-0.267 \pm 0.067 \pm 0.192$

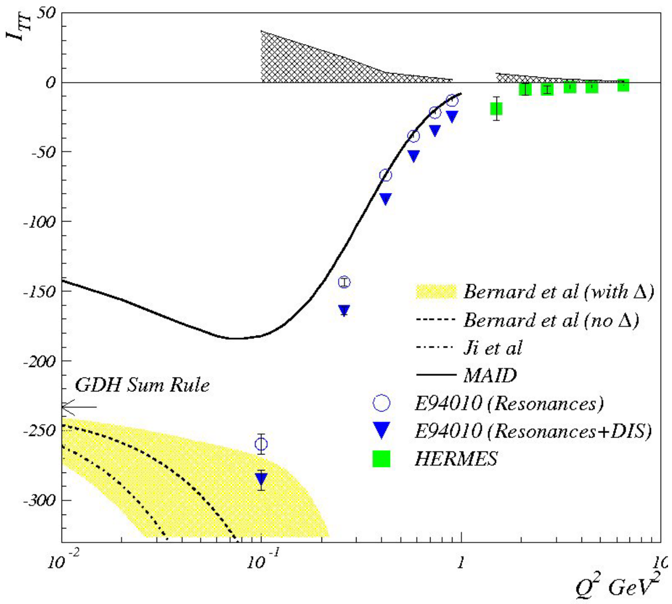
Measurements of Γ_1

Proton



Y. Prok et al. Phys. Lett. B672 12, 2009

Neutron

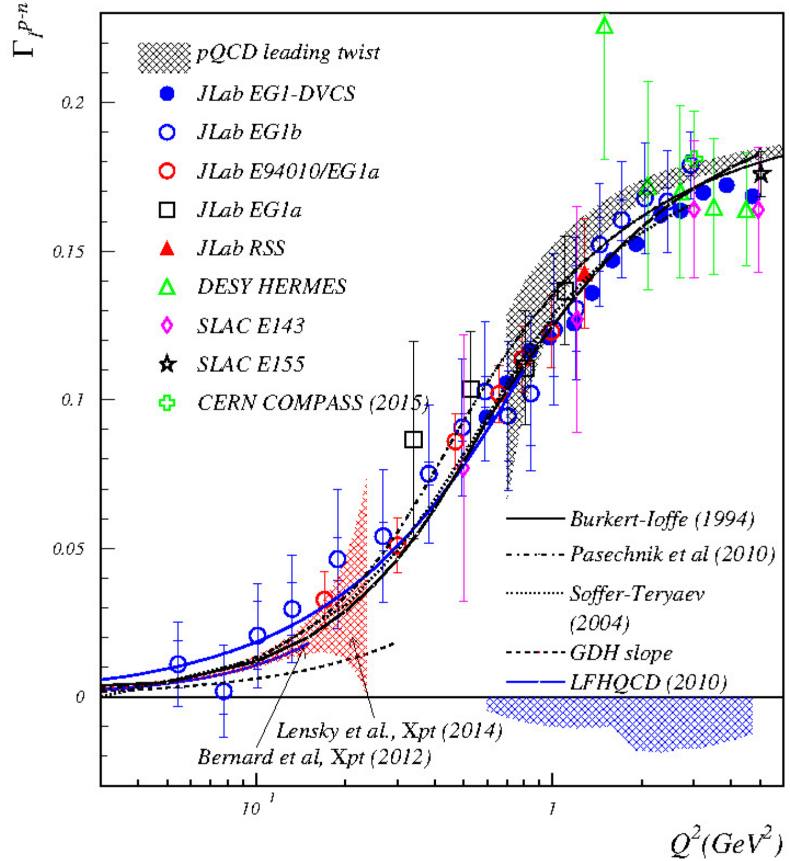


PRL 89, 242301 (2002)

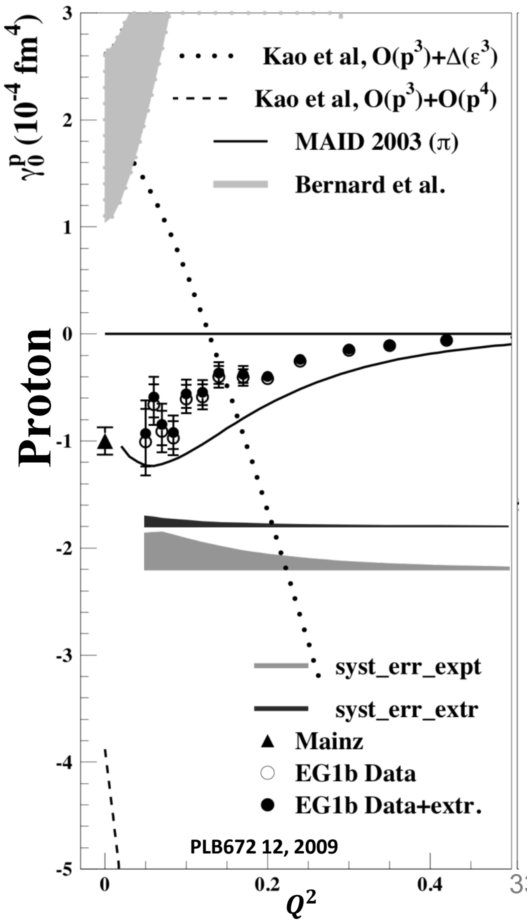
MAID: phenomenological model with only resonance contributions.

