JPAC, amplitudes & lattice QCD

Jozef Dudek

WILLIAM & MARY
CHARTERED 1693

Jefferson Lab
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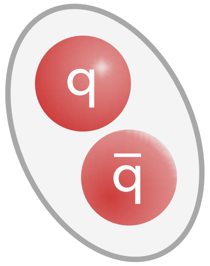
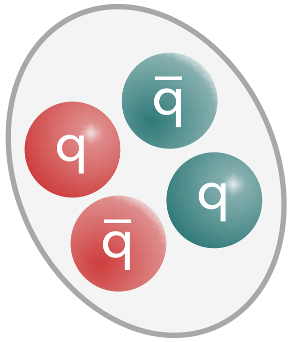
meson spectroscopy

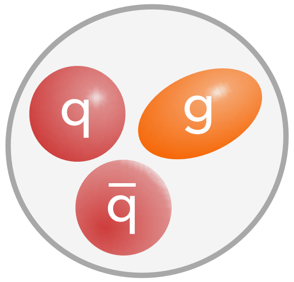
“pictures”



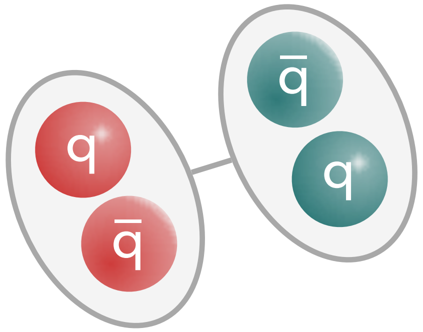
conventional meson



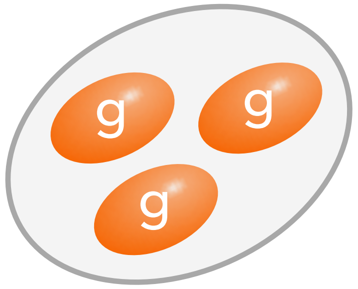
tetraquark



hybrid meson



meson-meson molecule



glueball

+ other suggestions …

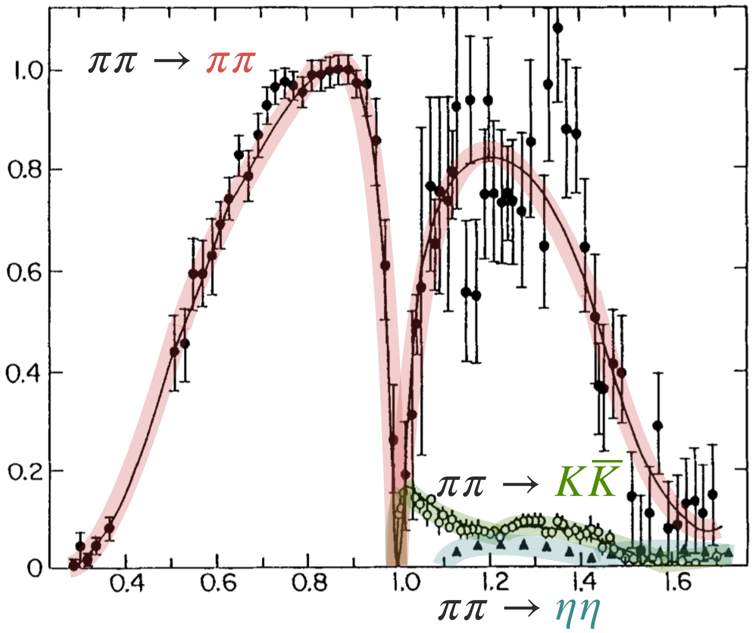
traditionally the preserve of **modeling**

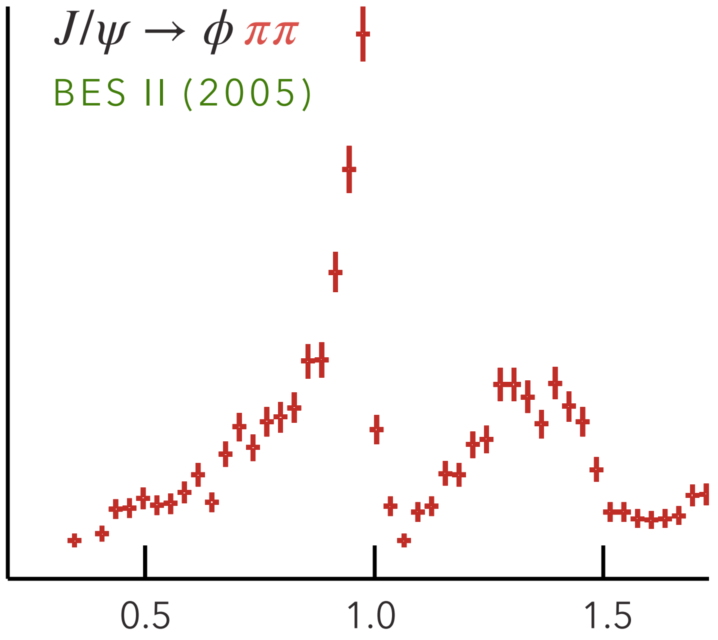
– connection to QCD unclear

“reality”

resonances in complicated production/decay

e.g. $f_0(980)$



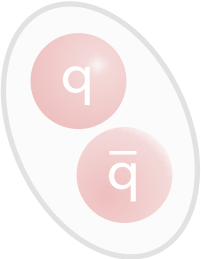


amplitudes in terms of scattering **stable hadrons**

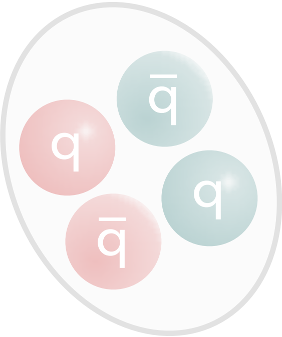
– connection to QCD unclear

meson spectroscopy

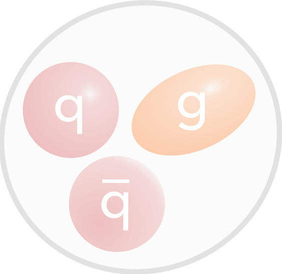
“pictures”



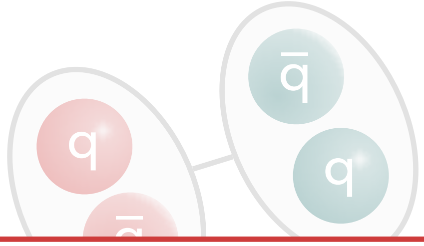
conventional
meson



tetraquark



hybrid meson

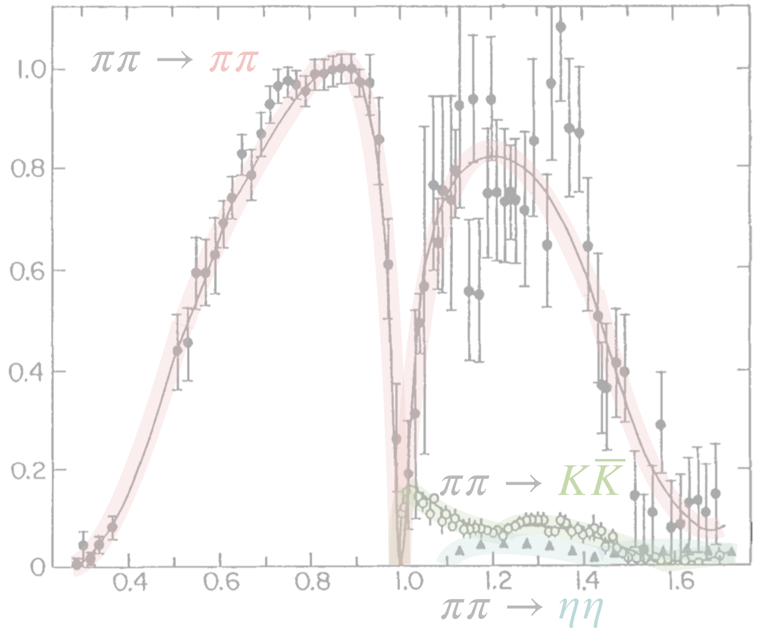


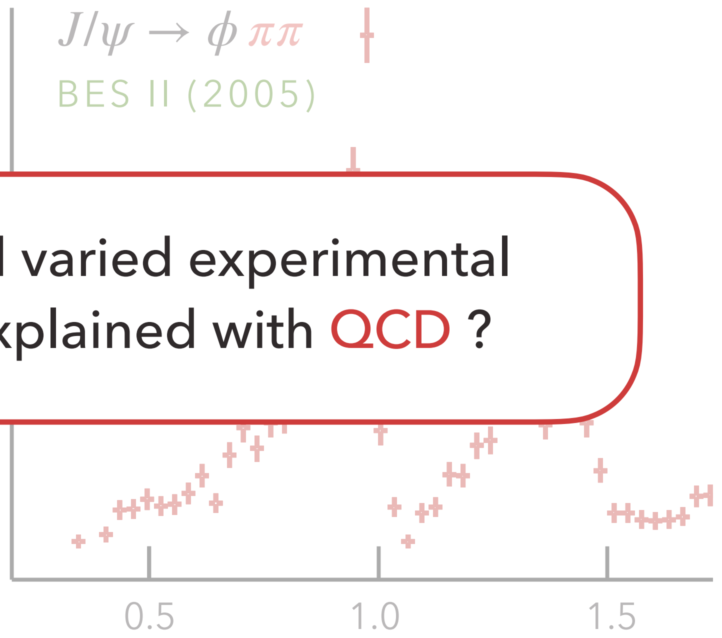
meson-meson
molecule

“reality”

resonances in complicated production/decay

e.g. $f_0(980)$



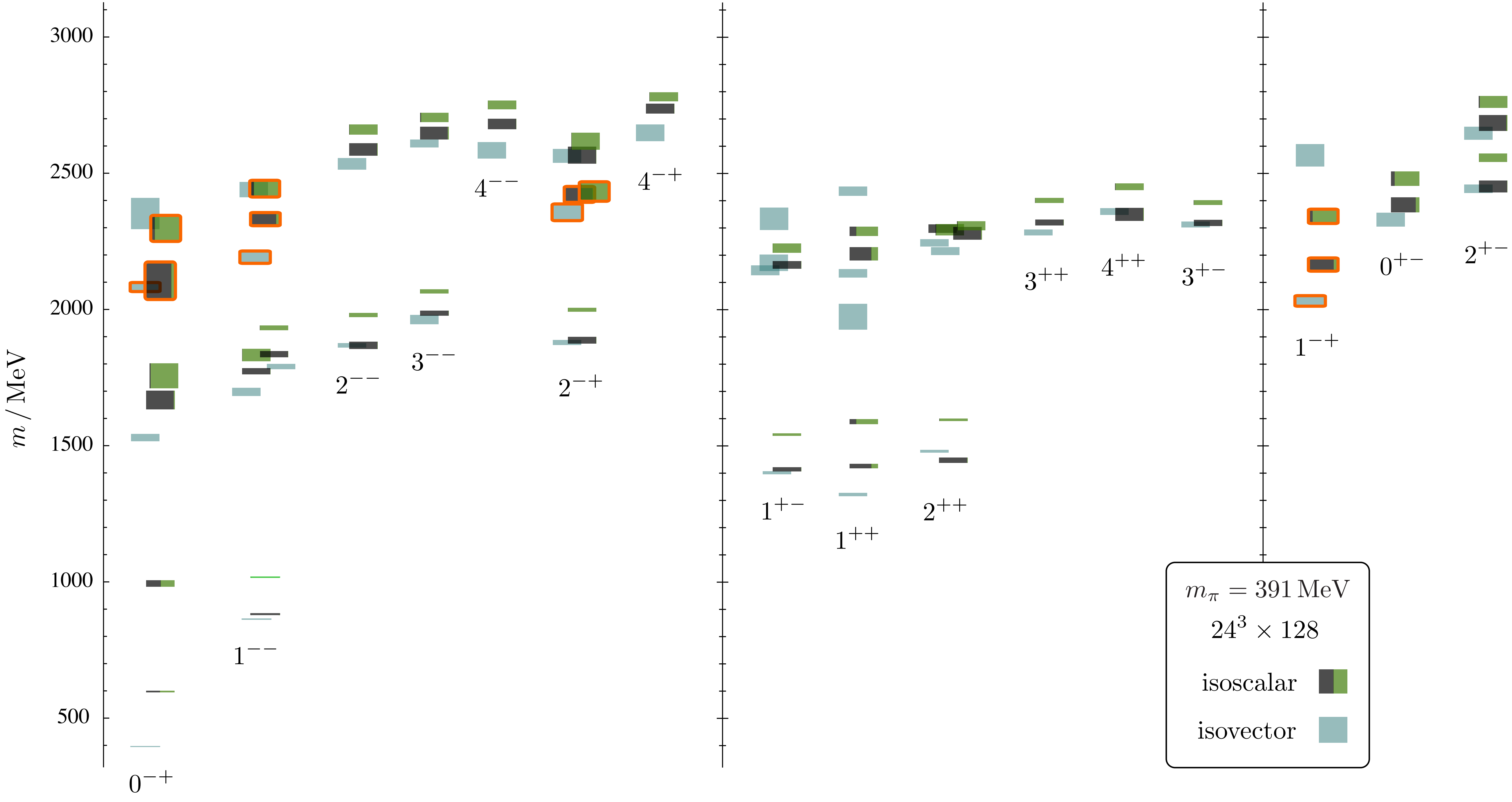


in what configurations does QCD
bind quarks and gluons ?

can complex and varied experimental
amplitudes be explained with QCD ?

progress in lattice QCD

2013



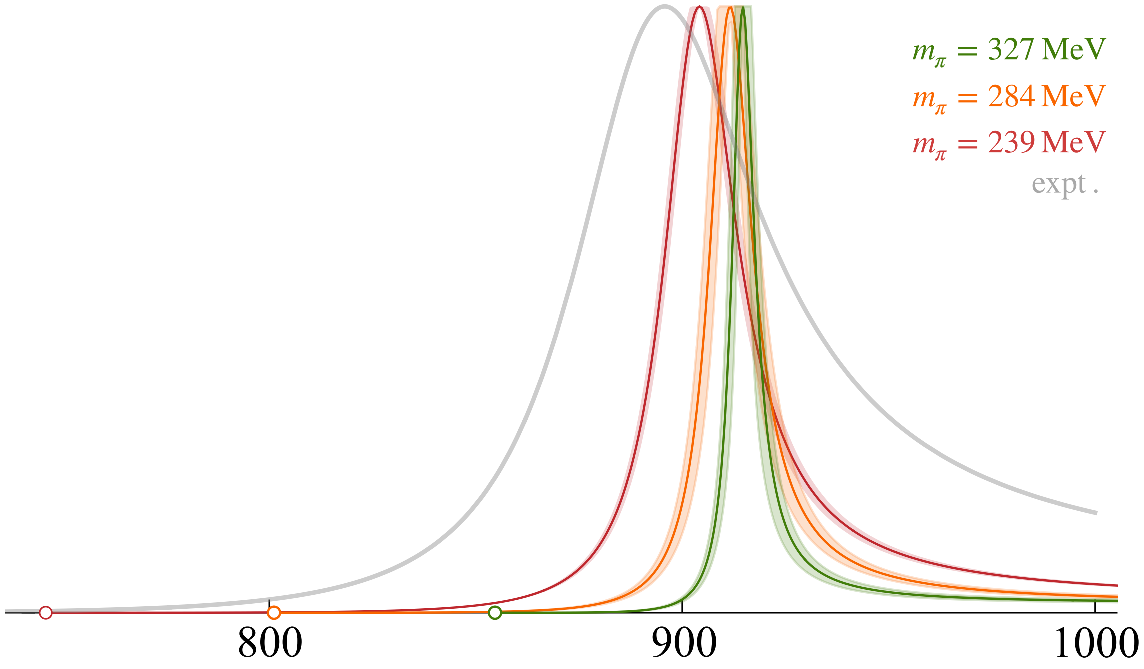
meson-meson scattering amplitudes in lattice QCD

rigorous relation between spectrum in a finite-volume & scattering matrix

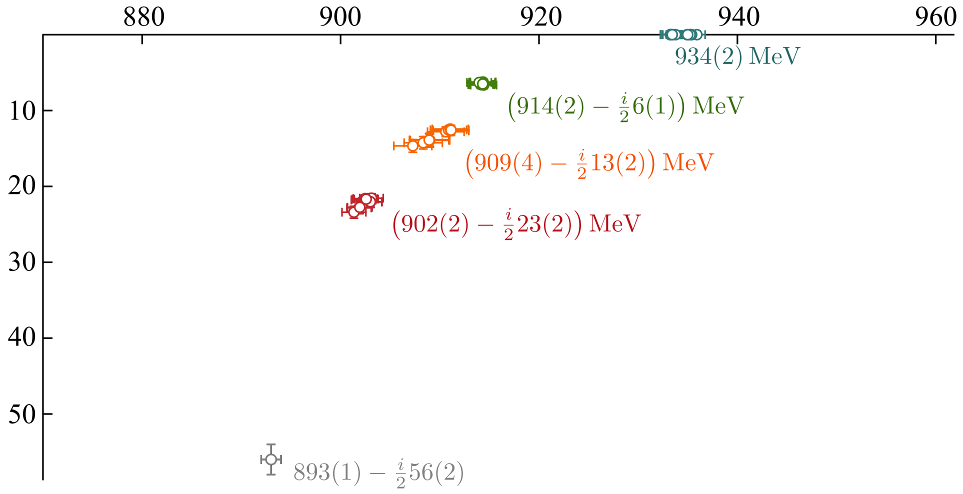
$E_n(L)$ are solutions of

$$\det\left[\underset{\text{kinematic functions}}{\underline{F^{-1}(E^\star;L)}}+\mathscr{M}(E^\star)\right]=0$$

e.g. $\pi K \to \pi K \;\; J^P = 1^-$ elastic scattering PRL 123 042002 (**2019**)



K^* resonance pole position



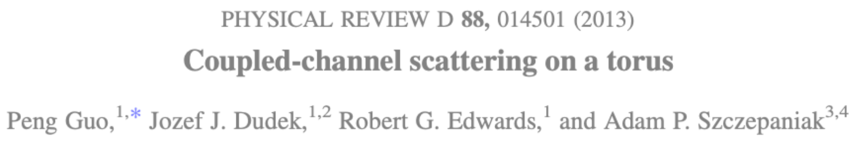
evolution of the K^* resonance as a function of varying light quark mass

coupled-channel meson-meson scattering amplitudes

rigorous relation between spectrum in a finite-volume & scattering matrix

$E_n(L)$ are solutions of

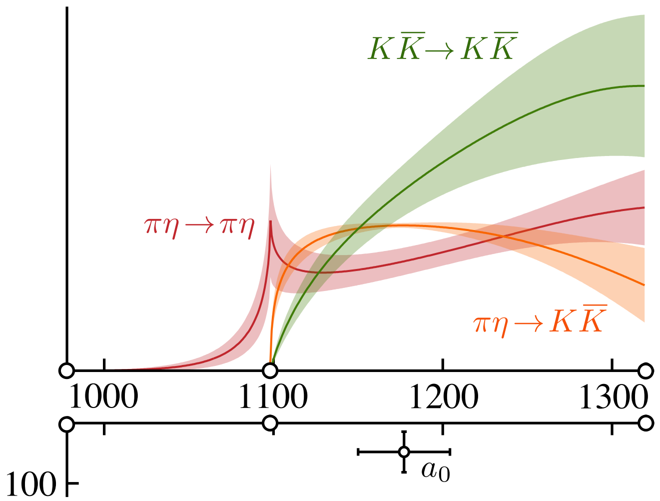
$$\det\left[\underbrace{F^{-1}(E^\star;L)}_{\text{kinematic functions}}+\mathscr{M}(E^\star)\right]=0$$


PHYSICAL REVIEW D **88,** 014501 (2013)

Coupled-channel scattering on a torus

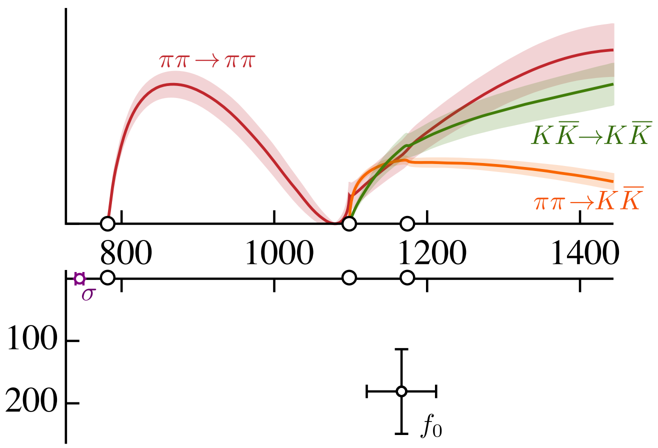
Peng Guo,[1,*] Jozef J. Dudek,[1,2] Robert G. Edwards,[1] and Adam P. Szczepaniak[3,4]

$J^P=0^+\ I^G=1^-\ (\eta\pi,K\bar{K})$

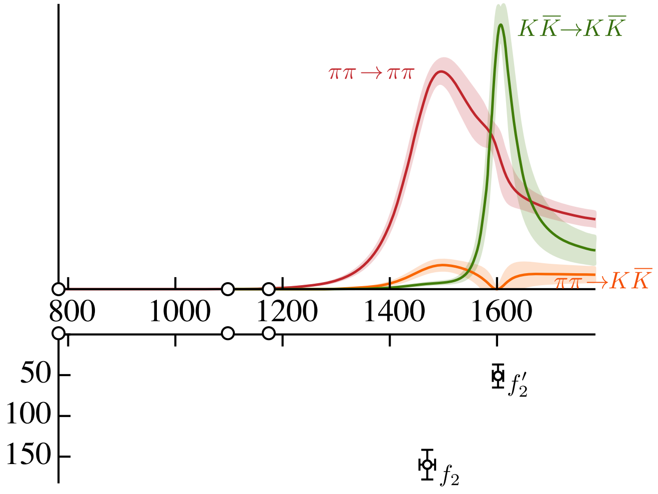


PRD 93 094506 (2016)

$J^P=0^+\ I^G=0^+\ (\pi\pi,K\bar{K},\eta\eta)$

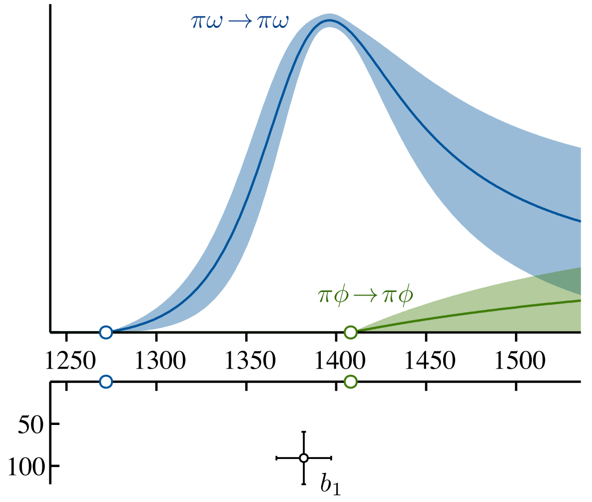


$J^P=2^+\ I^G=0^+\ (\pi\pi,K\bar{K},\eta\eta)$



PRD 97 054513 (2018)

$J^P=1^+\ I^G=1^-\ (\pi\omega,\pi\phi)$



PRD 100 054506 (2019)

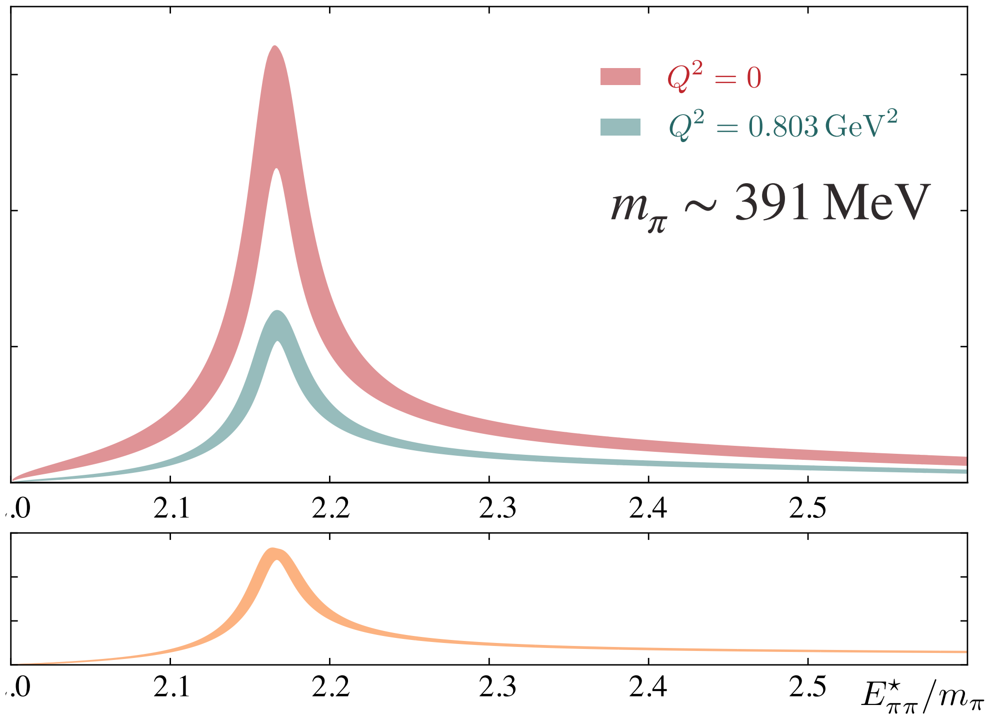
$m_\pi\sim 391\,\mathrm{MeV}$

rigorous definition of a resonance as pole singularity in scattering matrix

coupling to currents

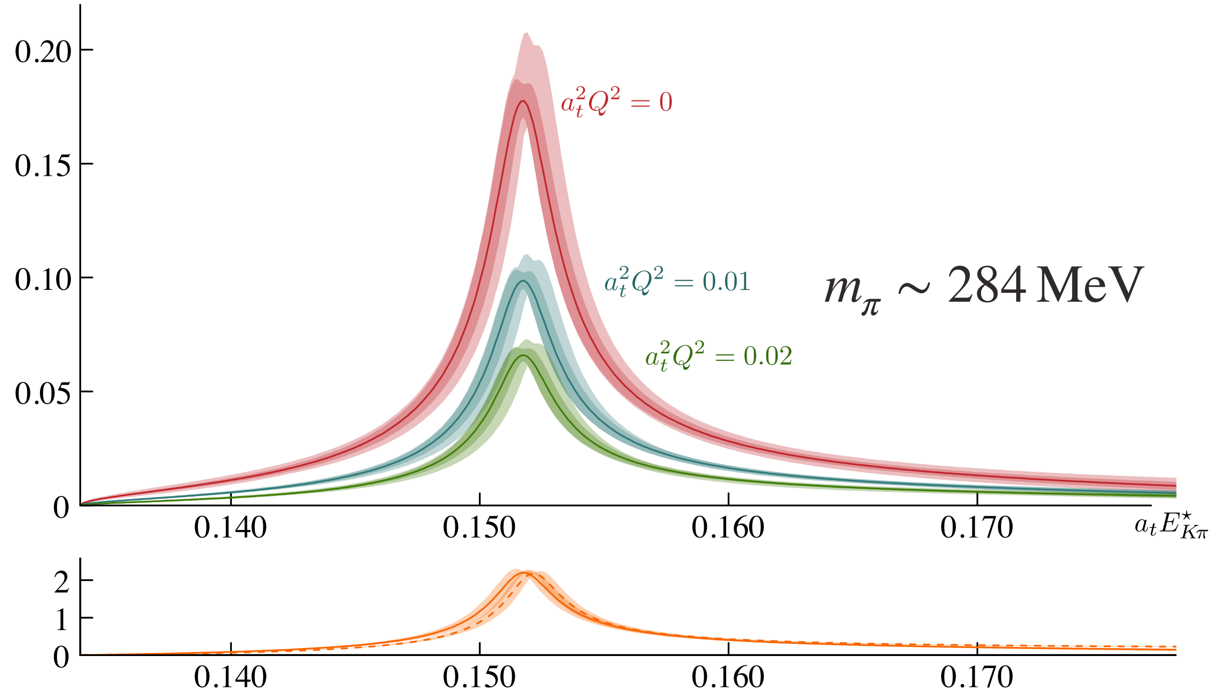
$\gamma\pi \to \pi\pi$

PRL 115 242001 (2015)
PRD 93 114508 (2016)



$\gamma K \to K\pi$

arXiv:2208.13755



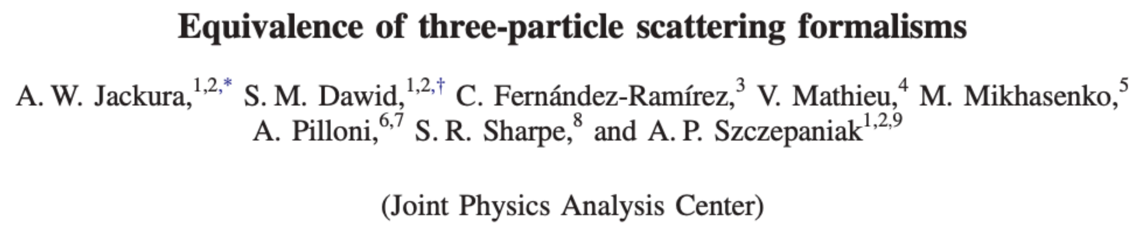
three-body systems

to study higher-lying resonances or lower pion masses, need three-body formalism

difficult even in infinite volume

has been healthy exchange of ideas between amplitudes and lattice communities

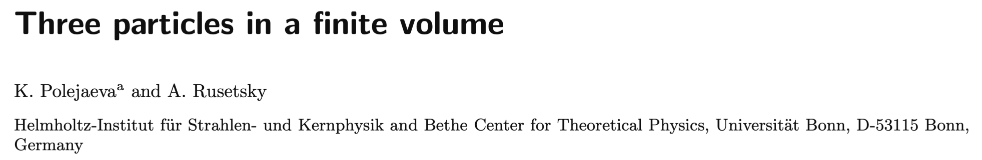
e.g.



