Longitudinal spin transfer of semi-inclusive Λ production in deep inelastic scattering

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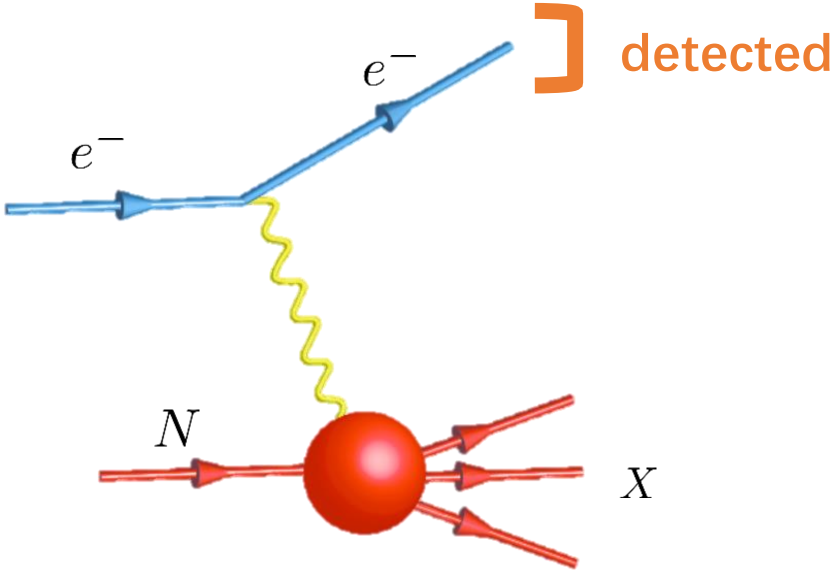
How to “see” the nucleon structure ?

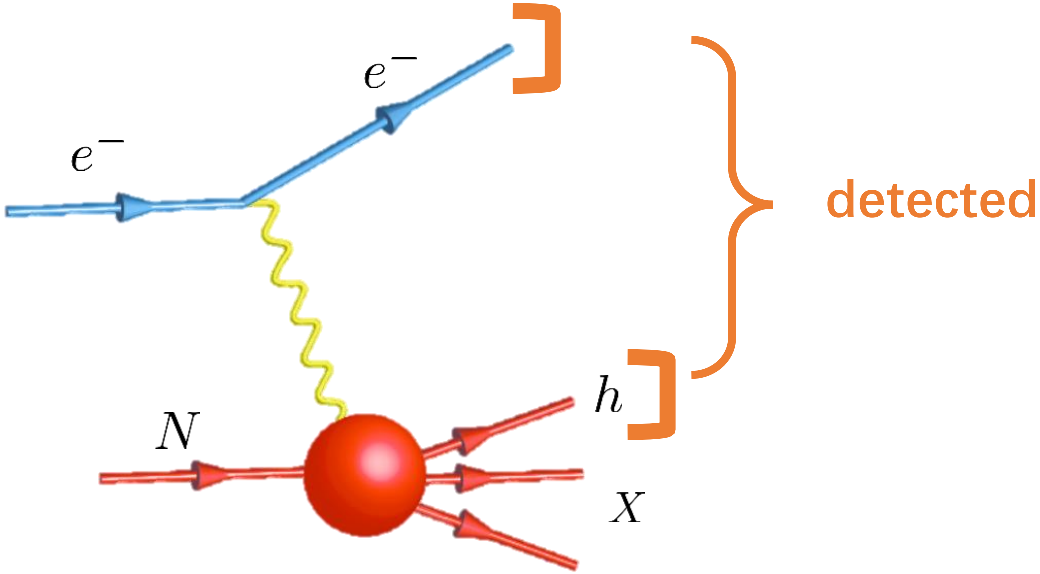
- Deep inelastic scattering (DIS)

$$e^- + N \to e^- + X$$



- Semi-inclusive deep inelastic scattering (SIDIS)

$$e^- + N \to e^- + h + X$$



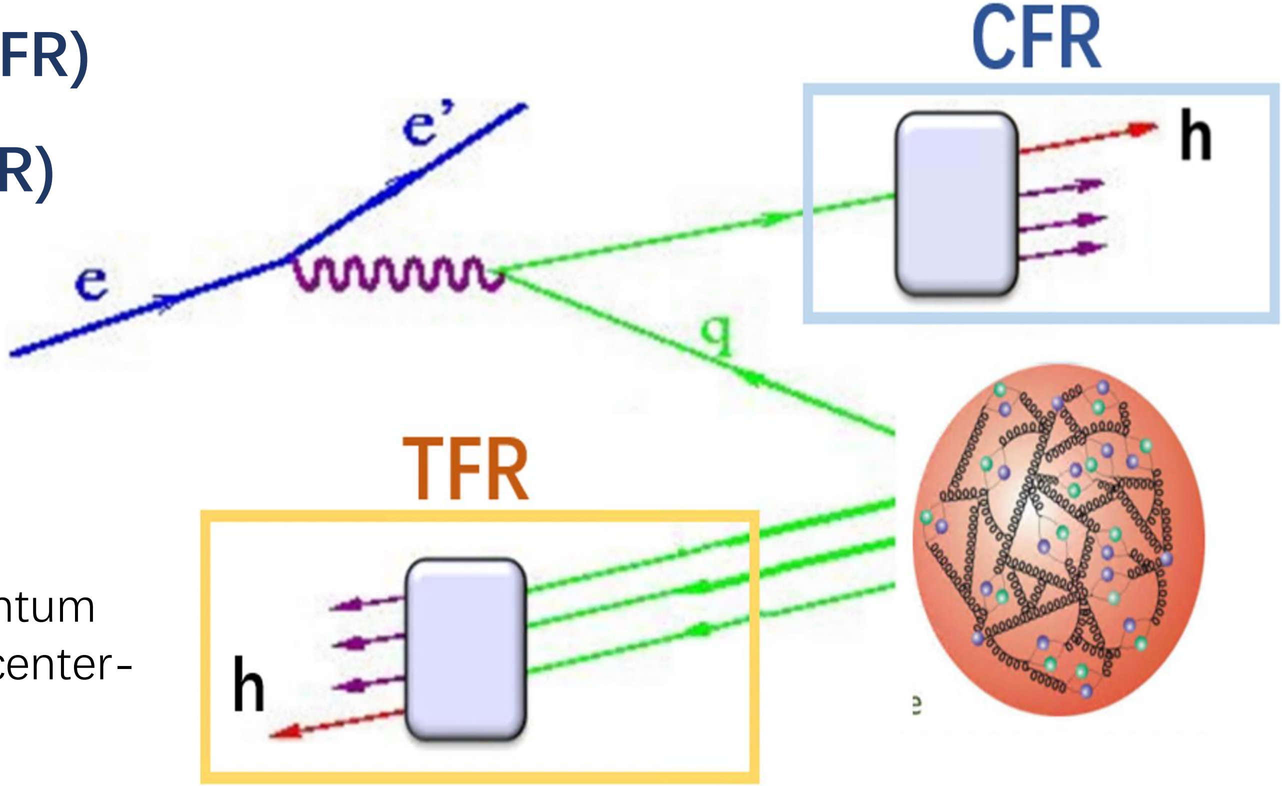
- **The structure of Λ particle & production process** $e^- P \to e^- \Lambda X$

$$\Lambda: uds;\ \ s = \frac{1}{2};\quad \Lambda \to p\pi, (BR = (64.1 \pm 0.5)\%);\quad M = 1.116 GeV$$

- Current Fragmentation Region (CFR)
- Target Fragmentation Region (TFR)

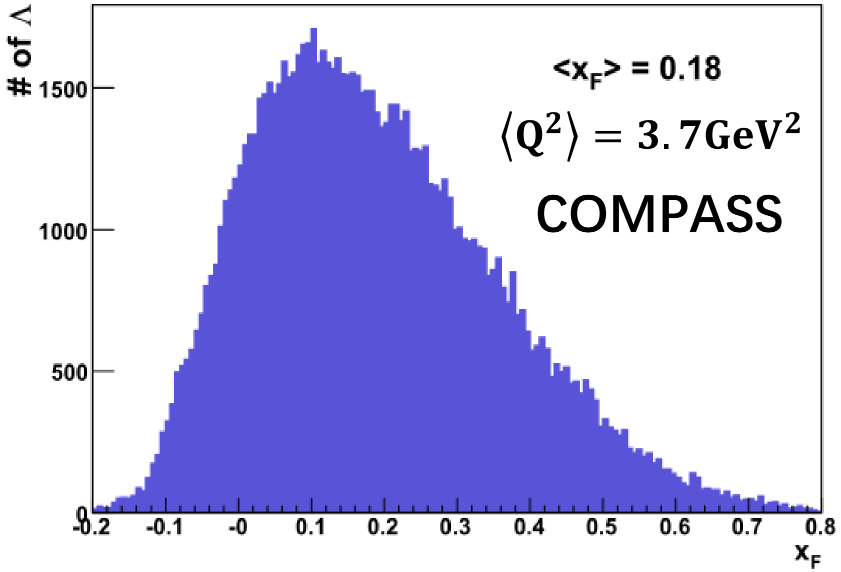
$$x_F = \frac{2P_{hL}}{W}$$

P_{hL} is the projection of the final-state hadron momentum onto the direction of the γ^*-momentum in the $\gamma^* N$ center-of-mass frame, W is invariant mass.

