Nucleon Spin Sum Rules and Spin Polarizabilities at low Q^2

A. Deur

Thomas Jefferson National Accelerator Facility

Obervables

Sum rule: relation between an integral of a dynamical quantity (cross section, structure function,...) and a global property of the target (mass, spin,…).

Can be used to:

- Test theory (e.g. QCD, χEFT) and/or hypotheses with which they are derived. Ex: GDH, Ellis-Jaffe, Bjorken sum rules.
- Measure the global property. Ex: spin polarizability sum rules.

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Here, we will discuss spin sum rules, in which the integral is over spin structure function(s).

- Gerassimov-Drell-Hearn (GDH) sum rule and its generalization to $Q^2>0$,
- Bjorken sum rule,
- Schwinger sum rule,
- Burkhardt–Cottingham (BC) sum rule,
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Electromagnetic polarizabilities were discussed in the previous talk.

We will discuss here polarizabilities *generalized* to electroproduction (Q^2-dependent).

Generalized forward spin polarizability $\gamma_0(Q^2)$;
Generalized Longitudinal-transverse spin polarizability $\delta_{LT}(Q^2)$.

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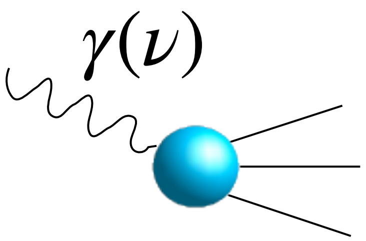
The GDH and Generalized GDH Sum Rules

GDH sum rule: derived for real photons ($Q^2=0$):



$$\int_{\nu_{thr}}^{\infty} \frac{\sigma_A(\nu)-\sigma_P(\nu)}{\nu}\,d\nu = \frac{-4\pi^2 S\alpha\kappa^2}{M^2}$$

QED coupling constant (α)

target anomalous magnetic moment (κ)

target spin (S)

target mass (M)

photoprod. cross section with photon spin anti-parallel to S (σ_A)

photon spin parallel to S (σ_P)

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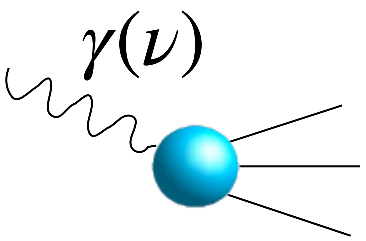

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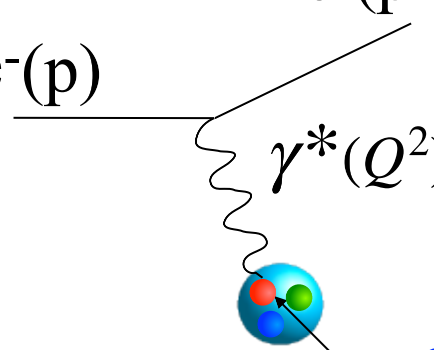
Generalized GDH sum rule: valid for any Q^2. Recover the original GDH sum rule as $Q^2 \rightarrow 0$



$$\Gamma_1(Q^2) \triangleq \int_0^{x_{th}} g_1(x,Q^2)dx = \frac{Q^2}{2M^2} I_1(0,Q^2)$$

$g_1(x,Q^2)$: first spin structure function (mostly a longit. target pol. observable)

$I_1(\nu,Q^2)$: first covariant polarized VVCS amplitude

$x= Q^2/2pq$ (Bjorken scaling variable)

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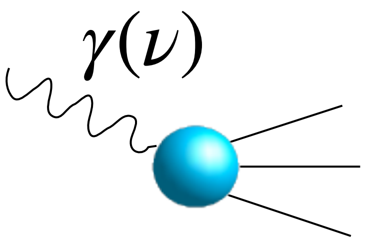

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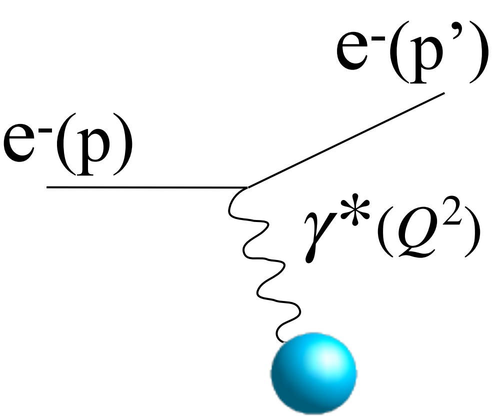

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⇒Study QCD at any scale

Hadronic degrees of freedoms — Partonic degrees of freedoms → Q^2

$I_1(0,Q^2)$:

Chiral Effective field theory (χEFT) — OPE, pQCD

Lattice QCD, SDE, AdS/QCD

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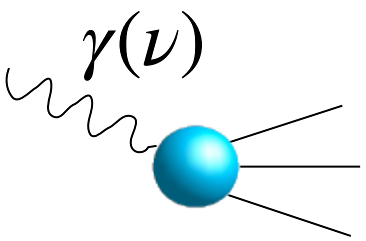

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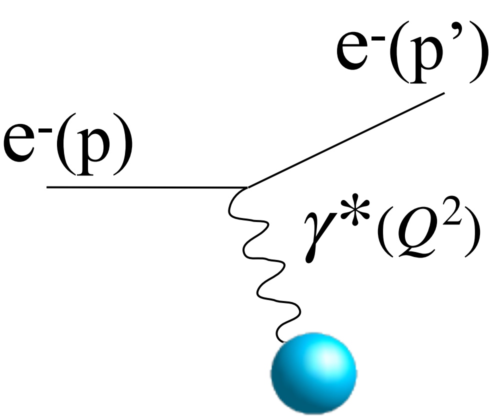

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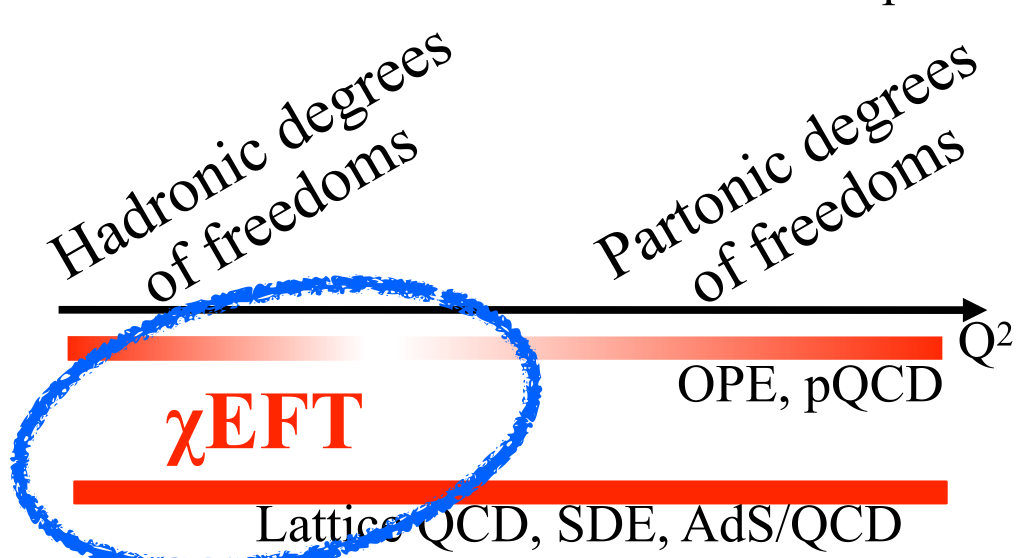

<u>Bjorken sum rule</u> = Generalized GDH sum rule on proton - neutron: $\Gamma_1^{p-n} \equiv \int g_1^p - g_1^n dx$

- Derived (1966) independently from GDH sum rule (1965/1966) and using different formalisms.
- Connection with generalized GDH sum rule occurred much later (Anselmino:1989 ….. Ji-Osborne:1999)
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Schwinger sum rule: $I_{LT}(Q^2) = \frac{8M^2}{Q^2} \int_0^{1^-} \left(g_1 + g_2\right) dx \xrightarrow[Q^2 \to 0]{} \kappa e_t$

anomalous magnetic moment×charge

$g_2(x,Q^2)$: second spin structure function (mostly a perp. target pol. observable)

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Spin polarizability sum rules involve higher moments:

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2M^2}{\pi Q^6} \int x^2 (g_1 - \frac{4M^2}{Q^2} x^2 g_2) dx$$

Longitudinal-Transverse polarizability:

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We do not know how to measure directly generalized spin polarizabilities. The sum rules are used to access them.

First measured in the 1990s at Jefferson Lab:

- $\gamma_0(Q^2)$ on proton & neutron,
- $\delta_{LT}(Q^2)$ on neutron.

Testing χEFT

Important to test χEFT: the leading effective theory dealing with the first level of complexity emerging from the Standard Model.

Nuclear physics
d.o.f: hadrons
↑
hadronic physics
d.o.f: hadrons ← χEFT
↑
Fundamental forces: electromagnetic, weak, strong, gravitation
Fundamental particles: quarks, electrons, neutrinos…

⇒ Crucial piece of our global understanding of Nature.

χEFT has been very successful in describing many hadronic and nuclear phenomena. However, the late 1990s JLab experiments suggested that it did not describe well nucleon spin observables, or/and that the Q^2 range of validity of χEFT was smaller than expected for spin observables.

JLab's first generation of χEFT tests/polarizability measurements at low Q^2

Results from JLab 1990's experiments (Hall A E94010, CLAS EG1a,b):

A: ~agree
X: ~disagree
- : No prediction available

	Generalized GDH		Bjorken SR	Generalized GDH	generalized spin polarizabilities					
Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X

1990s-2000s χEFT predictions in tension with
spin observable data more often than not.

Testing χEFT

Yet, some of the spin observables were expected to be well suited for testing χEFT :

Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
		😃		😃	😃	😃 😃	😃	😃 😃	😃 😃

More robust measurements (no significant missing low-x contribution. More on this later)

Nucleon resonance Δ_{1232} contribution suppressed (More robust χEFT calculations)

Testing χEFT

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The discrepancies for δ_{LT} was particularly puzzling:

- Expected to be a robust χEFT prediction;
- Expected to be a robust measurement.

χEFT calculation problem? Or were the experiments not reaching well enough into the χEFT applicability domain, i.e., at low Q^2?

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- Refined χEFT calculations, with improved expansion schemes & including the Δ_{1232} .
- New experimental program at JLab reaching well into the χEFT applicability domain & with improved precision.

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This talk: results from the new experimental program at JLab.

Estimating sum rules at low Q^2:

Low Q^2 + covering large ν range so that sum rule's integrals can be formed$\Rightarrow$forward angles

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Low Q^2 + covering large ν range so that sum rule's integrals can be formed⇒forward angles

E97-110 (neutron, using longitudinally and transversally polarized ^{3}He):

Spokespeople: **J.P. Chen,** A.D., F. Garibaldi

Students: C. Peng (Duke U.), J. Singh (UVa), V. Sulkosky (W&M), J. Yuan (Rutgers U.)

E08-027 (NH_3, longitudinally and transversally polarized):

Spokespeople: A. Camsonne, J.P. Chen, D. Crabb, **K. Slifer**

E03-006 (NH_3, longitudinally polarized):

Spokespeople: **M. Ripani**, M. Battaglieri, A.D., R. de Vita

Students: H. Kang (Seoul U.), K. Kovacs (UVa)

E06-017 (ND_3, longitudinally polarized):

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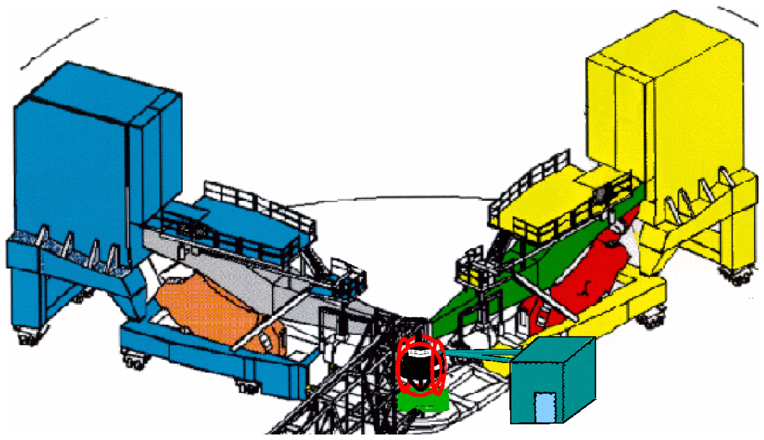

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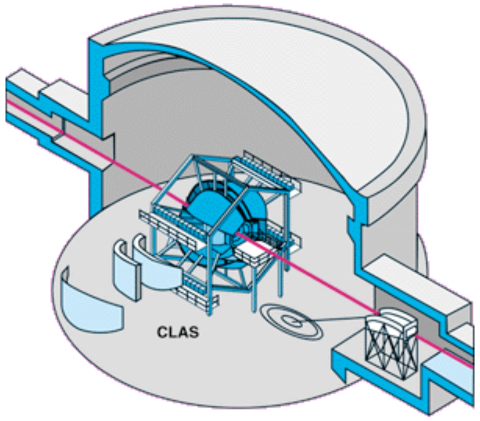
EG4 run group

JLab Hall B:



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Covered in J.P. Chen overview talk, Friday 14h-14h30.

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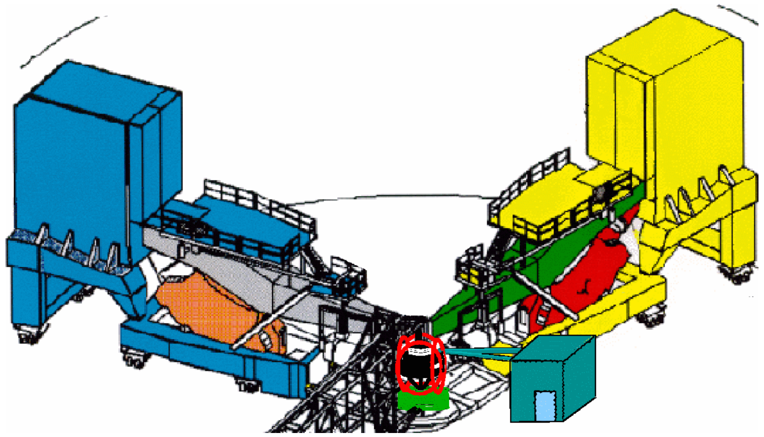

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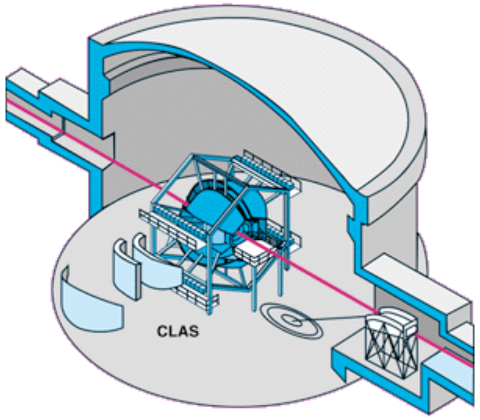

JLab Hall A experiment E97-110

V. Sulkosky et al.
Nature Physics, **17** 687 (2021);
Phys.Lett.B 805 135428 (2020)

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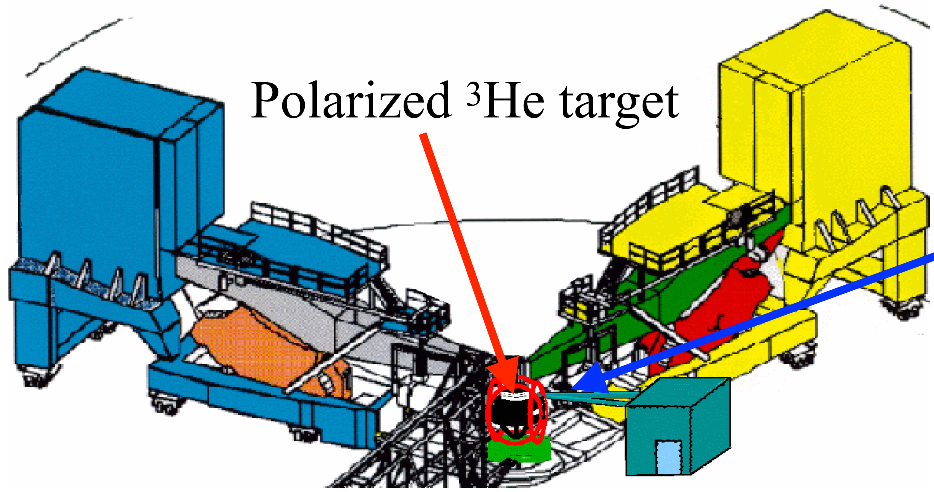
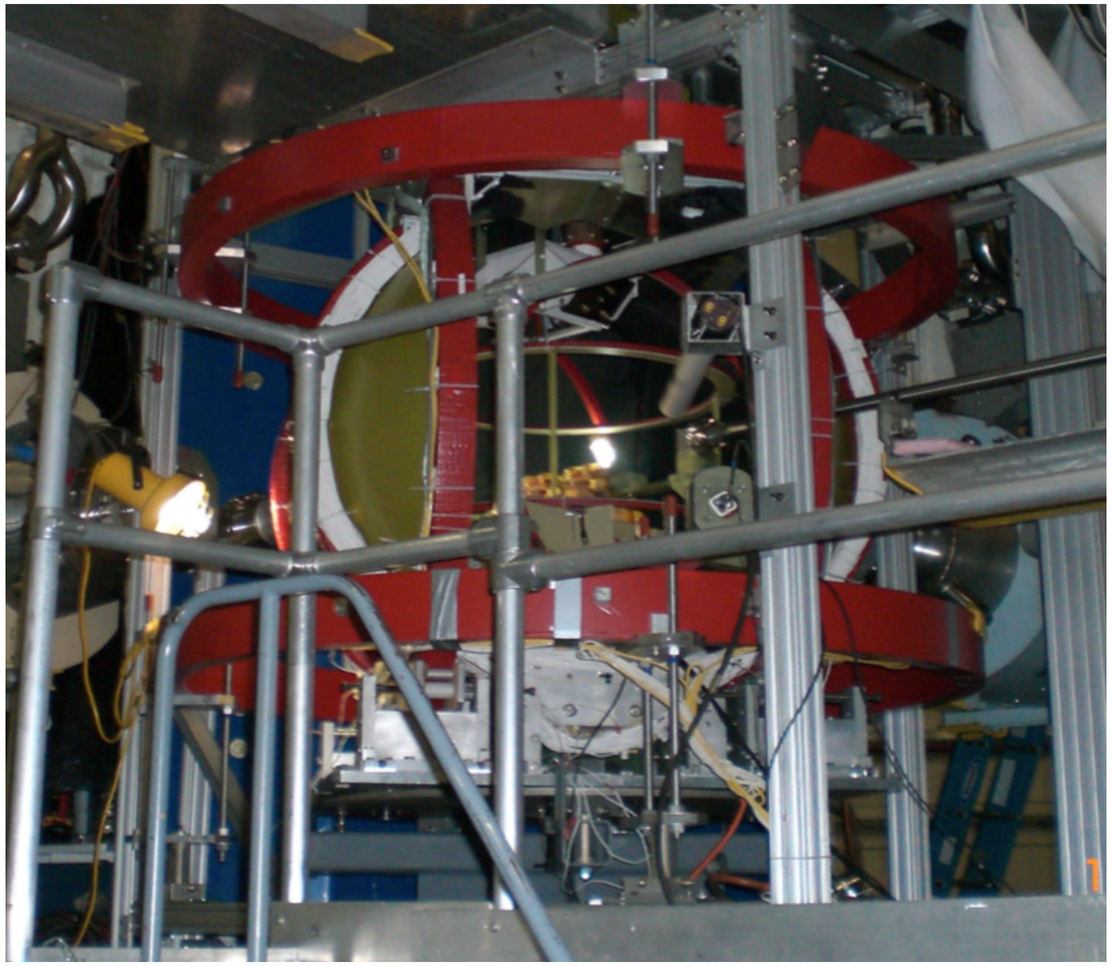


New magnet to allow to reach down to ~5° (12.5° otherwise).

JLab Polarized ³He target



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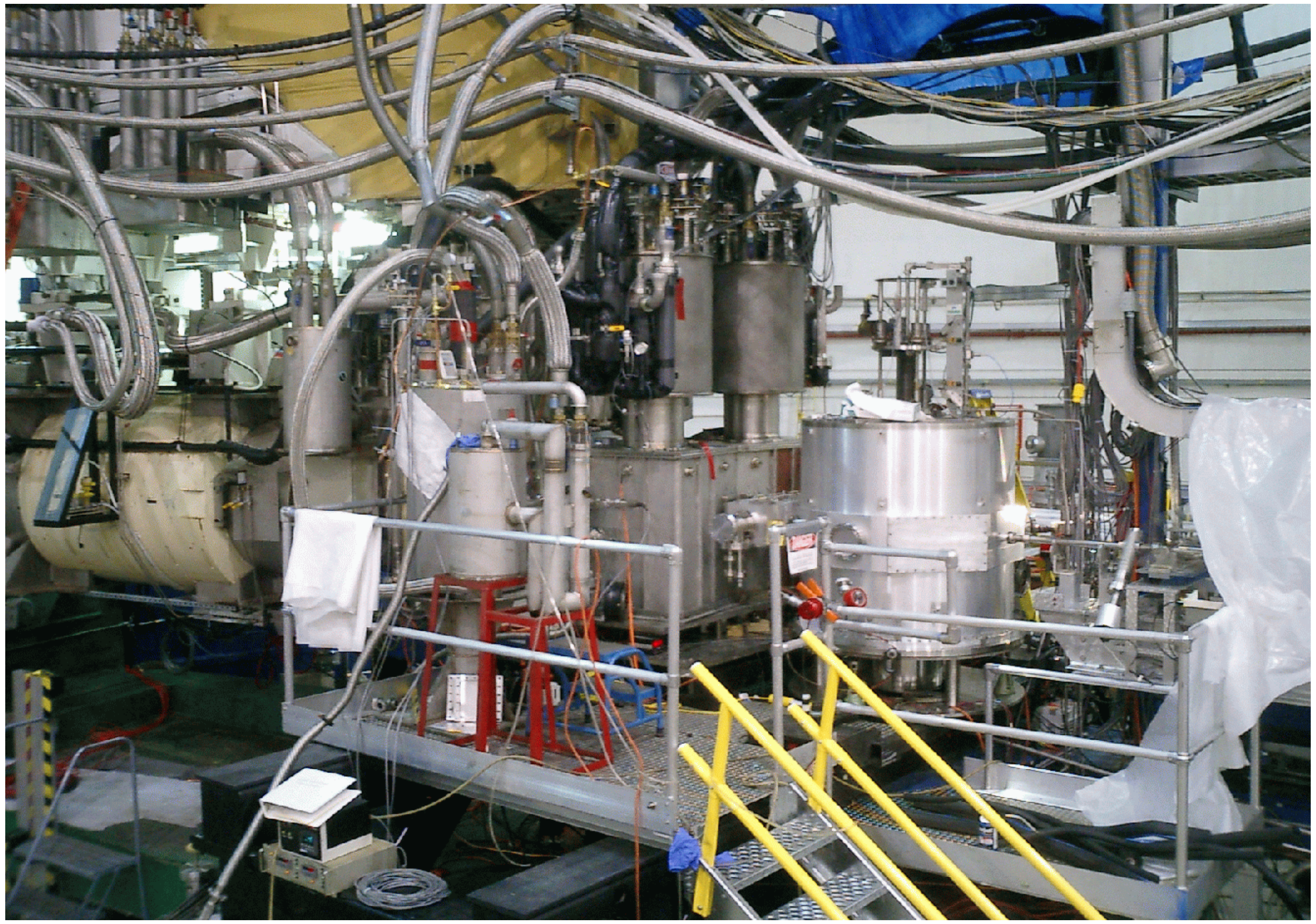
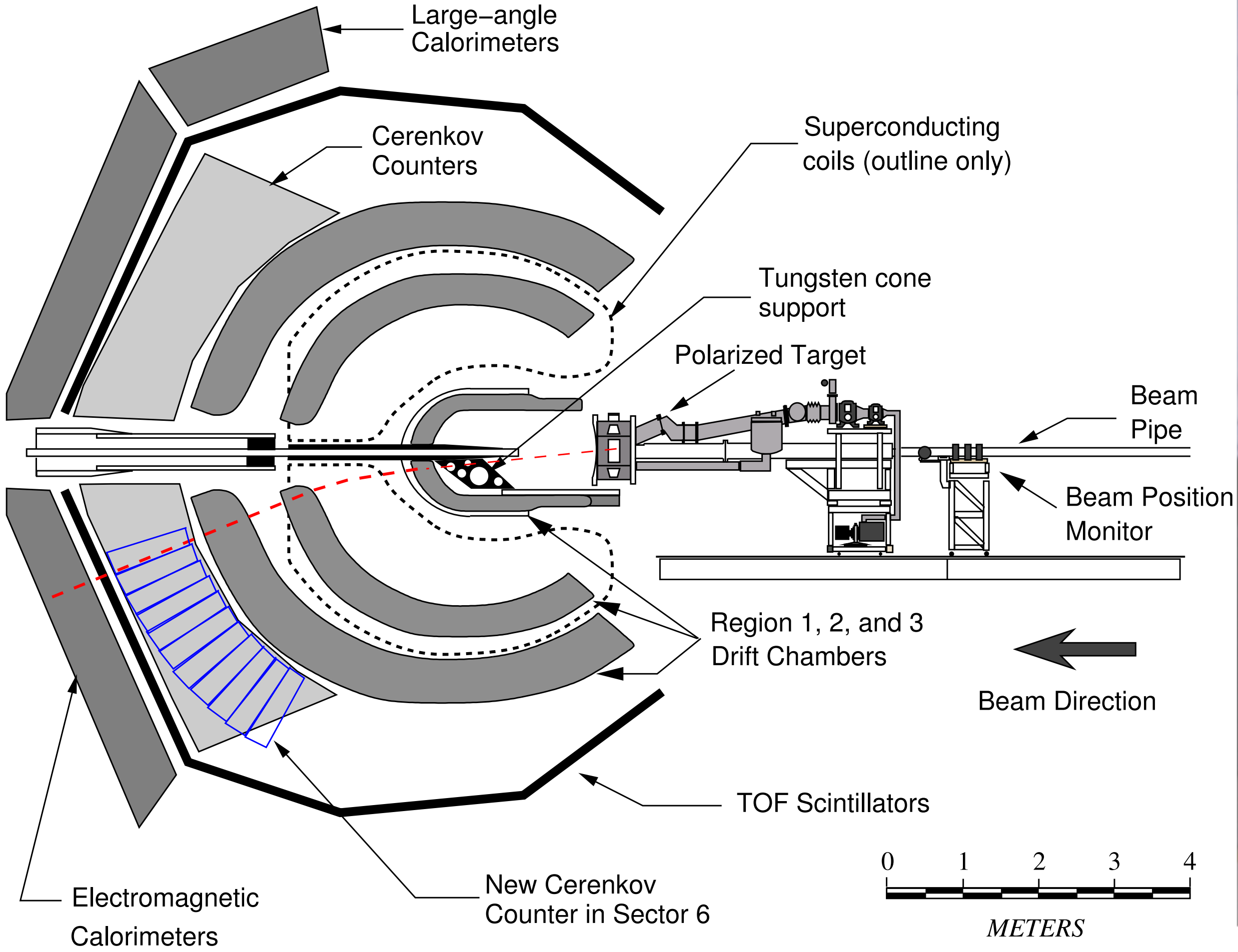


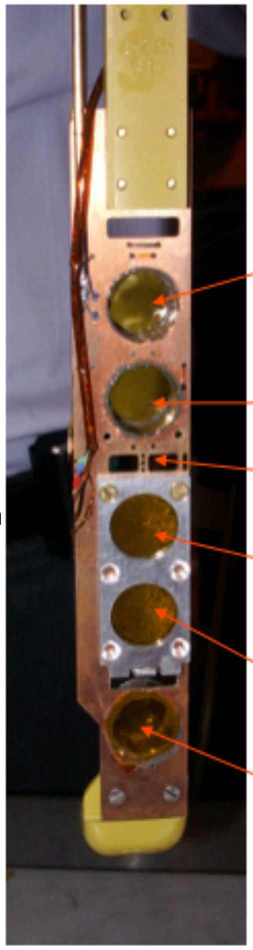
JLab Hall B EG4 experiment Group

X. Zheng et al., Nature Physics, 17 736 (2021)
K.P. Adhikari *et al.* PRL 120, 062501 (2018)
X. Zheng et al., PRC 94, 045206 (2016)

- $g_1^{p,n}$: ~longitudinally polarized target

DNP NH_3 and ND_3 target:





