Theoretical Model for double pion photoproduction: I

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Joint Physics Analysis Center

Outline

- Introduction and Motivation
- Double pion photoproduction:
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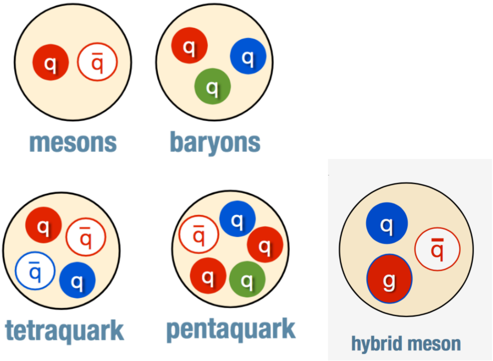
Introduction-Ordinary and Exotic Hadrons



The gluonic field excitations in hybrids

↓

Certain sets of quantum numbers cannot be formed from a quark and antiquark pair, such as:

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} ...$$

Motivation

- Studying the exotic mesons is an ideal way to confirm QCD, gain a better understanding of the fundamental quark-antiquark interactions, the role of gluons , and the origin of color confinement...
- Photoproduction process is an ideal way to produce hybrids, e.g. $\eta\pi$ photoproduction which has been applied at JLab.
- Understanding the non exotic channel such as $\pi\pi$-channel will be the start up point in our journey to fully understand and study the exotic channels.

➡ Our aim is to do a complete analysis of the $\pi\pi$ photoproduction and hence computing distributions and moments .

Model Description

Deck Mechanism :

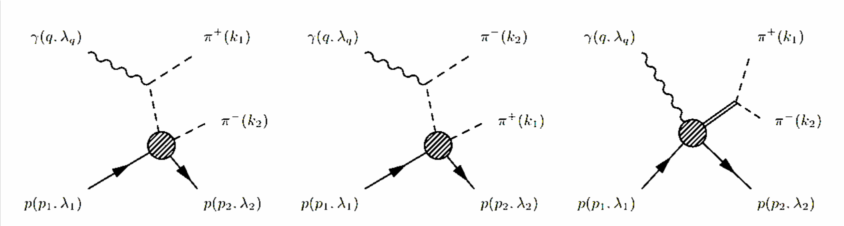


Figure: The Deck Mechanism for two pion photoproduction. Note that either charged pions may couple with the incoming photon.

Model Description

For $\gamma(q, \lambda_\gamma) + p(p_1, \lambda_1) \to \pi^+(k_1) + \pi^-(k_2) + p(p_2, \lambda_2)$.

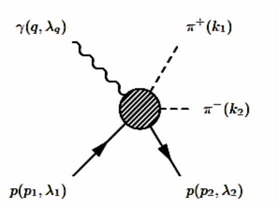


Figure: Kinematics

We have worked with the following kinematic invariants:

$$s = (p_1 + q)^2, \tag{1}$$

$$s_i = (k_i + p_2)^2, \tag{2}$$

$$t = (p_1 - p_2)^2, \tag{3}$$

$$s_{12} = (k_1 + k_2)^2. \tag{4}$$

Model Description- Deck Model

The guage invariant Deck Model amplitude *can be written as:

$$M^{\text{Deck}}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12},\Omega) = \sqrt{4\pi\alpha}\Bigg[\left(\frac{\epsilon(q,\lambda_q)\cdot k_1}{q\cdot k_1} - \frac{\epsilon(q,\lambda_q)\cdot(p_1+p_2)}{q\cdot(p_1+p_2)}\right)\beta(t_1)M^{-}_{\lambda_1\lambda_2}(s_2,t,t_1)$$
$$-\left(\frac{\epsilon(q,\lambda_q)\cdot k_2}{q\cdot k_2} - \frac{\epsilon(q,\lambda_q)\cdot(p_1+p_2)}{q\cdot(p_1+p_2)}\right)\beta(t_2)M^{+}_{\lambda_1\lambda_2}(s_1,t,t_2)\Bigg] \quad (5)$$

Where $\beta(t_i) = \exp((t_i - t_i^{\min})/\Lambda_\pi^2)$, $\Lambda_\pi = 0.9GeV$, $t_i = (q-k_i)^2$ and

$$t_1^{\min} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_2+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2))\lambda^{1/2}(s,s_2,m_\pi^2)\Big], \quad (6)$$
$$t_2^{\min} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_1+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2)\lambda^{1/2}(s,s_1,m_\pi^2)\Big] \quad (7)$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$.

