New Developments in Di-Hadron Theory and Phenomenology

(Andreas Metz, Temple University)

Part 1 Definition, interpretation and evolution of di-hadron fragmentation functions (DiFFs)

(D. Pitonyak, C. Cocuzza, A. Metz, A. Prokudin, N. Sato, 2305.11995)

Part 2 Simultaneous global analysis of DiFFs, transversity PDFs, and tensor charges

(C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl, 2306.12998 / 2308.14857)

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Motivation

- In spin physics, DiFFs relevant for extraction of transversity h_1^q
- Access to transversity using chiral-odd spin-dependent FFs
  - Single-hadron fragmentation (Collins effect) (Collins, 1992)

    $$h_1^q \otimes H_1^{\perp\, h/q}$$

    correlation btw transverse quark spin and transverse momentum of hadron,
    TMD factorization

  - Single-hadron fragmentation using collinear twist-3 factorization
    (Kang, Yuan, Zhou, 2010 / Metz, Pitonyak, 2012)

  - Di-hadron fragmentation (Collins, Heppelmann, Ladinsky, 1993)

    $$h_1^q \otimes H_1^{\sphericalangle\, h_1 h_2/q}$$

    correlation btw transverse quark spin and relative transverse momentum of (h_1, h_2),
    collinear twist-2 factorization

- Previous work on di-hadron production related to spin physics almost exclusively by Pavia Group

- Extraction of h_1^q from global analysis of di-hadron data (Radici, Bacchetta, 2018)
  - tensor charge

$$\delta q = \int_0^1 dx \left(h_1^q(x) - h_1^{\bar{q}}(x) \right) \qquad g_T = \delta u - \delta d$$

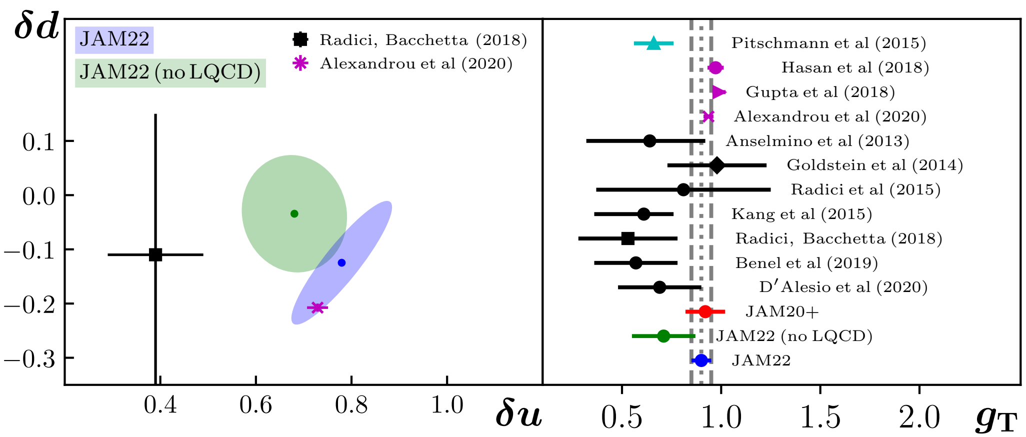


figure modified from arXiv:2205.00999 (JAM-3D)

for δu, some tension between di-hadron channel on the one hand, and single-hadron channel and lattice QCD on the other

- Independent numerical analysis of di-hadron channel well motivated

- We also revisited the definition, interpretation and evolution of DiFFs

Lessons from Single-Hadron Fragmentation Functions

- Process and frames

$$q(k) \to h(P_h) + X \qquad\qquad P_h^- = z k^- \ \text{large}$$

hadron frame: $\vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$

parton frame: $\vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \qquad\qquad (\vec{P}_{h\perp} = -z\vec{k}_T)$

- Definition and interpretation

$$D_1^{h/q}(z, z^2\vec{k}_T^2) = \frac{1}{4z}\int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h, X\rangle\langle h, X|\bar{\psi}_q(0)|0\rangle\,\gamma^-\Big]_{\xi^-=0}$$

$$= D_1^{h/q}(z, \vec{P}_{h\perp}^2)$$

  - $D_1^{h/q}(z, \vec{P}_{h\perp}^2)$ is number density (see, e.g., Collins, *Foundations of Perturbative QCD*)
  - $D_1^{h/q}(z, \vec{P}_{h\perp}^2)\, dz d^2\vec{P}_{h\perp}$ is number of hadrons h in $[z, z{+}dz]$, $[\vec{P}_{h\perp}, \vec{P}_{h\perp}{+}d^2\vec{P}_{h\perp}]$
  - factor $1/4z$ is crucial for interpretation as number density

- Collinear FF

$$D_1^{h/q}(z) = \int d^2\vec{P}_{h\perp}\, D_1^{h/q}(z, \vec{P}_{h\perp}^2)$$

- Number of hadrons in quark q

$$\sum_h \int dz\, D_1^{h/q}(z) = \langle N^q \rangle$$

- Momentum sum rule (Collins, Soper, 1981 / Meissner, Metz, Pitonyak, 2010)

$$\sum_h \int dz\, z\, D_1^{h/q}(z) = 1$$

- Leading-order cross section for $e^- e^+ \to hX$

$$\frac{d\sigma}{dz} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h/q}(z) \quad \text{with} \;\; \hat{\sigma}^q = \hat{\sigma}^{\bar{q}} = \hat{\sigma}(e^- e^+ \to \gamma^{(*)} \to q\bar{q}) = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

- Recent work on sum rules for FFs (Collins, Rogers, 2023)
  - does not put into question the definition of FFs

Definition and Interpretation of DiFFs

- Process and frames

$$q(k) \to h_1(P_1) + h_2(P_2) + X \qquad\qquad P_h^- = z k^- \ \ \text{large}$$

$$P_h = P_1 + P_2 \qquad R = \frac{1}{2}(P_1 - P_2) \qquad P_{1,2}^- = z_{1,2}\, k^- \qquad z = z_1 + z_2$$

hadron frame: $\vec{P}_{hT} = 0 \quad \vec{k}_T \neq 0$

parton frame: $\vec{P}_{h\perp} \neq 0 \quad \vec{k}_\perp = 0 \qquad\qquad (\vec{P}_{h\perp} = -z\vec{k}_T)$

- Definition and interpretation

$$\frac{1}{64\pi^3 z_1 z_2} \int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h_1, h_2, X\rangle\langle h_1, h_2, X|\bar{\psi}_q(0)|0\rangle\, \gamma^-\Big]_{\xi^-=0}$$

$$= D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \equiv D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

  - $D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$ is number density for hadron pairs (h_1, h_2)
  - factor $1/64\pi^3 z_1 z_2$ is crucial for interpretation as number density
  - previously defined/used DiFFs in spin physics have no number density interpretation
    (starting from pioneering work by Bianconi, Boffi, Jakob, Radici, 1999)

- Collinear DiFFs (see also, Majumder, Wang, 2004)

$$D_1^{h_1 h_2/q}(z_1, z_2) = \int d^2\vec{P}_{1\perp}\, d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

- Number of hadron pairs in quark q

$$\sum_{h_1,h_2} \int dz_1 dz_2\, D_1^{h_1 h_2/q}(z_1, z_2) = \langle N^q(N^q - 1)\rangle$$

- Momentum sum rule

$$\sum_{h_1} \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\, z_1\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = (1 - z_2)\, D_1^{h_2/q}(z_2, \vec{P}_{2\perp}^2)$$

  - sum rule after $\int d^2\vec{P}_{2\perp}$ already in previous literature (de Florian, Vanni, 2003 / ...)
  - similar (integrated) sum rule for double-parton distributions (Gaunt, Stirling, 2009 / ...)

