SBS nTPE Theory, Analysis Status, and Prospective

Neutron Two-Photon Exchange Contribution to Elastic e-n Scattering

Sebastian Seeds
On behalf of the nTPE analysis group

August 29, 2022
Jefferson Lab

Jefferson Lab UCONN

Topics to Cover

- Basic Theory
- Experimental Context
- Measurement
- Analysis Status
- Prospective

The Born Approximation

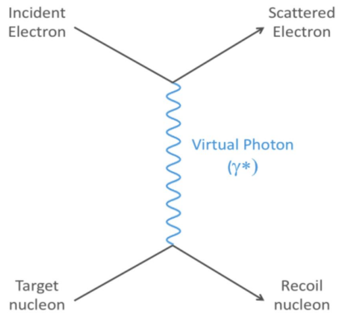
- Assuming single photon exchange (OPE) on the Born approximation, the elastic electron-neutron cross section can be parameterized with the point-like Mott term and Sachs form factors, G_M and G_E
- Isolating the form factors, the reduced cross section (σ_r) combines the differential cross section and the Mott term
  - Linear in ϵ when Q^2 is fixed
  - Further reparameterization relates σ_r to the transverse and longitudinal cross sections σ_T and σ_L respectively
- ϵ is the longitudinal polarization of the virtual photon and depends on the experimentally controlled parameters Q^2 and electron scattering angle θ

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{Mott}}{\epsilon(1-\tau)}\left(\tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)\right)$$

$$\sigma_r \equiv \left(\frac{d\sigma}{d\Omega}\right) \cdot \frac{\epsilon(1+\tau)}{\sigma_{Mott}} = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)$$

$$\sigma_r = \sigma_T + \epsilon \sigma_L$$

OPE



*

$$\tau \equiv \frac{Q^2}{4M^2}$$

$$\epsilon = 1/\left(1 + 2(1+\tau)\tan^2(\theta/2)\right)$$

*diagram credit: nTPE PAC Presentation, Sheren Alsalmi

Rosenbluth Technique (LT)

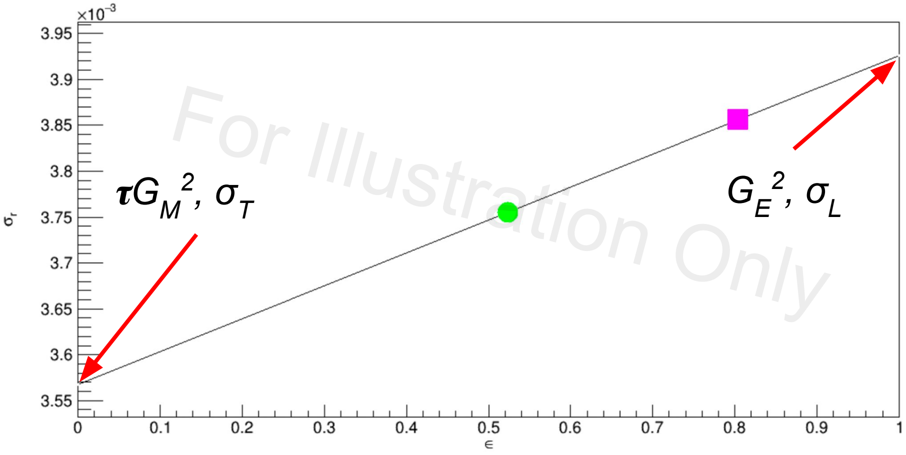
- By exploiting the *linearity of σ_r in ϵ* on the Born approximation and allowing only ϵ to vary, the form factors can be extracted
    - G_E^2 is the slope
    - τG_M^2 is the y-axis intercept
- The Rosenbluth Slope (RS) is the related directly to the ratio of the Sachs form factors (FFR)

$$\boxed{\sqrt{\tau \cdot RS} = \sqrt{\frac{\tau|\sigma_L|}{\sigma_T}} = \frac{G_E}{G_M}}$$

- By choosing the beam energy and Θ, Q^2 can be fixed between two different ϵ points
- **With $Q^2 = 4.5\ \mathrm{GeV}^2$, σ_r was measured at two such ϵ points during the SBS GMn run group, winter 2021-2022**
    - **SBS-8**
    - **SBS-9**

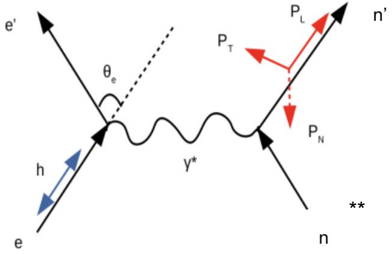
$$\sigma_r = \tau G_M^2 + \epsilon G_E^2$$

Example Reduced Cross Section with SBS Points



Polarization Transfer (PT)

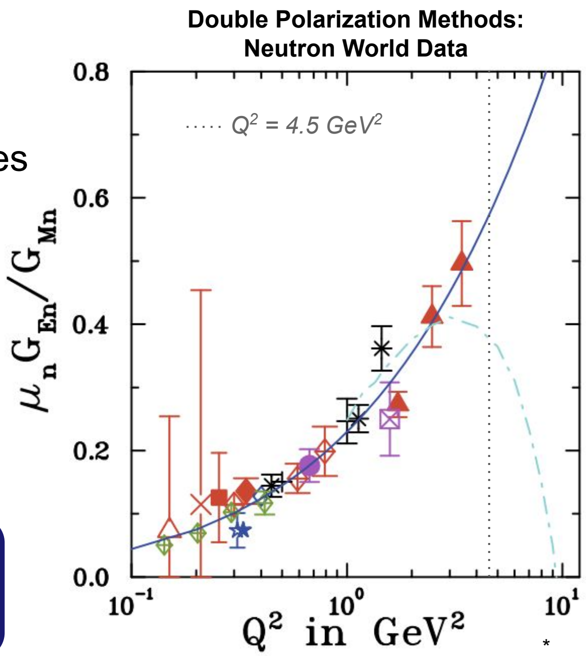
- The FFR is measurable also with polarization transfer methods employing polarized quasi-elastic electron-nucleon scattering
- Can relate the form factors without OPE from polarization observables
  - the transverse polarization (P_T)
  - longitudinal polarization (P_L)



$$I_0 P_T = -2\sqrt{\tau(1+\tau)} G_M^n G_E^n \tan(\theta/2)$$

$$I_0 P_L = \frac{E+E'}{M_n}\sqrt{\tau(1+\tau)} {G_M^n}^2 \tan^2(\theta/2)$$

$$\boxed{\frac{G_E}{G_M} = -\frac{P_T}{P_L}\left(\frac{E+E'}{2M}\right)\tan(\theta/2)}$$