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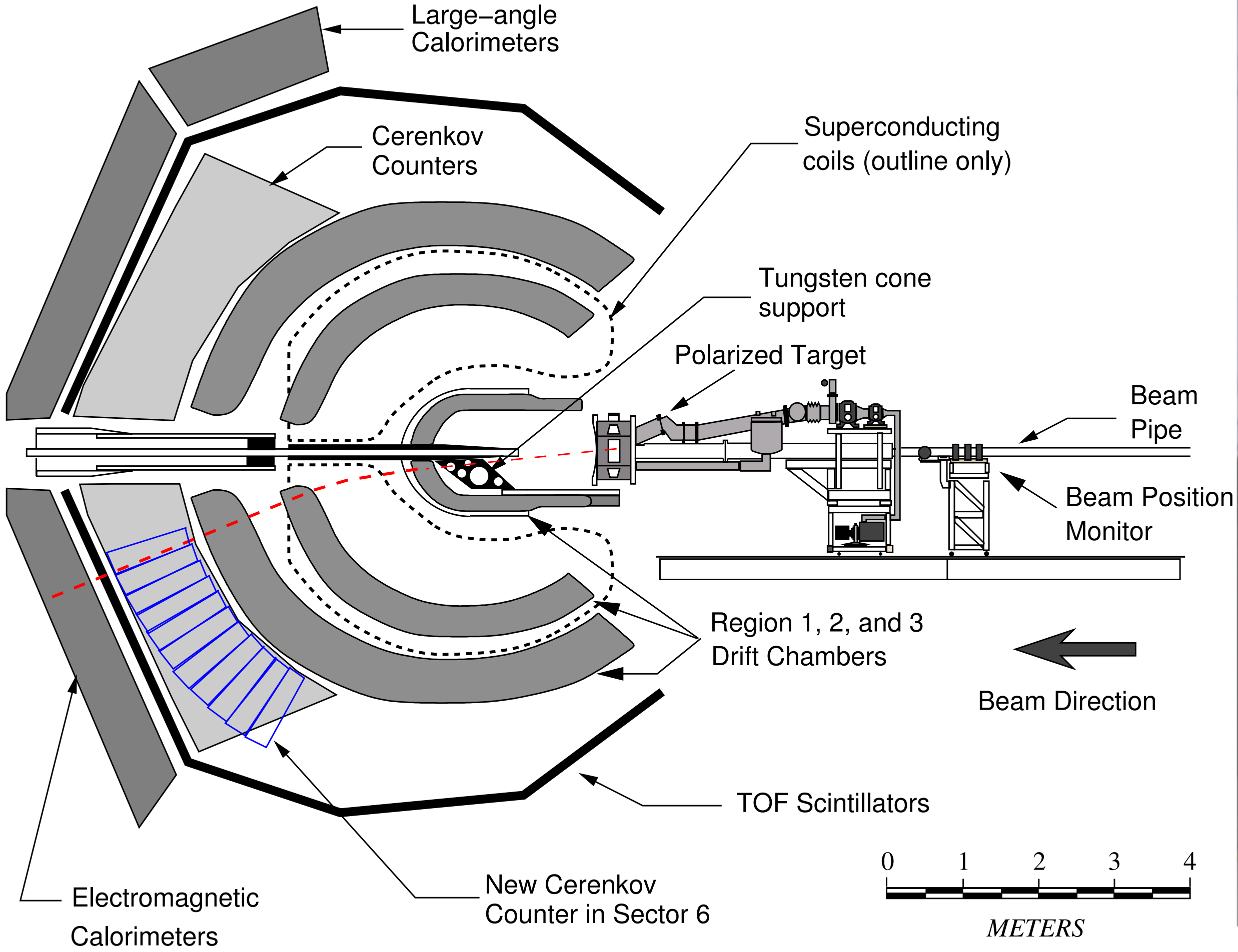
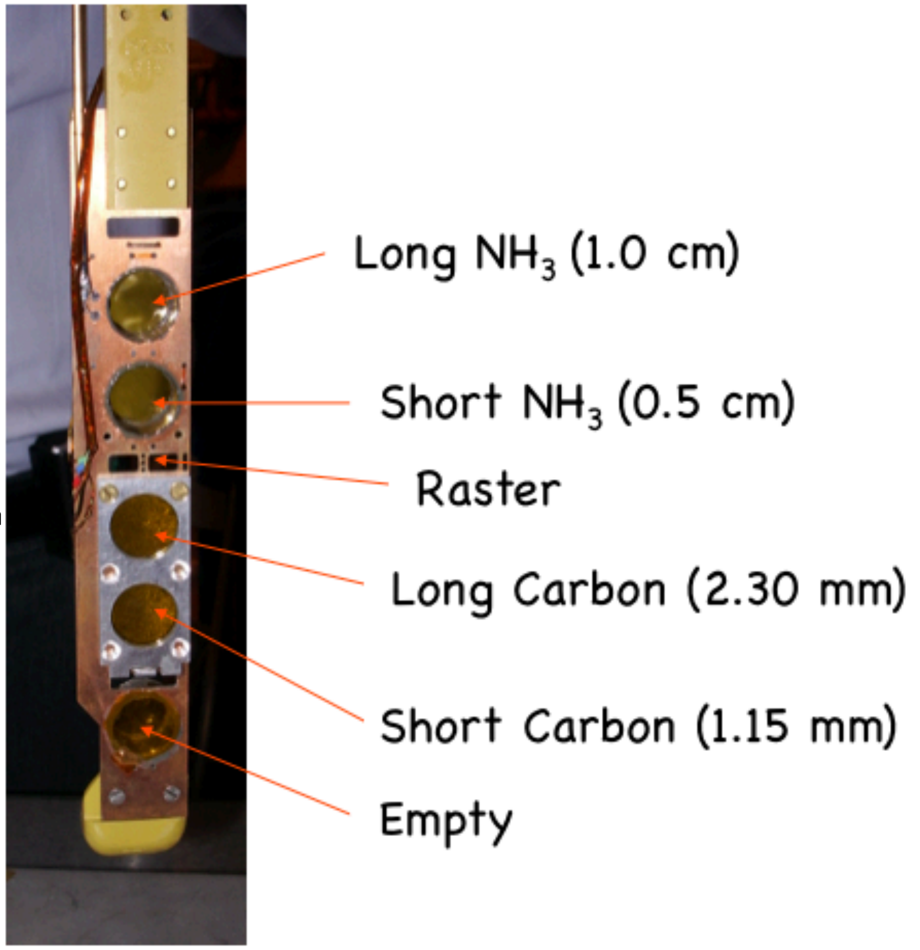
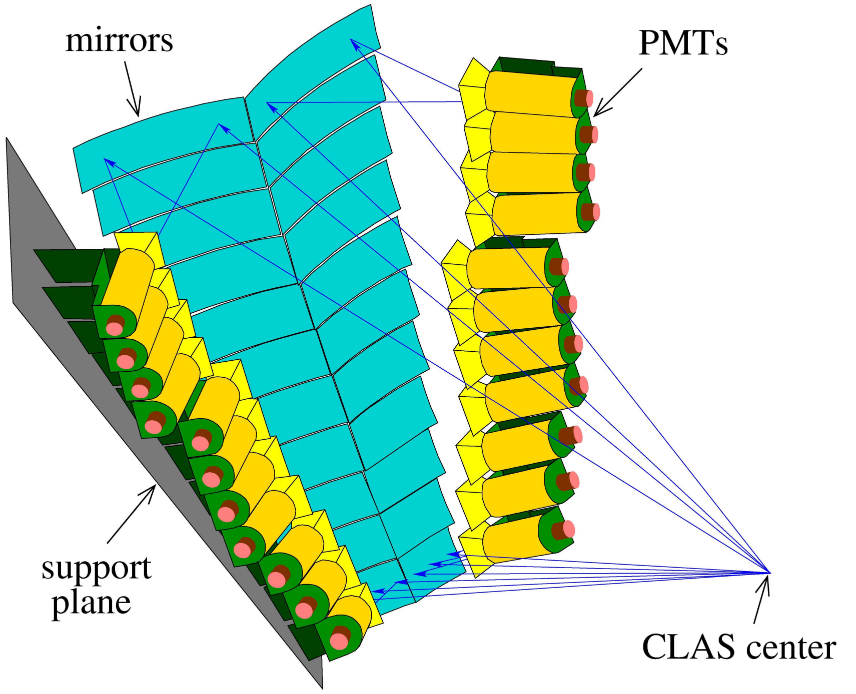




•g_1 from inclusive $eN \rightarrow e'X$ pol. cross-section differences. Advantage: dilution from unpol. target material cancels out

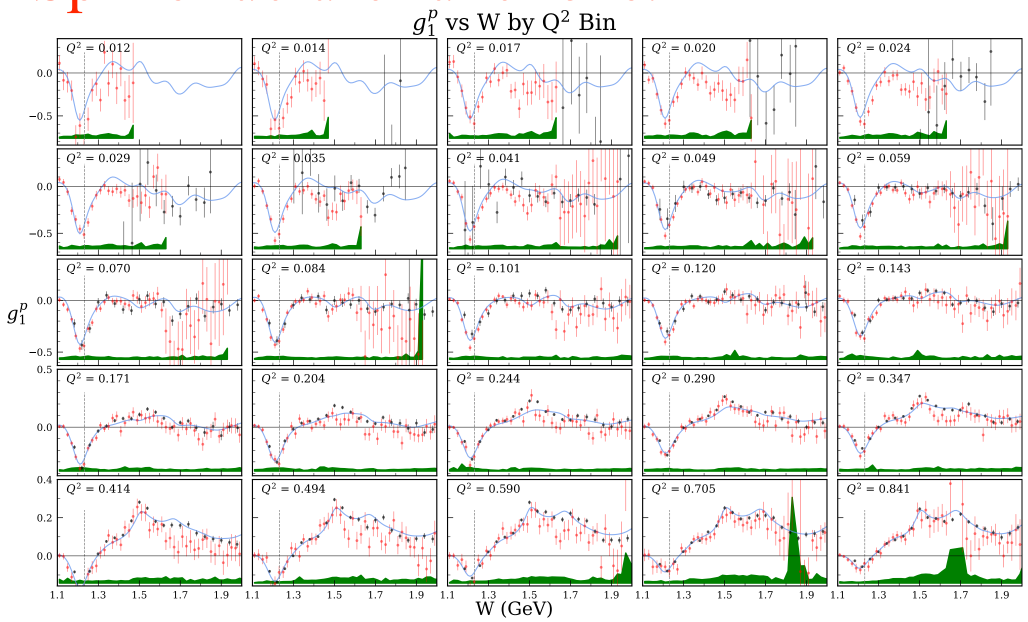
•Small angles: outbending torus field, target at -1m, new Möller shield

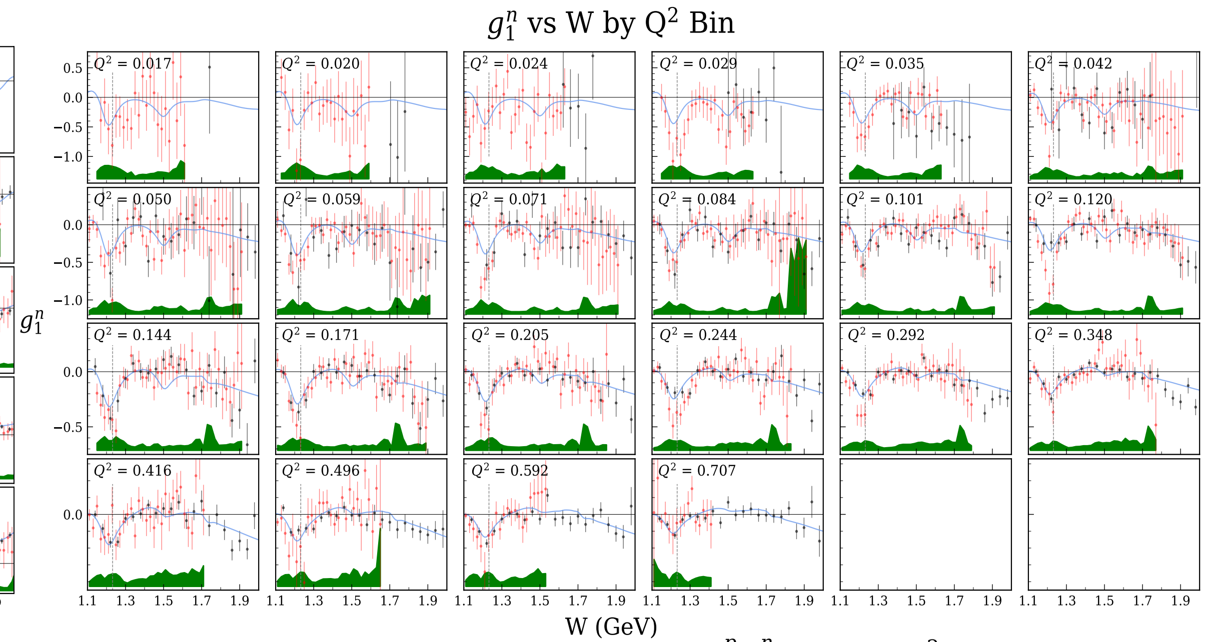
•Cross-sections ⇒ controlled (i.e. high) efficiency at small angles. New Cerenkov detector (INFN). Installed in sector 6. Covered down to 6°.

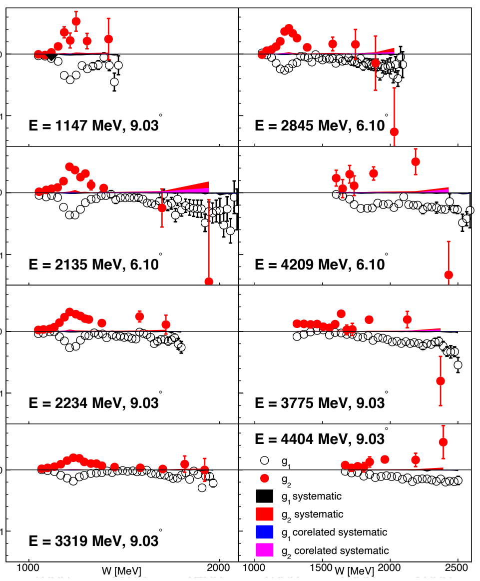


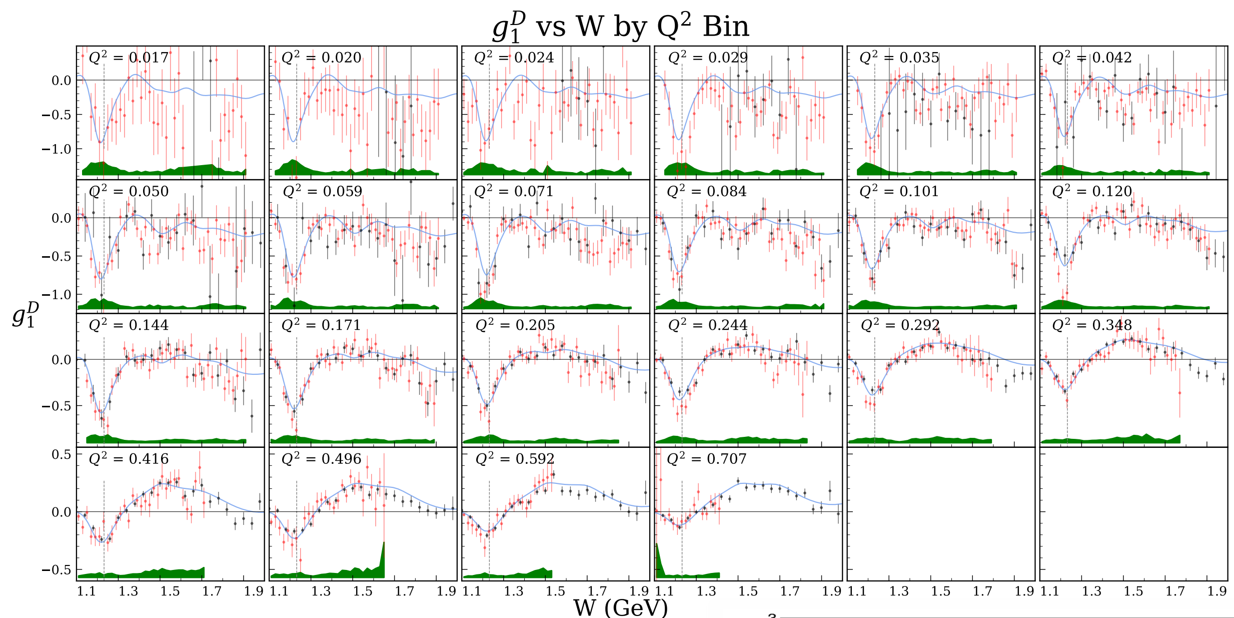
Lots of data on spin structure functions and their moments from E97-110, E03-006 and E05-111

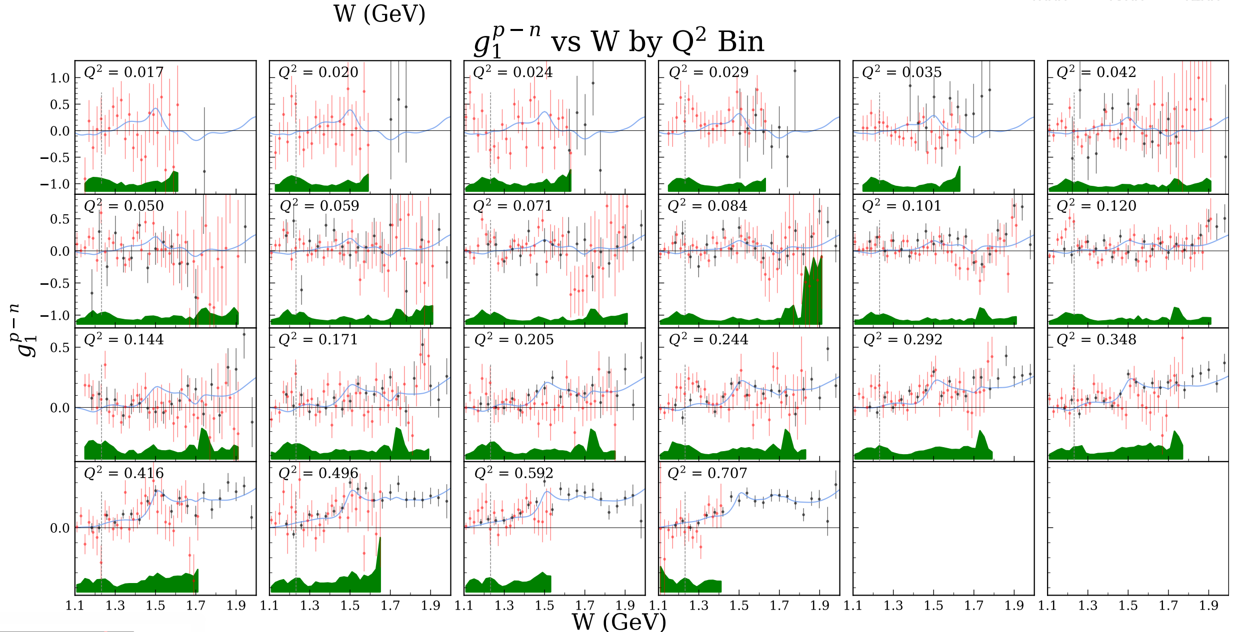
Spin structure functions:

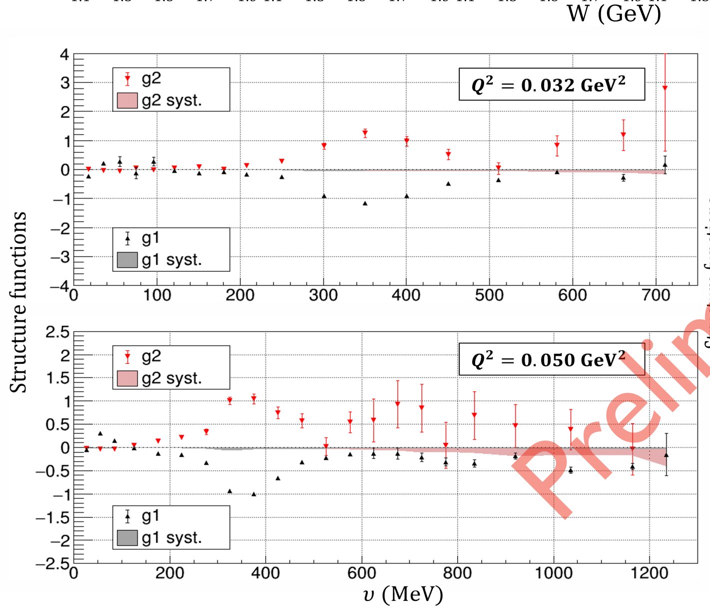


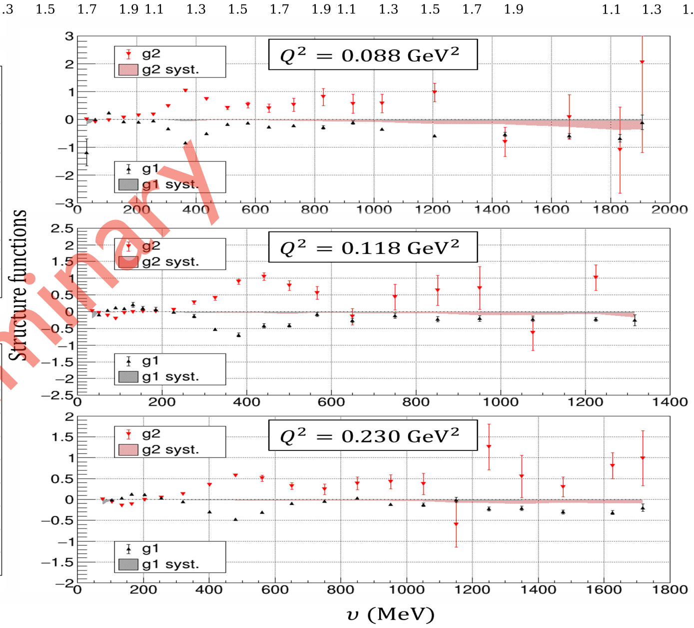








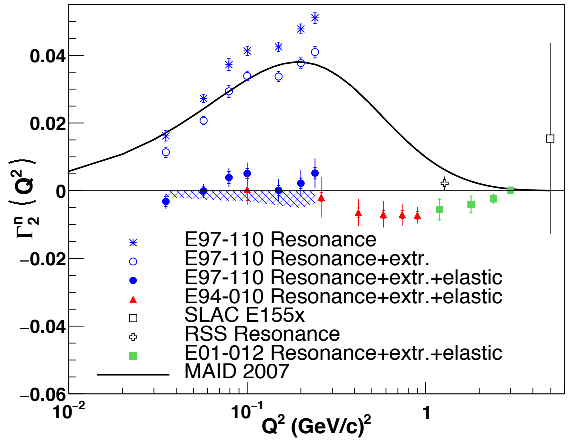


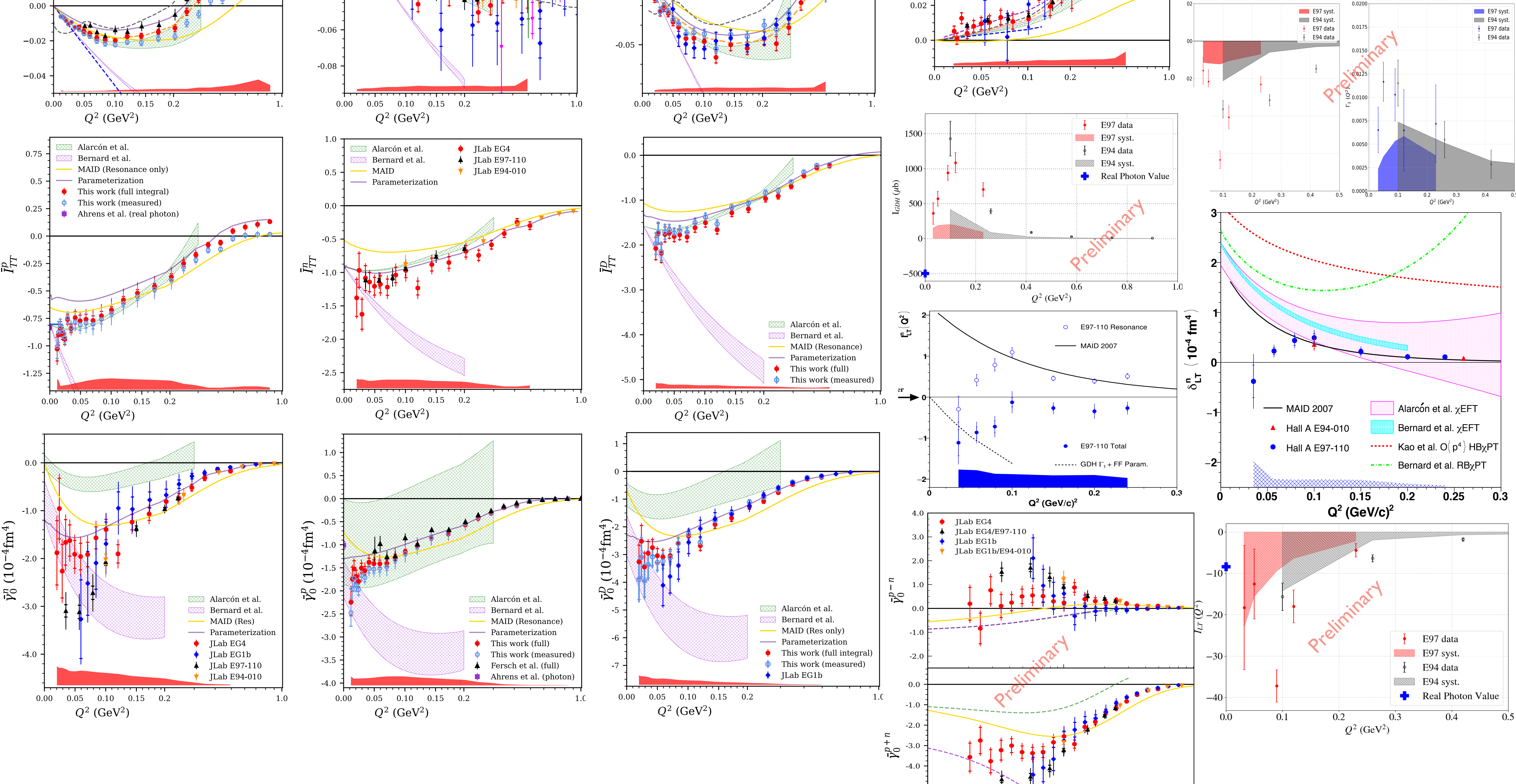


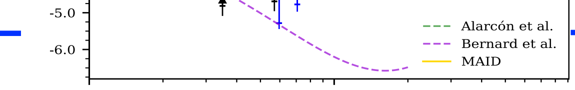
Preliminary

Lots of data on spin structure functions and their moments from E97-110, E03-006 and E05-111

Moments for spin sum rules

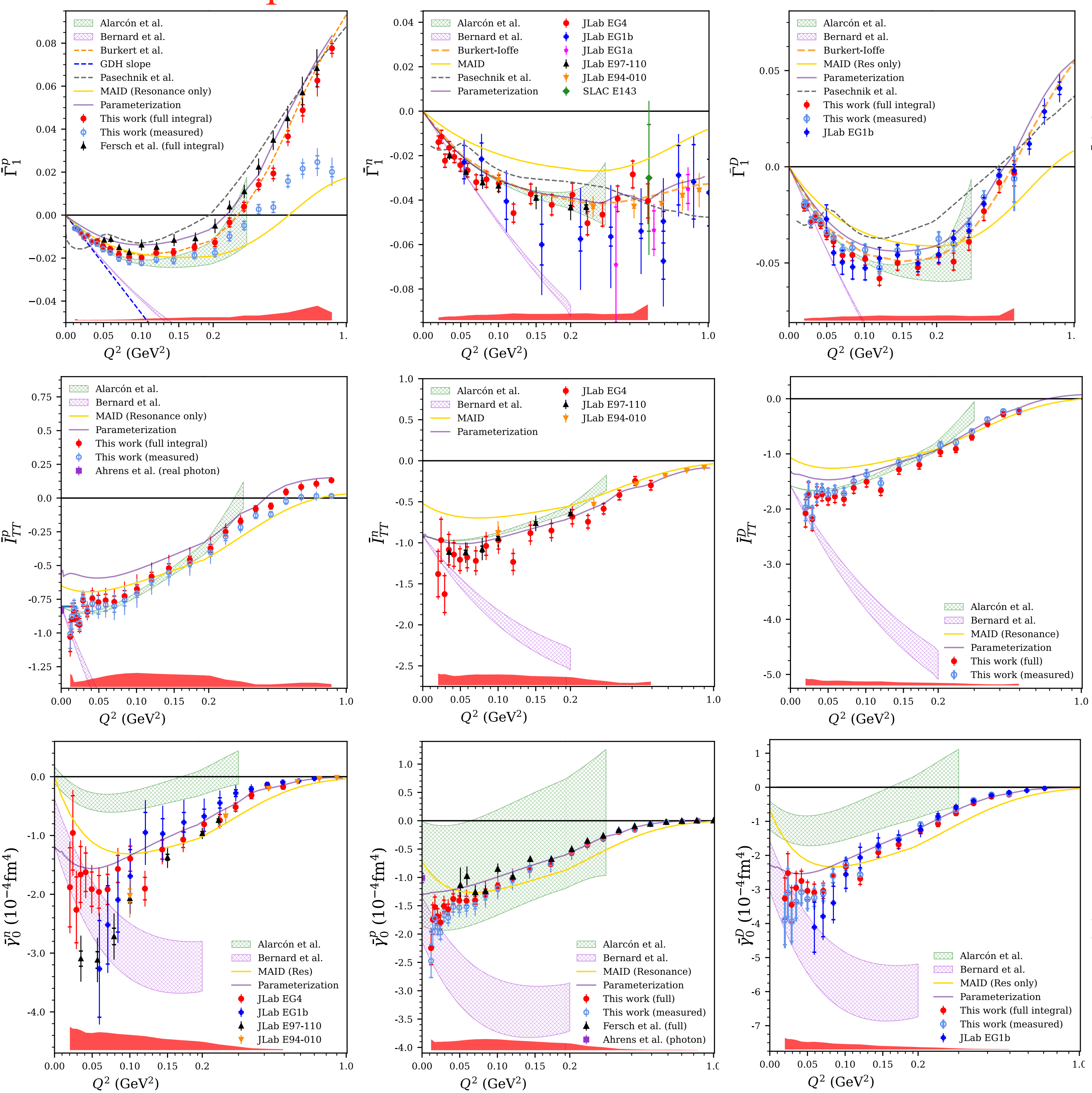


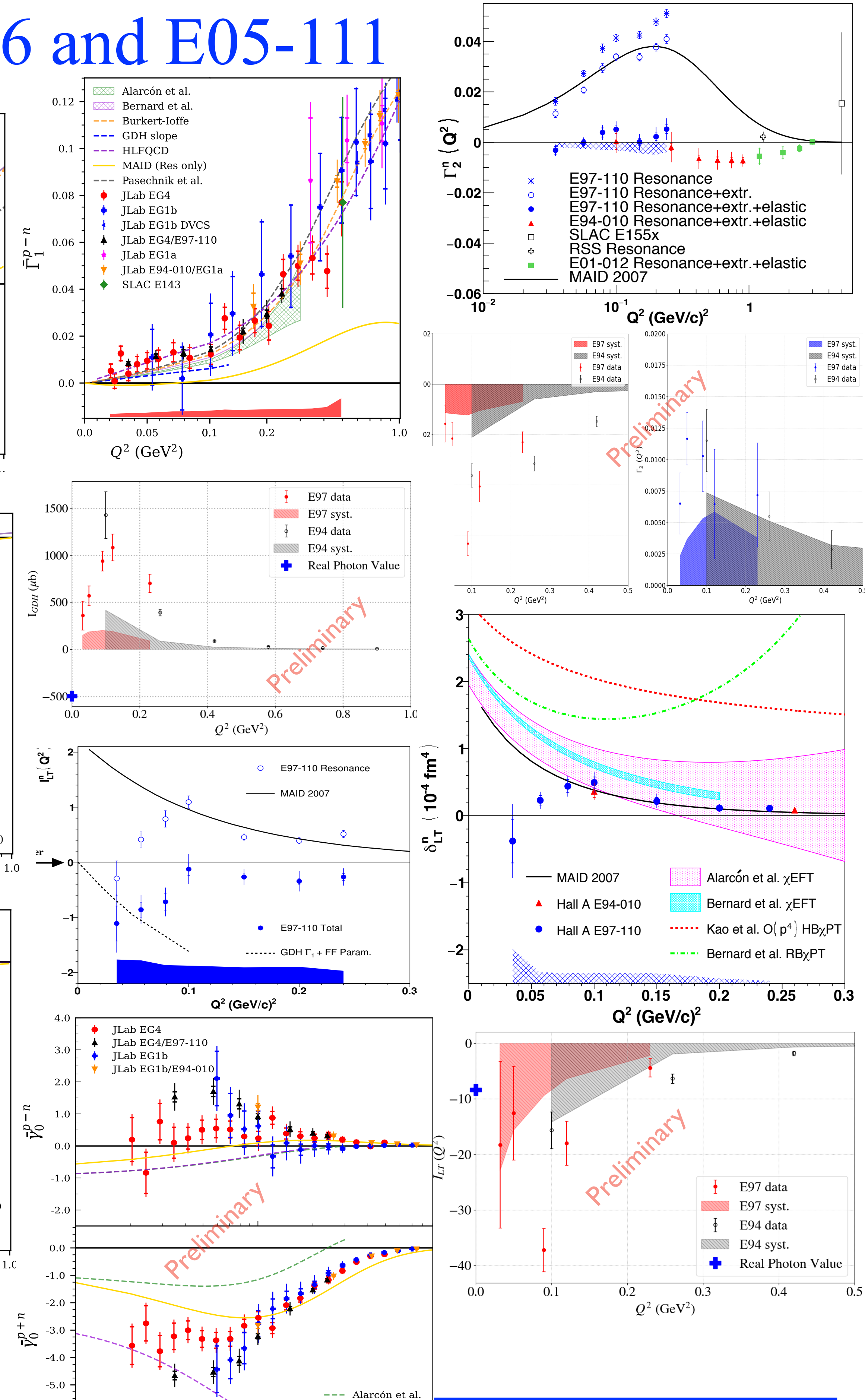




Lots of data on spin structure functions and their moments from E97-110, E03-006 and E05-111

