TMD phenomenology with the HSO approach

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OLD DOMINION UNIVERSITY

Based on

- The resolution to the problem of consistent large transverse momentum in TMDs (PhysRevD.107.094029)
  - (J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers)
- Combining nonperturbative transverse momentum dependence with TMD evolution (PhysRevD.106.034002)
  - (J. O. Gonzalez-Hernandez, T. C. Rogers, N. Sato)
- Basics of factorization in a scalar Yukawa field theory (PhysRevD.107.074031)

(F. Aslan, L. Gamberg, J.O. Gonzalez-Hernandez, T. Rainaldi, and T.C. Rogers)

Why TMDs?

Drell-Yan

SIDIS

$e^+ e^-$ --> H_a H_b X

Studying the role of intrinsic or **nonperturbative effects** in hadrons

Predicting transverse momentum distributions in **cross sections** after evolution to **high energies**

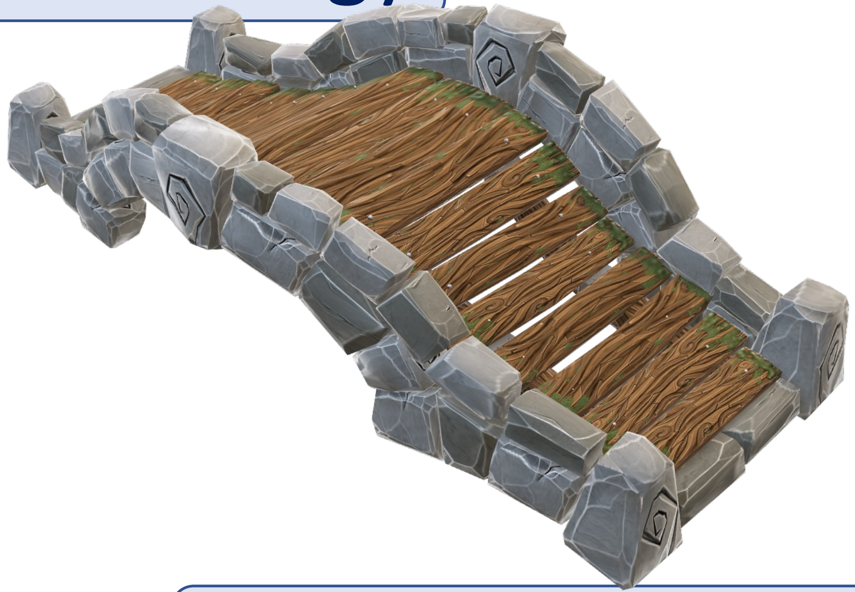
Factorization theorems

Universality

Evolution equations

Where TMDs?

High energy phenomenology



Nonperturbative hadron structure

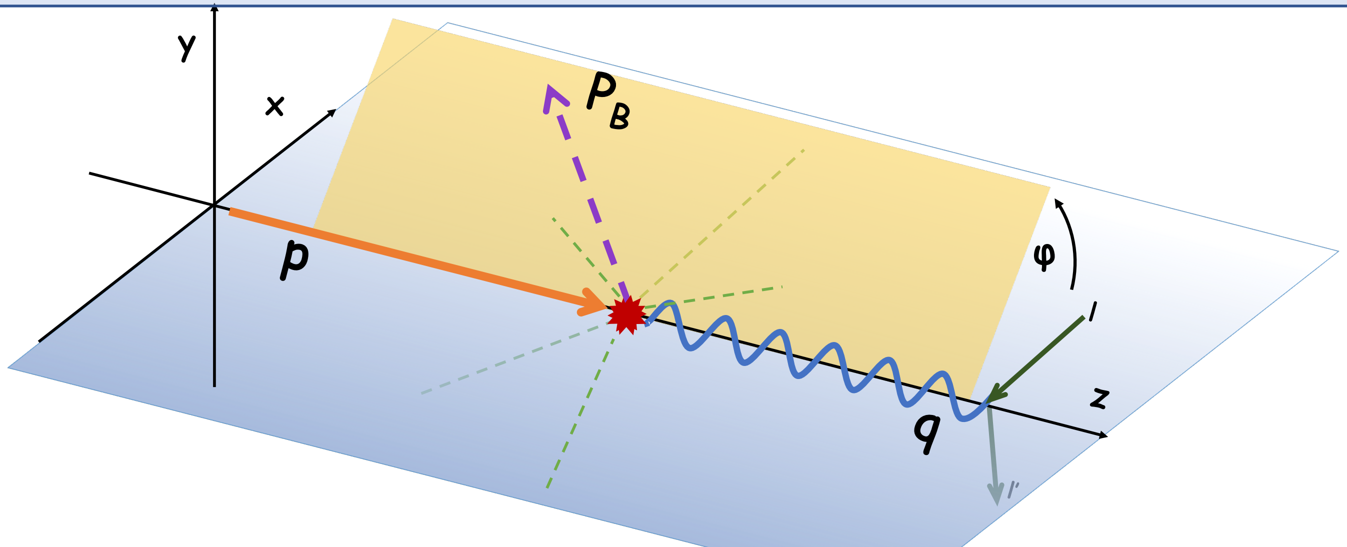
SIDIS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}q_{\mathrm{T}}^{2}} = \underbrace{W_{\mathrm{SIDIS}}}_{q_{\mathrm{T}} \ll Q} + \underbrace{Y_{\mathrm{SIDIS}}}_{q_{\mathrm{T}} \sim Q} + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

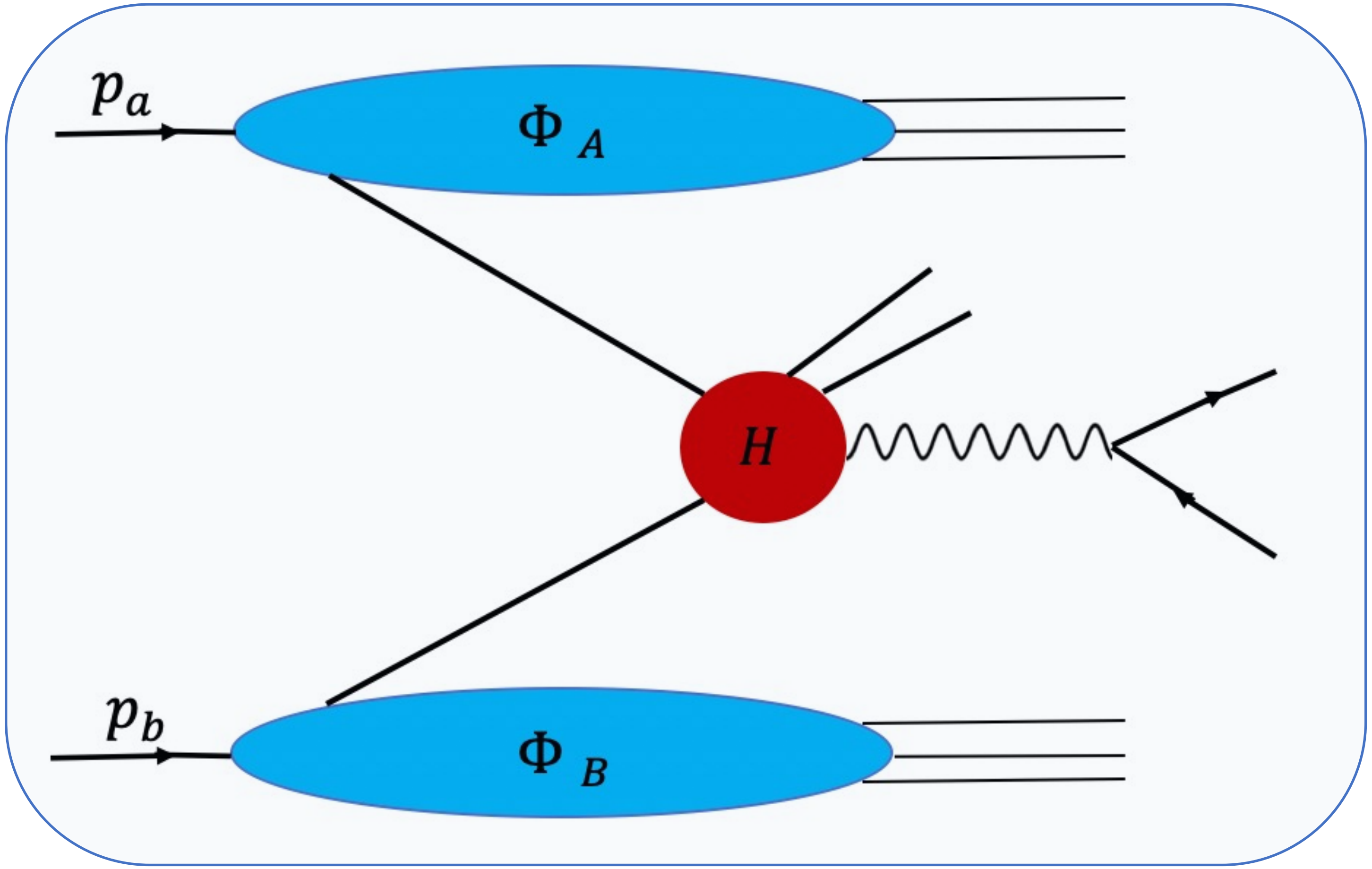
FO collinear factorization

$$\boxed{\mathrm{FO}_{\mathrm{SIDIS}} - \mathrm{ASY}_{\mathrm{SIDIS}}}$$

$$\boxed{H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}} \mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right) D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right) \delta^{(2)}\left(\boldsymbol{q}_{\mathrm{T}} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)}$$



Drell-Yan



$$W^{\mu\nu}_{\mathrm{DY}}\left(x_a, x_b, Q, \boldsymbol{q}_T\right) =$$

$$= \sum_j H^{\mu\nu}_{j,\mathrm{DY}} \int \frac{\mathrm{d}^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \tilde{f}_{j/h_a}\left(x_a, \boldsymbol{b}_T; \mu_Q; Q^2\right) \tilde{f}_{\bar{j}/h_b}\left(x_b, \boldsymbol{b}_T; \mu_Q; Q^2\right)$$

$$+ (a \longleftrightarrow b) + \mathcal{O}\left(\frac{q_T}{Q}, \frac{m}{Q}\right)$$

Conventional approach

Final parametrization of a TMD

$$
\begin{aligned}
\tilde{f}_{j/p}\left(x; \boldsymbol{b}_{\mathrm{T}}; \mu_Q, Q\right) = {} & \tilde{f}^{\mathrm{OPE}}_{j/p}\left(x; \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}\right) \times \\
& \times \exp\left\{ \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[\gamma\left(\alpha_S(\mu'); 1\right) - \ln\left(\frac{Q}{\mu'}\right) \gamma_K\left(\alpha_S(\mu')\right) \right] + \ln\left(\frac{Q}{\mu_{b_*}}\right) \tilde{K}\left(\boldsymbol{b}_*; \mu_{b_*}\right) \right\} \\
& \times \exp\left\{ -g_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}\right) - g_K\left(\boldsymbol{b}_{\mathrm{T}}\right) \ln\left(\frac{Q}{Q_0}\right) \right\}
\end{aligned}
$$

Nonperturbative

Perturbatively calculable

Drop this

$$
\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}\right) = \widetilde{C}_{j/j'}\left(x/\xi, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}\right) \otimes \tilde{f}_{j'/p}\left(\xi; \mu_{b_*}\right) + \mathcal{O}\left(\cancel{m^2 b^2_{\mathrm{max}}}\right)
$$