- Leading-order cross section for $e^- e^+ \to (h_1 h_2)X$

$$\frac{d\sigma}{dz_1 dz_2} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h_1 h_2/q}(z_1, z_2) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

Definition and Interpretation of Extended DiFFs

- More on kinematics
  - invariant mass of di-hadron pair, and alternative variable for longitudinal momentum

$$M_h^2 = P_h^2 = (P_1 + P_2)^2 \qquad\qquad \zeta = \frac{z_1 - z_2}{z}$$

  - momenta P_1 and P_2 in hadron frame ($\vec{P}_{hT} = 0$)

$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^2}{(1+\zeta)P_h^-}, \frac{1+\zeta}{2} P_h^-, \vec{R}_T \right) \qquad P_2 = \left(\frac{M_2^2 + \vec{R}_T^2}{(1-\zeta)P_h^-}, \frac{1-\zeta}{2} P_h^-, -\vec{R}_T \right)$$

  - important relation

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4} M_h^2 - \frac{1-\zeta}{2} M_1^2 - \frac{1+\zeta}{2} M_2^2$$

- Extended DiFFs (extDiFFs)
  - in contrast to $D_1^{h_1 h_2/q}(z_1, z_2)$, extDiFFs (also) depend on M_h (or $\vec{R}_T$)
  - extDiFFs appear in transversity-related observables

- Number density interpretation for (properly defined) extDiFFs
  - when changing variables, include Jacobian of transformation in definition of DiFFs
  - example

$$D_1^{h_1 h_2/q}(z, \zeta, \vec{k}_T, \vec{R}_T) = \frac{z^3}{2}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

  - further extDiFFs

$$\begin{aligned} D_1^{h_1 h_2/q}(z, M_h) &= \int d\zeta\, D_1^{h_1 h_2/q}(z, \zeta, M_h) \\ &= \int d\zeta\, \frac{\pi}{2} M_h\, (1-\zeta^2)\, D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2) \end{aligned}$$

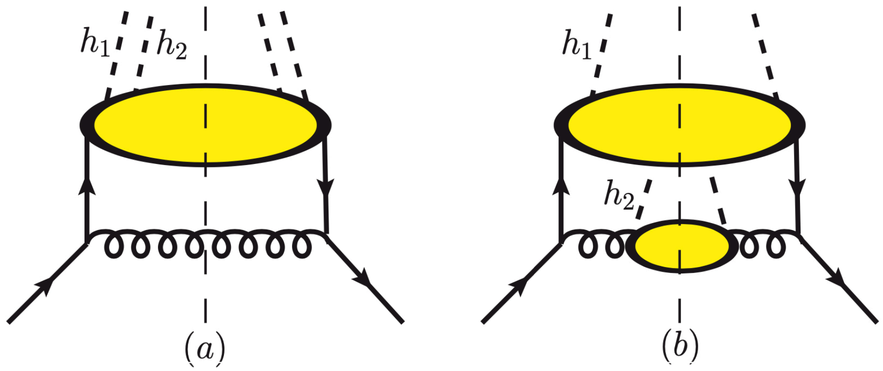
  - experimental information on $D_1^{h_1 h_2/q}(z, M_h)$ from Belle (Seidl et al, 2017)

- Leading-order cross section for $e^- e^+ \to (h_1 h_2) X$ (example)

$$\frac{d\sigma}{dz\, dM_h} = \sum_{q,\bar{q}} \hat{\sigma}^q\, D_1^{h_1 h_2/q}(z, M_h) \quad \text{with} \quad \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\text{em}}^2 N_c}{3Q^2}$$

Evolution of DiFFs

- Homogeneous and in-homogeneous contributions to evolution (sample diagrams)



- Evolution of extDiFFs (quark non-singlet)

$$\frac{\partial}{\partial \ln \mu^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu) = \int_z^1 \frac{dw}{w} D_1^{h_1 h_2/q}\Big(\frac{z}{w}, \zeta, \vec{R}_T^2; \mu\Big) P_{q\to q}(w)$$

  - evolution of extDiFFs only contains homogeneous term (standard DGLAP) (see also Ceccopieri, Bacchetta, Radici, 2007)
  - corresponding evolution equation for $H_1^{\sphericalangle\, h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu)$

- Upon $\int d^2\vec{R}_T$, we recover evolution of $D_1^{h_1 h_2/q}(z_1, z_2; \mu)$ where in-homogeneous term contributes as well (de Florian, Vanni, 2003 / ...)

Simultaneous Extraction of DiFFs and Transversity PDFs

- Main observables/input for DiFFs
  - unpolarized cross section in $e^- e^+ \to (h_1 h_2) X$ (data from Belle)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi\alpha_{\rm em}^2 N_c}{3Q^2} \sum_{q,\bar{q}} e_q^2\, D_1^q(z, M_h)$$

  - PYTHIA event generator

  - Artru-Collins asymmetry in $e^- e^+ \to (h_1 h_2)(\bar{h}_1 \bar{h}_2) X$ (data from Belle)

$$A^{e^- e^+}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_{q,\bar{q}} e_q^2\, H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1 + \cos^2\theta) \sum_{q,\bar{q}} e_q^2\, D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

- Further constraints on DiFFs from transverse single-spin asymmetries in semi-inclusive DIS and proton-proton collisions (simultaneous analysis)

- Observables for transversity PDFs
  - transverse SSA in SIDIS (data from HERMES and COMPASS)

$$A_{UT}^{\mathrm{SIDIS}} = c(y)\frac{\sum_{q,\bar{q}} e_q^2\, h_1^q(x)\; H_1^{\sphericalangle,q}(z, M_h)}{\sum_{q,\bar{q}} e_q^2\, f_1^q(x)\; D_1^q(z, M_h)}$$