*J. Pumplin, Phys. Rev. D 2, 1859 (1970).

Momentum-vectors in the Helicity and Gottfried-Jackson (GJ) Frames:

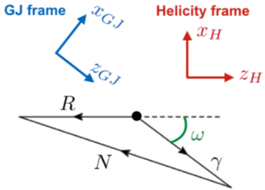
$$\boldsymbol{p}_{\boldsymbol{1}}^{H} = |\vec{p}_1|(\sin\theta_1, 0, \cos\theta_1) \quad ; \quad \boldsymbol{p}_{\boldsymbol{1}}^{GJ} = |\vec{p}_1|(-\sin\theta_1, 0, \cos\theta_1) \tag{8}$$

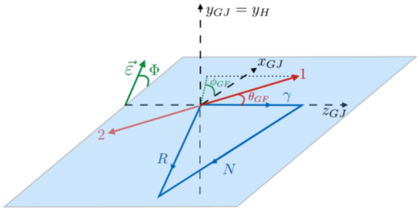
$$\boldsymbol{p}_{\boldsymbol{2}}^{H} = |\vec{p}_2|(0, 0, -1) \quad ; \quad \boldsymbol{p}_{\boldsymbol{2}}^{GJ} = |\vec{p}_2|(-\sin\theta_2, 0, \cos\theta_2) \tag{9}$$

$$\boldsymbol{q}^{H} = |\vec{q}|(-\sin\theta_q, 0, \cos\theta_q) \quad ; \quad \boldsymbol{q}^{GJ} = |\vec{q}|(0, 0, 1) \tag{10}$$

$$\boldsymbol{k}_{\boldsymbol{1}}^{H} = |\vec{k}_1|(\sin\theta^H\cos\phi^H, \sin\theta^H\sin\phi^H, \cos\theta^H) = -\boldsymbol{k}_{\boldsymbol{2}}^{H} \tag{11}$$

$$\boldsymbol{k}_{\boldsymbol{1}}^{GJ} = |\vec{k}_1|(\sin\theta^{GJ}\cos\phi^{GJ}, \sin\theta^{GJ}\sin\phi^{GJ}, \cos\theta^{GJ}) = -\boldsymbol{k}_{\boldsymbol{2}}^{GJ} \tag{12}$$





V. Mathieu [arXiv:1909.09473]

Pion-proton Scattering:

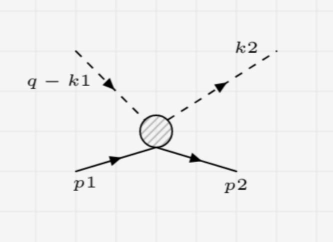


Figure: Feynman diagram for $\pi^- p \rightarrow \pi^- p$

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

$$M_\lambda^- = \bar{u}_\lambda(p_2)\left[A^-(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q - k_1 + k_2)^\mu B^-(s,t,t_\pi)\right]u_\lambda(p_1), \quad (13)$$

where $t_\pi = (q - k_1)$

Pion-proton Scattering:

Similarly for the positive exchanged pion:

$$M^+_\lambda = \bar{u}_\lambda(p_2)\left[A^+(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q-k_2+k_1)^\mu B^+(s,t,t_\pi)\right]u_\lambda(p_1), \quad (14)$$

where $t_\pi = (q-k_2)$.
In the πN center of mass frame the t-channel A and B defined as follows:

$$\frac{1}{4\pi}A^\pm = \frac{\sqrt{s}+m_p}{Z_1^+ Z_2^+}f_1^\pm - \frac{\sqrt{s}-m_p}{Z_1^- Z_2^-}f_2^\pm, \quad (15)$$

$$\frac{1}{4\pi}B^\pm = \frac{1}{Z_1^+ Z_2^+}f_1^\pm - \frac{1}{Z_1^- Z_2^-}f_2^\pm. \quad (16)$$

Where f_1 and f_2 are called the reduced helicity amplitudes, $Z_i^\pm = \sqrt{E_i \pm m_p}$.