Same for FF

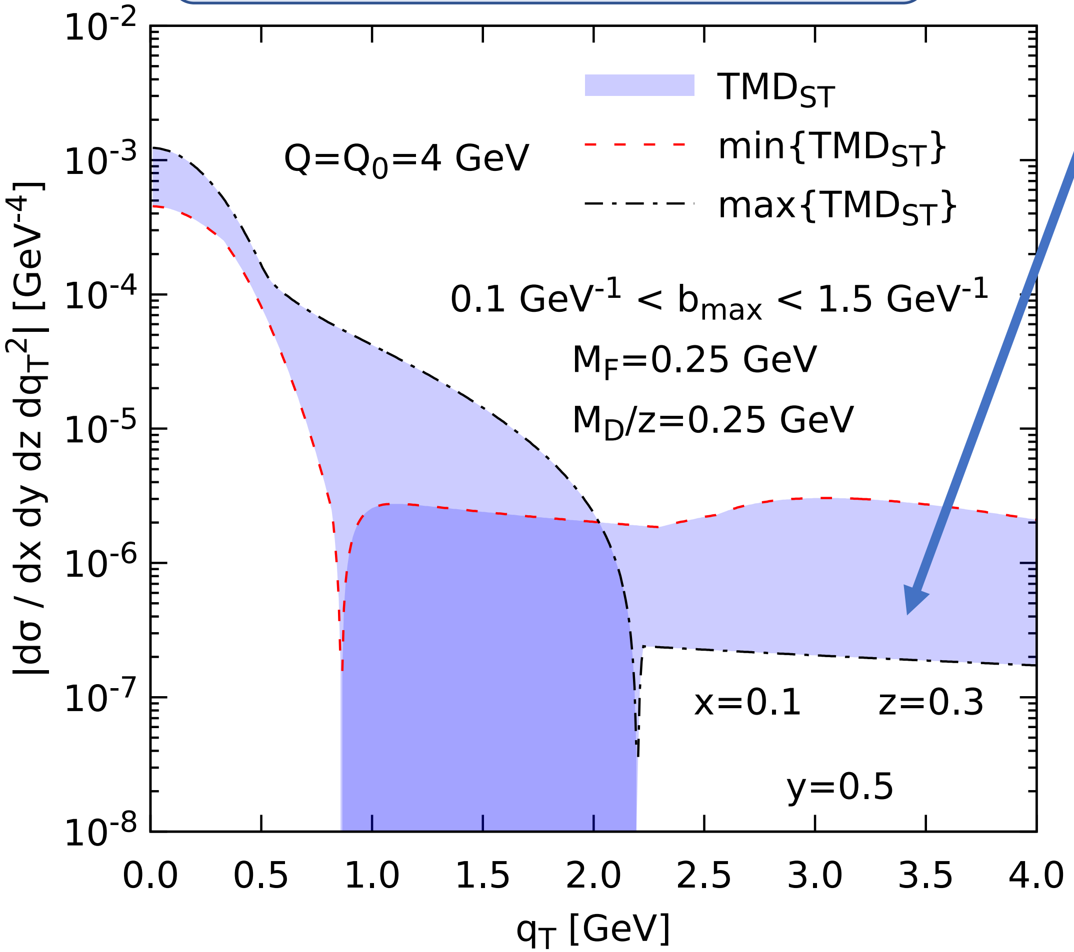
Fixed order collinear factorization

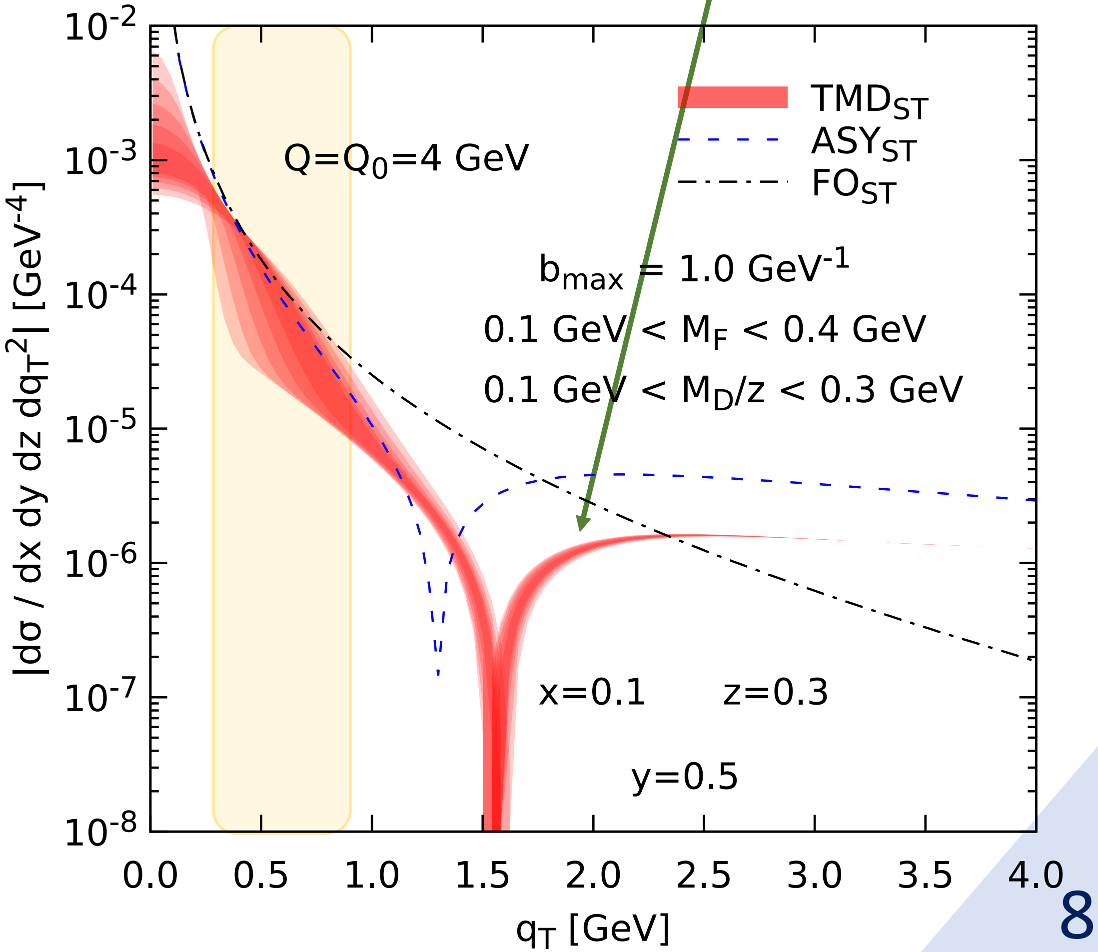
(Some) Issues with conventional approach

Large b_{max} dependence

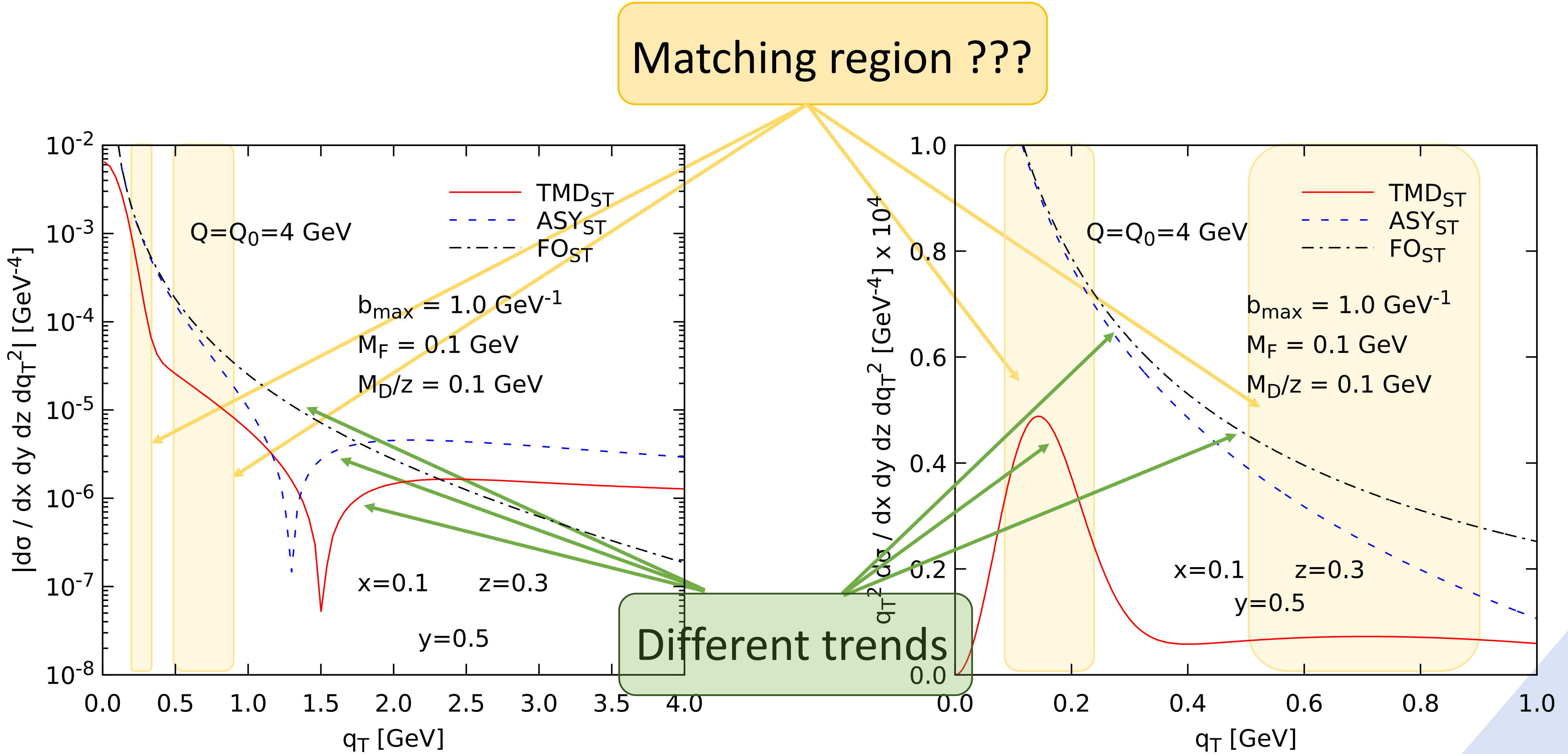
What is going on?

