Theoretical Model for double pion photoproduction: I

Nadine Hammoud

Institute of Nuclear Physics, PAS, Kraków, Poland

with:
Adam Szczepaniak, Łukasz Bibrzycki, Vincent Mathieu, Robert Perry

4th Workshop on Future Directions in Spectroscopy Analysis 2022
Outline

• Introduction and Motivation
• Double pion photoproduction:
  • Model description:
    • Deck Model
    • pion-proton Scattering
    • Resonance Production
  • Differential cross Section
  • Moments of angular distribution
• Summary
Certain sets of quantum numbers cannot be formed from a quark and antiquark pair, such as:

\[ J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots \]
Motivation

- Studying the exotic mesons is an ideal way to confirm QCD, gain a better understanding of the fundamental quark-antiquark interactions, the role of gluons, and the origin of color confinement...
- Photoproduction process is an ideal way to produce hybrids, e.g. $\eta \pi$ photoproduction which has been applied at JLab.
- Understanding the non exotic channel such as $\pi\pi$-channel will be the start up point in our journey to fully understand and study the exotic channels.

➡️ Our aim is to do a complete analysis of the $\pi\pi$ photoproduction and hence computing distributions and moments.
Deck Mechanism:

Figure: The Deck Mechanism for two pion photoproduction. Note that either charged pions may couple with the incoming photon.
Model Description

For $\gamma(q, \lambda_\gamma) + p(p_1, \lambda_1) \rightarrow \pi^+(k_1) + \pi^-(k_2) + p(p_2, \lambda_2)$.

Figure: Kinematics

We have worked with the following kinematic invariants:

\[ s = (p_1 + q)^2, \]  
\[ s_i = (k_i + p_2)^2, \]  
\[ t = (p_1 - p_2)^2, \]  
\[ s_{12} = (k_1 + k_2)^2. \]
The gauge invariant Deck Model amplitude *can be written as:

\[
\mathcal{M}^{\text{Deck}}_{\lambda_1 \lambda_2 \lambda_q}(s, t, s_{12}, \Omega) = \sqrt{4\pi \alpha} \left[ \left( \frac{\epsilon(q, \lambda_q) \cdot k_1}{q \cdot k_1} - \frac{\epsilon(q, \lambda_q) \cdot (p_1 + p_2)}{q \cdot (p_1 + p_2)} \right) \beta(t_1) \mathcal{M}^-_{\lambda_1 \lambda_2}(s_2, t, t_1) \right. \\
- \left. \left( \frac{\epsilon(q, \lambda_q) \cdot k_2}{q \cdot k_2} - \frac{\epsilon(q, \lambda_q) \cdot (p_1 + p_2)}{q \cdot (p_1 + p_2)} \right) \beta(t_2) \mathcal{M}^+_{\lambda_1 \lambda_2}(s_1, t, t_2) \right]
\]

(5)

Where \( \beta(t_i) = \exp((t_i - t_i^{\min})/\Lambda_\pi^2), \Lambda_\pi = 0.9\text{GeV}, t_i = (q - k_i)^2 \) and

\[
t_i^{\min} = m_\pi^2 - \frac{1}{2s} \left[ (s - m_p^2)(s - s_1 + m_\pi^2) - \lambda^{1/2}(s, 0, m_p^2) \lambda^{1/2}(s, s_1, m_\pi^2) \right],
\]

(6)

\[
t_2^{\min} = m_\pi^2 - \frac{1}{2s} \left[ (s - m_p^2)(s - s_2 + m_\pi^2) - \lambda^{1/2}(s, 0, m_p^2) \lambda^{1/2}(s, s_2, m_\pi^2) \right]
\]

(7)

where \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca) \).

---

Momentum-vectors in the Helicity and Gottfried-Jackson (GJ) Frames:

\[ p_1^H = |\vec{p}_1| (\sin \theta_1, 0, \cos \theta_1) \quad ; \quad p_1^{GJ} = |\vec{p}_1| (-\sin \theta_1, 0, \cos \theta_1) \]  

(8)

\[ p_2^H = |\vec{p}_2| (0, 0, -1) \quad ; \quad p_2^{GJ} = |\vec{p}_2| (-\sin \theta_2, 0, \cos \theta_2) \]  

(9)

\[ q^H = |\vec{q}| (-\sin \theta_q, 0, \cos \theta_q) \quad ; \quad q^{GJ} = |\vec{q}| (0, 0, 1) \]  

(10)

\[ k_1^H = |\vec{k}_1| (\sin \theta^H \cos \phi^H, \sin \theta^H \sin \phi^H, \cos \theta^H) = -k_2^H \]  