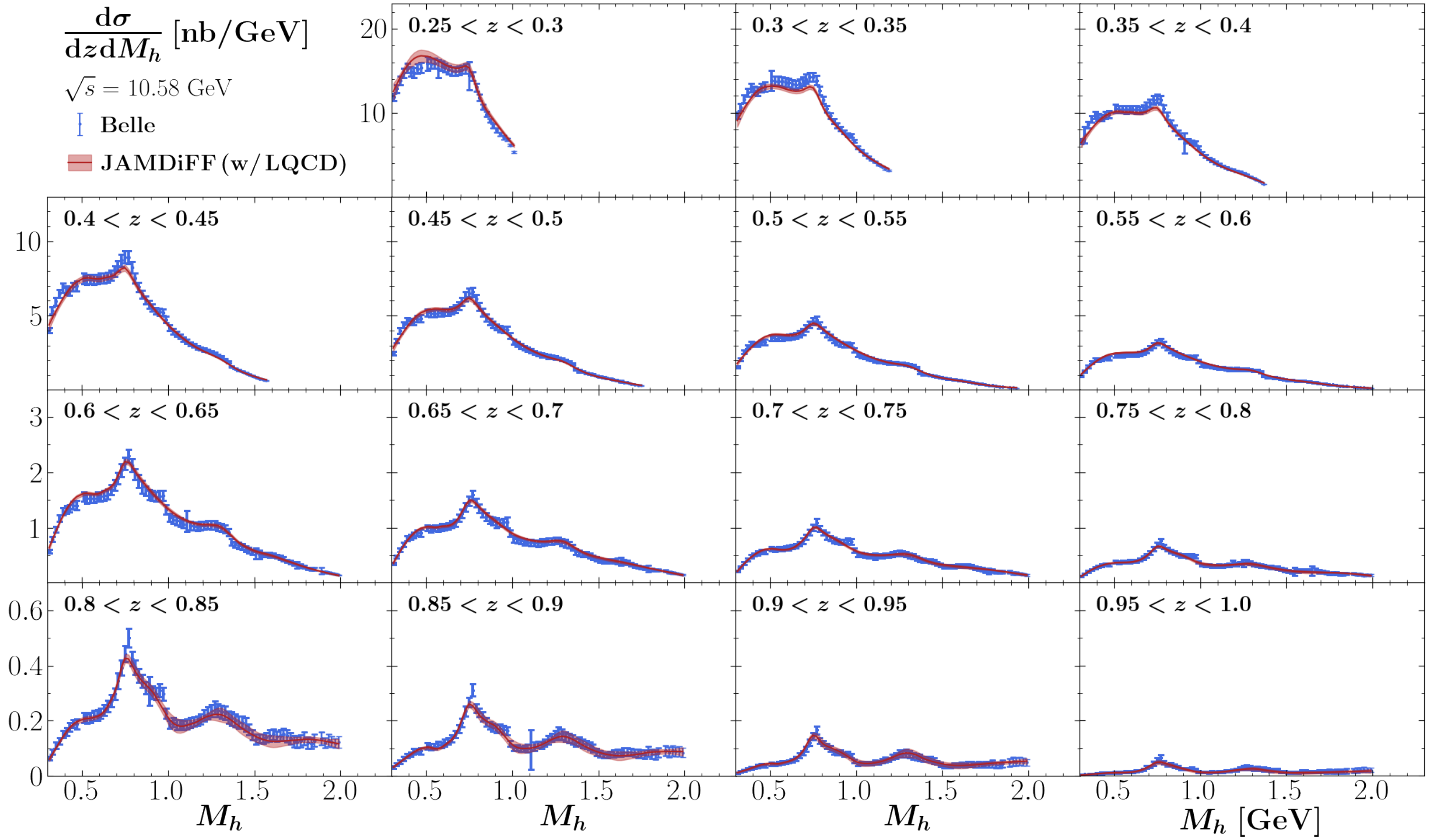
  - transverse SSA in pp collisions (data from STAR)

$$A_{UT}^{pp} = \frac{\mathcal{H}(M_h, P_{hT}, \eta)}{\mathcal{D}(M_h, P_{hT}, \eta)}$$

$$\mathcal{H} = 2P_{hT}\sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z}\, h_1^a(x_a)\; f_1^b(x_b) \frac{\mathrm{d}\Delta\hat{\sigma}_{a^\uparrow b \to c^\uparrow}}{\mathrm{d}\hat{t}}\, H_1^{\sphericalangle,c}(z, M_h)$$

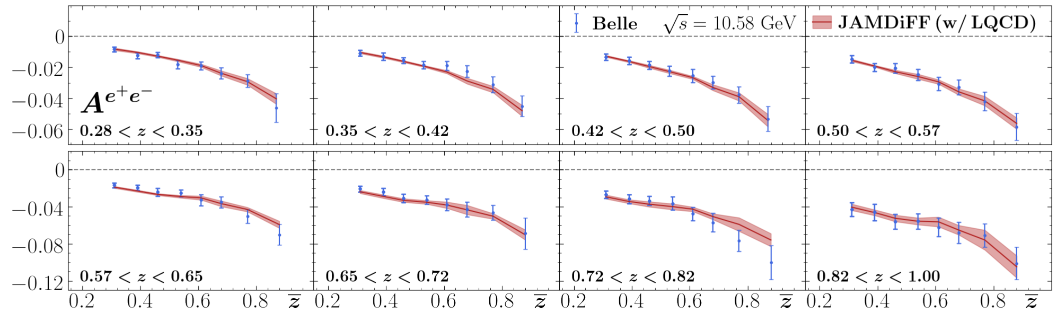
$$\mathcal{D} = 2P_{hT}\sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z}\, f_1^a(x_a)\; f_1^b(x_b) \frac{\mathrm{d}\hat{\sigma}_{a^\uparrow b \to c^\uparrow}}{\mathrm{d}\hat{t}}\, D_1^c(z, M_h)$$

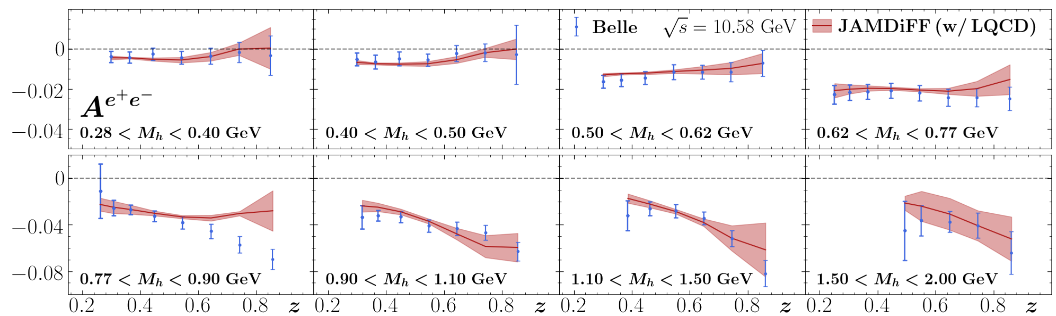
- Quality of fit: unpolarized cross section for $e^-e^+ \to (h_1 h_2)X$

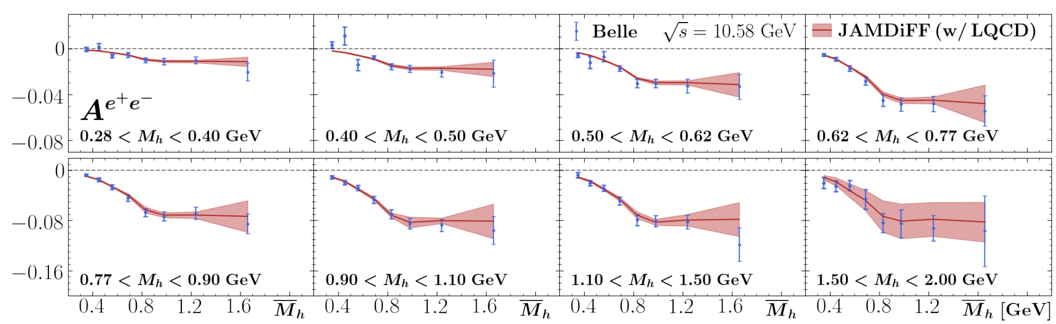


(data from Belle, 2017)