Large q_T inconsistency



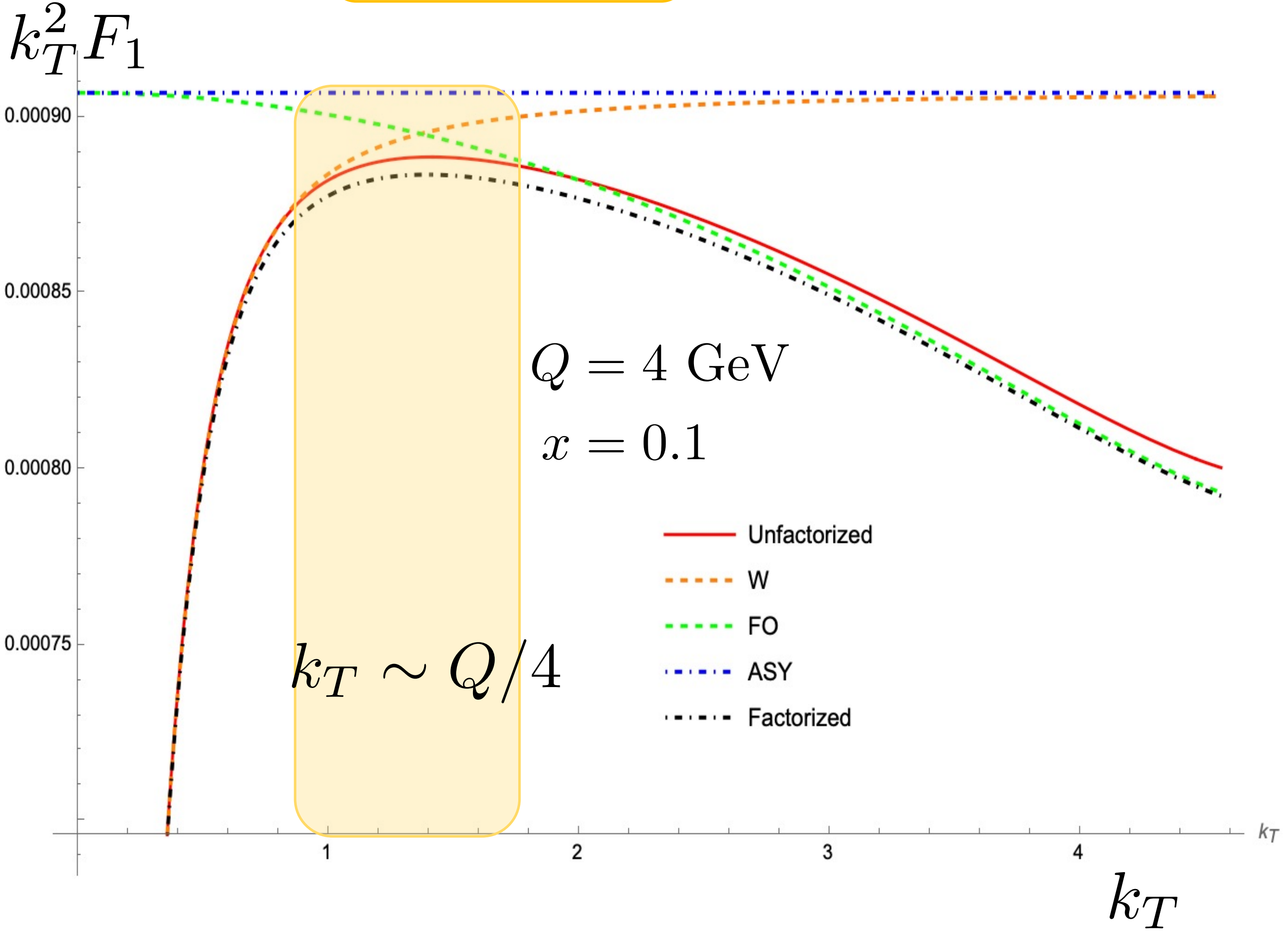


Conventional approach results for SIDIS

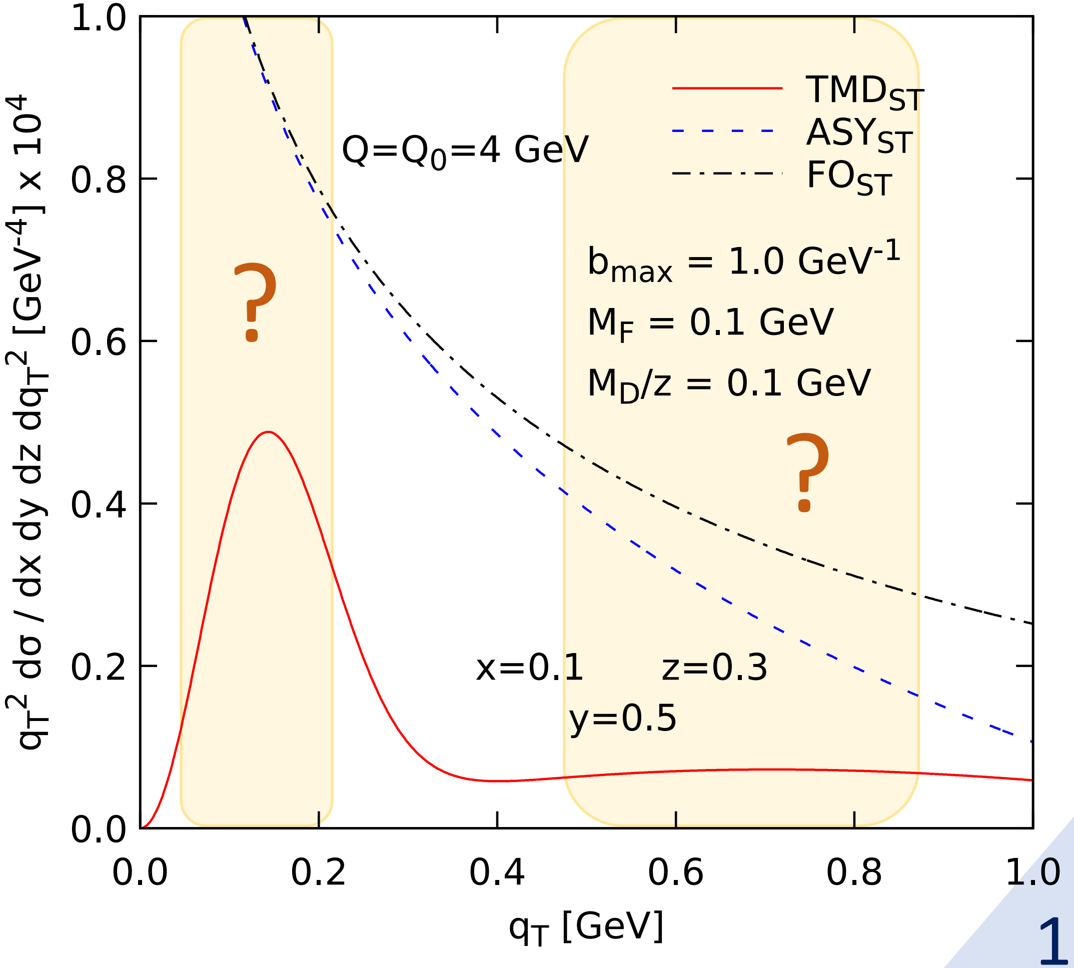


Matching region in SIDIS cross section?

Yukawa

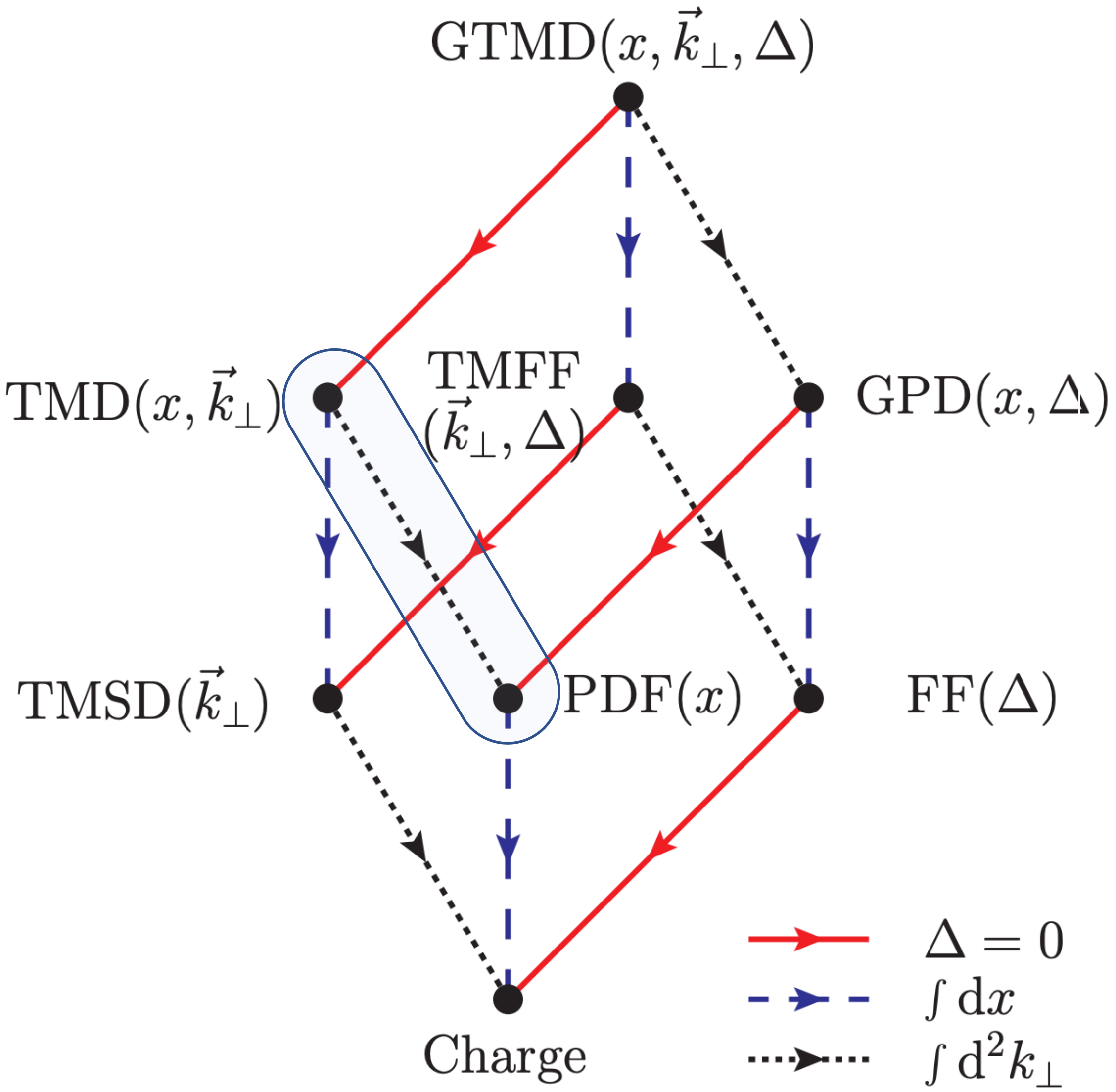


QCD (conventional)



Integral relations?