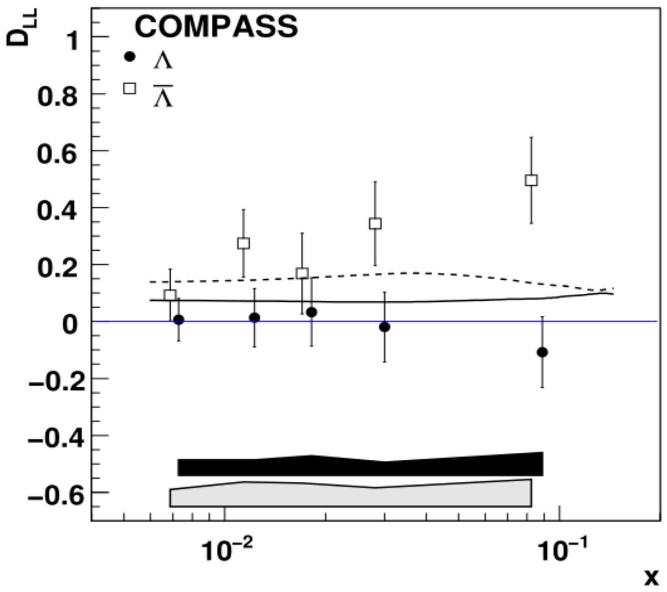


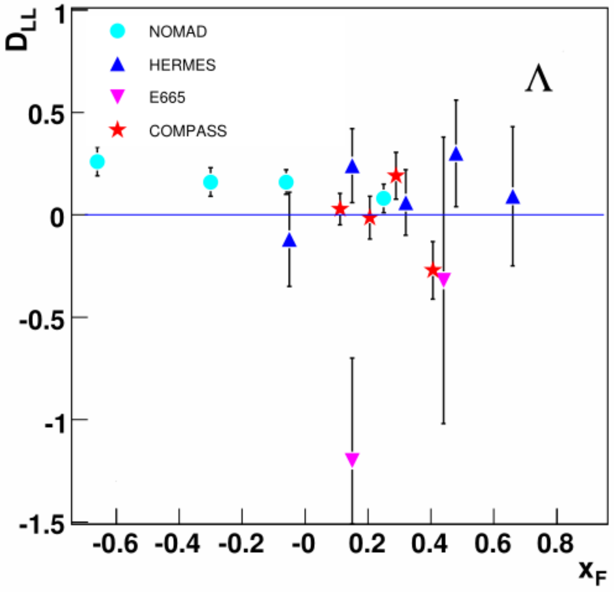
Introduction

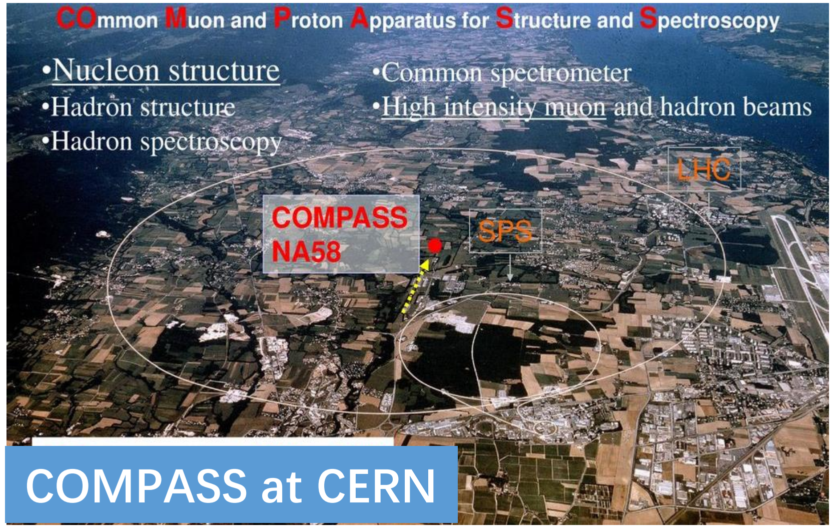
Distribution of Λ in SIDIS:

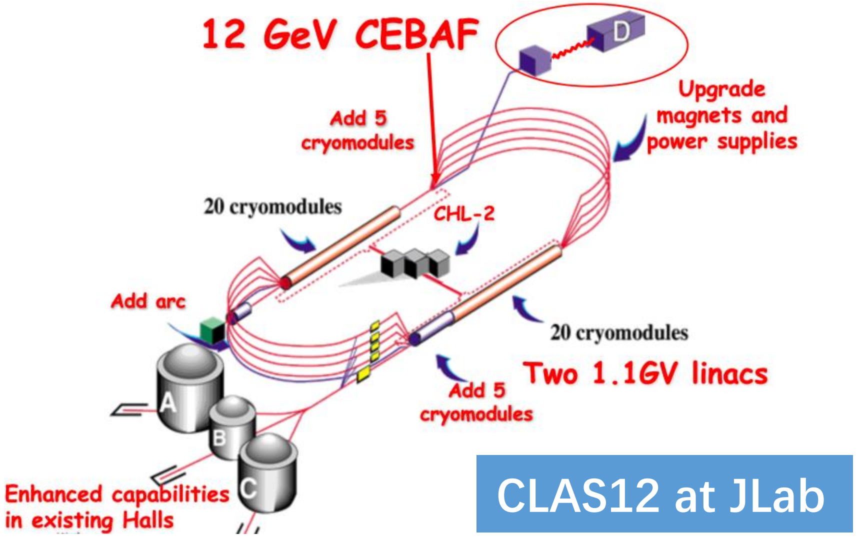


Λ longitudinal spin transfer: *Eur.Phys.J.C 64 (2009) 171-179*







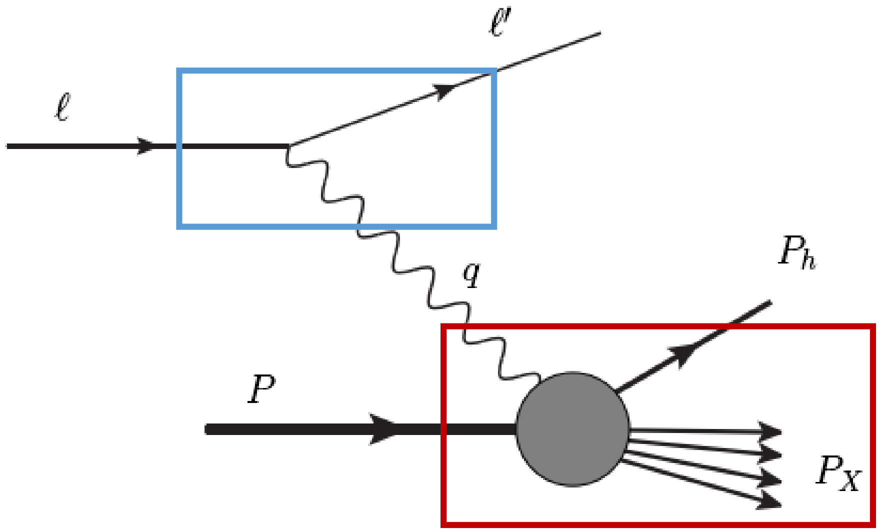


Longitudinal spin transfer of Λ Polarization in SIDIS:

$$e^- P \to e^- \Lambda X$$

- The kinematic analysis
- Spectator diquark model calculation
- Numerical estimate

Differential Cross Section

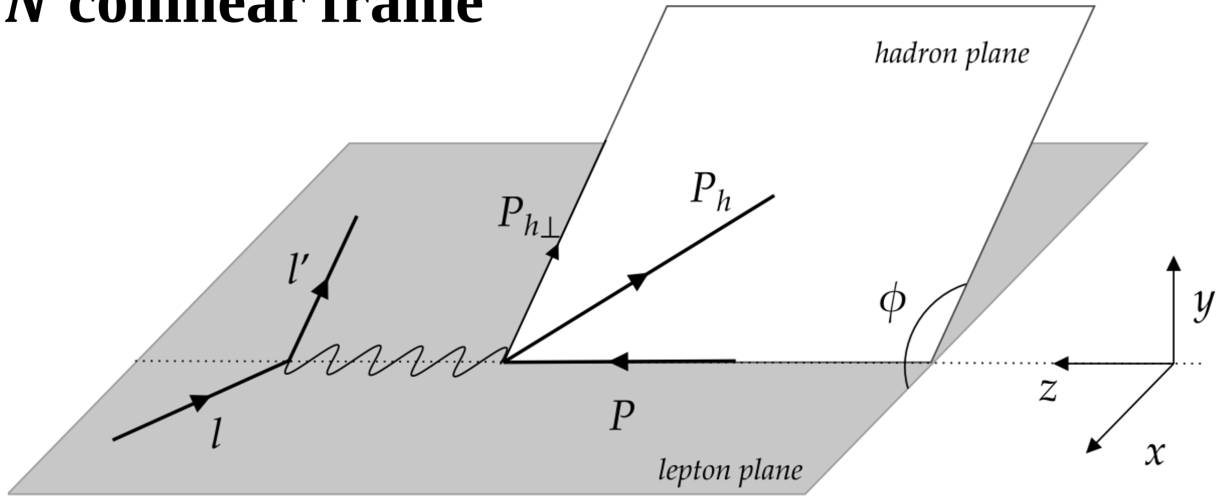


$$\frac{d\sigma^{\rm SIDIS}}{dx\,dy\,dz_\Lambda\, d^2\boldsymbol{P}_{\Lambda\perp}} = \frac{\pi\alpha_{\rm em}^2}{2Q^4}\,\frac{y}{z_\Lambda}\,L_{\mu\nu}W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu}\, l\cdot l' + i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma\right)$$

$$W^{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2P_X^0}\,(2\pi)^4\,\delta^4\left(q + P - P_\Lambda - P_X\right) < P, S|J^\mu(0)|P_\Lambda S_\Lambda; P_X >< P_\Lambda S_\Lambda; P_X|J^\nu(0)|P, S >$$

$\gamma^* N$ collinear frame



$$\cos\phi = -\frac{g_\perp^{\mu\nu} l_\mu P_{h\nu}}{|l_\perp|\,|P_{h\perp}|} \qquad \sin\phi = -\frac{\epsilon_\perp^{\mu\nu} l_\mu P_{h\nu}}{|l_\perp||P_{h\perp}|}$$

$$\cos\phi_S = -\frac{g_\perp^{\mu\nu} l_\mu S_\nu}{|l_\perp|\,|S_\perp|} \qquad \sin\phi_S = -\frac{\epsilon_\perp^{\mu\nu} l_\mu S_\nu}{|l_\perp||S_\perp|}$$

$$\cos\phi_{Sh} = -\frac{g_{h\perp}^{\mu\nu} P_{h\mu} S_{h\nu}}{|P_{h\perp}|\,|S_{h\perp}|} \qquad \sin\phi_{Sh} = -\frac{\epsilon_{h\perp}^{\mu\nu} P_{h\mu} S_{h\nu}}{|P_{h\perp}|\,|S_{h\perp}|}$$

Differential Cross Section

$$\frac{d\sigma}{dxdydzd^2P_{\Lambda\perp}} = \frac{2\pi\alpha_{em}^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x}\right)\{$$

Unpolarized P and Λ — 5

$$A(y)F_{UUU}^{T} + B(y)F_{UUU}^{L} + C(y)\cos\phi F_{UUU}^{\cos\phi} + B(y)\cos 2\phi F_{UUU}^{\cos 2\phi} + \lambda_e E(y)\sin\phi F_{LUU}^{\sin\phi}$$

Polarized P, Unpolarized Λ — 13

$$+\lambda C(y)\sin\phi F_{ULU}^{\sin\phi} + \lambda B(y)\sin 2\phi F_{ULU}^{\sin 2\phi} + \mathbf{S}_\perp A(y)\sin(\phi-\phi_S)F_{UTU}^{T\sin(\phi-\phi_S)}$$
$$+\mathbf{S}_\perp B(y)\sin(\phi-\phi_S)F_{UTU}^{L\sin(\phi-\phi_S)} + \mathbf{S}_\perp C(y)\sin(2\phi-\phi_S)F_{UTU}^{\sin(2\phi-\phi_S)}$$
$$+\mathbf{S}_\perp C(y)\sin\phi_S F_{UTU}^{\phi_S} + \mathbf{S}_\perp B(y)\sin(3\phi-\phi_S)F_{UTU}^{\sin(3\phi-\phi_S)} + \mathbf{S}_\perp B(y)\sin(\phi+\phi_S)F_{UTU}^{\sin(\phi+\phi_S)}$$
$$+\lambda_e\lambda D(y)F_{LLU}^{T} + \lambda_e\lambda E(y)\cos\phi F_{LLU}^{\cos\phi} + \lambda_e\mathbf{S}_\perp D(y)\cos(\phi-\phi_S)F_{LTU}^{T\cos(\phi-\phi_S)}$$
$$+\lambda_e\mathbf{S}_\perp E(y)\cos\phi_S F_{LTU}^{\cos\phi_S} + \lambda_e\mathbf{S}_\perp E(y)\cos(2\phi-\phi_S)F_{LTU}^{\cos(2\phi-\phi_S)}$$