Equivalence of three-particle scattering formalisms

A. W. Jackura,[1,2,*] S. M. Dawid,[1,2,†] C. Fernández-Ramírez,[3] V. Mathieu,[4] M. Mikhasenko,[5]
A. Pilloni,[6,7] S. R. Sharpe,[8] and A. P. Szczepaniak[1,2,9]

(Joint Physics Analysis Center)



Three particles in a finite volume

K. Polejaeva[a] and A. Rusetsky

Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, D-53115 Bonn, Germany

Equivalence of relativistic three-particle quantization conditions

Tyler D. Blanton[*] and Stephen R. Sharpe[†]

Physics Department, University of Washington, Seattle, Washington 98195-1560, USA

Unitarity of the infinite-volume three-particle scattering amplitude arising from a finite-volume formalism

Raúl A. Briceño,[1,2,*] Maxwell T. Hansen,[3,†] Stephen R. Sharpe,[4,‡] and Adam P. Szczepaniak[5,6,1,§]

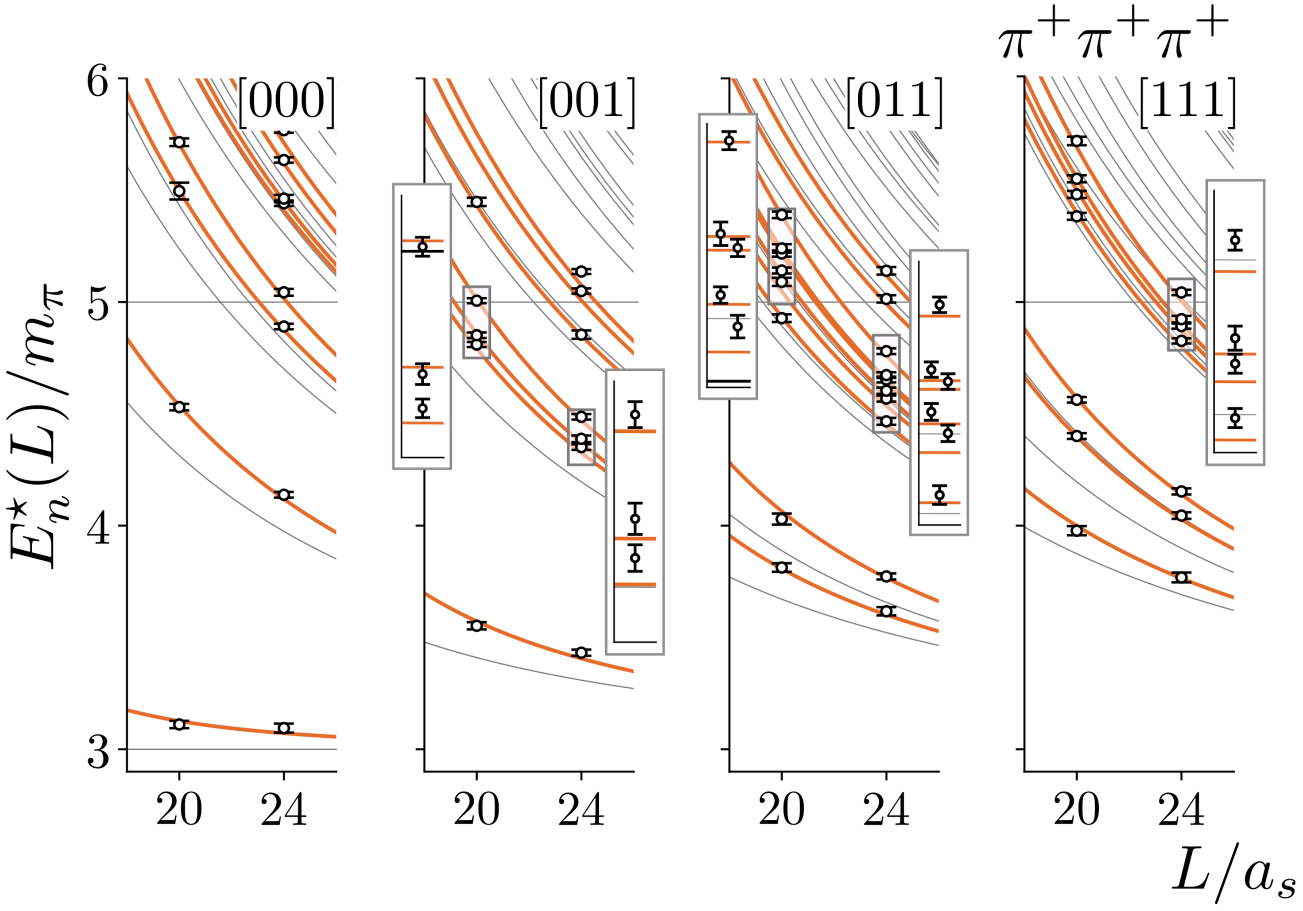
converged on formalism for three-body scattering in a finite-volume

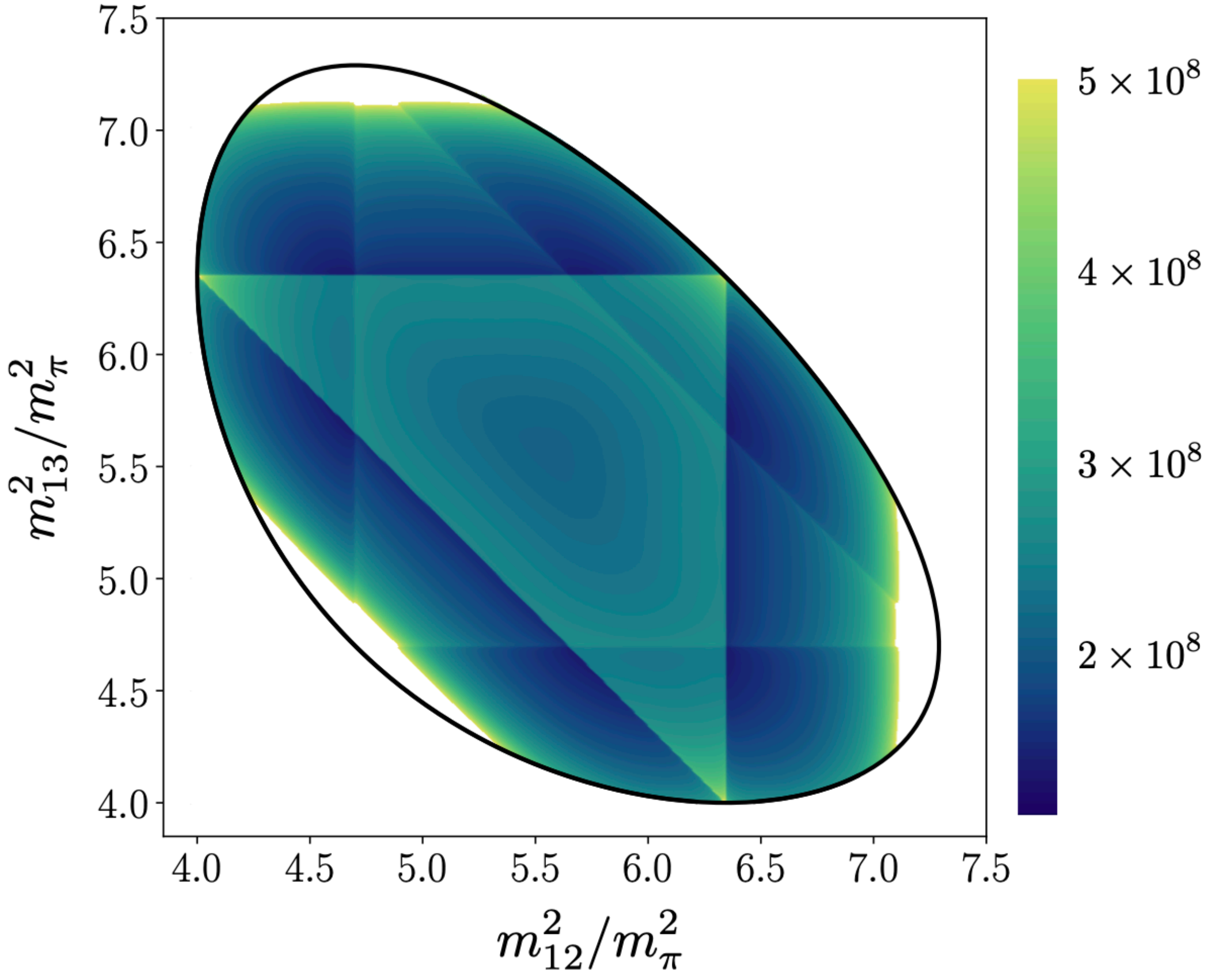
three-body systems – state of the art

mainly non-resonant maximal isospin systems so far, e.g. $\pi^+\pi^+\pi^+$

The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,[1,*] Raul A. Briceño,[2,3,†] Robert G. Edwards,[2,‡] Christopher E. Thomas,[4,§] and David J. Wilson[4,¶]
(for the Hadron Spectrum Collaboration)





three-body systems – where next ?

extend to systems with resonances in the two-body sub-channels

imposing unitarity in amplitudes,
without damaging analyticity ?

integral equations relating ‘K-matrix-like’ objects to full three-body amplitudes

analytic continuation into complex plane (for resonances) much more complicated

having a resource like JPAC containing accessible people who are familiar with amplitudes is invaluable

in addition, relationships to effective field theories, in regions of applicability

the communities overlap

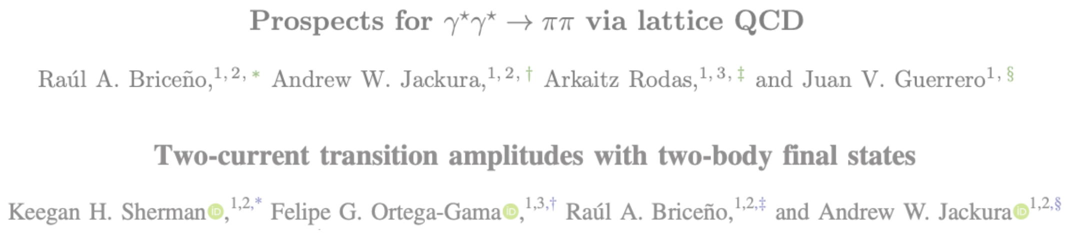
hadspec and *JPAC* have come to overlap in personnel – career opportunities expand

Andrew Jackura (ODU)

got his amplitude training in JPAC, now applying to lattice projects

Arkaitz Rodas (JLab)

member of both JPAC and hadspec

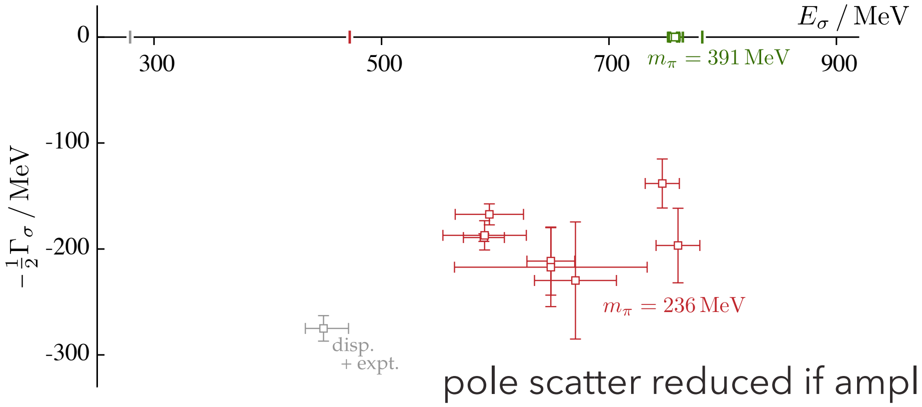


Prospects for $\gamma^\star\gamma^\star \to \pi\pi$ via lattice QCD

Raúl A. Briceño,[1,2,*] Andrew W. Jackura,[1,2,†] Arkaitz Rodas,[1,3,‡] and Juan V. Guerrero[1,§]

Two-current transition amplitudes with two-body final states

Keegan H. Sherman,[1,2,*] Felipe G. Ortega-Gama,[1,3,†] Raúl A. Briceño,[1,2,‡] and Andrew W. Jackura[1,2,§]



pole scatter reduced if amplitudes subject to constraints (analyticity, crossing)?
dispersion relations

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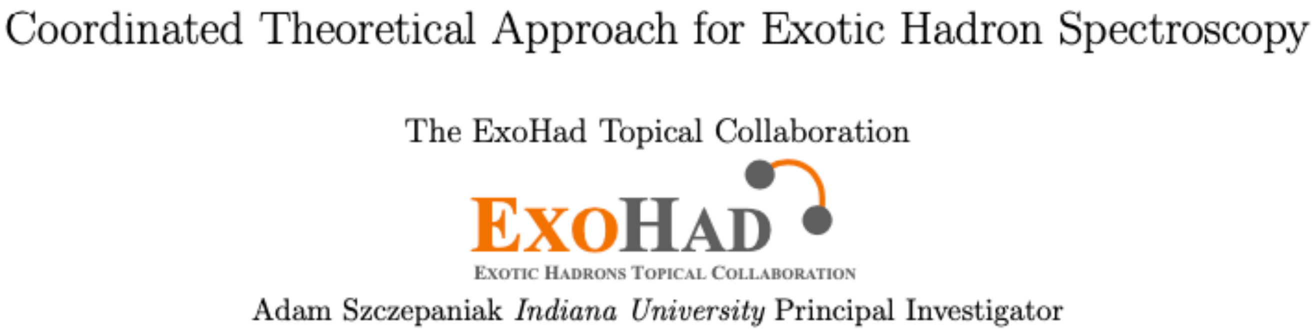
a DoE Topical Collaboration proposal
was built on a base of hadspec/JPAC collaboration



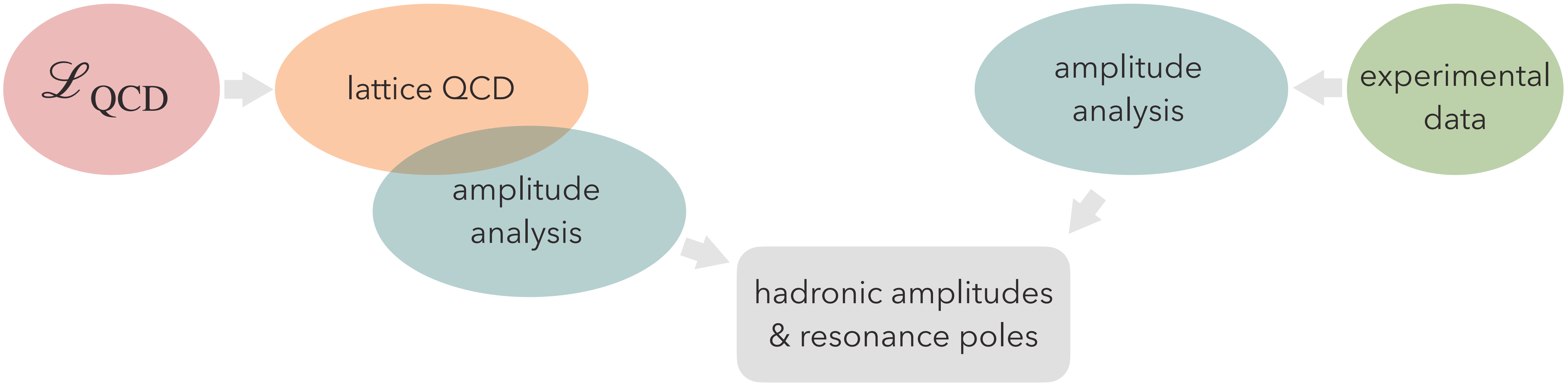
Coordinated Theoretical Approach for Exotic Hadron Spectroscopy

The ExoHad Topical Collaboration

EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

Adam Szczepaniak *Indiana University* Principal Investigator

modern hadron spectroscopy theory



robust connection made between real experiment & QCD

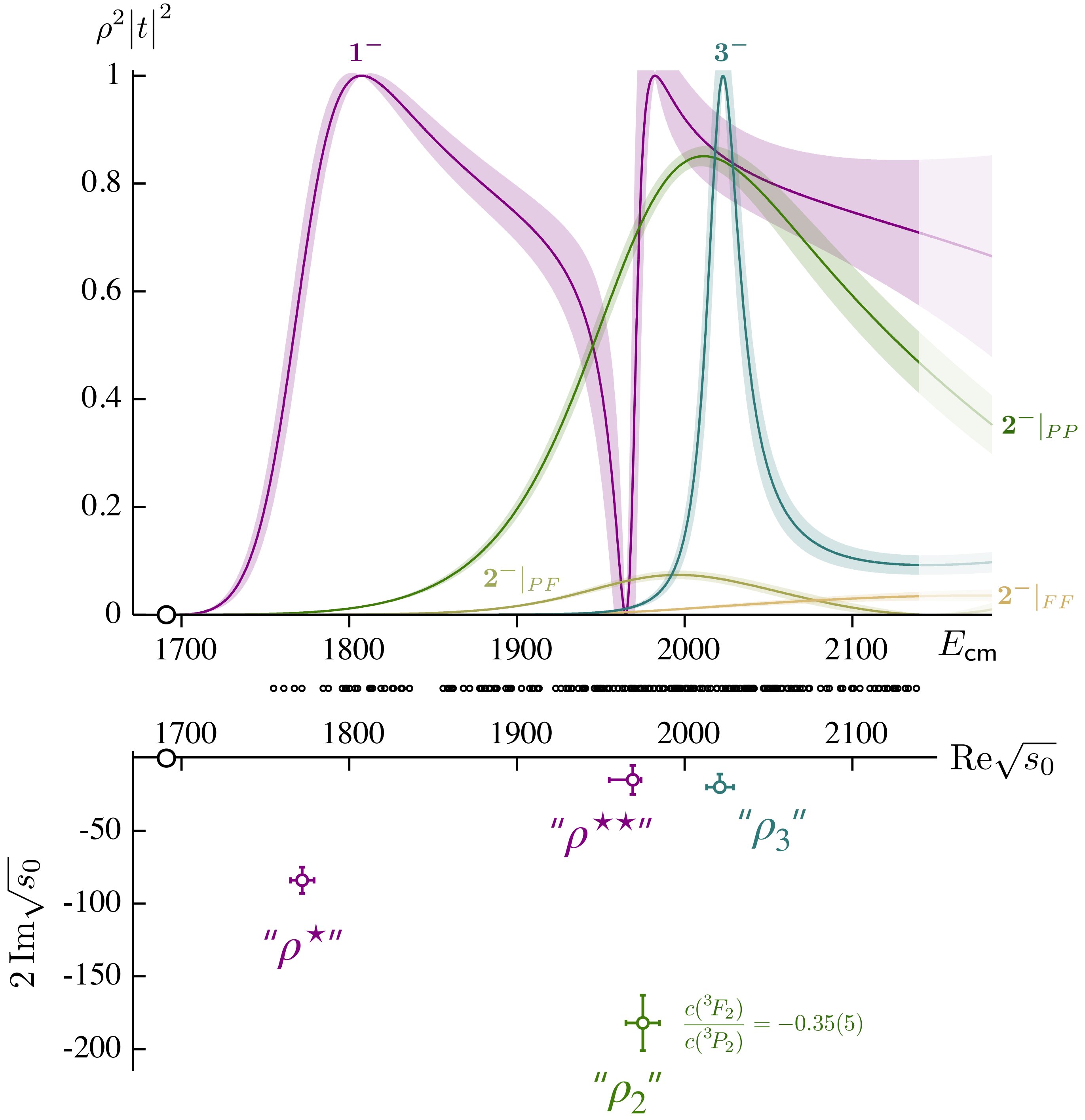
reliant upon a workforce having experience with properly constructed hadronic amplitudes

J^{--} resonances at SU(3)$_F$ point

PRD 103 074502 (**2021**)

$m_\pi \sim 700\,\text{MeV}$

decay to pseudoscalar + vector



actually the SU(3)$_F$ **singlet** states,
so don't take the names seriously

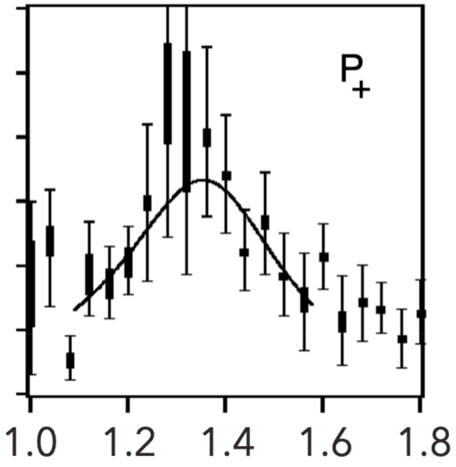
example of π_1

$\pi_1(1400)$ $I^G(J^{PC}) = 1^-(1^{-+})$

Mass $m = 1354 \pm 25$ MeV
Full width $\Gamma = 330 \pm 35$ MeV

BNL e852 (1997)

$\pi^- p \to \eta\pi^- p$

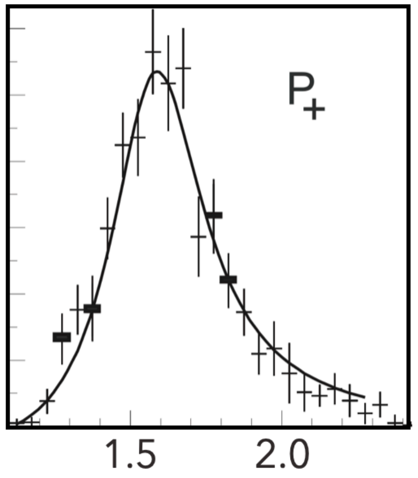


$\pi_1(1600)$ $I^G(J^{PC}) = 1^-(1^{-+})$

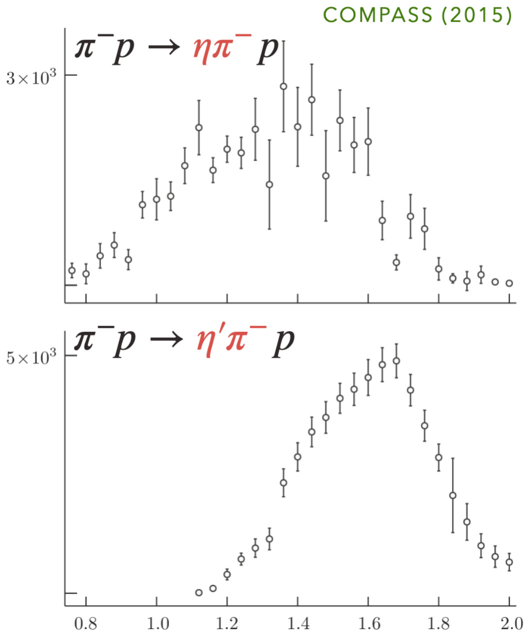
Mass $m = 1661^{+15}_{-11}$ MeV
Full width $\Gamma = 240 \pm 50$ MeV

BNL e852 (2001)