Moments for spin sum rules

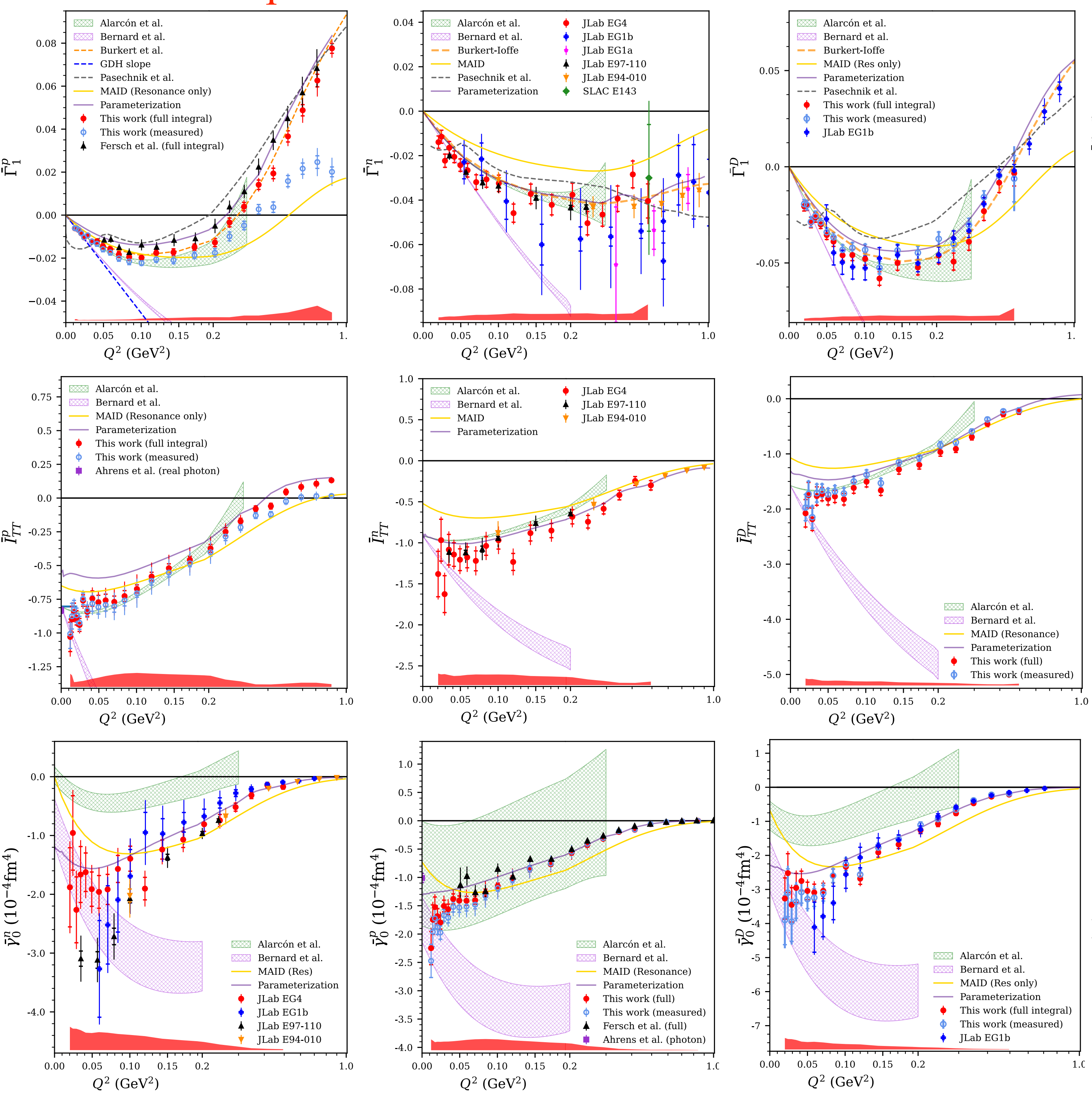


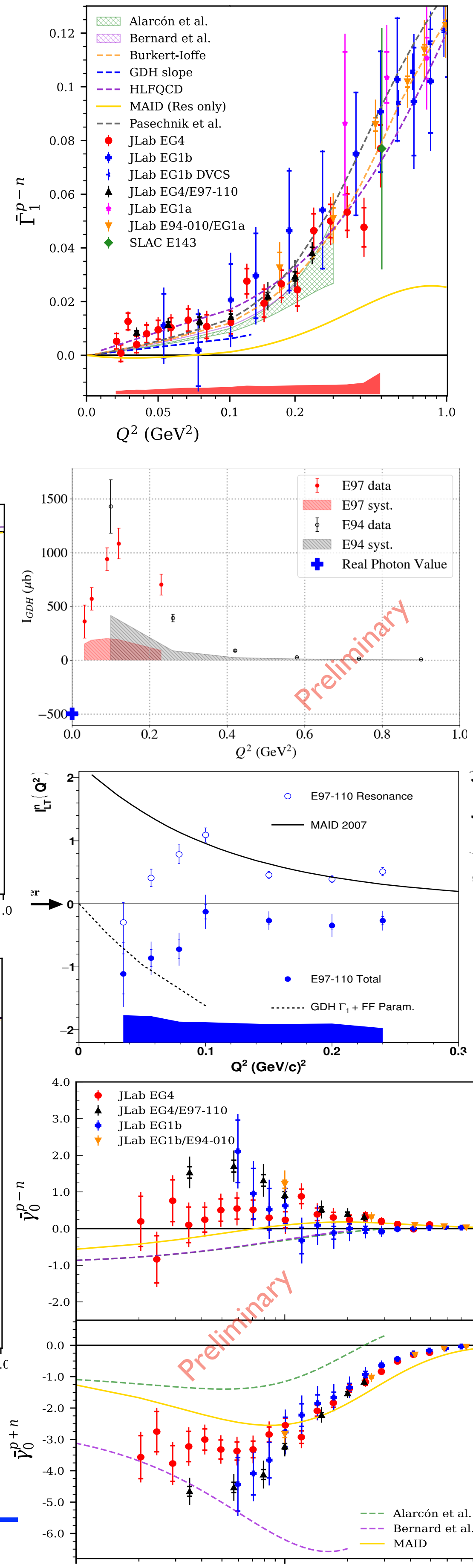


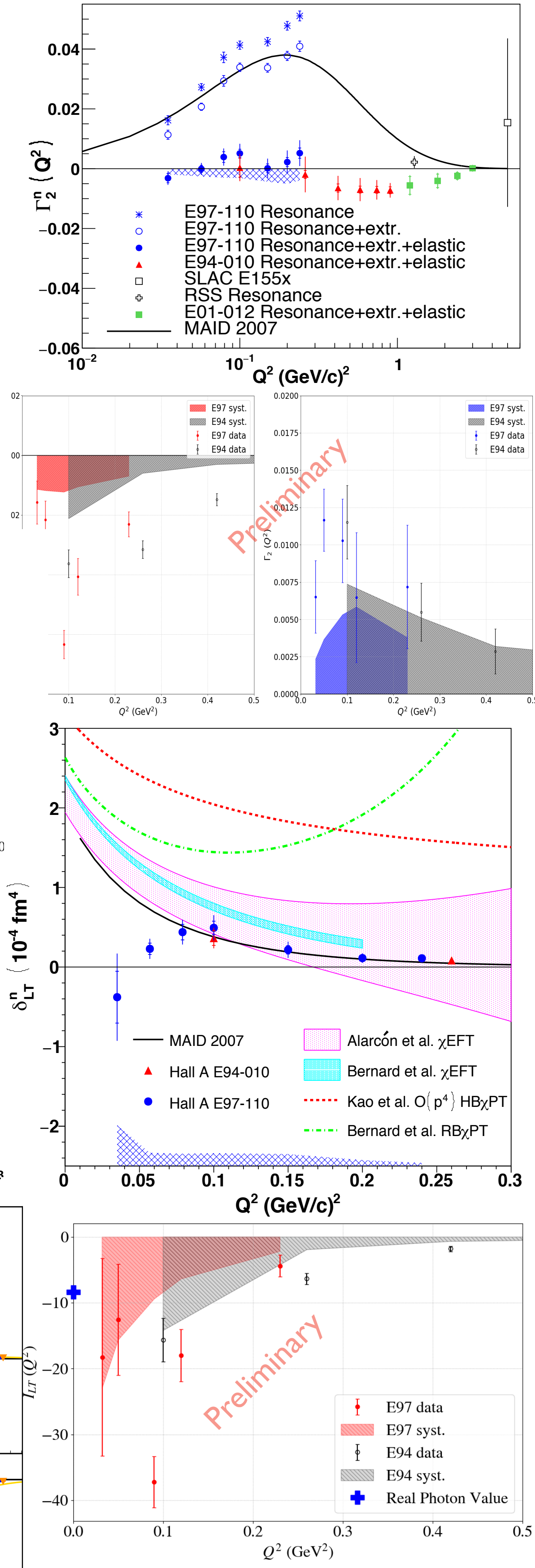
I will show only a few representative results. (The others are in the back-up slides, please ask about them.)

Lots of data on spin structure functions and their moments from E97-110, E03-006 and E05-111

Moments for spin sum rules



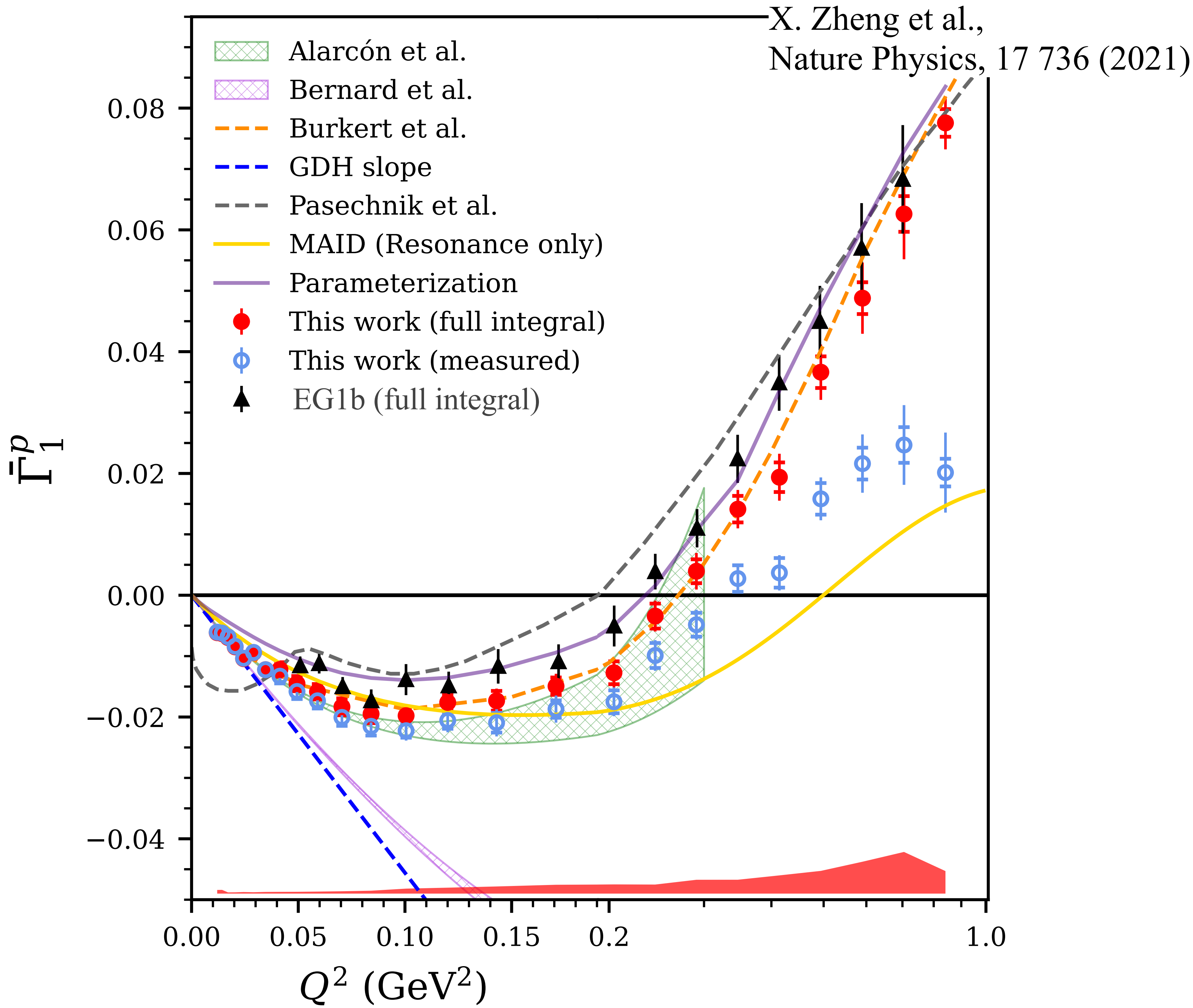




C. Peng talk Monday: ³He results from E97-110

First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

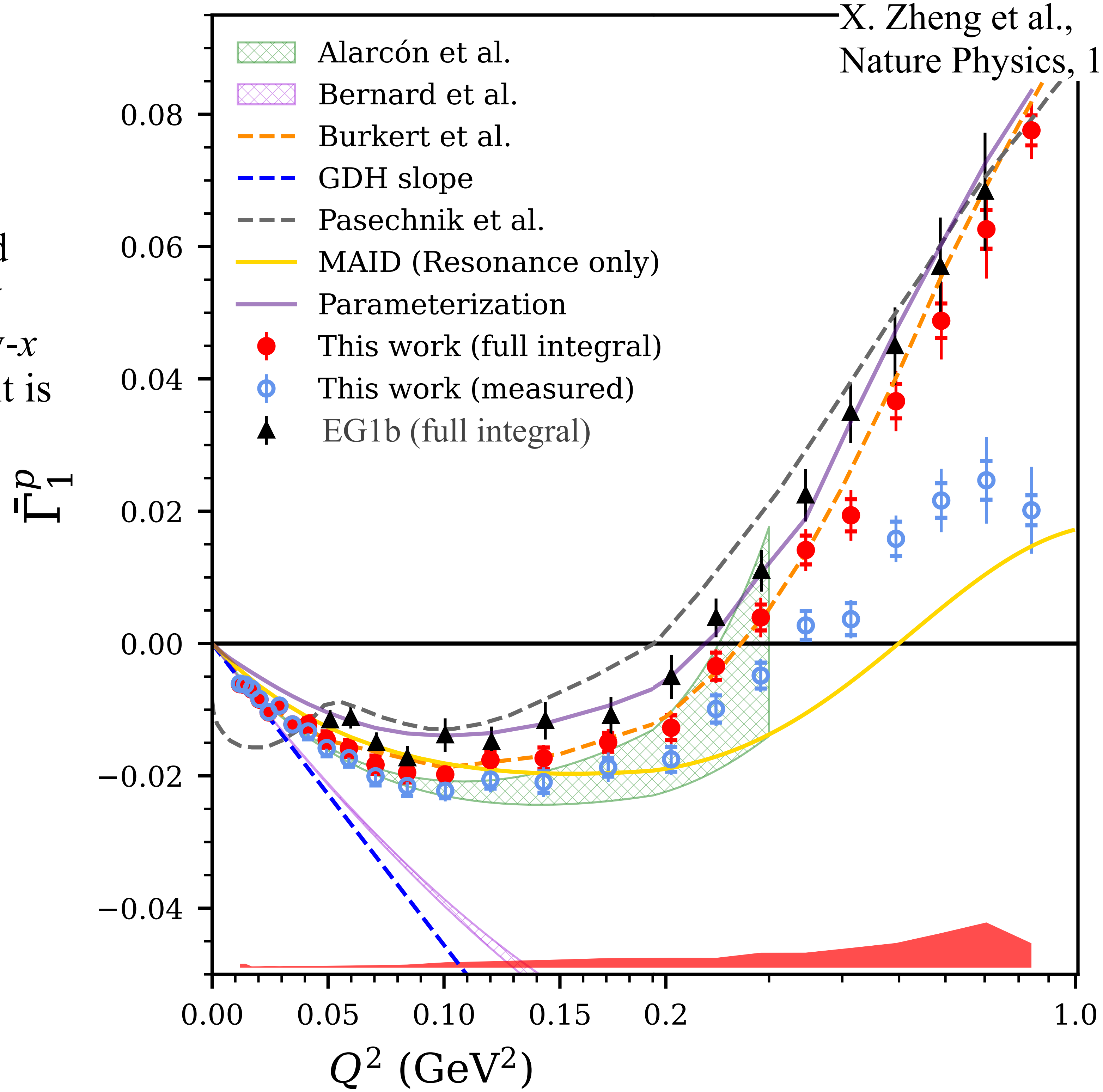


First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

To get to x=0 would demand infinite beam energy ⇒ Any measured moment has a low-x limit. For EG4 & E97-110, it is $x_{\min} \simeq 5 \times 10^{-3}$ typically.

X. Zheng et al., Nature Physics, 17 736 (2021)



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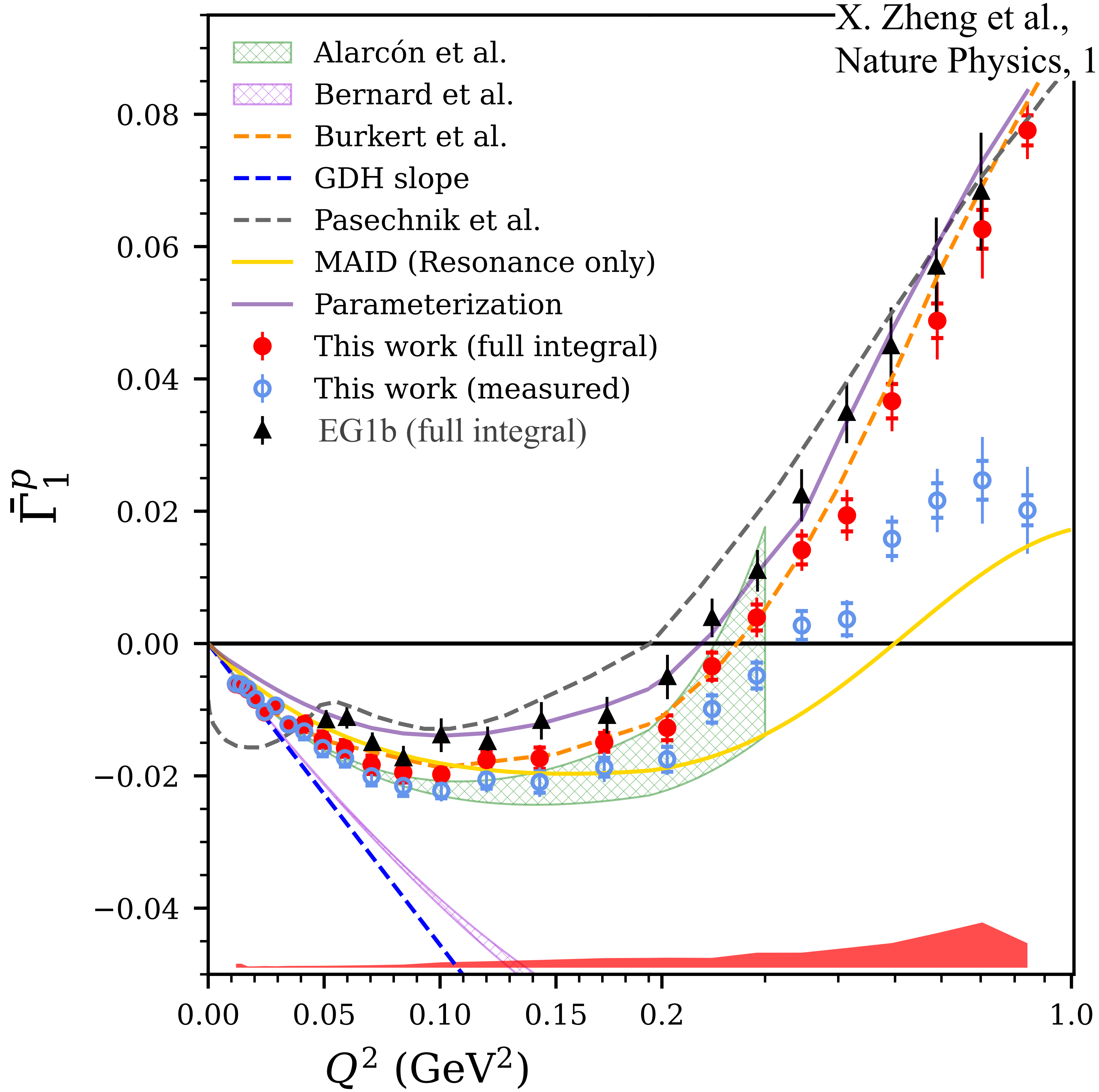
X. Zheng et al.,
Nature Physics, 17 736 (2021)

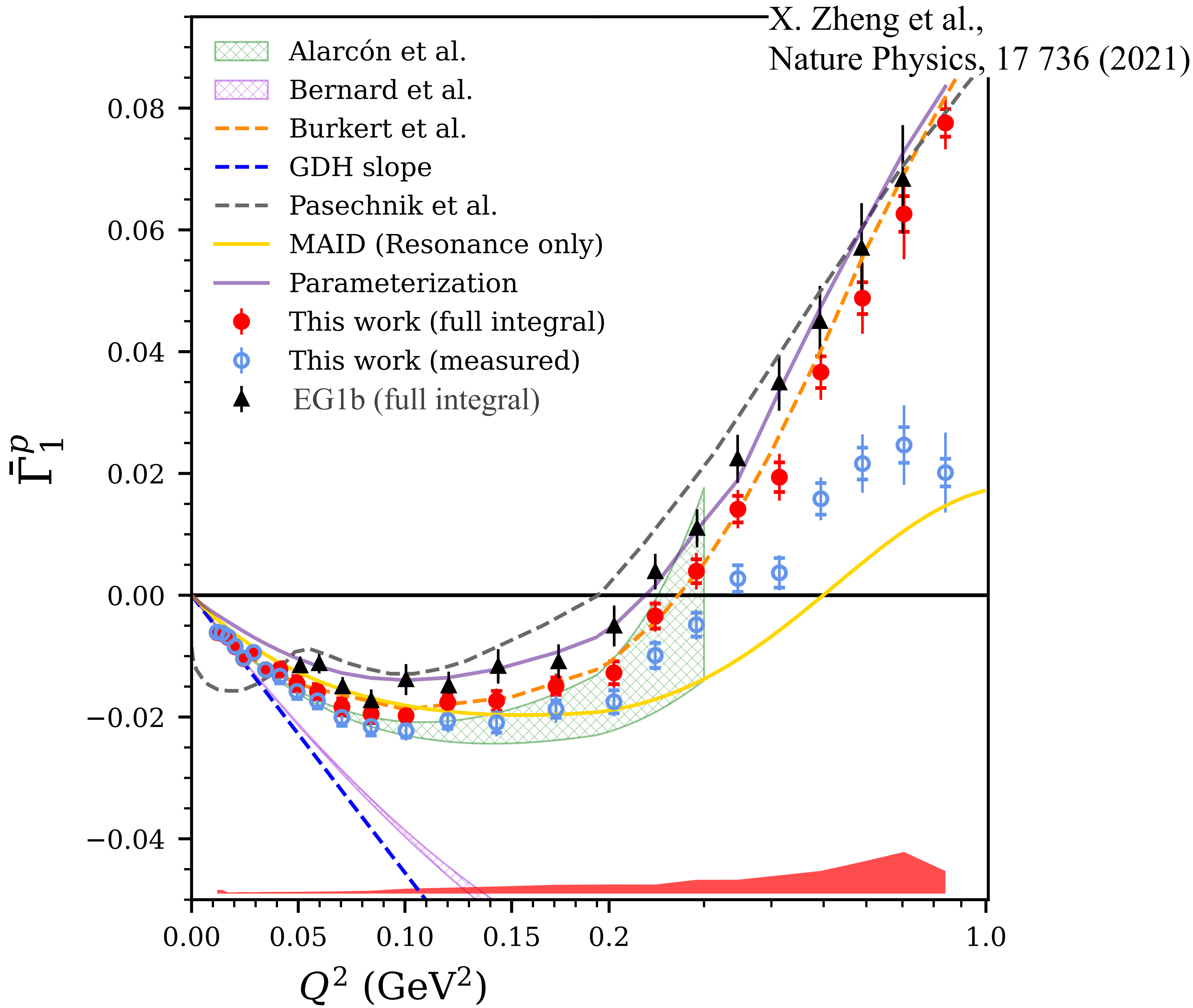


- Small unmeasured low-x contribution
- Lowest Q^2 decreased by factor of ~4
- Much improved precision

⇒Clean test of χEFT

First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

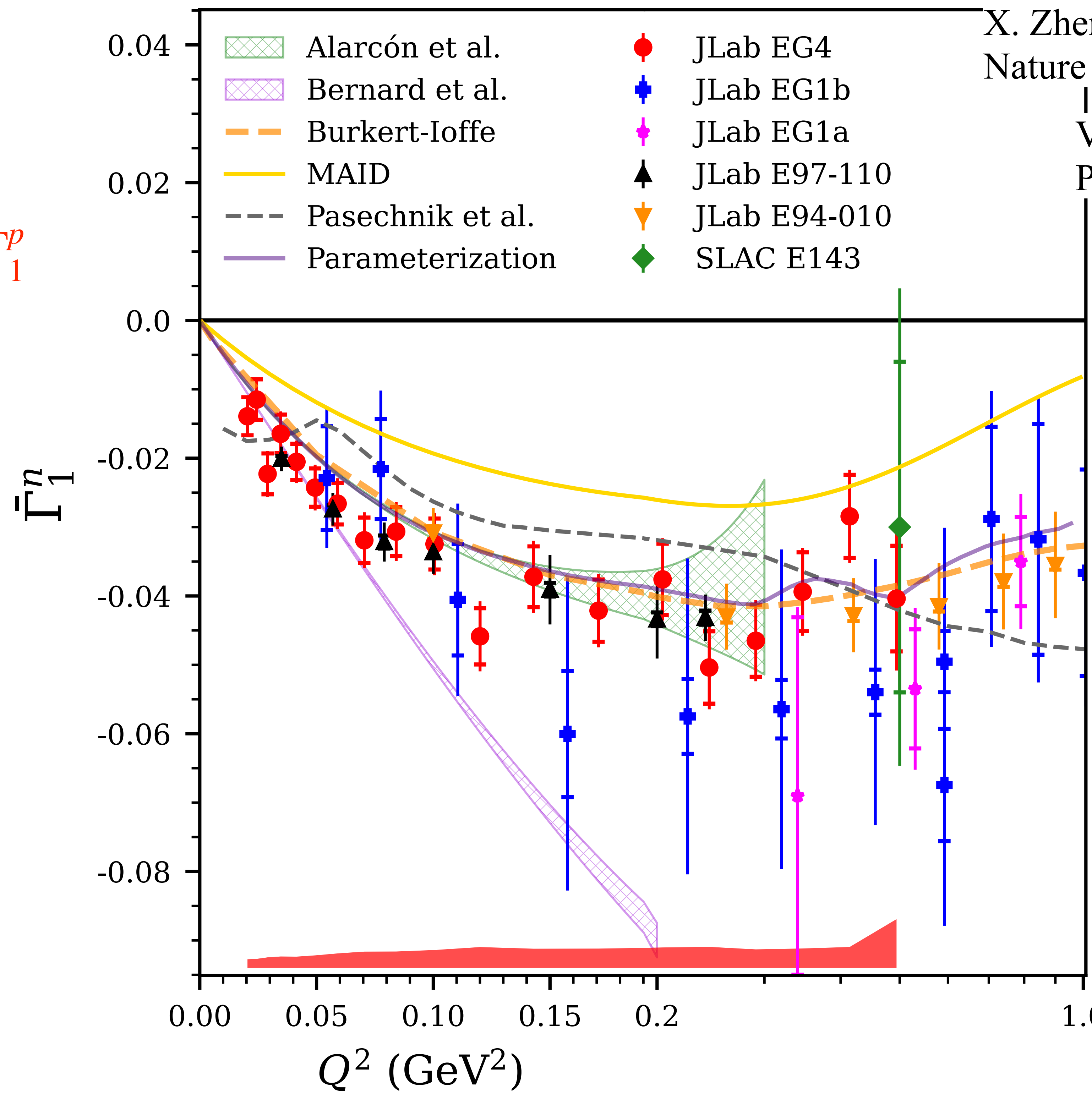


- Slight tension between EG4 and EG1b above Q^2 ~0.1 GeV^2. EG4: improved elastic radiative tail subtraction.
- EG4 and χEFT agree up to Q^2 ~0.04 GeV^2 (Bernard et al) or Q^2 >0.2 GeV^2 (Alarcón et al.)
- Some phenomenological models (Burkert-Ioffe, MAID) agree with data, other (Pasechnik et al) not as much.

