

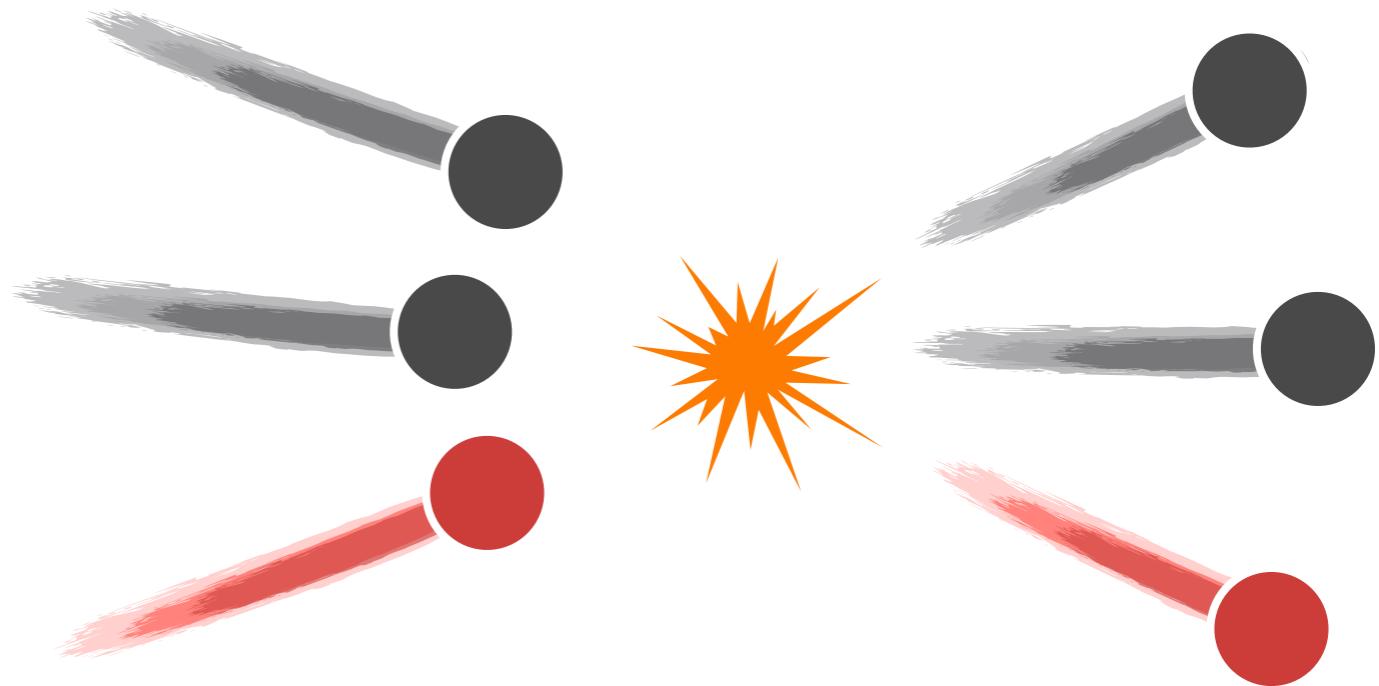
Few-Body Dynamics from QCD

An Overview of Three-Body Scattering from Lattice QCD

Andrew W. Jackura

4th Workshop on Future Directions in Spectroscopy Analysis (FDSA2022)