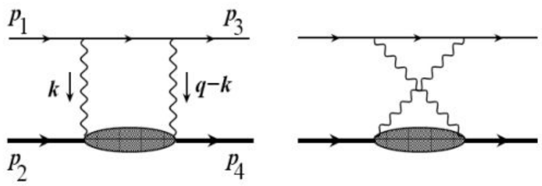


- For $Q^2 = 4.5$ GeV^2, $\mu_n G_E^n/G_M^n$ can be extrapolated from data to give the expected value of 0.55 ± 0.05

*credit: V. Punjabi, C. F. Perdrisat, M. K. Jones, E. J. Brash, and C. E. Carlson, Eur. Phys. J. A51, 510 79 (2015), arXiv:1503.01452 [nucl-ex].
**credit: E. Christy, 2019 Hall A/C Summer Workshop Slides

The Form Factor Ratio Puzzle

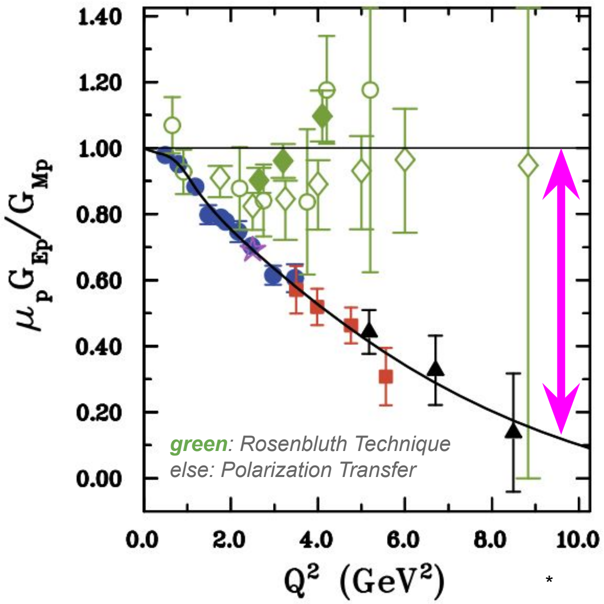
- Proton data show a discrepancy between FFR via LT and PT, and is well documented
  - LT relies on OPE, PT does not
  - Standard radiative corrections (Mo-Tsai, etc.) applied here do not account for hard Two-Photon-Exchange (TPE)
  - Difference expected to be from this hard TPE contribution!



Contribution expected to be small - around 5 - 8%

- No adequate neutron measurement of RS exists to date
  - Most recent FFR measurement *50 years ago* by Bartel et al. up to Q^2 = 2.7 GeV^2
- nTPE measured this difference!

Proton FFR Data: Both Methods



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*credit: V. Punjabi, C. F. Perdrisat, M. K. Jones, E. J. Brash, and C. E. Carlson, Eur. Phys. J. A51, 510 79 (2015), arXiv:1503.01452 [nucl-ex].

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Measurement

- With proton-neutron separation with the SBS magnet, measure quasi-elastic yields in HCal simultaneously
  - *D(e,e'n)p*
  - *D(e,e'p)n*
- Apply correction factor accounting for variation in hadron efficiencies with simulated data
- Obtain experimental observable A from SBS8 (ϵ_1) and SBS9 (ϵ_2)
- Use experimental observable B from world data from the proton
- Evaluate RS and nTPE via comparison with PT data

- Durand technique employed to reduce error on measurement where correlated sources of systematic error cancel on the ratio G_E/G_M
  - Nucleon momentum and binding cancelled
  - Inelastic e-n contamination and nucleon charge exchange partially cancelled

$$R_{observed} = \frac{N_n}{N_p}$$

$$R_{corrected} = f_{corr} \cdot R_{observed}$$

$$A = \frac{R_{corrected,\epsilon_1}}{R_{corrected,\epsilon_2}}$$

$$A = \frac{(\sigma_{e-n}/\sigma_{e-p})_{\epsilon_1}}{(\sigma_{e-n}/\sigma_{e-p})_{\epsilon_2}}$$

$$B = \frac{1+\epsilon_2 RS^p}{1+\epsilon_1 RS^p}; \quad RS^p \approx 0.087 \pm 0.010$$

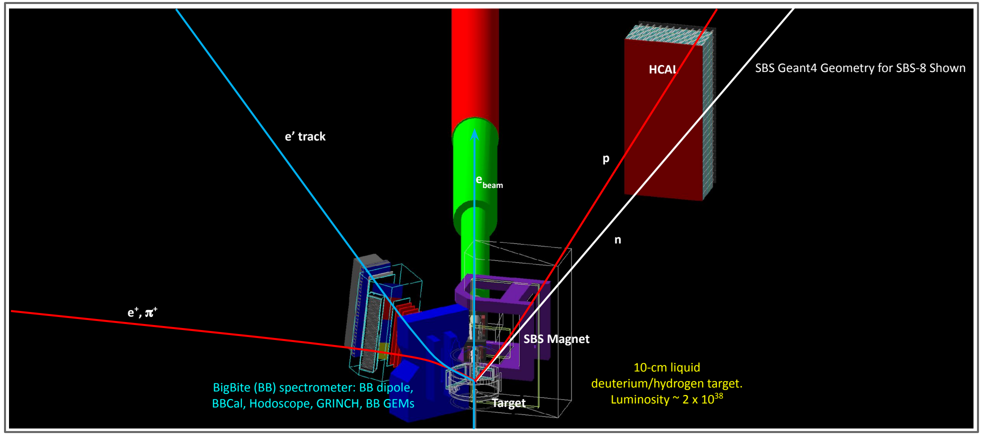
$$A = B \times \frac{1+\epsilon_1 RS^n}{1+\epsilon_2 RS^n} \approx B \times (1 + \Delta\epsilon RS^n)$$