- Quality of fit: Artru-Collins asymmetry

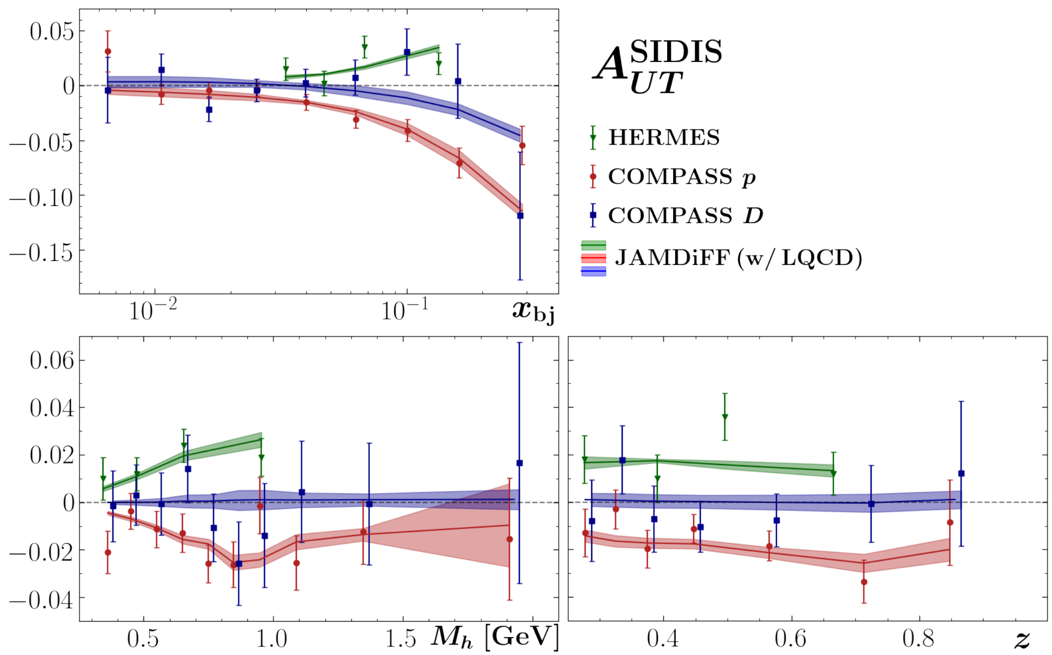




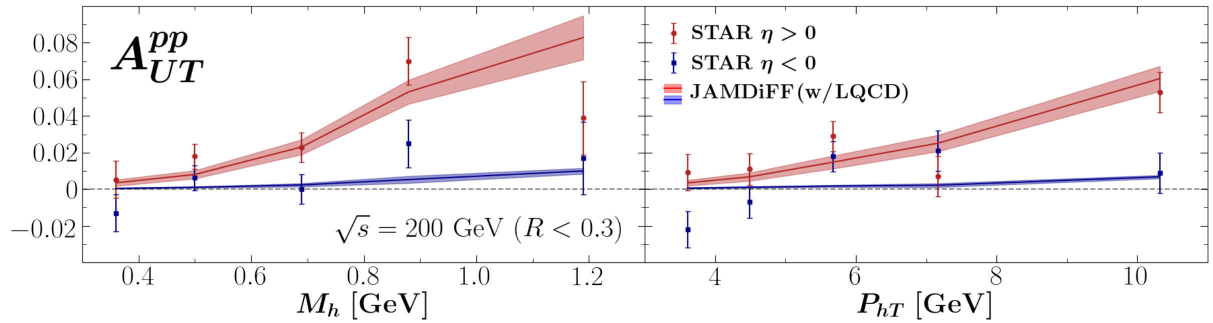


(data from Belle, 2011)

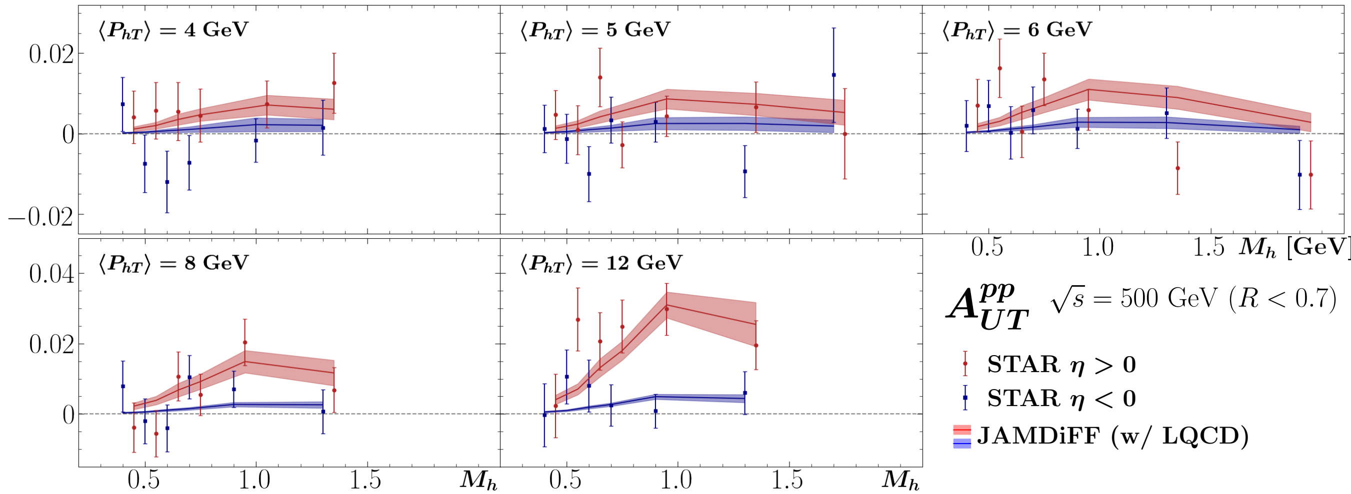
- Quality of fit: A_{UT}^{SIDIS} (data from HERMES, 2008 / COMPASS, 2023)