The Bjorken Sum

Proton – Neutron



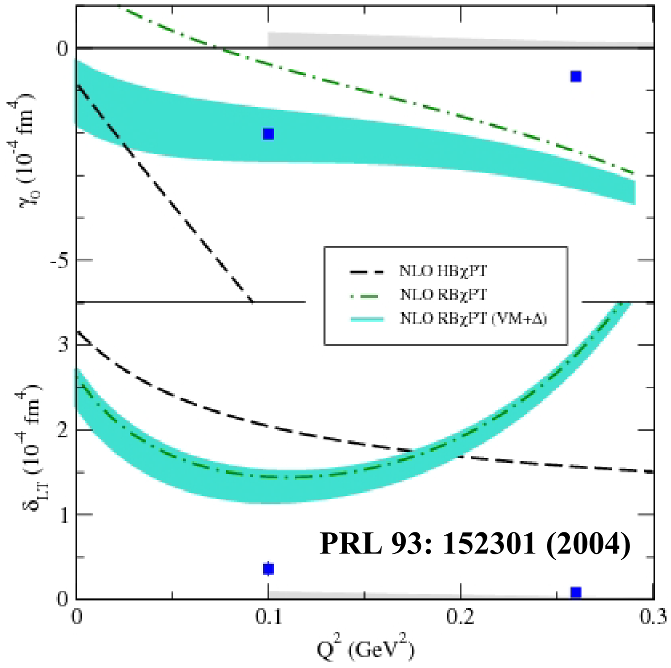
Proton Spin Polarizability: γ_0



$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

Calculations also fail for proton γ_0

Neutron Forward Spin Polarizabilities



$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

$$\delta_{LT} = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 + g_2\right]$$

Failure of χPT?

Heavy Baryon χPT Calculation
Kao, Spitzenberg, Vanderhaeghen
PRD 67:016001(2003)

Relativistic Baryon χPT
Bernard, Hemmert, Meissner
PRD 67:076008(2003)

Summary of Data Comparison with χPT

State of χpt affairs (2006):

no unmeasured low-x data

Ref.	Γ_1^p	Γ_1^n	Γ_1^{p-n}	Γ_1^{p+n}	γ_0^p	γ_0^n	γ_0^{p-n}	γ_0^{p+n}	δ_{LT}^n	d_2^p	d_2^n
Bernard et al.	X	X	A	X	X	A	X	X	X	-	X
Ji et al.	X	X	A	X	-	-	-	-	-	-	-
Kao et al.	-	-	-	-	X	A	X	X	X	-	X

Δ suppressed (Γ_1^{p-n})

Δ suppressed (γ_0^{p-n})

Δ suppressed (expected) "δ_{LT}crisis" (δ_{LT}^n)