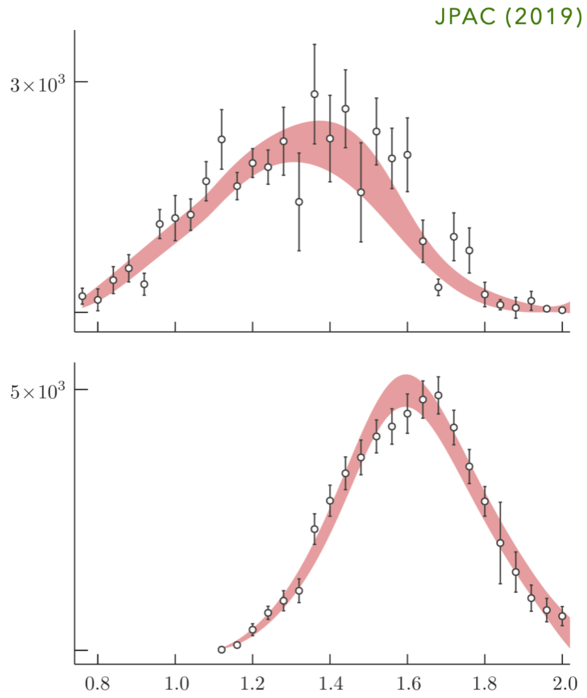
$\pi^- p \to \eta'\pi^- p$



also seen in $\pi\rho$



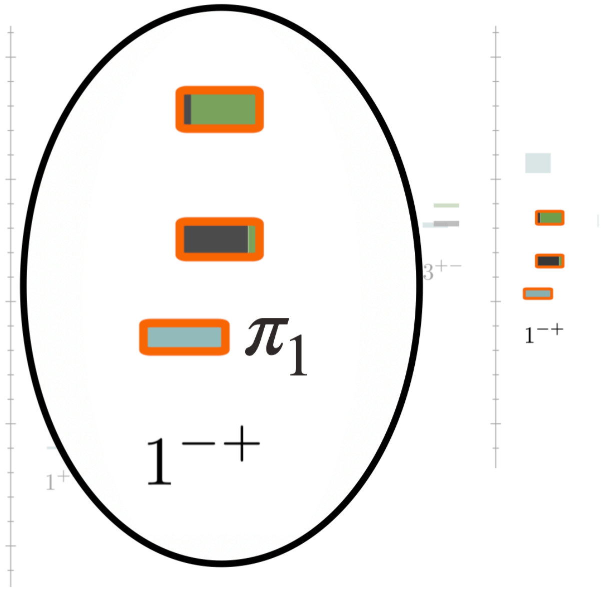
compatible with structures seen at BNL



amplitudes have only one pole singularity

$m = 1564(89)\,\text{MeV}$
$\Gamma = 492(115)\,\text{MeV}$

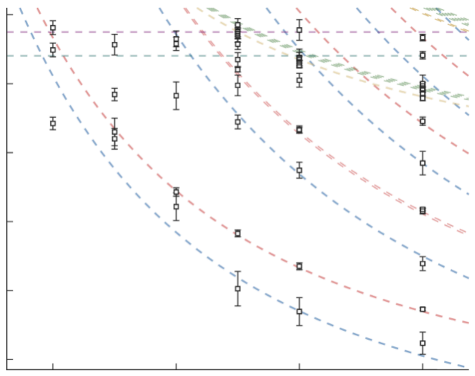
a broad resonance …



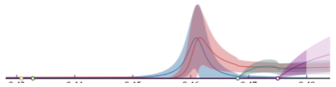
SU(3)$_F$ point

$m_u = m_d = m_s^{\text{phys}}$
$m_\pi \sim 700\,\text{MeV}$

spectrum in six lattice volumes



amplitudes in eight channels



extrapolated to physical quark mass

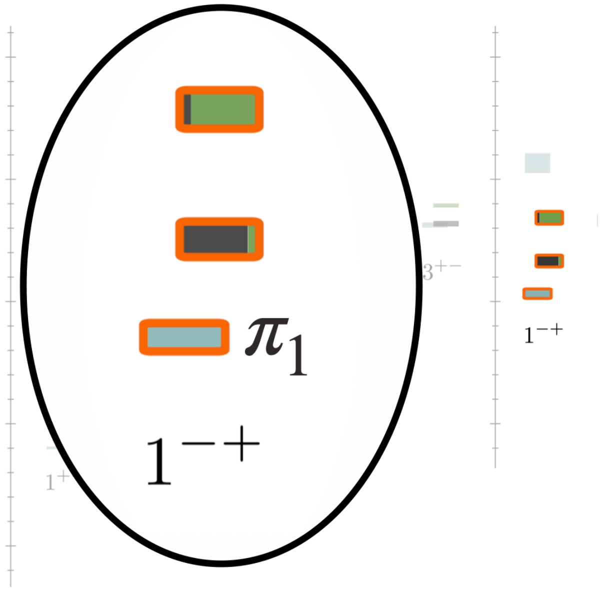
decay couplings of one resonance pole

for a $\pi_1(1564)$

$\Gamma \sim 140 - 600\,\text{MeV}$

$\Gamma(\pi\eta) \lesssim 1\,\text{MeV}$
$\Gamma(\pi\eta') \lesssim 12\,\text{MeV}$
$\Gamma(\pi\rho) \lesssim 20\,\text{MeV}$
$\Gamma(\pi b_1) \sim 139 - 529\,\text{MeV}$

example of π_1



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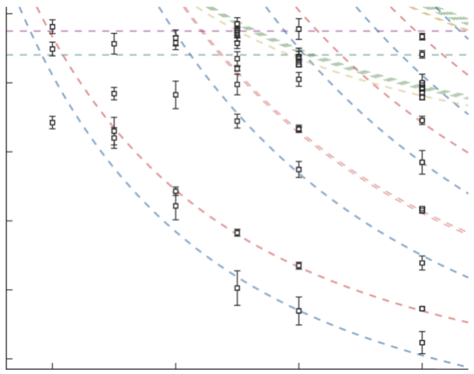
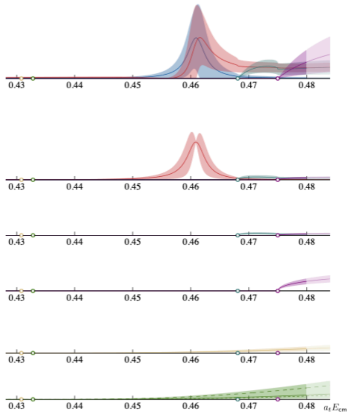
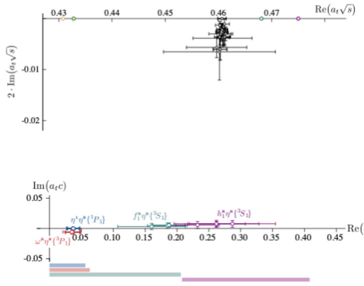


amplitudes in eight channels



decay couplings of
one resonance pole



narrow resonance at
this heavy quark mass

extrapolated to
physical quark mass

for a $\pi_1(1564)$

$\Gamma \sim 140 - 600\,\text{MeV}$

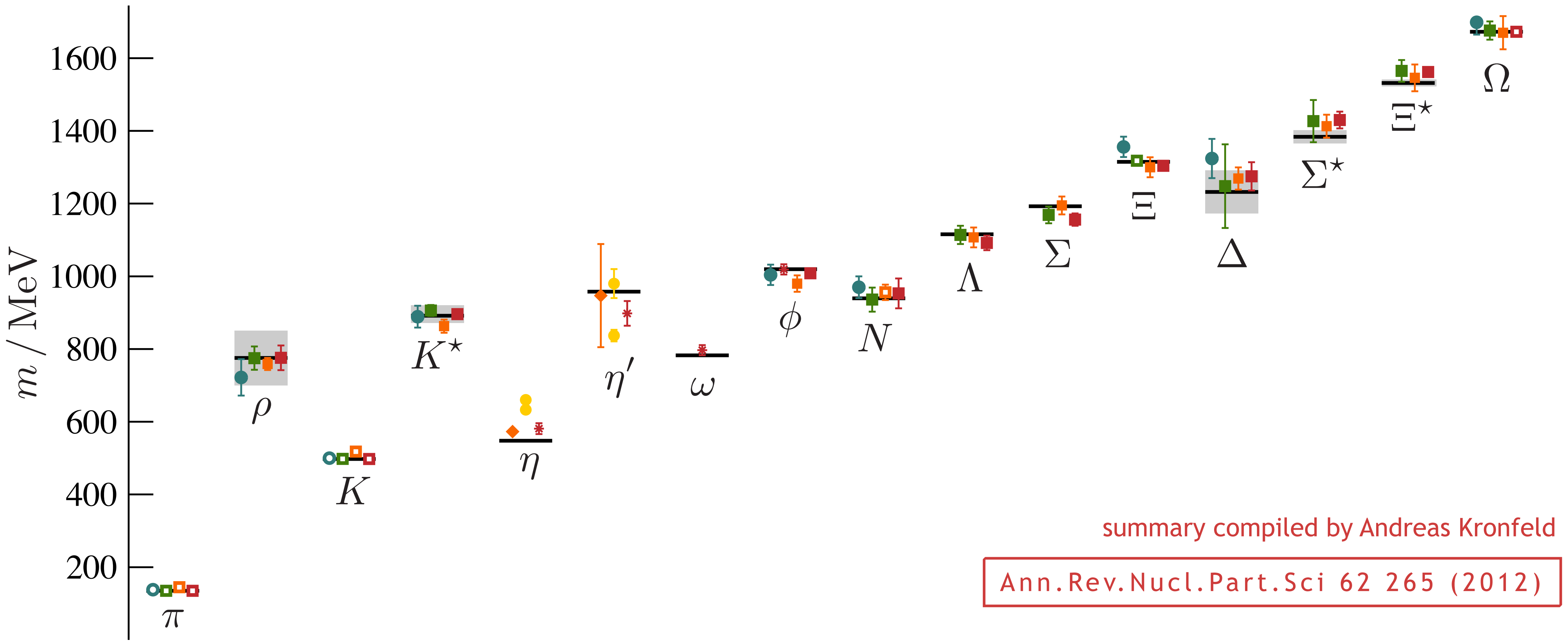
$\Gamma(\pi\eta) \lesssim 1\,\text{MeV}$

$\Gamma(\pi\eta') \lesssim 12\,\text{MeV}$

$\Gamma(\pi\rho) \lesssim 20\,\text{MeV}$

$\Gamma(\pi b_1) \sim 139 - 529\,\text{MeV}$

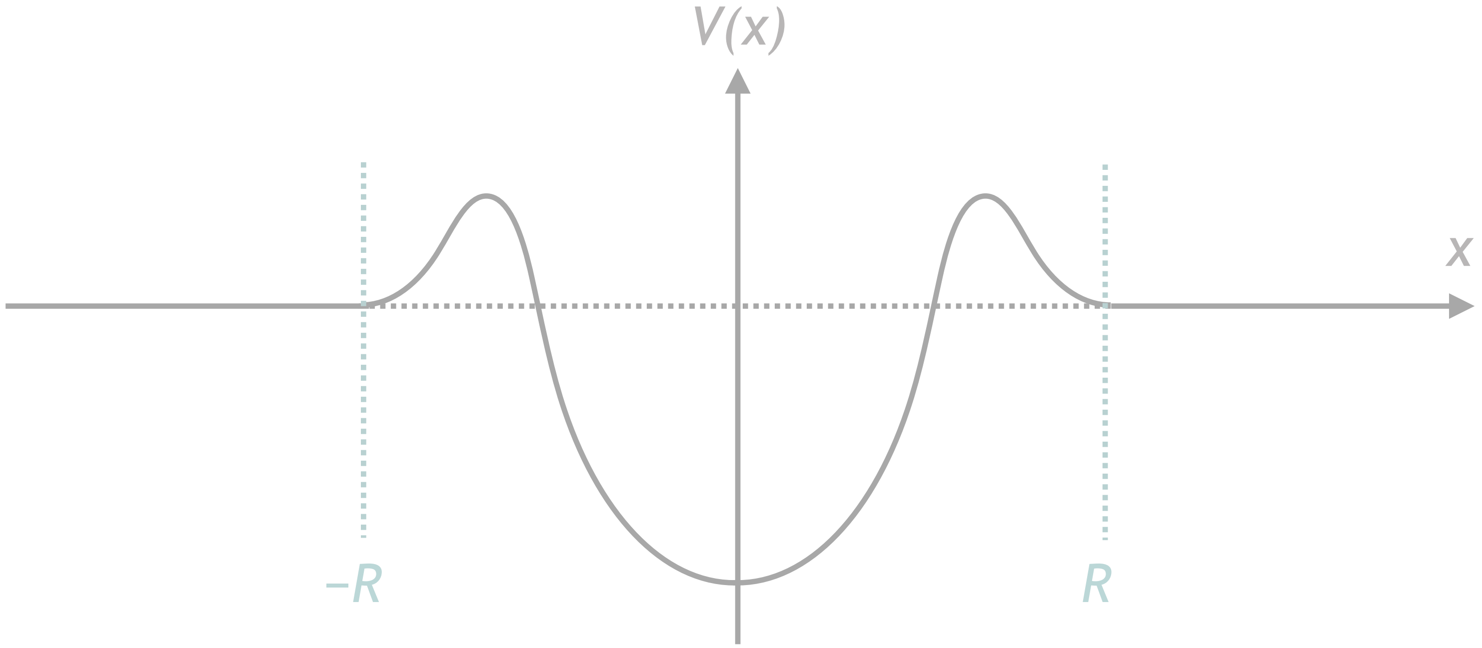
'stable' lightest hadrons with systematics assessment

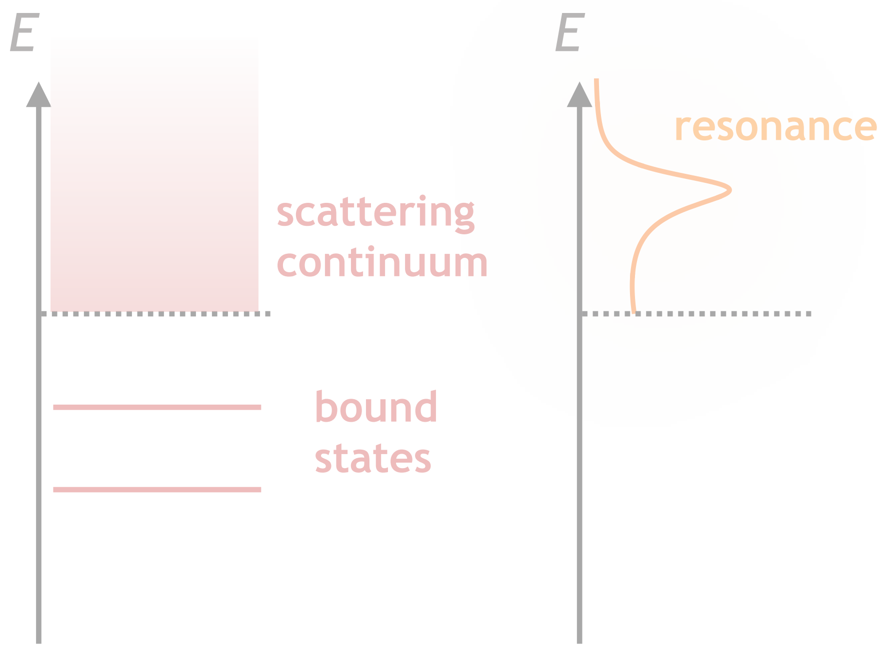


summary compiled by Andreas Kronfeld

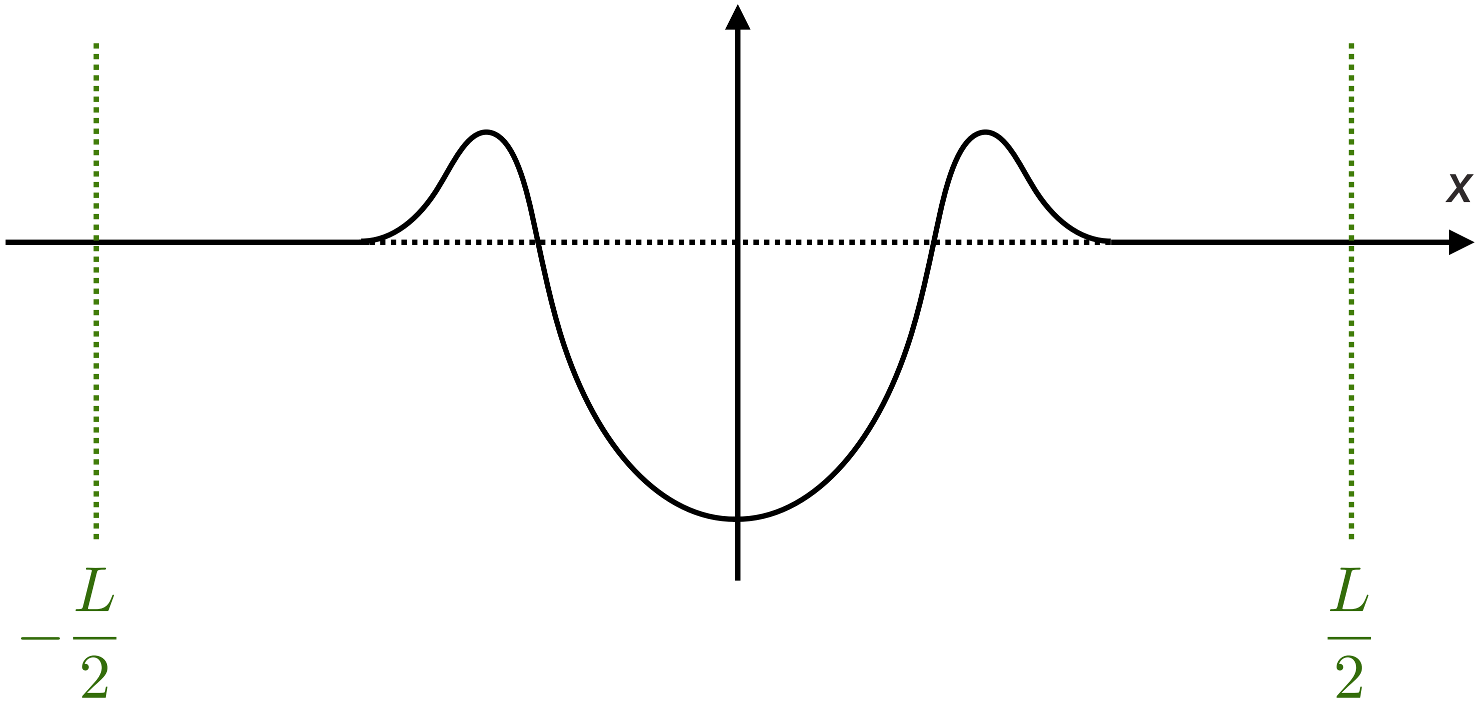
Ann.Rev.Nucl.Part.Sci 62 265 (2012)

resonances in a finite volume ?

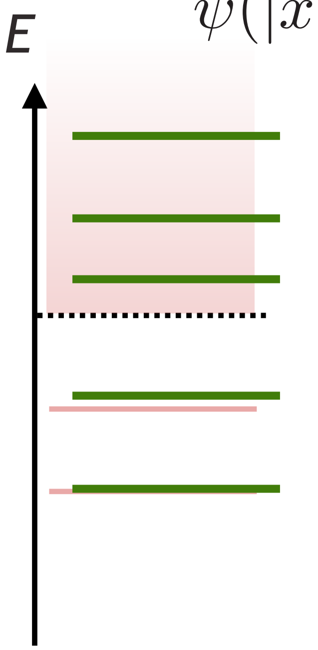




but in a periodic volume ...



$$\psi(|x| > R) \sim \cos\big(p|x| + \delta(p)\big)$$



applying the boundary conditions

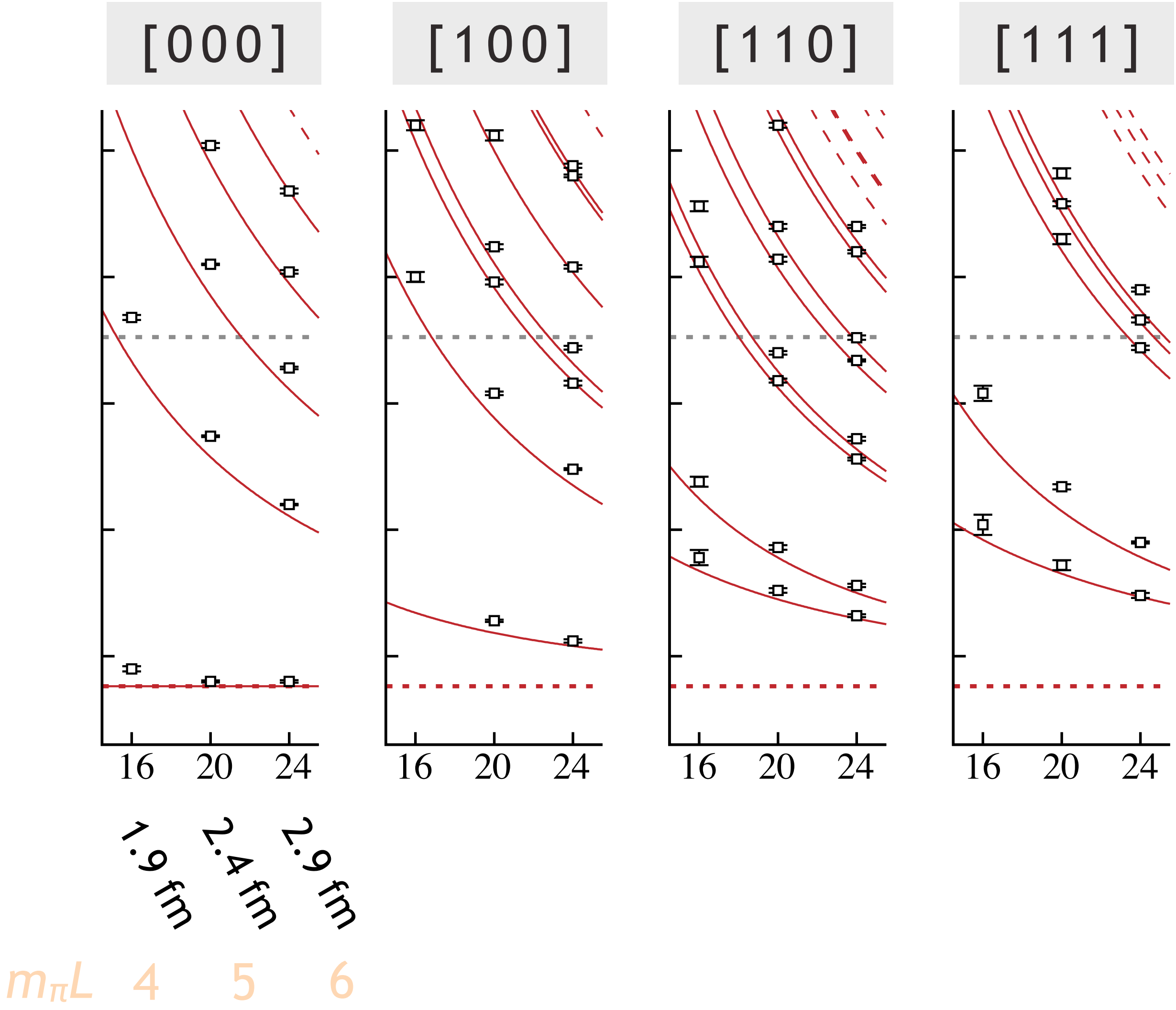
$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

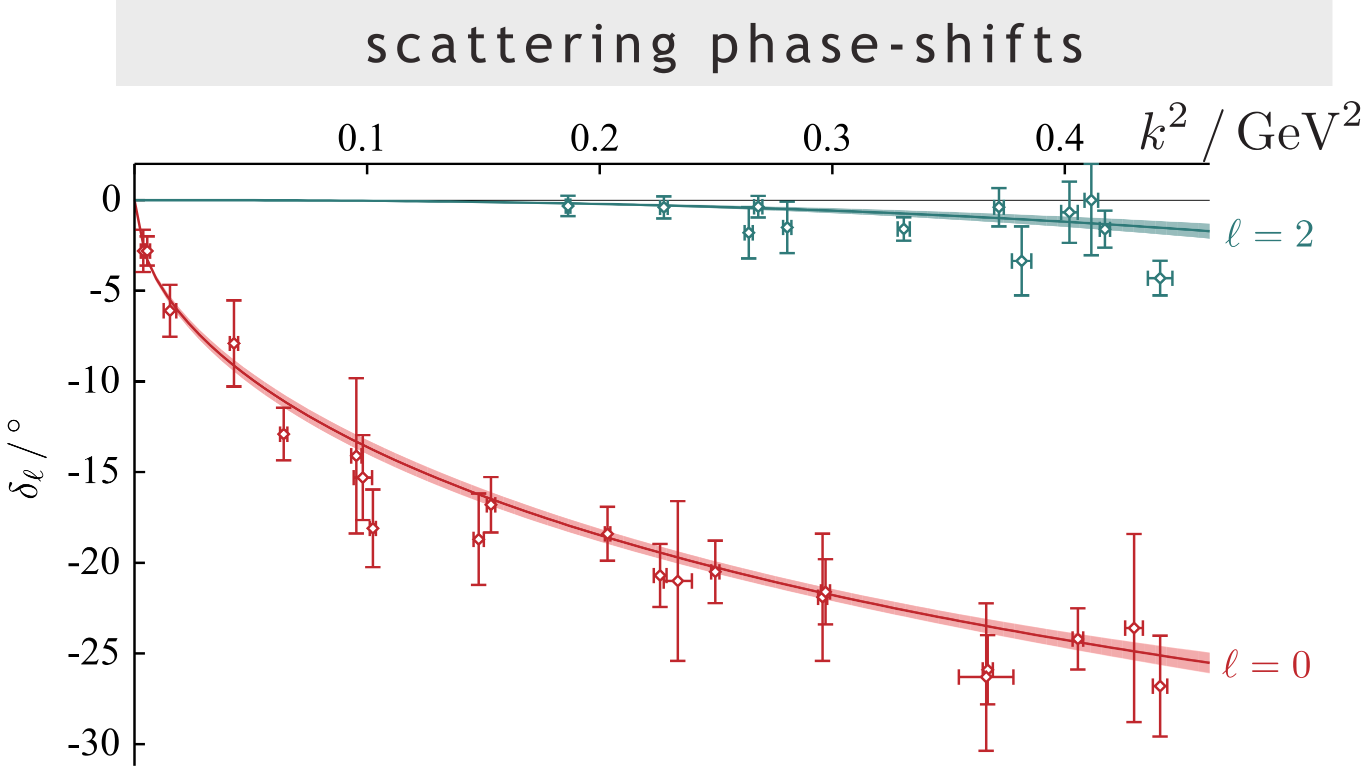
solved by **discrete** $p_n(L)$

non-resonant elastic scattering – $\pi^+\pi^+$ (isospin=2)

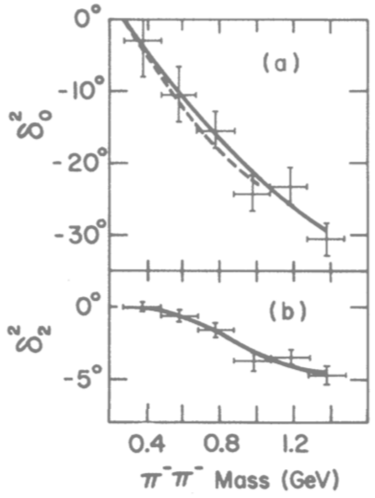
PRD86 034031 (2012)

m_π ~ 391 MeV





repulsive scattering, no resonances

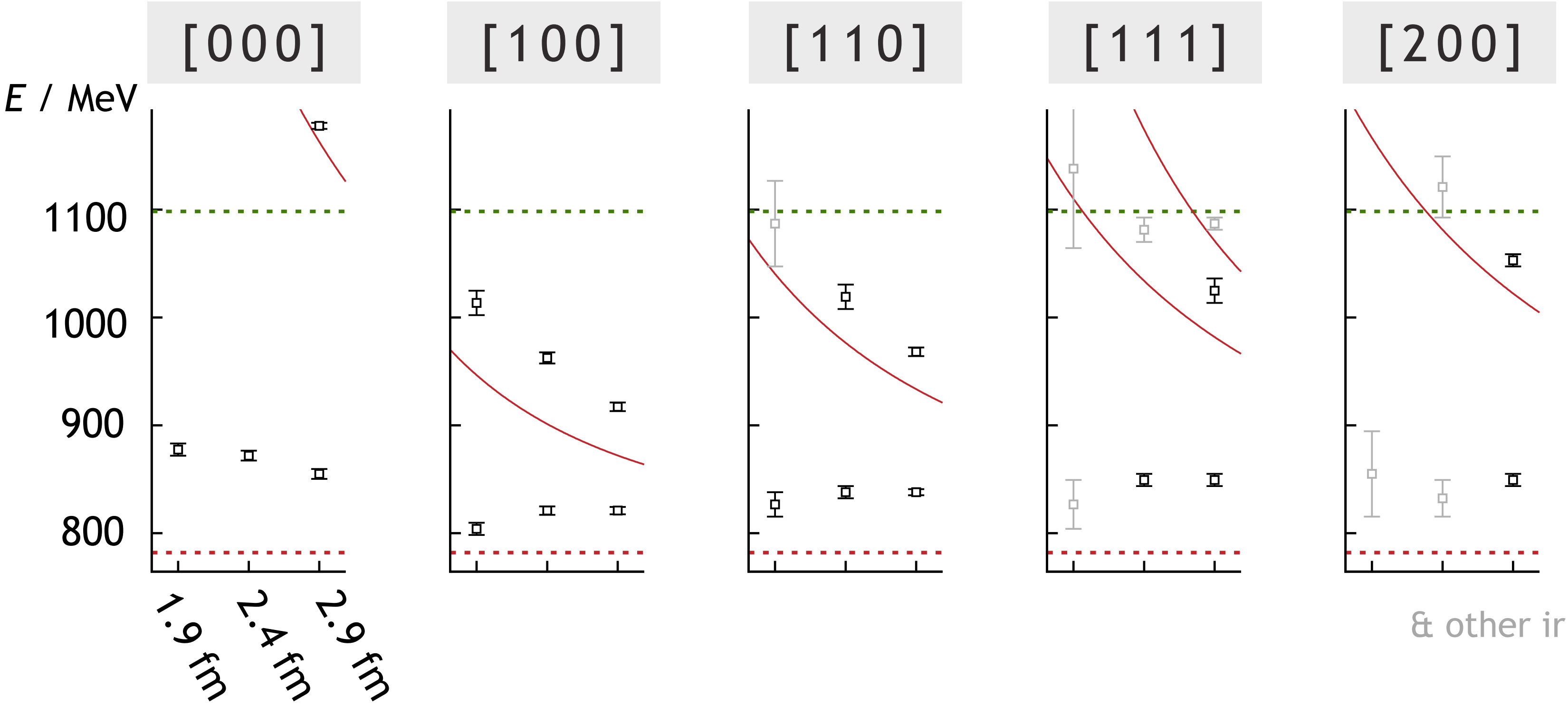


Cohen 1972

an elastic resonance – the ρ in $\pi\pi$ (isospin=1)