First moments: generalized GDH sum $\Gamma_1^n(Q^2)$ from E97-110 & EG4

$$\Gamma_1^n = \int_0^{1^-} g_1^n(x, Q^2)dx$$

$$\Gamma_1^n = 2\Gamma_1^d/(1 - 1.5\omega_d) - \Gamma_1^p$$



X. Zheng et al., Nature Physics, 17 736 (2021)

V. Sulkosky et al. PLB 805 135428 (2020)

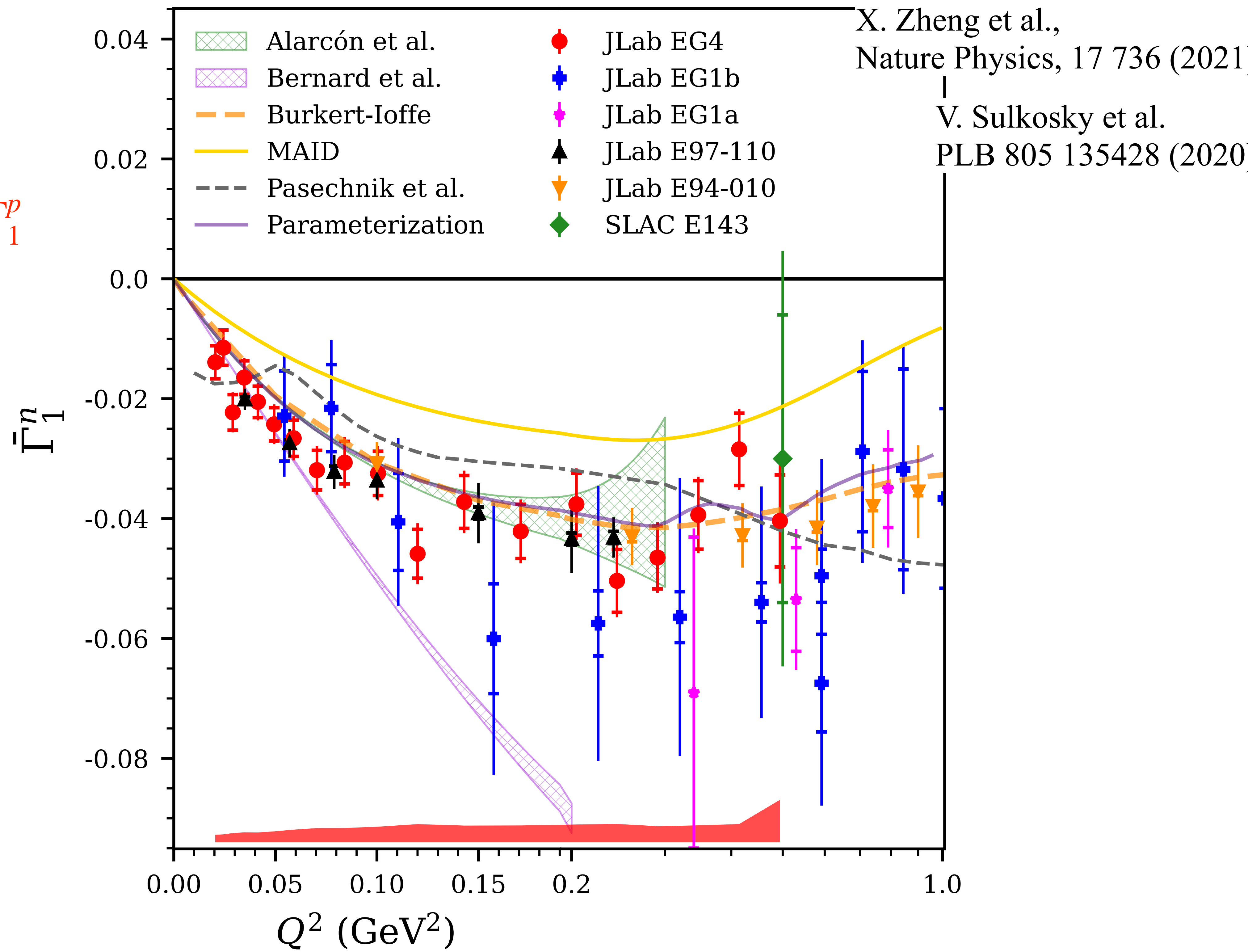
- Lowest Q² decreased by factor of ~4 (EG4) and ~2 (E97-110)
- Much improved precision, noticeably E97-110

⇒Clean test of χEFT

First moments: generalized GDH sum $\Gamma_1^n(Q^2)$ from E97-110 & EG4

$$\Gamma_1^n = \int_0^{1^-} g_1^n(x, Q^2)dx$$

$$\Gamma_1^n = 2\Gamma_1^d/\left(1 - 1.5\omega_d\right) - \Gamma_1^p$$



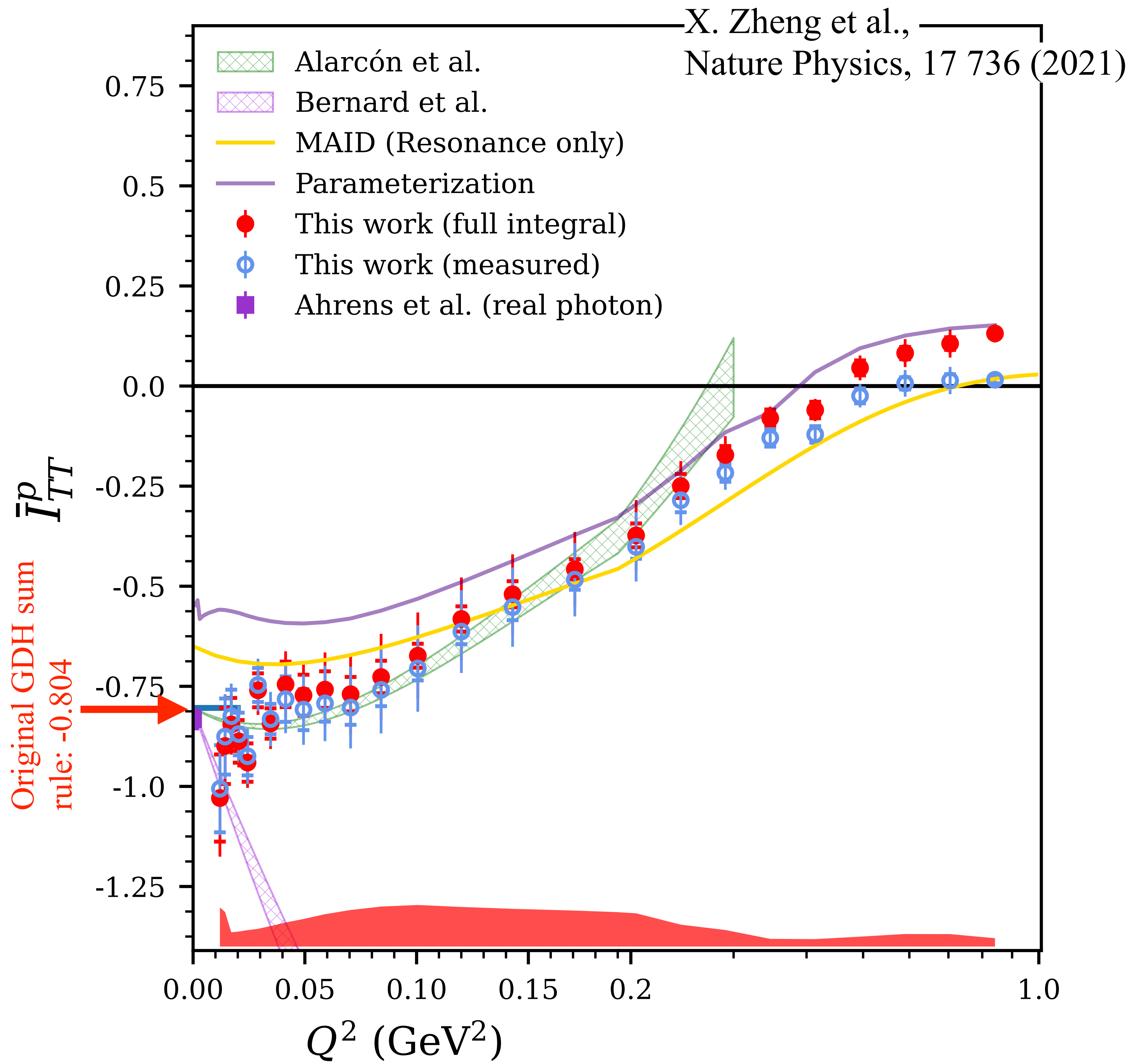
- E97-110 and EG4 agree well. They also agree with older data at larger Q^2 (EG1b, E94-010).
- E97-110 and EG4 agree with χEFT up to Q^2~0.06 GeV^2 (Bernard et al) or Q^2>0.4 GeV^2 (Alarcón et al.)
- Some phenomenological models (Burkert-Ioffe) agree with data, others (MAID, Pasechnik et al) not as much.

Another generalization of GDH sum: $I_{TT}^{p}(Q^2)$. EG4 Data

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (not measured by EG4)



Extrapolating the (very low Q^2) data to Q^2=0 provides an independent check of the GDH SR validity, with a different method (inclusive data) than photoproduction experiments (exclusive data).

$$I_{TT}^{p\ \mathrm{EG4}}(0) = -0.798 \pm 0.042$$

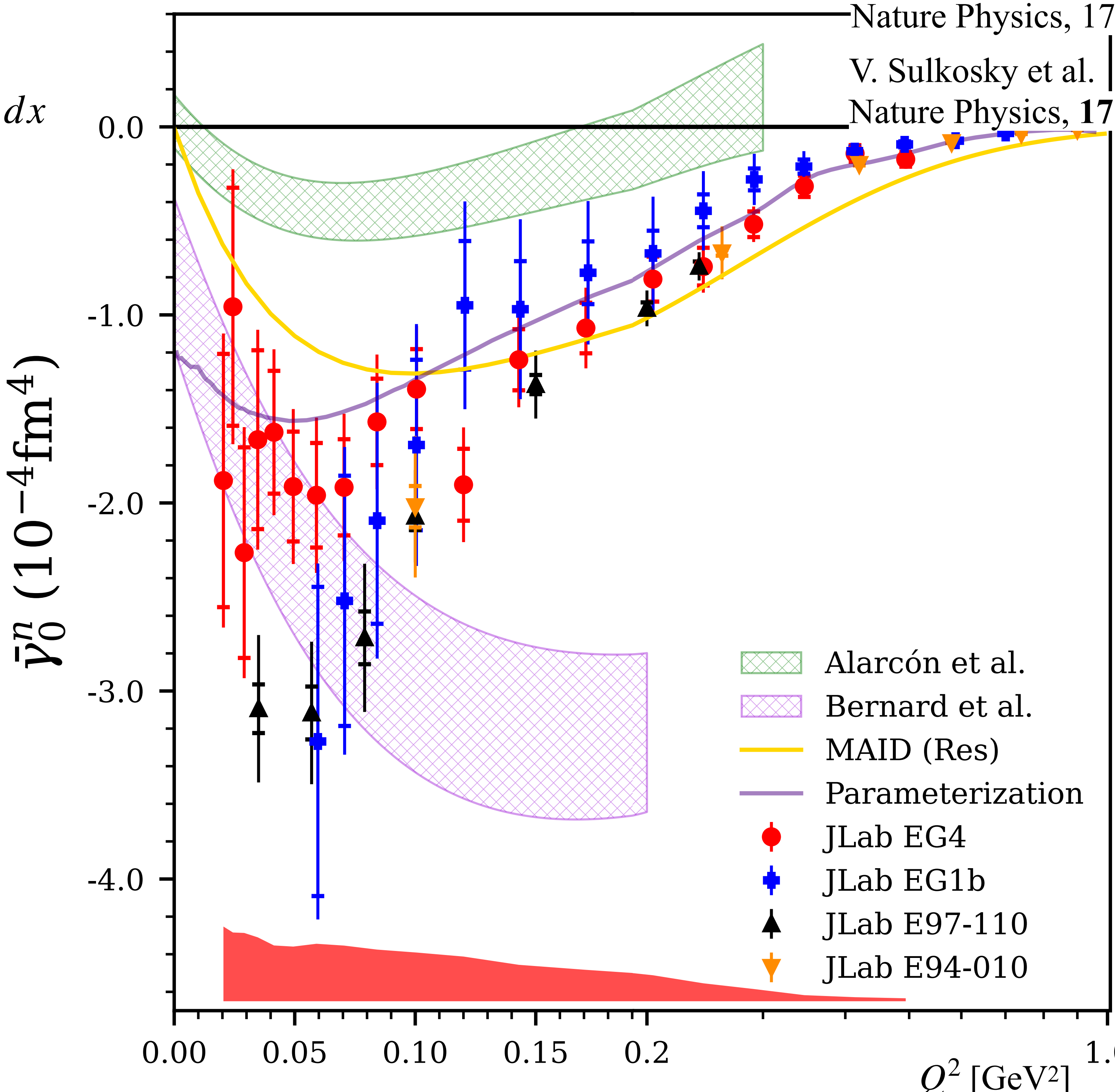
Agrees with the GDH SR, with precision similar to photoproduction method: $I_{TT}^{p\ \mathrm{MAMI}}(0) = -0.832 \pm 0.023(stat)$\$ $\pm 0.063(syst)$

Higher moments: Generalized forward spin polarizability γ_0^n from E97-110 and EG4

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]dx$$

X. Zheng et al.,
Nature Physics, 17 736 (2021)

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)

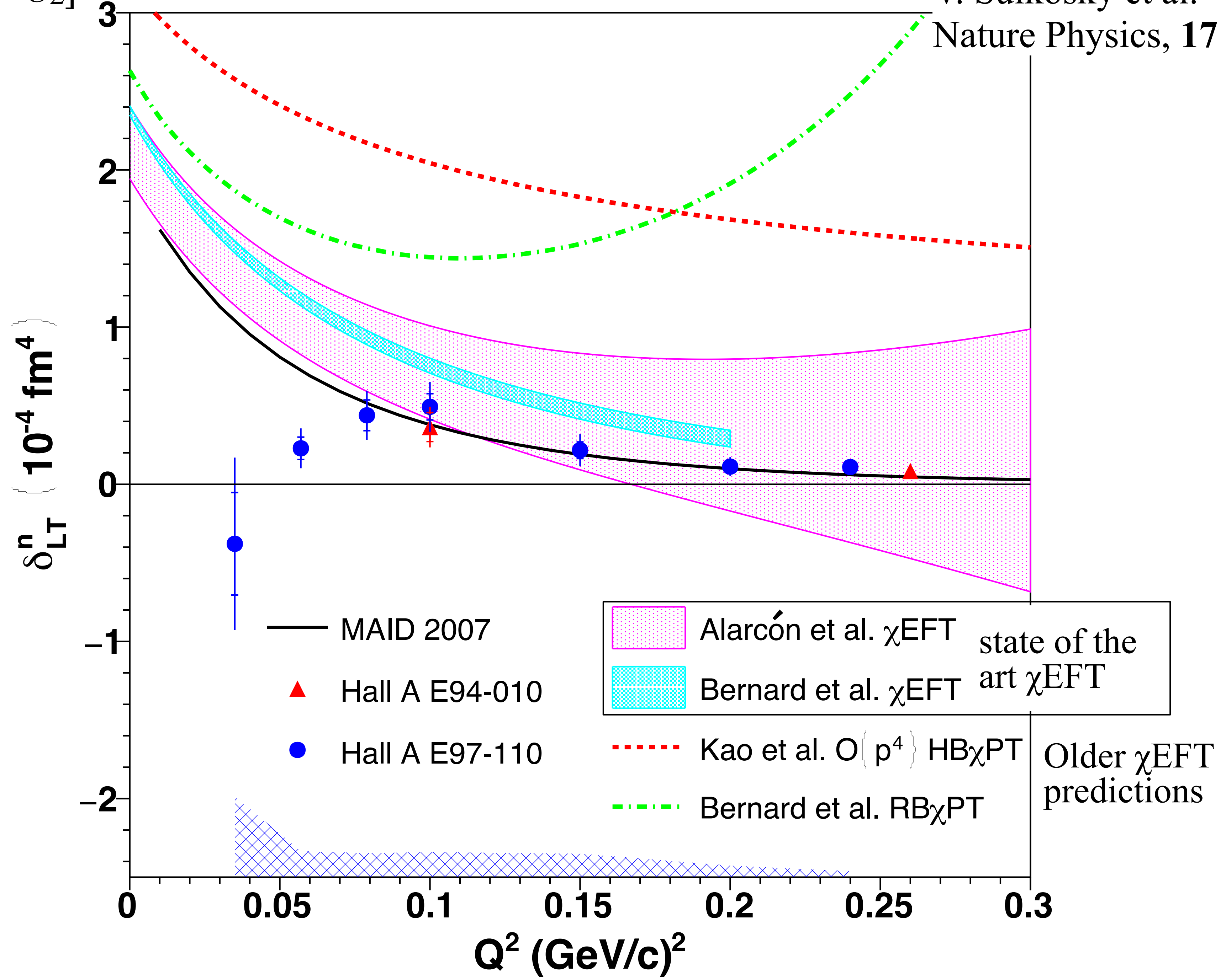


- E97-110 and EG4 agree with older data at larger Q^2 (EG1b, E94-010).
- Marginal agreement between EG4 and E97-110 in the lower Q^2 range. (Better agreement if the EG4 systematic errors are added linearly rather than in quadratures)
- χEFT result of Alarcón et al disagrees with data. Bernard et al. agrees for lowest Q^2 points.
- Maid disagrees with the data.

Higher moments: Longitudinal-transverse spin polarizability δ_{LT} from E97-110

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1 + g_2\right] dx$$

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)



- Good agreement with older data at larger Q^2 and with χEFT & MAID there.
- Disagreement with χEFT & MAID at lower Q^2, although first moment $\int_0^{1^-} x^2[g_1 + g_2]dx$ agrees with Schwinger sum rule, see back-up slides.
- $\Rightarrow$"$\delta^n_{LT}(Q^2)$ puzzle" still remains.

Summary: testing/using sum rules

Sum rule: relation between an integral of a dynamical quantity (cross section, structure function,...) and a global property of the target (mass, spin,…).

Can be used to:

- Test theory (e.g. QCD, χEFT) and/or hypotheses with which they are derived. Ex: GDH, Ellis-Jaffe, Bjorken sum rules.
  - **Gerassimov-Drell-Hearn sum rule**:
    - $I^p_{TT}(Q^2 \to 0)$ agrees with GDH expectation,
    - $I^n_{TT}(Q^2 \to 0)$ and $I^d_{TT}(Q^2 \to 0)$ ~agree with GDH expectations,
    - $I^{^3He}_{TT}(Q^2)$: Q^2-behavior too steep for $Q^2 \to 0$ extrapolation, but no sign that anything is wrong.
  - $\Gamma^n_2(Q^2) = 0$ and $\Gamma^{^3He}_2(Q^2) = 0$ with uncertainty, in agreement with **Burkhardt–Cottingham sum rule**.
  - $I^n_{LT}(Q^2 \to 0)$ agrees with **Schwinger sum rule**. $I^{^3He}_{LT}(Q^2 \to 0)$ unclear, but no sign that anything is wrong.

- Measure the global property.
  - Generalized forward spin polarizability: $\gamma_0(Q^2)$, Q^2-map for proton, neutron, $p \pm n$ and deuteron.
  - Generalized Longitudinal-transverse spin polarizability δ_{LT}. Q^2-map for neutron (and proton from E08-007, see J.P. Chen talk Friday)

Testing χEFT

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
			😃		😃	😃	😃 😃	😃	😃 😃	😃 😃

More robust measurements (no significant missing low-x contribution).

Nucleon resonance Δ_{1232} contribution suppressed (More robust χEFT calculations)

Testing χEFT

A: ~agree
X: ~disagree
- : No prediction available

State of χEFT affairs before E97-110/EG4 runs:

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X

Testing χEFT

A: agree over range $0<Q^2 \lesssim 0.1$ GeV^2

X: disagree over range $0<Q^2 \lesssim 0.1$ GeV^2

- : No prediction available

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X
Bernard 2012	X	X	~A	X	X	A	X	X	X	X
Alarcón 2020	A	A	~A	A	~A	X	X	X	A	X

state of the art χEFT (Bernard 2012, Alarcón 2020)

More robust measurements (no significant missing low-x contribution. More on this later)

Nucleon resonance Δ_{1232} contribution suppressed (More robust χEFT calculations)

Testing χEFT

A: agree over range $0<Q^2\lesssim 0.1$ GeV^2

X: disagree over range $0<Q^2\lesssim 0.1$ GeV^2

- : No prediction available

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X
Bernard 2012	X	X	~A	X	X	A	X	X	X	X
Alarcón 2020	A	A	~A	A	~A	X	X	X	A	X

Improvement compared to the state of affaire of early 2000s.

Despite χEFT refinements (new expansion scheme, including the Δ_{1232} d.o.f,…) and despite data now being well into the expected validity domain of χEFT, it remains challenged by results from dedicated polarized experiments at low Q^2.

Conclusion

E97-110 and EG4 provide high precision nucleon spin structure data at very low Q^2, in the domain where χEFT is expected to be valid.

General good agreement with other experiments. Good agreement between E97-110 and EG4 for most observables. Marginal agreement for γ_0 for the lowest Q^2 points

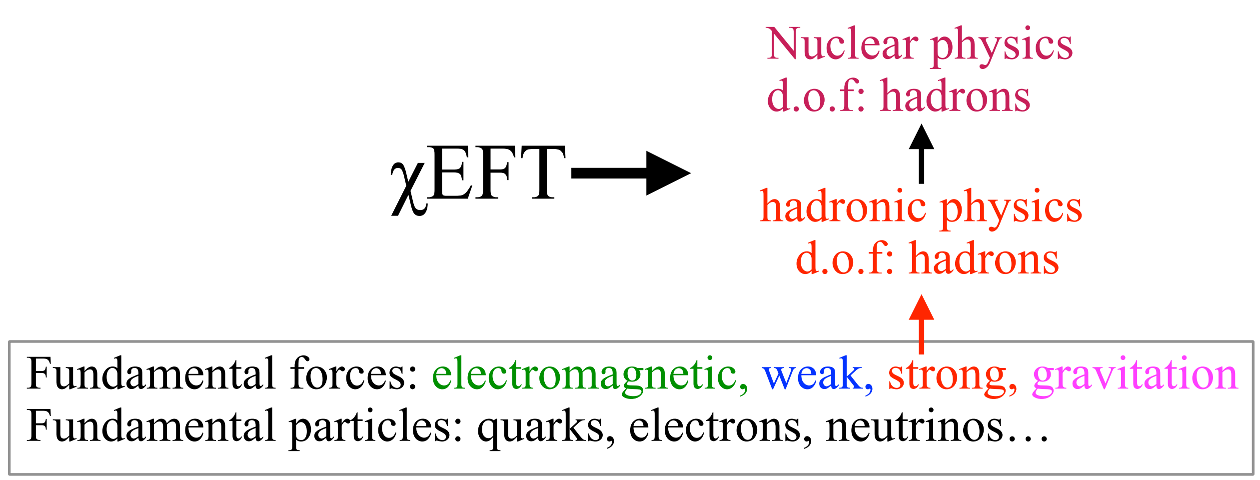
The data agree within uncertainties with the spin sum rules studied: GDH, BC, Schwinger.

Mixed of agreement/disagreement with χEFT, depending on observable, Q^2 range and calculations. "$\delta_{LT}^n(Q^2)$ puzzle", remains and γ_0^{p-n} disagree with χEFT expectation.

⇒ χEFT, although successful in many instances, is challenged by polarized low Q^2 data.

Surely, low Q^2 sum rule measurements are challenging (low-x extrapolation, high-x contamination). But the experiments are independent: very different detectors and methods. Yet, consistent experiment message. Also, some χEFT predictions disagree with each other

This is a problem in our endeavor for a complete description of Nature at all levels: χEFT is the leading approach to manage the first level of complexity arising above the Standard Model, in the strong force sector. Just as if atomic physics could not provide the theoretical foundations of chemistry.



It would be helpful to see what other non-perturbative approaches to QCD would predict: Dyson-Schwinger Eqs. , AdS/QCD…

Back-up slides

Bjorken sum rule

Bjorken sum rule = Generalized GDH sum rule on proton - neutron

- Derived (1966) independently from GDH sum rule (1965/1966) and using different formalisms.
- Connection with generalized GDH sum rule occurred much later (Anselmino:1989 ….. Ji-Osborne:1999)
- Provided crucial test that QCD works also when spin d.o.f. are explicit.

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 - \ldots \right] + \frac{\text{HT}}{Q^2} + \ldots$$

Nucleon's First spin structure function

Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

Bjorken sum rule

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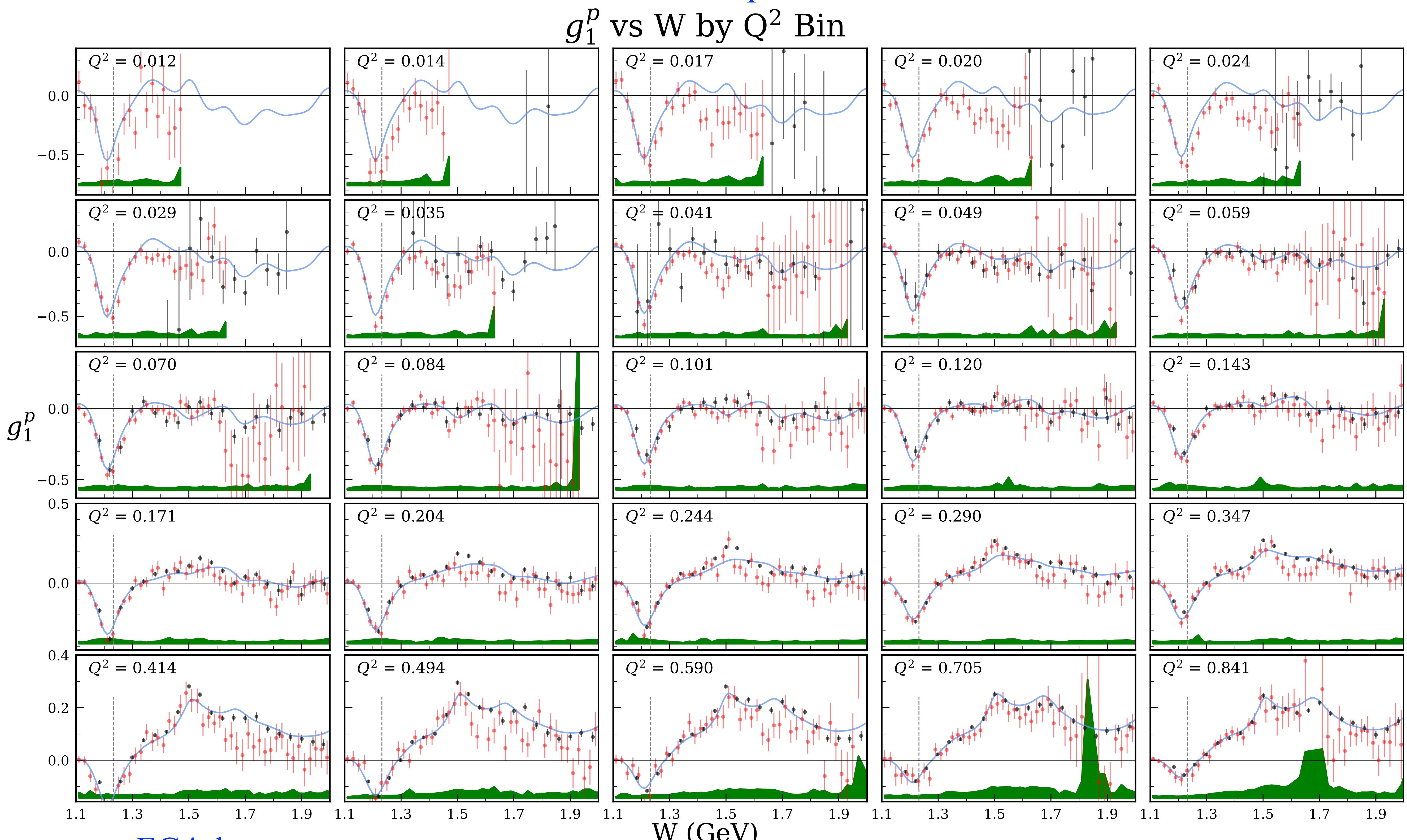
Valid in pQCD domain only (not at low Q^2)

Spin structure function $g_1^p(W, Q^2)$ data from EG4



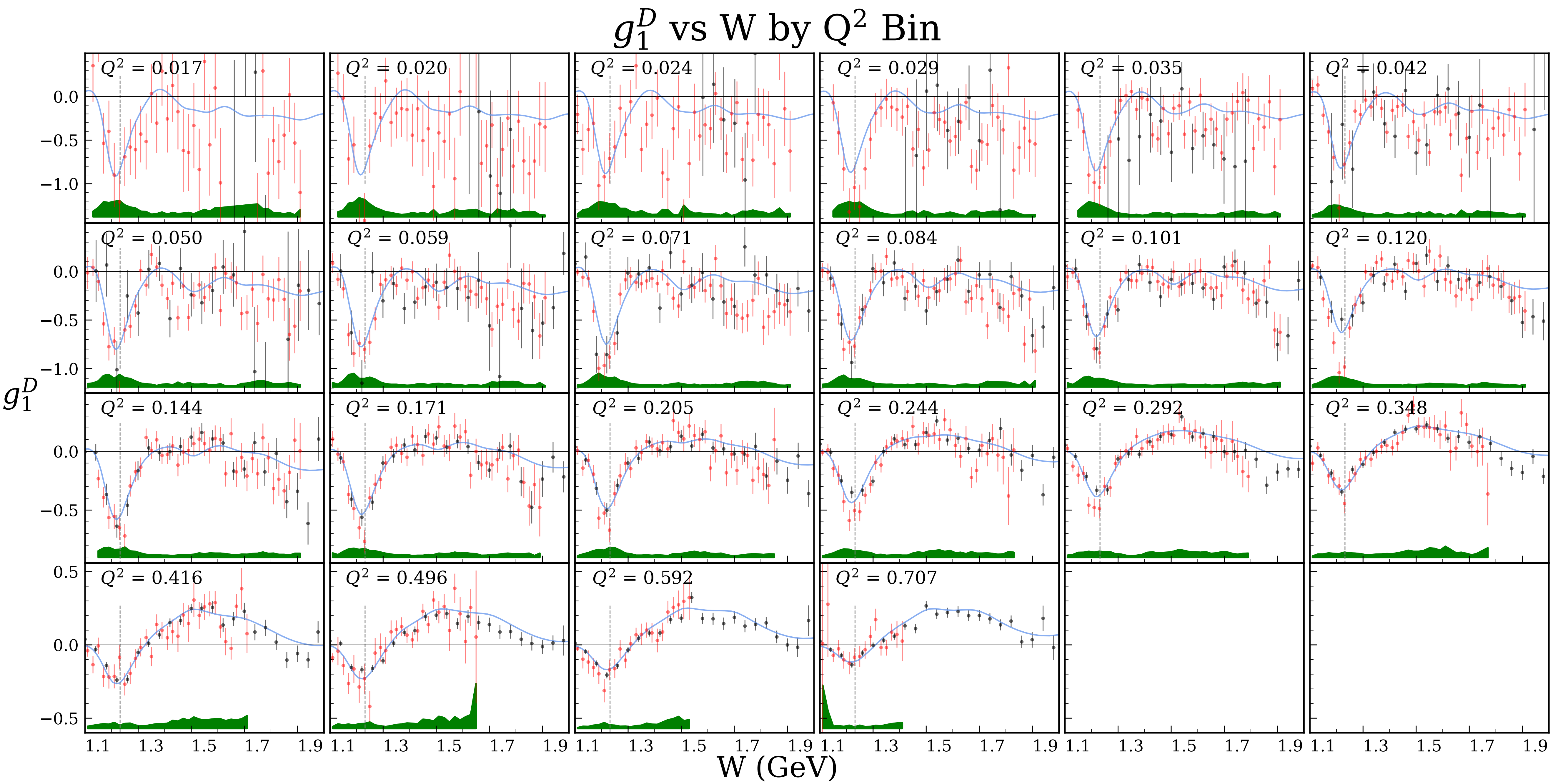
● EG4 data

● EG1b data

— "Model" (Fit to EG1b + other published data).