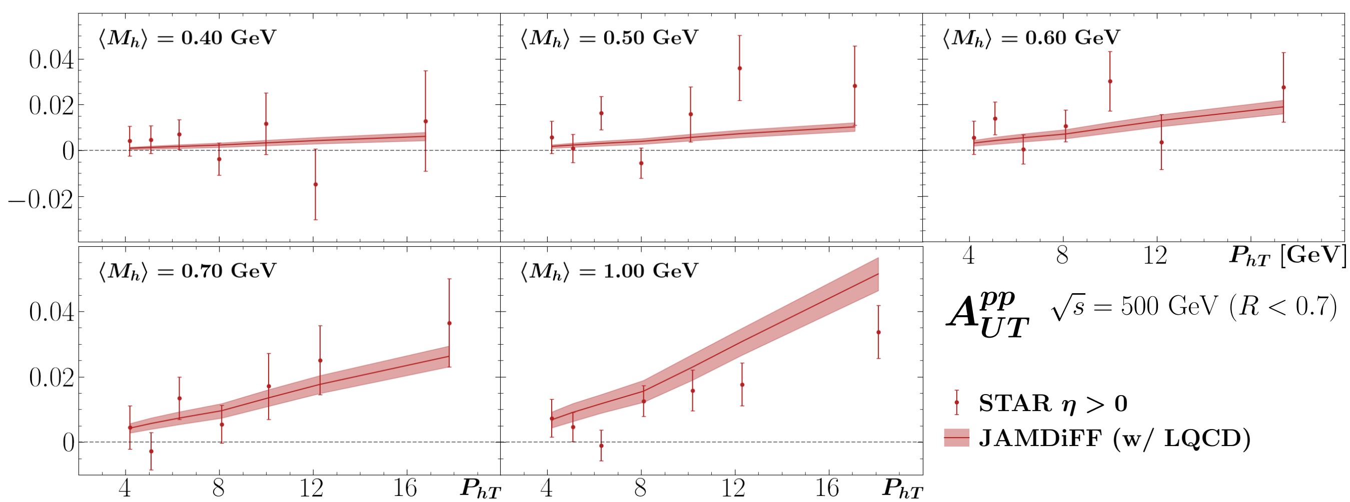


- Quality of fit: A_{UT}^{pp} (sample data for $\sqrt{s} = 200\,\mathrm{GeV}$ from STAR, 2015)



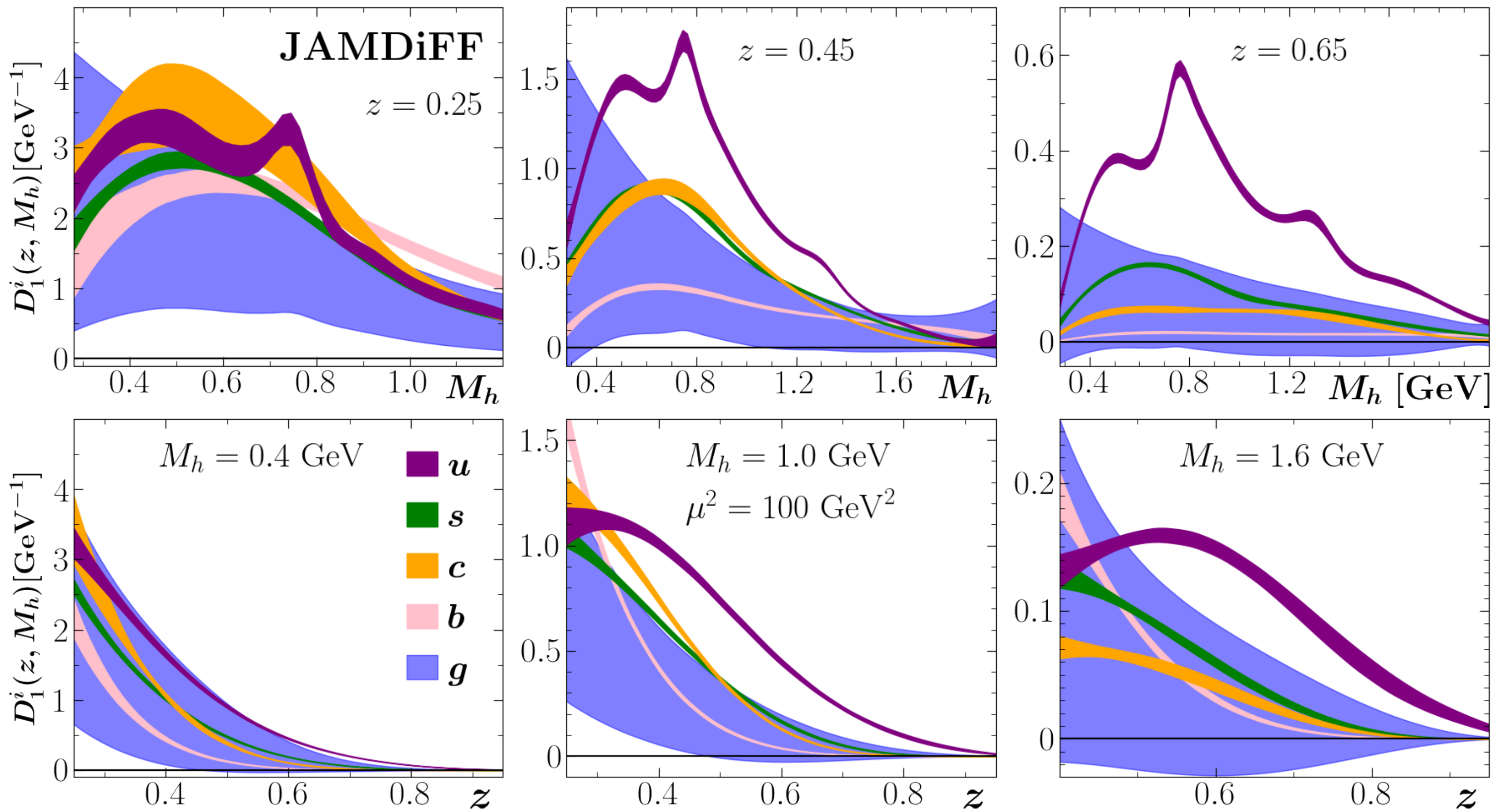
- Quality of fit: A_{UT}^{pp} (sample data for $\sqrt{s} = 500\,\text{GeV}$ from STAR, 2017)