(11)

\[ k_1^{GJ} = |\vec{k}_1| (\sin \theta^{GJ} \cos \phi^{GJ}, \sin \theta^{GJ} \sin \phi^{GJ}, \cos \theta^{GJ}) = -k_2^{GJ} \]  

(12)
Pion-proton Scattering:

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

\[
M_{\lambda}^{-} = \bar{u}_{\lambda}(p_{2}) \left[ A^{-}(s, t, t_{\pi}) + \frac{1}{2} \gamma_{\mu}(q - k_{1} + k_{2})^{\mu}B^{-}(s, t, t_{\pi}) \right] u_{\lambda}(p_{1}), \quad (13)
\]

where \( t_{\pi} = (q - k_{1}) \)
Pion-proton Scattering:

Similarly for the positive exchanged pion:

\[ M^+_\lambda = \bar{u}_\lambda(p_2) \left[ A^+(s, t, t_\pi) + \frac{1}{2} \gamma_\mu(q - k_2 + k_1)\mu B^+(s, t, t_\pi) \right] u_\lambda(p_1), \tag{14} \]

where \( t_\pi = (q - k_2) \).

In the \( \pi N \) center of mass frame the t-channel \( A \) and \( B \) defined as follows:

\[ \frac{1}{4\pi} A^\pm = \frac{\sqrt{s} + m_p}{Z^+_1 Z^+_2} f^\pm_1 - \frac{\sqrt{s} - m_p}{Z^-_1 Z^-_2} f^\pm_2, \tag{15} \]

\[ \frac{1}{4\pi} B^\pm = \frac{1}{Z^+_1 Z^+_2} f^\pm_1 - \frac{1}{Z^-_1 Z^-_2} f^\pm_2. \tag{16} \]

Where \( f_1 \) and \( f_2 \) are called the reduced helicity amplitudes, \( Z^\pm_i = \sqrt{E_i \pm m_p} \).
Pion-proton scattering

The partial wave decomposition:

\[ f_1 = \frac{1}{\sqrt{|p_1||p_2|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos \theta) - \frac{1}{\sqrt{|p_1||p_2|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos \theta), \]

(17)

\[ f_2 = \frac{1}{\sqrt{|p_1||p_2|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos \theta). \]

(18)

In our model the pion virtuality appears clearly in the incoming proton energy \(E_1\), momentum \(P_1\) as well as our scattering angle \(\cos \theta\). Hence we can say that the scalar functions \(A\) and \(B\) in our case depends on the pion virtuality.

\[ E_1 = \frac{s_i - t_\pi + m_p^2}{2 \sqrt{s_i}}, \]

(19)

\[ \cos \theta = \frac{2s_i(t - 2m_p^2) + (s_i - t_\pi + m_p^2)(s_i - m_\pi^2 + m_p^2)}{\sqrt{\lambda(s_i, t_\pi, m_p^2)} \sqrt{\lambda(s_i, m_\pi^2, m_p^2)}}. \]

(20)
Resonance Production:

Adding the $P$ wave to our amplitude i.e. the $\rho$ resonance which comes as a result of the exchange of both $P$ and $f_2$. \(^\dagger\)

The amplitude can be defined as:

$$M_{P,\lambda,\lambda_1,\lambda_2} = -\frac{1}{s} g_{\rho \pi} \beta_N^\rho BW(m_\pi) R(s, t) \bar{u}(p_2, \lambda_2) \gamma^\alpha u(p_1, \lambda_1) \nu^\lambda_\alpha \ast e^{\beta_N^t},$$

(21)

$$M_{P-Tot,\lambda,\lambda_1,\lambda_2} = M_P \ast e^{\beta_P^t} + M_{f_2}((1 - \epsilon) \ast e^{\beta_{f_2}^t_1} + \epsilon \ast e^{\beta_{f_2}^t_2}).$$

(22)

Where $\nu^\lambda_\alpha = k_\alpha \epsilon^\lambda \cdot (k_1 - k_2) - q \cdot (k_1 - k_2) \epsilon^\lambda_\alpha$, and $k = k_1 + k_2$.