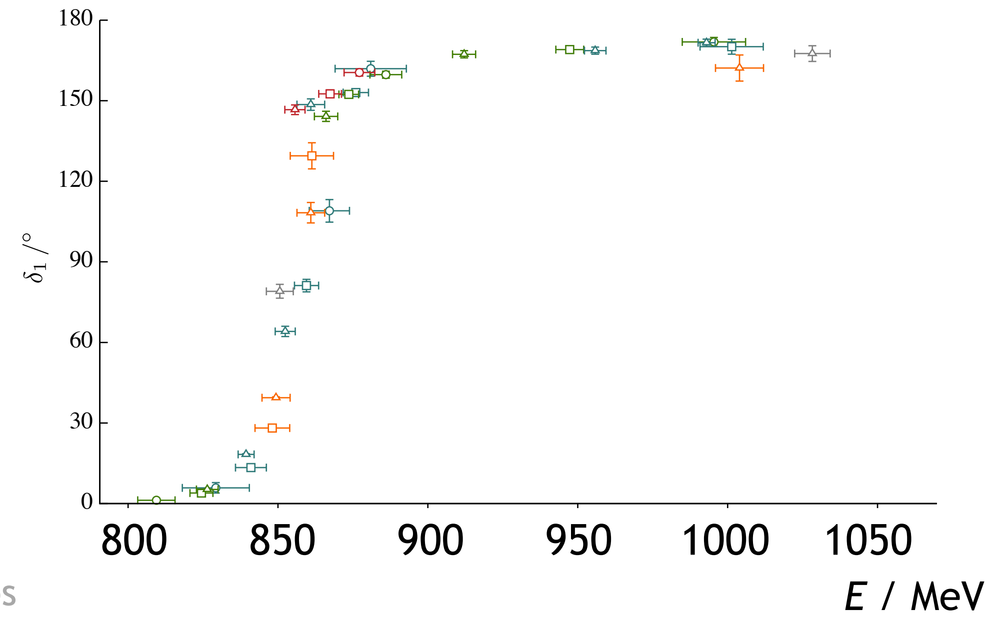
PRD87 034505 (2013)

m_π ~ 391 MeV



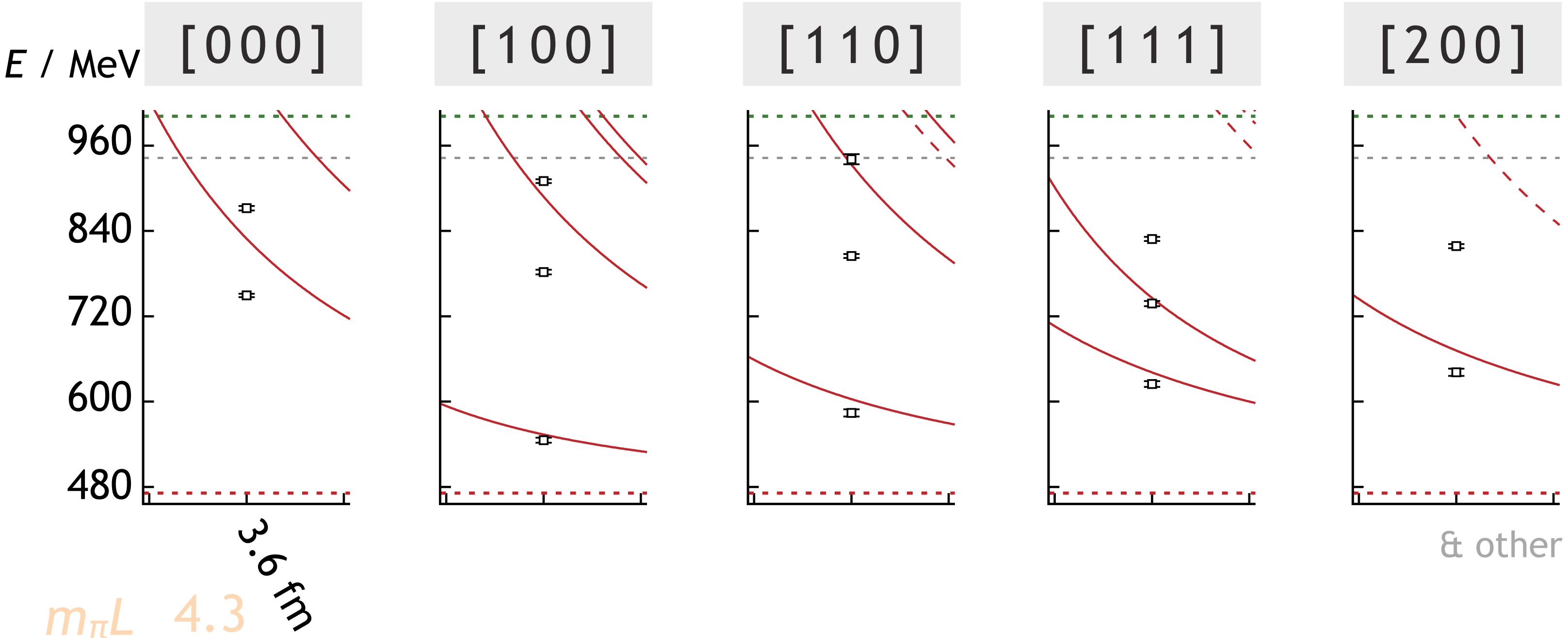
& other irreps

scattering phase-shift



PRD92 094502 (2015)

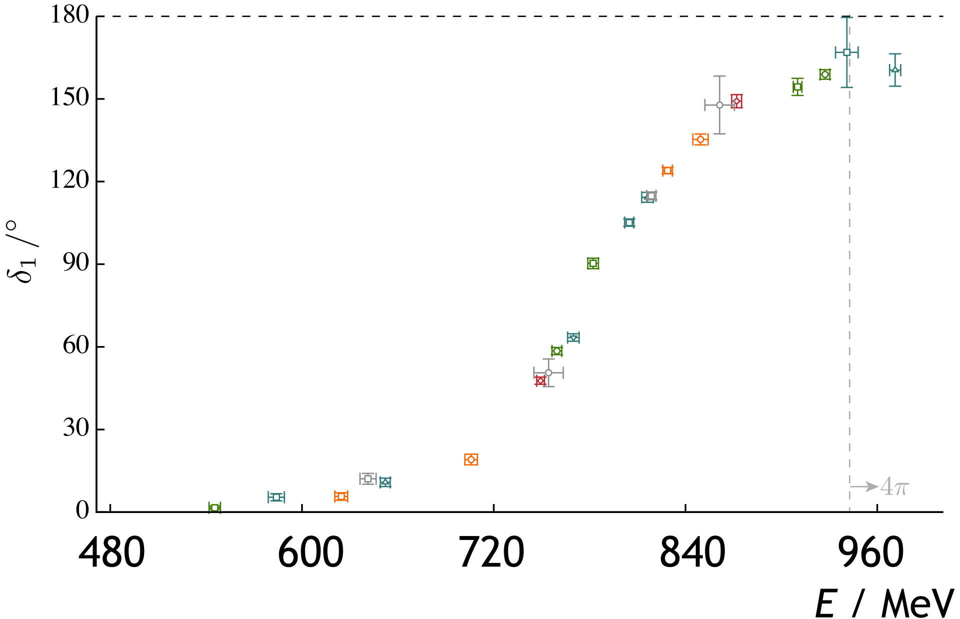
m_π ~ 236 MeV



$m_\pi L$ 4.3

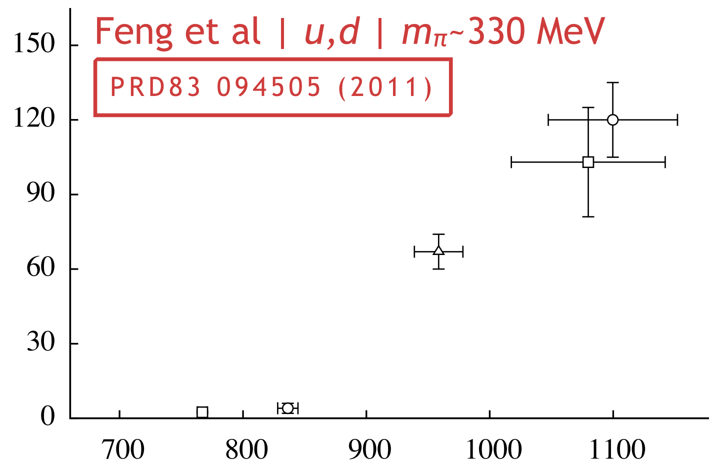
& other irreps

scattering phase-shift



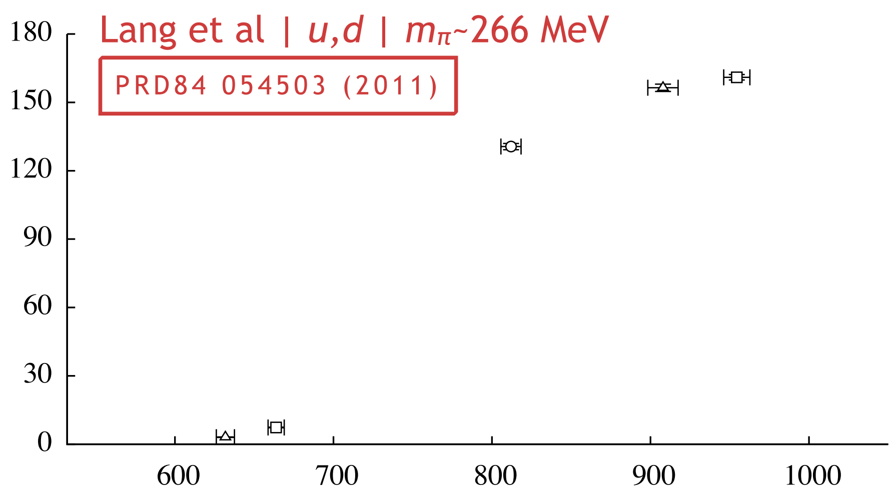
an elastic resonance – the ρ in $\pi\pi$ (isospin=1)

has become a common quantity to compute in lattice QCD …



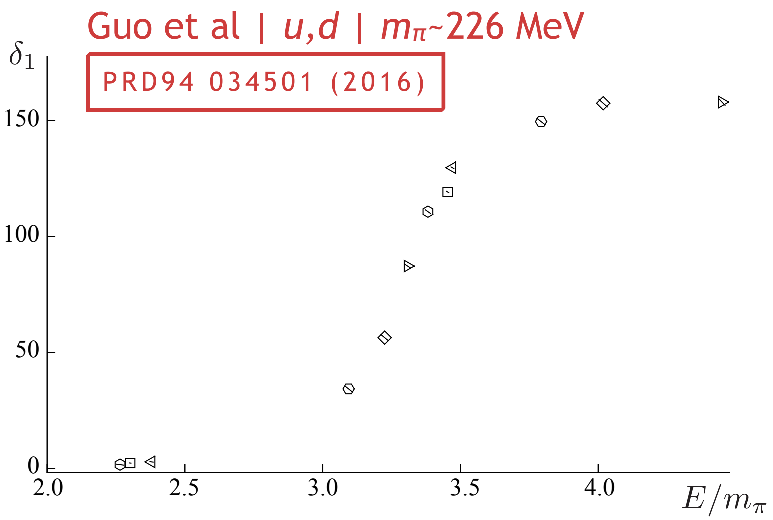
Feng et al | *u,d* | m_π~330 MeV

PRD83 094505 (2011)



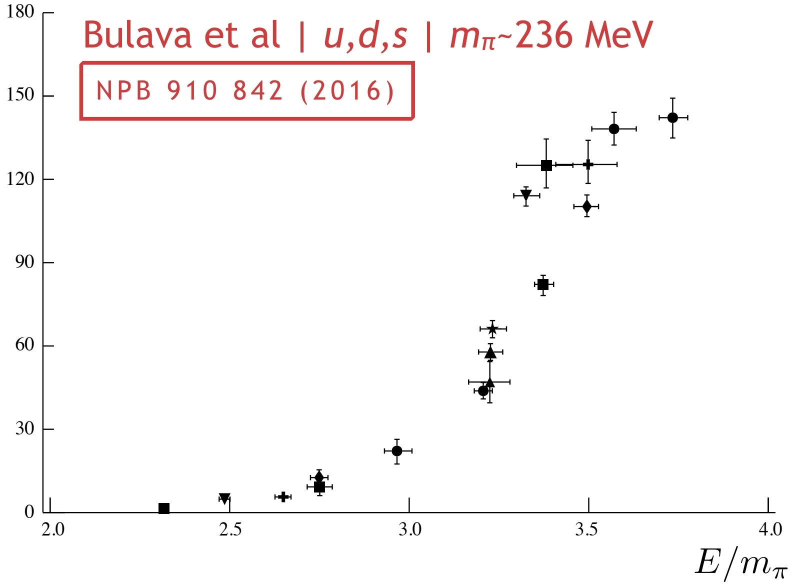
Lang et al | *u,d* | m_π~266 MeV

PRD84 054503 (2011)



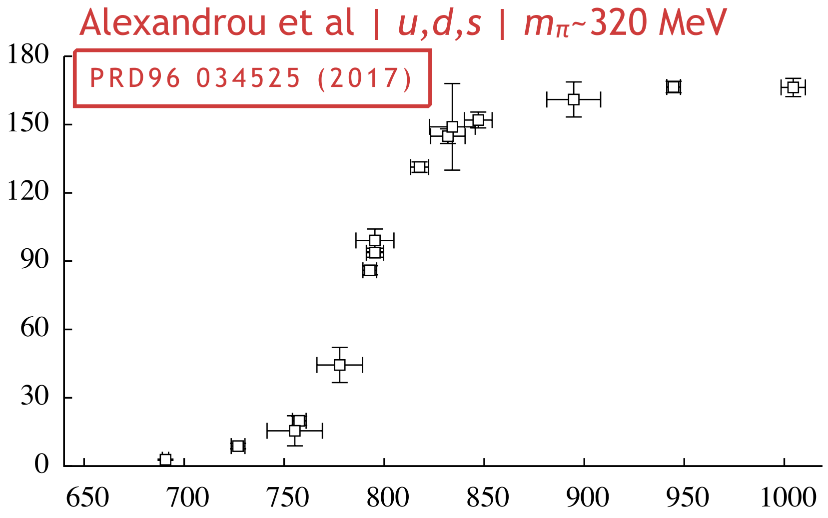
Guo et al | *u,d* | m_π~226 MeV

PRD94 034501 (2016)



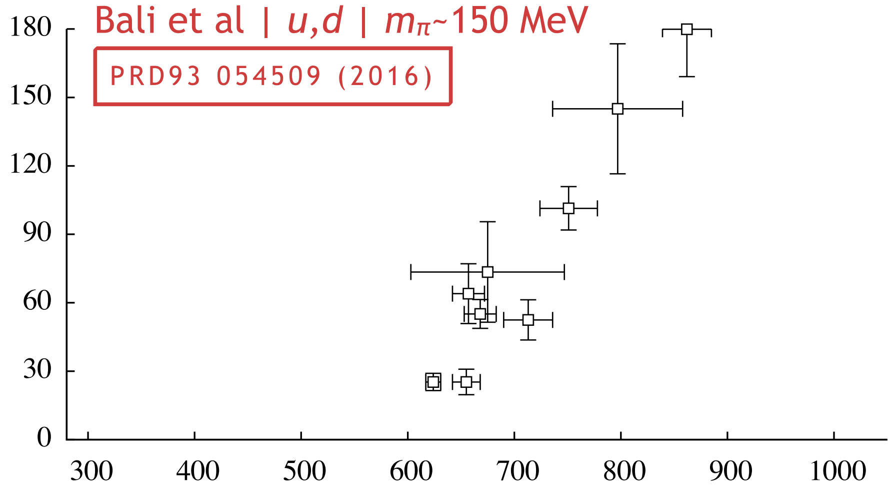
Bulava et al | *u,d,s* | m_π~236 MeV

NPB 910 842 (2016)



Alexandrou et al | *u,d,s* | m_π~320 MeV

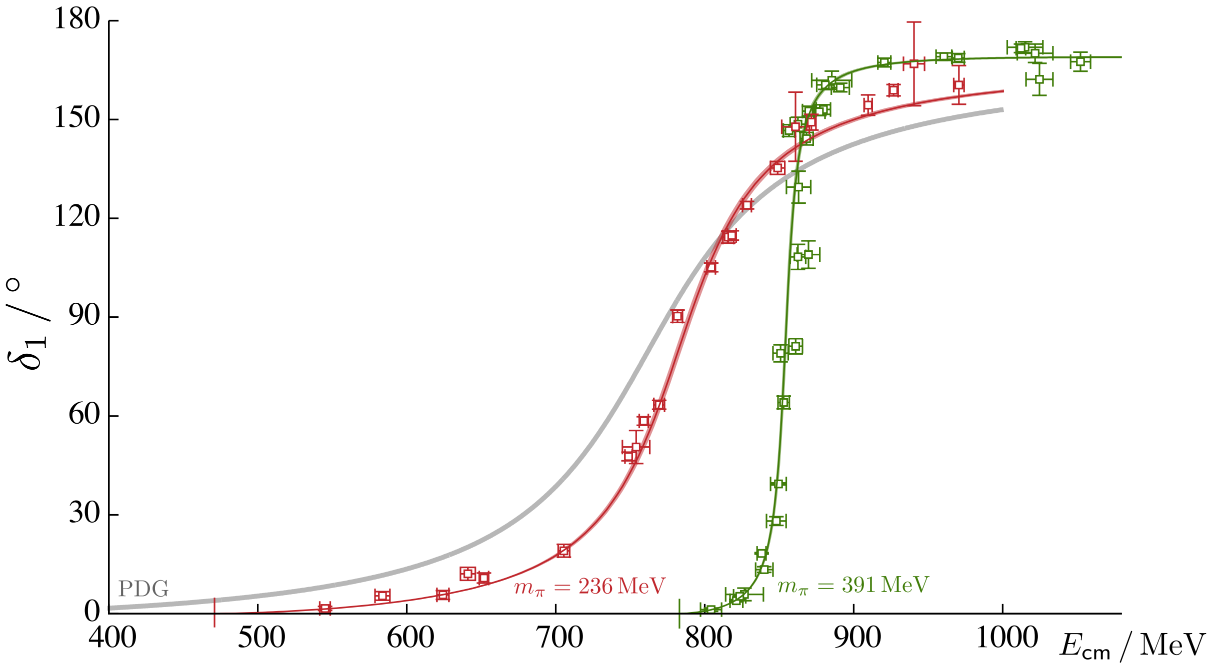
PRD96 034525 (2017)

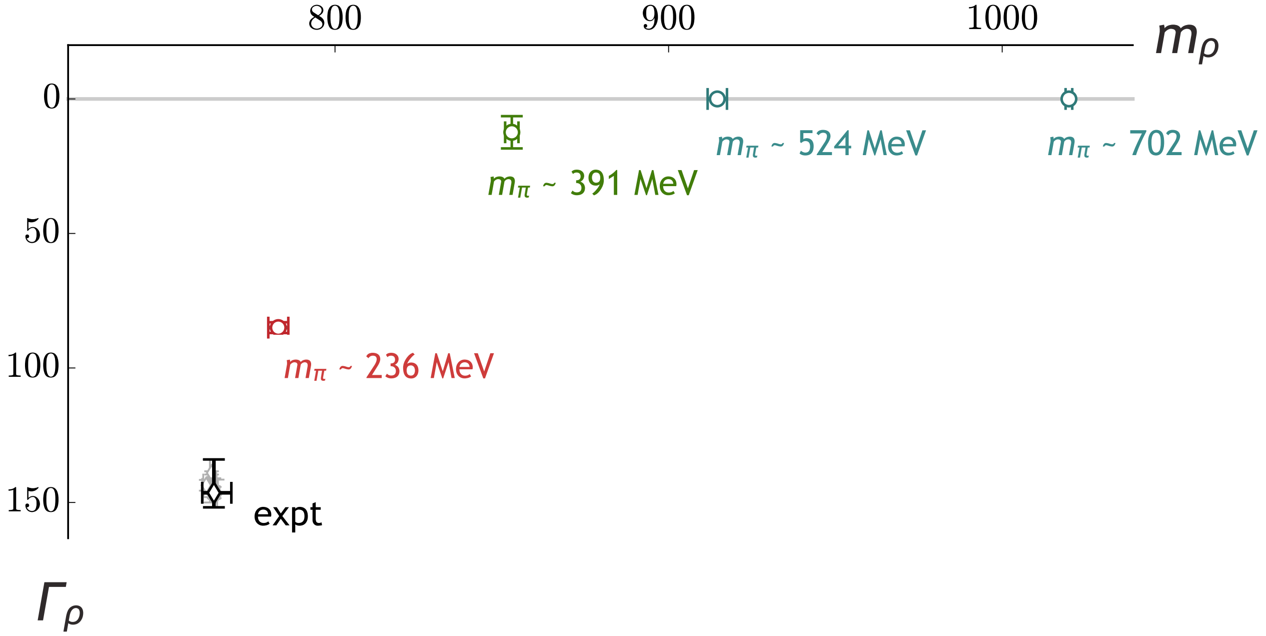


Bali et al | *u,d* | m_π~150 MeV

PRD93 054509 (2016)

ρ pole with changing quark mass from lattice QCD

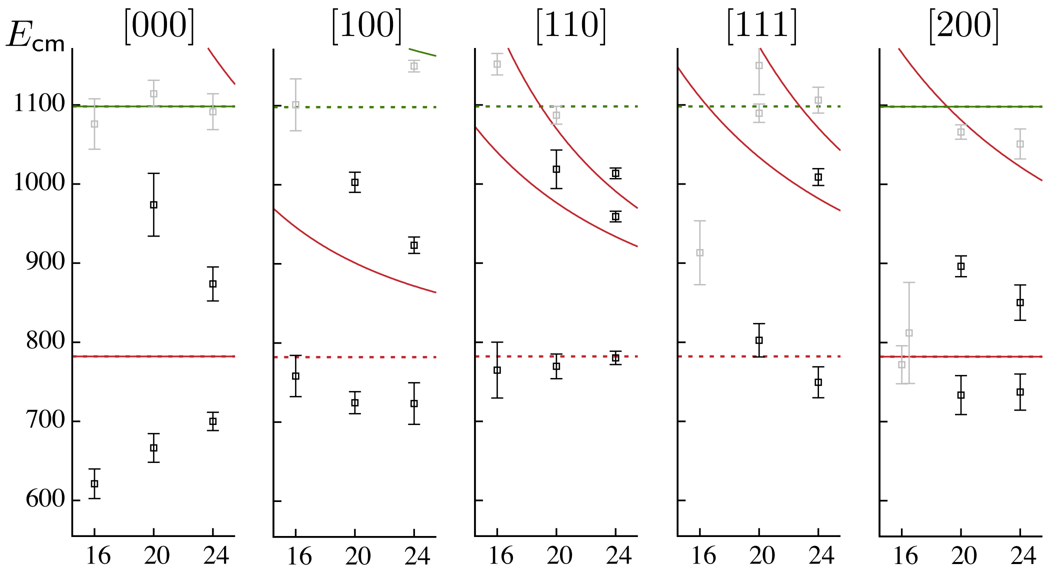




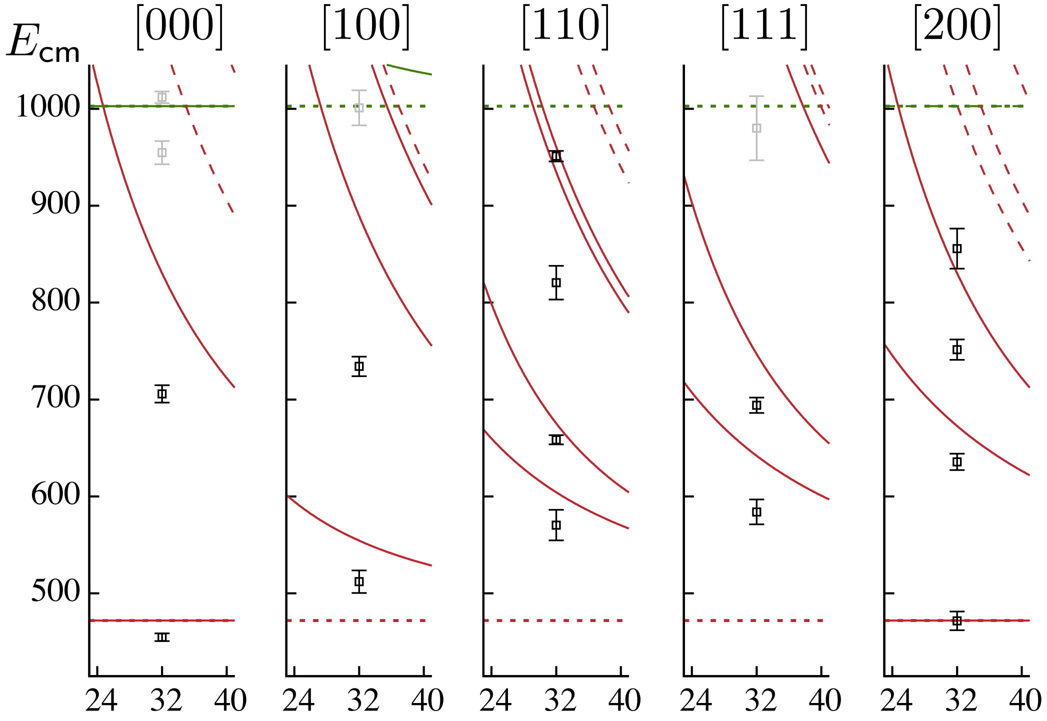
$\pi\pi$ isospin=0

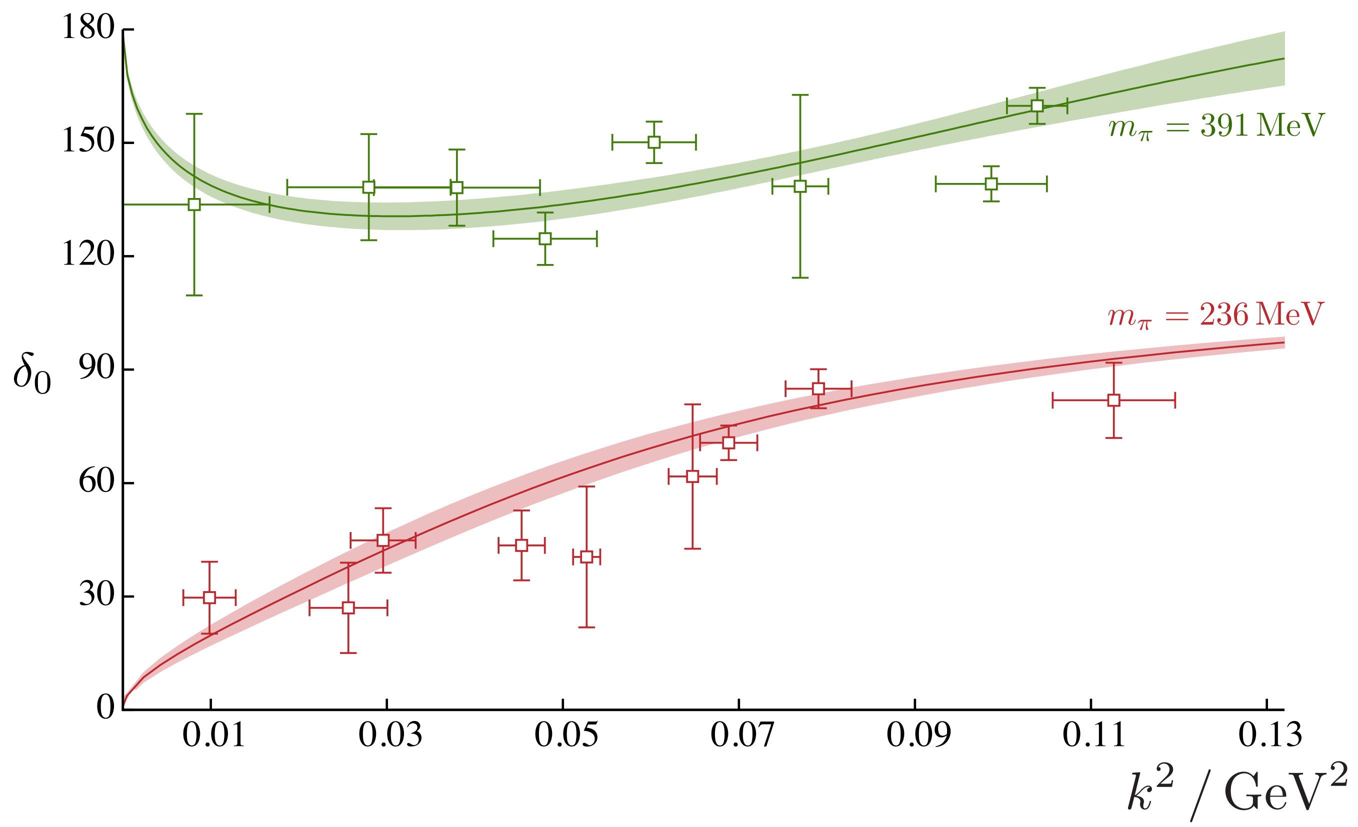
PRL118 022002 (2017)