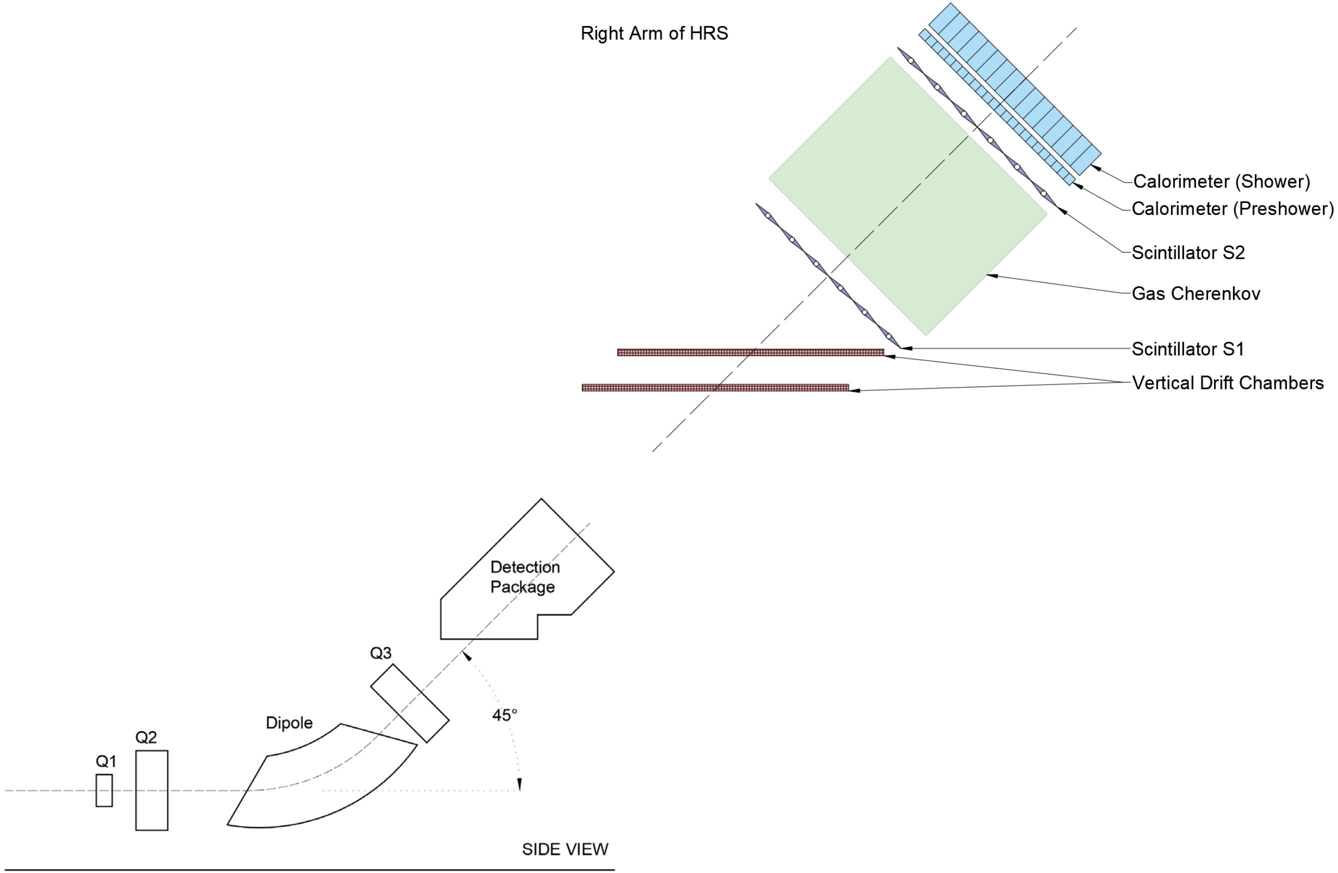
A: agree with data
X: disagree with data
-: no calculation available

Recent Experimental Efforts at JLab

1. EG4 (CLAS): NH_3 and ND_3 to measure g_1
2. E08-027 (Hall A): NH_3 to measure g_2
3. **E97-110 (Hall A): ^3He to measure g_1 and g_2**

All measurements down to at least
$Q^2 \sim 0.02$ GeV^2

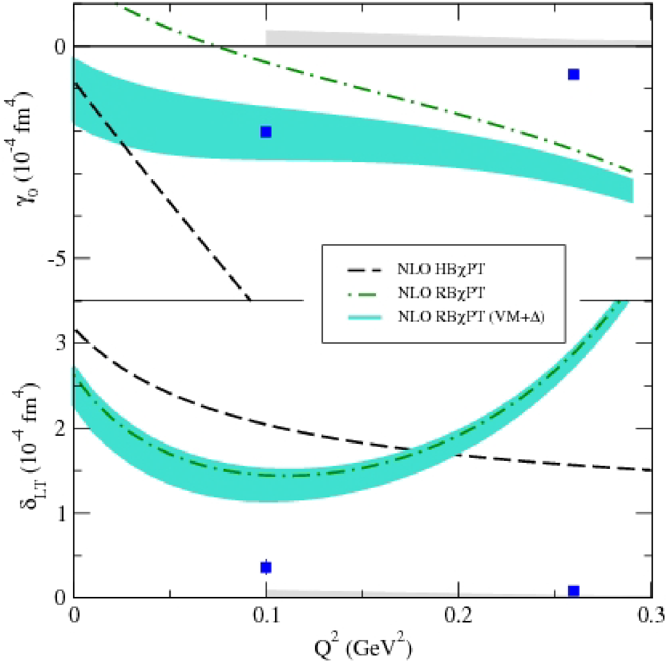
HRS


Right Arm of HRS
Calorimeter (Shower)
Calorimeter (Preshower)
Scintillator S2
Gas Cherenkov
Scintillator S1
Vertical Drift Chambers
Detection Package
Q3
45°
Dipole
Q2
Q1
SIDE VIEW

Systematic Table (V. Sulkosky)