Unpolarized P, Polarized Λ — 13

$$+\lambda_h C(y)\sin\phi F_{UUL}^{\sin\phi} + \lambda_h B(y)\sin 2\phi F_{UUL}^{\sin 2\phi} + \mathbf{S}_{h\perp}\sin\phi_{Sh}B(y)F_{UUT}^{L} + \mathbf{S}_{h\perp}\sin\phi_{Sh}A(y)F_{UUT}^{T}$$
$$+\mathbf{S}_{h\perp}B(y)\sin(2\phi+\phi_{Sh})F_{UUT}^{\sin(2\phi+\phi_{Sh})} + \mathbf{S}_{h\perp}B(y)\sin(2\phi-\phi_{Sh})F_{UUT}^{\sin(2\phi-\phi_{Sh})}$$
$$+\mathbf{S}_{h\perp}C(y)\sin(\phi+\phi_{Sh})F_{UUT}^{\sin(\phi+\phi_{Sh})} + \mathbf{S}_{h\perp}C(y)\sin(\phi-\phi_{Sh})F_{UUT}^{\sin(\phi-\phi_{Sh})}$$
$$+\lambda_e\lambda_h D(y)F_{LUL}^{T} + \lambda_e\mathbf{S}_{h\perp}\cos\phi_{Sh}D(y)F_{LUT}^{T} + \lambda_e\lambda_h E(y)\cos\phi F_{LUL}^{\cos\phi}$$
$$+\lambda_e\mathbf{S}_{h\perp}E(y)\cos(\phi-\phi_{Sh})F_{LUT}^{\cos(\phi-\phi_{Sh})} + \lambda_e\mathbf{S}_{h\perp}E(y)\cos(\phi+\phi_{Sh})F_{LUT}^{\cos(\phi+\phi_{Sh})}$$

F_{ZXY}

Z: lepton; X: nucleon; Y: Λ

U: unpolarized; L: longitudinal;

T: transvers

$$A(y) = \frac{y^2}{4}(2+\gamma^2) - y + 1$$

$$B(y) = 1 - y - \frac{1}{4}\gamma^2 y^2$$

……

Differential Cross Section

$$
\begin{aligned}
&+\lambda\lambda_h A(y) F_{ULL}^{T} + \lambda\lambda_h B(y) F_{ULL}^{L} + \lambda\lambda_h C(y)\cos\phi F_{ULL}^{T\cos\phi} + \lambda\lambda_h B(y)\cos 2\phi F_{ULL}^{T\cos 2\phi} + \lambda \mathbf{S}_{h\perp}\cos\phi_{Sh} B(y) F_{ULT}^{L} \\
&+\lambda \mathbf{S}_{h\perp}\cos\phi_{Sh} A(y) F_{ULT}^{T} + \lambda \mathbf{S}_{h\perp} C(y)\cos(\phi-\phi_{Sh}) F_{ULT}^{\cos(\phi-\phi_{Sh})} + \lambda \mathbf{S}_{h\perp} B(y)\cos(2\phi-\phi_{Sh}) F_{ULT}^{\cos(2\phi-\phi_{Sh})} \\
&+\lambda \mathbf{S}_{h\perp} C(y)\cos(\phi+\phi_{Sh}) F_{ULT}^{\cos(\phi+\phi_{Sh})} + \lambda \mathbf{S}_{h\perp} B(y)\cos(2\phi+\phi_{Sh}) F_{ULT}^{\cos(2\phi+\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\lambda_h C(y)\cos\phi_S F_{UTL}^{\phi_S} + \mathbf{S}_{\perp}\lambda_h A(y)\cos(\phi-\phi_S) F_{UTL}^{T\cos(\phi-\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(\phi-\phi_S) F_{UTL}^{L\cos(\phi-\phi_S)} \\
&+\mathbf{S}_{\perp}\lambda_h C(y)\cos(2\phi-\phi_S) F_{UTL}^{\cos(2\phi-\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(\phi+\phi_S) F_{UTL}^{\cos(\phi+\phi_S)} + \mathbf{S}_{\perp}\lambda_h B(y)\cos(3\phi-\phi_S) F_{UTL}^{\cos(3\phi-\phi_S)} \\
&+\lambda_e \mathbf{S}_{\perp}\lambda_h E(y)\sin\phi_S F_{LTL}^{\sin\phi_S} + \lambda_e \mathbf{S}_{\perp}\lambda_h E(y)\sin(2\phi-\phi_S) F_{LTL}^{\sin(2\phi-\phi_S)} + \lambda_e \mathbf{S}_{\perp}\lambda_h D(y)\sin(\phi-\phi_S) F_{LTL}^{T\sin(\phi-\phi_S)} \\
&+\lambda_e\lambda\lambda_h E(y)\sin\phi F_{LLL}^{\sin\phi} + \lambda_e\lambda \mathbf{S}_{h\perp} E(y)\left[\sin(\phi+\phi_{Sh}) F_{LLT}^{\sin(\phi+\phi_{Sh})} + \sin(\phi-\phi_{Sh}) F_{LLT}^{\sin(\phi-\phi_{Sh})}\right] + \lambda_e\lambda \mathbf{S}_{h\perp}\sin\phi_{Sh} D(y) F_{LLT}^{T} \\
&+\lambda_e \mathbf{S}_{\perp}\mathbf{S}_{h\perp} D(y)\left[\sin(\phi-\phi_S+\phi_{Sh}) F_{LTT}^{T\sin(\phi-\phi_S+\phi_{Sh})} + \sin(\phi-\phi_S-\phi_{Sh}) F_{LTT}^{T\sin(\phi-\phi_S-\phi_{Sh})}\right] \\
&+\lambda_e \mathbf{S}_{\perp}\mathbf{S}_{h\perp} E(y)\Big[\sin(\phi_S+\phi_{Sh}) F_{LTT}^{\sin(\phi_S+\phi_{Sh})} + \sin(\phi_S-\phi_{Sh}) F_{LTT}^{\sin(\phi_S-\phi_{Sh})} \\
&\qquad + \sin(2\phi-\phi_S-\phi_{Sh}) F_{LTT}^{\sin(2\phi-\phi_S-\phi_{Sh})} + \sin(2\phi-\phi_S+\phi_{Sh}) F_{LTT}^{\sin(2\phi-\phi_S+\phi_{Sh})}\Big] \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp}\cos(\phi-\phi_S-\phi_{Sh}) A(y) F_{UTT}^{T\cos(\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\sin(\phi-\phi_S-\phi_{Sh}) F_{UTT}^{L\sin(\phi-\phi_S-\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(3\phi-\phi_S-\phi_{Sh}) F_{UTT}^{\cos(3\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(\phi+\phi_S-\phi_{Sh}) F_{UTT}^{\cos(\phi+\phi_S-\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(2\phi-\phi_S-\phi_{Sh}) F_{UTT}^{\cos(2\phi-\phi_S-\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(\phi_S-\phi_{Sh}) F_{UTT}^{\cos(\phi_S-\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp}\cos(\phi-\phi_S+\phi_{Sh}) A(y) F_{UTT}^{T\cos(\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\sin(\phi-\phi_S+\phi_{Sh}) F_{UTT}^{L\sin(\phi-\phi_S+\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(3\phi-\phi_S+\phi_{Sh}) F_{UTT}^{\cos(3\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} B(y)\cos(\phi+\phi_S+\phi_{Sh}) F_{UTT}^{\cos(\phi+\phi_S+\phi_{Sh})} \\
&+\mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(2\phi-\phi_S+\phi_{Sh}) F_{UTT}^{\cos(2\phi-\phi_S+\phi_{Sh})} + \mathbf{S}_{\perp}\mathbf{S}_{h\perp} C(y)\cos(\phi_S+\phi_{Sh}) F_{UTT}^{\cos(\phi_S+\phi_{Sh})}
\end{aligned}
$$

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Polarized P
Polarized Λ

CFR

Spin asymmetry： $A = \frac{d\sigma(\uparrow) - d\sigma(\downarrow)}{d\sigma(\uparrow) + d\sigma(\downarrow)}$ ⇒ $A_{XY}^{\omega(\phi_h,\phi_S)} \equiv \frac{F_{XY}^{\omega(\phi_h,\phi_S)}}{F_{UU}}$

QCD factorization： $d\sigma \sim d\hat{\sigma} \otimes PDF \otimes FF$

Unpolarized P&Λ

$$\frac{d\sigma(\ell H \to \ell' h X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\Big\{A(y)\mathcal{I}[f_1 D_1] + B(y)cos2\phi\mathcal{I}\Big[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_1^\perp H_1^\perp\Big]\Big\}$$

Polarized P, Unpolarized Λ

$$\frac{d\sigma(\ell\vec{H} \to \ell' h X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda\lambda_e C(y)\mathcal{I}[g_1 D_1] + \lambda B(y)\sin(2\phi)\mathcal{I}\Big[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_{1L}^\perp H_1^\perp\Big] + \cdots$$

Unpolarized P, Polarized Λ

$$\frac{d\sigma(\ell H \to \ell' \vec{h} X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda_e\lambda_h C(y)\mathcal{I}[f_1 G_1] + \lambda_h B(y)\sin 2\phi\mathcal{I}\Big[\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_1^\perp H_{1L}^\perp\Big] + \cdots$$

Polarized P, Polarized Λ

$$\frac{d\sigma(\ell\vec{H} \to \ell' \vec{h} X)}{dx dz dy d^2 \boldsymbol{P}_{\Lambda\perp}} = \frac{2\pi\alpha^2}{yQ^2}\{\lambda\lambda_h A(y)\mathcal{I}[g_1 G_1] + \lambda\lambda_h B(y)\cos 2\phi\mathcal{I}\Big[-\frac{2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T}{MM_h}h_{1L}^\perp H_{1L}^\perp\Big] + \lambda|\boldsymbol{S}_{h\perp}|A(y)\cos\phi_{S_h}\mathcal{I}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}g_1 G_{1T}\Big] + \lambda_h|\boldsymbol{S}_T|B(y)\cos(3\phi - \phi_S)\mathcal{I}\Big[-\frac{4\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - 2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\boldsymbol{p}_T\cdot\boldsymbol{k}_T - \hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\boldsymbol{p}_T^2}{2M_h M^2}h_{1T}^\perp H_{1L}^\perp\Big] + \cdots$$

Structure Function &PDFs⊗FFs

$$F_{UUU}^{T} = \mathcal{I}\left[f_1 D_1\right]$$

$$F_{UUU}^{\cos 2\phi} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{h_1^{\perp}H_1^{\perp}}{MM_h}\right]$$

$$F_{ULU}^{\sin 2\phi} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{h_{1L}^{\perp}H_1^{\perp}}{MM_h}\right]$$

$$F_{UTU}^{T\sin(\phi-\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{f_{1T}^{\perp}D_1}{M}\right]$$

$$F_{UTU}^{T\sin(\phi+\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{h_1 H_1^{\perp}}{M_h}\right]$$

$$F_{UTU}^{T\sin(3\phi-\phi_S)} = \mathcal{I}\left[-\left(4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - 2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\boldsymbol{p}_T\cdot\boldsymbol{k}_T - \boldsymbol{p}_T^2\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\frac{h_{1T}^{\perp}H_1^{\perp}}{2M^2M_h}\right]$$