m_π~391 MeV



m_π~236 MeV



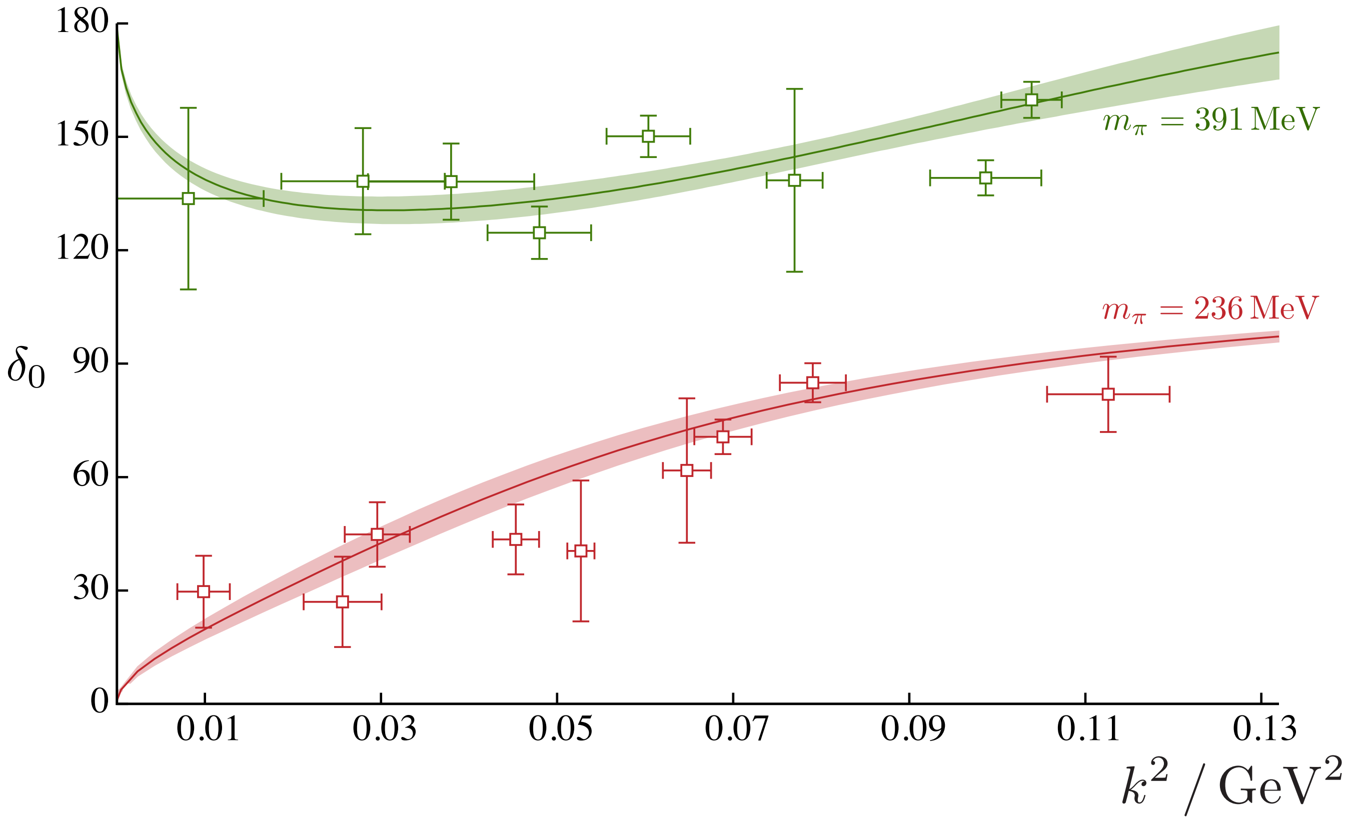


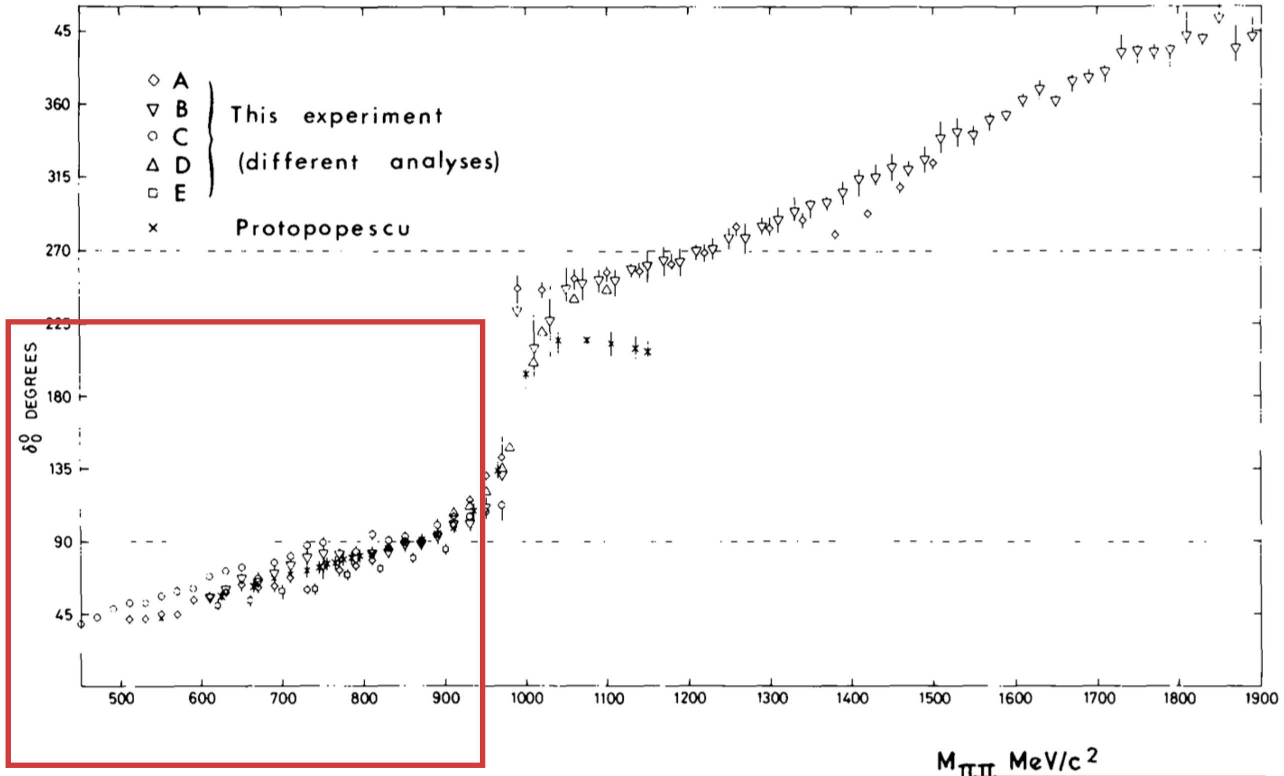
heavier quark mass — **a bound-state**

lighter quark mass — attraction, maybe a **broad resonance** ?

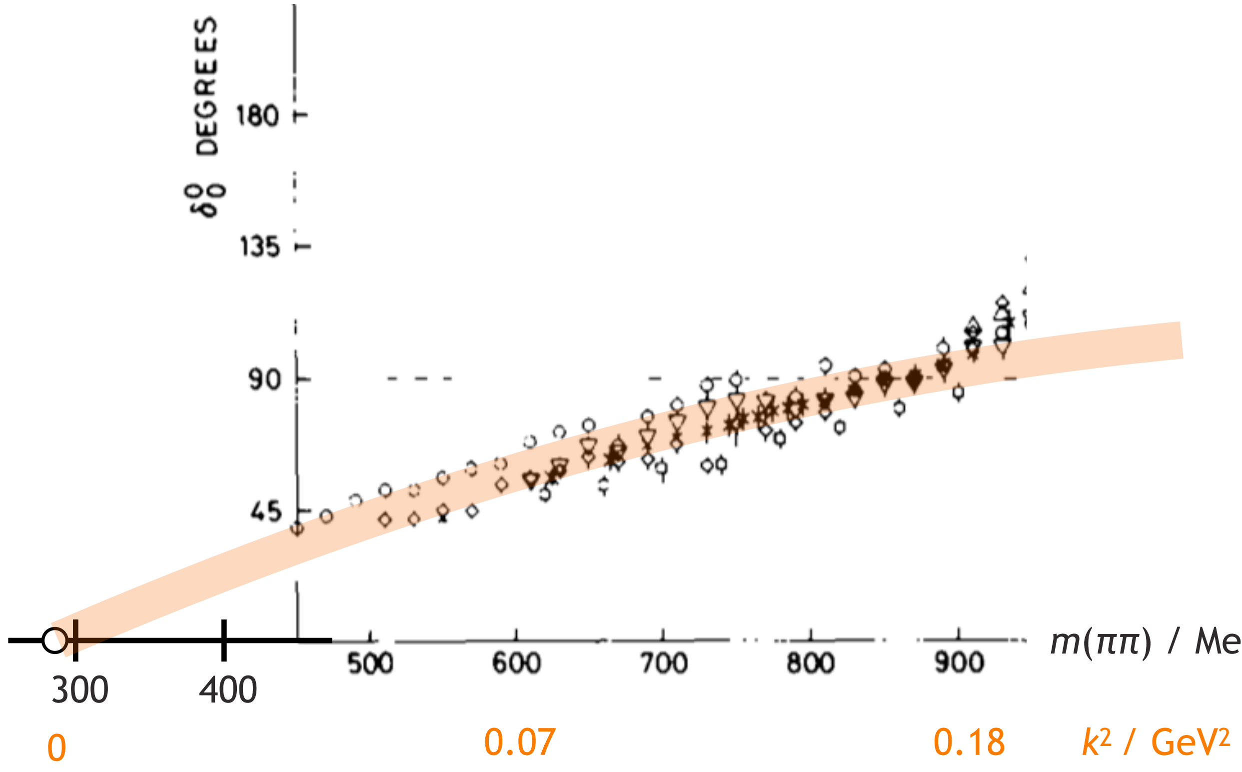
c.f. the experimental σ resonance …

ππ isospin=0



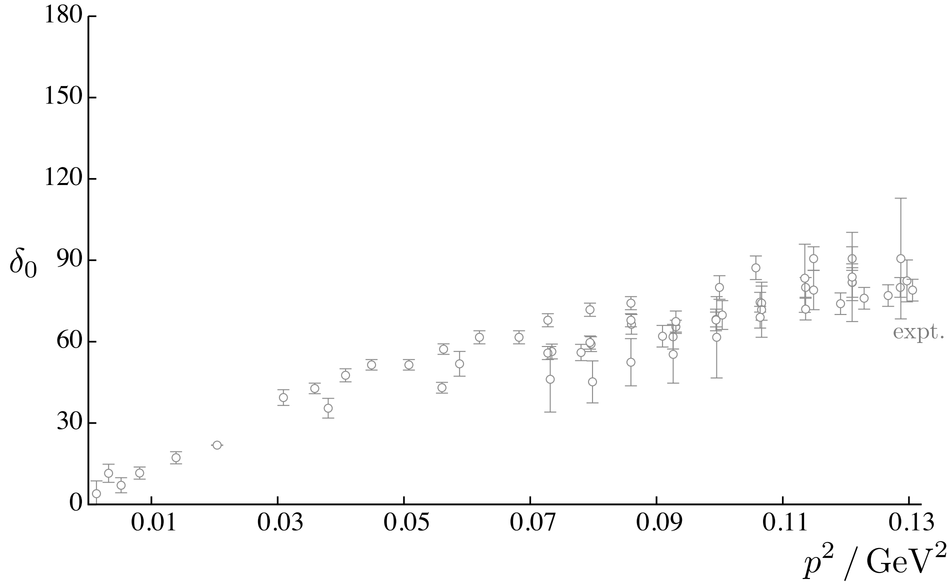


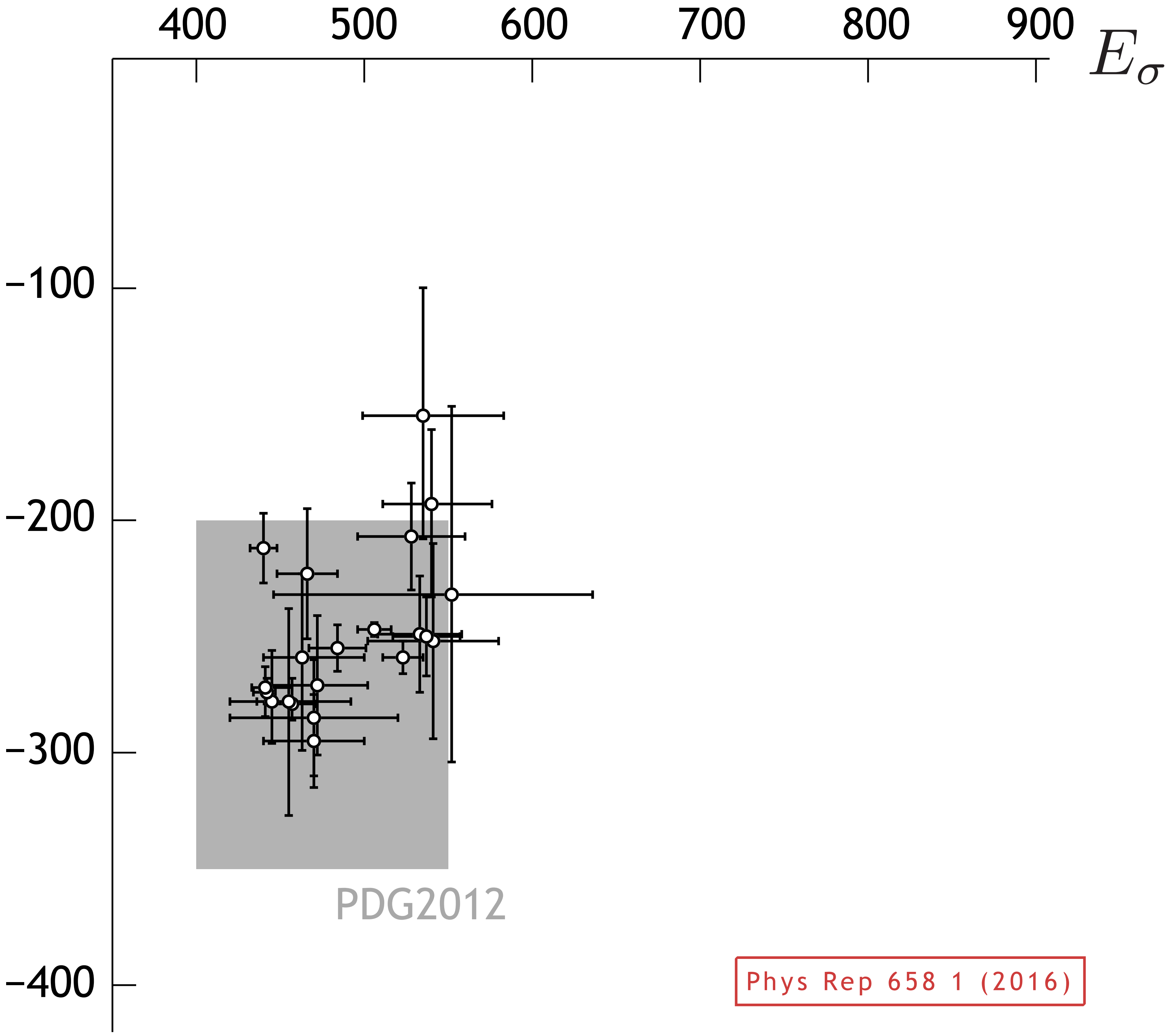
Grayer 1974



σ pole scatter from experimental phase-shift data

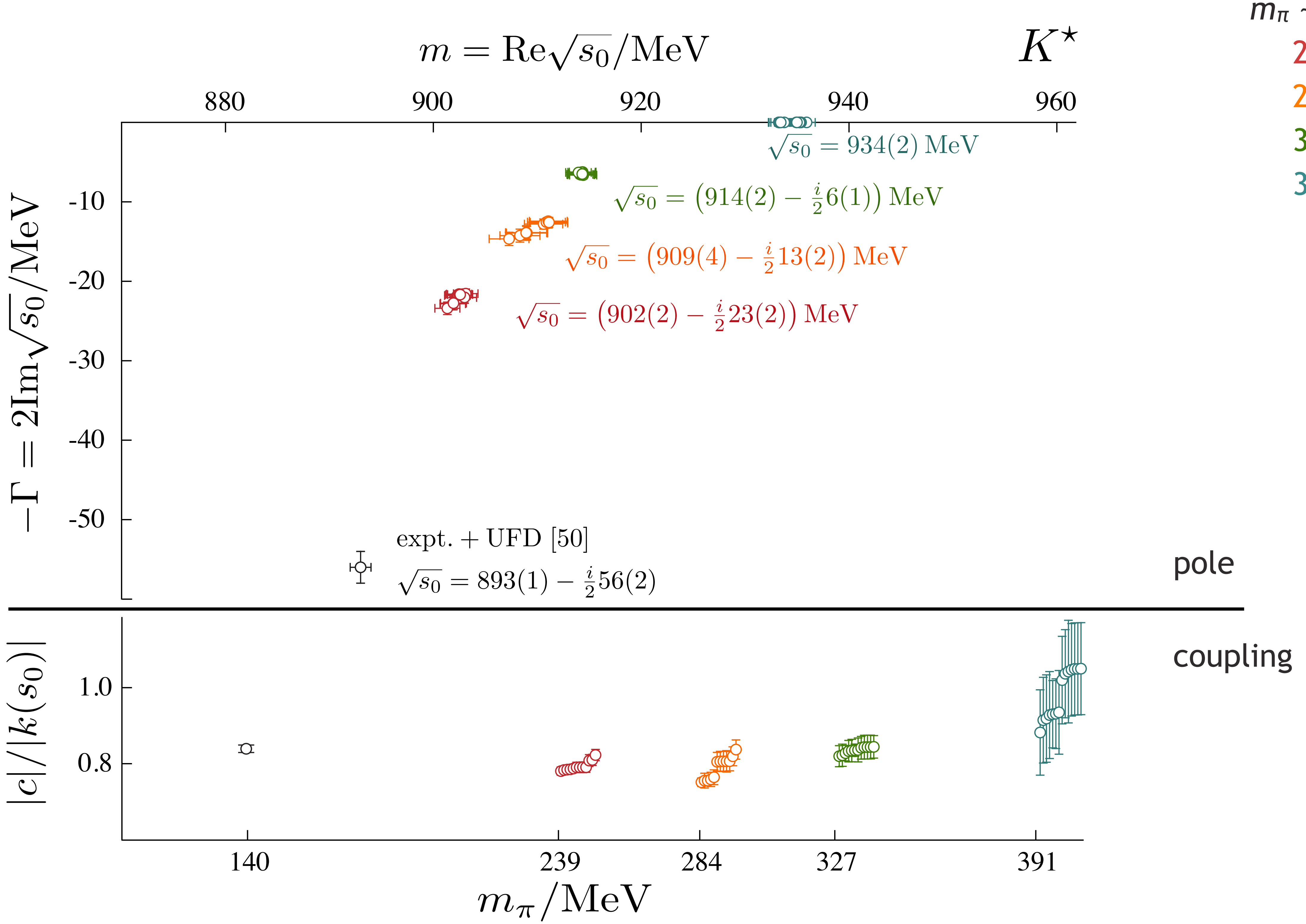
actually same problem using
experimental scattering data …





Phys Rep 658 1 (2016)

πK elastic scattering at four light quark masses



m_π ~

239 MeV

284 MeV

327 MeV

391 MeV

‘dropping’ ops in $\rho \to \pi\pi$

exclude the $K\bar{K}$ operator

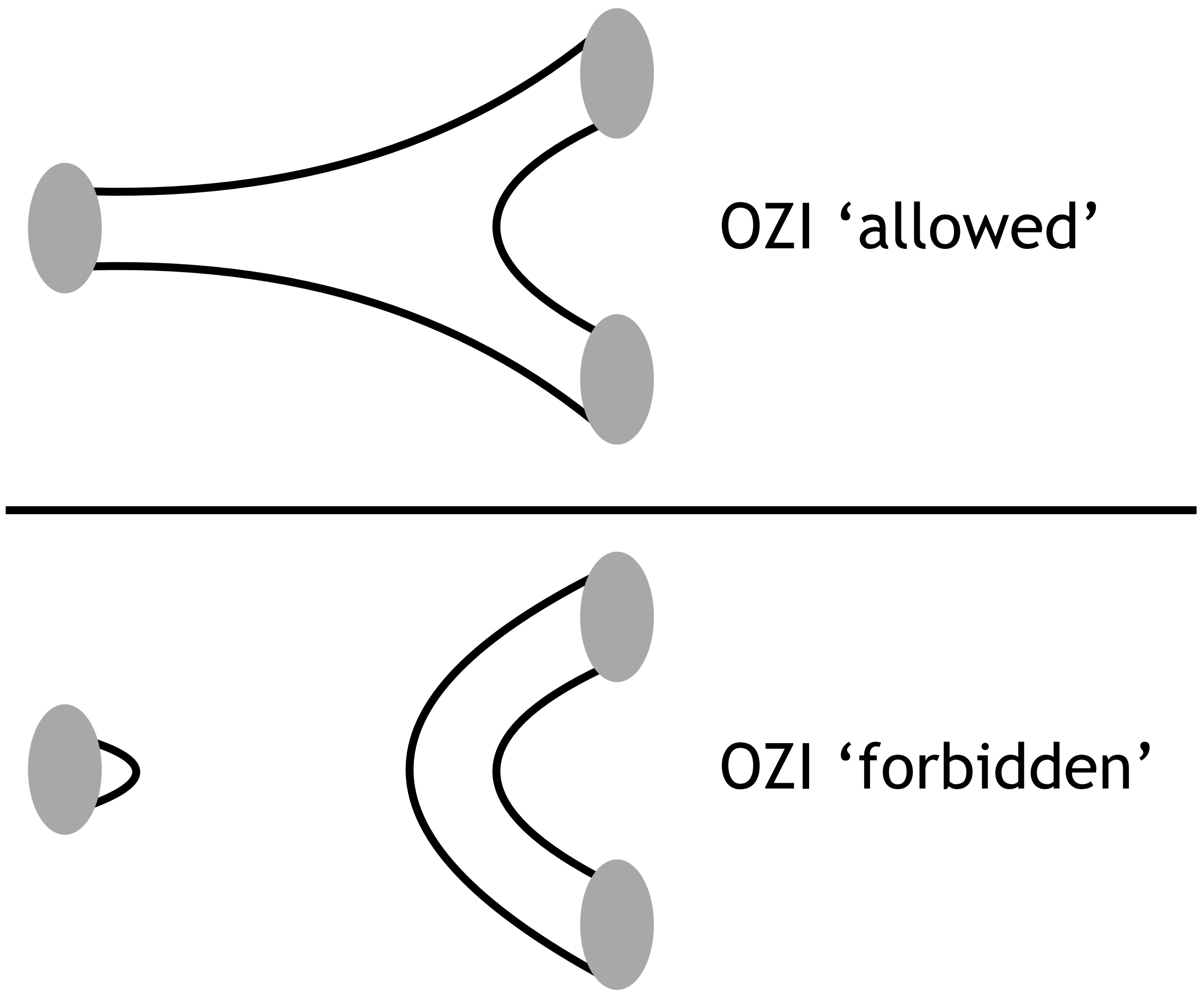
exclude all meson-meson

exclude all $\bar{\psi}\Gamma\psi$

$m_\pi = 0.039$
$m_K = 0.083$

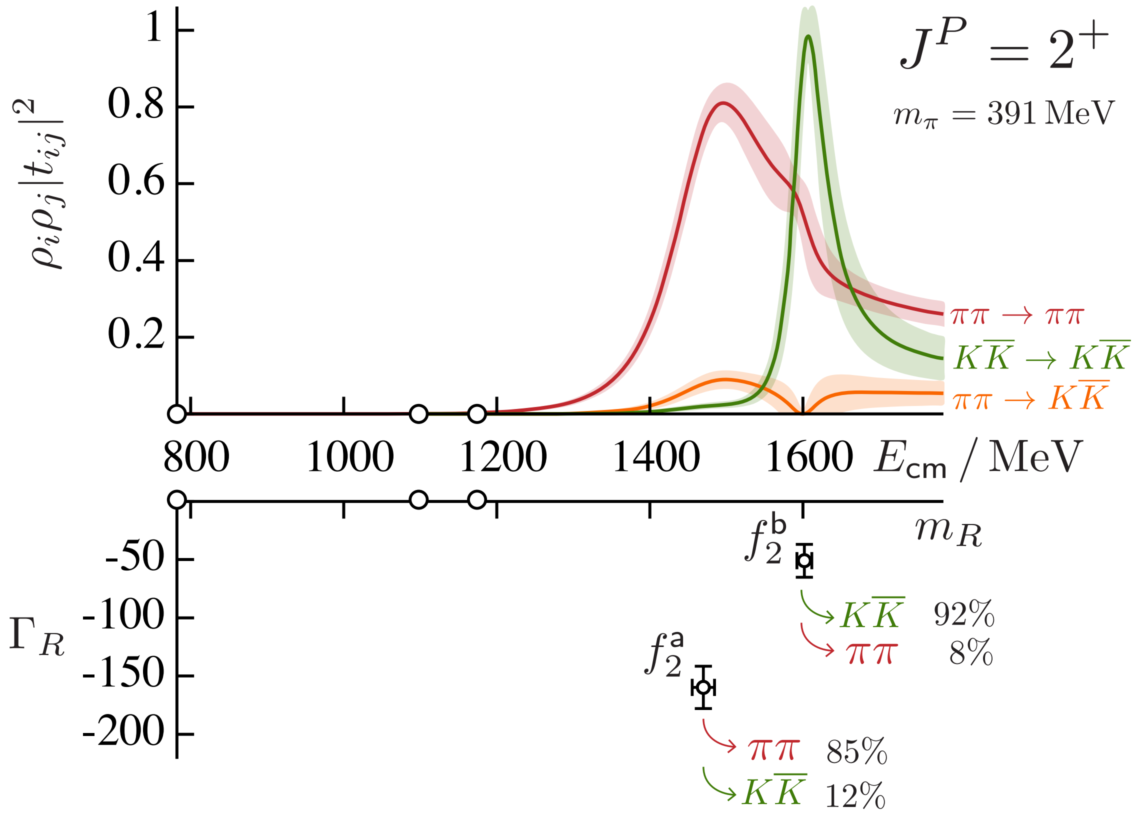
L ~ 3.8 fm

f_2 resonances – decay couplings & OZI



OZI 'allowed'

OZI 'forbidden'



$f_2^{\mathsf{a}} \sim u\bar{u} + d\bar{d}$ $\quad f_2^{\mathsf{b}} \sim s\bar{s}$