The Regge propagator is given by:

$$R(s, t) = \frac{\alpha_N(t)}{\alpha_N(0)} \frac{1 + e^{i\pi \alpha_N(t)}}{\sin(\pi \alpha(t))} \left( \frac{s}{s_0} \right)^{\alpha_N(t)}$$

(23)

with the following linear trajectories for both $P$ and $f_2$ exchanges:

$$\alpha_P(t) = 1.08 + 0.2t$$

(24)

$$\alpha_{f_2}(t) = 0.5 + 0.9t$$

(25)

\(^\dagger\)V. Mathieu et al. Phys. Rev. D 97, 094003
We used the energy dependent width BW distribution:

\[
BW(s) = \frac{1}{m_\rho^2 - m_{\pi\pi}^2 - im_\rho \Gamma(s)} \quad \text{with} \quad \Gamma(s) = \left(\frac{s - 4m_{\pi}^2}{m^2 - 4m_{\pi}^2}\right)^{3/2}
\]  

(26)

And the following constants:\‡

\[
g_{\rho\pi} = 5.96 \quad \beta_{\rho}^{\gamma\rho} = 2.506 \quad \beta_{\rho} = 3.6 \quad \beta_{f_2}^{\gamma\rho} = 2.47 \quad \beta_{f_2}^{1} = 0.55 \quad s_0 = 1 \text{ GeV}^2
\]  

(27)

The fitted parameters were§:

\[
\beta_{f_2}^{2} = -0.20923732, \quad \epsilon = 1.3710881
\]  

(28)

\‡V.Mathieu et al. Phys. Rev. D 97, 094003

§Credits to Robert Perry
Differential Cross Section

The $P$-wave differential cross section can be written as:

$$\frac{d\sigma}{dT d\Omega} = \frac{1}{2} (2\pi) \kappa \sum_{\lambda_q \lambda_1 \lambda_2} \left| \sum_{l=0}^{\infty} \sum_{m=-l}^{l} M_{\lambda_1 \lambda_2 \lambda_q}^{lm}(s, t, s_{12}) Y_{lm}(\Omega) \right|^2,$$  \hspace{1cm} (29)

where

$$\kappa = \frac{1}{(2\pi)^3} \frac{1}{4\pi} \frac{1}{2\pi} \frac{\lambda^{1/2}(s_{12}, m_{\pi}^2, m_{\pi}^2)}{16 \sqrt{s_{12} (s - m^2)^2}}.$$  \hspace{1cm} (30)

Thus we obtain:

$$\frac{d\sigma_l}{dt} = \sum_{m=-l}^{l} \int_{m_{\pi\pi}^{\min}}^{m_{12}^{\max}} dm_{\pi\pi} \frac{1}{2} (2\pi) \kappa \sum_{\lambda_q \lambda_1 \lambda_2} \left| M_{\lambda_1 \lambda_2 \lambda_q}^{lm}(s, t, s_{12}) \right|^2.$$  \hspace{1cm} (31)
**Figure:** Differential cross section for our improved model in the $M_{\pi\pi}$ range 0.4 – 1.2 GeV and beam energy $E_\gamma = 3.4$ GeV.

Credits to Robert Perry
Moments of Angular Distribution

We used the following moments convention\textsuperscript{\textcopyright}

\[ Y_{LM}(s, t, m_{\pi\pi}) = \sqrt{4\pi} \int d\Omega \frac{d\sigma}{dtdm_{\pi\pi}d\Omega} \text{Re} Y_{LM}(\Omega), \quad (32) \]

this normalization ensures that

\[ Y_{00}(s, t, m_{\pi\pi}) = \frac{d\sigma}{dtdm_{\pi\pi}}. \quad (33) \]

One can use the other definition of the moments given in [Phys.Rev.D 100, 054017], they are related to the above definition via

\[ Y_{LM} = 2\pi \sqrt{2L + 1} H_{LM}^{0} \]

\textsuperscript{\textcopyright}Phys.Rev.D80:072005,2009
Moments of Angular Distribution

\[ E_\gamma = 4.7 \text{ GeV}, \; t = -0.087 \text{ GeV}^2 \]

\[ E_\gamma = 4.7 \text{ GeV}, \; t = -0.275 \text{ GeV}^2 \]

Figure: Comparing the prediction of JPAC model with experimental data from Ballam and CLAS.

Credits to Robert Perry
Summary

In this talk I have presented:

- the definition of our model which composed of two mechanisms i.e Deck model arises from diffractive scattering of photon on the target proton via one pion exchange and the $\rho$ resonance production piece.
- the computation of both Deck and the $P$ wave contribution amplitudes.
- the results of:
  - the $P$- wave differential cross section of Deck model, $\rho$ resonance ($\mathcal{P} + f_2$) as well as the full model.
  - the moments of angular distribution $< Y_{00} >$ at different momentum transfer $t$.
- **Currently** we are working on the calculations of the remaining moments. We also are in the last steps of finalizing our paper to be published soon. So stay tuned!
Thank you
Figure: Amplitude with energy dependent versus energy independent BW distribution.