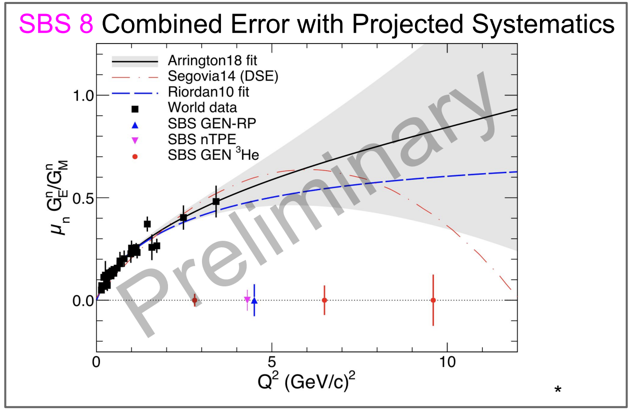
$$and \quad with \quad \Delta\epsilon = \epsilon_1 - \epsilon_2 \approx 0.281$$

$$\boxed{RS^n = \frac{A-B}{B\Delta\epsilon}}$$

SBS Program and FFR Error

- Super BigBite Spectrometer
  - Electron arm (beam left)
    - Single-arm electron trigger
    - Scattered electron tracks
  - Hadron arm (beam right)
    - Super BigBite magnet for proton-neutron separation
    - Hadron Calorimeter (HCal) to measure scattered nucleons





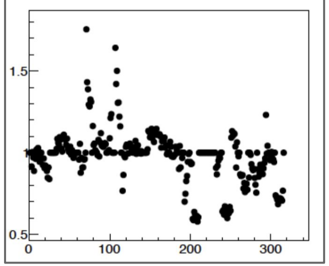
- SBS began with GMn run group - Fall 2021
- FFR Uncertainty
  - Systematic, limited with Durand technique: ± 0.012 (*Projected*)
  - Statistical: ± 0.010 (*Projected*)

*plot credit: Andrew Puckett

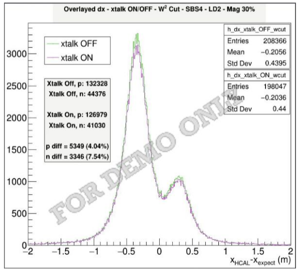
Calibrations and Corrections

- SBS is a *new* spectrometer and brings many challenges to understand and optimize each of its subsystems!
    - All pass 1 cooking is complete, but calibrations are still underway
- GEMs
    - APV gain matching corrects for variations in amplification (tracking efficiency)
        - Included for pass 1. WIP for pass 2. See Zeke's slides.
    - Deconvolution recovers BG suppressed hits very close in time (tracking eff.)
        - WIP for pass 2. See Anu's slides.
    - Cross-talk corrections remove false hits from adjacent channels (tracking eff.)
        - WIP for pass 2. See John's slides.
- GRINCH (heavy gas data available for SBS8/9)
    - Timing alignment and clustering (improve PID)
        - WIP for pass 2. See Maria's slides.
- Hodoscope
    - TDC mean time (improve timing res.)
        - Included for pass 1. Ready for pass 2

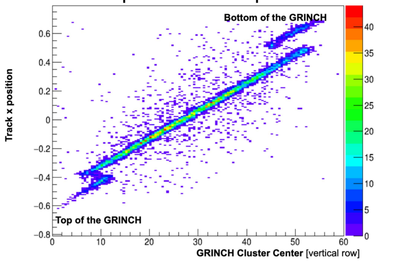
Current APV Gain Coefficients



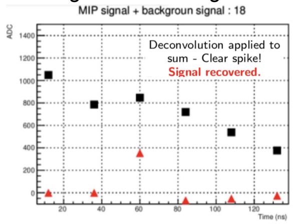
Difference in yield with/ without Crosstalk Correction



Grinch Cluster Center vs Dispersive Track position

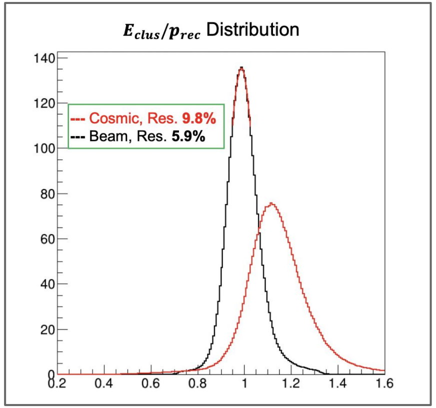


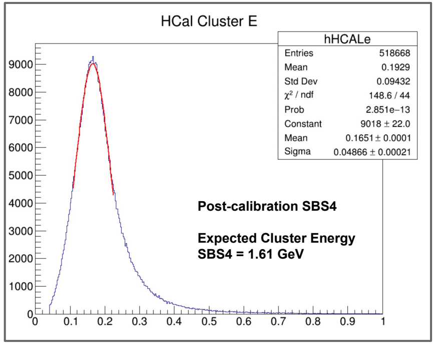
Deconvolution separating signal from background



Calibrations and Corrections

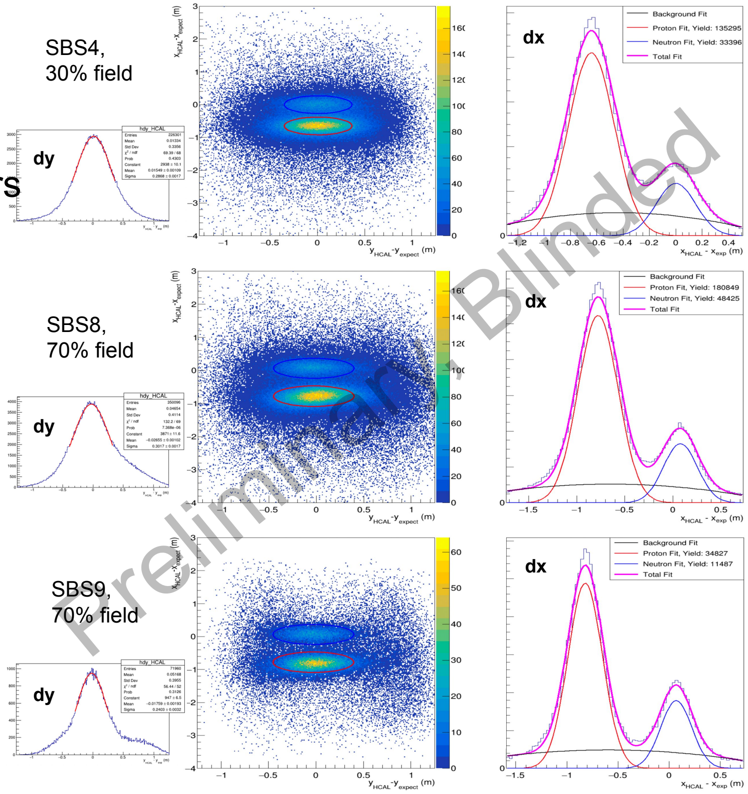
- BBCal
  - Energy maps integrated ADC (pC) to energy (GeV) calculated from e’ tracks (trigger integrity, tracking)
    - Included for pass 1. WIP for pass 2. See Provakar’s slides.
- HCal
  - Energy maps integrated ADC (pC) from many blocks in clusters to scattered nucleon energy (GeV) calculated from e’ tracks (cluster positions, PID)
    - Included for pass 1. WIP for pass 2.
  - Timing improves TDC resolution with TOF, timewalk, and trigger jitter corrections; and aligns signals by channel (elastic selection)
    - Included for pass 1. WIP for pass 2.
  - See my slides for HCal calibrations.