•
•
•

$$F_{LLU}^{T} = \mathcal{I}\left[g_1 D_1\right]$$

$$F_{LTU}^{T\cos(\phi-\phi_S)} = \mathcal{I}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{g_{1T}^{\perp}D_1}{M}\right]$$

$$F_{LUL}^{T} = \mathcal{I}\left[f_1 G_{1L}\right]$$

$$F_{LUT}^{T\cos\phi_{Sh}} = \mathcal{I}\left[\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{f_1 G_{1T}}{M_h}\right]$$

$$F_{LLT}^{T\sin\phi_{Sh}} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\frac{g_{1L}D_{1T}^{\perp}}{M_h}\right]$$

$$F_{LTL}^{T\sin(\phi-\phi_S)} = \mathcal{I}\left[-\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\frac{f_{1T}^{\perp}G_{1L}}{M}\right]$$

$$F_{LTT}^{T\sin(\phi-\phi_S-\phi_{Sh})} = \mathcal{I}\left[-\left(2\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T - \boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)\frac{f_{1T}^{\perp}G_{1T} - g_{1T}D_{1T}^{\perp}}{2MM_h}\right]$$

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•
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Numerical Estimate

◆ Longitudinal Spin Transfer D_{LL} in current fragmentation region:

$$P_L(x,z,Q^2) = \frac{d\sigma_{\Downarrow} - d\sigma_{\Uparrow}}{d\sigma_{\Downarrow} + d\sigma_{\Uparrow}} = \frac{F_{UL}}{F_{UU}} = D(y)D_{LL'}$$

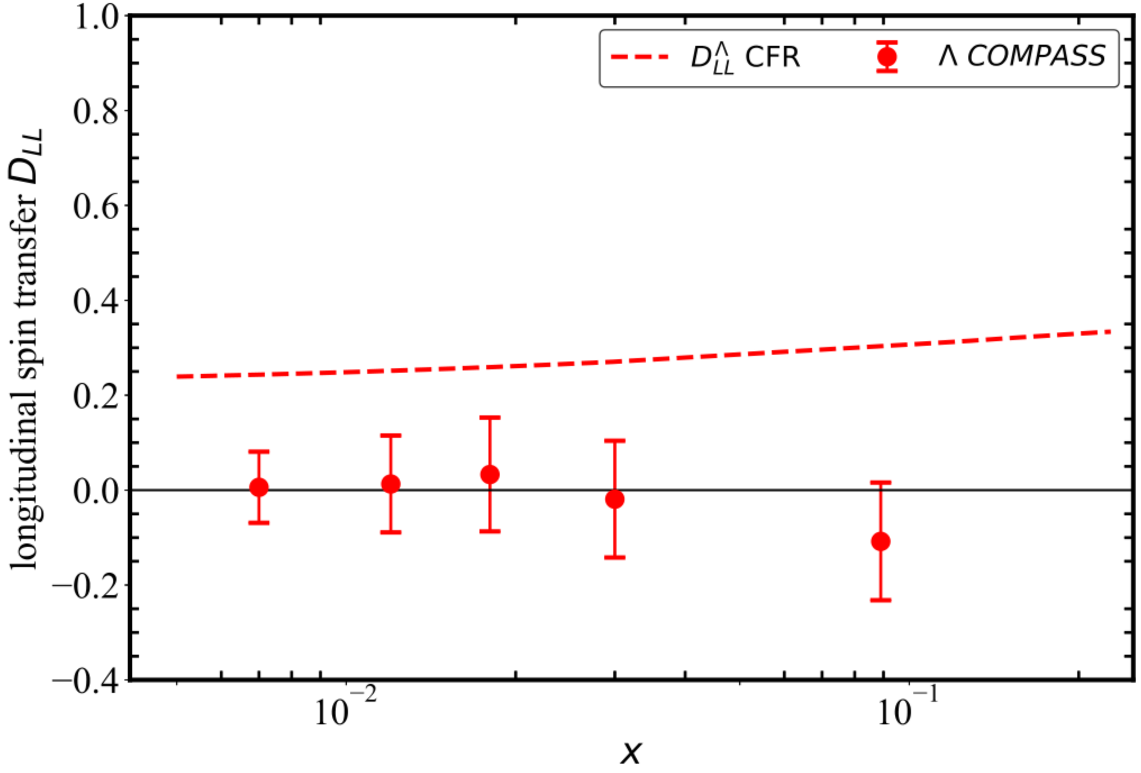
$$D_{LL'} = \frac{\sum_a e_a^2 f_a(x,Q^2) G_a(z,Q^2)}{\sum_a e_a^2 f_a(x,Q^2) D_a(z,Q^2)} \; \omega_q(z_\Lambda, P_{h\perp})$$

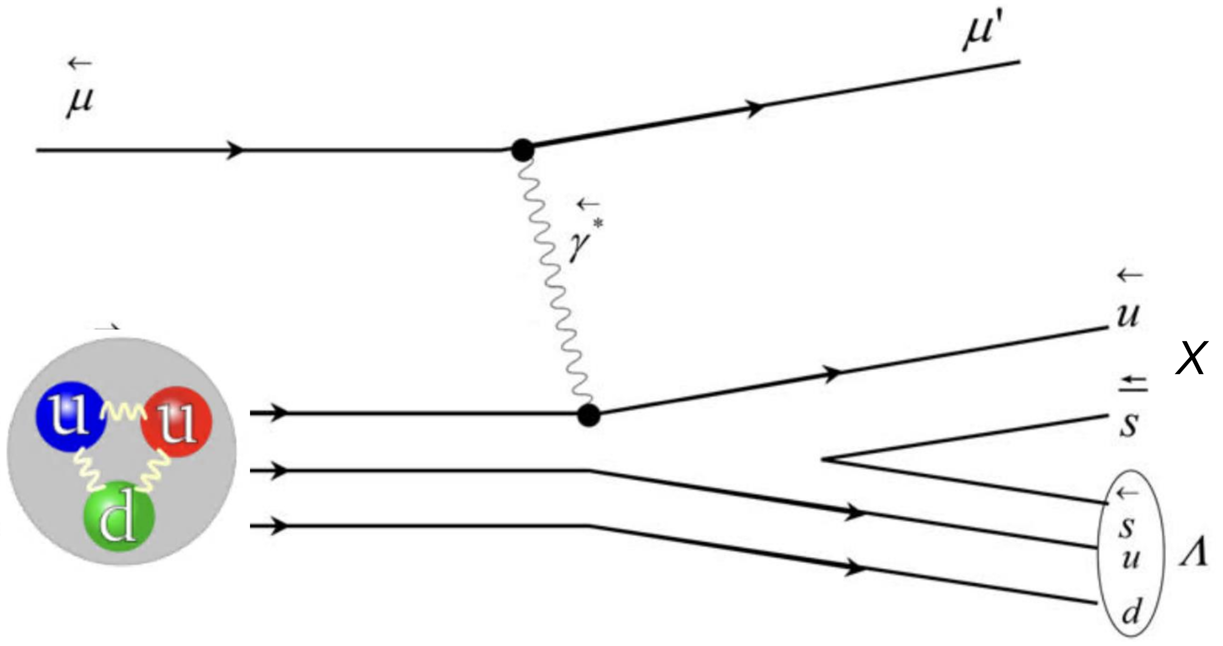
$$f_{1q}(x,k_\perp) = f_{1q}(x)\frac{1}{\pi\Delta_p^2}e^{-k_\perp^2/\Delta_p^2}, \quad \Delta_p^2 = 0.61\ \mathrm{GeV}^2$$

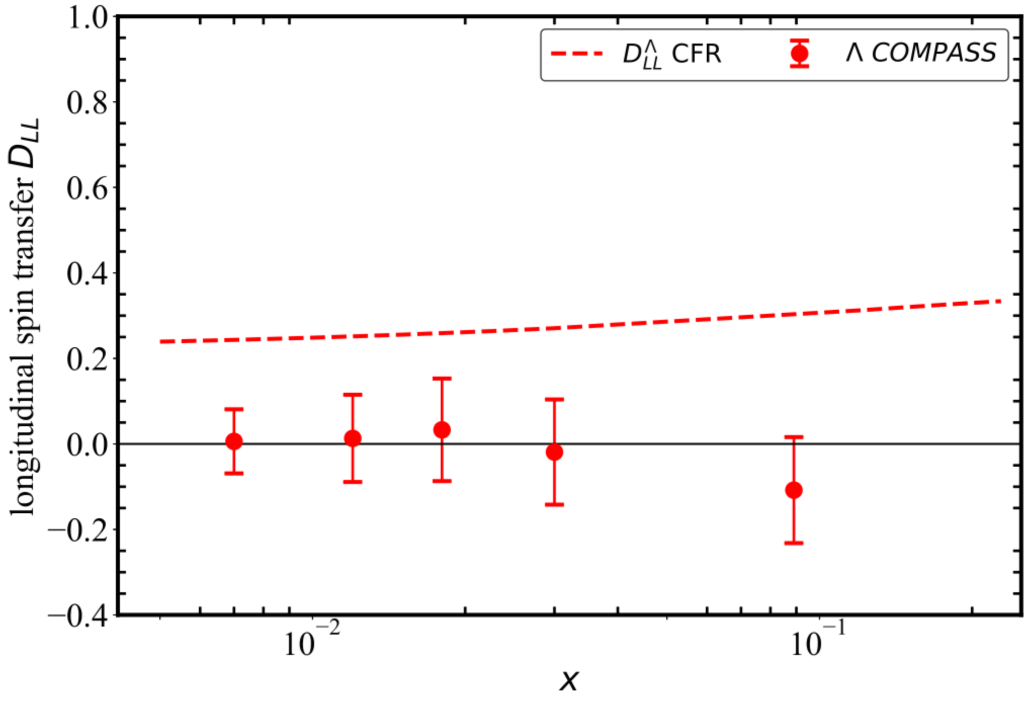
$$D_{1q}^\Lambda(z,p_T) = D_{1q}^\Lambda(z)\frac{1}{\pi\Delta_\Lambda^2}e^{-p_T^2/\Delta_\Lambda^2}, \quad \Delta_\Lambda^2 = 0.118\ \mathrm{GeV}^2$$

$f_{q/p}(x_B, Q)$ — **CT18NLO PDF**
Progress in the CTEQ-TEA NNLO global QCD analysis