Tuesday, November 15th, 2022



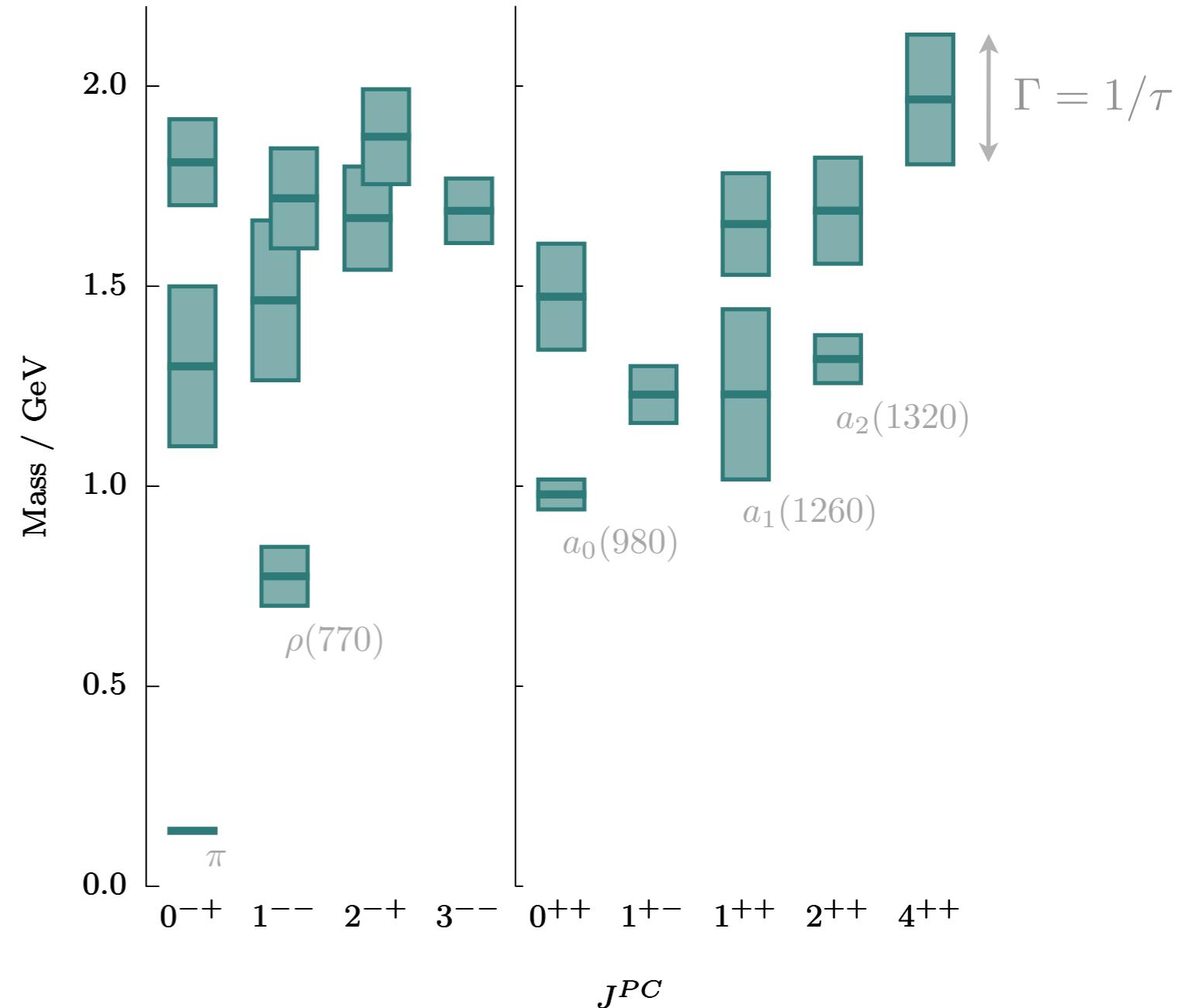
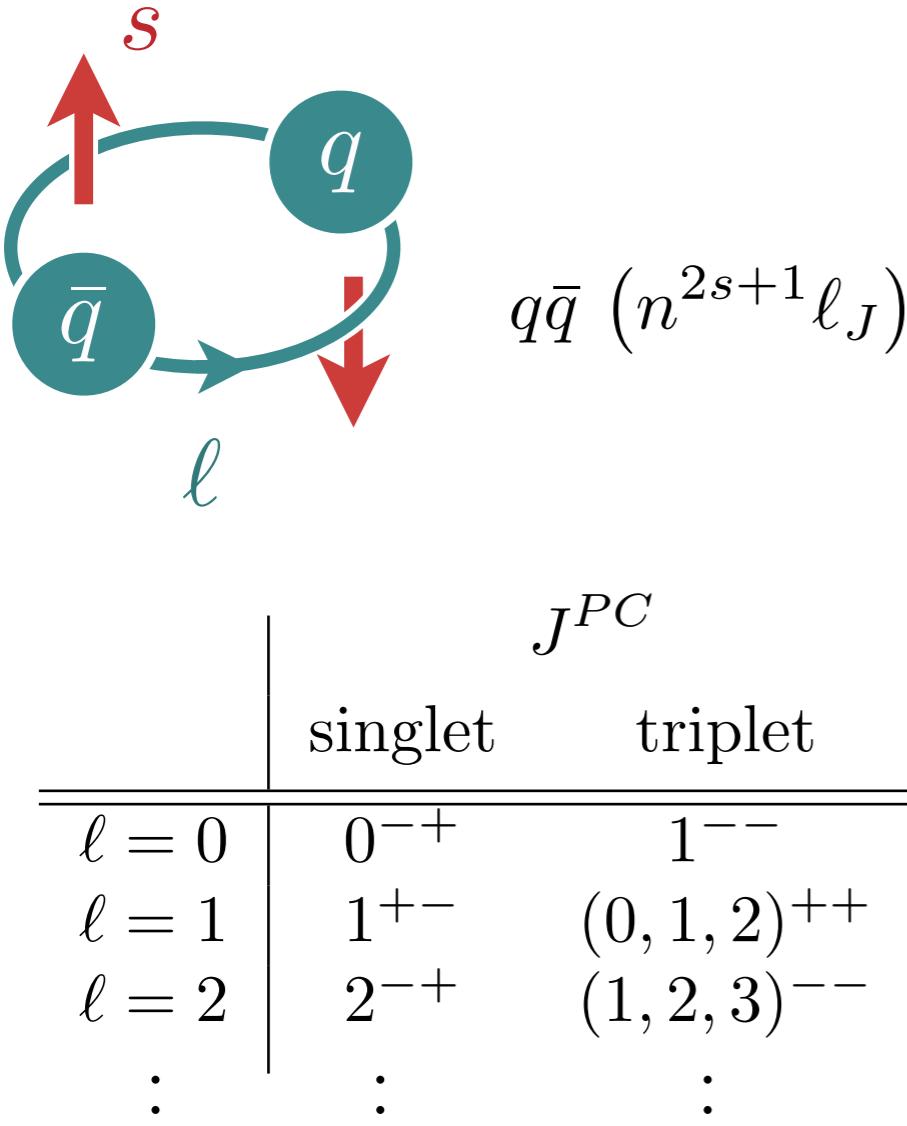
**OLD DOMINION
UNIVERSITY**