The net of distribution functions



Most of these integrals are divergent.
A more careful treatment is necessary

Credits: Lorcé, Pasquini and Vanderhaeghen

Consistency is the way

TMDs are uniquely determined by their operatorial definition

$$H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} \boxed{f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right)} \boxed{D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right)} \delta^{(2)}\left(\boldsymbol{q}_\mathrm{T} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)$$

Consistency is the way

At large $q_T \sim Q$ the cross section is determined solely by fixed order collinear factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\boldsymbol{q}_{\mathrm{T}}\dots} \overset{q_T \sim Q}{\sim} H(q_T) \otimes f \otimes d$$

Collinear PDFs/FFs

Consistency is the way

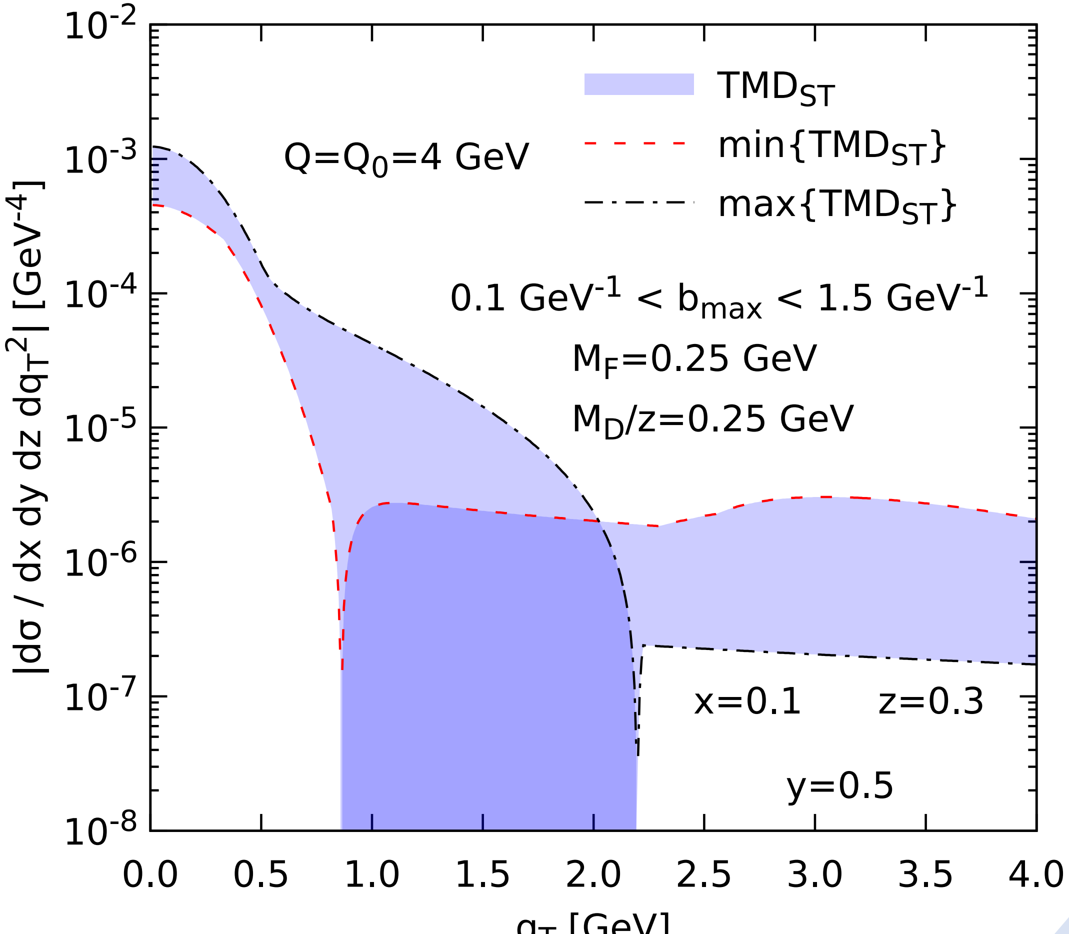
Similarly, at large TM (k_T)/ small b_T the TMDs are uniquely determined by an OPE expansion in terms of collinear PDFs/FFs

$$f_{i/H}(x, b_T; \mu, \zeta) = \widetilde{C}_{ij}(x, b_T; \mu, \zeta) \otimes f_{j/H}(x; \mu) + \mathcal{O}(mb_T)$$

Consistency is the way

Any auxiliary parameter cannot change the Physics (b_{max} /b_{min})

$$\frac{d}{db_{\max}} \frac{d\sigma}{dq_T^2 \dots} = 0$$



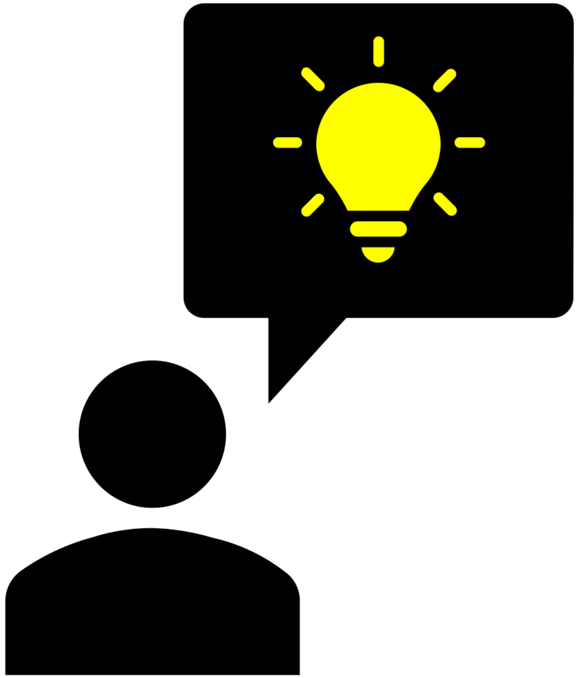
So what?

The former are generally either not imposed or violated

Why don’t we impose them in the parametrization at the **input scale**?

Evolution factor

$$f_{i/H}(x, b_T; \mu, \mu^2) = f_{i/H}(x, b_T; \mu_0, \mu_0^2) E(b_T; \mu/\mu_0)$$



The key points of the HSO approach

- Consistency: **integral relation** connecting TMD with collinear distributions even at moderate Q
- **Match large transverse momentum asymptotic behavior** that is dictated by the operator definitions
- Controlled transition between perturbative and nonperturbative descriptions of transverse momentum dependence. **No b_{max}**
- Fit at input scale and then evolve to higher scales to **minimize uncertainties**
- No global fits: experimental **data not all on the same footing**

TMD PDF HSO parametrization at input scale

Fixed order collinear factorization

$\mathcal{O}\left(\alpha_S\right)$

Large k_T OPE coefficients

$$f_{\mathrm{inpt,i/p}}(x, \boldsymbol{k}_T; \mu_{Q_0}; Q_0^2) = \frac{1}{2\pi}\frac{1}{k_T^2 + m^2}\left[A^f_{i/p}(x;\mu_{Q_0}) + B^f_{i/p}(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_T^2+m^2}\right] + \frac{1}{2\pi}\frac{1}{k_T^2+m^2}A^{f,g}_{i/p}(x;\mu_{Q_0})$$
$$+ \boxed{C^f_{i/p}} f_{\mathrm{core,i/p}}(x, \boldsymbol{k_T}; Q_0^2)$$

Small k_T model

NP parameters

Such that

$$f^c_{j/p}(x;\mu_Q) \equiv 2\pi\int_0^{k_c} \mathrm{d}k_\mathrm{T}\, k_\mathrm{T} f_{j/p}\left(x, \boldsymbol{k}_\mathrm{T}; \mu_Q, \sqrt{\zeta}\right)$$
$$= f_{j/p}(x;\mu_Q) + \Delta_{j/p}(x;\mu_Q, k_c) + \mathrm{p.s.}$$

Choose "core" models (examples)

$$f^{\text{Gauss}}_{\text{core},i/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right)=\frac{e^{-k_{\text{T}}^2/M_F^2}}{\pi M_F^2}$$

$$D^{\text{Gauss}}_{\text{core},h/j}\left(z,z\boldsymbol{k}_{\text{T}};Q_0^2\right)=\frac{e^{-z^2k_{\text{T}}^2/M_D^2}}{\pi M_D^2}$$