- Extracted DiFFs $D_1^{\pi^+\pi^-/a}$ $(a = q, g)$

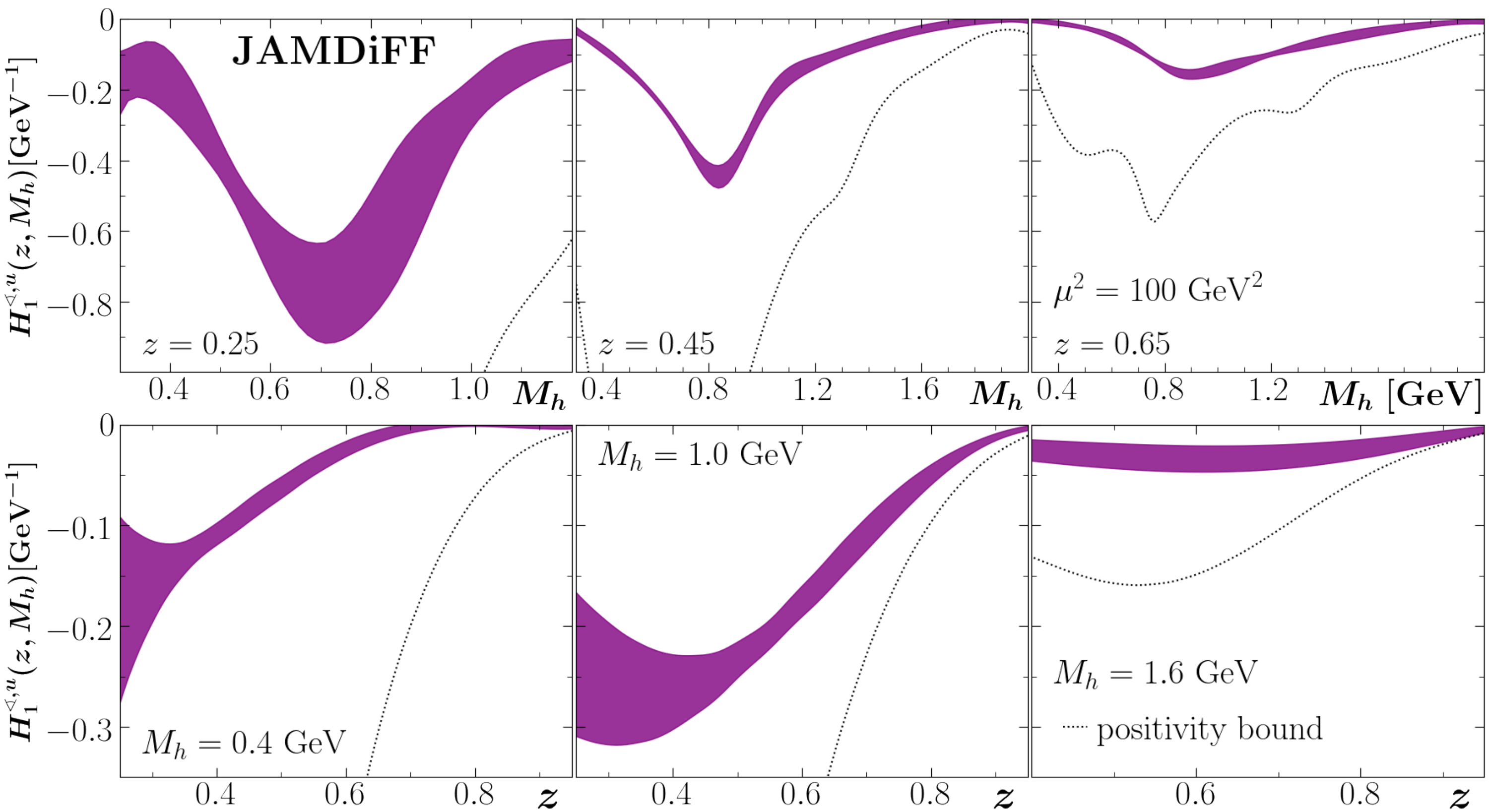
$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}}$ $D_1^s = D_1^{\bar{s}}$ $D_1^c = D_1^{\bar{c}}$ $D_1^b = D_1^{\bar{b}}$ D_1^g



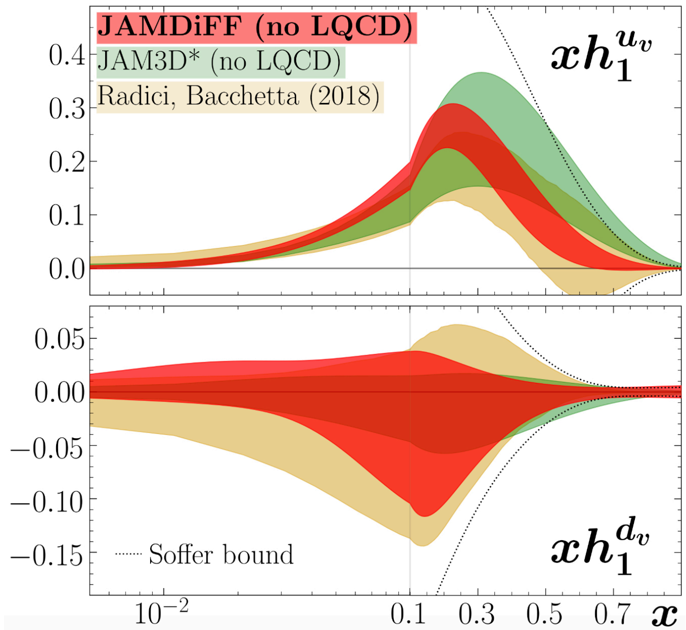
  - with our new definition of DiFFs we can compute $\langle M_h \rangle$ and $\langle z \rangle$

- Extracted DiFF $H_1^{\sphericalangle\,\pi^+\pi^-/u}$

$$H_1^{\sphericalangle\,u} = -H_1^{\sphericalangle\,d} = -H_1^{\sphericalangle\,\bar{u}} = H_1^{\sphericalangle\,\bar{d}} \qquad H_1^{\sphericalangle\,q} = 0 \text{ for } q = s, c, b$$



- Extracted transversity PDFs



– fit of $h_1^{u_v}$, $h_1^{d_v}$, $h_1^{\bar{u}} = -h_1^{\bar{d}}$
large-N_c constraint for antiquarks (Pobylitsa, 2003)

– Soffer bound (Soffer, 1995)

$$h_1^q(x) \leq \tfrac{1}{2}\left|f_1^q(x) + g_1^q(x)\right|$$

– small-x constraint (Kovchegov, Sievert, 2019)

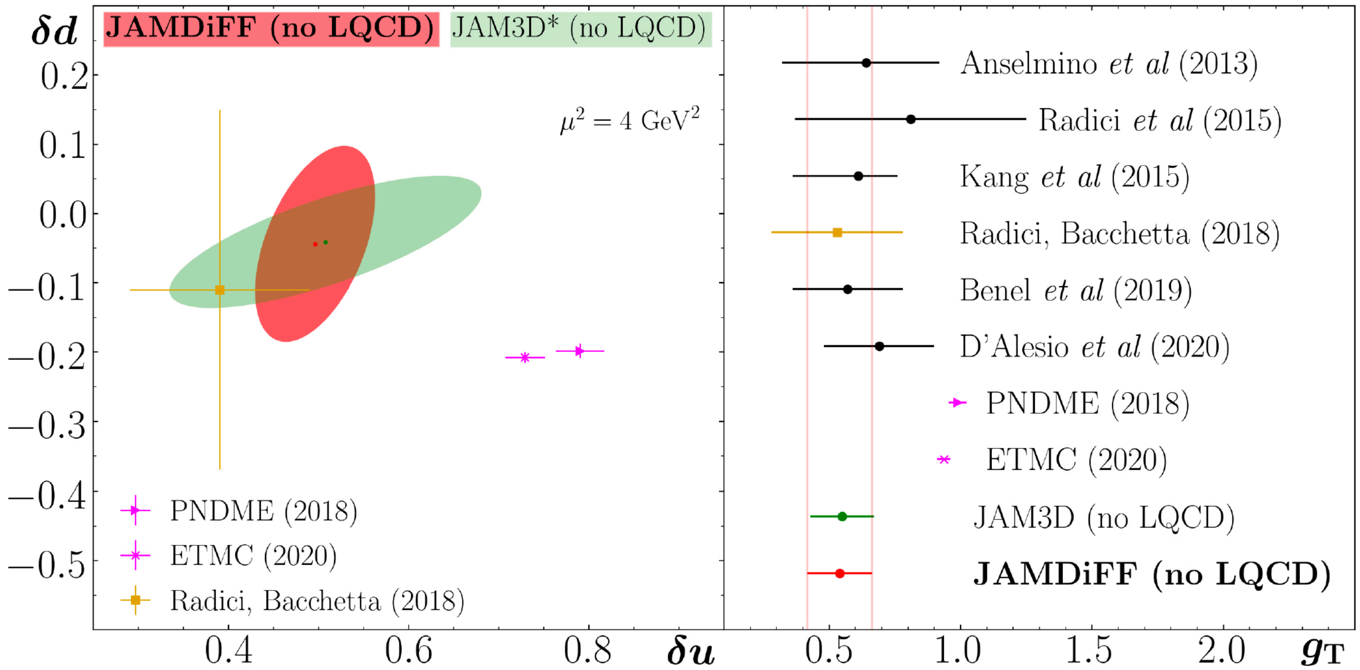
$$h_1^q \xrightarrow{x\to 0} x^{\alpha_q} \qquad \alpha_q \approx 0.17 \pm 0.085$$