Source	Systematic Uncertainty (%)
Target density	1.6
VDC tracking efficiency	1.0
Beam charge	1.0
Detector efficiencies	1.4-1.9
ν-dependent stability	0.0-1.25
Spectrometer acceptance	2.0-3.0
Beam polarization	3.5
Target polarization	3.0-5.2

E94-010 Results



$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

$$\delta_{LT} = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 + g_2\right]$$

Need further tests at low Q^2

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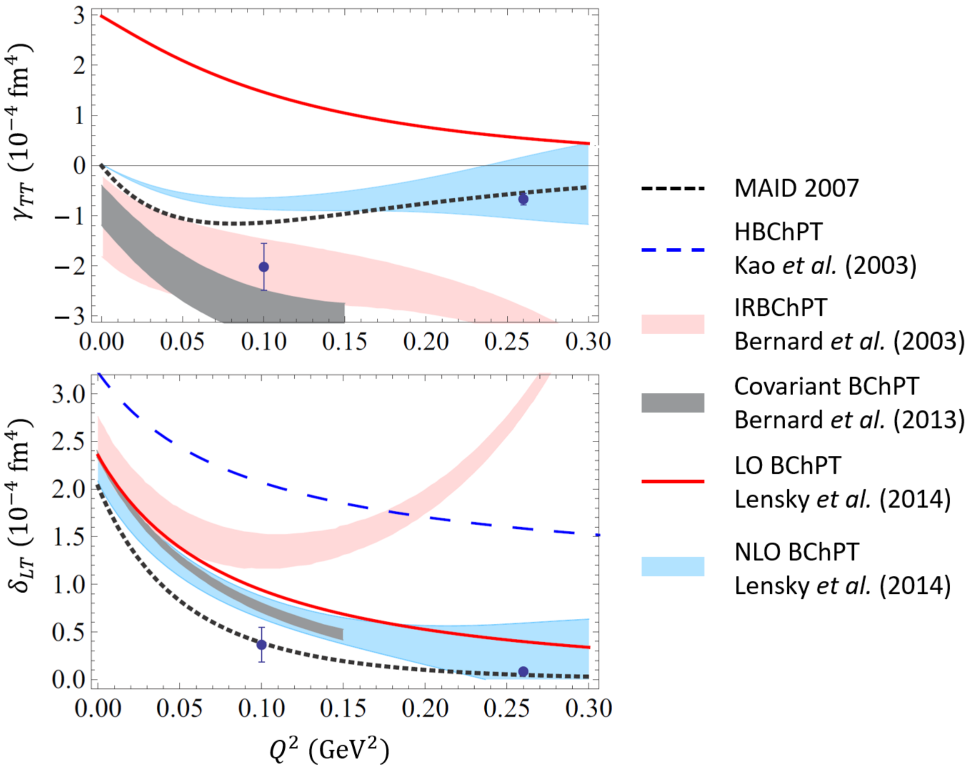
Theoretical developments



Radiative Correction

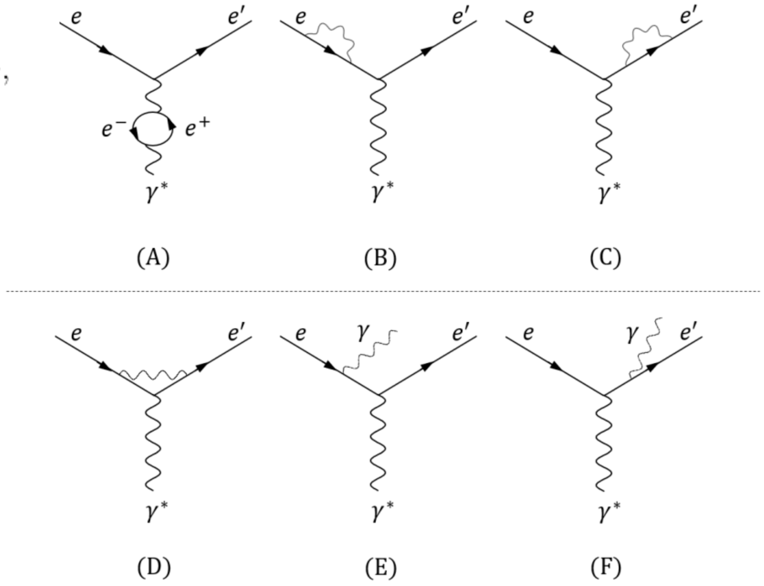
- Born level cross sections are retrieved after radiative corrections

$$\sigma_0^{Born} = \sigma_0^{exp} + \Delta\sigma_{RC},$$

$$\Delta\sigma_{\parallel,\perp}^{Born} = \Delta\sigma_{\parallel,\perp}^{exp} + \delta(\Delta\sigma_{\parallel,\perp})_{RC},$$