R-observed

- Quasi-elastics
  - Cut on e' track and HCal energy and timing parameters to select quasi-elastic events
- "Delta" plots
  - From e' track variables calculate expected HCal x and y positions
  - Make fiducial cuts to ensure Durand technique via exclusion of events w/undetectable partner nucleons
  - Take difference "delta" between this position (x_{exp}, y_{exp}) and energy-weighted cluster center in HCal (x_{HCAL}, y_{HCAL})
- Quasi-elastic yields
  - Fit background, proton, and neutron peak to obtain yields (N_p and N_n)
  - Blinding
    - Applies random factor to yields (N_p/N_n)
      - Blinding improvements expected for pass 2



$$R_{observed} = \frac{N_n}{N_p}$$

Simulated Events, R-corrected

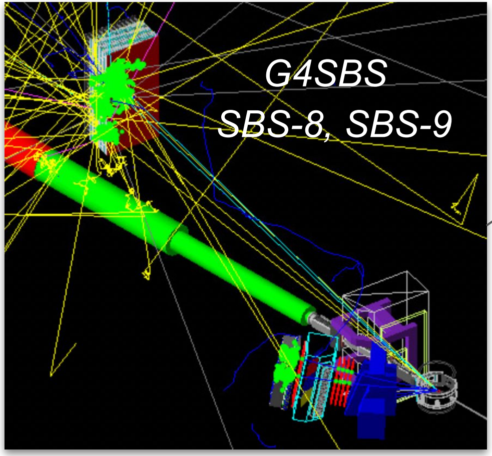
- Simulations with MC event generator G4SBS over SBS8 and SBS9 experimental configurations
- Digitize the data and cook using the same method as the data (replay with SBS Offline)
- MC/data delta plots comparisons and χ^2 minimization to get FFR
  - Radiative corrections not yet implemented in G4SBS. WIP

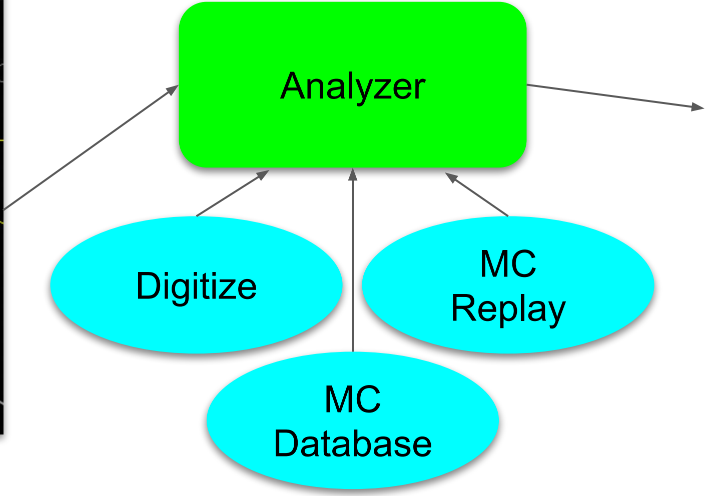
$$\Delta data_i = \frac{\sqrt{data_i}}{N_{data}}$$

Tuned to minimize χ^2

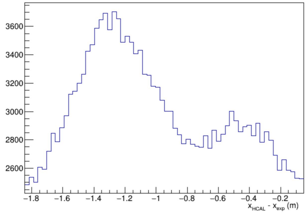
$$sim_i = N_i(protons_{hist} + \mathcal{R} \cdot neutrons_{hist})$$

$$\boxed{\chi^2 = \sum_{i=0}^{N_{bins}} \frac{(data_i - sim_i)^2}{(\Delta data_i)^2}}$$

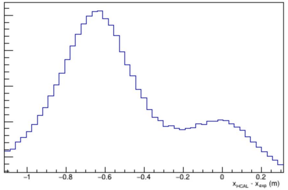




dx from G4SBS (very low statistics)



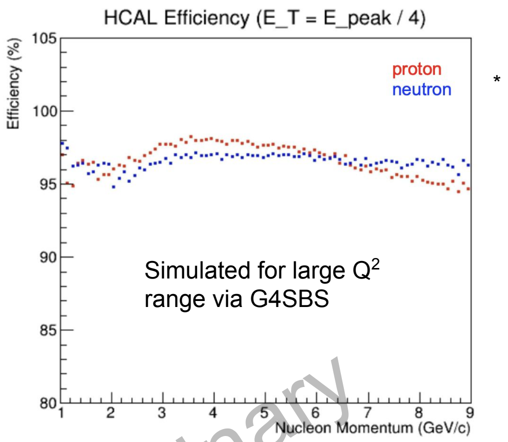
dx from SBS-4

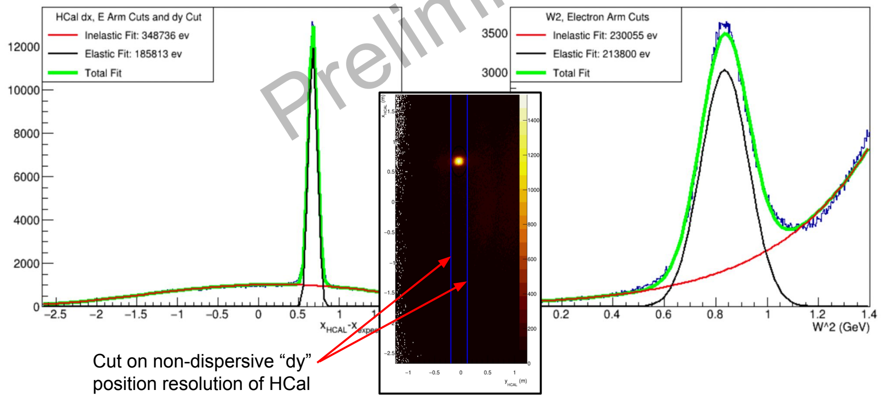


HCal Detection Efficiency

- Simulate *expected* efficiencies
  - Threshold: E_peak/4
  - Complete for all kinematics
- Proton (LH2 target)
  - Extract expected elastics using e' track cuts and HCal active area cuts only
  - Extract detected elastics from HCal dispersive delta plot "dx" fits
  - Ratio detected/expected is *observed* eff.
    - WIP
- We will check these results against the MC detection efficiencies for $f_{corrected}$