X. Zheng et al.,
Nature Physics, 17 736 (2021)

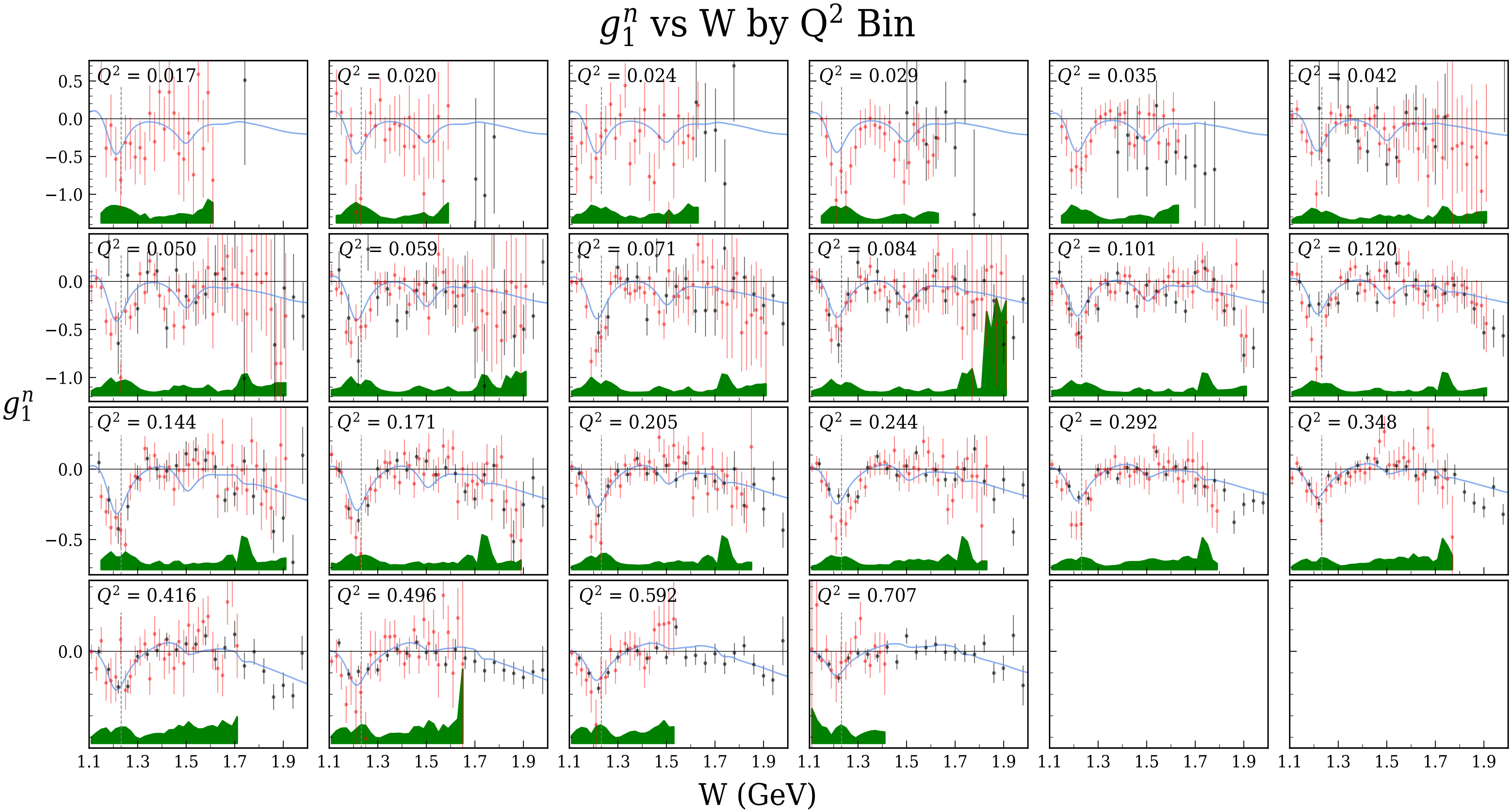
Spin structure function $g_1^d(W, Q^2)$ data from EG4



K. Adhikari et al.
PRL **120**, 062501 (2018)

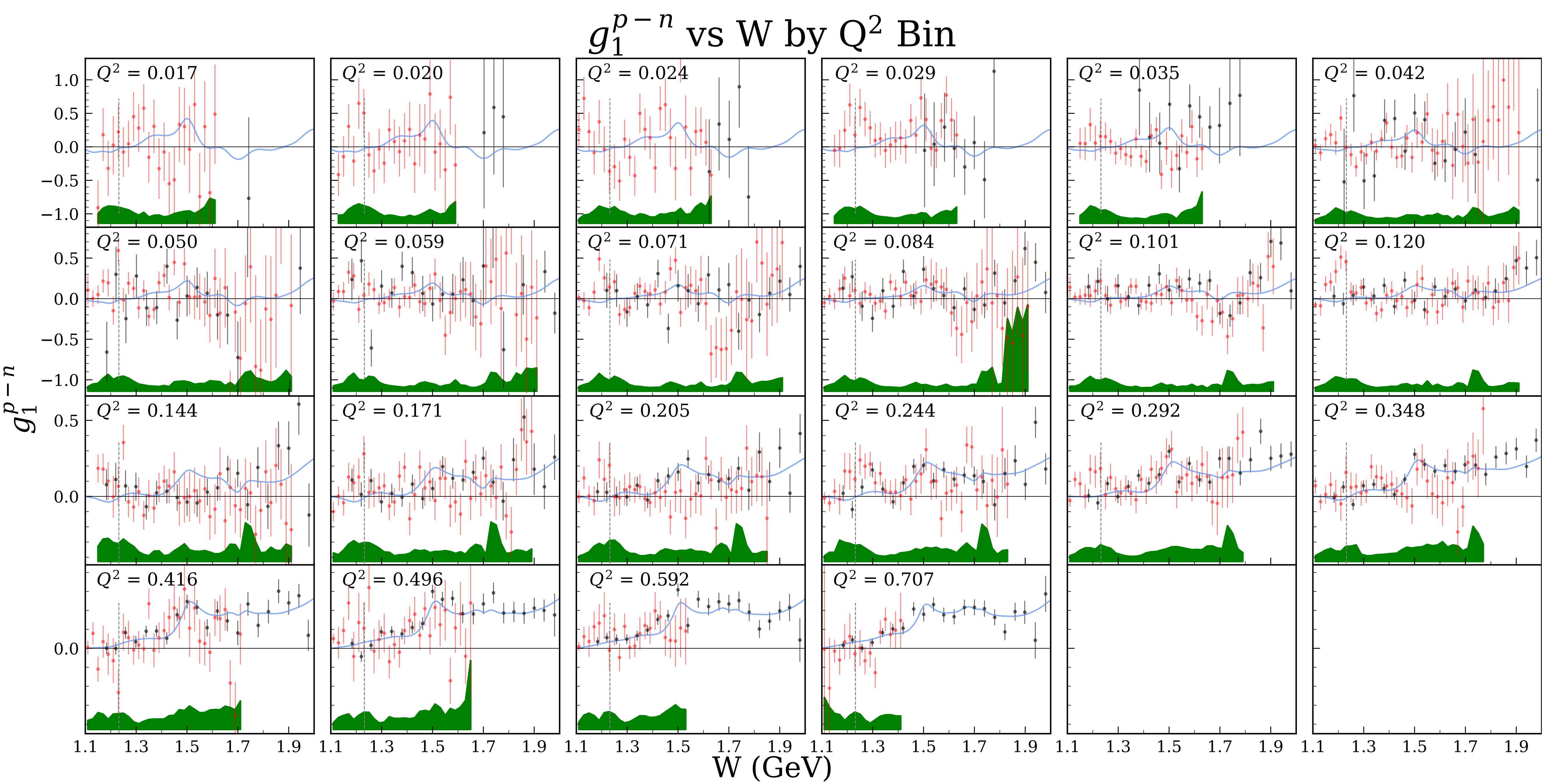
- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

Spin structure function $g_1^n(W, Q^2)$ data from EG4



- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

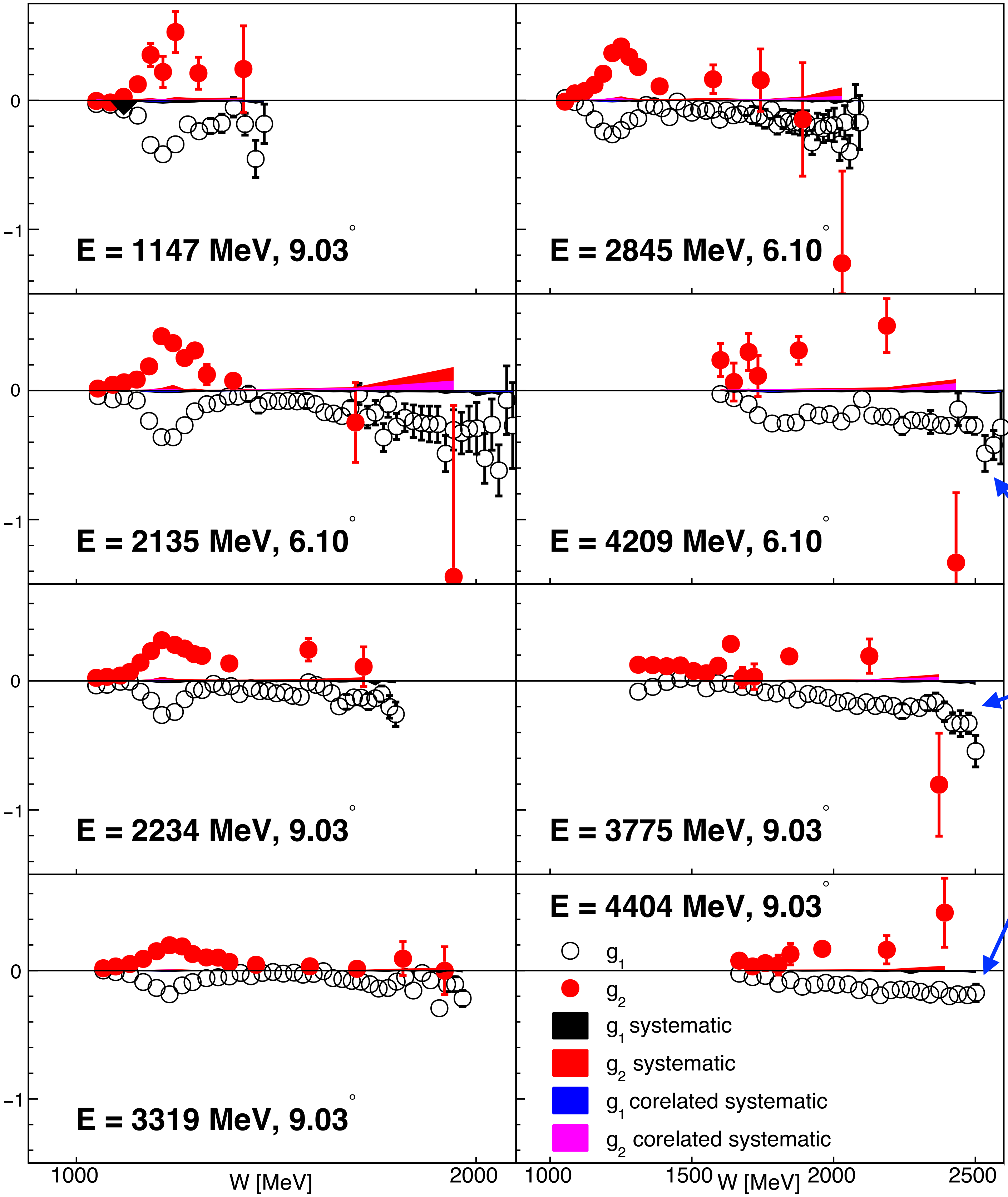
Spin structure function $g_1^{p-n}(W, Q^2)$ data from EG4



- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

Spin structure function $g_1^{^3He}(W,Q^2)$ and $g_2^{^3He}(W,Q^2)$ data from E97-110

We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)



V. Sulkosky et al.
PLB 805 135428 (2020)

We observe the expected $g_1 \simeq -g_2$ symmetry near the Δ_{1232}.
Δ: ~M1 transition $\Rightarrow \sigma_{LT} \propto g_1 + g_2 \simeq 0$

Large W coverage to test sum rule convergency

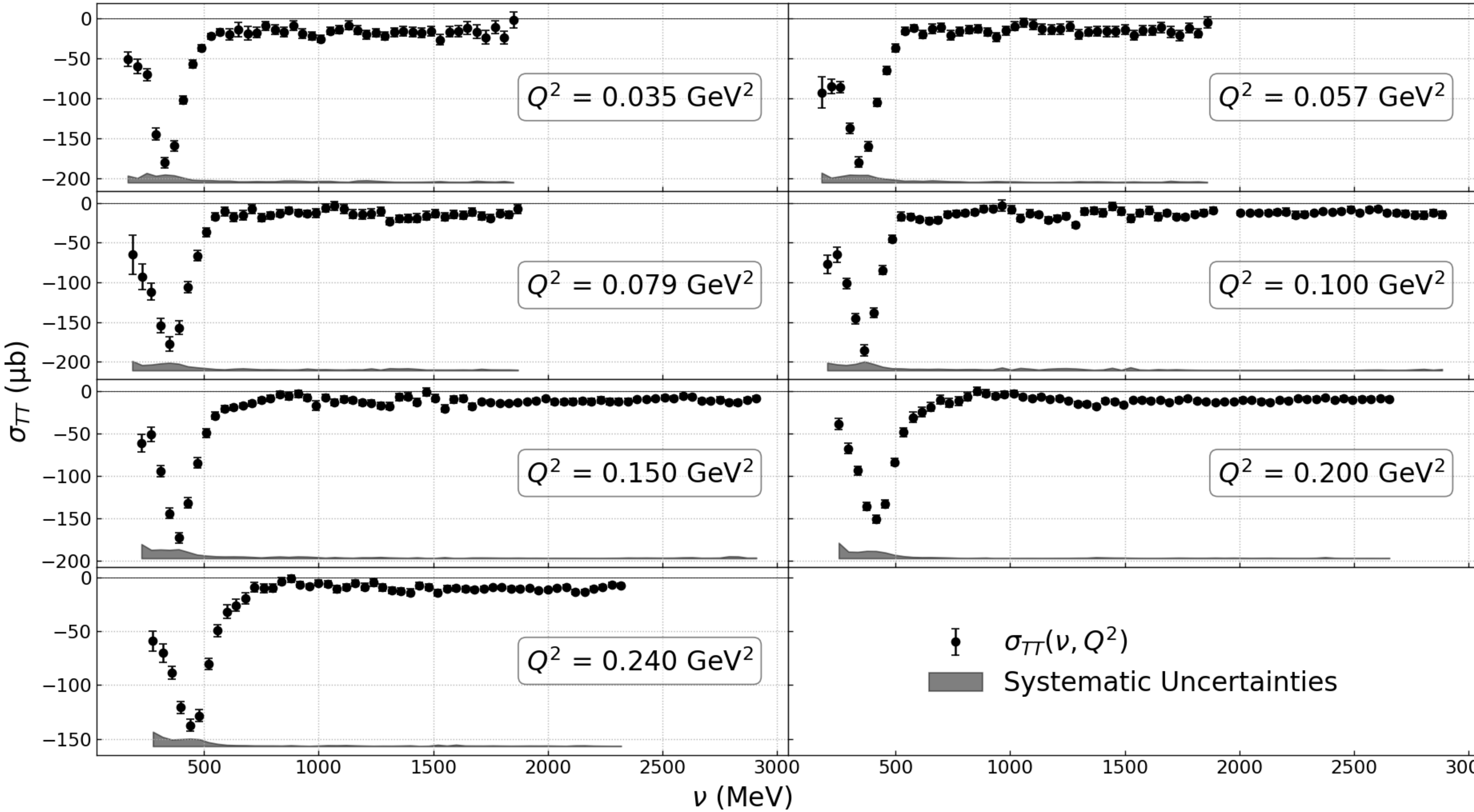
Polarized cross-section $\sigma_{TT}^{^3He}(\nu, Q^2)$ data from E97-110

We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)

$$\sigma_{TT} = \frac{\sigma_A - \sigma_p}{2} = \frac{4\pi^2\alpha}{MK}(g_1 - \gamma^2 g_2)$$

K: virtual photon flux

V. Sulkosky et al.
PLB 805 135428 (2020)

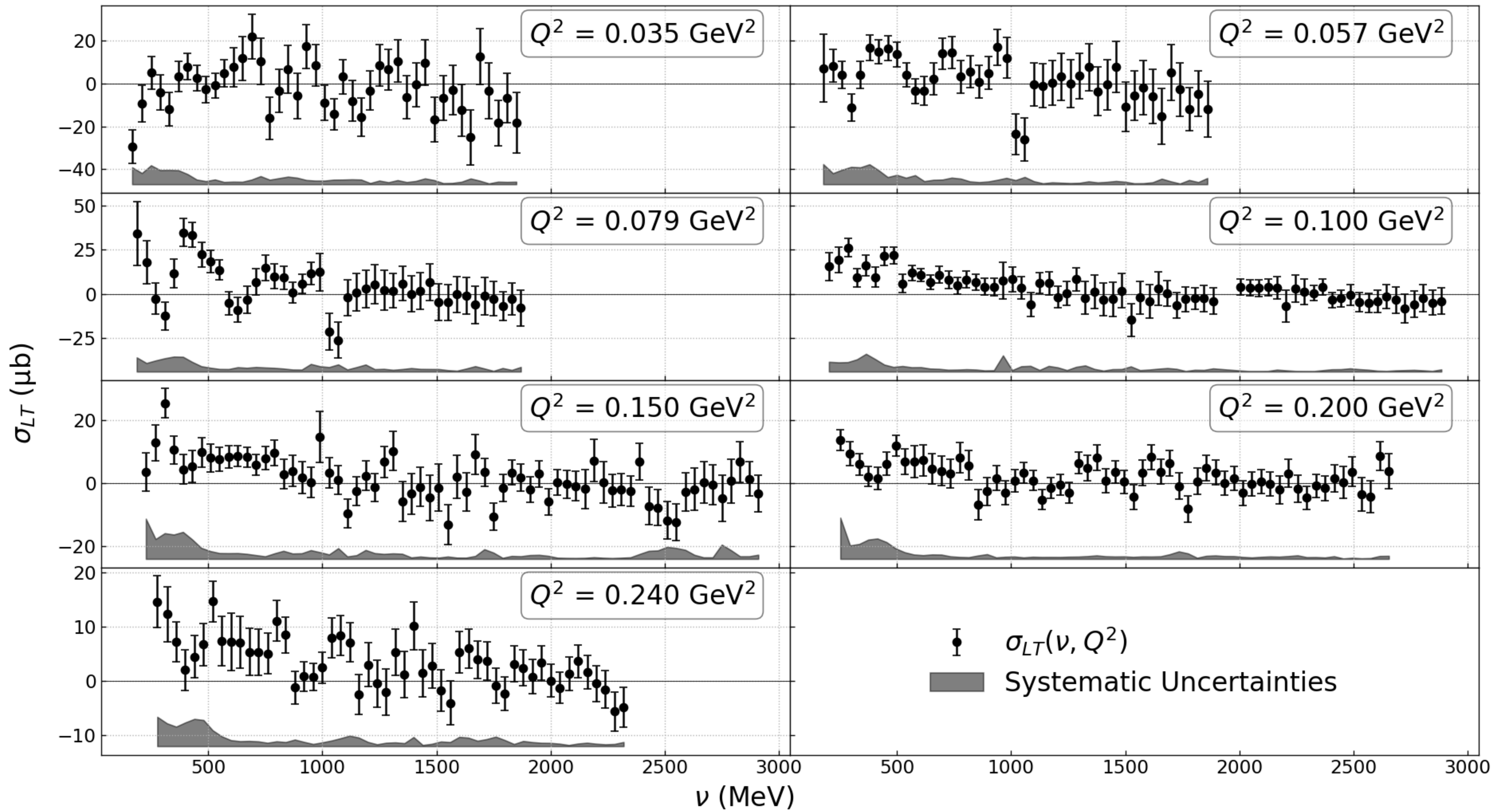


Polarized cross-section $\sigma_{LT}^{^3He}(\nu, Q^2)$ data from E97-110

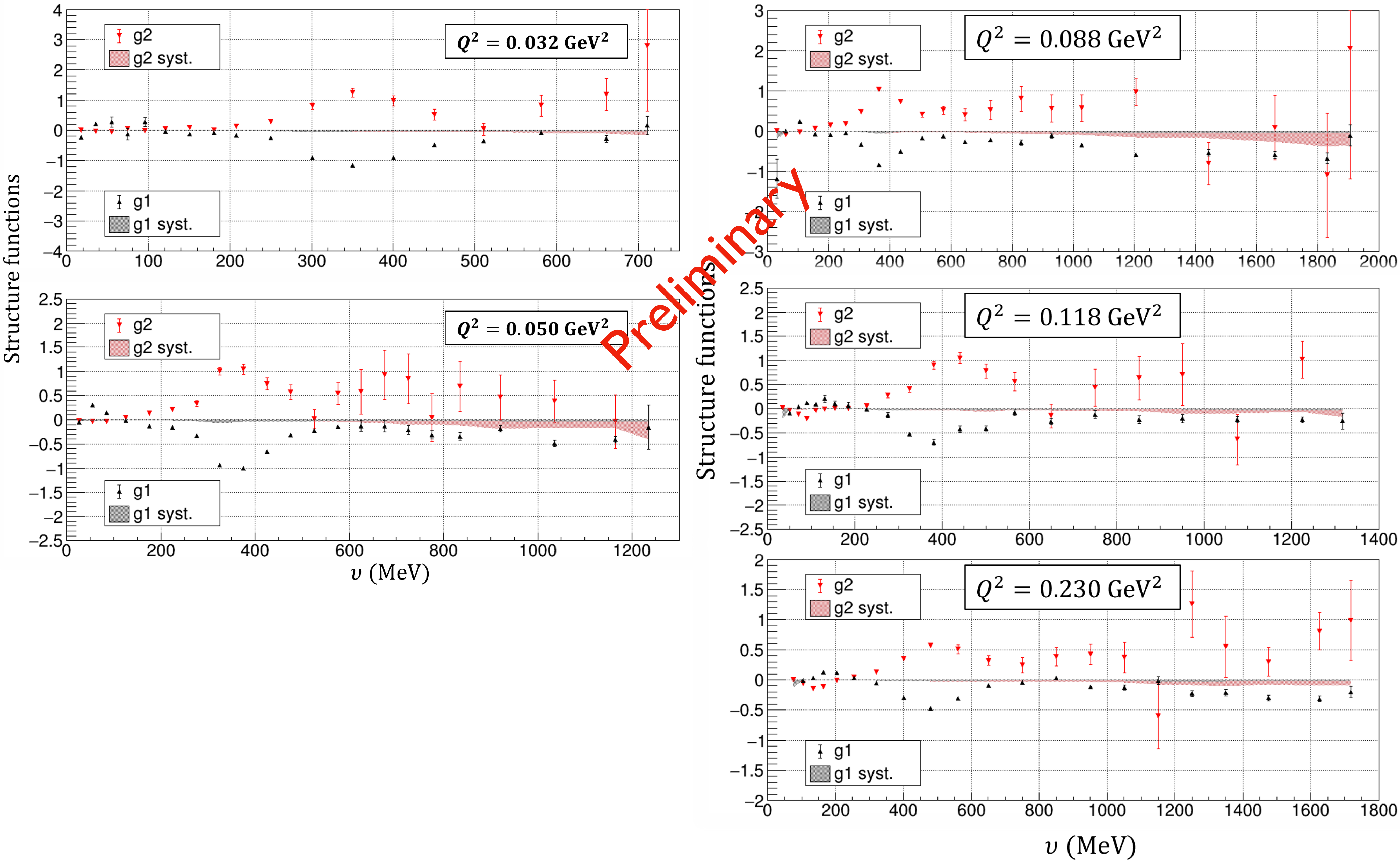
We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)

$$\sigma_{LT} = \frac{4\pi^2\alpha}{MK}\gamma(g_1 + g_2)$$

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)



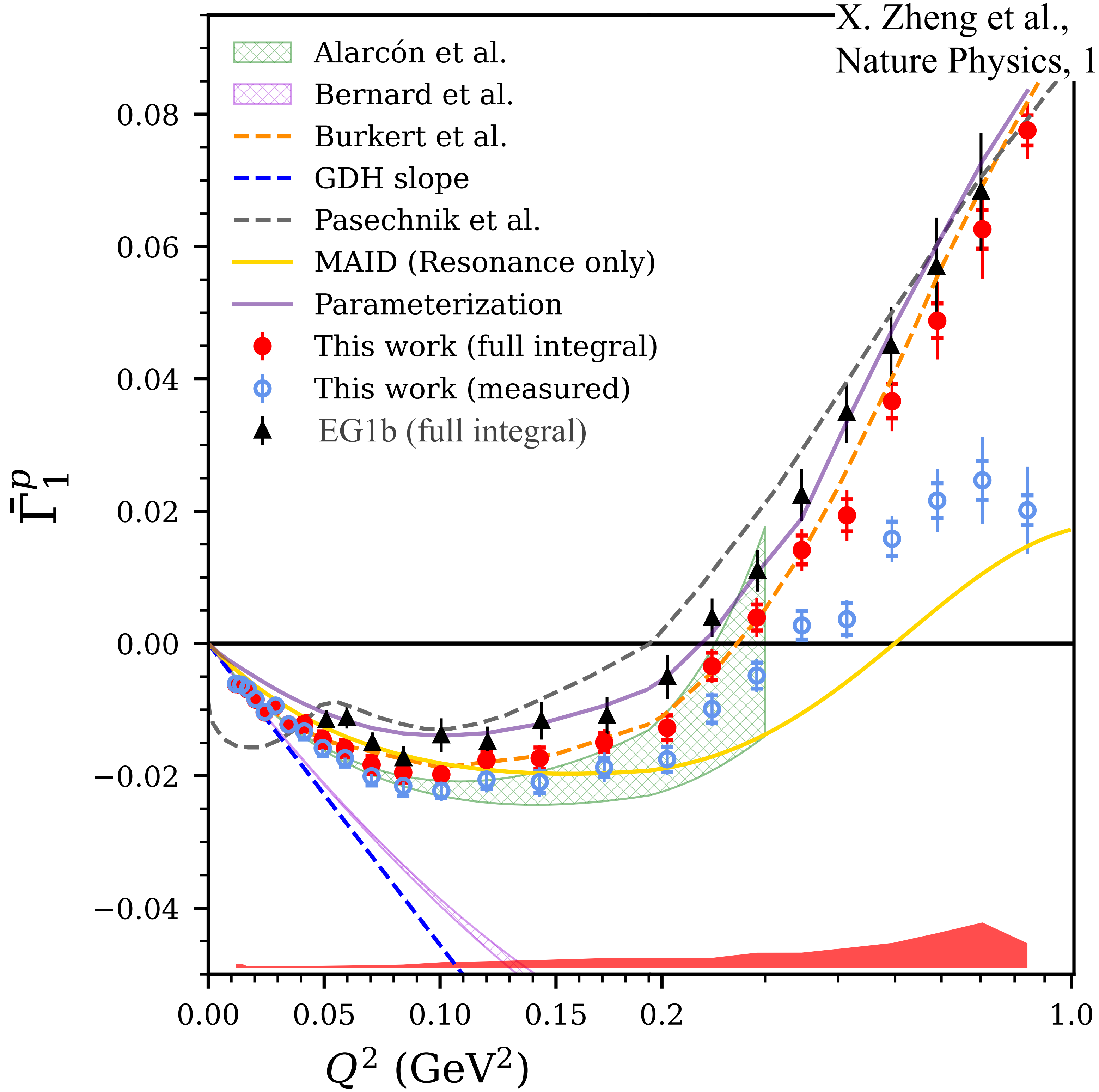
$g_1^{^3He}(\nu, Q^2)$ and $g_2^{^3He}(\nu, Q^2)$ with quasi-elastic, from E97-110



First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

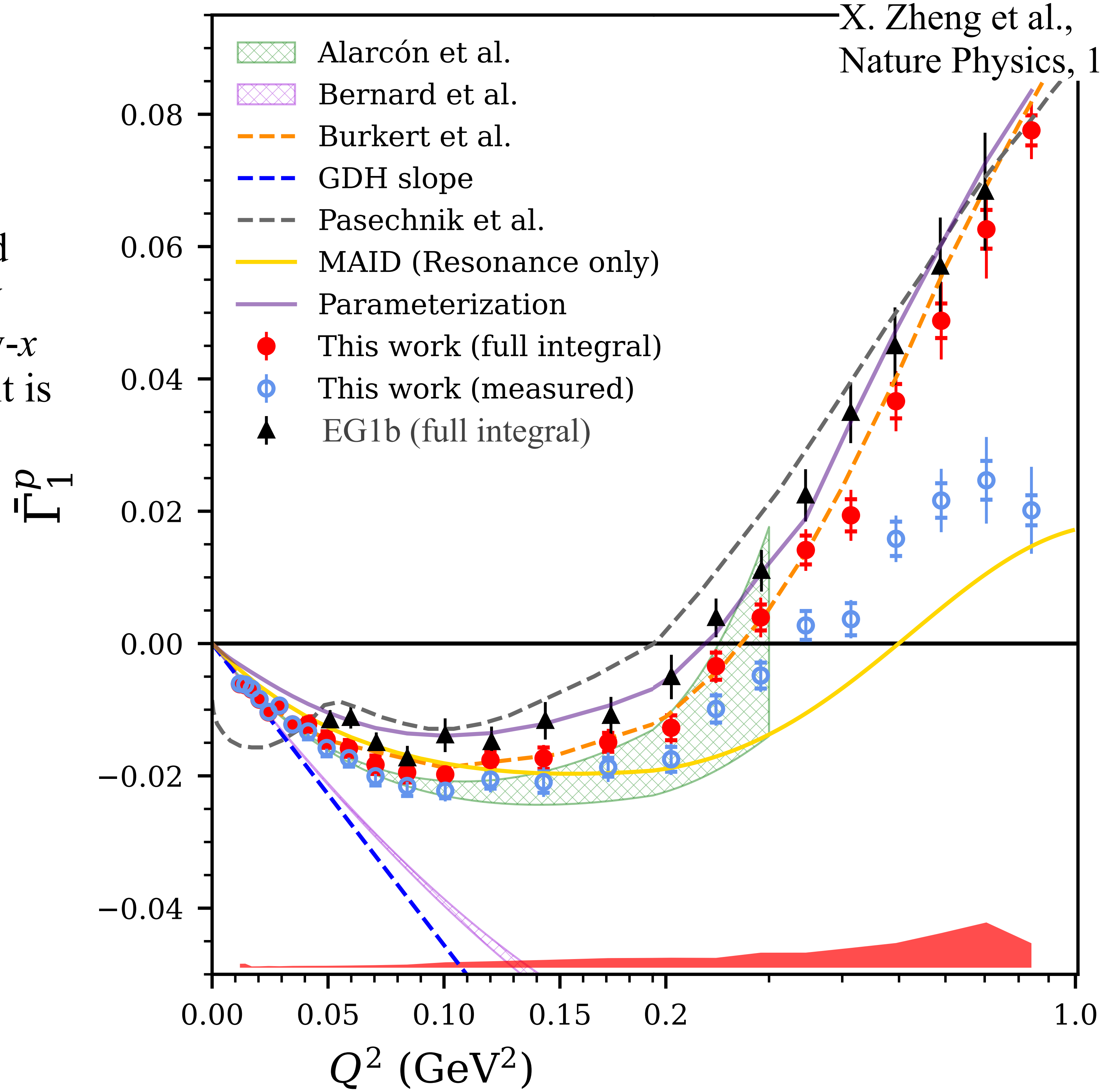
X. Zheng et al., Nature Physics, 17 736 (2021)



First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

To get to x=0 would demand infinite beam energy ⇒ Any measured moment has a low-x limit. For EG4 & E97-110, it is $x_{\min} \simeq 5 \times 10^{-3}$ typically.