The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos\theta) - \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos\theta), \tag{17}$$

$$f_2 = \frac{1}{\sqrt{|\boldsymbol{p_1}||\boldsymbol{p_2}|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos\theta). \tag{18}$$

In our model the pion virtuality appears clearly in the incoming proton energy (E_1), momentum (P_1) as well as our scattering angle ($\cos\theta$). Hence we can say that the scalar functions A and B in our case depends on the pion virtuality.

$$E_1 = \frac{s_i - t_\pi + m_p^2}{2\sqrt{s_i}}, \tag{19}$$

$$\cos\theta = \frac{2s_i(t - 2m_p^2) + (s_i - t_\pi + m_p^2)(s_i - m_\pi^2 + m_p^2)}{\sqrt{\lambda(s_i, t_\pi, m_p^2)}\sqrt{\lambda(s_i, m_\pi^2, m_p^2)}}. \tag{20}$$

Resonance Production:

Adding the P wave to our amplitude i.e. the ρ resonance which comes as a result of the exchange of both $\mathcal{P}$ and f_2. [†]

The amplitude can be defined as:

$$\mathcal{M}^{\mathrm{P}}_{\lambda,\lambda_1,\lambda_2} = \frac{-1}{s} g_{\rho\pi}\beta_N^{\gamma\rho} BW(m_{\pi\pi}) R(s,t)\bar{u}(p_2,\lambda_2)\gamma^{\alpha}u(p_1,\lambda_1)v^{\lambda}_{\alpha} * e^{\beta_N t}, \tag{21}$$

$$\mathcal{M}^{\mathrm{P\text{-}Tot}}_{\lambda,\lambda_1,\lambda_2} = M_{\mathcal{P}} * e^{\beta_{\mathcal{P}} t} + M_{f_2}((1-\epsilon) * e^{\beta^1_{f_2} t} + \epsilon * e^{\beta^2_{f_2} t}). \tag{22}$$

Where $v^{\lambda}_{\alpha} = k_{\alpha}\epsilon^{\lambda} \cdot (k_1 - k_2) - q \cdot (k_1 - k_2)\epsilon^{\lambda}_{\alpha}$, and $k = k_1 + k_2$.

The Regge propagator is given by:

$$R(s,t) = \frac{\alpha_N(t)}{\alpha_N(0)} \frac{1 + e^{i\pi\alpha_N(t)}}{\sin(\pi\alpha(t))} \left(\frac{s}{s_0}\right)^{\alpha_N(t)} \tag{23}$$

with the following linear trajectories for both $\mathcal{P}$ and f_2 exchanges:

$$\alpha_{\mathcal{P}}(t) = 1.08 + 0.2t \tag{24}$$

$$\alpha_{f_2}(t) = 0.5 + 0.9t \tag{25}$$

[†] V.Mathieu et al. Phys. Rev. D 97, 094003

Resonance Production:

We used the energy dependent width BW distribution:

$$BW(s) = \frac{1}{m_\rho^2 - m_{\pi\pi}^2 - i m_\rho \Gamma(s)} \quad \text{with} \quad \Gamma(s) = \left(\frac{s - 4m_\pi^2}{m^2 - 4m_\pi^2}\right)^{3/2} \qquad (26)$$

And the following constants:‡

$$g_{\rho\pi} = 5.96 \quad \beta_{\mathcal{P}}^{\gamma\rho} = 2.506 \quad \beta_{\mathcal{P}} = 3.6 \quad \beta_{f_2}^{\gamma\rho} = 2.47 \quad \beta_{f_2}^{1} = 0.55 \quad s_0 = 1 \;\; GeV^2 \qquad (27)$$

The fitted parameters were§:

$$\beta_{f_2}^2 = -0.20923732, \qquad \epsilon = 1.3710881 \qquad (28)$$

‡V.Mathieu et al. Phys. Rev. D 97, 094003

§Credits to Robert Perry

Differential Cross Section

The *P*-wave differential cross section can be written as:

$$\frac{d\sigma}{dt\,dm_{\pi\pi}\,d\Omega} = \frac{1}{2}(2\pi)\kappa \sum_{\lambda_q\lambda_1\lambda_2} \left| \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathcal{M}^{lm}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12}) Y_{lm}(\Omega) \right|^2, \quad (29)$$

where

$$\kappa = \frac{1}{(2\pi)^3} \frac{1}{4\pi} \frac{1}{2\pi} \frac{\lambda^{1/2}(s_{12}, m_\pi^2, m_\pi^2)}{16\sqrt{s_{12}}(s-m^2)^2} \frac{1}{2}. \quad (30)$$

Thus we obtain:

$$\frac{d\sigma_l}{dt} = \sum_{m=-l}^{l} \int_{m_{\pi\pi}^{\min}}^{m_{12}^{\max}} dm_{\pi\pi} \frac{1}{2}(2\pi)\kappa \sum_{\lambda_q\lambda_1\lambda_2} |\mathcal{M}^{lm}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12})|^2 \quad (31)$$