$$eff. = \frac{N_{ev\_over\_threshold}}{N_{ev}}$$

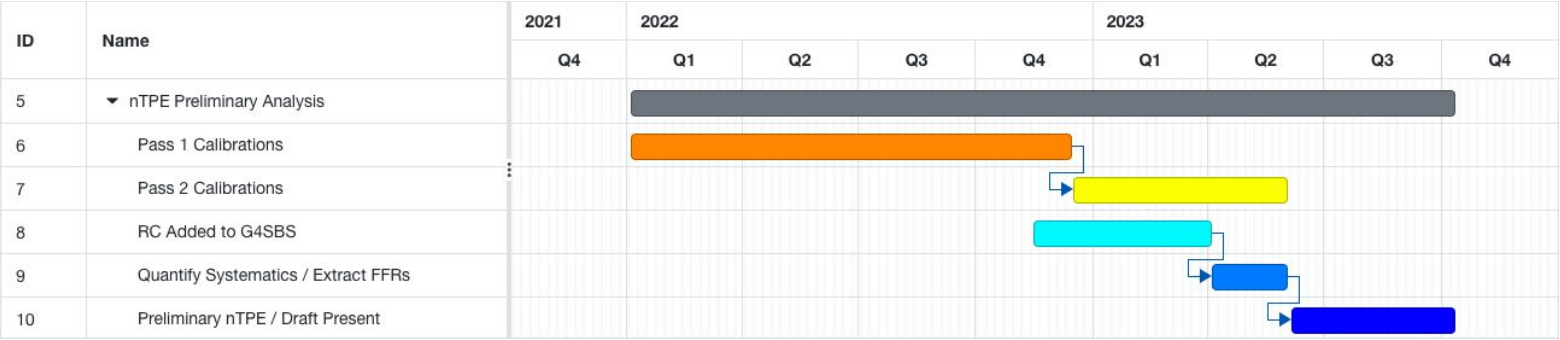




*plot credit: Provakar Datta, GMn Analysis Meeting Slides 11.18.22

Prospective

- Next Steps
  - Continue to improve calibrations and resultant elastic selection over additional cooking passes
  - Add RC model from simc to simulation in G4SBS
  - Begin to quantify known systematics
  - Calculate nTPE with systematic error over-estimate
- Target nTPE preliminary results by Summer 2023
  - Hope to be ready for DNP in the Fall!



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The nTPE Graduate Analysis Group

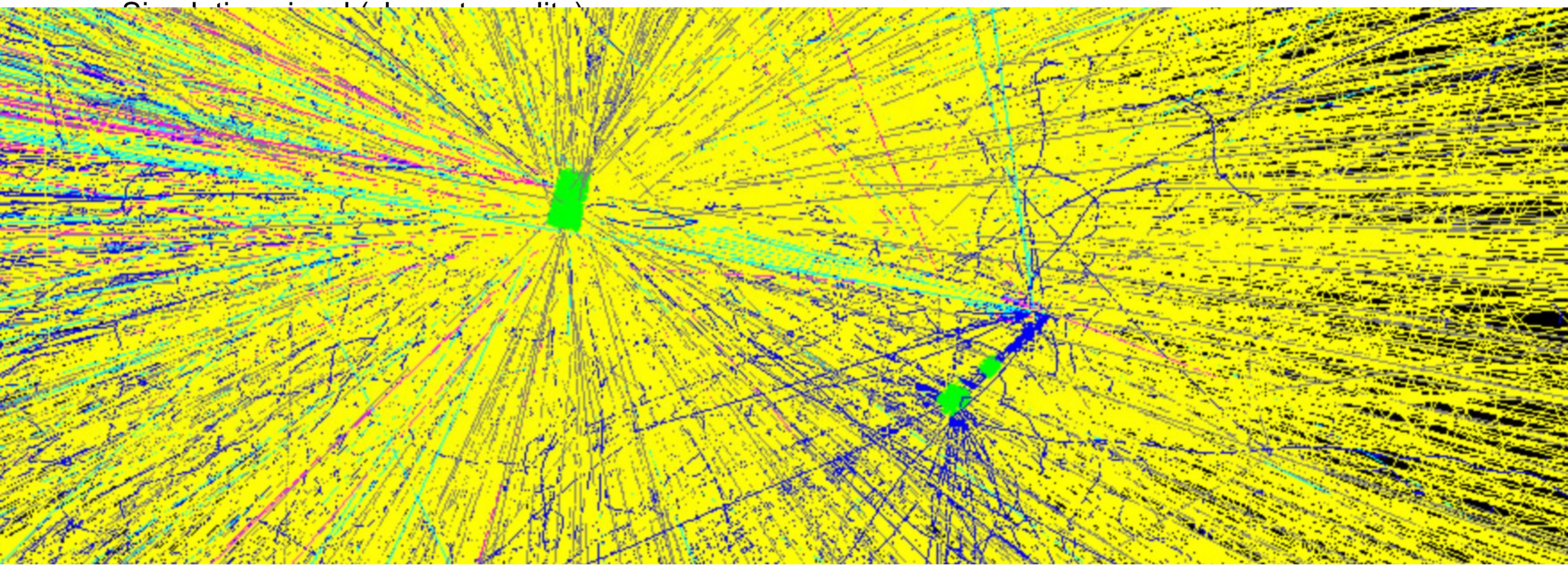
John Boyd

Zeke Wertz

Sebastian Seeds

I’m happy to answer any questions!