- External Energy loss
  - Bremsstrahlung
  - Ionization

- Internal effects
  - Bremsstrahlung, energy loss
  - Virtual photon, no loss



Elastic Radiative Tail

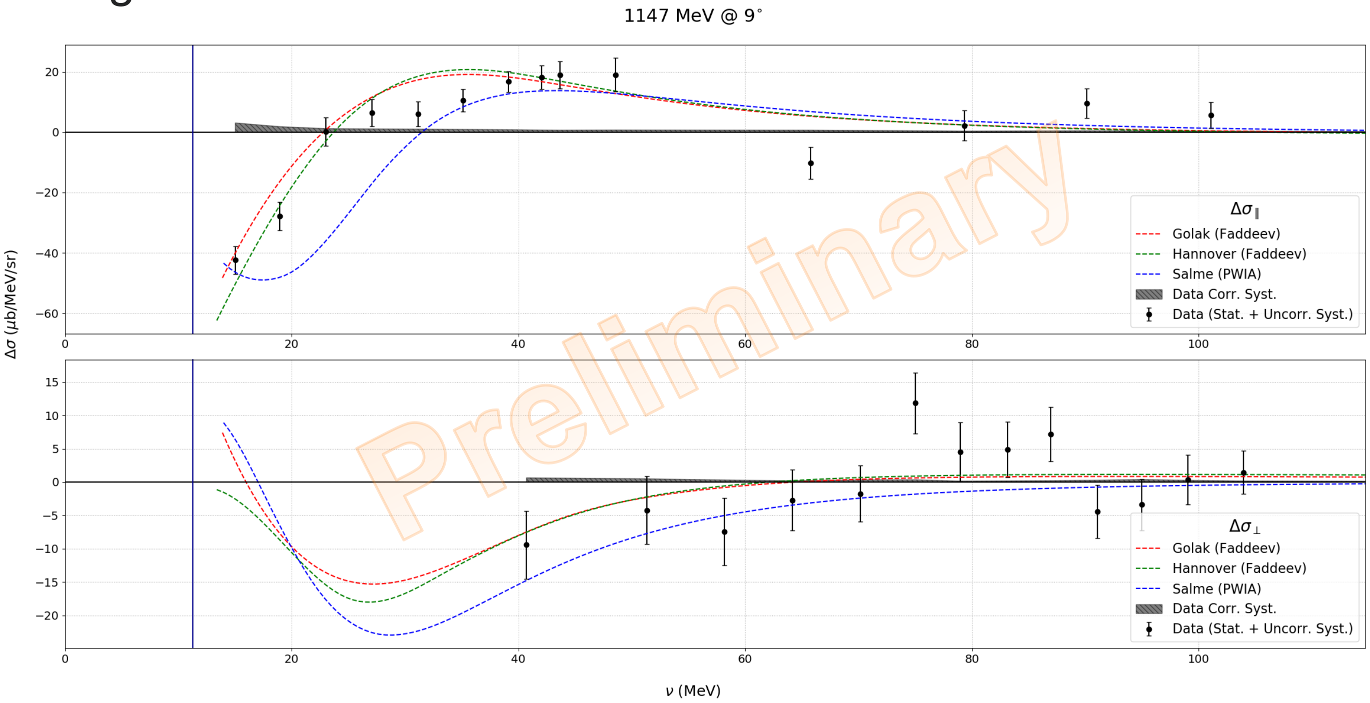
- Generated with Monte-Carlo simulation
  - 3He form factor parameterization from Amroun, Nuclear Physics A, 579(3):596 – 626, 1994

- Internal effects
  - Akushevich & Shumeiko (AS), Journal of Physics G, 20(4):513, 1994.
  - Mo & Tsai (MT), Rev. Mod. Phys., 41:205–235, 1969.
  - Main difference lies in Bremsstrahlung calculation, MT – modified Bethe-Heitler formula, AS – QED calculation

- External effects
  - Bremsstrahlung, Tsai's formula (SLAC-PUB-848)
  - Ionization, Landau theory