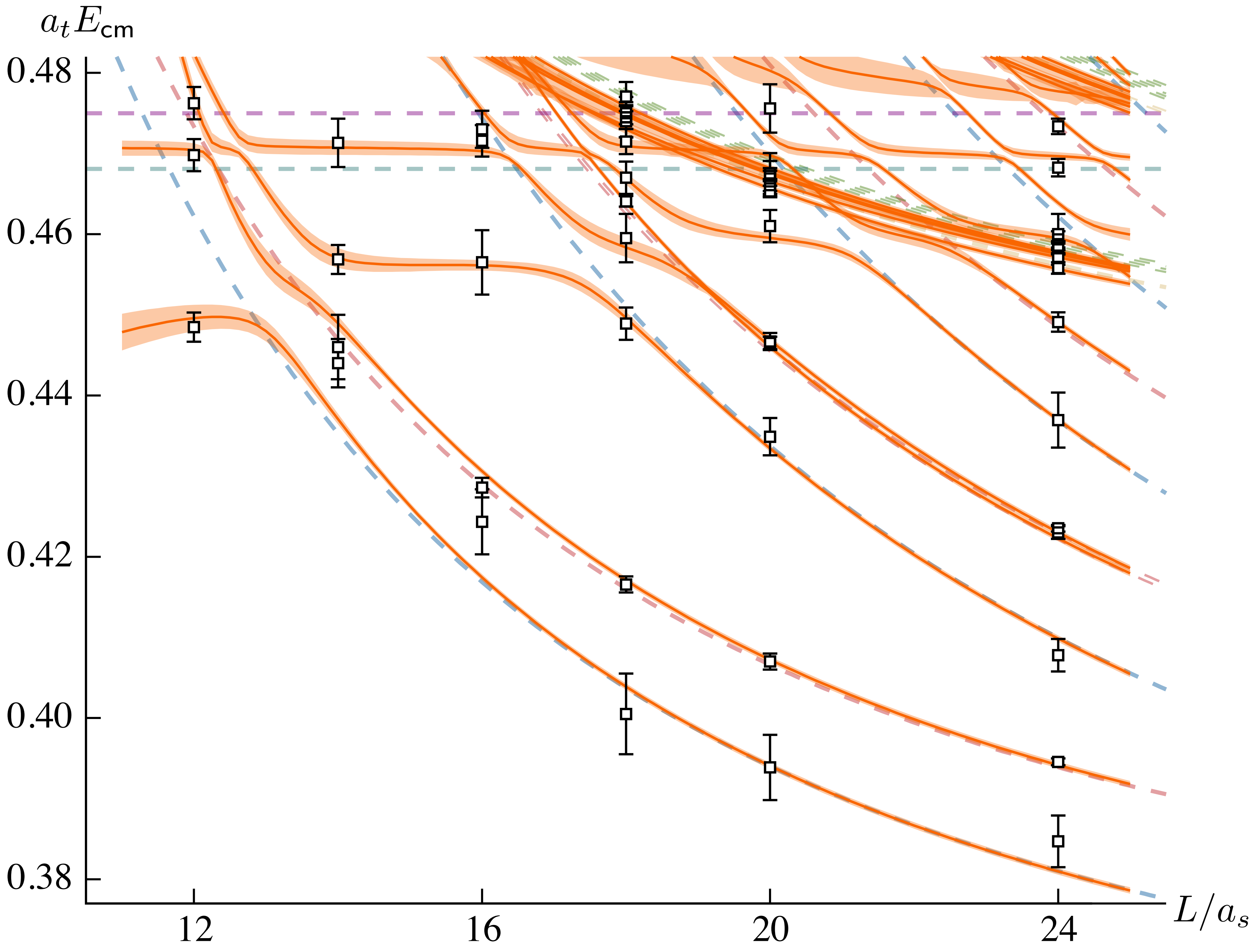
couplings from pole residue

	$\frac{a_t \lvert c_{\pi\pi} \rvert}{(a_t k_{\pi\pi})^2}$	$\frac{a_t \lvert c_{K\bar{K}} \rvert}{(a_t k_{K\bar{K}})^2}$
f_2^{a}	$7.1(4)$	$4.8(9)$
f_2^{b}	$1.0(3)$	$5.5(8)$

zero in 'OZI' limit
— requires $s\bar{s}$ annihilation

an ‘eight’ channel scattering amplitude

describe scattering by a unitarity-preserving *K*-matrix featuring a pole (11 free parameters)



$$\chi^2/N_{\rm dof} = \frac{43.6}{53-11} = 1.04$$

a good description of the spectrum ...

extrapolation

$$|c|^{\mathrm{phys}} = \left| \frac{k^{\mathrm{phys}}(m_R^{\mathrm{phys}})}{k(m_R)} \right|^{\ell} |c|.$$

$$\Gamma(R \to i) = \frac{|c_i^{\mathrm{phys}}|^2}{m_R^{\mathrm{phys}}} \cdot \rho_i(m_R^{\mathrm{phys}}).$$

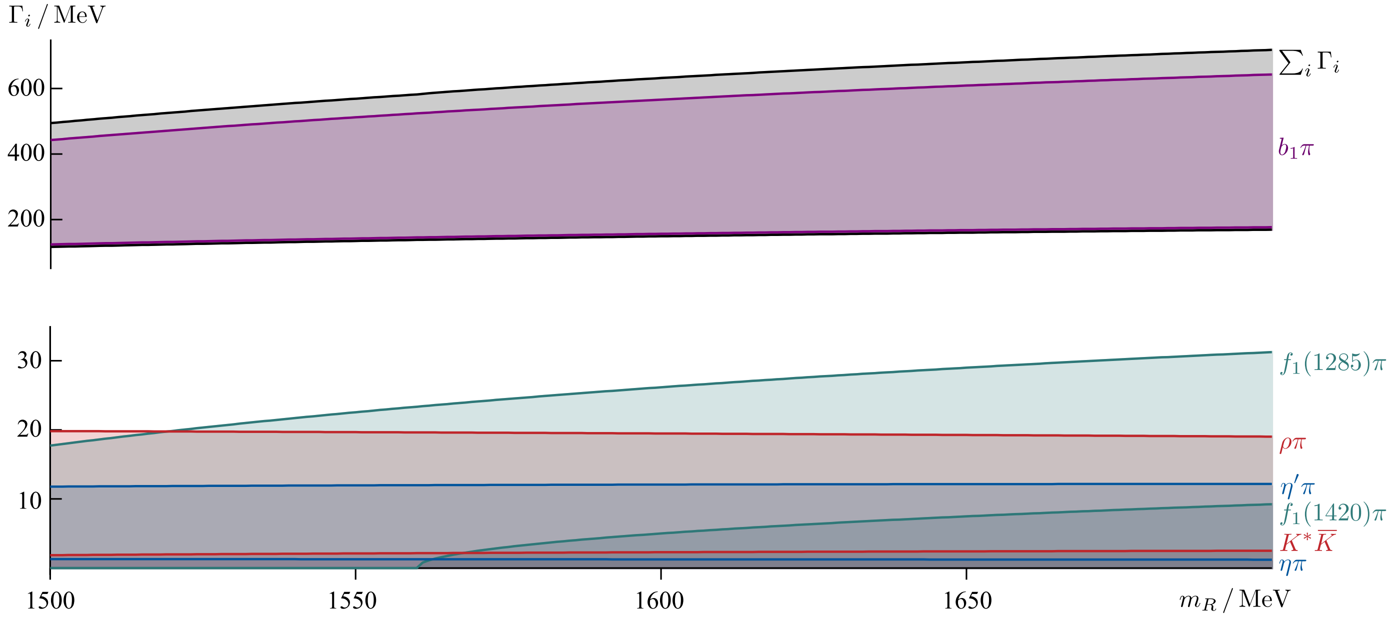
example ‘success’ – f_2, f_2' calculated at m_π~400 MeV

	Scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	488(28)	453^{+9}_{-4},
$\lvert c(f_2 \to K\bar{K})\rvert$	139(27)	132(7),
$\lvert c(f_2' \to \pi\pi)\rvert$	103(32)	33(4),
$\lvert c(f_2' \to K\bar{K})\rvert$	321(50)	389(12),

$$\frac{1}{\sqrt{3}}(\pi^+\rho^0 - \pi^0\rho^+) + \frac{1}{\sqrt{6}}(K^+\bar{K}^{*0} - \bar{K}^0K^{*+}),$$

$$-\sqrt{\frac{3}{10}}(K_{1A}^+\bar{K}^0 + \bar{K}_{1A}^0K^+) + \frac{1}{\sqrt{5}}(a_1^+\eta_8 + (f_1)_8\pi^+),$$

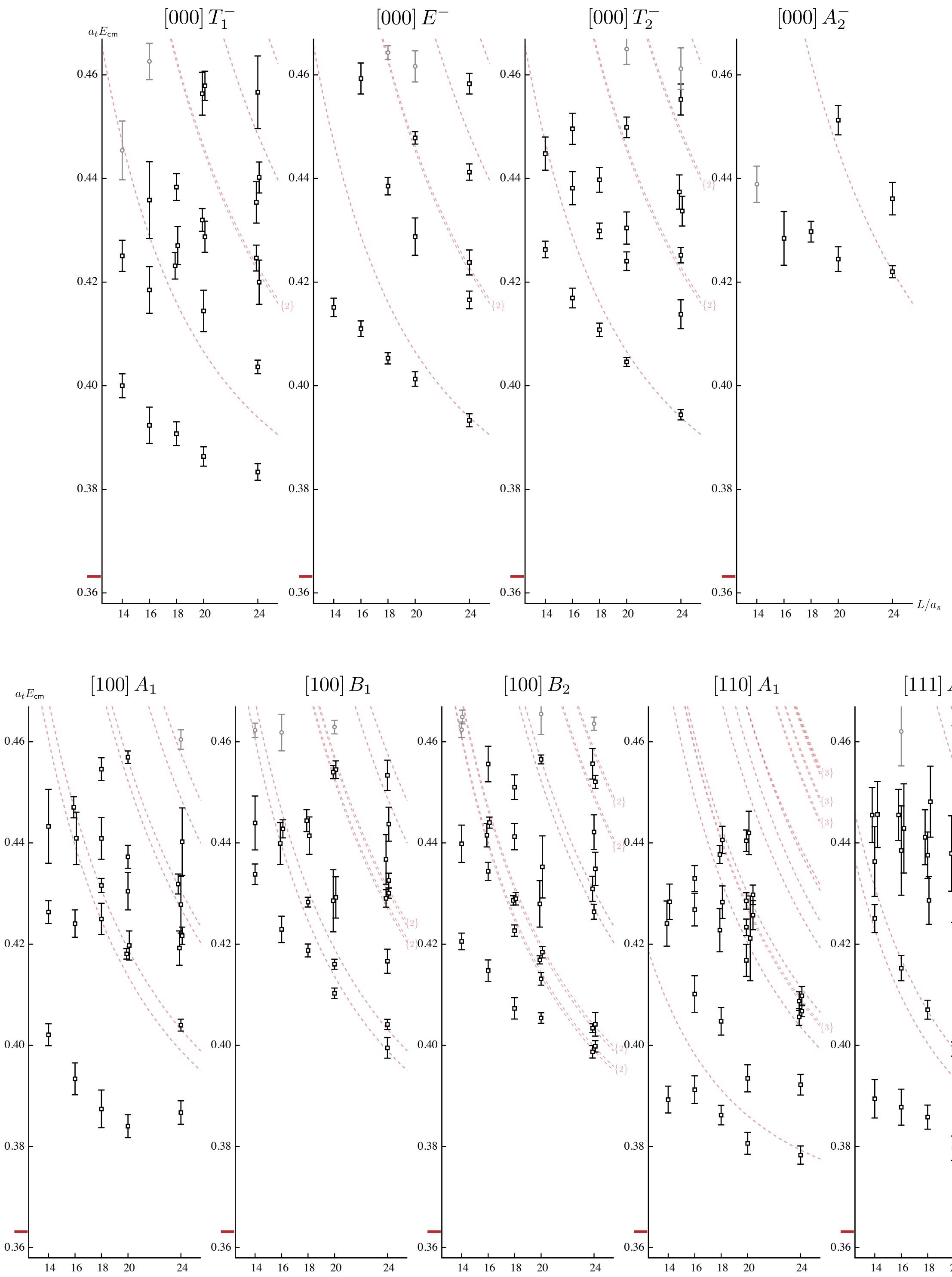
$$\frac{1}{\sqrt{6}}(K_{1B}^+\bar{K}^0 - \bar{K}_{1B}^0K^+) + \frac{1}{\sqrt{3}}(b_1^+\pi^0 - b_1^0\pi^+),$$



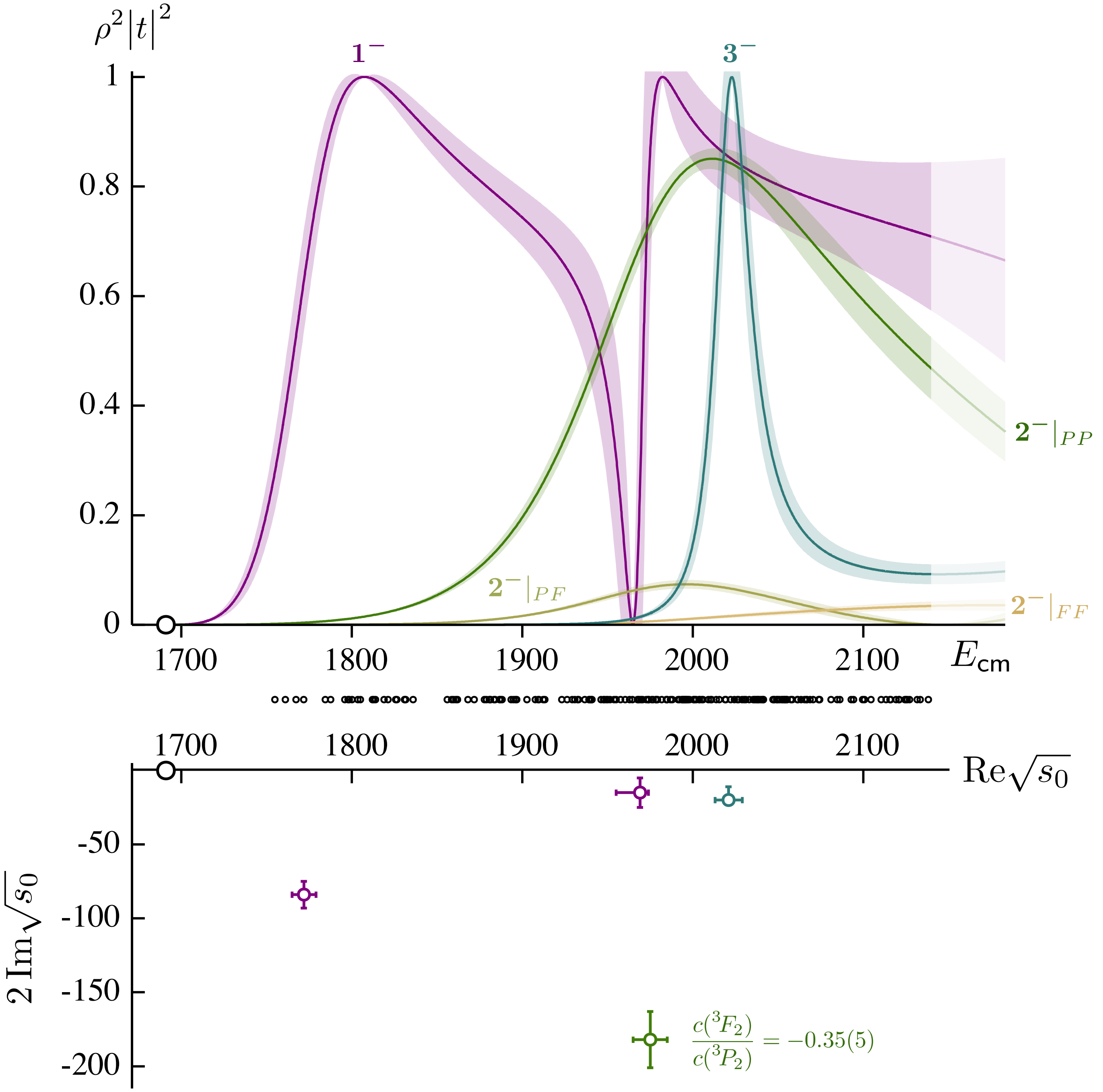
excited J^{--} meson resonances – m_π~700 MeV

PRD103 074502 (2021)

exact SU(3) flavor symmetry $\omega_J^{\mathbf{1}} \to \eta^{\mathbf{8}}\omega^{\mathbf{8}}$

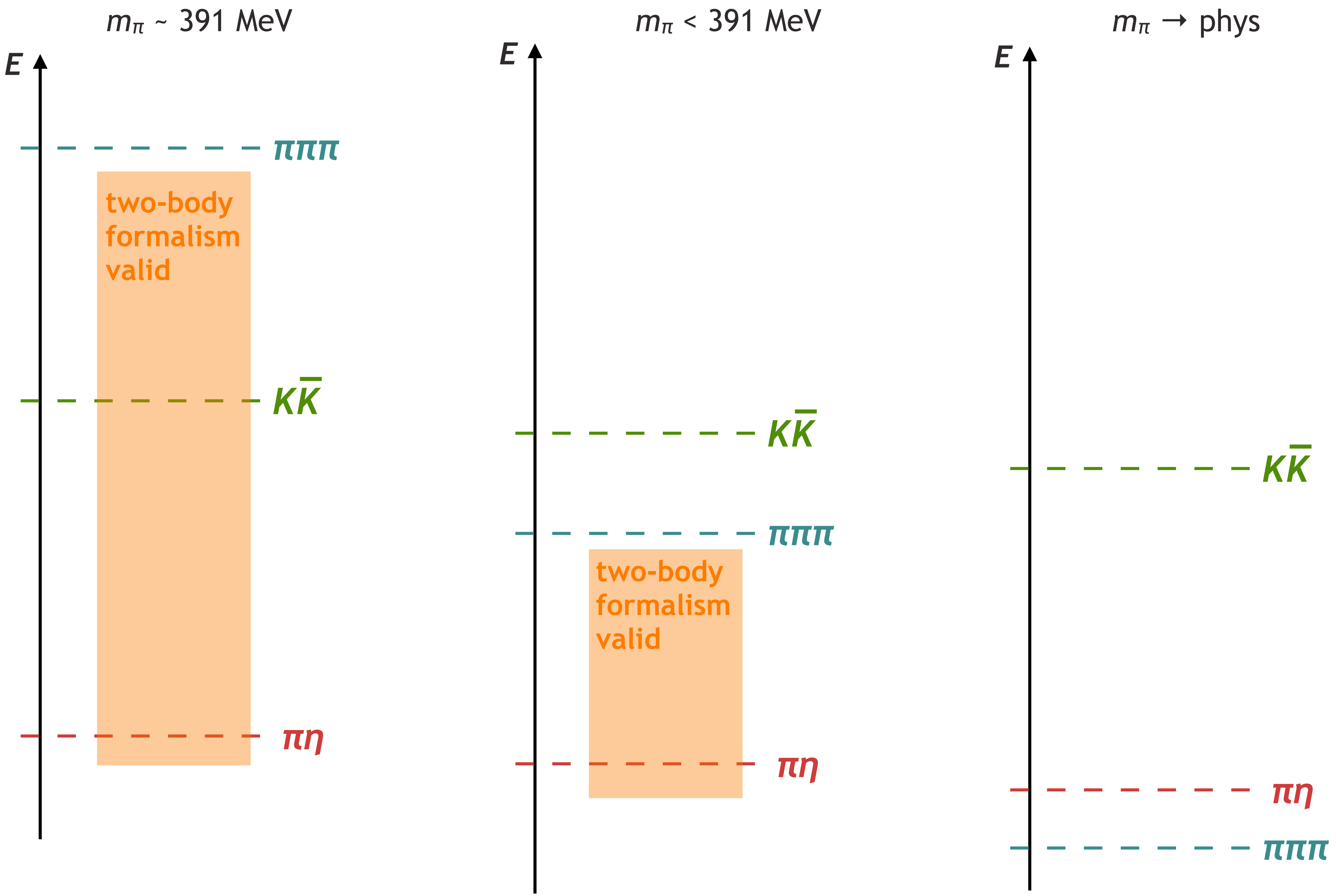


unprecedented number of energy levels



lowering the quark mass toward physical limit

challenge of **reducing quark mass** really the challenge of **including three-meson scattering**

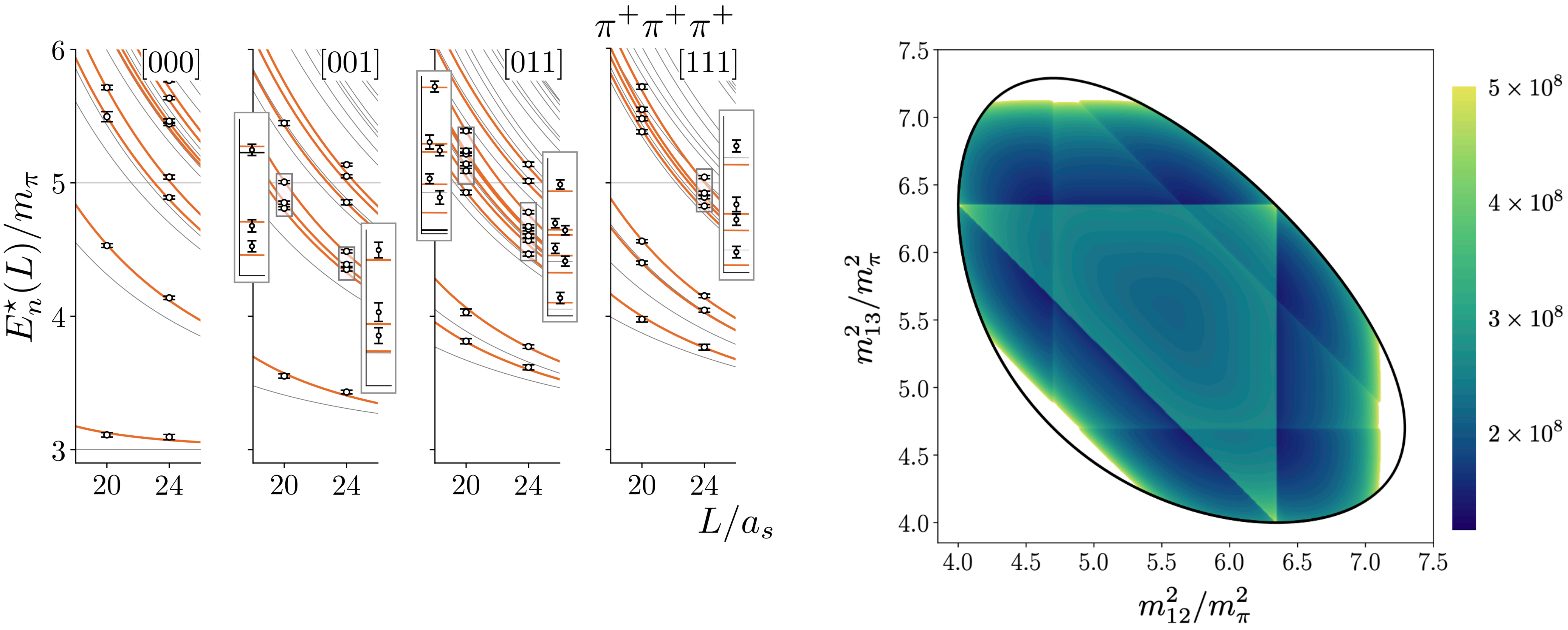


accounting for three hadron scattering

including three-meson scattering is starting to become practical

The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,[1,*] Raul A. Briceño,[2,3,†] Robert G. Edwards,[2,‡] Christopher E. Thomas,[4,§] and David J. Wilson[4,¶]
(for the Hadron Spectrum Collaboration)



coupling scattering systems to external currents

e.g. consider the process in which
a pion absorbs a photon* to become two pions $\gamma\pi \to \pi\pi$

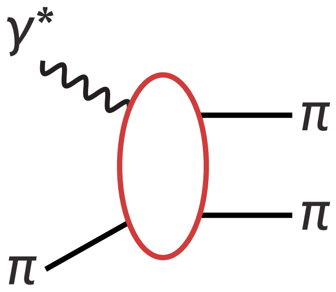
* could be virtual

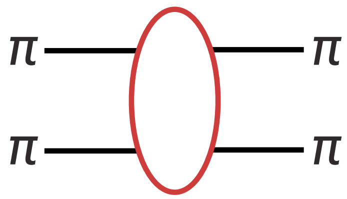
in infinite volume, described by a matrix element $\big\langle \pi\pi(E_{\mathsf{cm}}, \mathbf{P}) \big| j^\mu(0) \big| \pi(\mathbf{p}) \big\rangle$

ππ state can be projected
into a partial wave, e.g. ℓ=1

$$\propto F(E_{\mathsf{cm}}, Q^2)$$

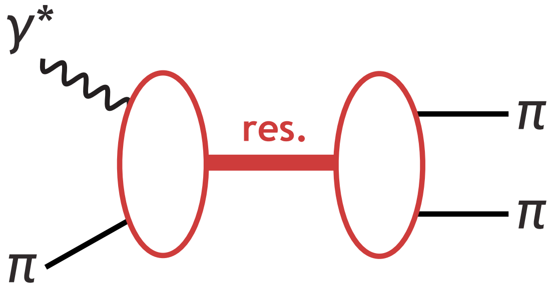
after the current produces ππ … ππ will rescatter strongly





⇒ the matrix element is proportional to $t_\ell(E_{\mathsf{cm}})$

if there's a resonance $t_\ell(s \sim s_0) \sim \dfrac{c^2}{s_0 - s}$ and $F(s \sim s_0, Q^2) \sim \dfrac{c\, f(Q^2)}{s_0 - s}$



resonance transition form-factor $f(Q^2)$
rigorously defined at the complex pole position

e.g. ρ→πγ

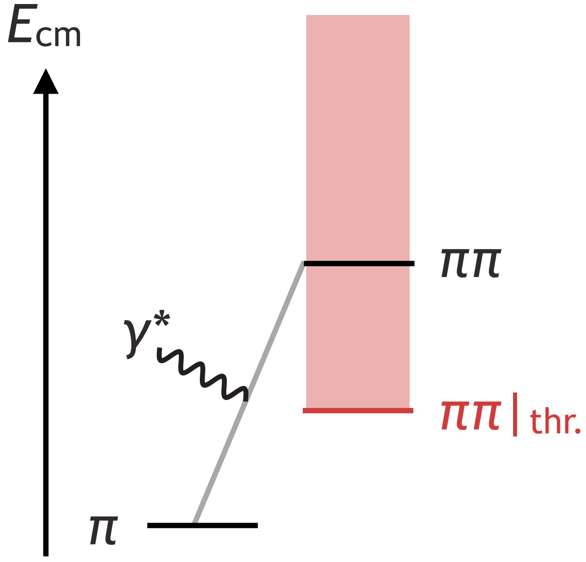
but what changes in a finite volume … ?

coupling scattering systems to external currents – fin. vol.

e.g. consider the process in which
a pion absorbs a photon to become two pions

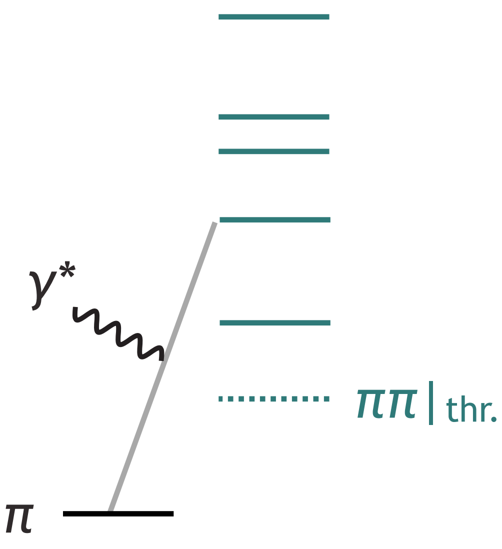
$$\gamma\pi \to \pi\pi$$

infinite volume



can transition
to any energy
in the *ππ* continuum

finite volume



can only transition
to one of the discrete
f.v. eigenstates

finite-volume matrix element

$${}_L\big\langle \pi\pi(E_n(L),\mathbf{P})\big|j^\mu(0)\big|\pi(\mathbf{p})\big\rangle_L$$

single hadron state

$$\big|\pi(\mathbf{p})\big\rangle_L = \big|\pi(\mathbf{p})\big\rangle_\infty + O(e^{-m_\pi L})$$

hadron-hadron state

$$\big|\pi\pi(E_n(L),\mathbf{P})\big\rangle_L \sim \sqrt{\mathcal{R}_n}\,\big|\pi\pi(E_{\mathsf{cm}}=E_n(L),\mathbf{P})\big\rangle_\infty$$

effective f.v.
normalization

$$\mathcal{R}_n = 2E_n \lim_{E\to E_n}(E-E_n)\Big(F^{-1}(E,\mathbf{P};L)+M(E)\Big)^{-1}$$