Backup



Backup

- Calculation of the RS used from Born appx

$$\tau \equiv -q^2/4M^2$$

$$Q^2 \equiv -q^2 \equiv 4EE' \sin^2\theta/2$$

$$\sigma_T + \epsilon\sigma_L = \tau G_M^2 + \epsilon G_E^2$$

$$G_M^2 \tau \left(1 + \epsilon \frac{G_E^2}{\tau G_M^2}\right) = \sigma_T \left(1 + \epsilon \frac{\sigma_L}{\sigma_T}\right)$$

$$= G_M^2 \tau (1 + \epsilon RS)$$

$$\sqrt{\tau \cdot RS} = \sqrt{\frac{\tau|\sigma_L|}{\sigma_T}} = \frac{G_E}{G_M}$$

Backup

- Calculation of the FF ratio from polarization transfer

$$I_0 P_T = -2\sqrt{\tau(1+\tau)}G_M^n G_E^n \tan(\theta/2)$$

$$I_0 P_L = \frac{E+E'}{M_n}\sqrt{\tau(1+\tau)}{G_M^n}^2 \tan^2(\theta/2)$$

$$I_0 = {G_E^n}^2 + \frac{\tau}{\epsilon}{G_M^n}^2$$

$$-\frac{2}{P_T}\sqrt{\tau(1+\tau)}G_M^n G_E^n \tan(\theta/2) = \frac{E+E'}{P_L M_n}\sqrt{\tau(1+\tau)}{G_M^n}^2 \tan^2(\theta/2)$$

$$-\frac{2}{P_T}G_E^n = \frac{E+E'}{P_L M_n}G_M^n \tan(\theta/2)$$

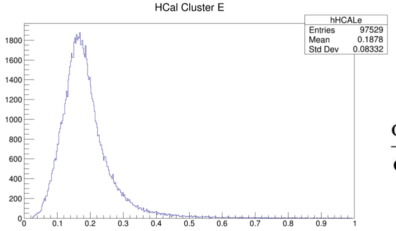
$$\frac{P_L}{P_T} = -\frac{G_M^n}{G_E^n}\left(\frac{E+E'}{2M_n}\right)\tan(\theta/2)$$

$$\frac{G_E^n}{G_M^n} = -\frac{P_T}{P_L}\left(\frac{E+E'}{2M_n}\right)\tan(\theta/2)$$

Credit for original polarization forms:
A. I. Akhiezer and M. P. Relanko, Sov. J. Part. Nucl. 3, (1974) 277 and Arnold, Carlson and Gross, Phys. Rev. C23 (1981) 363

Calibrations - HCal Energy

1. Energy Calibrations by channel from scattered protons at $Q^2 = 4.5\ \mathrm{GeV}^2$
   a. Relate ADC values (pC) to deposited energy (GeV)
      i. c_j in GeV/pC
         1. pC for integrated ADC waveforms
      ii. Indices *i, j* over hits within cluster
      iii. Energy E_i
         1. Kinetic energy of hadron incident to HCal
            a. Calculated assuming elastic scattering from BigBite track momentum and beam
         2. Apply sampling fraction of 7.95% for HCal
            a. Obtained from monte-carlo simulations
   b. Chi squared minimization with linear system of equations relating energy deposited on single channel to total deposited energy of elastically scattered hadron in cluster per event.
      i. Populate matrix with measured integrated ADC values (pC)
      ii. Reject cells with insufficient statistics
         1. Set diag element for cell to 1, all coupled set to 0
      iii. Solve for coefficients via inversion of matrix
   c. **Apply coefficients by channel to convert ADC values to energy deposited in HCal!**

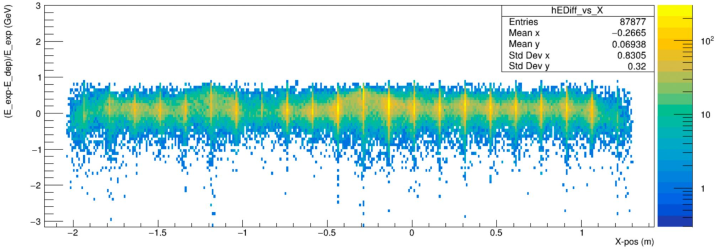


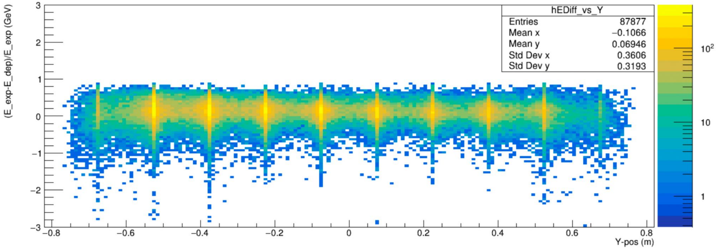
$$\chi^2 = \sum_{i=1}^{n} \left(E_i - \sum_j c_j A_j^i \right)^2 / \sigma_E^2$$

$$\sigma_E^2 \approx E_i$$

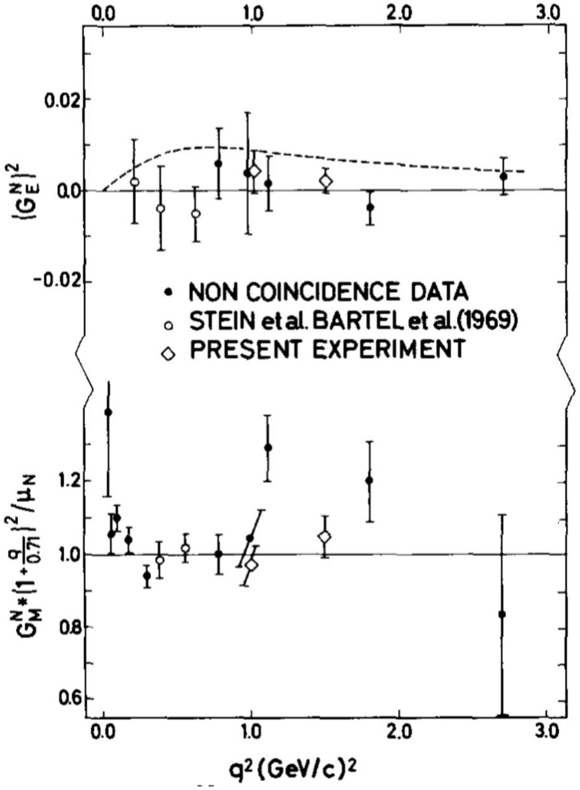
$$\frac{d\chi^2}{dc_k} = 2 \sum_{i=1}^{n} \frac{1}{E_i} \left(E_i - \sum_j c_j A_j \right) A_k = 0$$