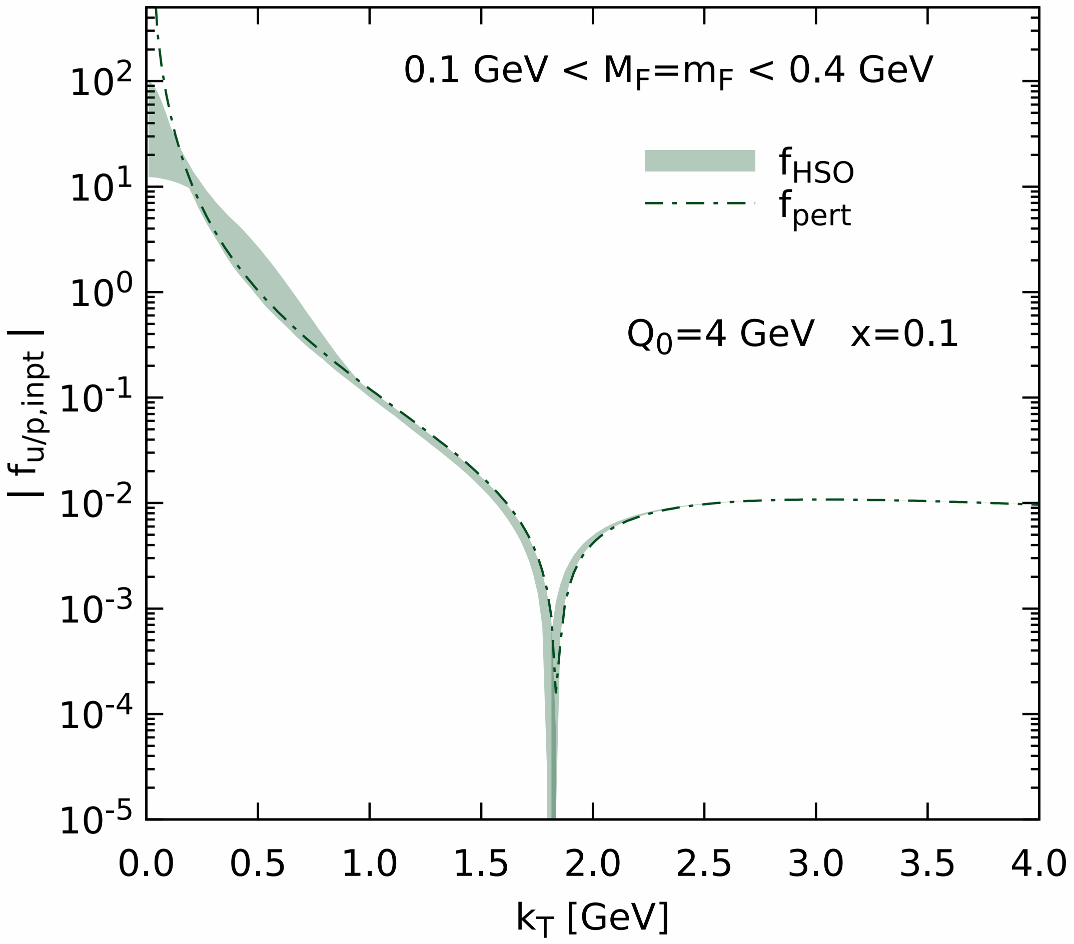
Gaussian "core" models

Spectator-like "core" models

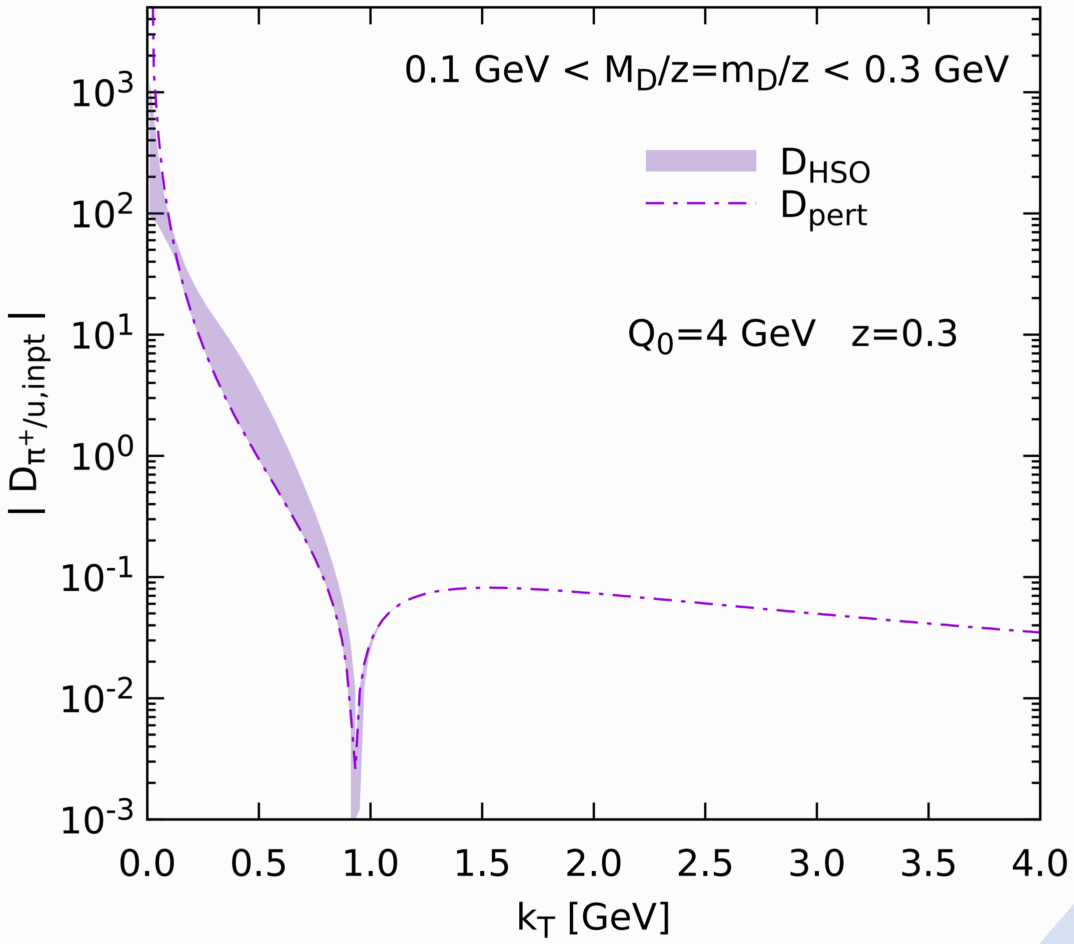
$$f^{\text{Spect}}_{\text{core},j/p}\left(x,\boldsymbol{k}_{\text{T}};Q_0^2\right)=\frac{6M_{0F}^6}{\pi\left(2M_F^2+M_{0F}^2\right)}\frac{M_F^2+k_{\text{T}}^2}{\left(M_{0F}^2+k_{\text{T}}^2\right)^4}$$

$$D^{\text{Spect}}_{\text{core},h/j}\left(z,z\boldsymbol{k}_{\text{T}};Q_0^2\right)=\frac{2M_{0D}^4}{\pi\left(M_D^2+M_{0D}^2\right)}\frac{M_D^2+z^2k_{\text{T}}^2}{\left(M_{0D}^2+z^2k_{\text{T}}^2\right)^3}$$

Up-quark from Proton TMD pdf



π^+ from Up-quark TMD ff



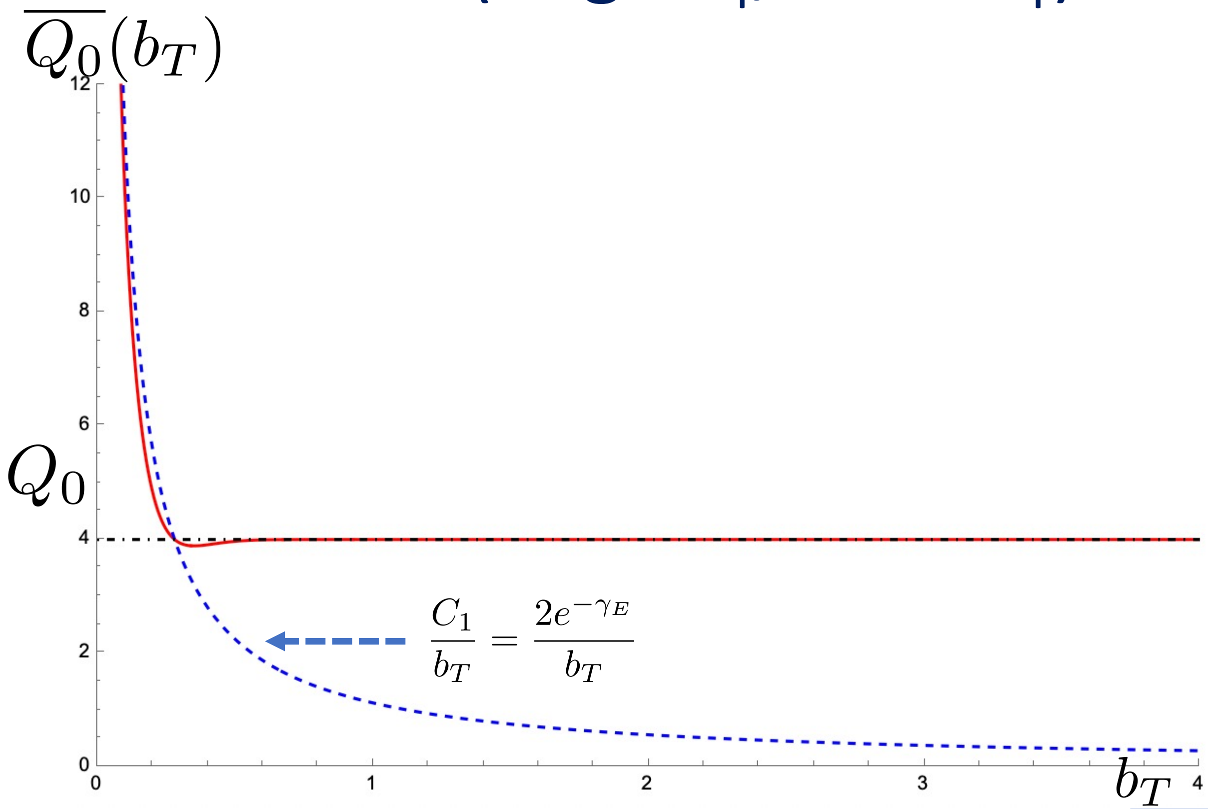
Evolution?

HSO Collins-Soper kernel at the input scale and RG improvement with $\overline{Q_0}(b_T)$ prescription.

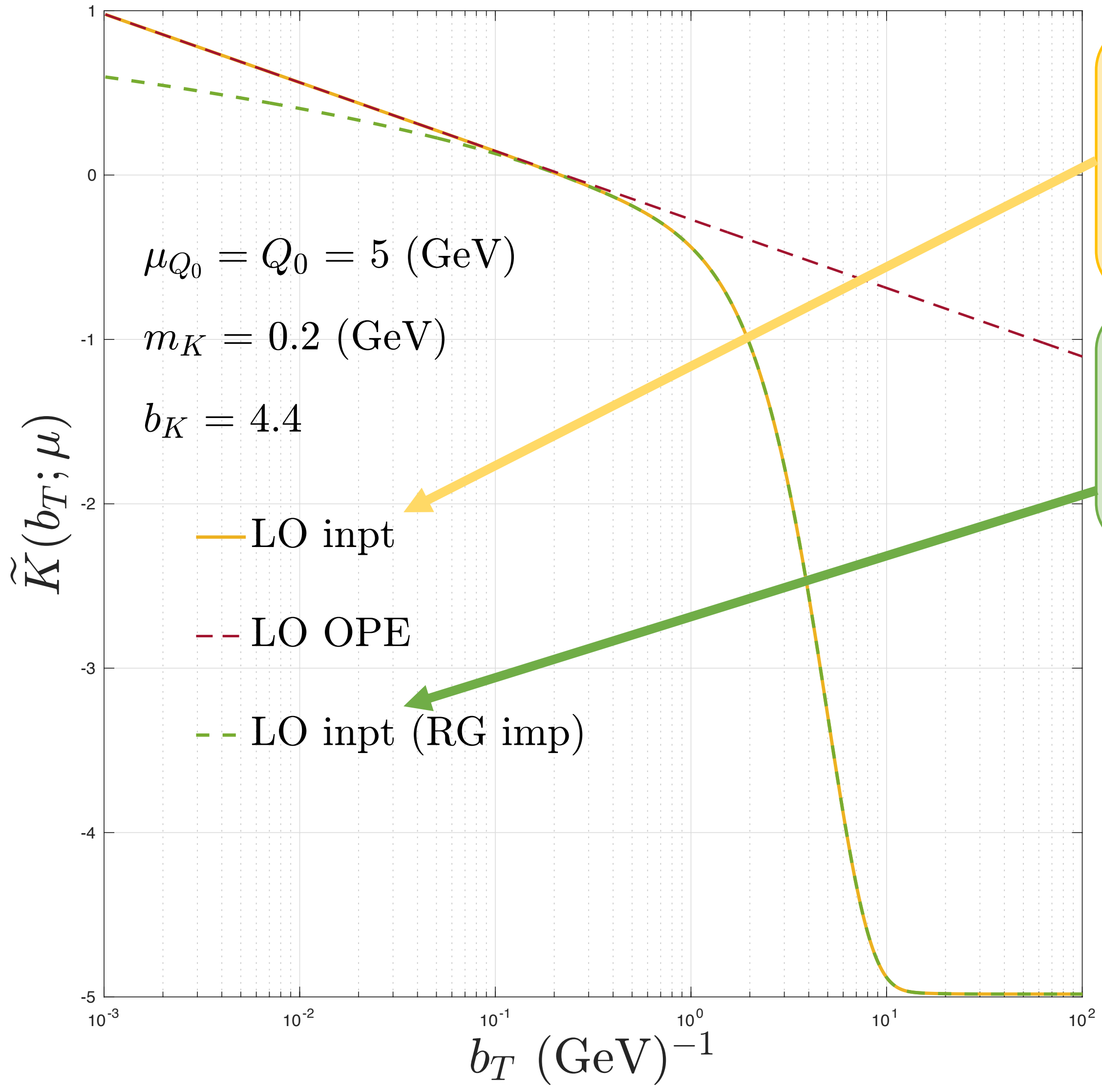
Match small b_T/large k_T with OPE and assign "core" model (large b_T/small k_T)

We need to change scheme

$$\overline{Q_0}\,(b_T, a) = Q_0 \left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$



RG improvements for CS-kernel (LO example)