– JAM3D* = JAM3D-22 (no LQCD)
+ antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$
+ small-x constraint

– agreement between all three analyses within errors

Extraction of Tensor Charges

- Tensor charges and comparison with results from LQCD



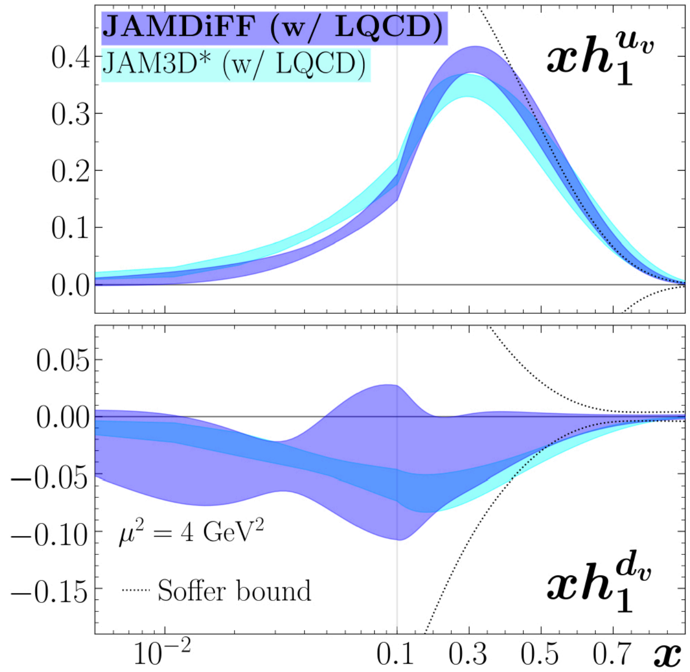
  - for δu, we find 3.2σ discrepancy with ETMC, 3.9σ discrepancy with PNDME
  - what happens if LQCD results for δu and δd are included in the fit ?

- Quality of fit: χ^2 values for various data sets

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [72]	12	1.13	1.10
COMPASS (p) [71]	26	1.24	0.75
COMPASS (D) [71]	26	0.78	0.76
STAR (2015) [94]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	—
ETMC δd [28]	1	1.02	—
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total $\boldsymbol{\chi^2_{\text{red}}}$ (N_{dat})		**1.01** (1475)	**0.98** (1471)

  - successful fit after inclusion of LQCD tensor charges

- Extracted transversity PDFs (w/ LQCD)



- Soffer bound (Soffer, 1995)

$$h_1^q(x) \leq \frac{1}{2}\left|f_1^q(x) + g_1^q(x)\right|$$

- small-x constraint (Kovchegov, Sievert, 2019)

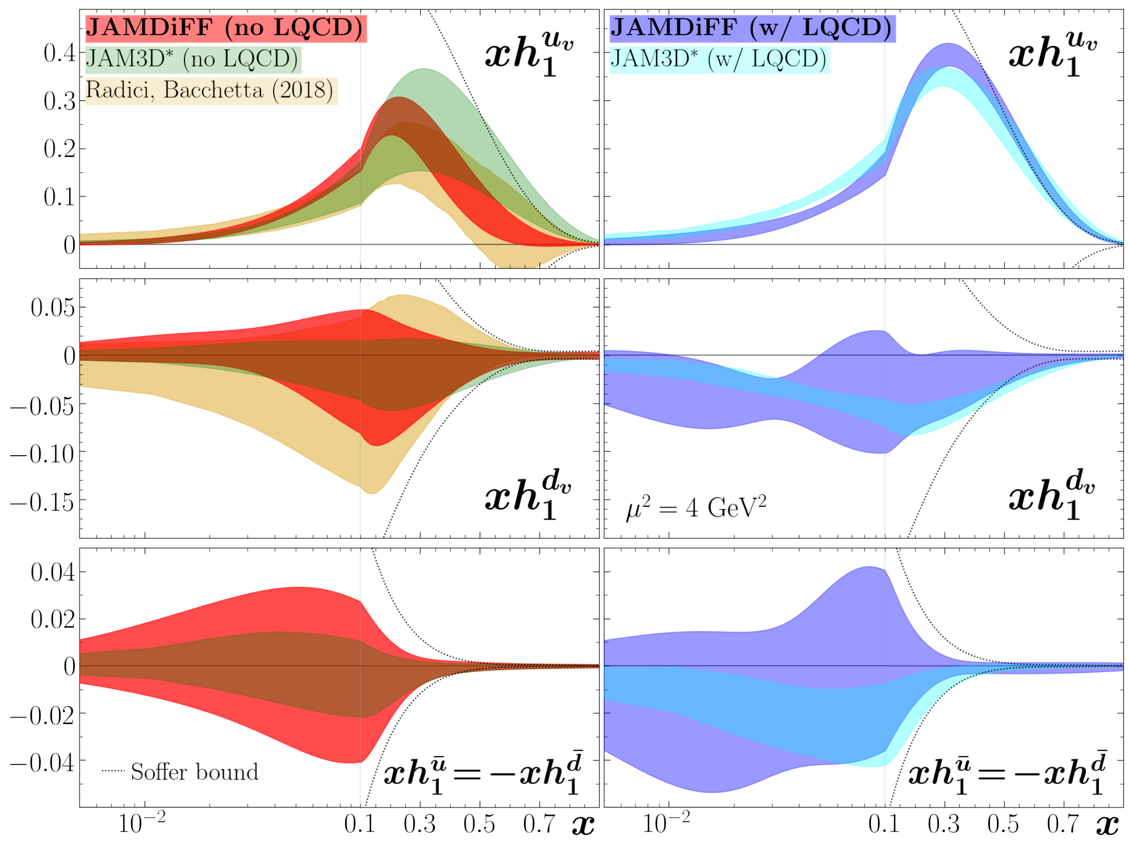
$$h_1^q \xrightarrow{x\to 0} x^{\alpha_q} \qquad \alpha_q \approx 0.17 \pm 0.085$$

- after inclusion of LQCD tensor charges:
  (1) increase of $h_1^{u_v}$ for $x \gtrsim 0.3$
  (2) $h_1^{d_v}$ tends to become negative

- JAM3D* = JAM3D-22 (w/ LQCD)
  + antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$
  + small-x constraint
  + δu, δd from ETMC & PNDME