Minimize



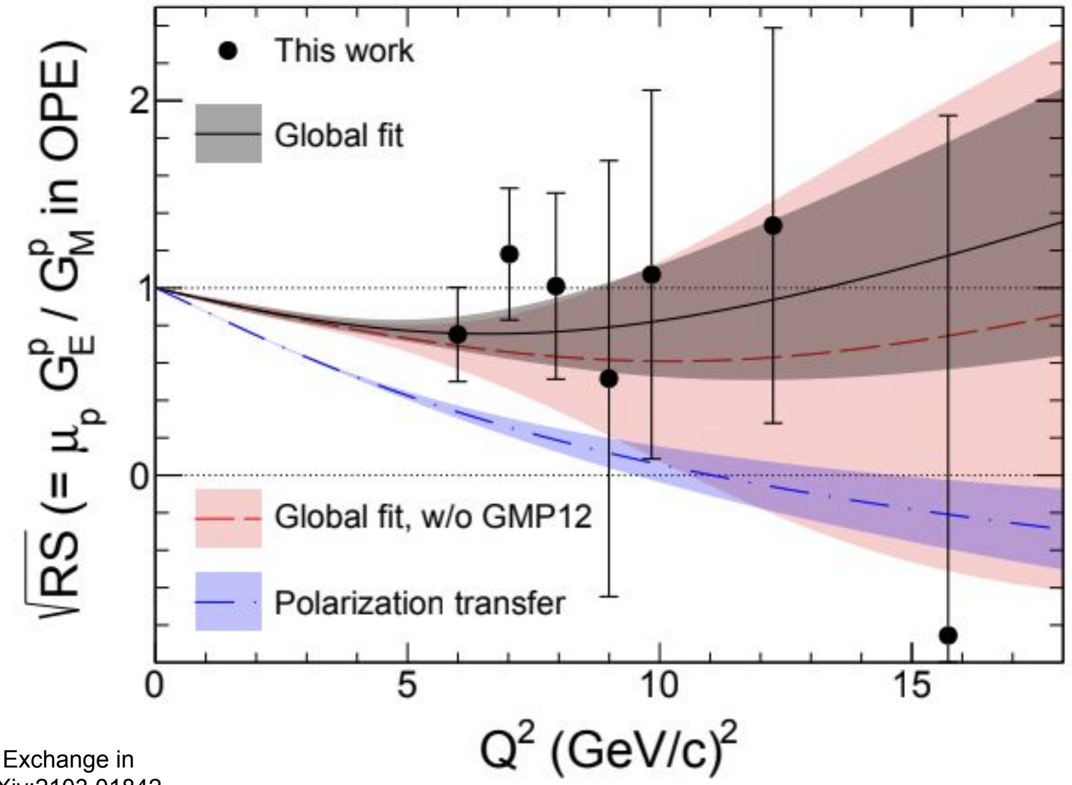


Neutron FFR
Most Recent Data



Credit: Bartel et al., Phys. Lett. 39B, 407 (1972)

$G_M{}^p$ Results - FFR RT vs PT



Credit: Christi et al, Form Factors and Two-Photon Exchange in High-Energy Elastic Electron-Proton Scattering, arXiv:2103.01842

nTPE experimental parameters and Error Budget

Kin	Q^2 (GeV/c)^2	E (GeV)	E' (GeV)	θ_{BB} (deg)	θ_{SBS} (deg)	ε
SBS9	4.45	4.015	1.63	49.0	22.5	0.524
SBS8	4.45	5.965	3.57	26.5	30.0	0.805

Systematic Uncertainty		
ε	0.599	0.838
Acceptance	0.5%	0.4%
Inelastic contamination	0.9%	0.6%
Nucleon misidentification	0.6%	
Syst. uncertainty on (quadratic sum of the above)	1.3%	1.0%

Syst. uncertainty on slope	±0.01
Projected systematic uncertainty	±0.01
, Eur. Phys. J. A51, 19 (2015)	±0.05
Combined uncertainty on TPE contribution to S^n	**± 0.016**

Two Photon Exchange Contribution to Elastic *e-n* Scattering (nTPE)

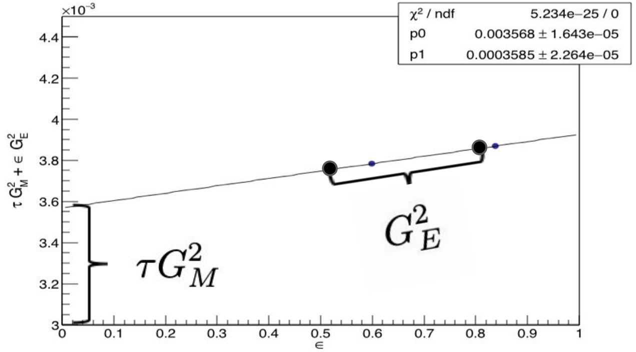
- In Born approx. separate Sach's FF with Rosenbluth technique
- Can extract FF from y-intercept and slope where reduced cross section $\tau G_M^2 + \varepsilon G_E^2$ linear in ε
- Obtain Rosenbluth slope (RS) for neutron at our kinematics with measurements at different ε (world data are sparse!)
- Discrepancy between recoil polarimetry result in one photon exchange (OPE) and Rosenbluth technique result can be explained by **TPE**

$$\left(\frac{d\sigma}{d\Omega}\right)_{eN\to eN} = \frac{\sigma_{Mott}}{\epsilon(1+\tau)}\left[\tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)\right]$$

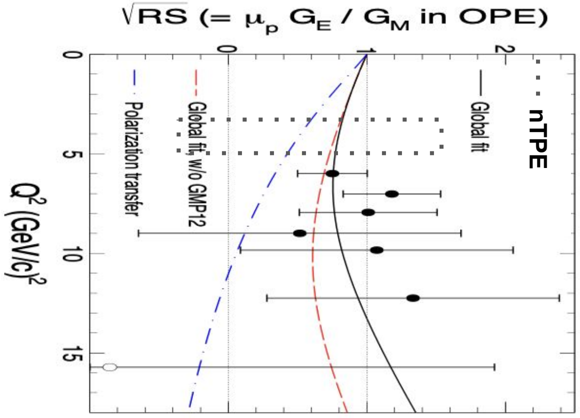
$$\epsilon = 1/\left(1 + 2\left(1 + \frac{Q^2}{4M_n}\right)\tan^2(\theta/2)\right)$$