X. Zheng et al., Nature Physics, 17 736 (2021)

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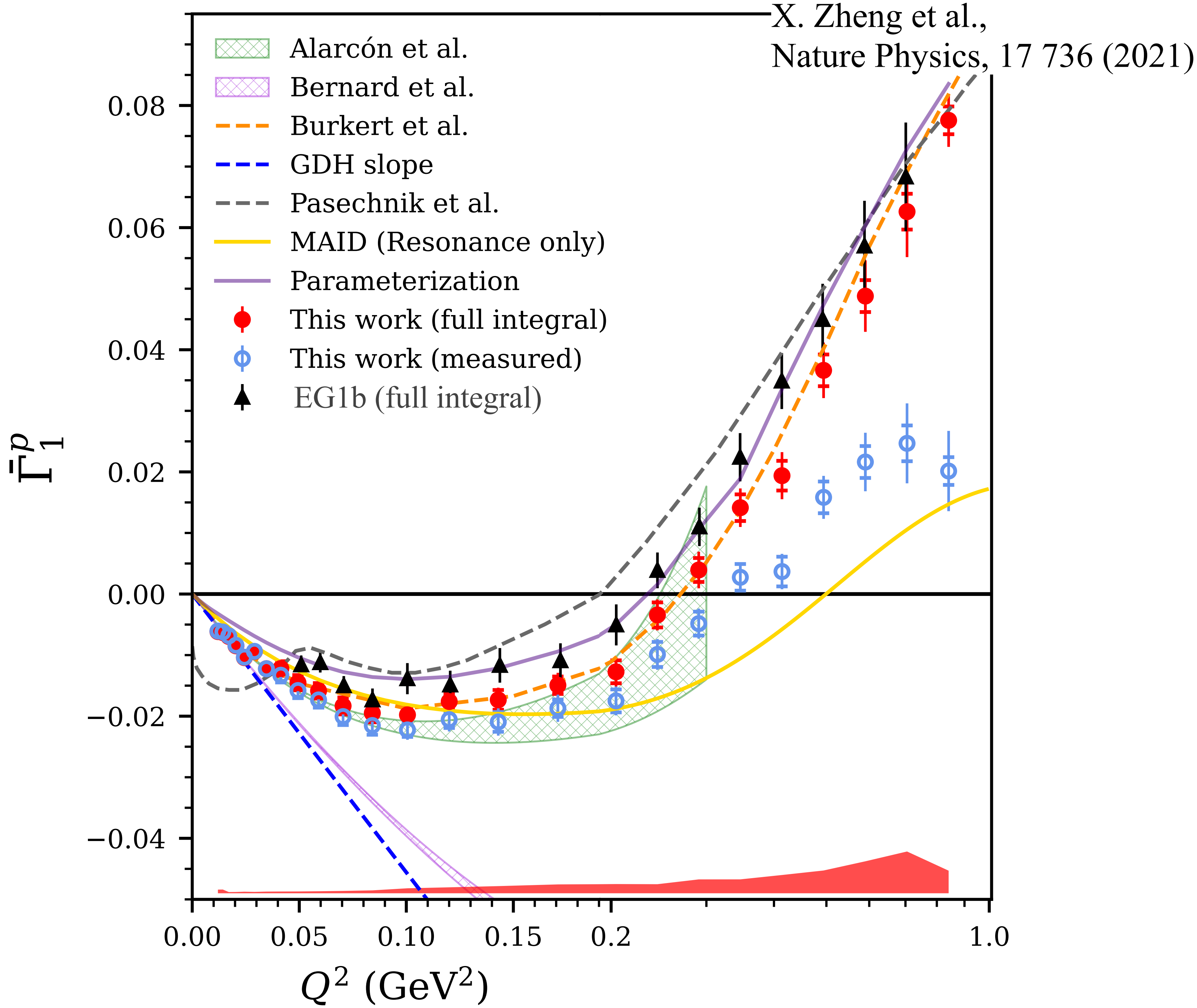
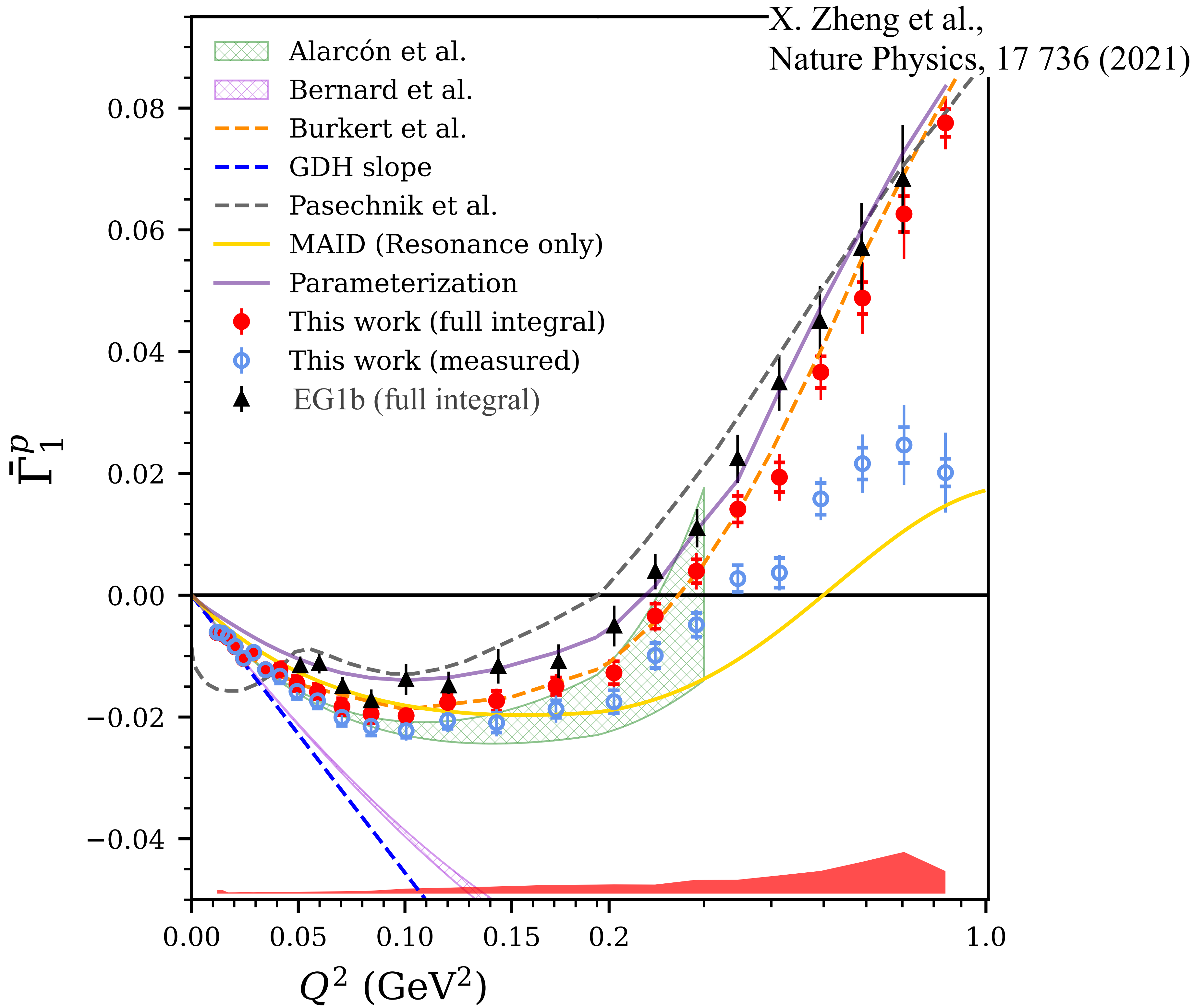


- Small unmeasured low-x contribution
- Lowest Q^2 decreased by factor of ~4
- Much improved precision

⇒Clean test of χEFT

First moments: generalized GDH sum $\Gamma_1^p(Q^2)$ measurement from EG4

$$\Gamma_1^p = \int_0^{1^-} g_1^p(x, Q^2)dx$$

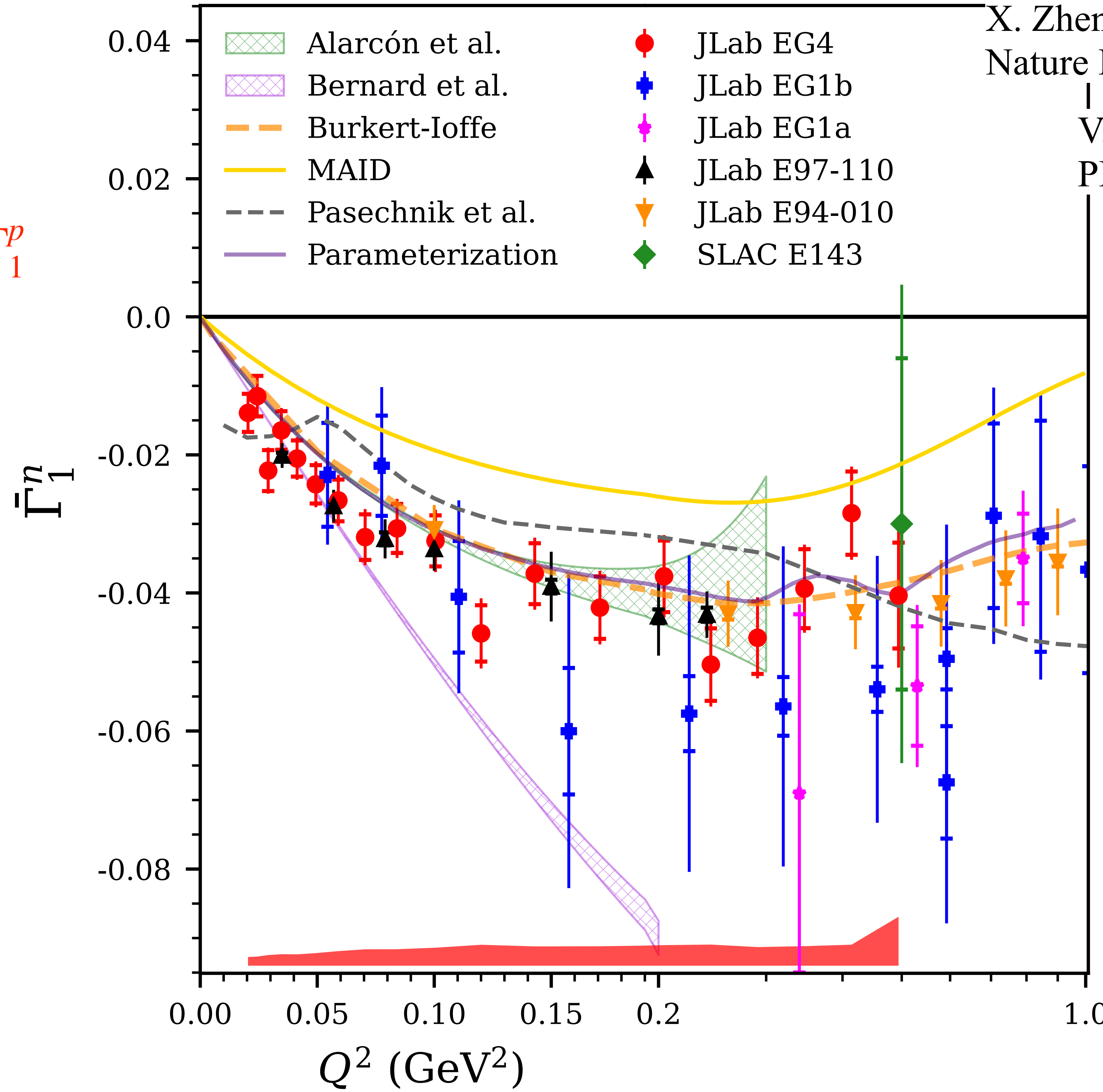


- Slight tension between EG4 and EG1b above Q^2 ~0.1 GeV^2. EG4: improved elastic radiative tail subtraction.
- EG4 and χEFT agree up to Q^2 ~0.04 GeV^2 (Bernard et al) or Q^2 >0.2 GeV^2 (Alarcón et al.)
- Some phenomenological models (Burkert-Ioffe, MAID) agree with data, other (Pasechnik et al) not as much.

First moments: generalized GDH sum $\Gamma_1^n(Q^2)$ from E97-110 & EG4

$$\Gamma_1^n = \int_0^{1^-} g_1^n(x, Q^2)dx$$

$$\Gamma_1^n = 2\Gamma_1^d/(1-1.5\omega_d) - \Gamma_1^p$$



X. Zheng et al., Nature Physics, 17 736 (2021)

V. Sulkosky et al. PLB 805 135428 (2020)

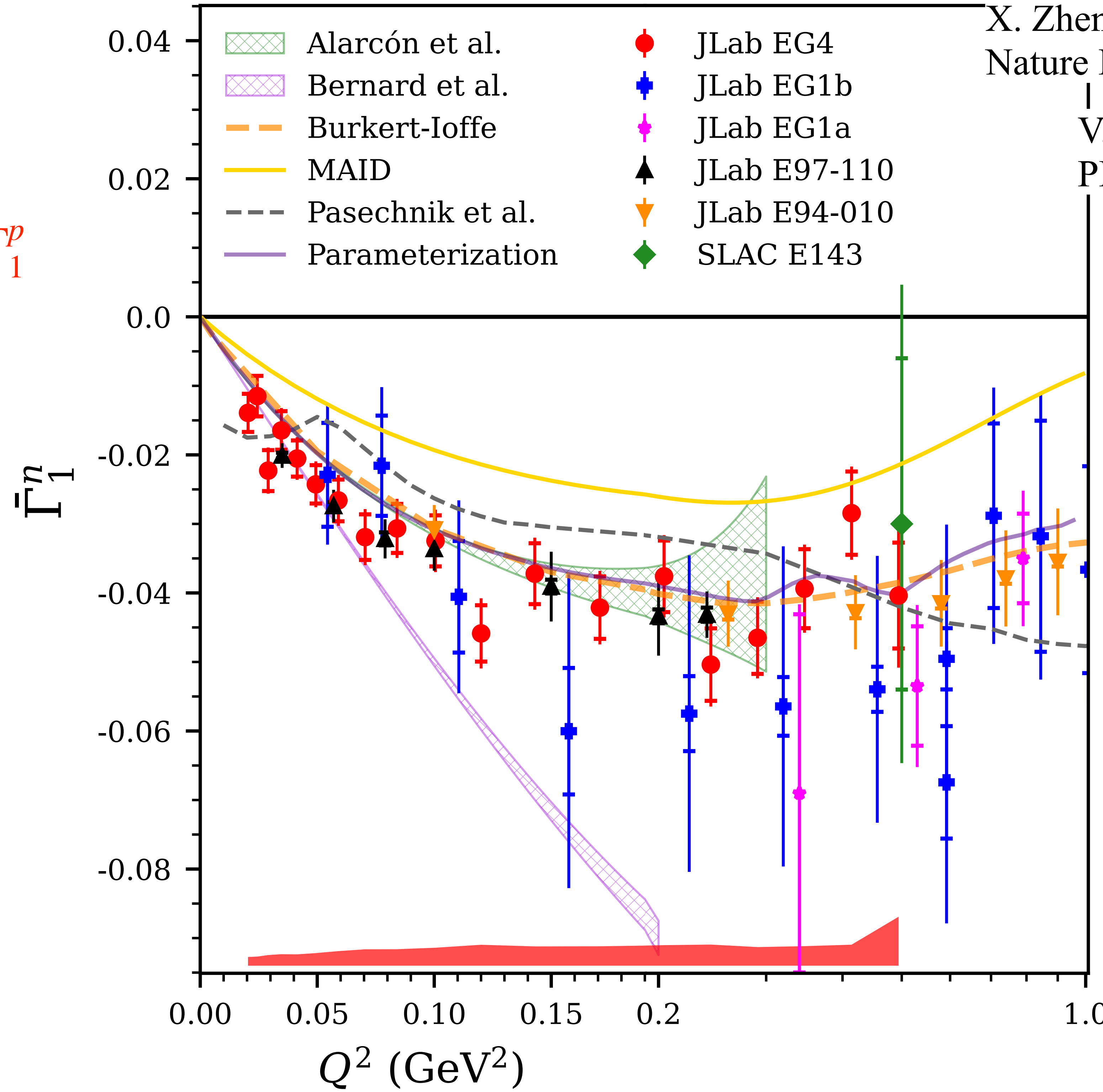
- Lowest Q^2 decreased by factor of ~4 (EG4) and ~2 (E97-110)
- Much improved precision, noticeably E97-110

⇒Clean test of χEFT

First moments: generalized GDH sum $\Gamma_1^n(Q^2)$ from E97-110 & EG4

$$\Gamma_1^n = \int_0^{1^-} g_1^n(x, Q^2)dx$$

$$\Gamma_1^n = 2\Gamma_1^d/\left(1 - 1.5\omega_d\right) - \Gamma_1^p$$



X. Zheng et al., Nature Physics, 17 736 (2021)

V. Sulkosky et al. PLB 805 135428 (2020)

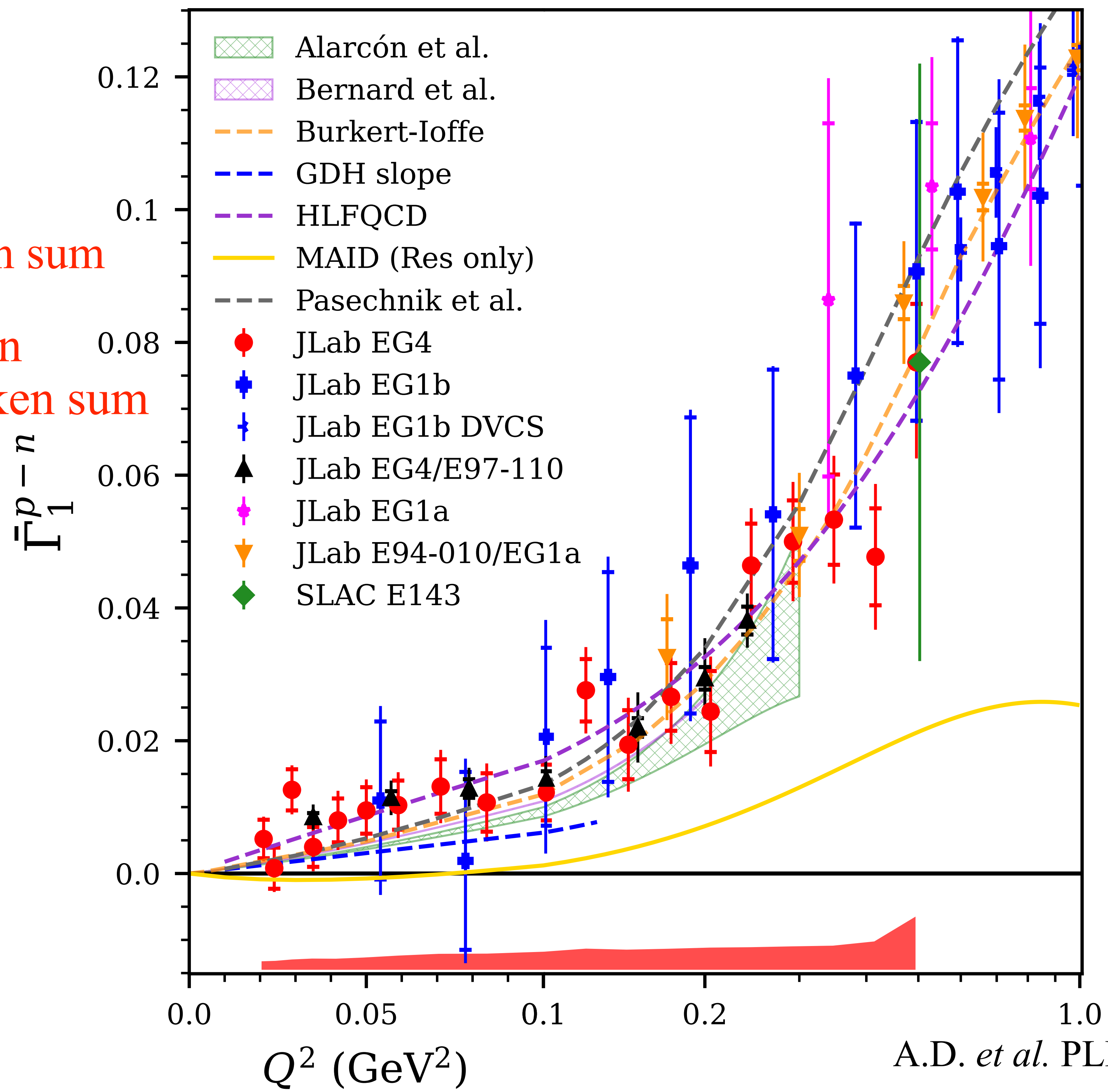
- E97-110 and EG4 agree well. They also agree with older data at larger Q^2 (EG1b, E94-010).
- E97-110 and EG4 agree with χEFT up to $Q^2 \sim 0.06$ GeV^2 (Bernard et al) or $Q^2 > 0.4$ GeV^2 (Alarcón et al.)
- Some phenomenological models (Burkert-Ioffe) agree with data, others (MAID, Pasechnik et al) not as much.

First moments: Bjorken sum $\Gamma_1^{p-n}(Q^2)$ from E97-110 and EG4

$$\Gamma_1^{p-n} = \int_0^{1^-} g_1^p - g_1^n dx$$

Proton-neutron = Bjorken sum

Δ-resonance contribution suppressed for the Bjorken sum

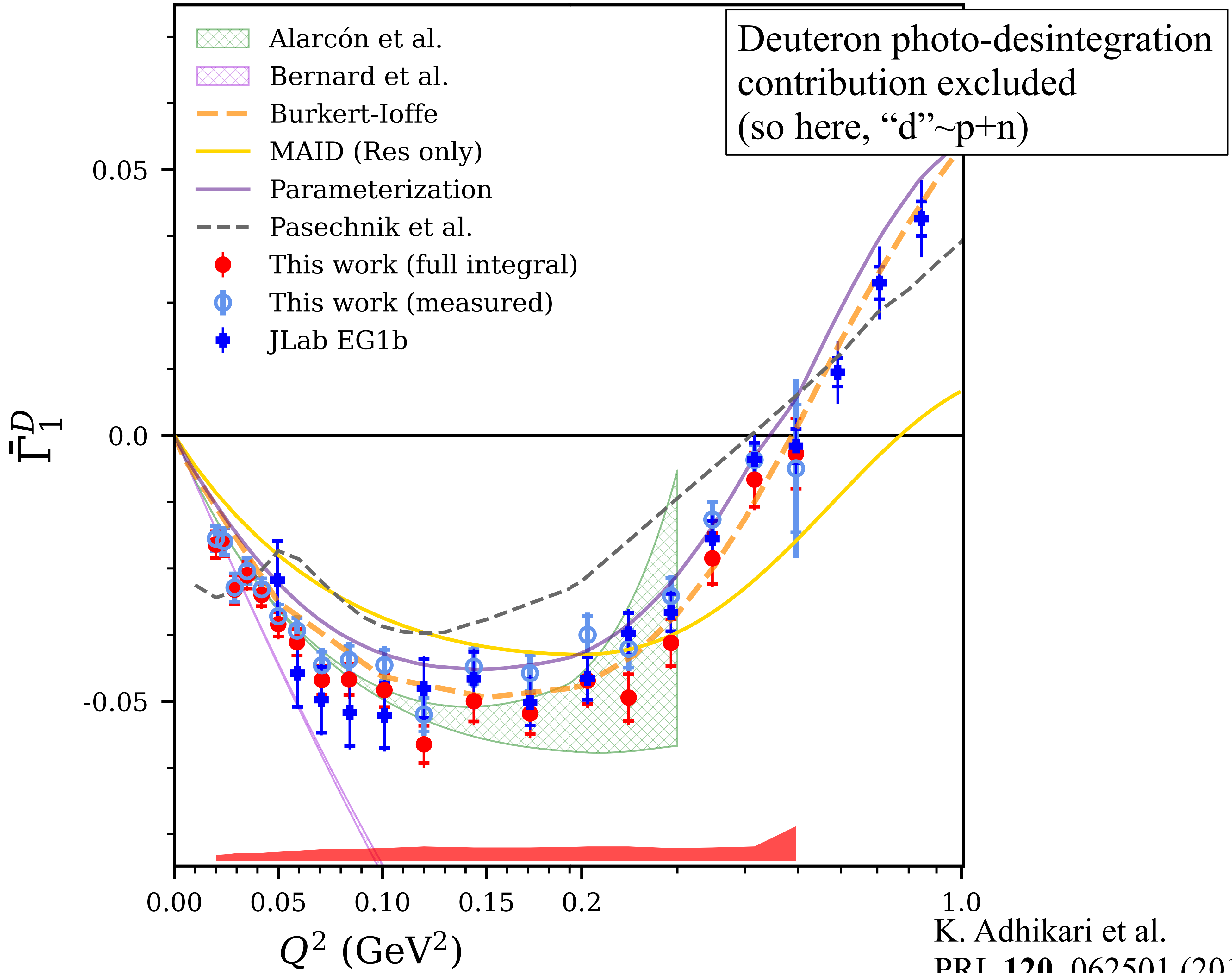


A.D. *et al.* PLB 825 136878 (2022)

E97-110 & EG4 somewhat above χEFT predictions and most phenom. models for $Q^2 < 0.1$ GeV^2.

First moments: generalized GDH sum $\Gamma_1^d(Q^2)$ measurement from EG4

$$\Gamma_1^d = \int_0^{1^-} g_1^d(x, Q^2)dx$$

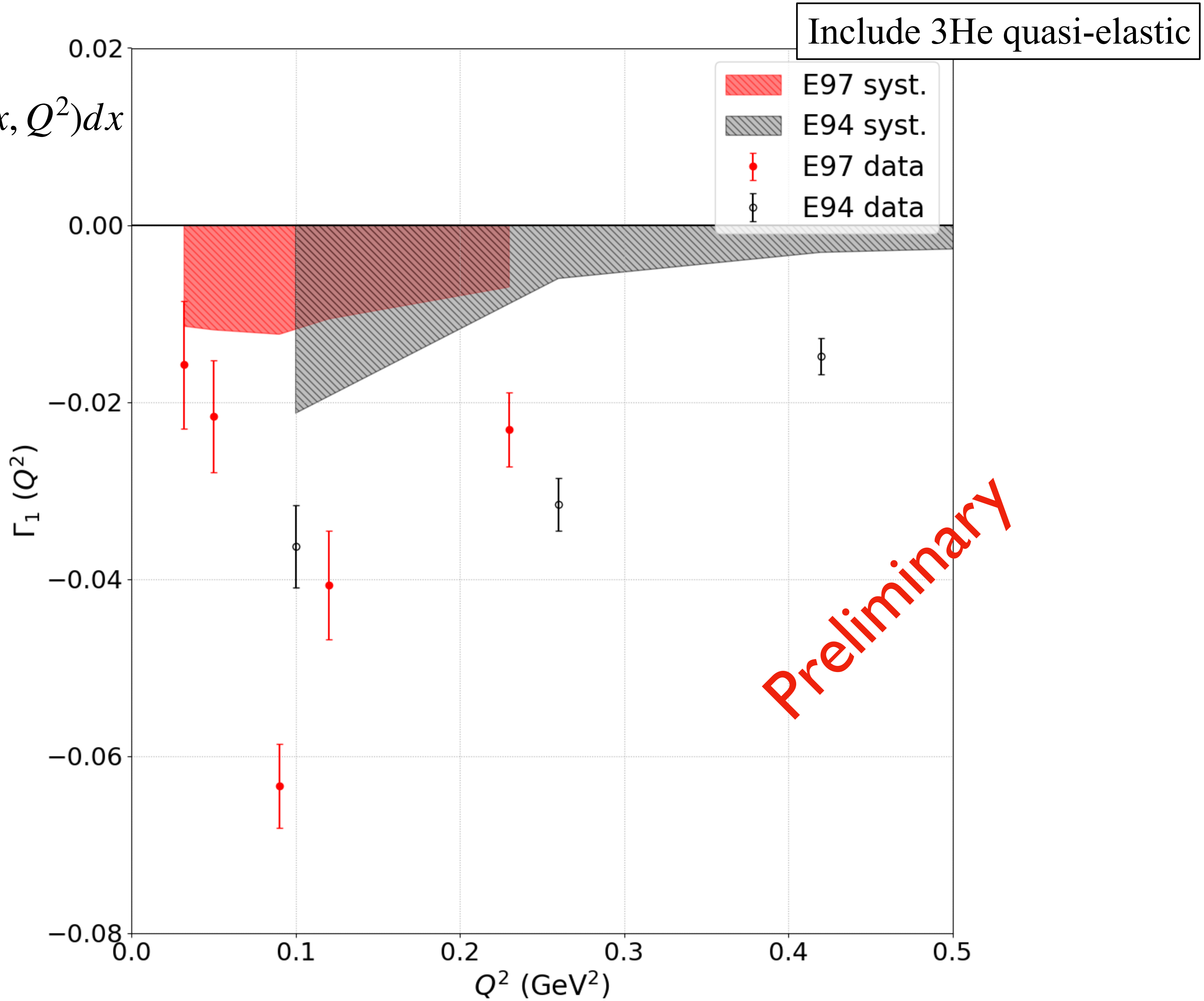


K. Adhikari et al.
PRL **120**, 062501 (2018)

- EG4 and EG1 agree well.
- EG4 and χEFT agree up to Q^2 ~0.04 GeV^2 (Bernard et al) or Q^2 >0.3 GeV^2 (Alarcón et al.)

First moments: generalized GDH sum $\Gamma_1^{3He}(Q^2)$ from E97-110

$$\Gamma_1^{3He} = \int_0^{1^-} g_1^{3He}(x, Q^2)dx$$

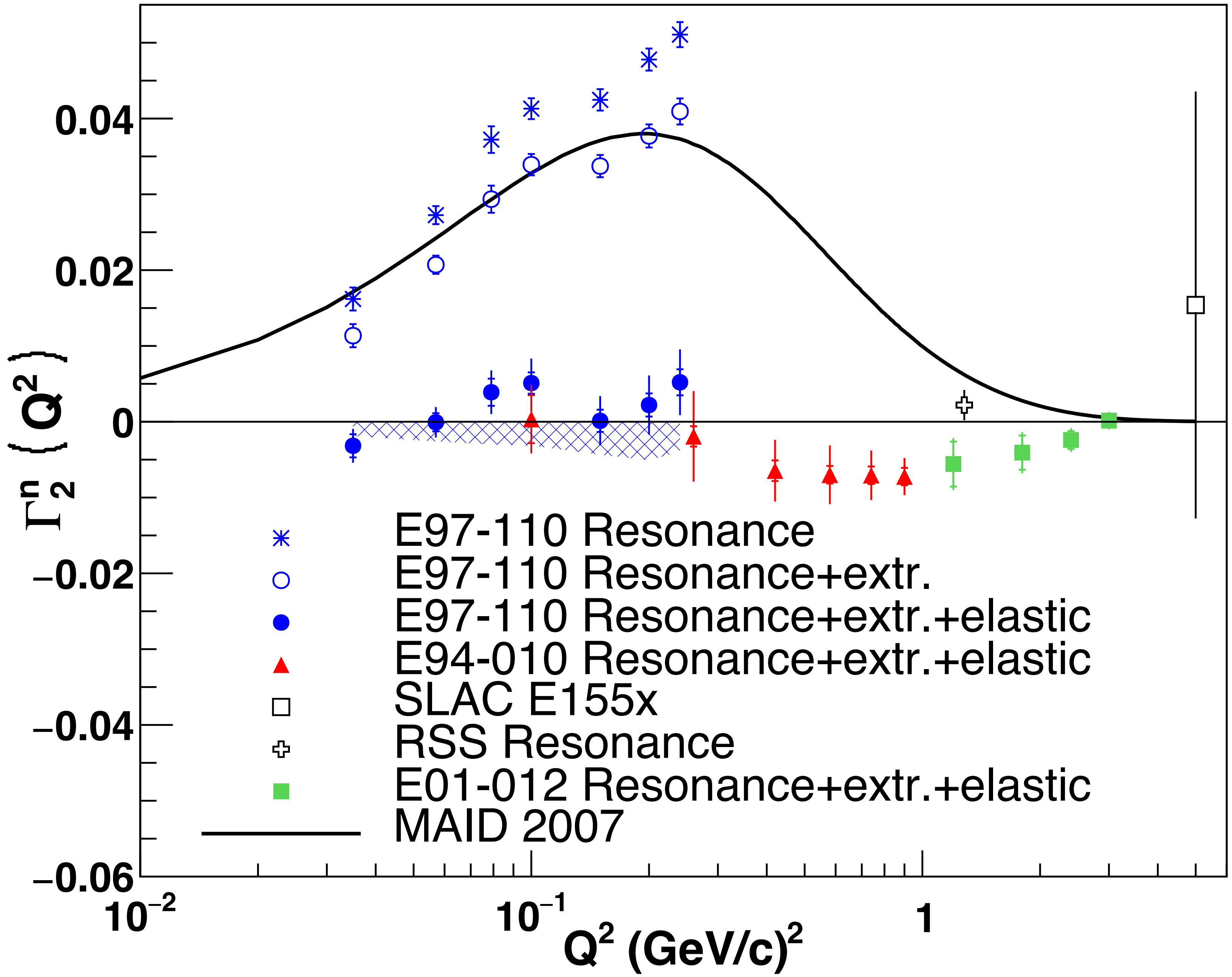


First moments: Burkhardt–Cottingham sum rule on neutron from E97-110

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2\, dx = 0$$

B-C sum rule

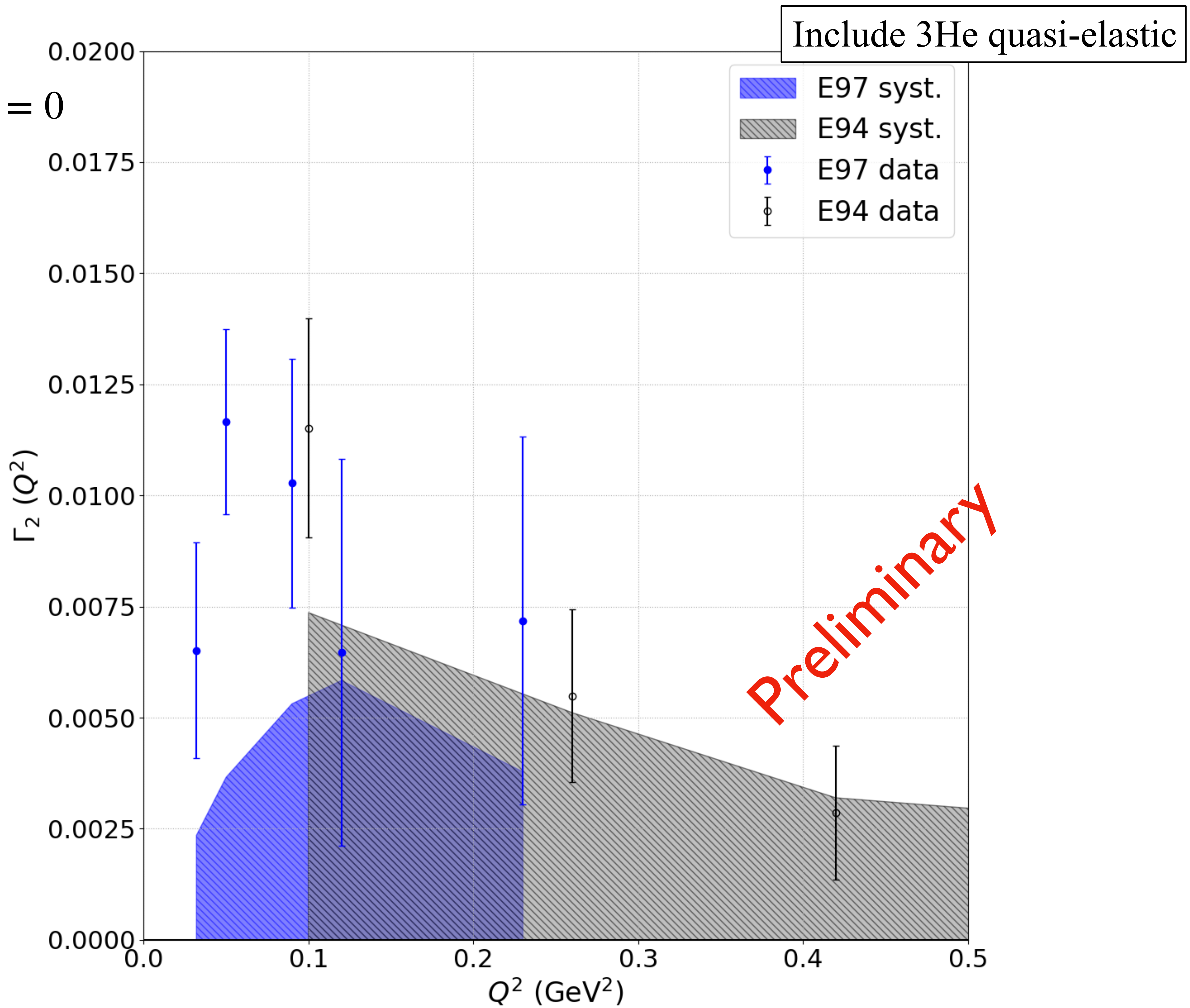
V. Sulkosky et al.
PLB 805 135428 (2020)



E97-110 verifies the B-C sum rule at low Q^2. Older experiments at higher Q^2 also verify it.