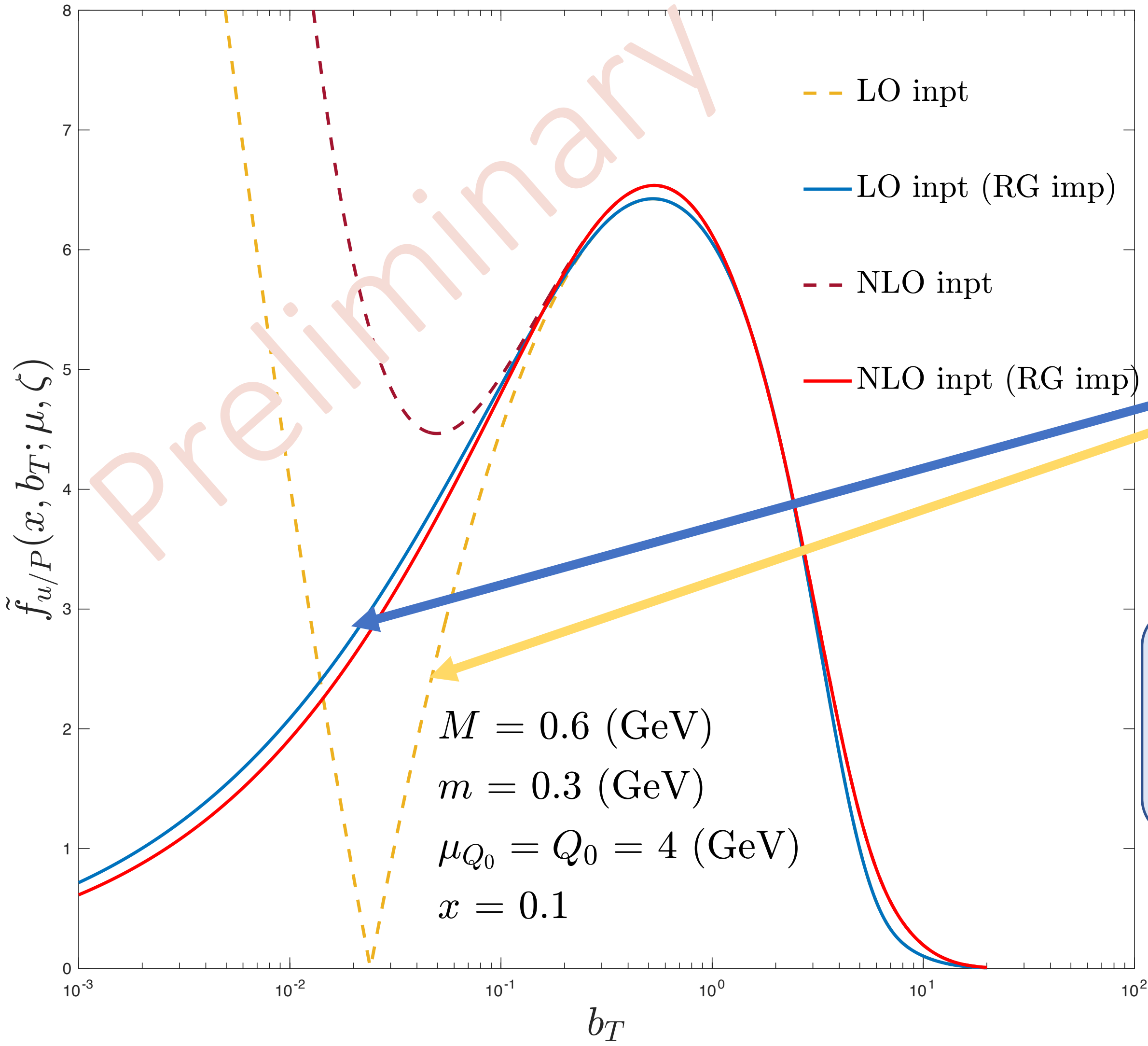
$$\widetilde{K}^{(LO)}_{\text{input}}(b_T;\mu_{Q_0}) = 2\pi A_K^{(1)}(\mu_{Q_0}) K_0(m_K b_T) + b_K\left(e^{-m_K^2 b_T^2} - 1\right) + D_K(\mu_{Q_0})$$

$$\underline{\widetilde{K}}(b_T;\mu_{Q_0}) \equiv \widetilde{K}(b_T;\mu_{\overline{Q_0}}) - \int_{\mu_{\overline{Q_0}}}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'}\gamma_K\left(a_S(\mu')\right)$$

A good approximation even for $b_T < 1/Q_0$

NO b_* and/or b_{max} / b_{min} necessary

Input scale RG improvement (no large logarithms)



INPUT SCALE 4 GeV

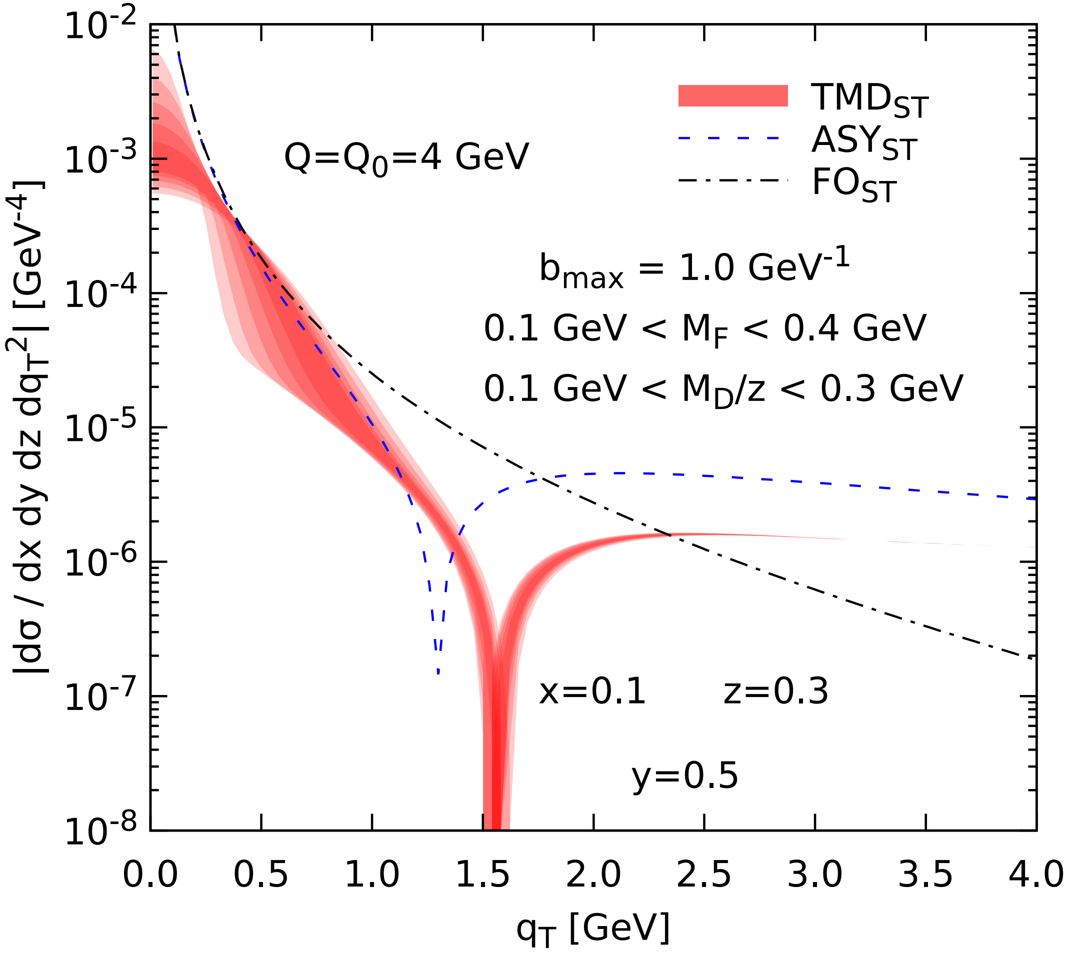
$$\overline{Q_0}\,(b_T, a) = Q_0 \left[1 - \left(1 - \frac{C_1}{Q_0 b_T}\right) e^{-a^2 b_T^2}\right]$$

$$\tilde{f}_i(x, b_T; \mu_{Q_0}, Q_0^2) = \tilde{f}_{i,\text{input}}(x, b_T; \mu_{\overline{Q_0}}, \overline{Q_0}^2)\times$$
$$\exp\left\{\int_{\mu_{\overline{Q_0}}}^{\mu_{Q_0}} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(a_S(\mu')\right) - \ln\frac{Q_0}{\mu'}\gamma_K\left(a_S(\mu')\right)\right] + \ln\frac{Q_0}{\overline{Q_0}}\widetilde{K}_{\text{input}}\left(b_T; \mu_{\overline{Q_0}}\right)\right\}$$

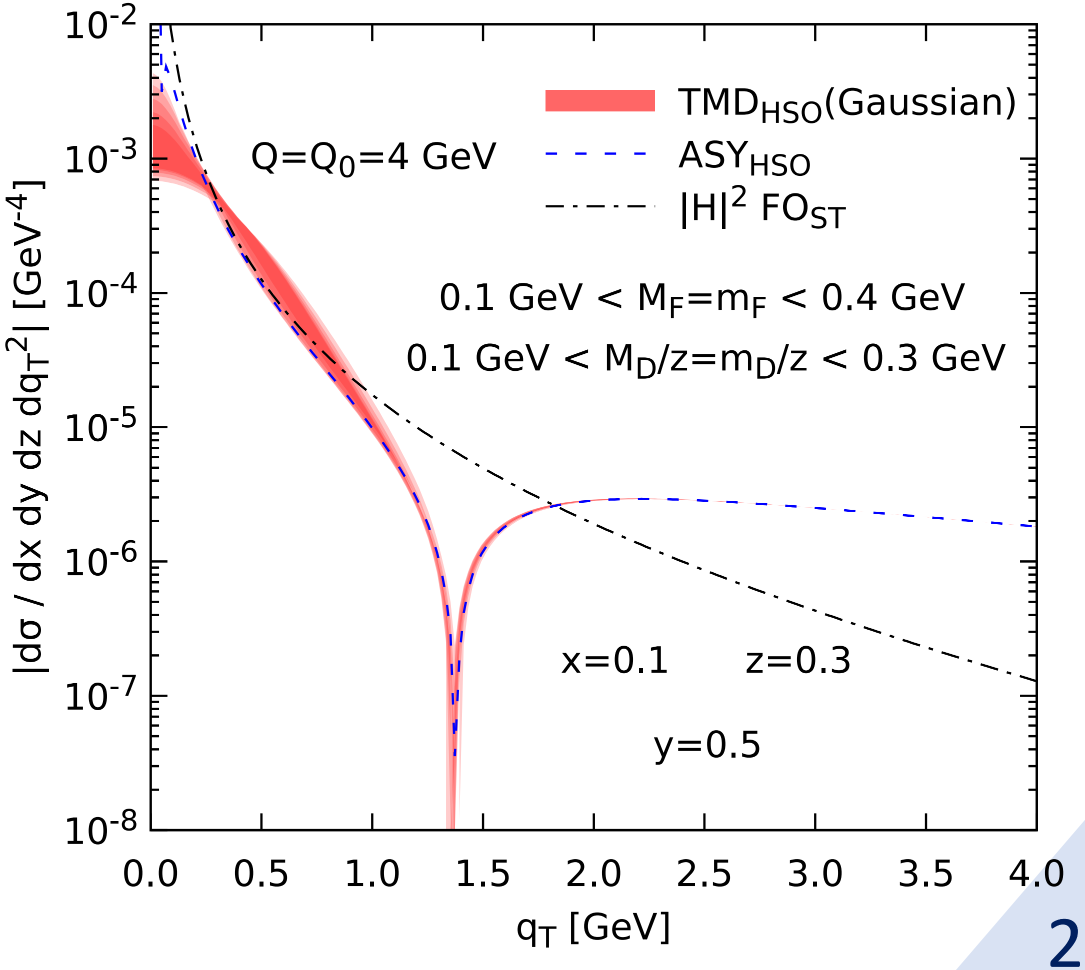
Some results

Conventional vs HSO - SIDIS cross section (not a fit)

Conventional



HSO (Gaussian)