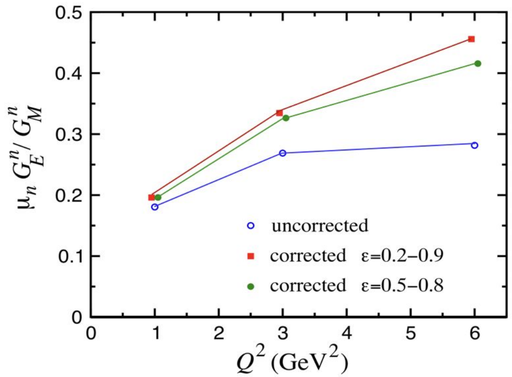
$$\sigma_r = \frac{d\sigma}{d\Omega}\frac{\epsilon(1+\tau)}{\sigma_{Mott}} = \tau G_M^2(Q^2) + \epsilon G_E^2(Q^2)$$

$$= \sigma_T + \epsilon\,\sigma_L$$



Will extract S^n, the Rosenbluth slope for the neutron, and be able to distinguish between theoretical TPE corrections





Corrected with TPE contribution between two hypothetical measurements at ε = 0.2 and 0.9

Corrected with TPE contribution between two hypothetical measurements at ε = 0.5 and 0.8

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Kinematics and Projected Uncertainty

- SBS8 and SBS9 provide the two measurements of ε (SBS9 data collection ongoing!)
- Will measure ε via ratio method for simultaneous measurement of D(e,e'n) and D(e,e'p) (Durand technique) reducing systematic uncertainties

Using the experimental observable A

$$A = R_{corrected,\epsilon_1} / R_{corrected,\epsilon_2}$$

where $$R_{corrected} = \frac{\sigma^n_{Mott}(1+\tau_p)}{\sigma^p_{Mott}(1+\tau_n)} \times \frac{\sigma^n_T + \epsilon\sigma^n_L}{\sigma^p_T + \epsilon\sigma^p_L}$$

and

$$B = (R_{Mott,\epsilon_1} / R_{Mott,\epsilon_2}) \times (1+\epsilon_2 S^p) / (1+\epsilon_1 S^p)$$

where $$R_{Mott} = \frac{\sigma^n_{Mott}(1+\tau_p)}{\sigma^p_{Mott}(1+\tau_n)}$$

And with $\Delta\epsilon = \epsilon_1 - \epsilon_2$ => $\boxed{S^n = \frac{A-B}{B\Delta\epsilon}}$

Kin	Q^2 (GeV/c)^2	E (GeV)	E' (GeV)	θ_{BB} (deg)	θ_{SBS} (deg)	**ε**
SBS9	4.5	4.03	1.63	49	22	0.523
SBS8	4.5	5.97	3.59	16.5	29.4	0.915

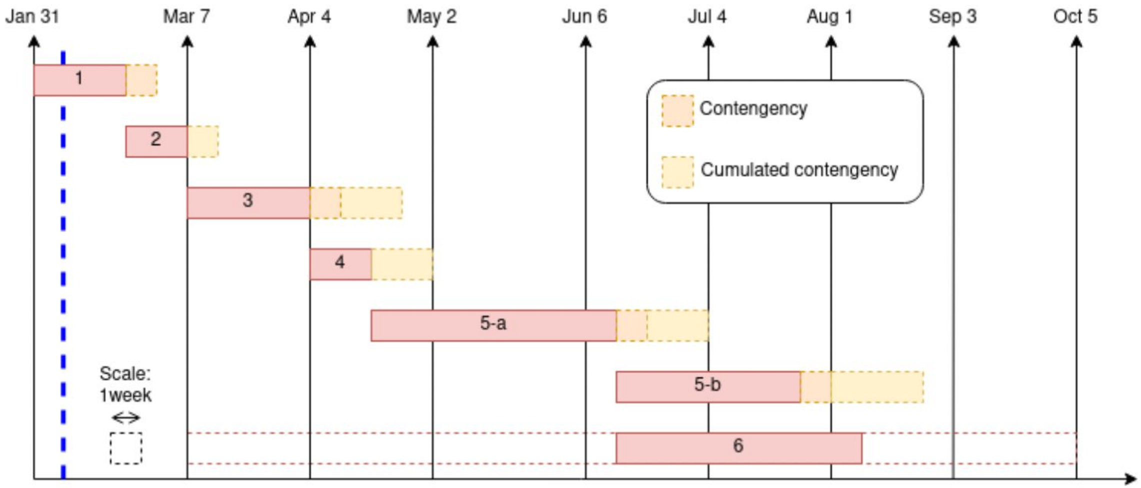
Systematic Uncertainty		
ε	0.599	0.838
Acceptance	0.5%	0.4%
Inelastic contamination	0.9%	0.6%
Nucleon misidentification	0.6%	
Syst. uncertainty on σ_{en}/σ_{ep} (quadratic sum of the above)	1.3%	1.0%

Syst. uncertainty on slope $S^p = \sigma^p_L/\sigma^p_T$	±0.01
Projected systematic uncertainty $S^n = \sigma^n_L/\sigma^n_T$	±0.01
$\mu_n G^n_E/G^n_M = 0.55$, Eur. Phys. J. A51, 19 (2015)	±0.05
Combined uncertainty on TPE contribution to S^n	**± 0.016**

We expect S^n **= 0.063 ± 0.010 (stat) ± 0.012 (syst)**

Analysis Timeline and Current Status

1. First pass calibrations of BBCal and HCal for all kinematics -*calibration scripts written and tested, most calibrations complete*
2. First mass replay and analysis -*mass replay and analysis shell scripts written and tested*
3. Refined calibrations (BBCal, HCal, Optics) -*pending second replay*
4. Second mass replay and analysis -*pending second replay*
5. Physics
    a. HCal uniformity and systematics analysis -*HCal uniformity analysis script written*
    b. Combination of kinematics and extraction of observables -*Ongoing*
6. Preparation of publication



We're nearly ready to do some serious analysis!

UCONN

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And with $\Delta\epsilon = \epsilon_1 - \epsilon_2$ => $\boxed{S^n = \frac{A-B}{B\Delta\epsilon}}$

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SBS9	4.5	4.03	1.63	49	22.5	0.523
SBS8	4.5	6.0	3.59	26.5	29.9	0.804

Systematic Uncertainty		
ε	0.599	0.838
Acceptance	0.5%	0.4%
Inelastic contamination	0.9%	0.6%
Nucleon misidentification	0.6%	
Syst. uncertainty on σ_{en}/σ_{ep} (quadratic sum of the above)	1.3%	1.0%

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