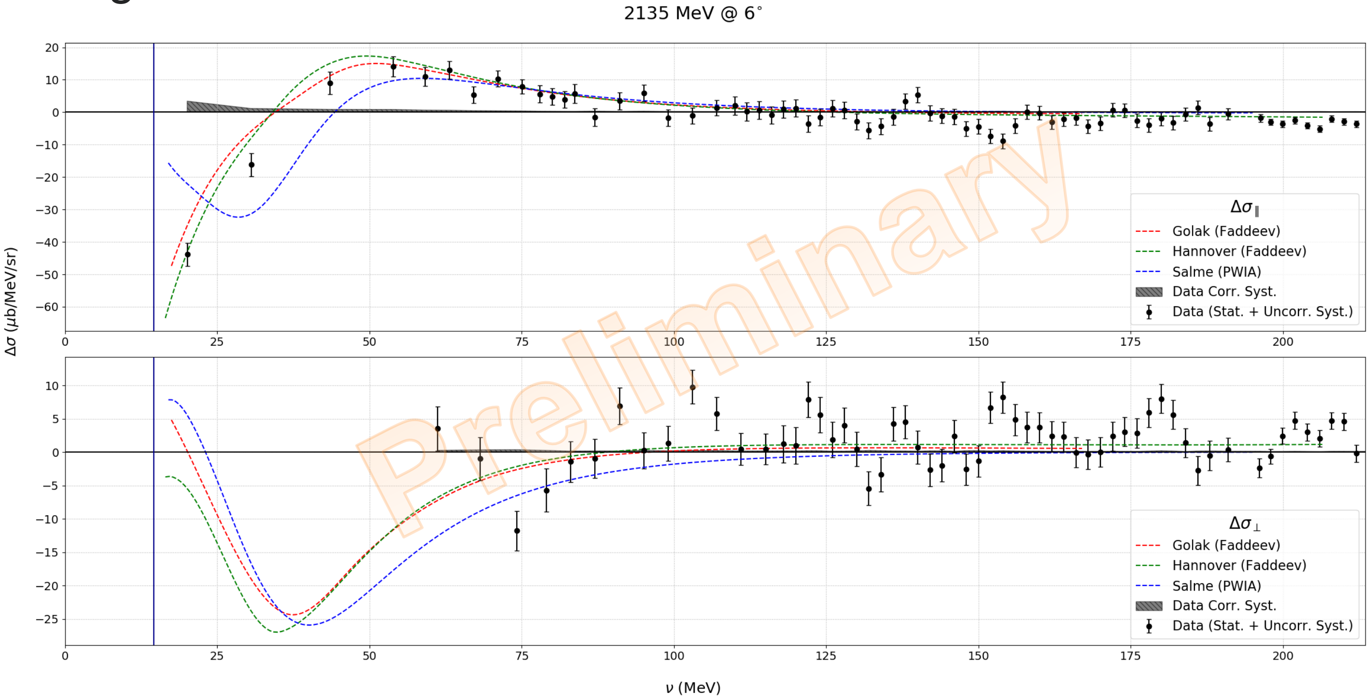
Faddeev Calculation Comparison

- Good agreement at lowest Q^2



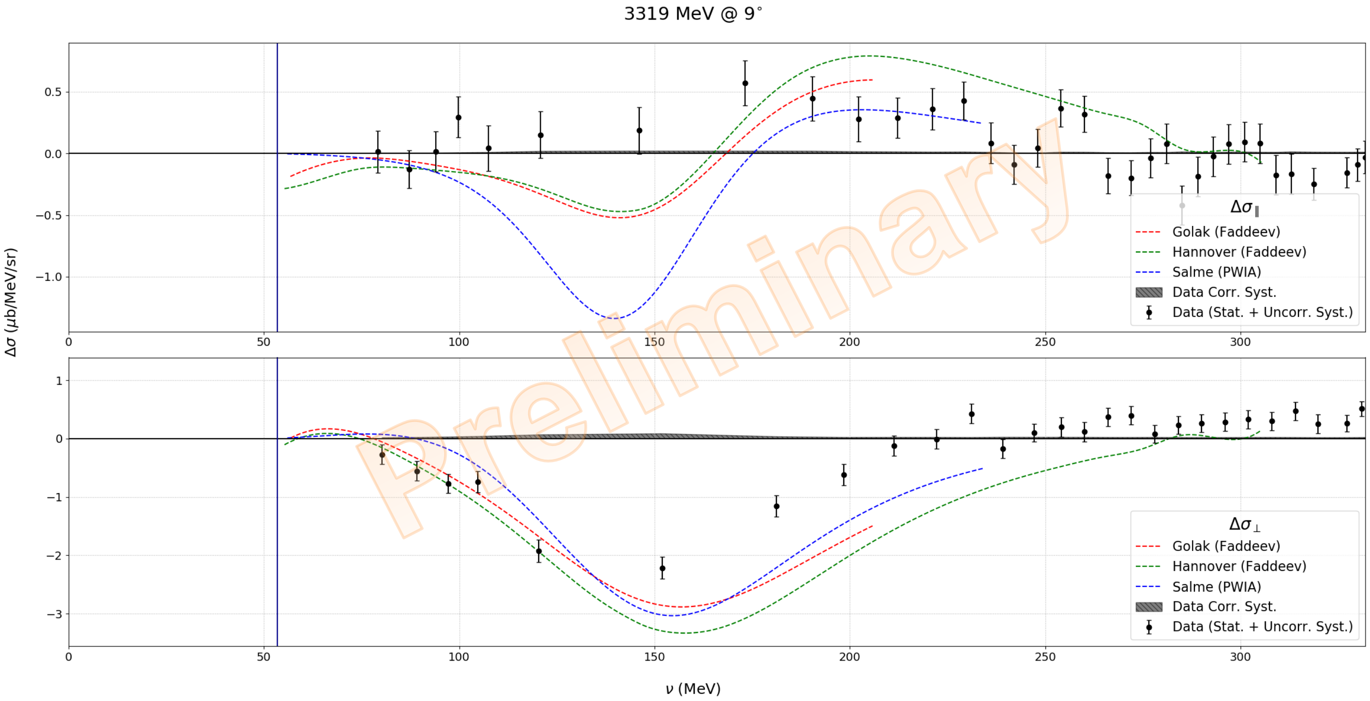
Faddeev Calculation Comparison

- Good agreement at lowest Q^2