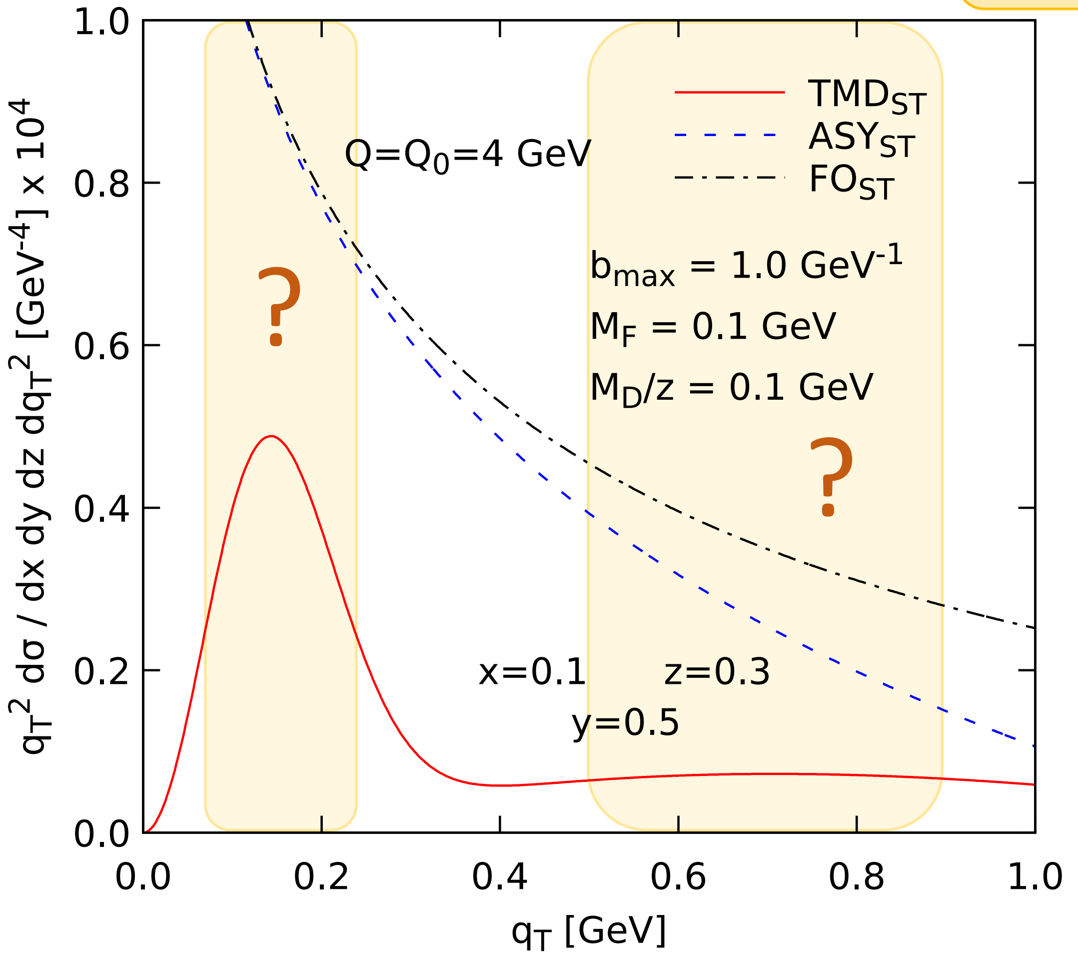


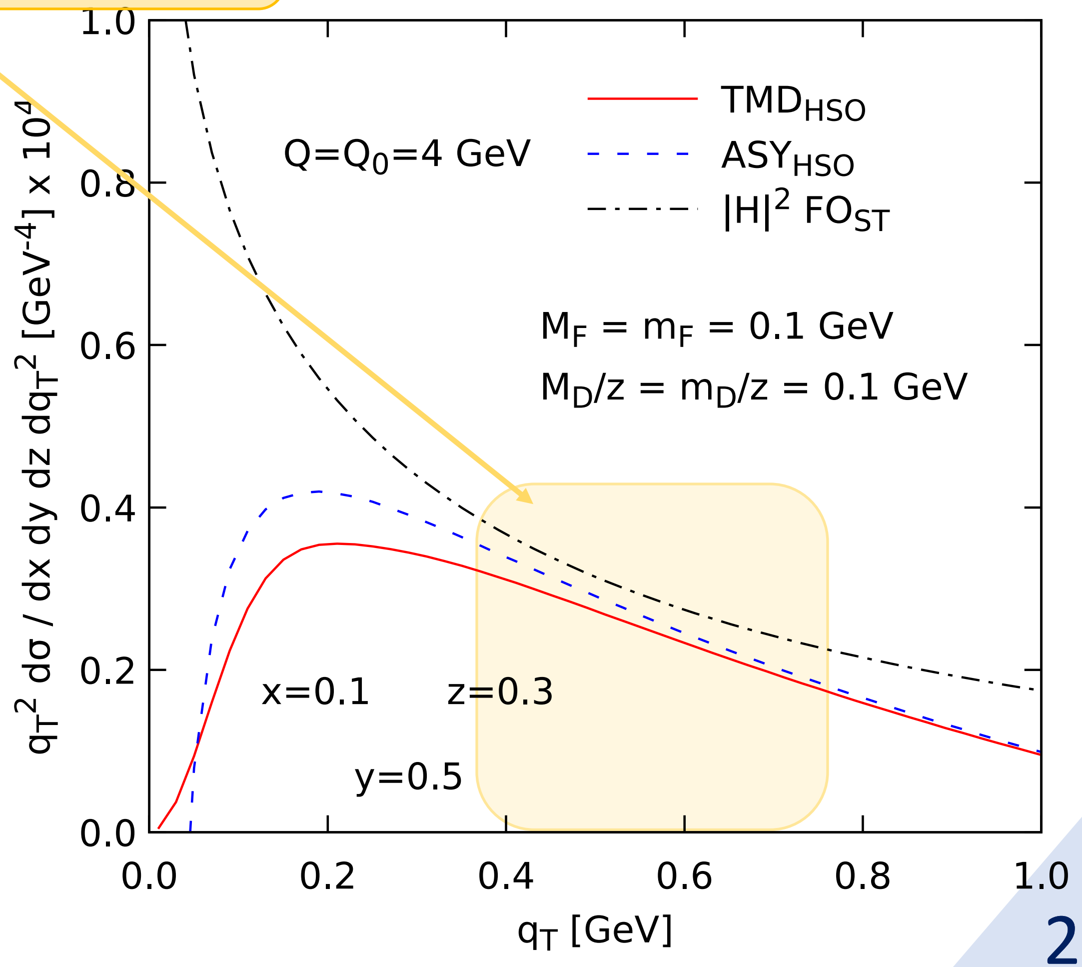
Conventional vs HSO - SIDIS cross section (not a fit)

Conventional

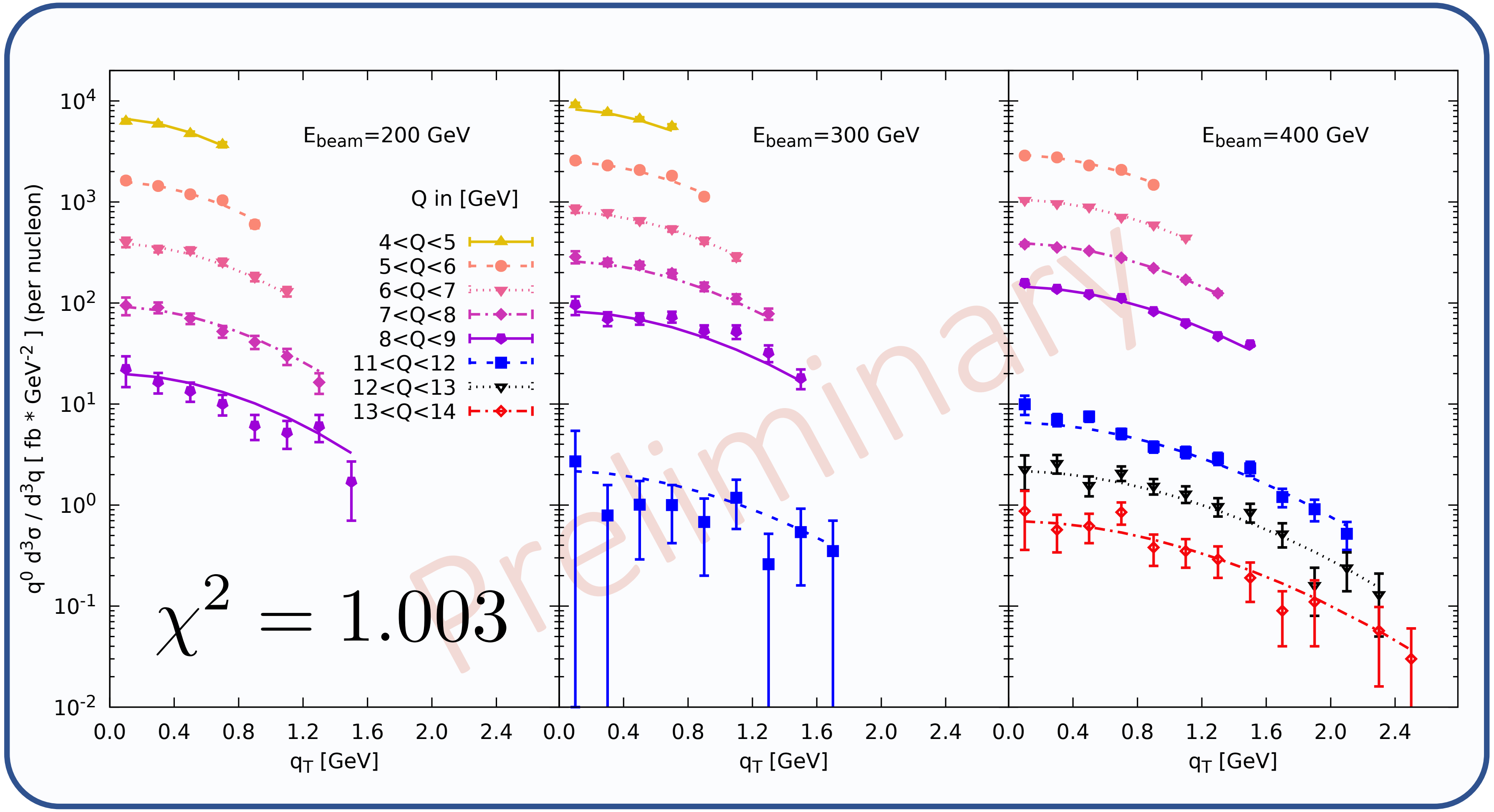
Matching region !!!