First moments: Burkhardt–Cottingham sum rule on ^{3}He from E97-110

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2 \, dx = 0$$

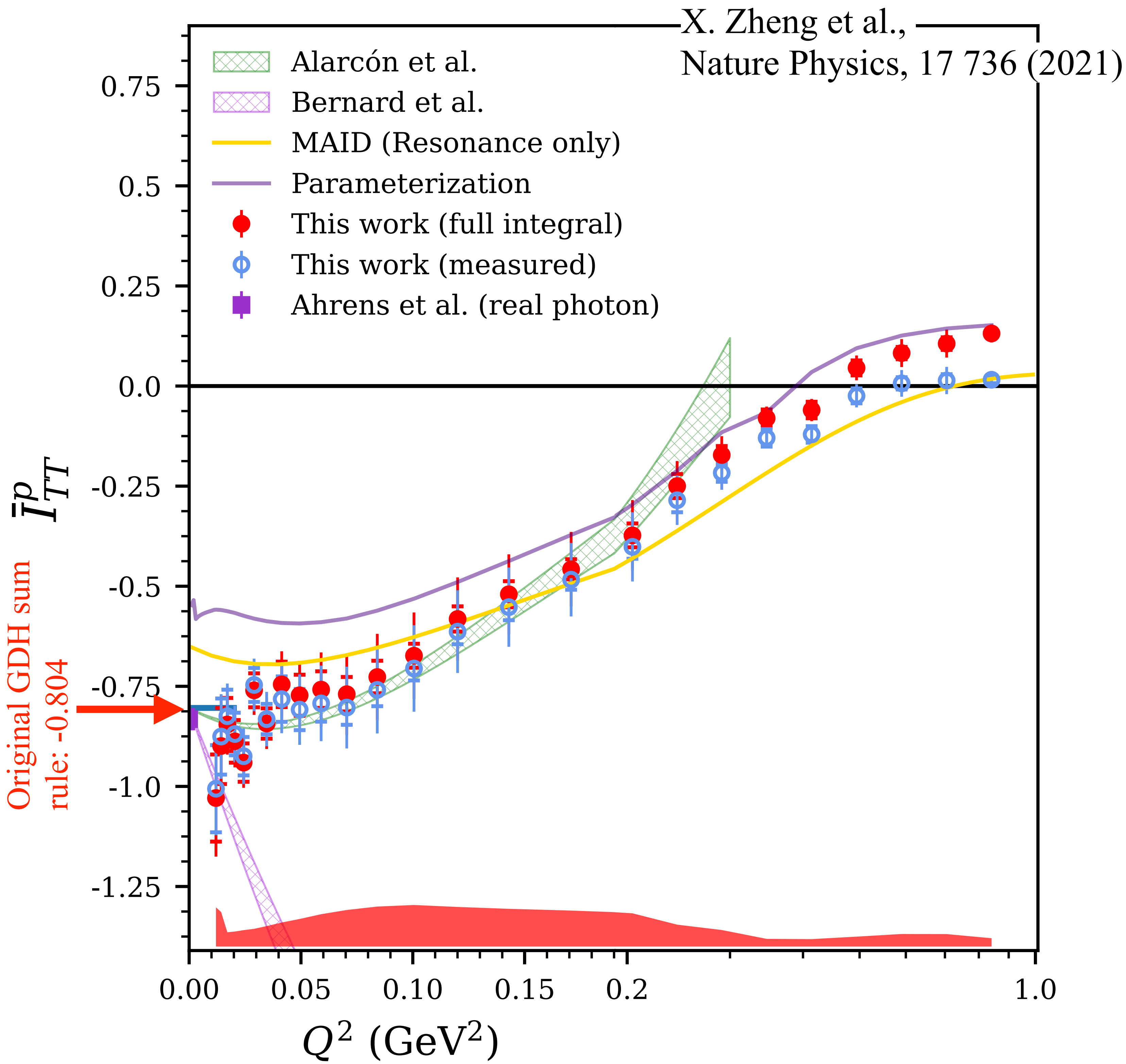


Another generalization of GDH sum: $I^p_{TT}(Q^2)$. EG4 Data

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A - \sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (not measured by EG4)



Extrapolating the (very low Q^2) data to Q^2=0 provides an independent check of the GDH SR validity, with a different method (inclusive data) than photoproduction experiments (exclusive data).

$$I^{p\ \mathrm{EG4}}_{TT}(0) = -0.798 \pm 0.042$$

Agrees with the GDH SR, with precision similar to photoproduction method: $I^{p\ \mathrm{MAMI}}_{TT}(0) = -0.832 \pm 0.023(stat)$\$ \pm 0.063(syst)$

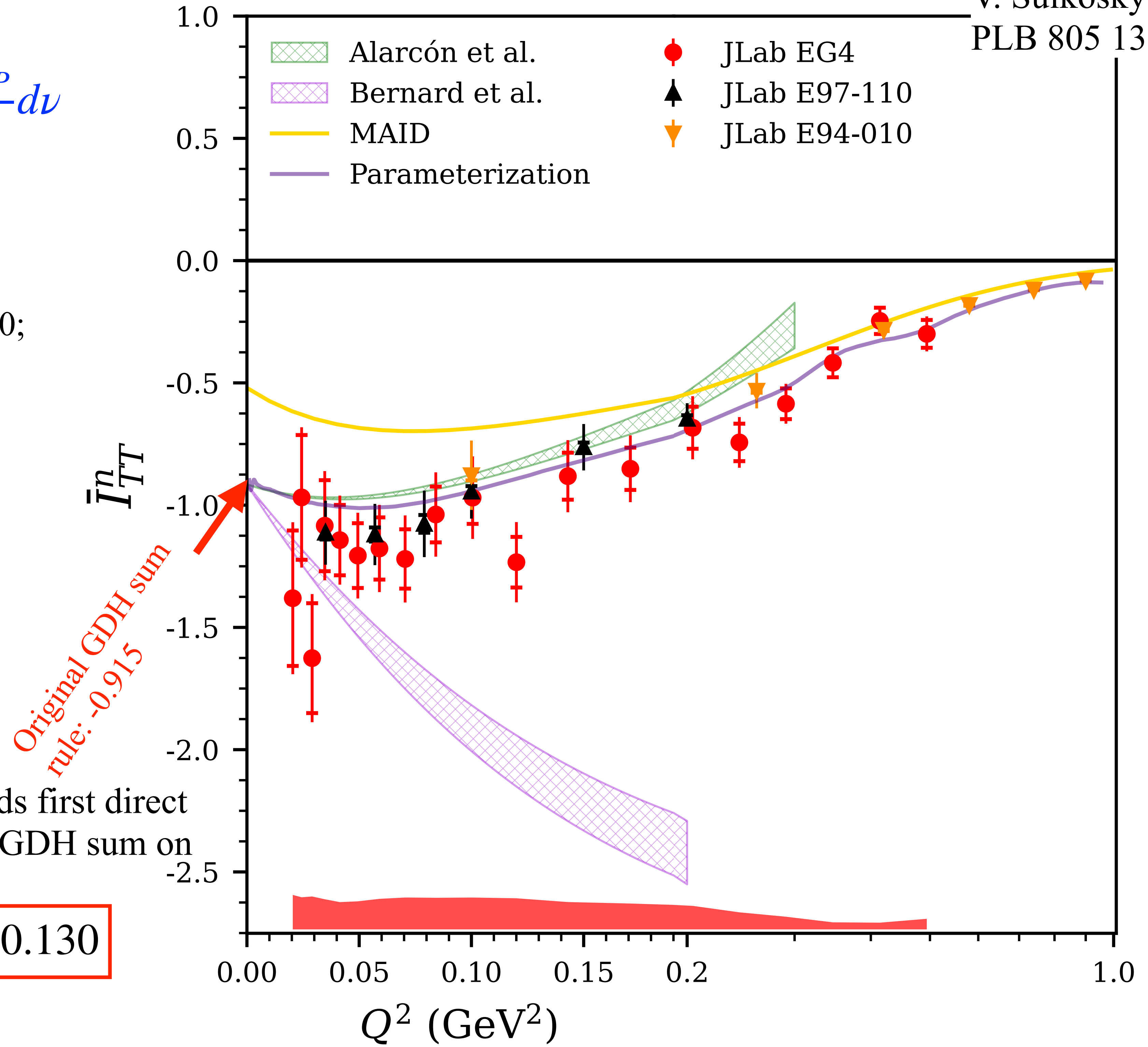
Another generalization of GDH sum: $I^n_{TT}(Q^2)$. E97-110 & EG4 Data

V. Sulkosky et al.
PLB 805 135428 (2020)

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (measured by E97-110; not measured by EG4)



Extrapolation (EG4 data only) yields first direct experimental check of the original GDH sum on the neutron.

$$I^{n\ \mathrm{EG4}}_{TT}(0) = -1.084 \pm 0.130$$

- E97-110 and EG4 agree with each other and with older data at larger Q^2.
- E97-110, EG4 and χEFT:
  - agree for lowest data point ($Q^2 \sim 0.04$ GeV^2) for Bernard *et al.*
  - disagree with Alarcón *et al*. except at the higher Q^2.
- Maid disagrees with the data.

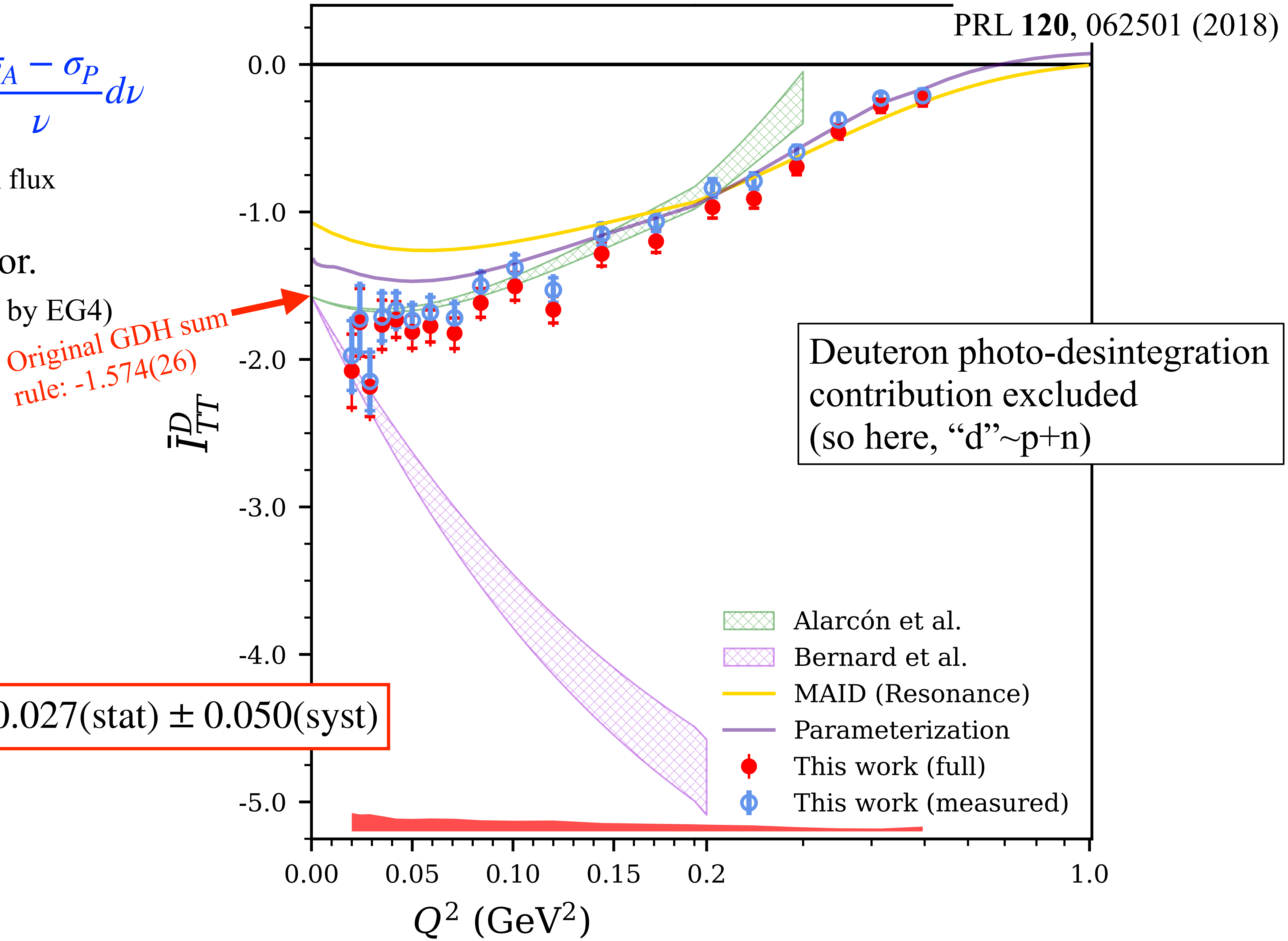
Another generalization of GDH sum: $\bar{I}^{d}_{TT}(Q^2)$. EG4 Data

K. Adhikari et al.
PRL **120**, 062501 (2018)

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (not measured by EG4)



Extrapolating EG4 yields:

$$\bar{I}^{d\ \mathrm{EG4}}_{TT}(0) = -1.724 \pm 0.027(\mathrm{stat}) \pm 0.050(\mathrm{syst})$$

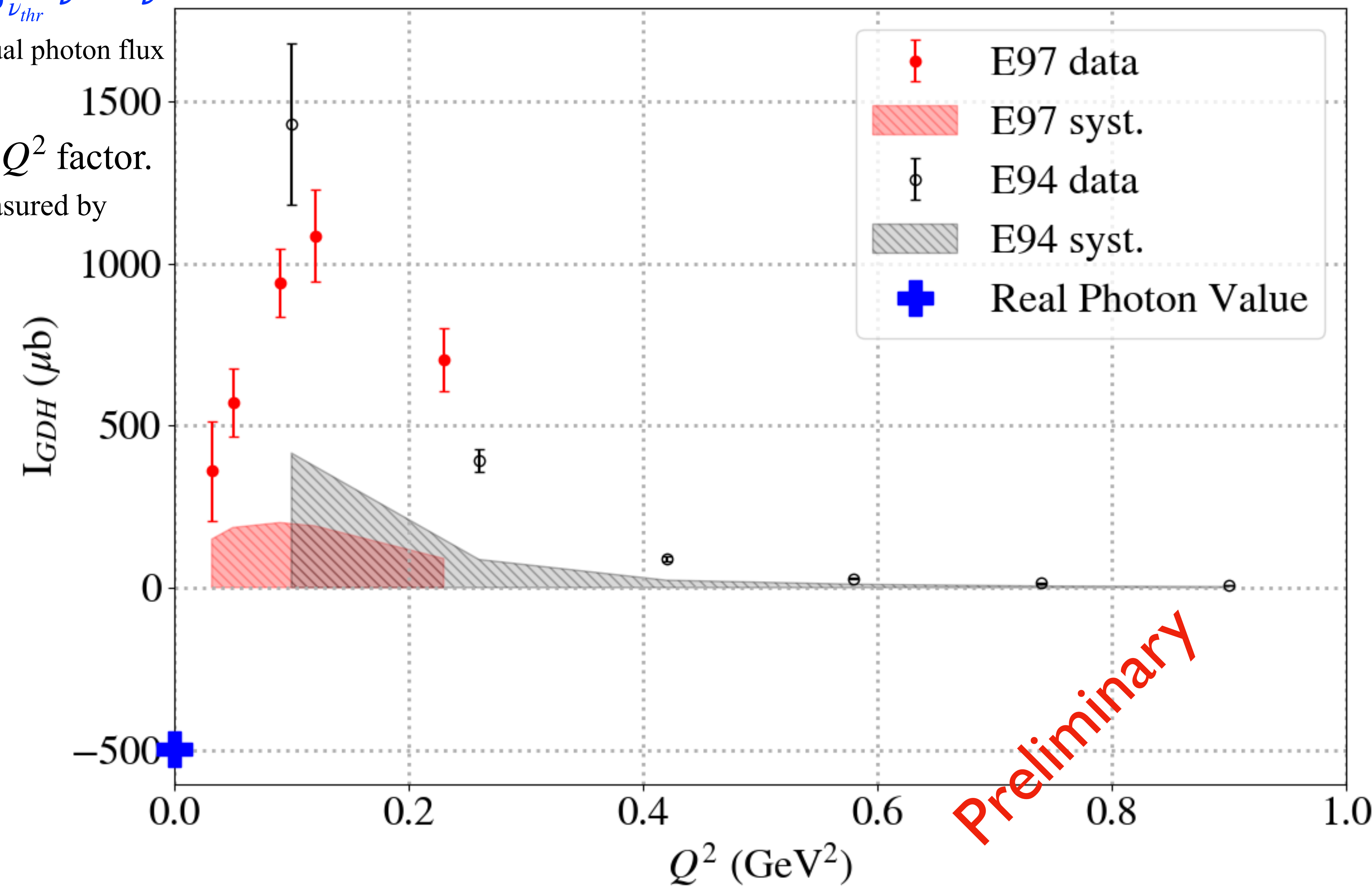
Another generalization of GDH sum: $\bar{I}_{TT}^{3He}(Q^2)$, E97-110 Data

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A - \sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (measured by E97-110)

$$I_{GDH}(Q^2) = \frac{8\pi^2\alpha}{M^2} I_{TT}(Q^2)$$

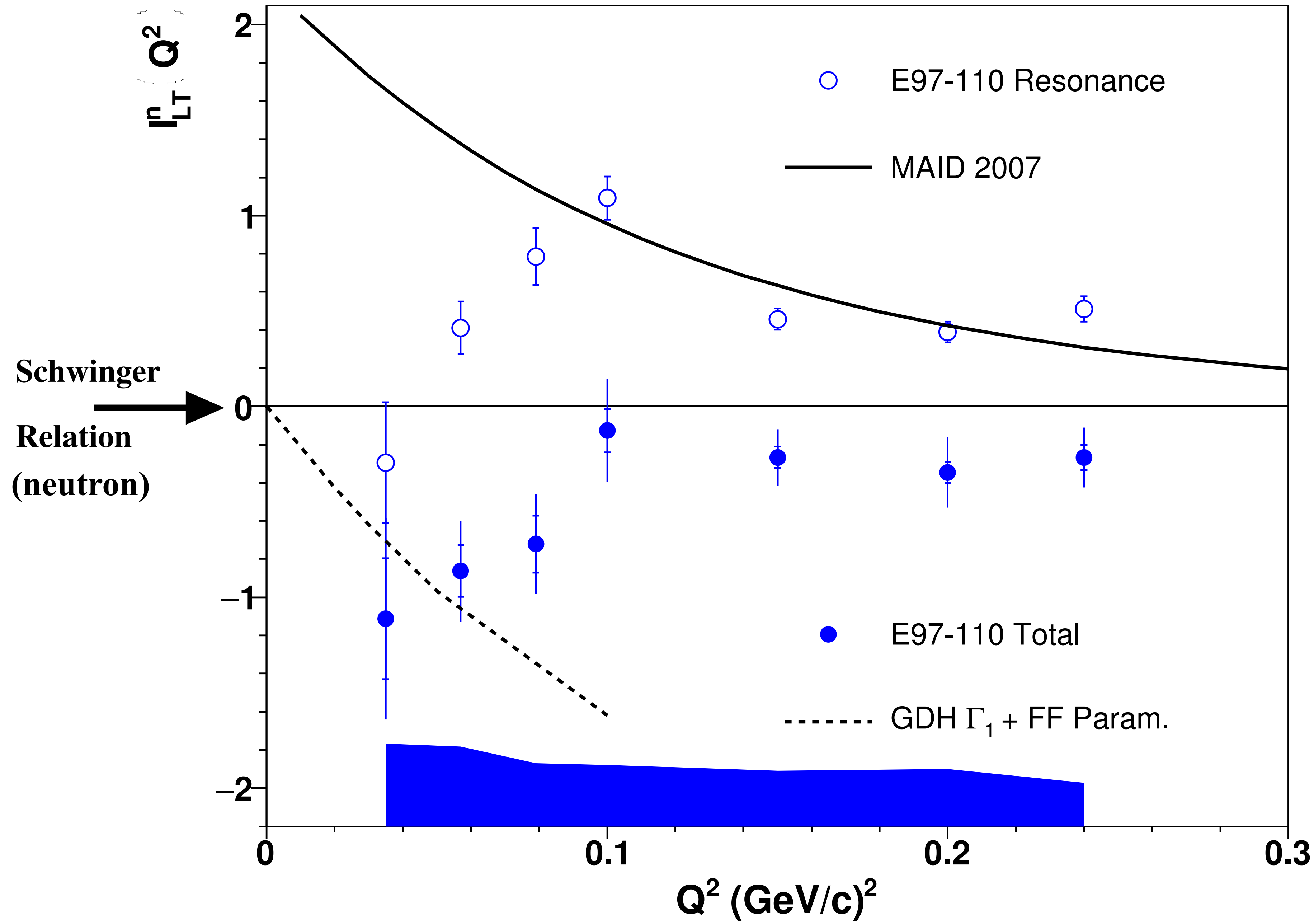


First moments: Schwinger sum rule on neutron from E97-110

$$I_{LT}(Q^2) = \frac{8M^2}{Q^2}\int_0^{1^-}(g_1+g_2)dx \xrightarrow[Q^2\to 0]{} \kappa e_t$$

anomalous magnetic moment×charge

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)

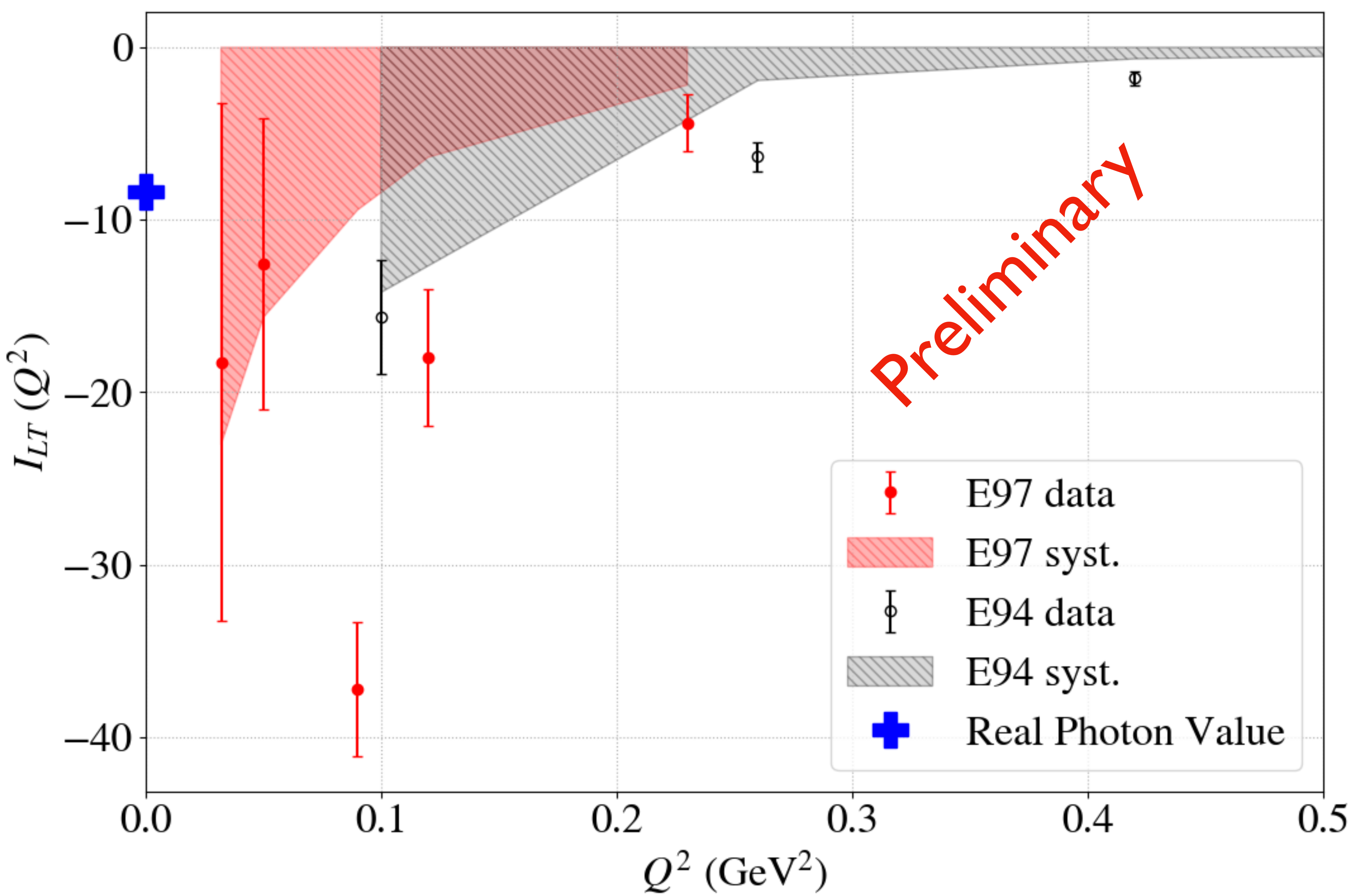


E97-110 (+GDH+BC sum rule+known neutron elastic form-factor) agrees with Schwinger sum rule.

First moments: Schwinger sum rule on ^{3}He from E97-110

$$I_{LT}(Q^2) = \frac{8M^2}{Q^2}\int_0^{1^-} (g_1 + g_2)dx \xrightarrow[Q^2\to 0]{} \kappa e_t$$

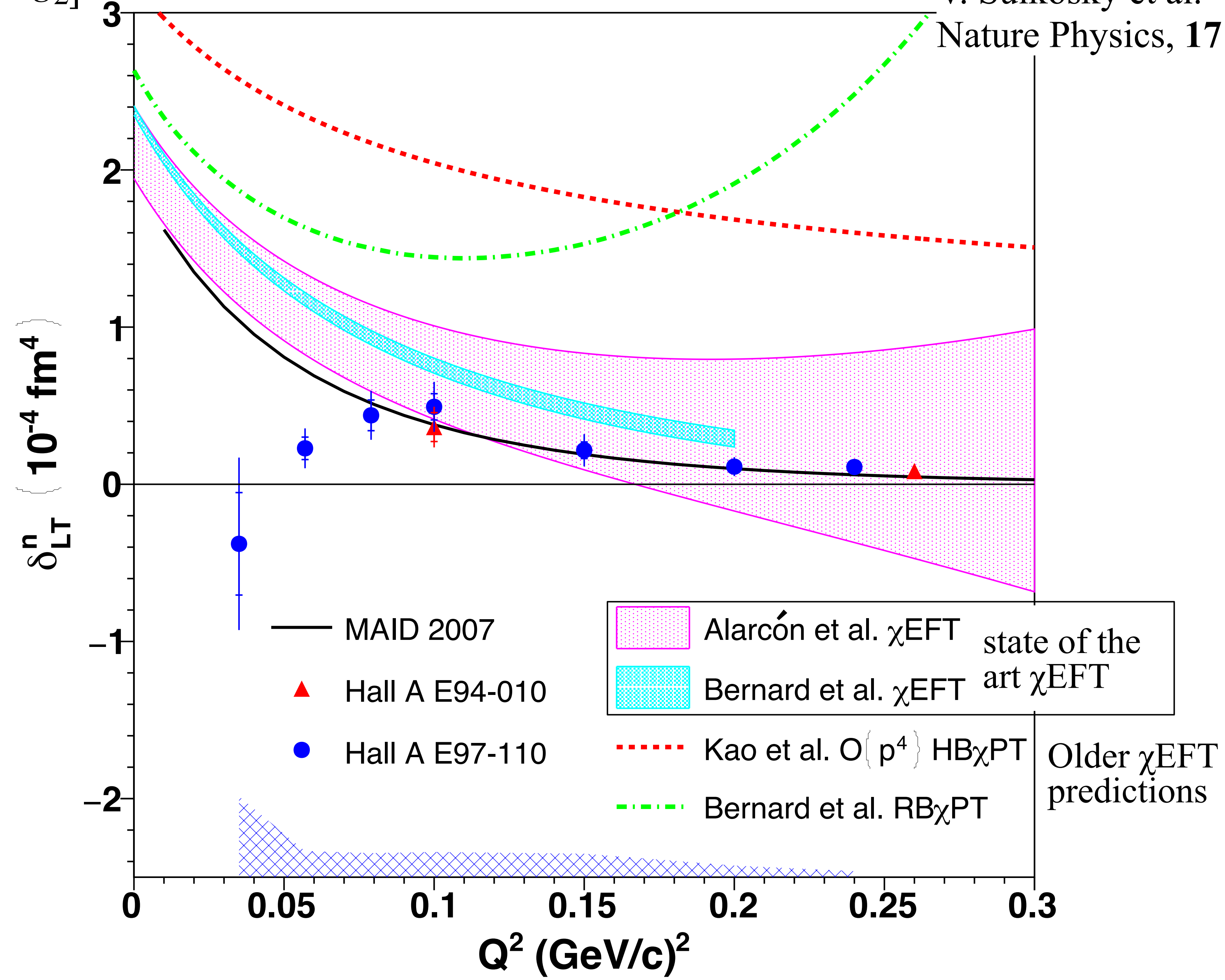
anomalous magnetic moment×charge



Higher moments: Longitudinal-transverse spin polarizability δ_{LT} from E97-110

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1+g_2\right]dx$$

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)

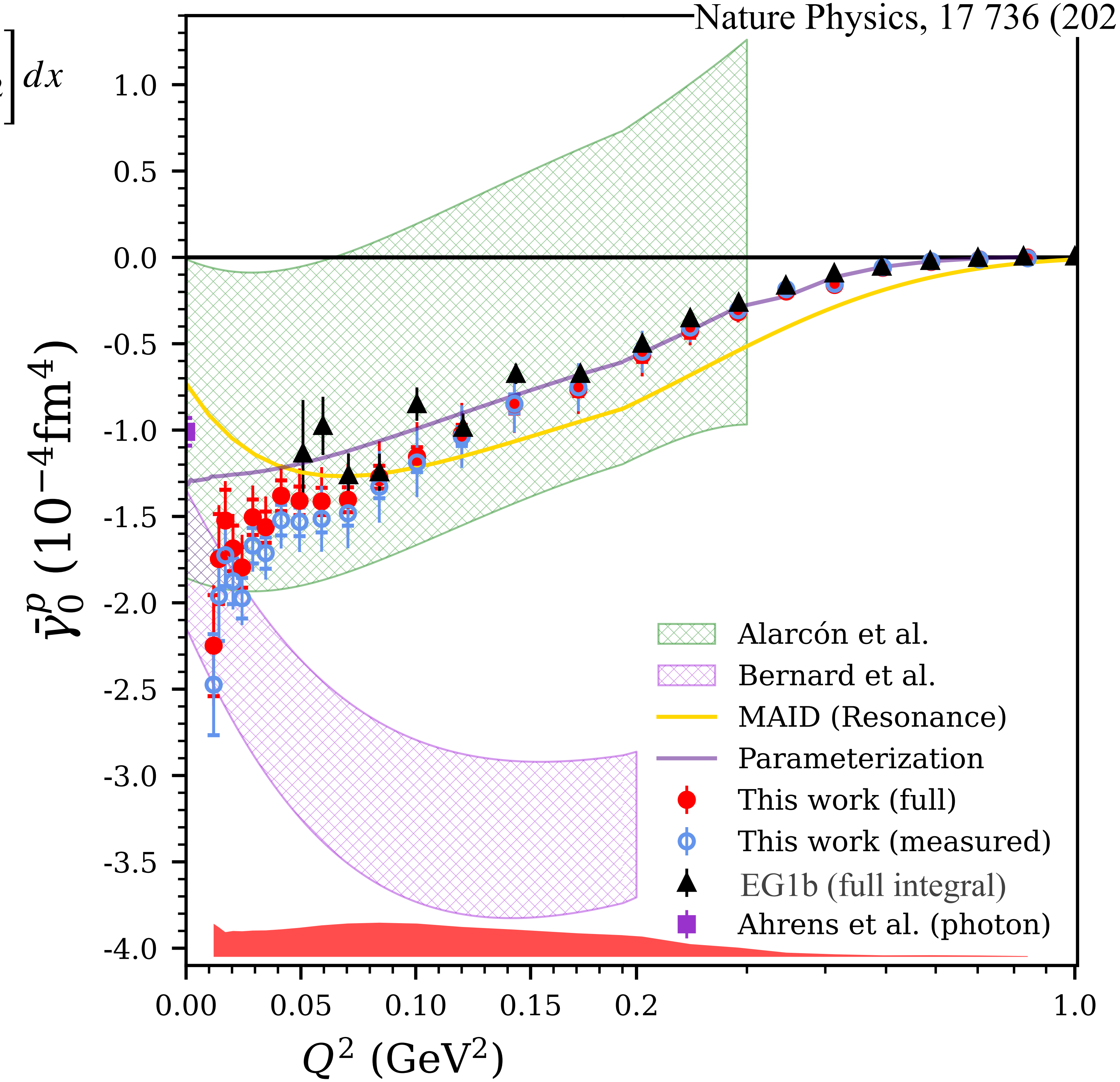


- Good agreement with older data at larger Q^2 and with χEFT & MAID there.
- Disagreement with χEFT & MAID at lower Q^2, although first moment $\int_0^{1^-} x^2[g_1+g_2]dx$ agrees with Schwinger sum rule.
- ⇒"$\delta^n_{LT}(Q^2)$ puzzle" still remains.