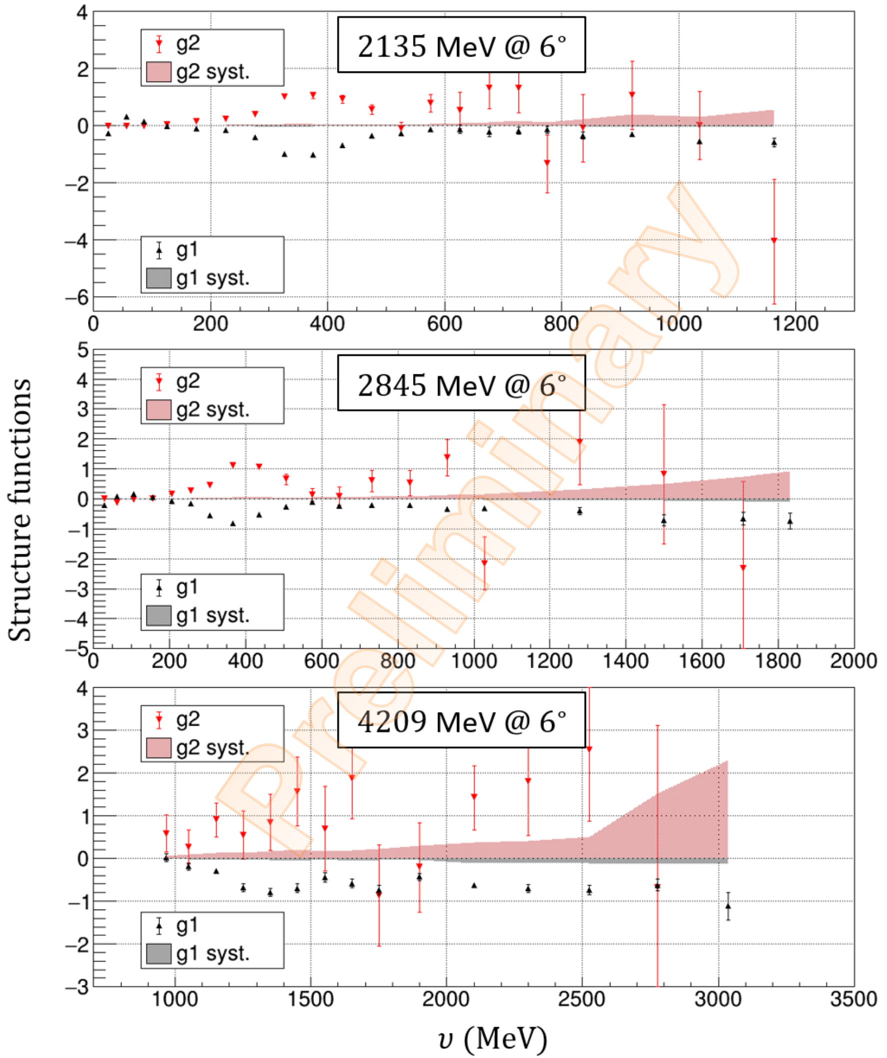
Faddeev Calculation Comparison

- Both Faddeev and PWIA are not working well at intermediate Q^2

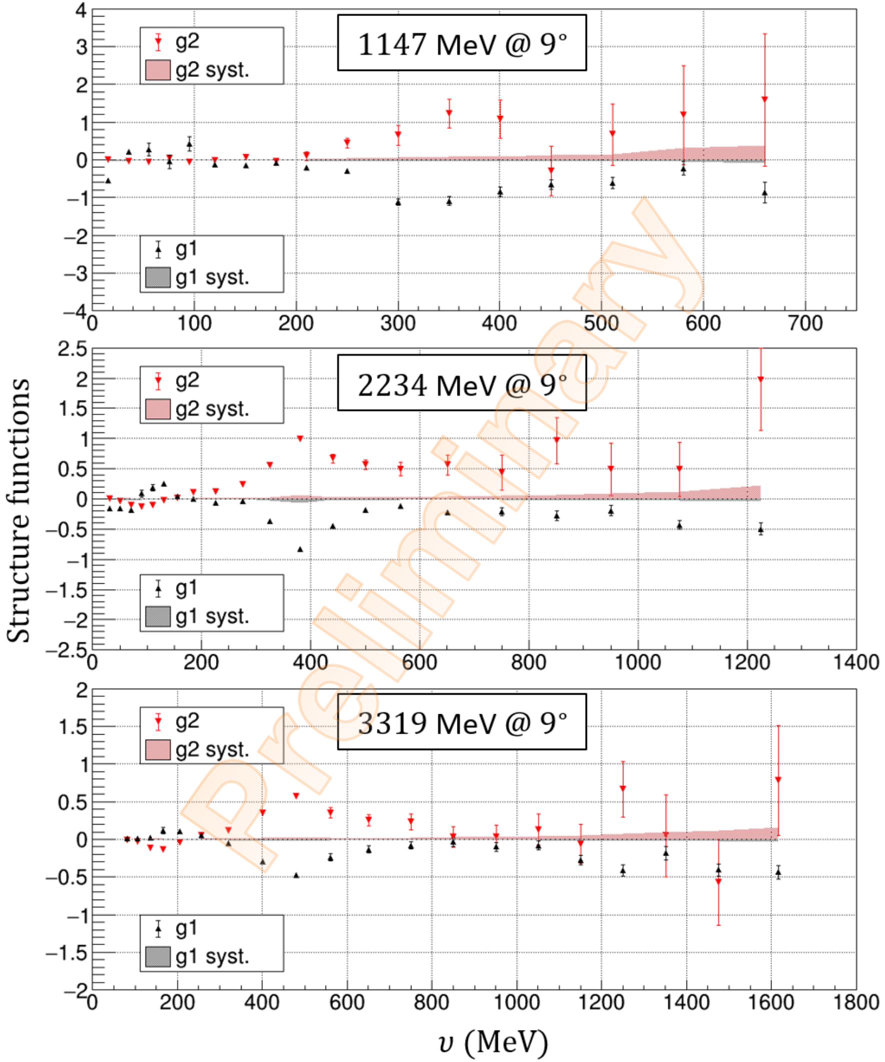


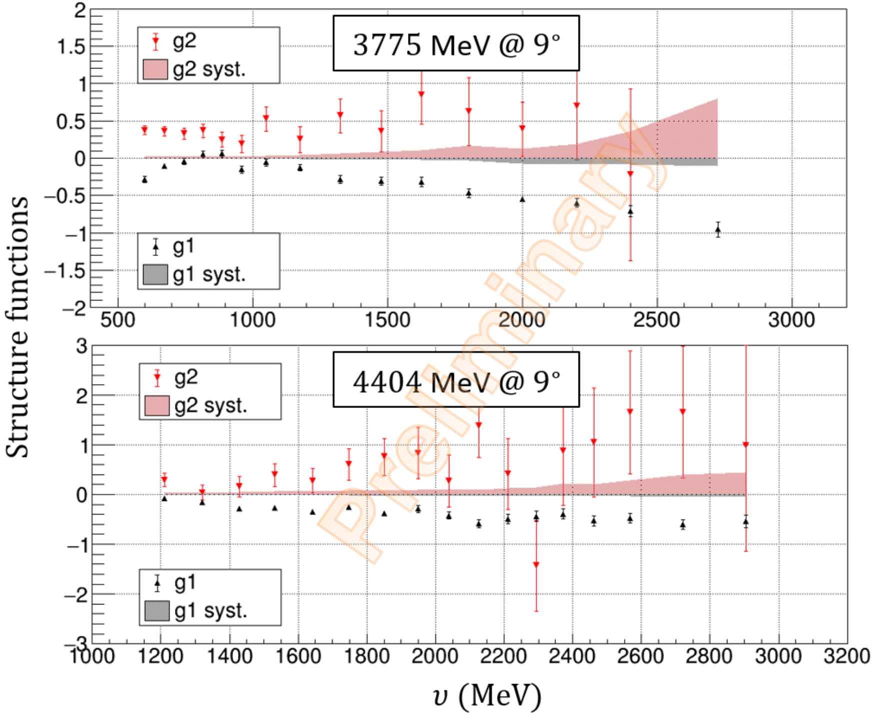
Spin-dependent Structure Function, 6 degree

- $g_1 + g_2 \approx 0$ for Delta resonance



Spin-dependent Structure Function, 9 degree





Interpolation to Constant Q2 (He3 results)

$Q^2 = 0.032 \sim 0.23\ \mathrm{GeV}^2$