HSO (Gaussian)





Work in progress… (DY fit)



Drell-Yan

E288 experiment

$$p + \{Cu, Pt, Be\} \rightarrow \gamma \rightarrow \mu^+ + \mu^-$$

Looks promising ☺

CS kernel: 2 parameters
Input TMD: 2-3 parameters
Normalization

Summary

- Consistent TMD parametrization for large TM at input scale
- HSO = CSS (with explicit constraints)
- No need of b_{max}
- Improved TM behavior in matching region
- Easily swappable NP models

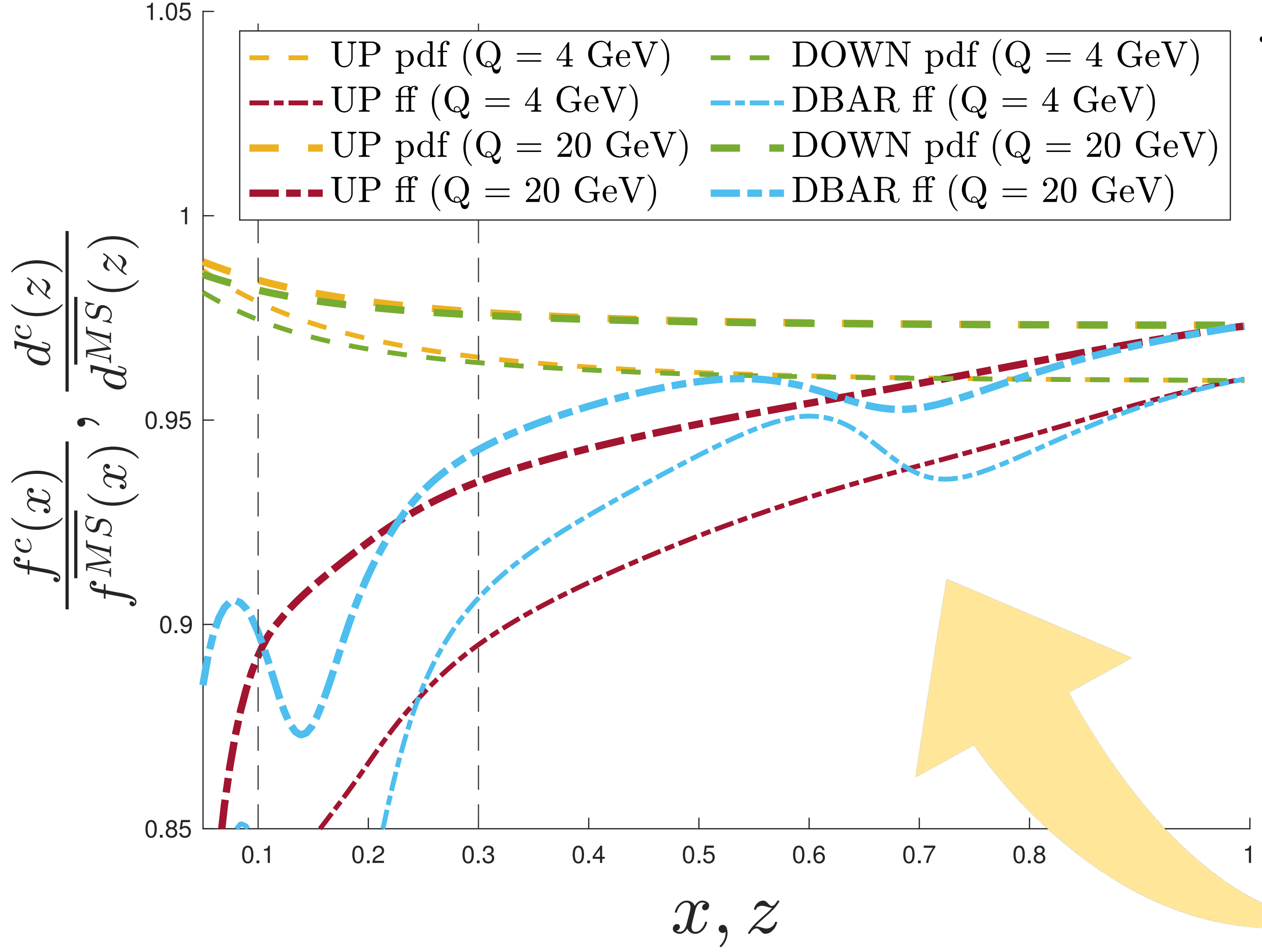
NEXT/SOON:

- Check with data (SIDIS, **DY**, DIA, …)
- Add higher orders (NLO done)
- Incorporate NP calculations (lattice, EFT, …)

Thank you

Backup slides

Pseudo-probability distribution property saved



$$f_i^c(x;\mu,k_c) \equiv \pi \int_0^{k_c^2} \mathrm{d}k_T^2 f_{i/p,\text{input}}(x,k_T;\mu,\zeta)$$

$$= f_i + C_{ij,\Delta}^c \otimes f_j + p.s.$$

$\overline{\text{MS}}$ PDF

Completely determined by OPE expansion coefficients

It might make a BIG difference

Check: the RG equations are satisfied

$$\frac{\partial \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\partial \ln \sqrt{\zeta}} = \tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\mathrm{d} \ln \mu} = \gamma\left(\alpha_S(\mu); \mu/\sqrt{\zeta}\right)$$

$$\frac{\mathrm{d}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)}{\mathrm{d} \ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$

Conventional approach :

1

$$H \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}} f_{j/p}\left(x, \boldsymbol{k}_{1\mathrm{T}}; \mu, \sqrt{\zeta}\right) D_{h/j}\left(z, z\boldsymbol{k}_{2\mathrm{T}}; \mu, \sqrt{\zeta}\right) \delta^{(2)}\left(\boldsymbol{q}_\mathrm{T} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}}\right)$$

Fourier Transform

$$H \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{-i\boldsymbol{b}_\mathrm{T}\cdot\boldsymbol{q}_\mathrm{T}} \tilde{f}_{j/p}\left(x, \boldsymbol{b}_\mathrm{T}; \mu, \sqrt{\zeta}\right) \tilde{D}_{h/j}\left(z, \boldsymbol{b}_\mathrm{T}; \mu, \sqrt{\zeta}\right)$$

Solve evolution equations relating input scale with SIDIS scale

2

$$\frac{\partial \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\partial \ln \sqrt{\zeta}} = \tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)$$

$$\frac{\mathrm{d} \ln \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right)}{\mathrm{d} \ln \mu} = \gamma\left(\alpha_S(\mu); \mu/\sqrt{\zeta}\right)$$

$$\frac{\mathrm{d}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu\right)}{\mathrm{d} \ln \mu} = -\gamma_K\left(\alpha_S(\mu)\right)$$

Same for FF

$$\mu = \sqrt{\zeta}$$
$$\mu_0 = \sqrt{\zeta_0}$$

$$\tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu, \sqrt{\zeta}\right) = \tilde{f}_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}; \mu_0, \sqrt{\zeta_0}\right) \times$$
$$\times \exp\left\{\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu'); 1\right) - \ln\left(\frac{\sqrt{\zeta}}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right] + \ln\left(\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\tilde{K}\left(\boldsymbol{b}_{\mathrm{T}}; \mu_0\right)\right)\right\}$$

Separate $b_T < b_{max}$ & $b_T > b_{max}$ regions with a b_* prescription

3

$$\tilde{f}_{j/p}\left(x;\boldsymbol{b}_\mathrm{T};\mu_Q,Q\right) = \boxed{\tilde{f}_{j/p}\left(x;\boldsymbol{b}_*;\mu_Q,Q\right)} \boxed{\underbrace{\frac{\tilde{f}_{j/p}\left(x;\boldsymbol{b}_\mathrm{T};\mu_Q,Q\right)}{\tilde{f}_{j/p}\left(x;\boldsymbol{b}_*;\mu_Q,Q\right)}}_{\exp\left\{-g_{j/p}(x,\boldsymbol{b}_\mathrm{T})\right\}}}$$

Same for FF

Perturbatively calculable with fixed order collinear factorization

Nonperturbative

$$g_K(\boldsymbol{b}_\mathrm{T}) \equiv \tilde{K}\left(\boldsymbol{b}_*;\mu\right) - \tilde{K}\left(\boldsymbol{b}_\mathrm{T};\mu\right)$$

Choose ansatzes for g functions

4

$$g_{j/p}\left(x, \boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{4} M_F^2 b_{\mathrm{T}}^2$$

$$g_{h/j}\left(z, \boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{4z^2} M_D^2 b_{\mathrm{T}}^2$$

$$g_K\left(\boldsymbol{b}_{\mathrm{T}}\right) = \frac{g_2}{2M_K^2} \ln\left(1 + M_K^2 b_{\mathrm{T}}^2\right)$$

$$g_K\left(\boldsymbol{b}_{\mathrm{T}}\right) = \frac{1}{2} M_K^2 b_{\mathrm{T}}^2$$

No constraints whatsoever

Relate μ_{b_*} with input scale Q_0 and get OPE expansion

5

$$\tilde{f}_{j/p}\left(x;\boldsymbol{b}_\mathrm{T};\mu_Q,Q\right) = \boxed{\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x;\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)} \times$$

$$\times \boxed{\exp\left\{\int_{\mu_{b_*}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma\left(\alpha_S(\mu');1\right) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K\left(\alpha_S(\mu')\right)\right] + \ln\left(\frac{Q}{\mu_{b_*}}\right)\tilde{K}\left(\boldsymbol{b}_*;\mu_{b_*}\right)\right\}}$$

$$\times \boxed{\exp\left\{-g_{j/p}\left(x,\boldsymbol{b}_\mathrm{T}\right) - g_K\left(\boldsymbol{b}_\mathrm{T}\right)\ln\left(\frac{Q}{Q_0}\right)\right\}}$$

Nonperturbative

Perturbatively calculable

Drop this

$$\boxed{\tilde{f}^{\mathrm{OPE}}_{j/p}\left(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right) = \widetilde{C}_{j/j'}\left(x/\xi,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}\right)\otimes\tilde{f}_{j'/p}\left(\xi;\mu_{b_*}\right) + \cancel{\mathcal{O}\left(m^2 b^2_{\max}\right)}}$$

Same for FF

Fixed order collinear factorization

Collinear Evolution

$$\text{Note}: \lim_{a_S \to 0} C^c_\Delta = 0$$

$$\frac{\mathrm{d} f^c_i}{\mathrm{d}\ln\mu} \equiv \boxed{2P^c_{ij} \otimes f^c_j} + \boxed{p.s.}$$

$$= \boxed{\boxed{2P_{ij} \otimes f_j} + \boxed{C^c_{\Delta,ij} \otimes 2P_{jk} \otimes f_k + \frac{\mathrm{d}C^c_{\Delta,ij}}{\mathrm{d}\ln\mu} \otimes f_j}} + \boxed{\frac{\mathrm{d}p.s.}{\mathrm{d}\ln\mu}}$$

Usual evolution

Additional term (scheme change)

Power suppressed

Asymptotic term

HSO

$$\mathrm{ASY_{HSO}} = \lim_{\frac{\mathrm{q_T}}{\mathrm{Q}} \to \sim 1, \frac{\mathrm{m}^2}{\mathrm{Q}^2} \to 0} \mathrm{W_{HSO}}$$

Conventional

$$\mathrm{ASY_{ST}} = \lim_{\frac{\mathrm{q_T}}{\mathrm{Q}} \to 0, \frac{\mathrm{m}^2}{\mathrm{Q}^2} \to 0} \mathrm{FO_{ST}}$$

$$\begin{aligned}
[f, D]_{\mathrm{ASY}} = {} & D^{\mathrm{pert}}\left(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) f^c\left(x; \mu_Q\right) + \frac{1}{z^2} f^{\mathrm{pert}}\left(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) d^c\left(z; \mu_Q\right) \\
& + \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}} \Big\{ f^{\mathrm{pert}}\left(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q, Q^2\right) D^{\mathrm{pert}}\left(z, z\left(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2\right); \mu_Q, Q^2\right) \\
& - D^{\mathrm{pert}}\left(z, z\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) f^{\mathrm{pert}}\left(x, \boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2; \mu_Q, Q^2\right) \Theta\left(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} - \boldsymbol{q}_{\mathrm{T}}/2|\right) \\
& - D^{\mathrm{pert}}\left(z, z\left(\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2\right); \mu_Q, Q^2\right) f^{\mathrm{pert}}\left(x, -\boldsymbol{q}_{\mathrm{T}}; \mu_Q, Q^2\right) \Theta\left(\mu_Q - |\boldsymbol{k}_{\mathrm{T}} + \boldsymbol{q}_{\mathrm{T}}/2|\right) \Big\}
\end{aligned}$$