$\dfrac{G_a(z,Q^2)}{D_a(z,Q^2)}$ — **DSV FFs**
Phys.Rev.D 57 (1998) 5811-5824



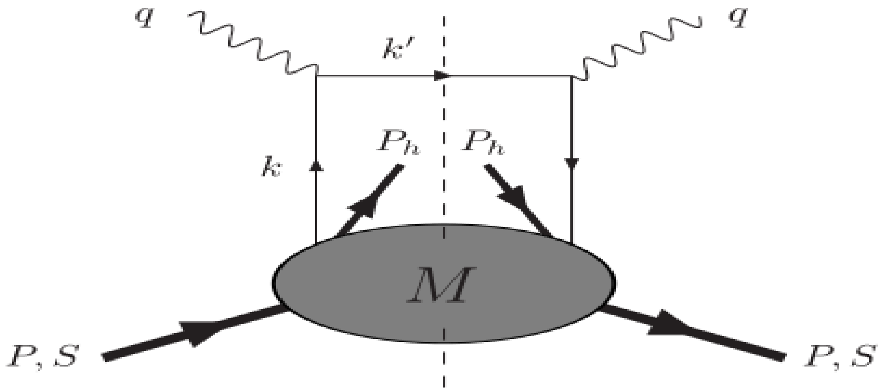




$$\psi = \eta_{\text{flavor}}\chi_{\text{spin}}\xi_{\text{color}}\phi_{\text{space}}$$

$$\Lambda^{\uparrow} = \frac{1}{\sqrt{3}}(ud)_{0,0}s^{\uparrow} + \frac{1}{\sqrt{12}}(us)_{0,0}d^{\uparrow} - \frac{1}{\sqrt{12}}(ds)_{0,0}u^{\uparrow}$$
$$+\frac{1}{2}\left(\sqrt{\frac{2}{3}}(us)_{1,1}d^{\downarrow} - \sqrt{\frac{1}{3}}(us)_{1,0}d^{\uparrow}\right) - \frac{1}{2}\left(\sqrt{\frac{2}{3}}(ds)_{1,1}u^{\downarrow} - \sqrt{\frac{1}{3}}(ds)_{1,0}u^{\uparrow}\right)$$

◆ The fracture matrix $\mathcal{M}$



$$W^{\mu\nu} = \sum_a e_a^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} 2\pi\, \delta[(k+q)^2]\, \mathrm{Tr}\, [\mathcal{M}\gamma^\mu (\not{k} + \not{q})\gamma^\nu]$$

$$\mathcal{M}_{ij}(k;P,S;P_\Lambda,S_\Lambda) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2E_X} \int \frac{d^4\xi}{(2\pi)^4} e^{ik\xi} \langle P,S|\overline{\Psi}_j(0)|P_\Lambda,S_\Lambda;X\rangle \langle P_\Lambda,S_\Lambda;X\,|\Psi_i(\xi)|\,P,S\rangle$$

decompose it on a basis of Dirac structures： $\mathcal{M} = \frac{1}{2}\left(\mathcal{S}I + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^\mu\gamma_5 + i\mathcal{P}\gamma_5 + i\mathcal{T}_{\mu\nu}\sigma^{\mu\nu}\gamma_5\right)$

$\mathcal{M}_{ij}(k;P,S;P_\Lambda,S_\Lambda)$ ⇨

Dirac Struture：$1, \gamma^\mu, \gamma^\mu\gamma_5, \gamma_5, \sigma^{\mu\nu}\gamma_5$

Five Vectors： $k^\mu, P^\mu, P^\mu_\Lambda, S^\mu, S^\mu_\Lambda.$

⇨ the most general decomposition of $\mathcal{M}$

Use $\gamma^+, \gamma^+\gamma_5, i\sigma^{i+}\gamma_5$ to pick out the leading-twist terms

Differential Cross Section

$$
\begin{aligned}
\frac{d\sigma^{(\mathrm{TFR})}}{dx\,dy\,d\zeta\,d^2\mathbf{P}_{h\perp}} = \frac{4\alpha^2}{yQ^2}\sum_a e_a^2 \Bigg\{ \Big(\frac{y^2}{2} - y + 1\Big) \Bigg[& M_{UU} + \lambda\lambda_h M_{LL} - S_\perp \frac{P_{h\perp}}{m_h} \sin(\phi - \phi_S)\, M_{TU}^h \\
& - S_{h\perp} \frac{P_{h\perp}}{m_h} \sin(\phi - \phi_{S_h})\, M_{UT}^h + S_\perp S_{h\perp} \frac{P_{h\perp}^2}{m_h^2} \big(\cos(2\phi - \phi_S - \phi_{S_h}) + \cos(\phi_S - \phi_{S_h})\big)\, M_{TT}^h \\
& + S_\perp S_{h\perp} \cos(\phi_S - \phi_{S_h})\, M_{TT} + \lambda S_{h\perp} \frac{P_{h\perp}}{m_h} \cos(\phi - \phi_{S_h})\, M_{LT}^h + S_\perp \lambda_h \frac{P_{h\perp}}{m_h} \cos(\phi - \phi_S)\, M_{TL}^h \Bigg] \\
+ \lambda_e y\Big(1 - \frac{y}{2}\Big) \Bigg[& \lambda \Delta M_{LU} + \lambda_h \Delta M_{UL} + S_\perp \frac{P_{h\perp}}{m_h} \cos(\phi - \phi_S)\, \Delta M_{TU}^h + S_{h\perp} \frac{P_{h\perp}}{m_h} cos(\phi - \phi_{S_h})\, \Delta M_{UT}^h \\
& - S_\perp S_{h\perp} \sin(\phi_S - \phi_{S_h}) \Delta M_{TT} - S_\perp S_{h\perp} \frac{P_{h\perp}^2}{m_h^2} \big(\sin(2\phi_h - \phi_S - \phi_{S_h}) + \sin(\phi_S - \phi_{S_h})\big)\, \Delta M_{TT}^h \\
& - \lambda S_{h\perp} \frac{P_{h\perp}}{m_h} \sin(\phi - \phi_{S_h})\, \Delta M_{LT}^h - S_\perp \lambda_h \frac{P_{h\perp}}{m_h} \sin(\phi - \phi_S)\, \Delta M_{TL}^h \Bigg] \Bigg\}
\end{aligned}
$$

M_{XY}^h X: nucleon; Y: Λ hyperon

ΔM: longitudinally polarized quark

Fracture Functions

$$F_{UUU}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{UU}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{ULL}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} M_{LL}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTT}^{\cos(\phi_S-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[M_{TT}(x,\zeta,\mathbf{P}_{h\perp}^2) + \frac{P_{h\perp}^2}{m_h^2} M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)\right]$$

$$F_{UTT}^{T\cos(2\phi-\phi_S-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTU}^{T\sin(\phi-\phi_S)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TU}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UUT}^{T\sin(\phi-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{UT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{ULT}^{\cos(\phi-\phi_S)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{LT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{UTL}^{\cos(\phi-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} M_{TL}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LLU}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{LU}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LUL}^{T} = \sum_a e_a^2 x_B \frac{\zeta}{z_h} \Delta M_{UL}(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTU}^{\cos(\phi-\phi_S)} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TU}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LUT}^{\cos(\phi-\phi_{S_h})} = \sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{UT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LLT}^{T\sin(\phi_\Lambda-\phi_{S_\Lambda})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{LT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTL}^{T\sin(\phi_\Lambda-\phi_S)} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}}{m_h} \Delta M_{TL}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

$$F_{LTT}^{\sin(\phi_S-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\left[\Delta M_{TT}(x,\zeta,\mathbf{P}_{h\perp}^2) + \frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)\right]$$

$$F_{LTT}^{\sin(2\phi-\phi_S-\phi_{S_h})} = -\sum_a e_a^2 x_B \frac{\zeta}{z_h}\frac{P_{h\perp}^2}{m_h^2} \Delta M_{TT}^h(x,\zeta,\mathbf{P}_{h\perp}^2)$$

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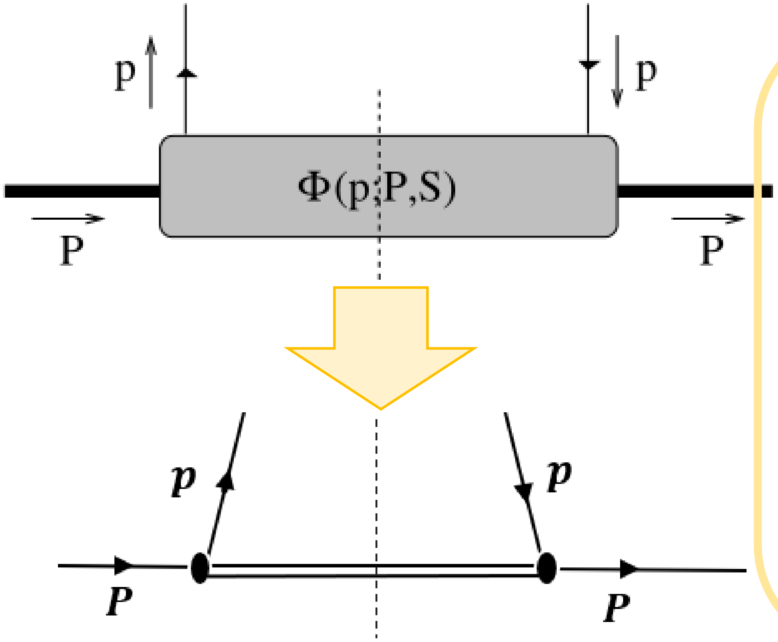
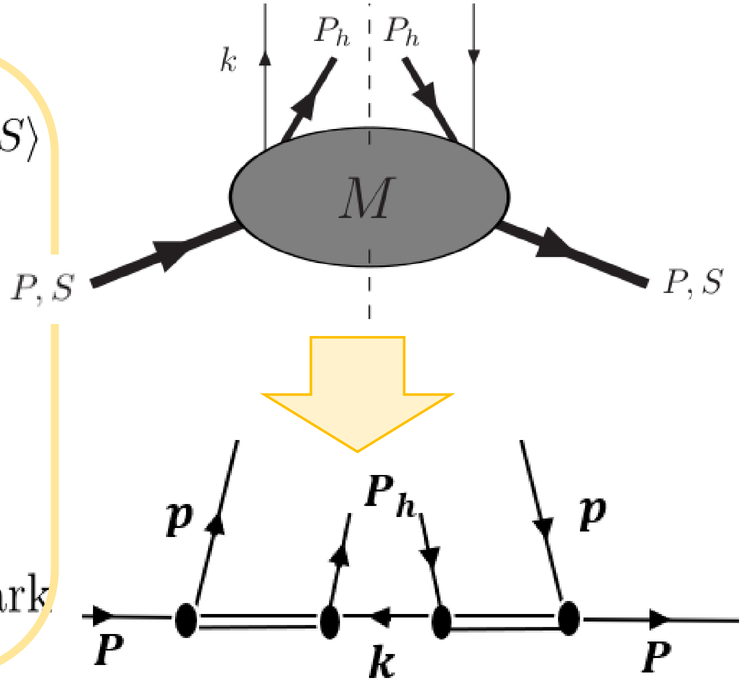
Spectator Diquark Model

R. P. Feynman, "Photon Hadron Interactions," New York 1972-01-01.



$$\Phi_{ij}^{R} = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2P_X^0} \langle P,S|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|P,S\rangle$$
$$* (2\pi)^4 \delta(P-p-P_X)$$

$$\langle P,S|\bar{\psi}(0)|X\rangle$$
$$= \begin{cases} \bar{U}(P,S)\Upsilon_s \frac{i}{\not{p}-m_q}, & \text{scalar diquark} \\ \bar{U}(P,S)\Upsilon_a^{\mu} \frac{i}{\not{p}-m_q}\epsilon_\mu, & \text{axial-vector diquark} \end{cases}$$