Jefferson Lab
Thomas Jefferson National Accelerator Facility

The Hadron Spectrum

Quark models give gross structure of the hadron spectrum

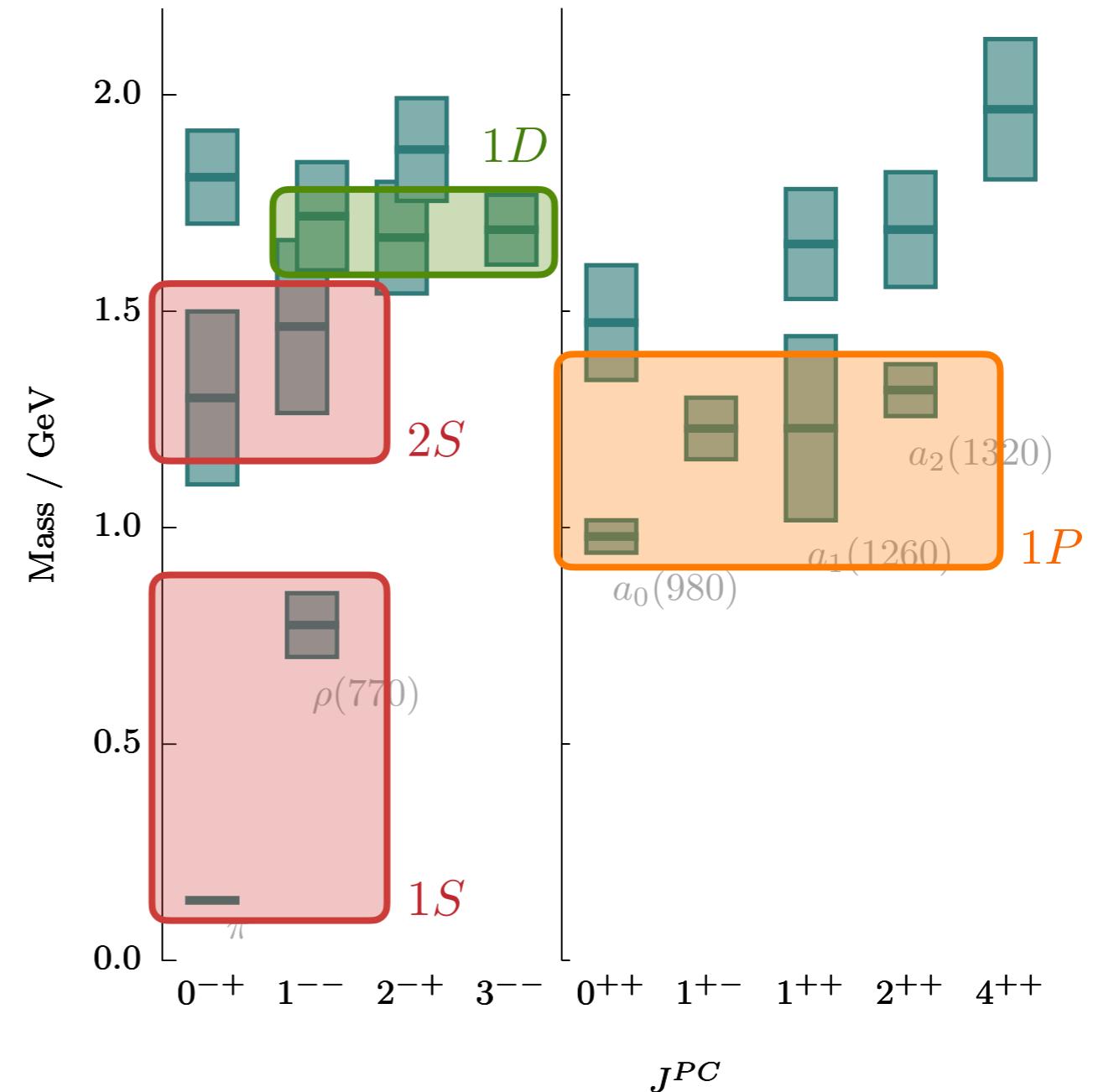
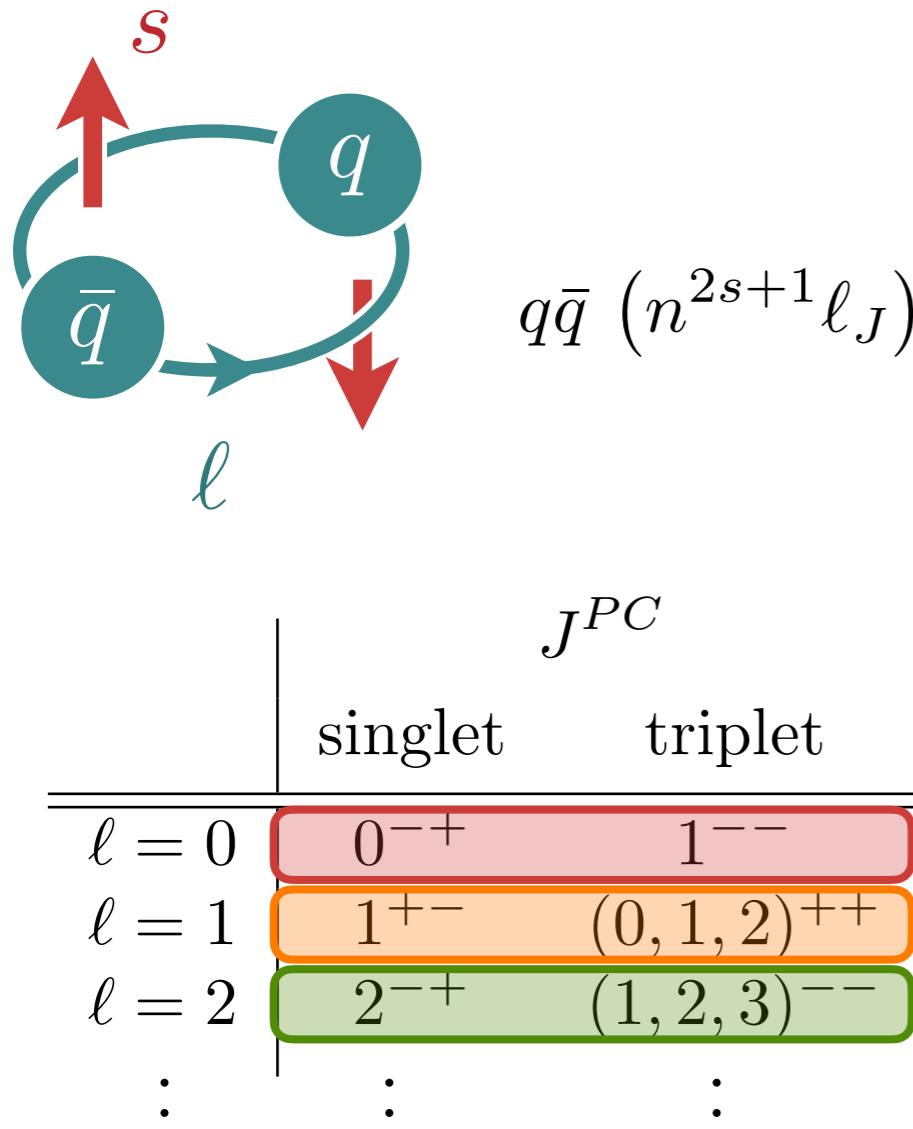
e.g. light isovector mesons