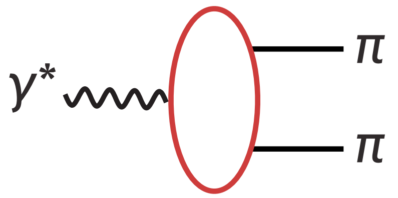
$$F = \tfrac{1}{16\pi}\,i\rho\,(1+i\mathcal{M})$$

$$M = 16\pi\, t$$

effective f.v. normalization depends
on the hadron-hadron scattering amplitude

coupling resonances to currents

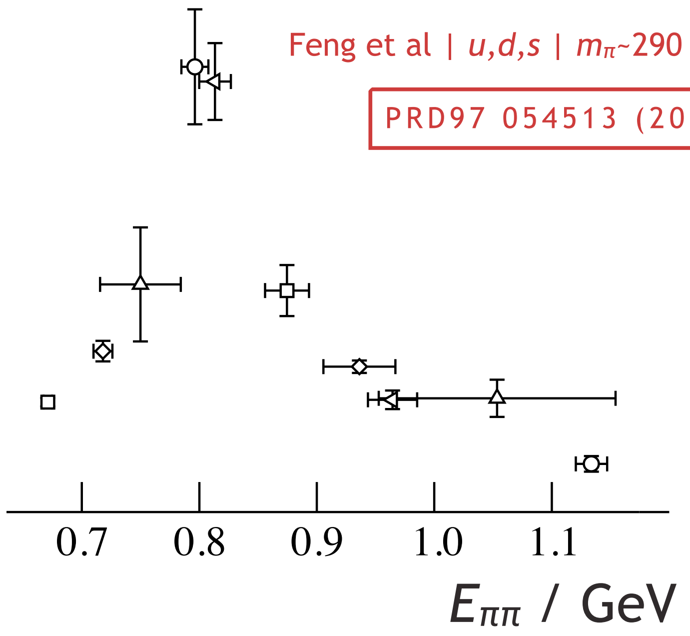
$\gamma^* \rightarrow \pi\pi$



$F_\pi(E_{\pi\pi})$

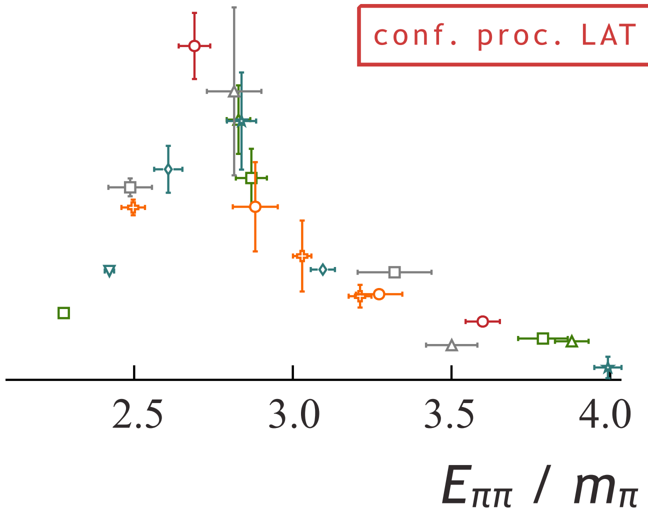
Feng et al | *u,d,s* | m_π~290 MeV

PRD97 054513 (2018)

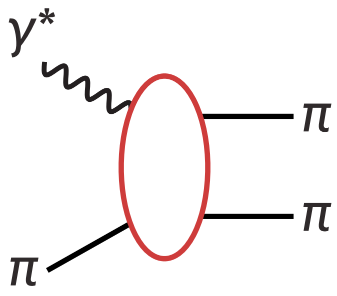


Bulava et al | *u,d,s* | m_π~280 MeV

conf. proc. LAT '15



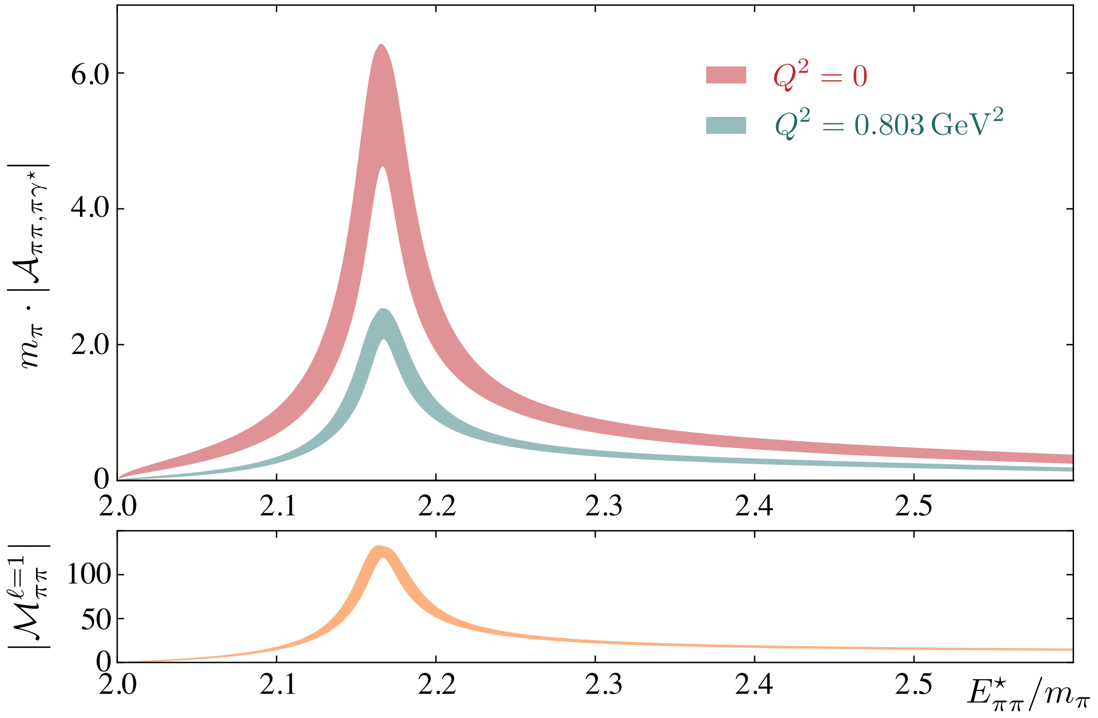
$\gamma^*\pi \rightarrow \pi\pi$

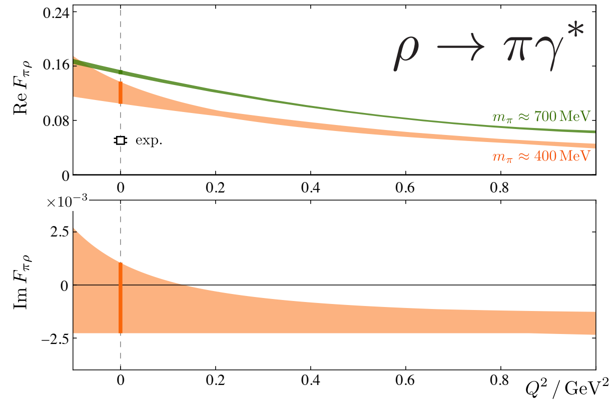


$\mathcal{A}(E_{\pi\pi}, Q^2)$

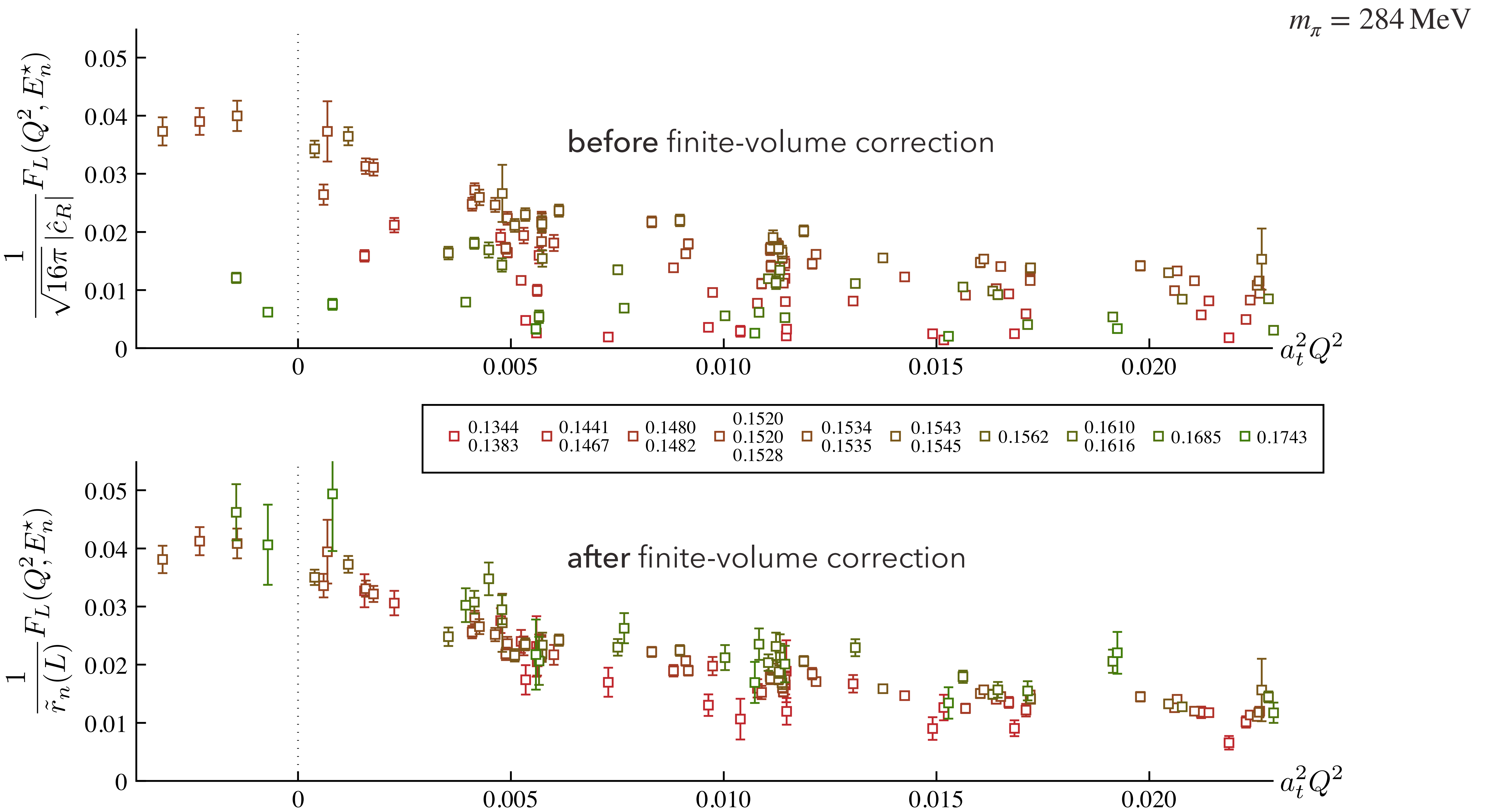
Briceno et al | *u,d,s* | m_π~391 MeV

PRL115 242001 (2015)





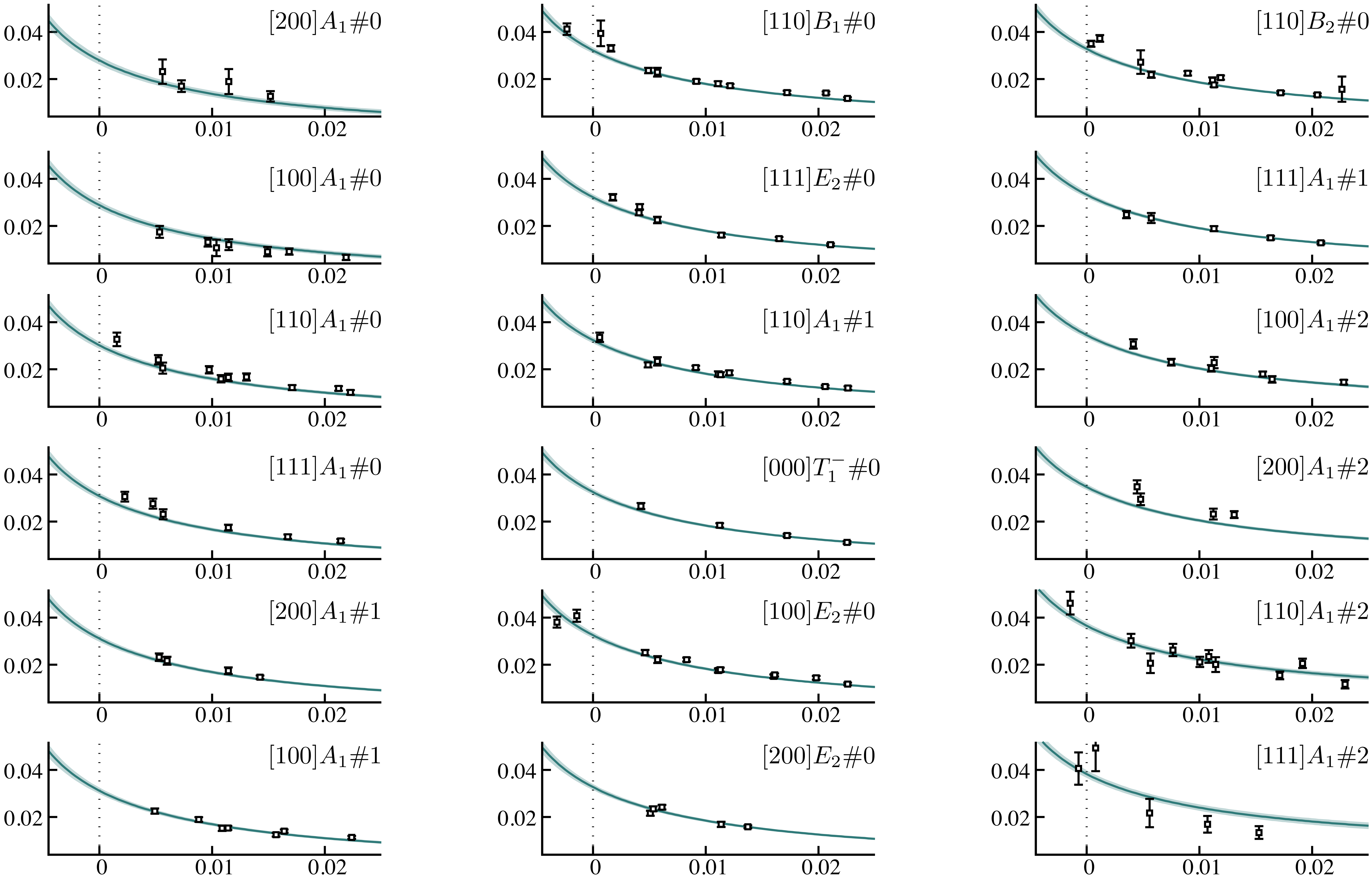
$\gamma K \to K\pi$ in finite-volume



$\gamma K \to K\pi$ as function of Q^2 at discrete energies

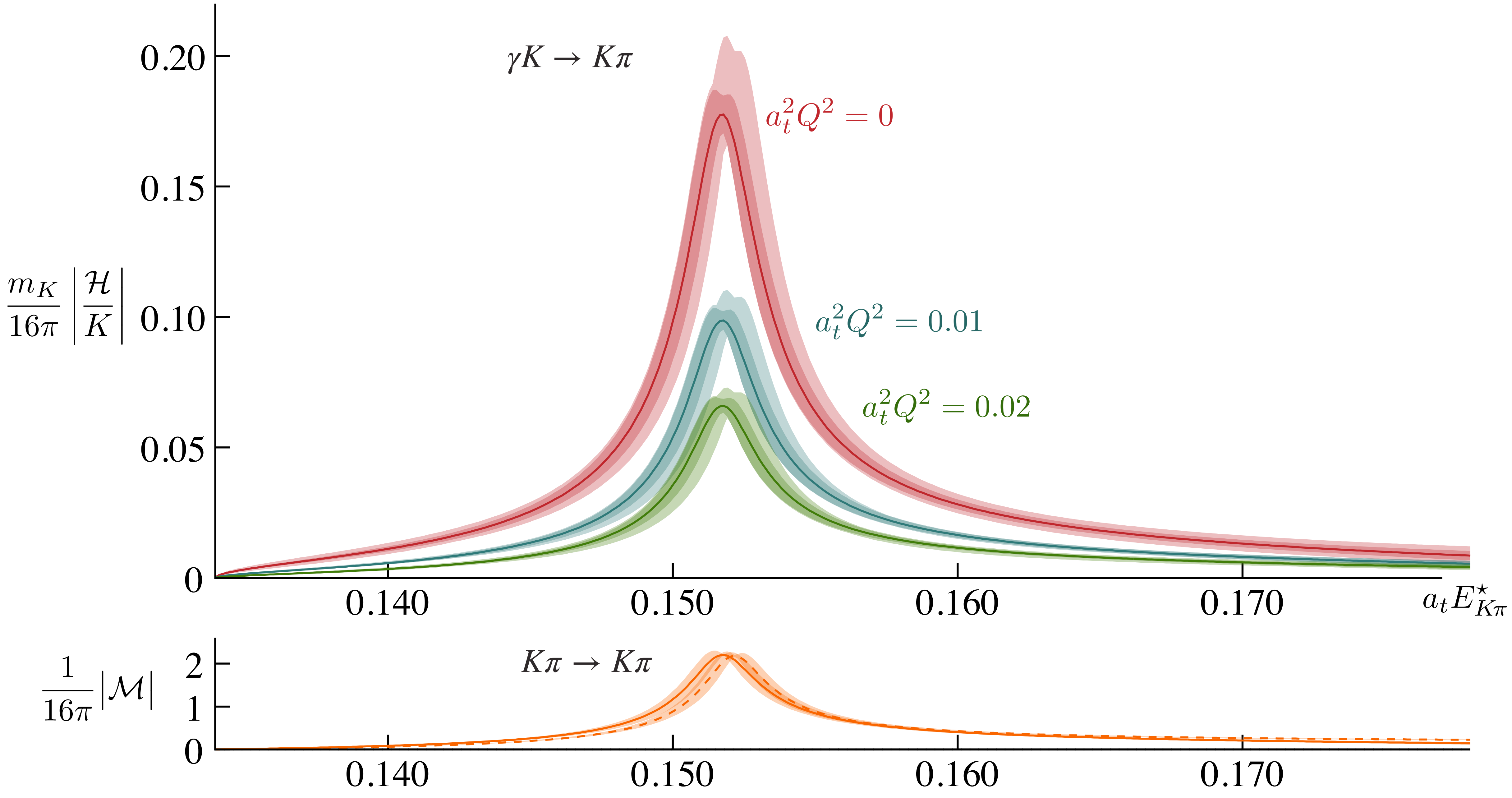
global fitting across scattering energy & photon virtuality

$m_\pi = 284\,\mathrm{MeV}$



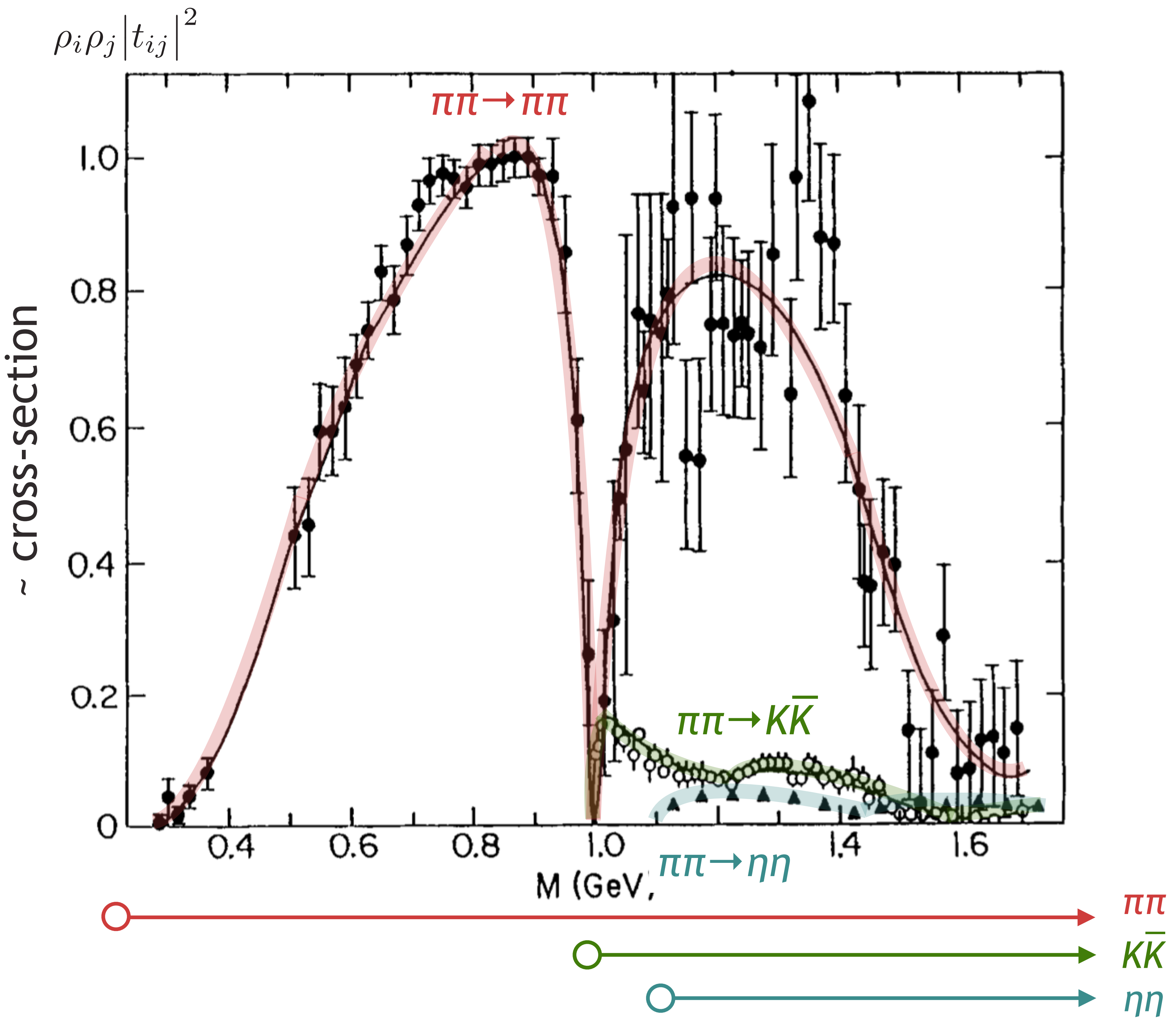
$\gamma K \to K\pi$ as function of scattering energy at several Q^2

$m_\pi = 284\,\mathrm{MeV}$



*K**('892') resonance

resonances aren’t always bumps



normalization of $\pi\pi \to KK$
slightly uncertain ...

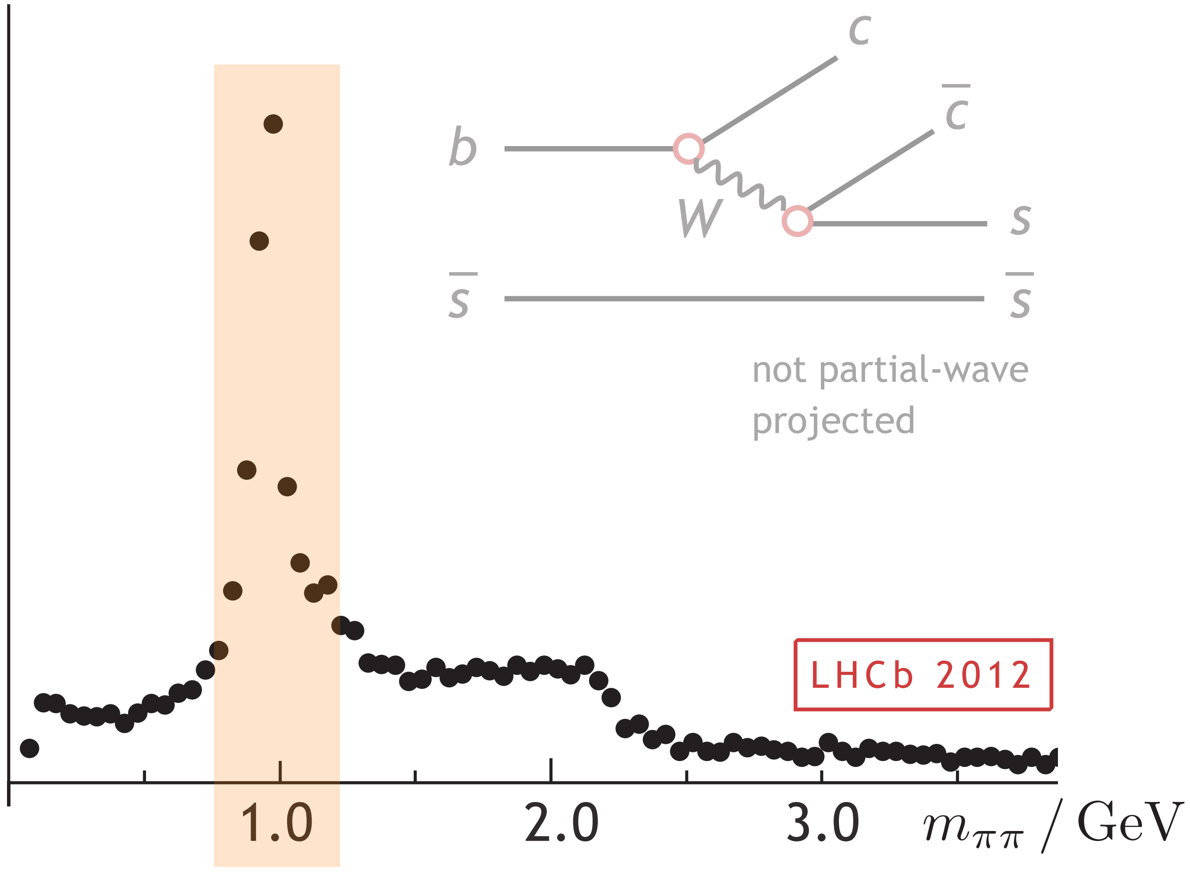
experimentally
quite difficult to fill out
the whole t-matrix

$$t = \begin{array}{cccc} \pi\pi & K\bar{K} & \eta\eta & \\ \blacksquare & \blacksquare & \blacksquare & \pi\pi \\ & \square & \square & K\bar{K} \\ & & \square & \eta\eta \end{array}$$

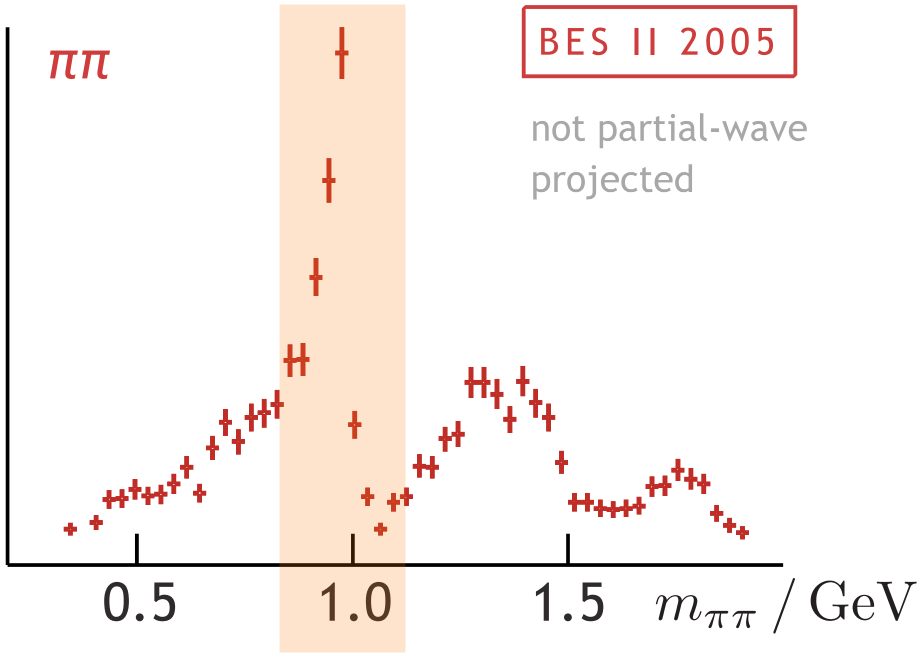
isolating kaon exchange hard
& η beams don’t exist

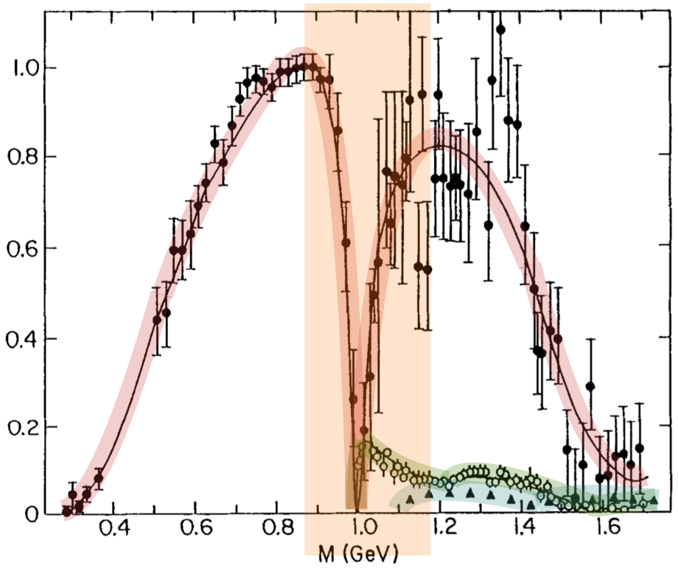
“production” of ππ (as opposed to scattering)

$\bar{B}^0_s \to J/\psi\ \pi^+\pi^-$



$J/\psi \to \varphi\ \pi\pi$

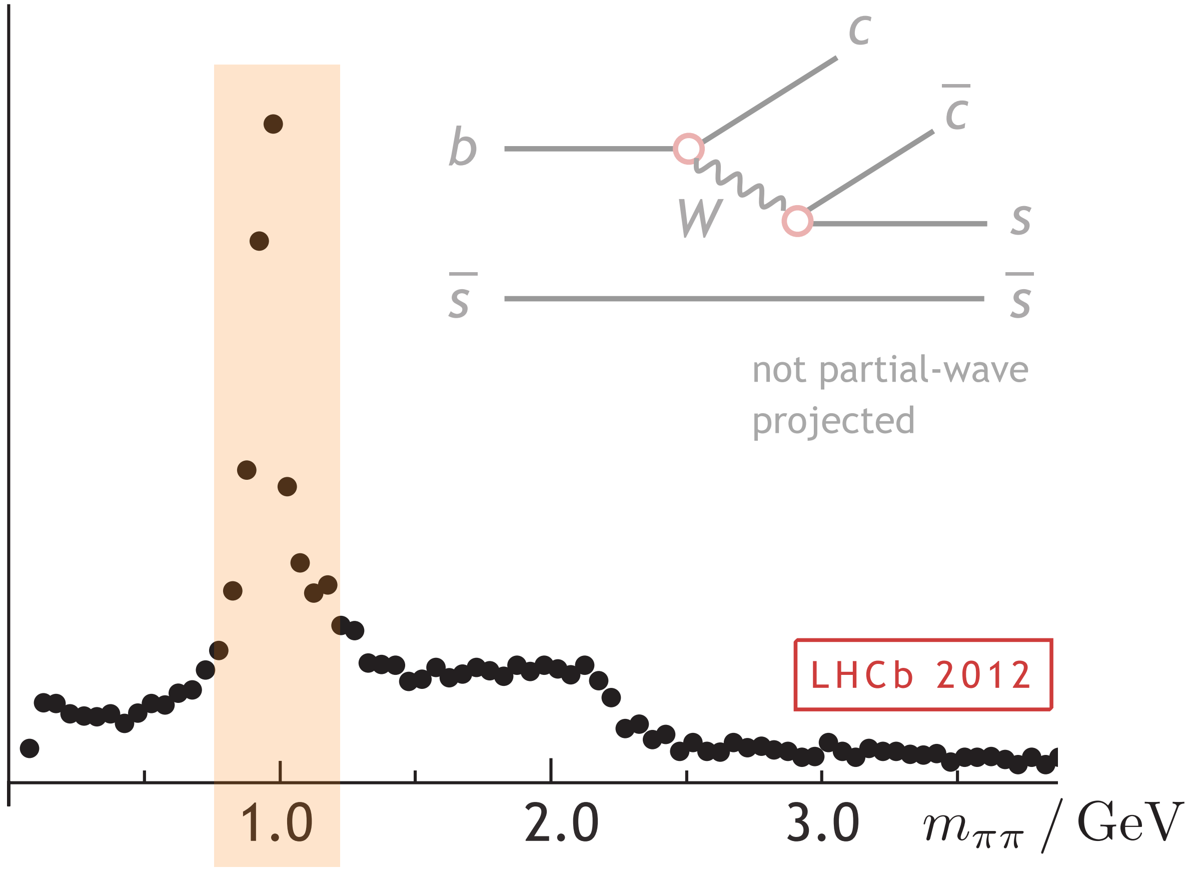




can ‘look’ drastically different to scattering !