Parton distribution function **Target Fragment**

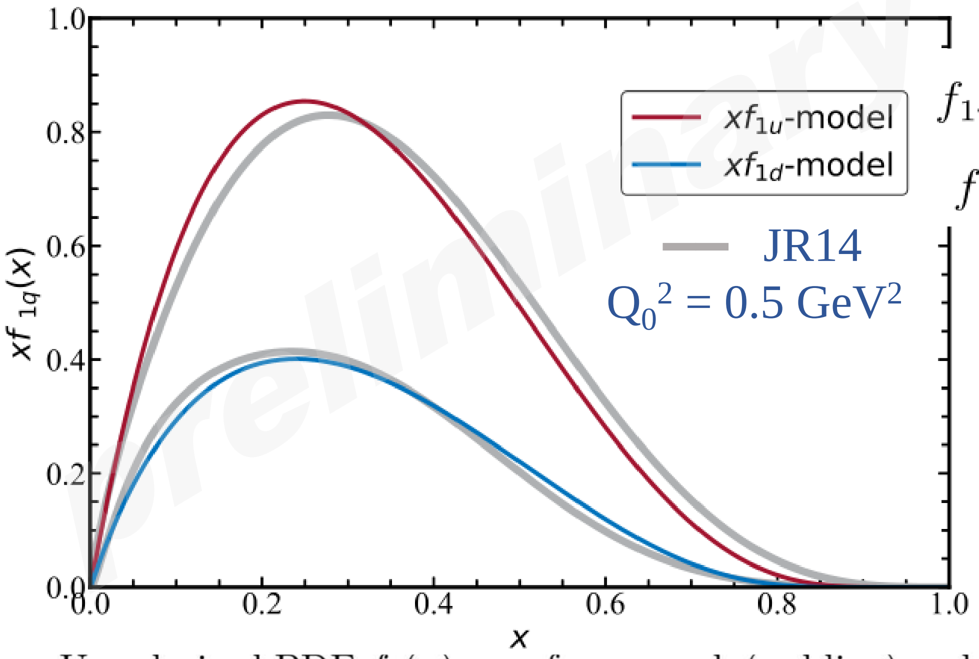
$$M_{UU}^{(s)}(x,\zeta,\mathbf{P}_{h\perp}) = \frac{|g(p,p')|^2 \, |g(k,p')|^2}{8(2\pi)^6 \zeta(1-x-\zeta)}$$
$$* \frac{Tr\left[(\not{p}+m)(\not{P}+M)(\not{p}+m)\gamma^+\right] Tr\left[(\not{k}-m_q)(\not{P}_h+M_h)\right]}{2(p^2-m^2)^2(p'^2-M_R^2)^2 P^+}$$

Model Calculation

$$f_1^{q(s)}(x, \boldsymbol{p}_T^2) = \frac{g_s^2}{(2\pi)^3} \frac{[(m + xM)^2 + \boldsymbol{p}_T^2](1-x)}{2[\boldsymbol{p}_T^2 + L_s^2(m^2)]^2}$$

$$f_1^{q(a)}(x, \boldsymbol{p}_T^2) = \frac{g_a^2}{(2\pi)^3} \frac{1}{4[\boldsymbol{p}_T^2 + L_a^2(m^2)]^2 M_a^2(1-x)} [\boldsymbol{p}_T^4 + xM_a^2(2\boldsymbol{p}_T^2 + xM_a^2) + (1-x)^2[\boldsymbol{p}_T^2(M^2 + m^2 + 2M_a^2) + 2m^2M_a^2 + 6xmMM_a^2 + 2x^2M^2M_a^2 + m^2M^2(1-x)^2]],$$

$$L_X^2(m^2) = xM_X^2 + (1-x)m^2 - x(1-x)M^2$$



Unpolarized PDF $f_1(x)$ vs x for u quark (red line) and d quark (blue line). The gray band from the parametrizations of JR14, and the curves represent the best fit obtained with our spectator model. *Eur. Phys. J. C75 (2015) 3, 132*

$$f_{1u} = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a$$

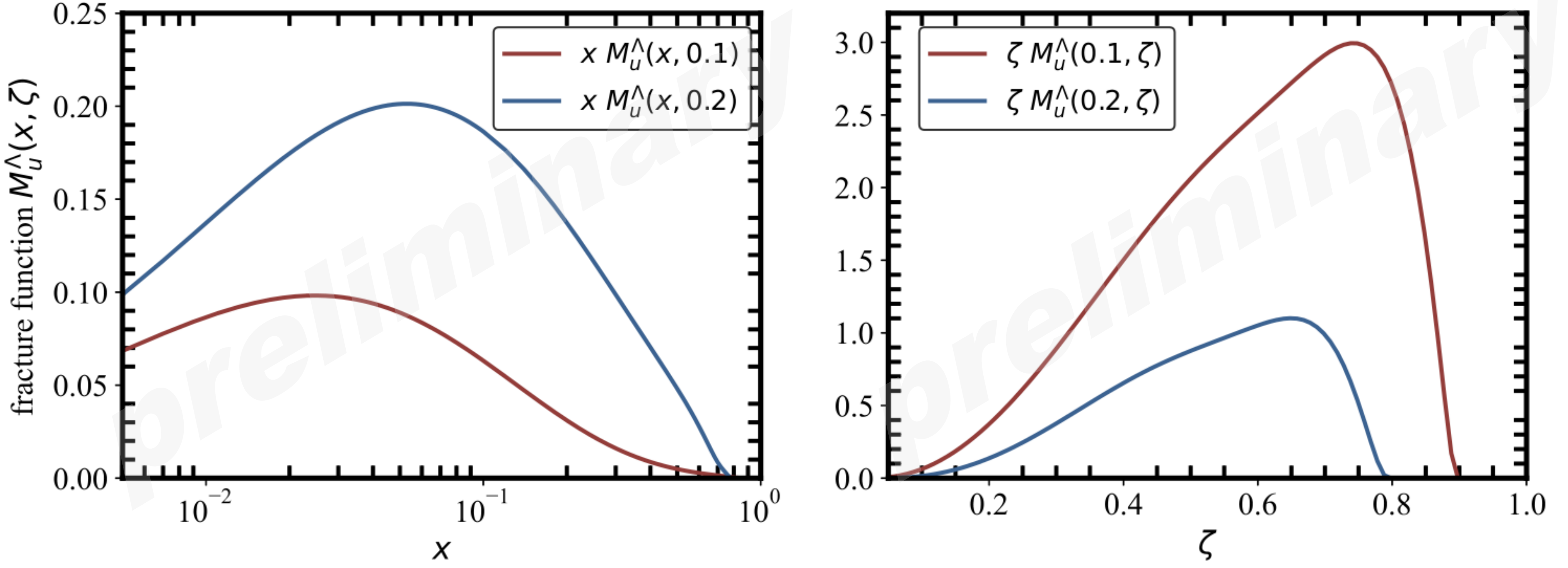
$$f_{1d} = f_1^a$$

$$m = 0.3 \text{ GeV},$$
$$M_s = 1.2 \text{ GeV}, \Lambda_s = 2.3 \text{ GeV},$$
$$M_a = 1.3 \text{ GeV}, \Lambda_a = 1.6 \text{ GeV},$$
$$g_s = 14.98, \quad g_a = 15.33.$$

Model Calculation

$$m = 0.3\ \text{GeV},\ \text{g}_\text{s} = 14.98,$$
$$M_s = 1.2\ \text{GeV},\ \Lambda_\text{s} = 2.3\ \text{GeV},$$

$$\hat{M} = \frac{g_{1s}^2 g_{2s}^2 x\left[(m+xM)^2 + \boldsymbol{p}_T^2\right]}{2(2\pi)^6 \zeta^2 (1-\zeta-x)^2 (p^2-m^2)^2} \times \frac{\left[(1-x-\zeta)M_h - \zeta m_{\bar{q}}\right]^2 + \left[(1-x)\boldsymbol{P}_{h\perp} + \zeta \boldsymbol{p}_T\right]^2}{x(1-x)M^2 - xM_s^2 - (1-x)p^2 + \boldsymbol{p}_T^2}.$$



The results for unploarized fracture function $xM_u^\Lambda(x)$ and $\zeta M_u^\Lambda(\zeta)$ on x and ζ dependences using the spectator diquark model.