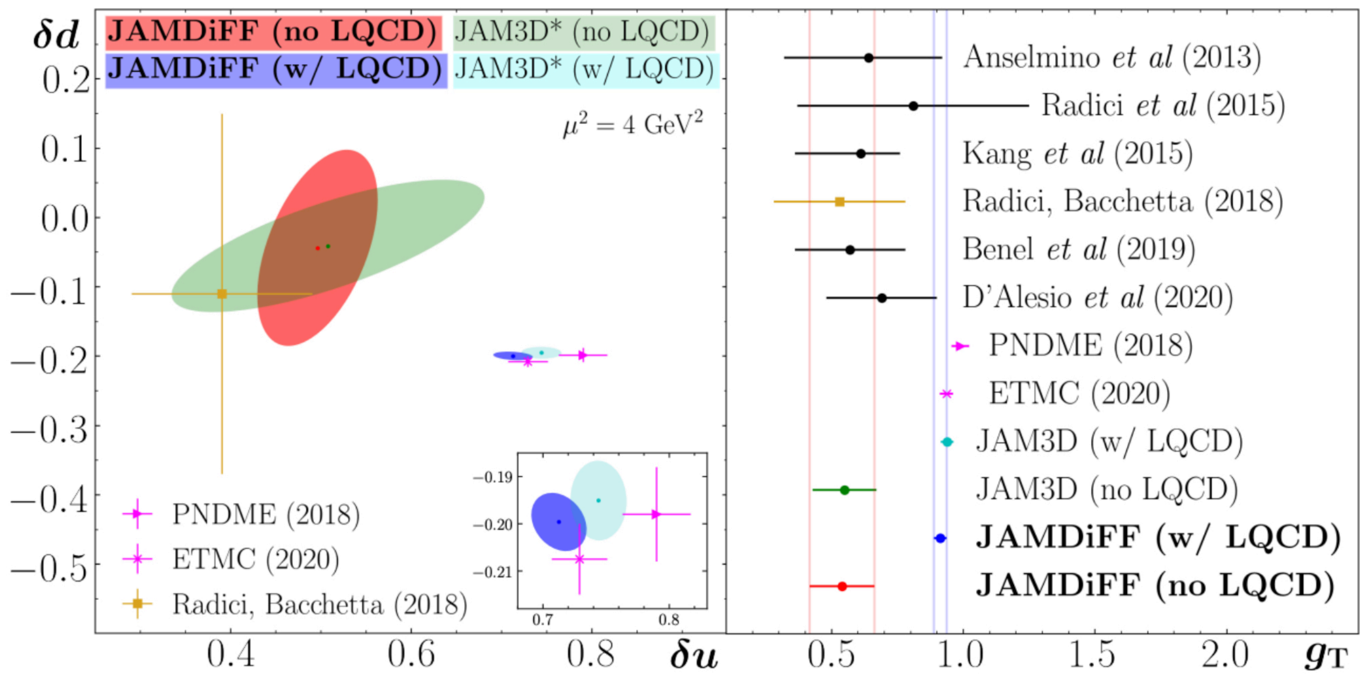
- agreement between two analyses within errors

- Extracted transversity PDFs: comp. no LQCD and w/ LQCD fits (with antiquarks)



  - $h_1^{u_v}$ is by far the largest distribution
  - Soffer bound implies $h_1^{\bar{q}} \approx 0$ for $x \gtrsim 0.4$

- Tensor charges (no LQCD vs w/ LQCD)

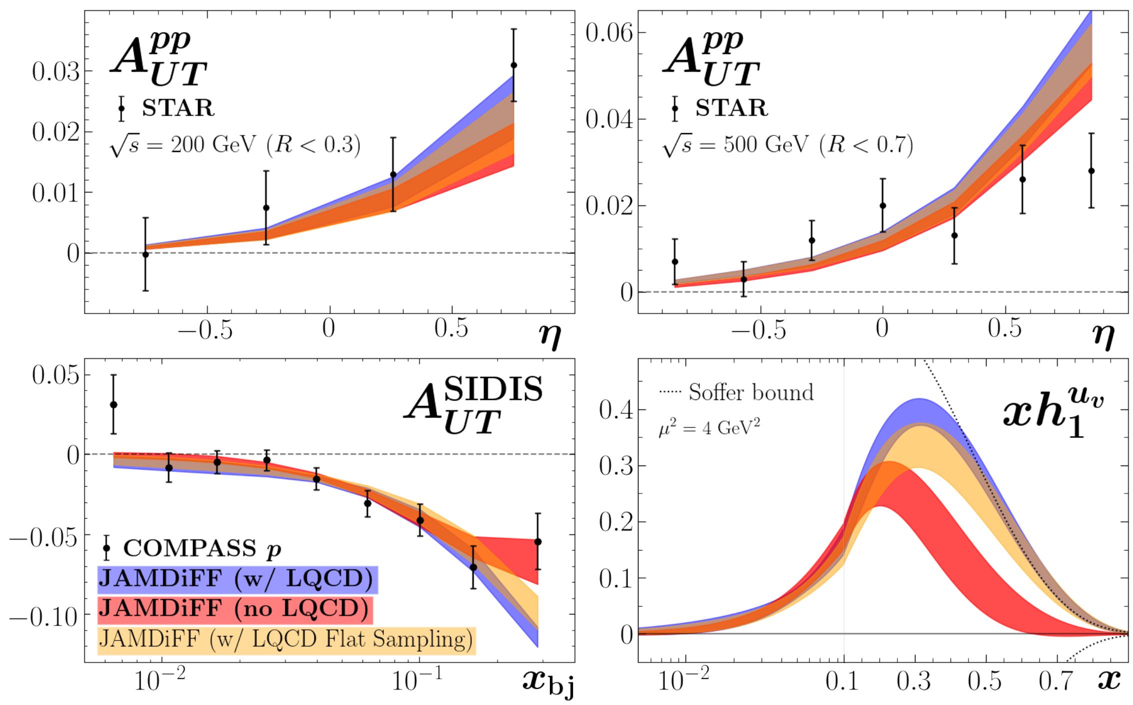


  - noticeable shift for δu after including LQCD results

Overall finding: universal nature of all available information on h_1^q —
(1) data for di-hadron production, (2) data for single-hadron production,
(3) LQCD results for tensor charge, (4) Soffer bound, (5) small-x constraint

Need for New Data

- Unpolarized data for di-hadron production in SIDIS and pp collision would help to constrain $D_1^{\pi^+\pi^-/a}$
- (Precise) data for A_{UT} at any x would better constrain $h_1^{q_v}(x)$, $h_1^{\bar{q}}(x)$
- For the tensor charge δq, (new) data for $x \gtrsim 0.2$ seems most important



Summary

- For both quarks and gluons, we propose a field-theoretic definition of DiFFs which have an interpretation as number densities

- Number density interpretation can be obtained for different variables of interest (including extDiFFs)

- Operator definition of DiFFs also allows one to "easily" obtain their evolution

- New numerical study of data on di-hadron production: Simultaneous global analysis of DiFFs, transversity PDFs, and tensor charges

- Main differences compared to previous analyses of di-hadron data: (1) inclusion of Belle cross section data, STAR $500\,\mathrm{GeV}$ data, all binnings for Artru-Collins and SIDIS asymmetries, (2) simultaneous fit of DiFFs and transversity PDFs,
(3) small-x theory constraint, (4) improved functional form for DiFF fit functions,
(5) inclusion of LQCD results for tensor charges, (6) fit of antiquark transversities

- We find compatibility of all available information on transversity PDFs