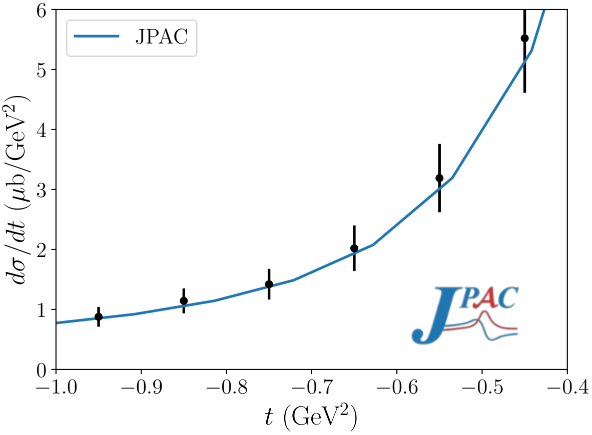


Figure: Differential cross section for our improved model in the $M_{\pi\pi}$ range 0.4 – 1.2 GeV and beam energy E_γ = 3.4 GeV.

Credits to Robert Perry

Moments of Angular Distribution

We used the following moments convention[¶]

$$Y_{LM}(s,t,m_{\pi\pi}) = \sqrt{4\pi}\int d\Omega \frac{d\sigma}{dt dm_{\pi\pi} d\Omega} \mathrm{Re}\, Y_{LM}(\Omega), \qquad (32)$$

this normalization ensures that

$$Y_{00}(s,t,m_{\pi\pi}) = \frac{d\sigma}{dt dm_{\pi\pi}}. \qquad (33)$$

One can use the other definition of the moments given in [Phys.Rev.D 100, 054017], they are related to the above definition via

$$Y_{LM} = 2\pi\sqrt{2L+1} H^0_{LM}$$

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[¶] Phys.Rev.D80:072005,2009

Moments of Angular Distribution

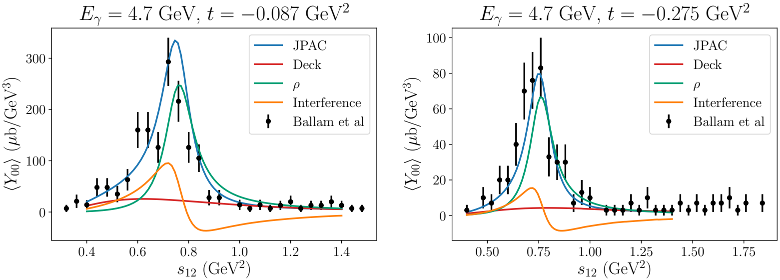

Figure: Comparing the prediction of JPAC model with experimental data from Ballam and CLAS.

Credits to Robert Perry

Summary

In this talk I have presented:

- the definition of our model which composed of two mechanisms i.e Deck model arises from diffractive scattering of photon on the target proton via one pion exchange and the ρ resonance production piece.
- the computation of both Deck and the P wave contribution amplitudes.
- the results of:
  - the P- wave differential cross section of Deck model, ρ resonance ($\mathcal{P}$ +f_2) as well as the full model.
  - the moments of angular distribution $< Y_{00} >$ at different momentum transfer t.
- **Currently** we are working on the calculations of the remaining moments. We also are in the last steps of finalizing our paper to be published soon. So stay tuned!


Thank
you

Back-Up

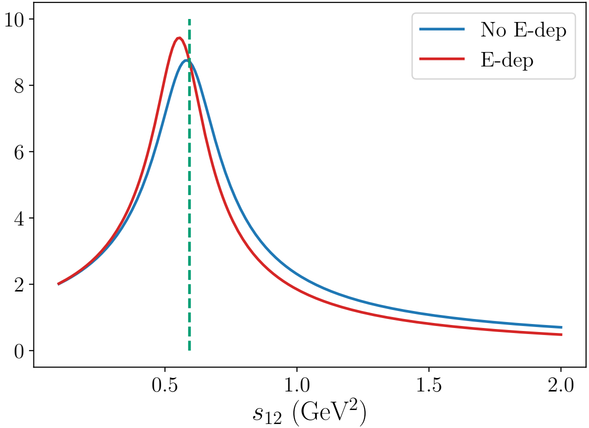

Figure: Amplitude with energy dependent versus energy independent BW distribution.