Higher moments: Generalized forward spin polarizability $\gamma_0^p(Q^2)$ from EG4

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]dx$$

X. Zheng et al.,
Nature Physics, 17 736 (2021)



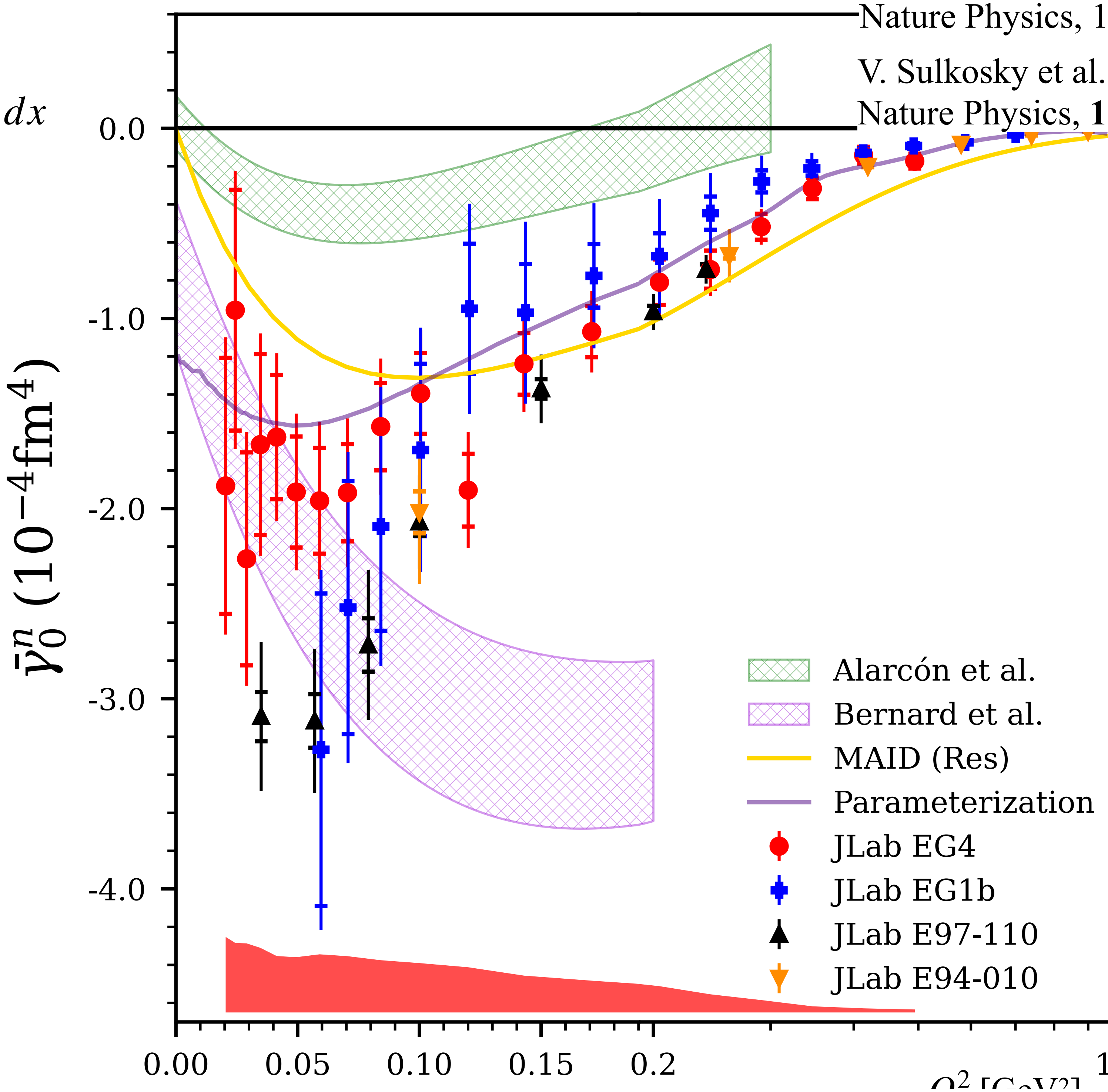
- χEFT result of Alarcón et al agrees with data.
- Bernard et al. χPT calculation agrees for lowest Q^2 points. Large slope at low Q^2 supported by the MAMI+EG4 data
- Maid disagrees with the data.

Higher moments: Generalized forward spin polarizability γ_0^n from E97-110 and EG4

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]dx$$

X. Zheng et al., Nature Physics, 17 736 (2021)

V. Sulkosky et al. Nature Physics, **17** 687 (2021)

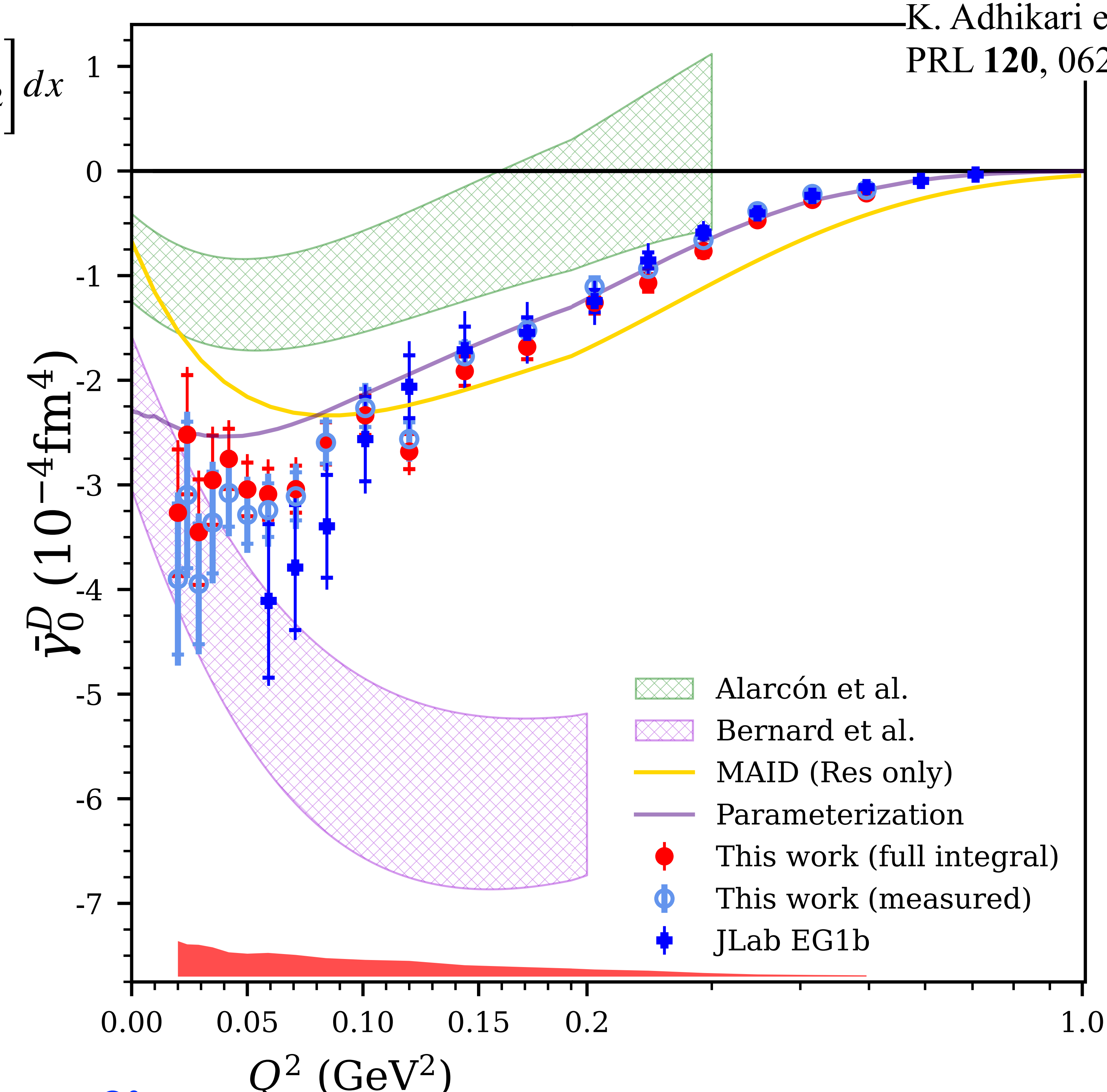


- E97-110 and EG4 agree with older data at larger Q² (EG1b, E94-010).
- Marginal agreement between EG4 and E97-110 in the lower Q² range. (Better agreement if the EG4 systematic errors are added linearly rather than in quadratures)
- χEFT result of Alarcón et al disagrees with data. Bernard et al. agrees for lowest Q^2 points.
- Maid disagrees with the data.

Higher moments: Generalized forward spin polarizability $\gamma_0^d(Q^2)$ from EG4

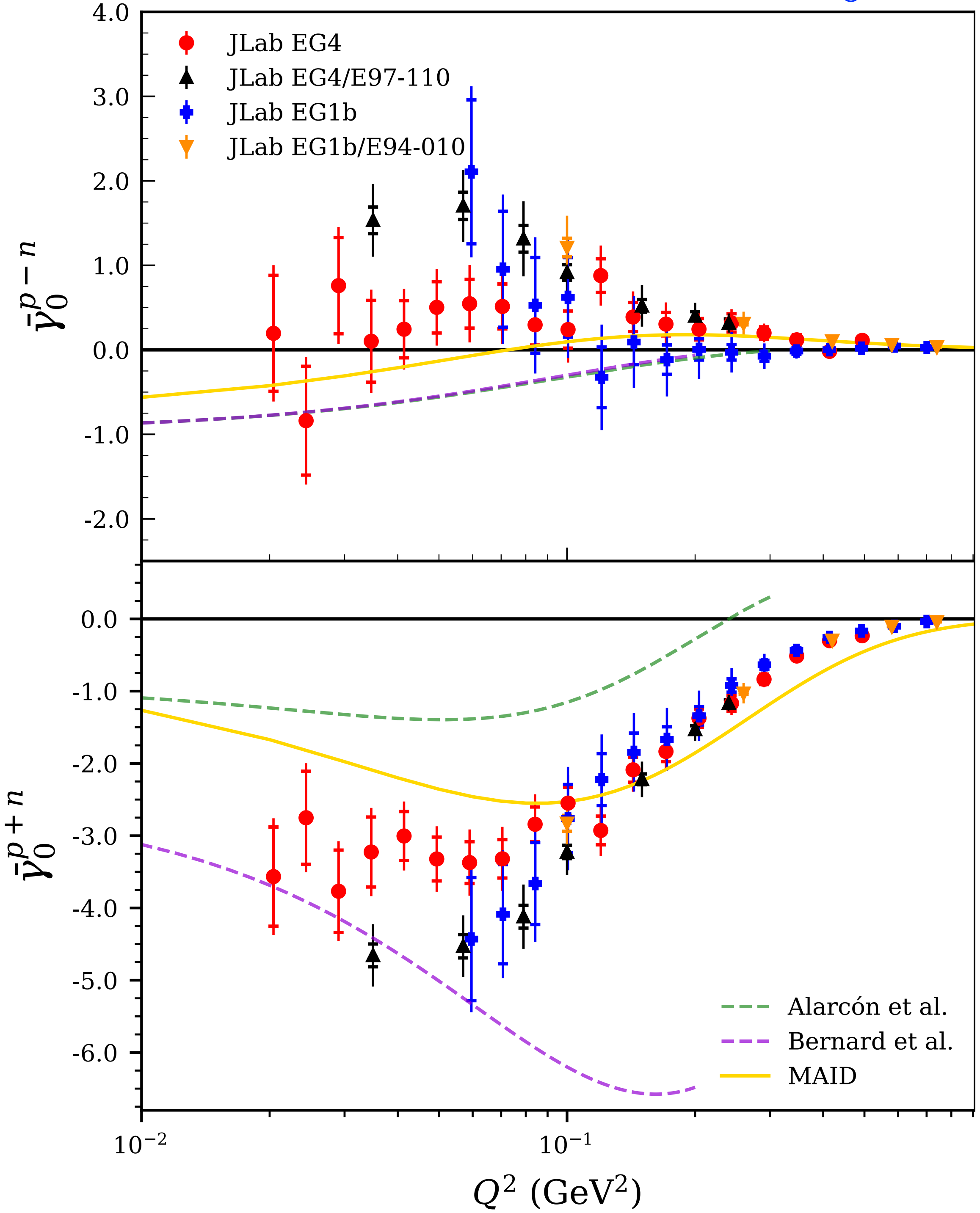
$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{1^-} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]dx$$

K. Adhikari et al.
PRL **120**, 062501 (2018)



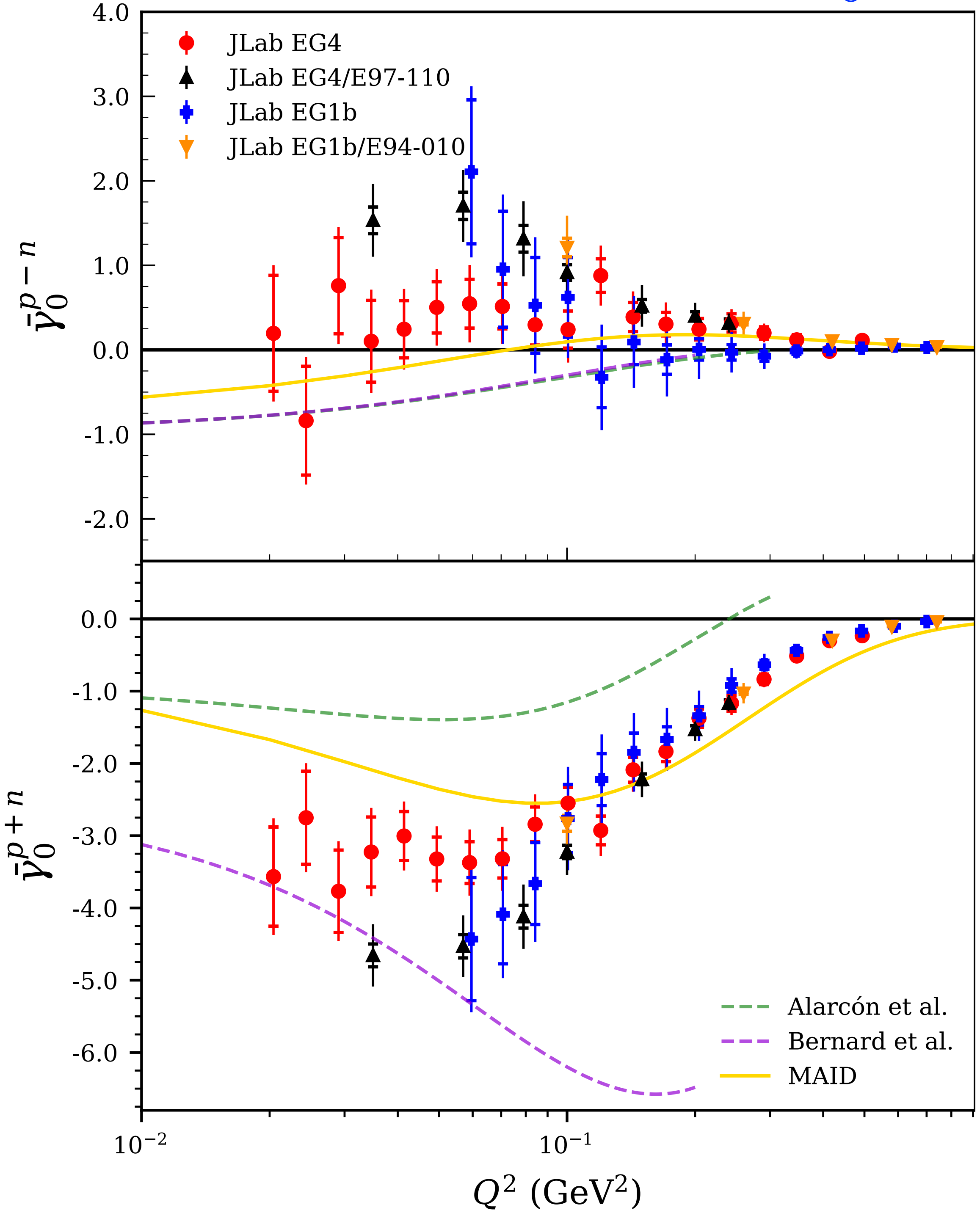
- EG4 agree with older EG1b data at larger Q^2.
- χEFT result of Alarcón et al disagrees with data. Bernard et al. calculation agrees for lowest Q^2 points.
- Maid disagrees with the data.

Isospin decomposition of $\gamma_0(Q^2)$



- Agreement with older (larger Q^2) experiment, EG1b, E94010.
- Tension between EG4 (p from H and D, n from D) and EG4/E97110 (p from H and n from ^{3}He).
- χEFT result of Alarcón et al disagrees with data.
- Bernard et al. χEFT calculation agrees for γ_0^{p+n} and for γ_0^{p-n} for lowest Q^2 points.
- Both new and old data (from 5 different experiments) indicate that γ_0^{p-n} is positive.

Isospin decomposition of $\gamma_0(Q^2)$



- Agreement with older (larger Q^2) experiment, EG1b, E94010.
- Tension between EG4 (p from H and D, n from D) and EG4/E97110 (p from H and n from ^{3}He).

Tension may come from adding systematic uncertainties from E03006, E05111 or E97110 quadratically. If we combine linearly the total systematic uncertainties of each experiments, there is not tension.

- Both new and old data (from 5 different experiments) indicate that γ_0^{p-n} is positive.

JLab's first generation of χEFT tests/polarizability measurements at low Q^2

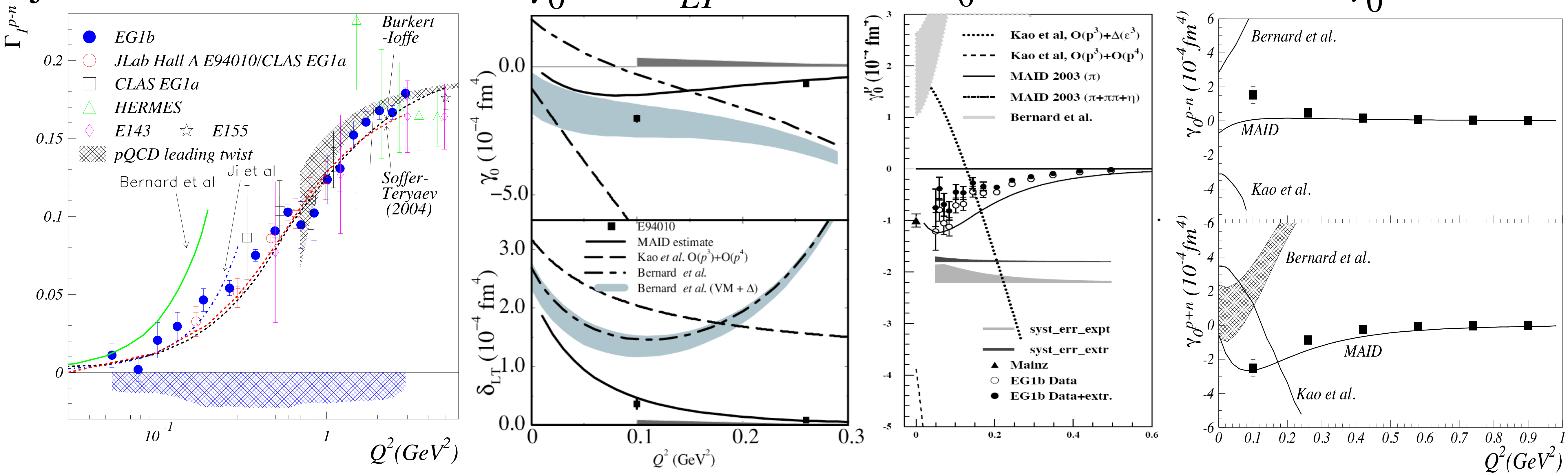
Results from JLab 1990's experiments (Hall A E94010, CLAS EG1a,b):

A: ~agree
X: ~disagree
- : No prediction available

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X

Ex:

Bjorken sum rule E94-010/EG1b — γ_0^n and δ_{LT}^n E94-010 — γ_0^p EG1b — $\gamma_0^{p\pm n}$



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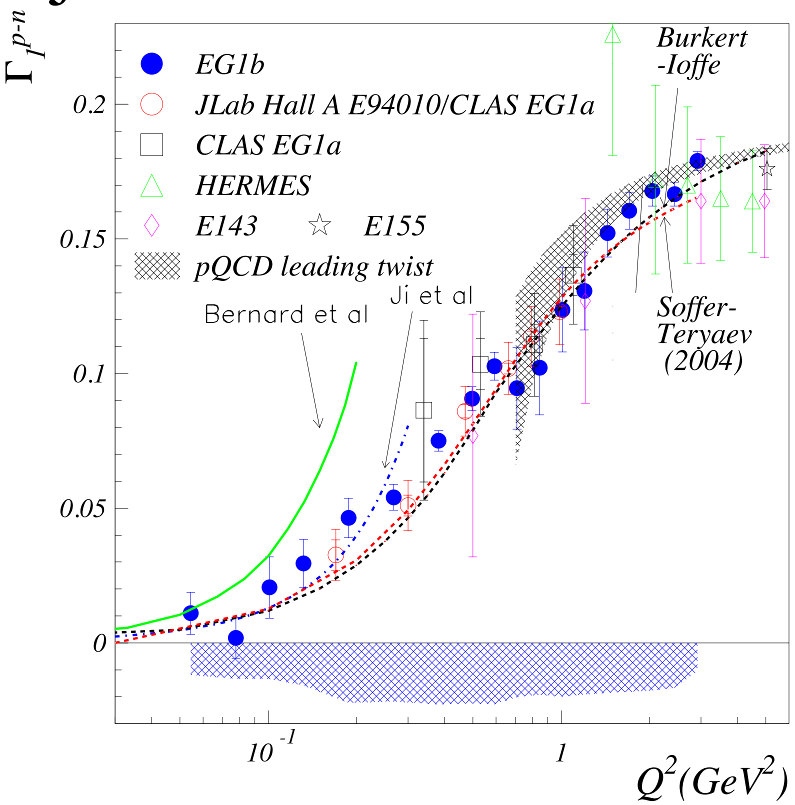
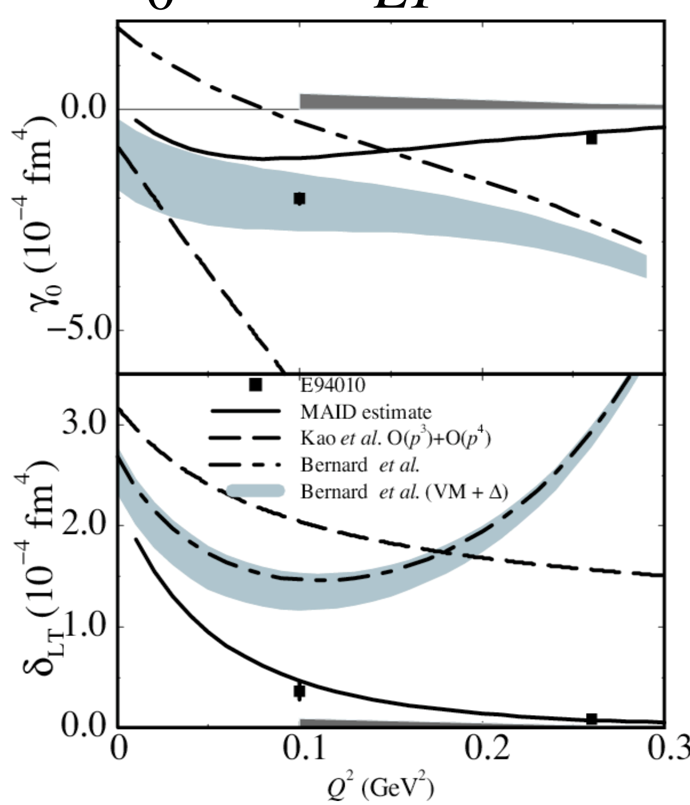
Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X

Ex:

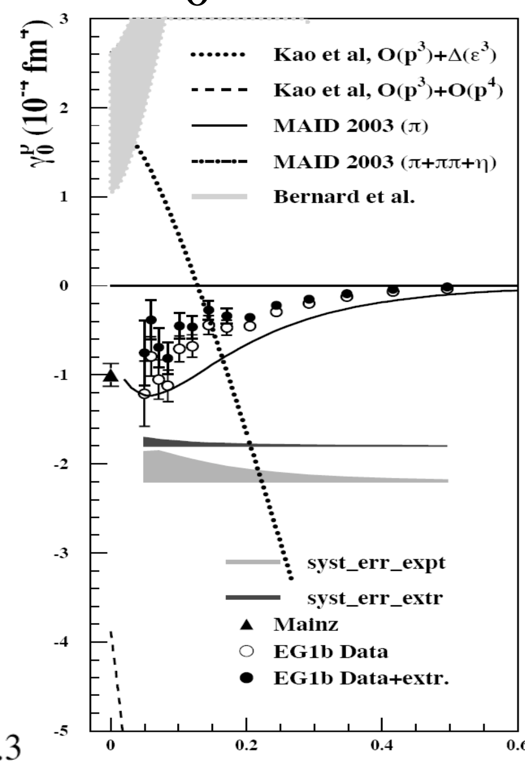
Bjorken sum rule E94-010/EG1a



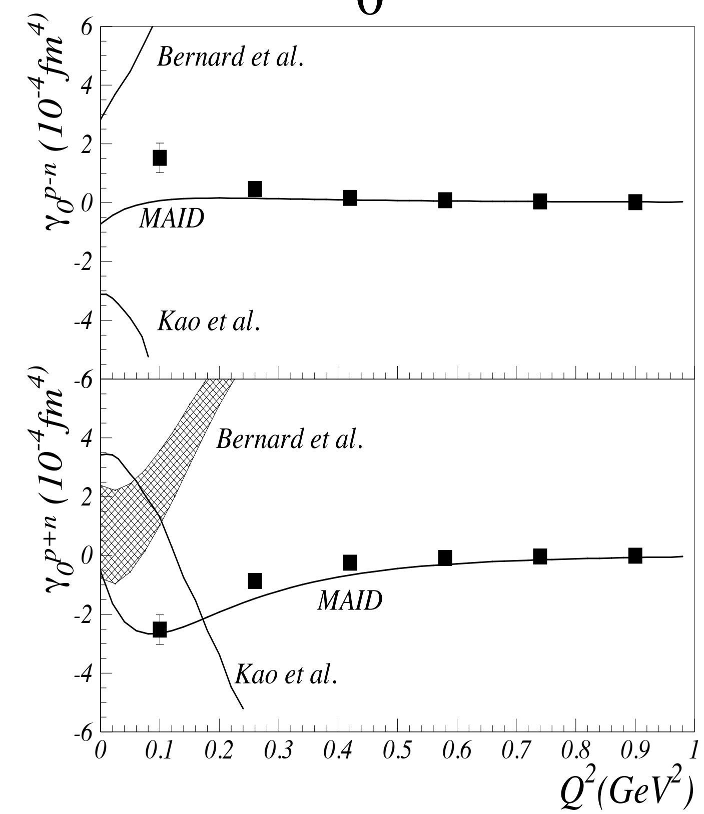
γ_0^n and δ_{LT}^n E94-010



γ_0^p EG1b



$\gamma_0^{p\pm n}$



1990s-2000s χEFT predictions in tension with spin observable data more often than not.

JLab Hall A experiment E97-110

V. Sulkosky et al.
Nature Physics, **17** 687 (2021);
Phys.Lett.B 805 135428 (2020)

Main goal: measurement of the generalized GDH sum for the neutron at very low Q^2.

Spokespeople: **J.P. Chen,** A.D., F. Garibaldi.

Students: C. Peng (Duke U), V. Laine (Clermont U), J. Singh (UVa), V. Sulkosky (W&M), N. Ton (UVa), J. Yuan (Rutgers U).

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Motivations for E97-110:

* Provide very low Q^2 nucleon spin data to test χEFT,
* Test original GDH sum rule with inclusive data.
* Observables of interest: spin sum rules, generalized spin polarizabilities.

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* Provide very low Q^2 nucleon spin data to test χEFT,
* Test original GDH sum rule with inclusive data.
* Observables of interest: spin sum rules, generalized spin polarizabilities.

E97-110 aimed at precision measurement of neutron spin structure (polarized ^3He target).
E97-110 in Hall A: high resolution, small solid angle detectors. (EG4: Hall B, lower resolution, large solid angle)

^3He target has transverse polarization capability:

* No need to model $g_2(x, Q^2)$ for $\Gamma_1(Q^2)$, $I_{TT}(Q^2)$ and $\gamma_0(Q^2)$,
* $g_2(x, Q^2)$ data and associated sum rules,
* $\delta^n_{LT}(Q^2)$ data.

The EG4 experiment Group

Main goal: generalized GDH sum for the proton, neutron & deuteron at very low Q^2.

E03-006 (NH_3):

Spokespeople: **M. Ripani**, M. Battaglieri, A.D., R. de Vita

Students: H. Kang (Seoul U.), K. Kovacs (UVa)

X. Zheng et al., Nature Physics, 17 736 (2021)

E05-111 (ND_3)

Spokespeople: **A.D.**, G. Dodge, M. Ripani, K. Slifer

Students: K. Adhikari (ODU)

K.P. Adhikari *et al.* (CLAS Collaboration), PRL 120, 062501 (2018)

Focus on inclusive analyses, but exclusive analysis ($\overrightarrow{e}\,\overrightarrow{p} \rightarrow e\pi^+(n)$) also available.

X. Zheng et al., PRC 94, 045206 (2016)