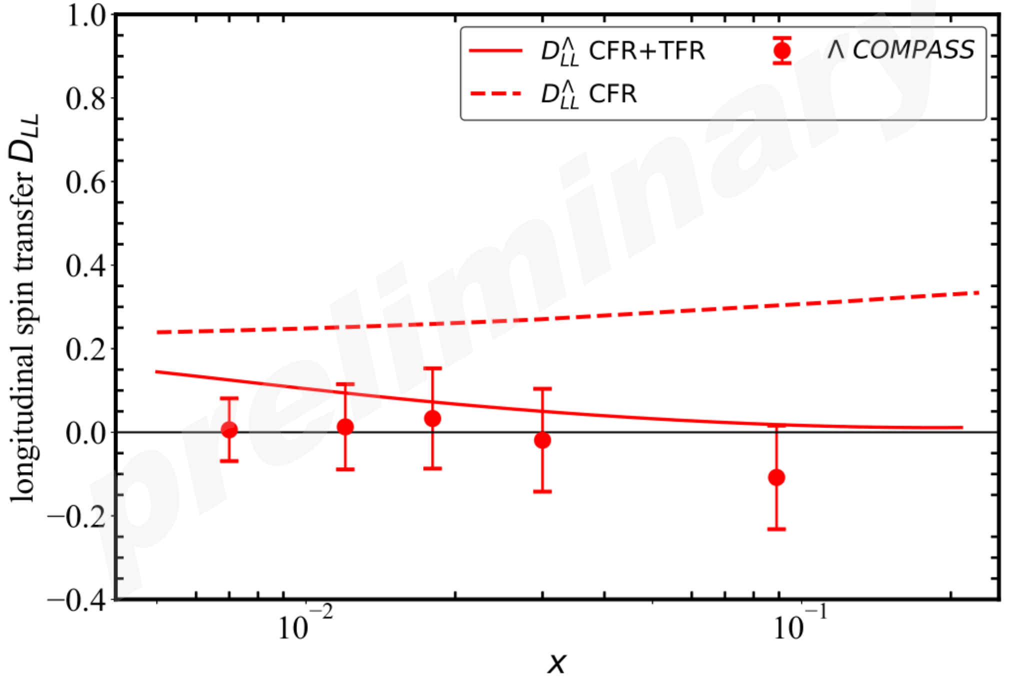
Numerical Estimate

- Longitudinal Spin Transfer $D_{LL}(x)$

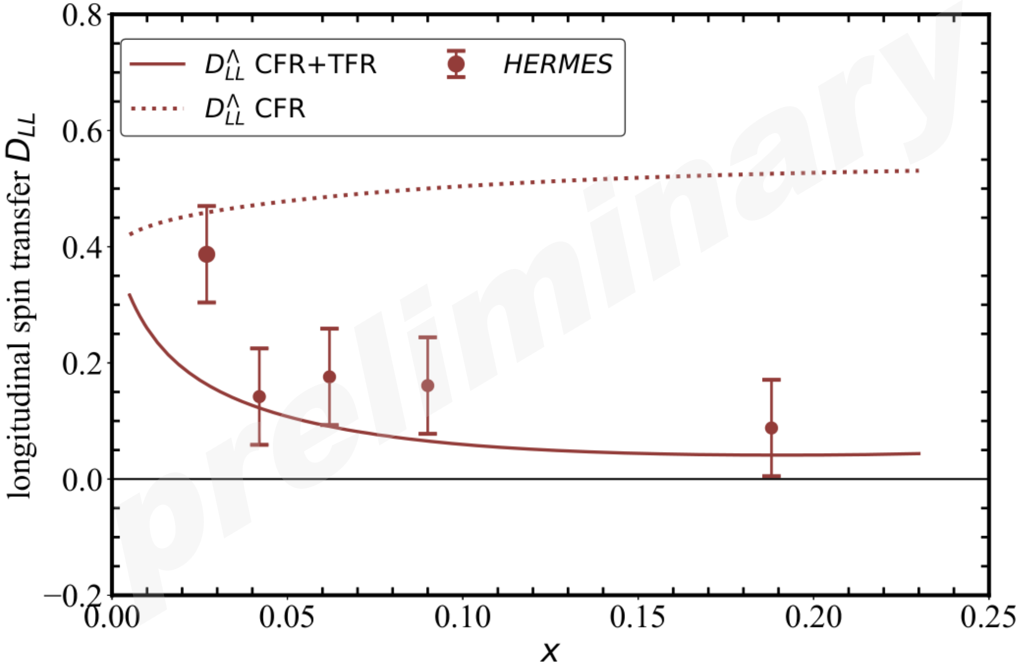
$$D_{LL}^{\Lambda}(x,z,Q^2)=\frac{\sum_q e_q^2 z^2 f_{1q}(x_B,Q^2)G_{1Lq}^{\Lambda}(z_\Lambda,Q^2)}{\sum_q e_q^2\left[z^2 f_{1q}(x_B,Q^2)D_{1q}^{\Lambda}(z_\Lambda,Q^2)+\frac{\zeta}{z}M_q^{\Lambda}(x_B,\zeta,Q^2)\right]}.$$

$$\zeta=\frac{2x_B(M_h^2+\boldsymbol{P}_{h\perp}^2)}{z_h Q^2+\sqrt{z_h^2Q^4-4x_B^2M^2(M_h^2+\boldsymbol{P}_{h\perp}^2)}}$$

D_{LL}^{Λ} in CFR (dashed line) and CFR+TFR (real line):



COMPASS data from *Eur. Phys. J. C64, 171-179 (2009)*
at $\bar{Q}^2=3.7\ GeV^2$, $\bar{z}=0.27$



HERMES data from *J. Phys. Conf. Ser, 295,02114 (2011)*
at $\bar{Q}^2=2.4\ GeV^2$, $\bar{z}=0.45$

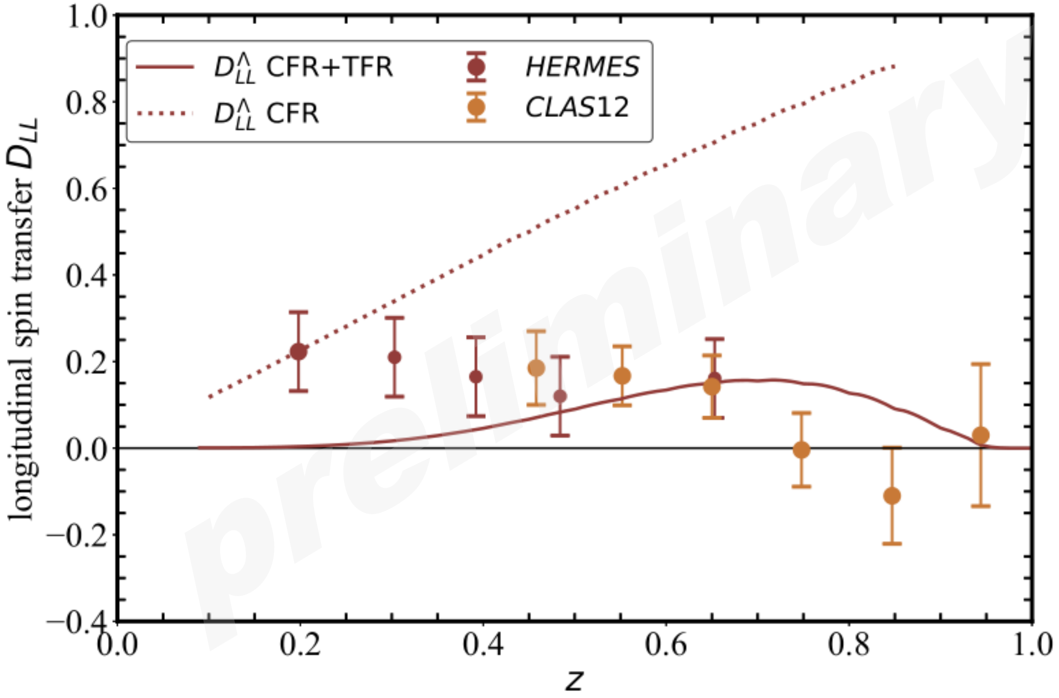
Numerical Estimate

◆ Longitudinal Spin Transfer $D_{LL}(z)$ $\sigma = \sigma^{CFR} + \sigma^{TFR}$

$$D^{\Lambda}_{LL}(x,z,Q^2) = \frac{\sum_q e_q^2 z^2 f_{1q}(x_B,Q^2) G^{\Lambda}_{1Lq}(z_\Lambda,Q^2)}{\sum_q e_q^2 \left[z^2 f_{1q}(x_B,Q^2) D^{\Lambda}_{1q}(z_\Lambda,Q^2) + \frac{\zeta}{z} M^{\Lambda}_q(x_B,\zeta,Q^2) \right]}.$$

D^{Λ}_{LL} in CFR (dashed line) and CFR+TFR (real line):

$$\zeta = \frac{2x_B(M_h^2 + \boldsymbol{P}_{h\perp}^2)}{z_h Q^2 + \sqrt{z_h^2 Q^4 - 4x_B^2 M^2 (M_h^2 + \boldsymbol{P}_{h\perp}^2)}}$$



HERMES data from *J. Phys. Conf. Ser, 295,02114 (2011)*
CLAS12 data from *JPS Conf. Proc. 37, 020304(2022)*
at $\bar{Q}^2 = 2.4\ GeV^2$, $\bar{x} = 0.088$

Numerical Estimate

$$x_F = \frac{z_\Lambda}{\frac{x_B M^2}{Q^2} + (1 - x_B)} \left[\left(1 + \frac{Q^2}{2x_B M^2}\right) \sqrt{1 - \frac{4x_B^2 M^2 (M_h^2 + \mathbf{P}_{h\perp}^2)}{z_\Lambda^2 Q^4}} - \sqrt{\frac{Q^4}{4x_B^2 M^4} + \frac{Q^2}{M^2}} \right]$$

D_{LL}^{Λ} in CFR (dashed line) and CFR+TFR (real line):

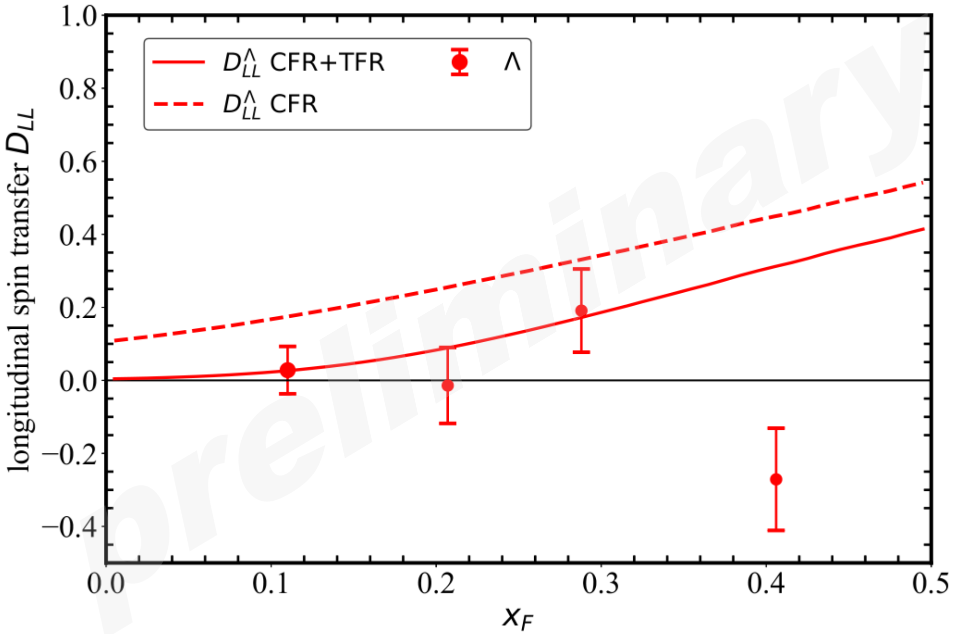


FIG. 8. The comparison of our result of the x_F-dependent longitudinal spin transfer and the experimental data at COMPASS. Inputs of TFR nonzero values are considered for Λ, as shown in the red line. The dashed and blue line represents the result in CFR for Λ and $\bar{\Lambda}$ respectively.

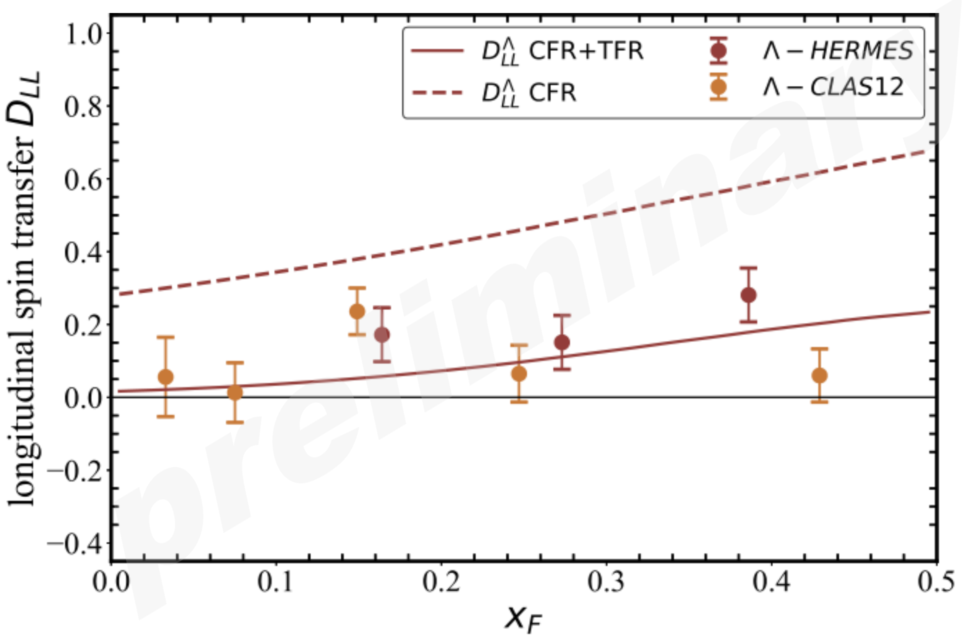


FIG. 9. Longitudinal spin transfer to Λ in SIDIS compared with HERMES and CLAS12. The red line represents the combined contributions of CFR and TFR, while dashed line solely from CFR.