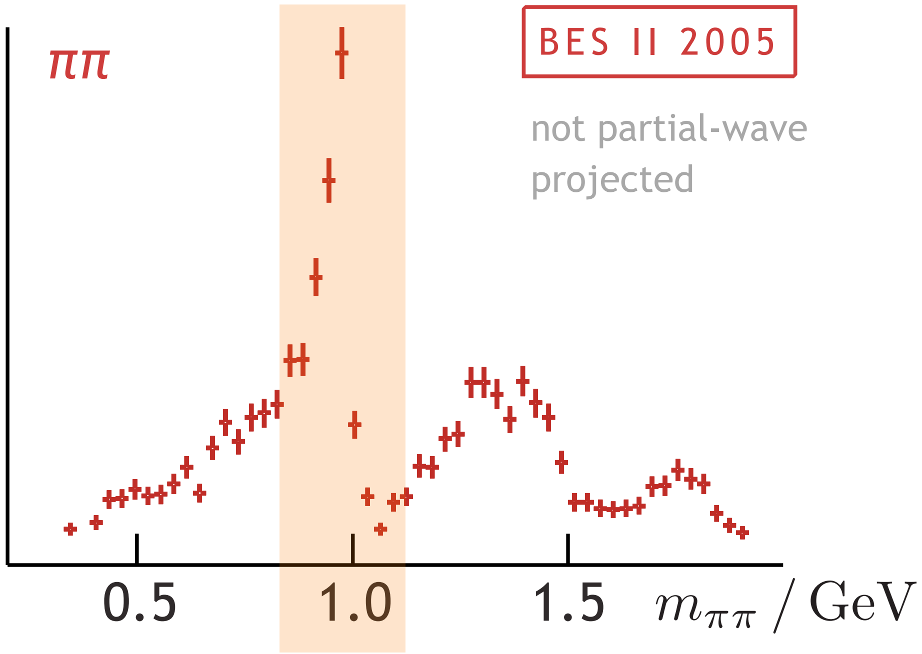
“production” of ππ (as opposed to scattering)

$\bar{B}^0_s \to J/\psi\ \pi^+\pi^-$

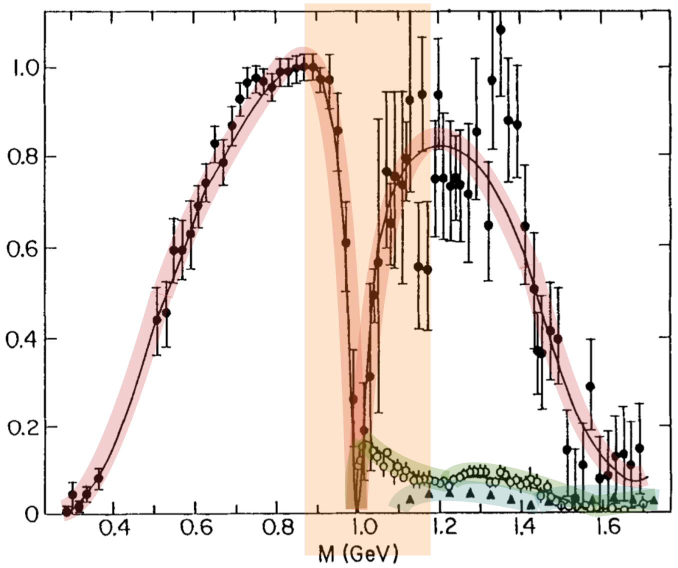


not partial-wave projected

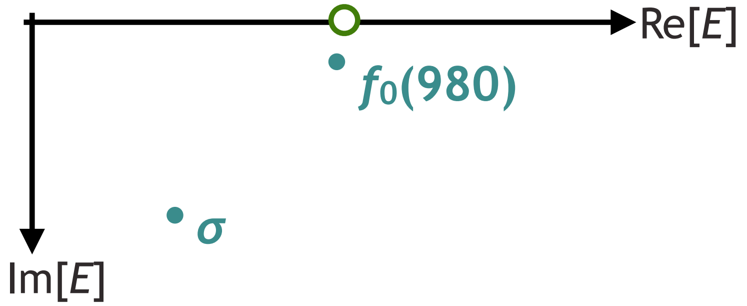
$J/\psi \to \varphi\ \pi\pi$



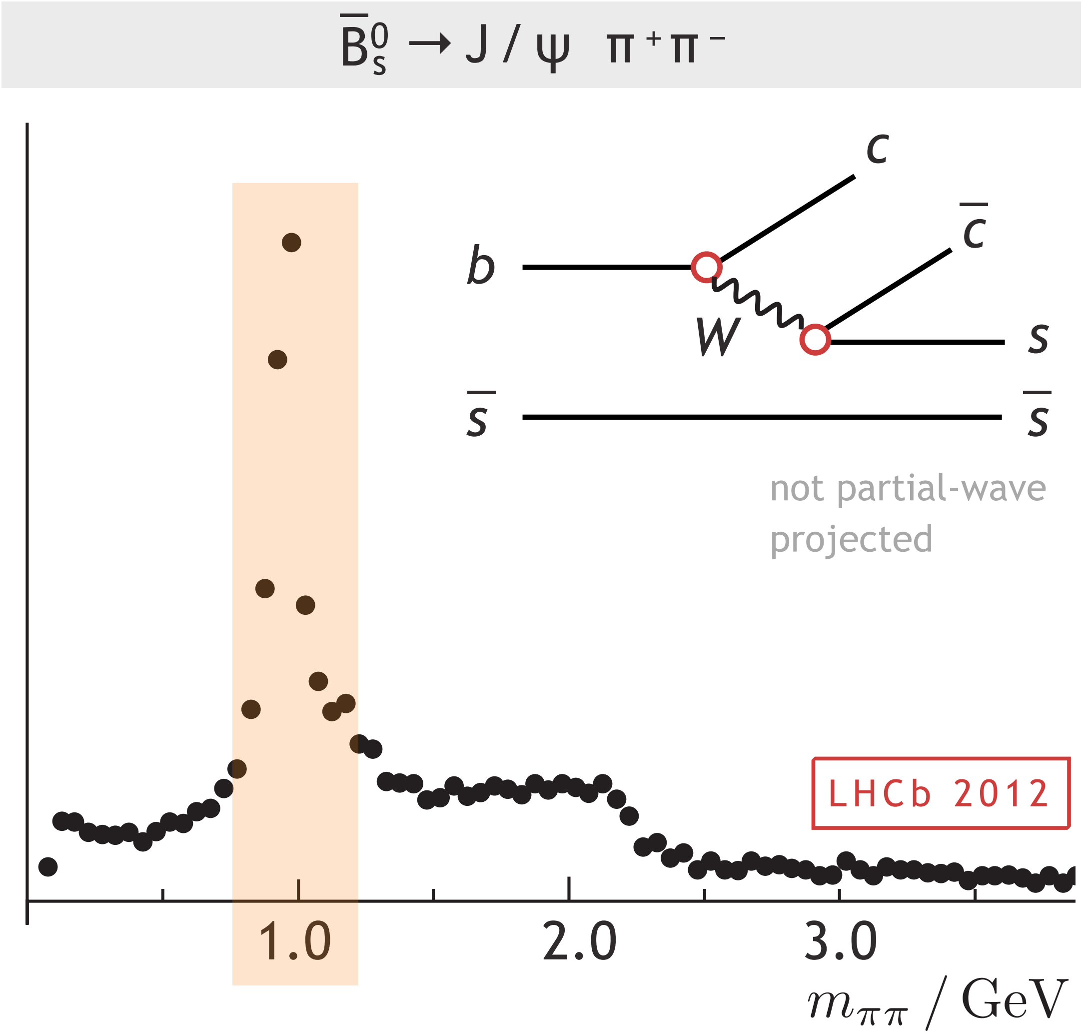
not partial-wave projected

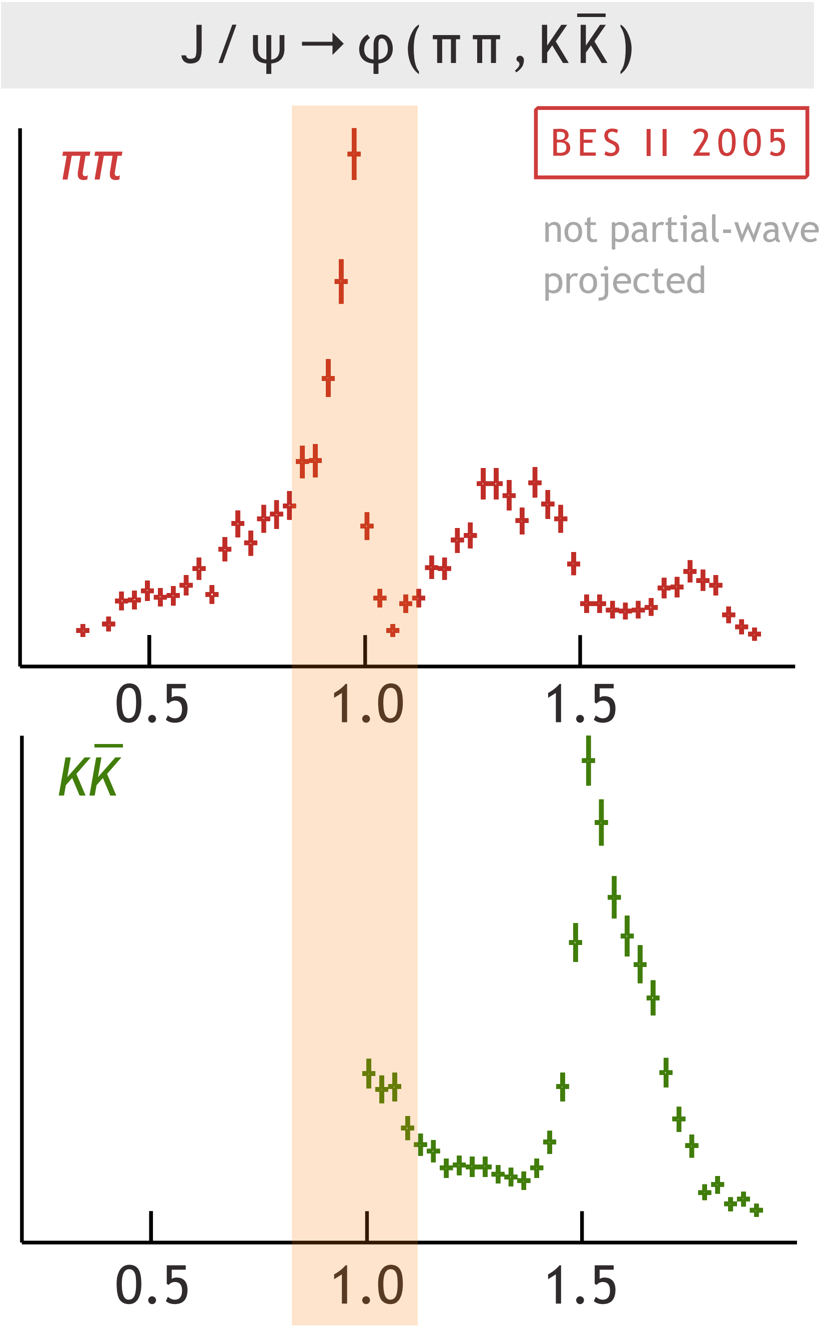


… **same poles** (σ, $f_0(980)$) — different couplings …



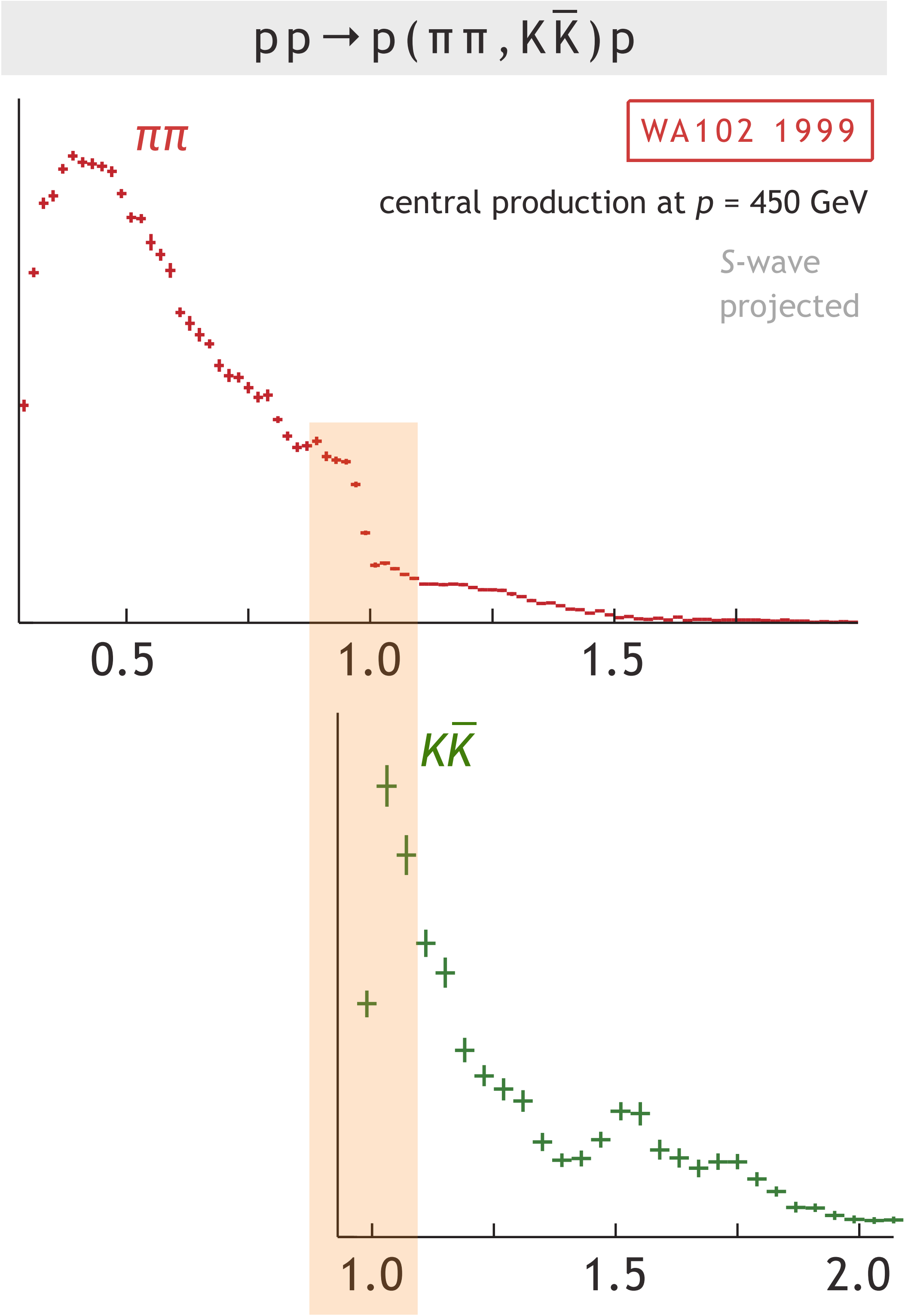
$f_0(980)$ as a peak in "$s\bar{s}$" production



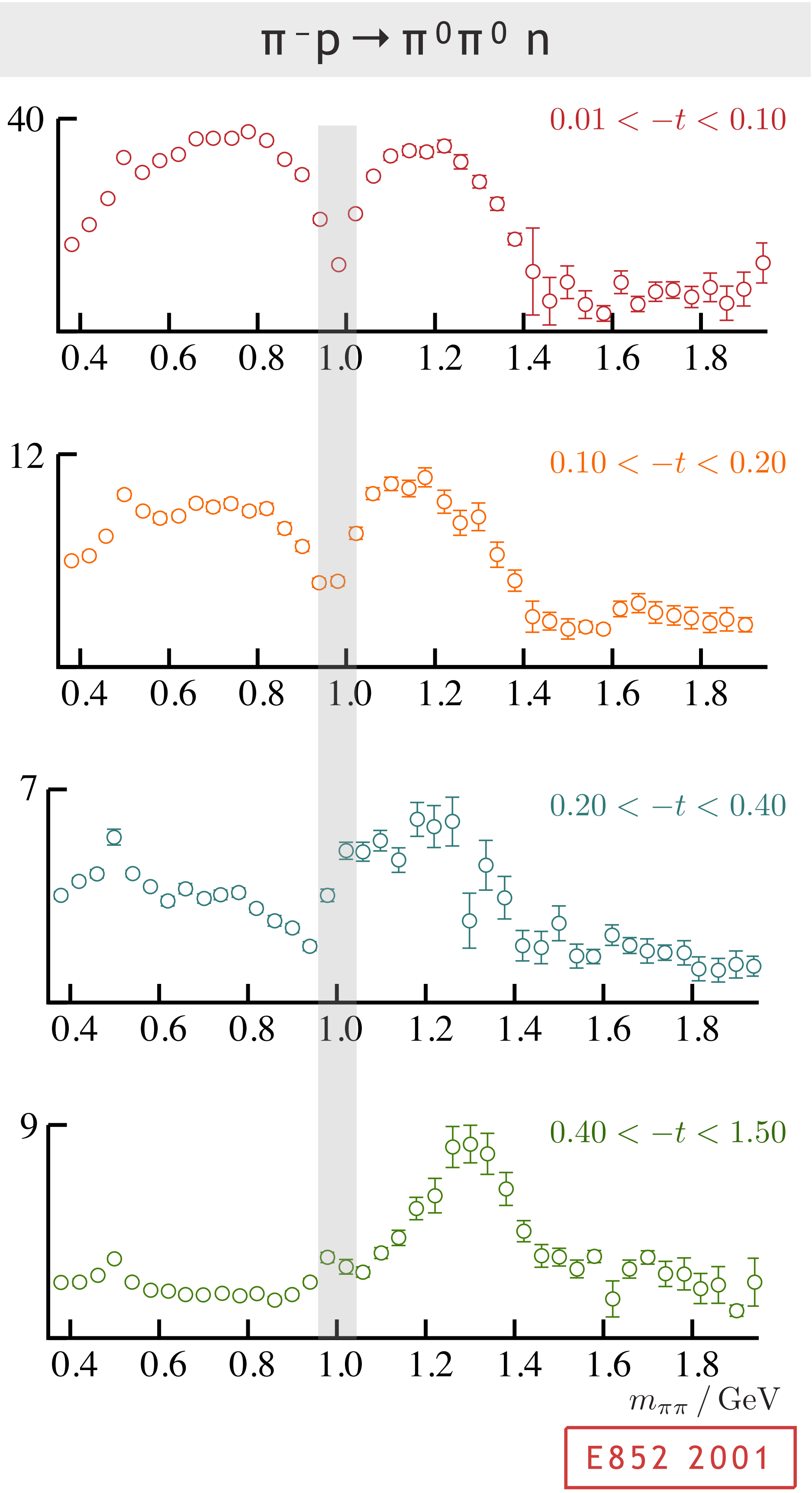


note the rapid turn-on of $K\bar{K}$ at threshold

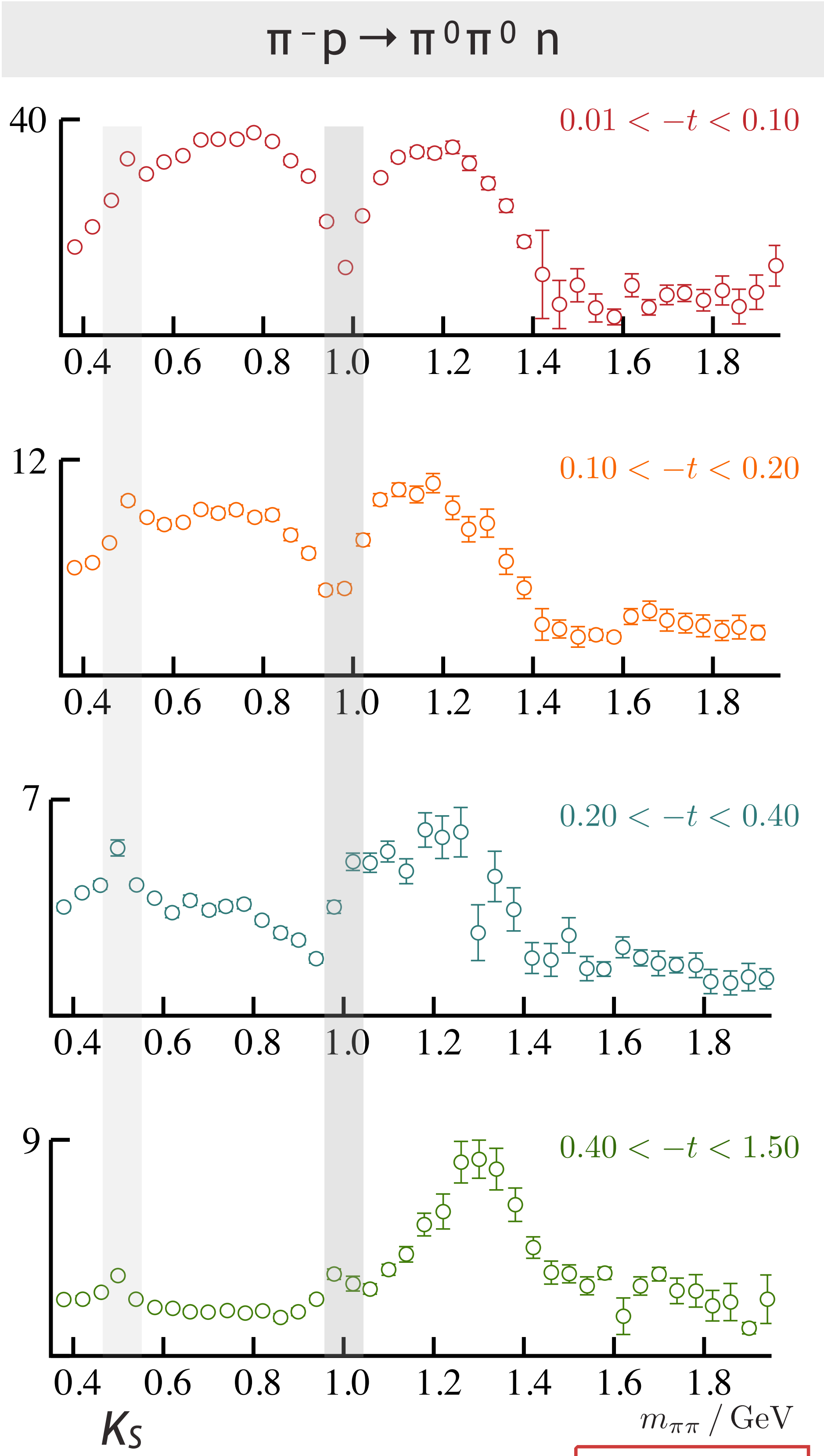
$f_0(980)$ as ?



$f_0(980)$ as a shoulder on a large σ ‘background’



S–wave $\pi\pi$ production



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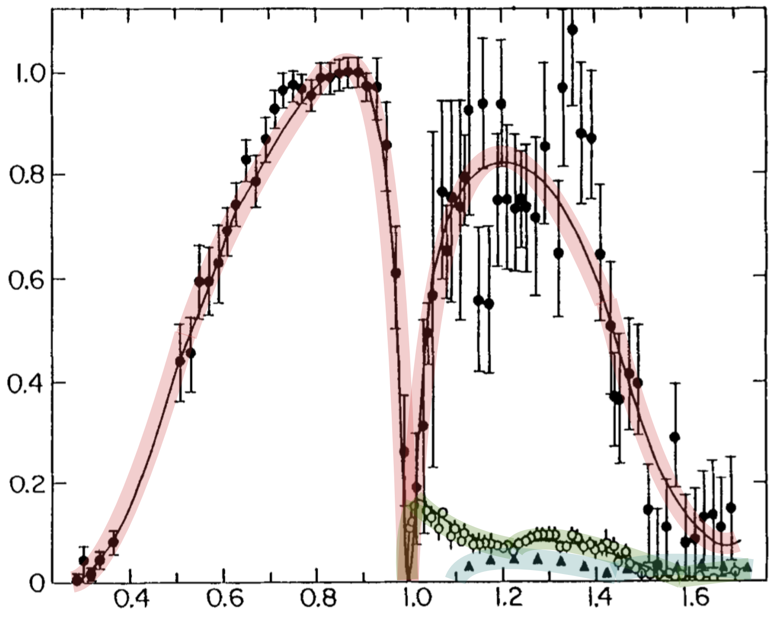
dominated by π exchange
— looks like the the 1970s
elastic phase-shift data →



other (non-π) exchanges
becoming significant,
$f_0(980)$ dip less pronounced

σ no longer large,
$f_0(980)$ starting to be a peak ?