Summary

- We derived the general form of cross section for spin-1/2 hadrons, and obtained expressions of structure functions at the leading twist in CFR and TFR.

- We studied the contribution from TFR to the Λ production in SIDIS and perform the estimation to quantitatively demonstrate the effect by diquark model.

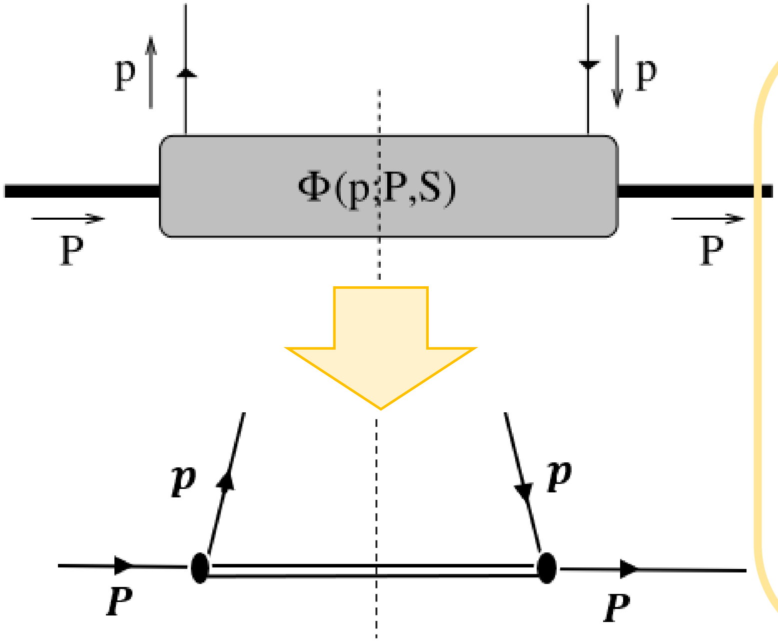
- We estimated the spin transfer D_{LL}, while considering the TFR, our calculation results can explain the COMPASS, HERMES and CLAS12 data reasonably.

Thank you!

Back up

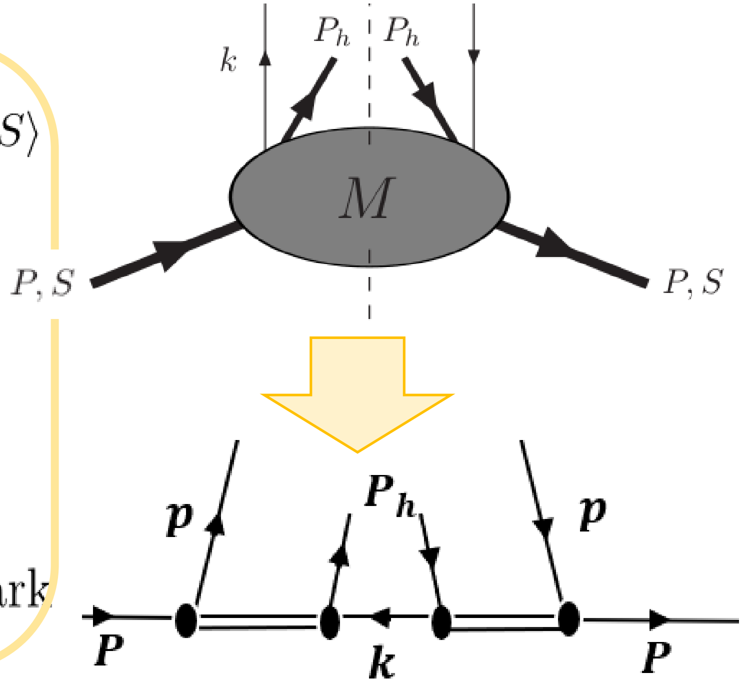
Spectator Model

R. P. Feynman, "Photon Hadron Interactions," New York 1972-01-01.



$$\Phi_{ij}^{R} = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3 2P_X^0} \langle P,S|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|P,S\rangle$$
$$* (2\pi)^4 \delta(P - p - P_X)$$

$$\langle P,S|\bar{\psi}(0)|X\rangle = \begin{cases} \bar{U}(P,S)\Upsilon_s \frac{i}{\not{p}-m_q}, & \text{scalar diquark} \\ \bar{U}(P,S)\Upsilon_a^\mu \frac{i}{\not{p}-m_q}\epsilon_\mu, & \text{axial-vector diquark} \end{cases}$$



nucleon-quark-diquark vertex: $\Upsilon_s = i g_s(p^2)\mathbf{1}, \quad \Upsilon_a^\mu = i\frac{g_a(p^2)}{\sqrt{2}}\gamma^\mu\gamma_5$

$$g_X(p^2) = g_X^{\exp} e^{(p^2-m^2)/\Lambda_X^2} \quad \text{exponential,}$$

the polarization sum: $d_{\mu\nu} = \sum \epsilon_\mu^*(P-p)\epsilon_\nu(P-p) = -g_{\mu\nu} + \frac{(P-p)_\mu (P-p)_\nu}{(P-p)^2}$

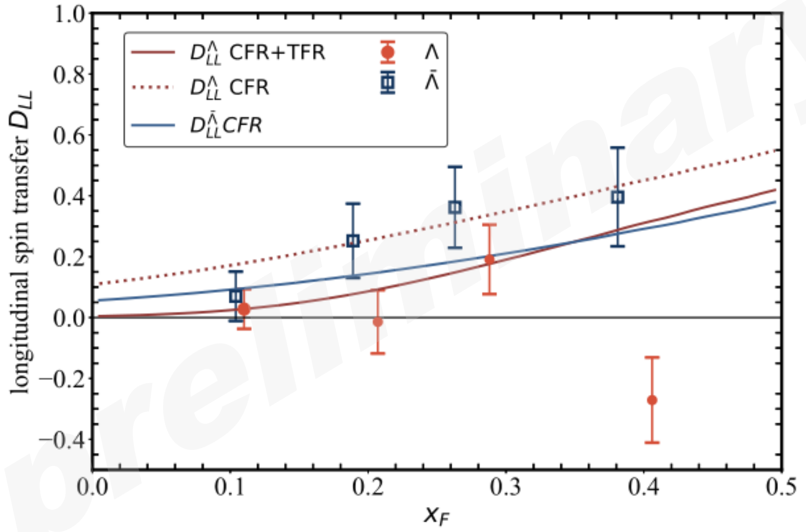
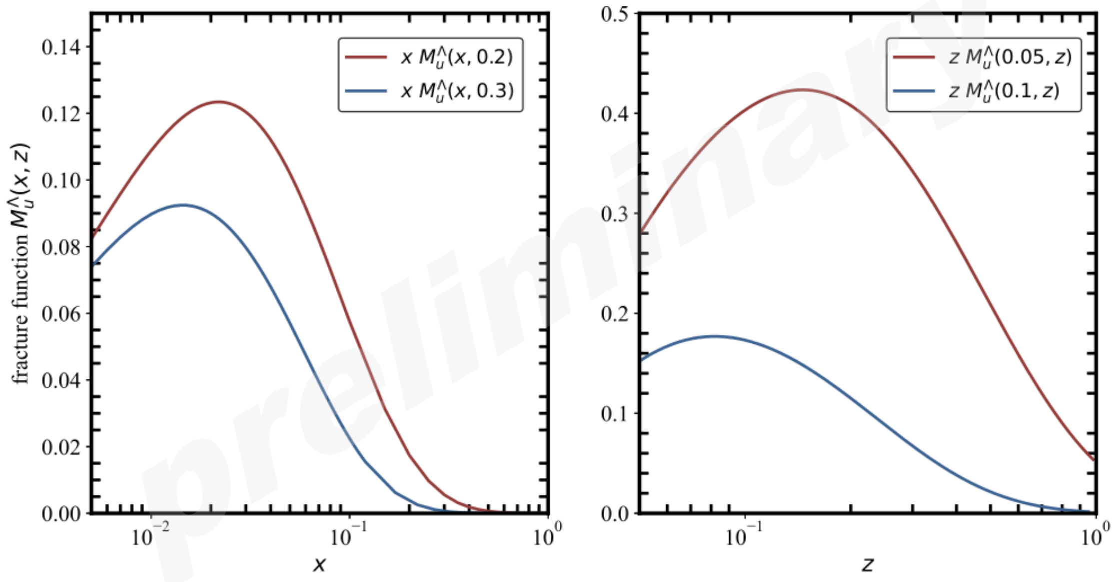


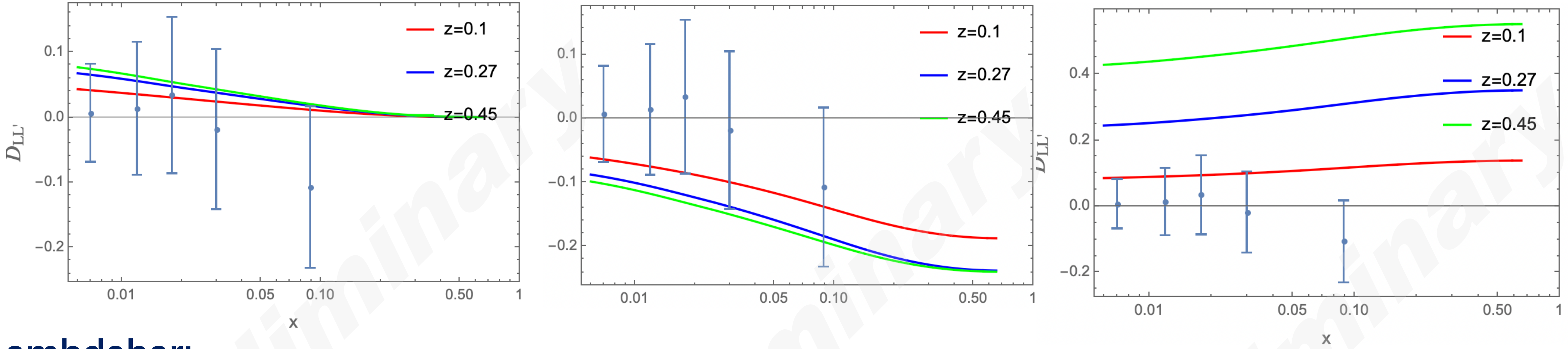
FIG. 8. The comparison of our result of the x_F-dependent longitudinal spin transfer and the experimental data at COMPASS. Inputs of TFR nonzero values are considered for Λ, as shown in the red line. The dashed and blue line represents the result in CFR for Λ and $\bar{\Lambda}$ respectively.



DSV scenario1 | DSV scenario2 | DSV scenario3

Lambda:

longitudinal spin tranfer $D_{LL'}$ CFR



Lambdabar:

longitudinal spin tranfer $D_{LL'}$ CFR