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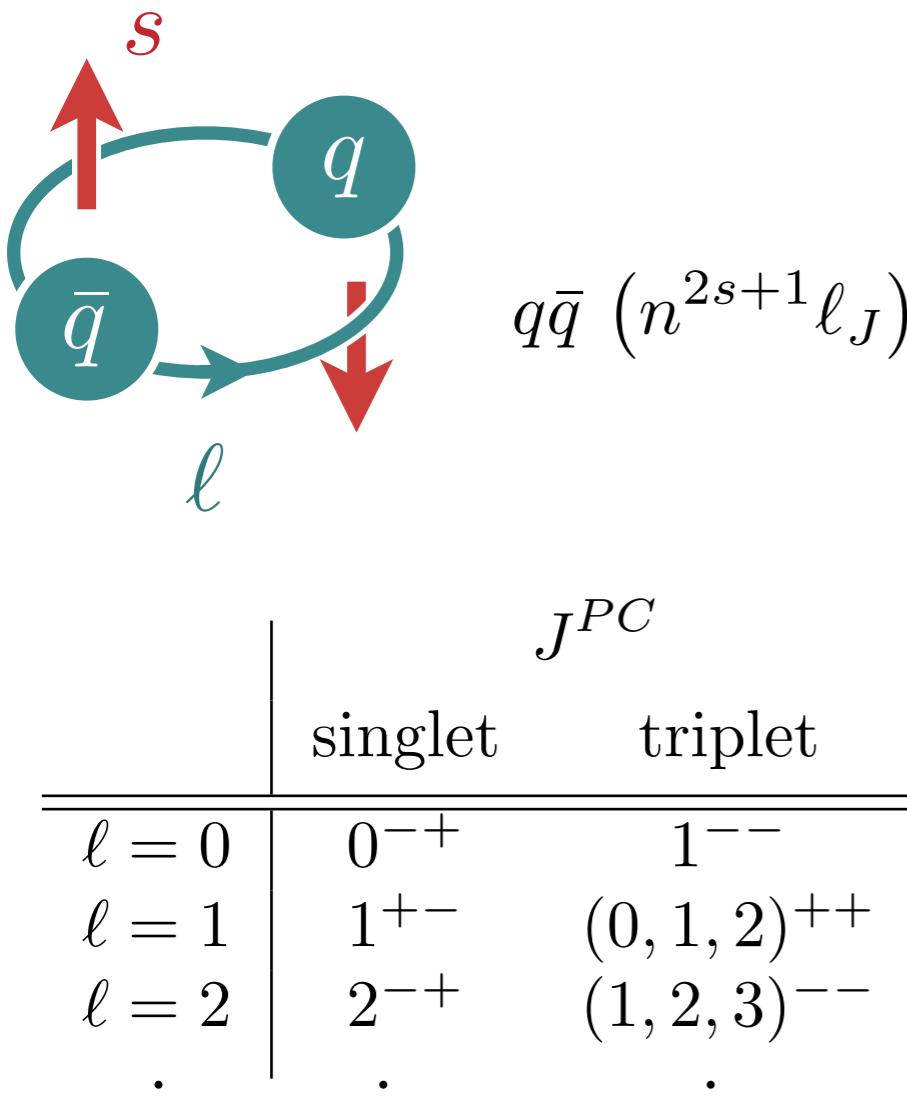
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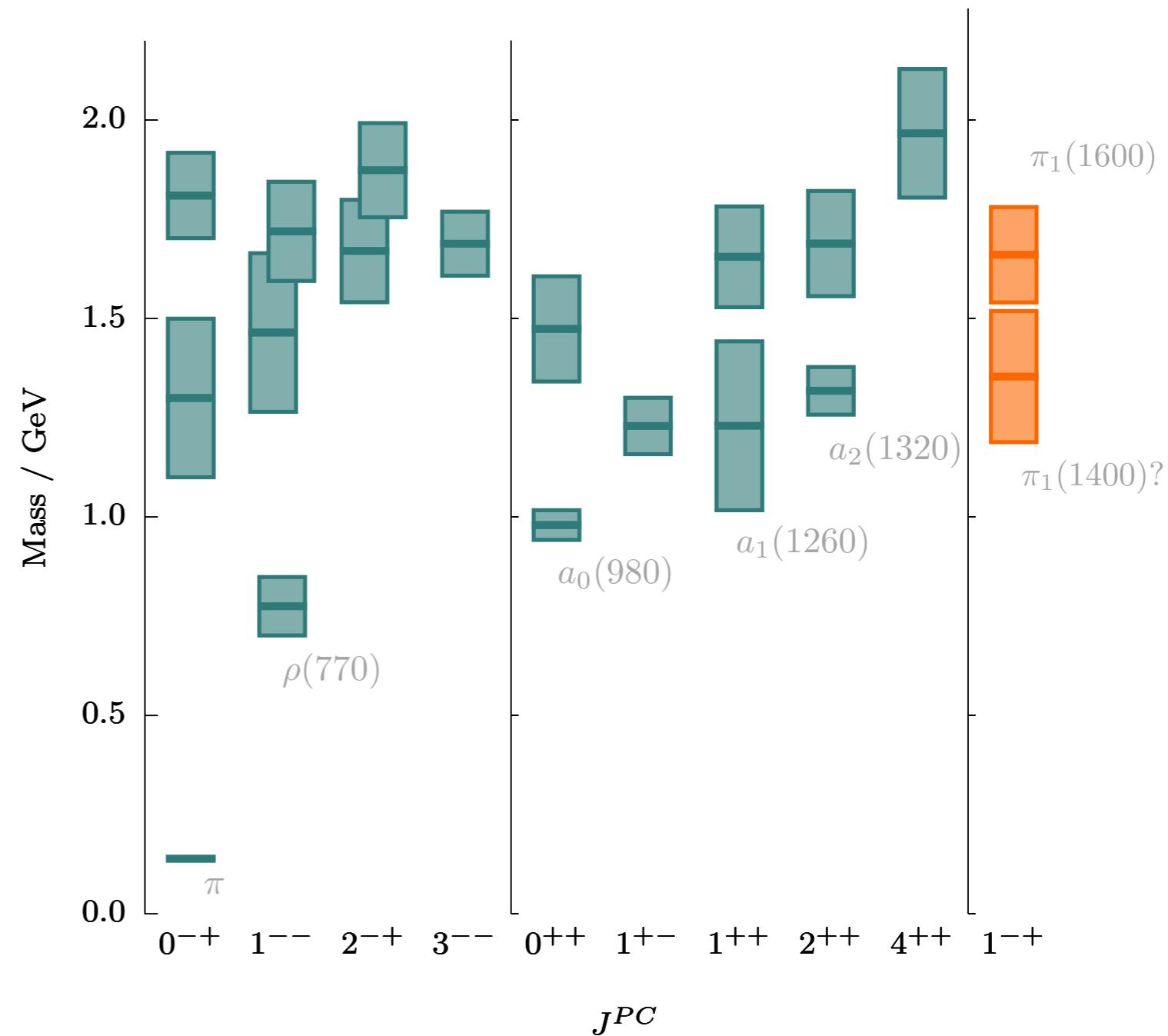
The Hadron Spectrum

Quark models give gross structure of the hadron spectrum

e.g. light isovector mesons



Forbidden quantum numbers: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \dots$

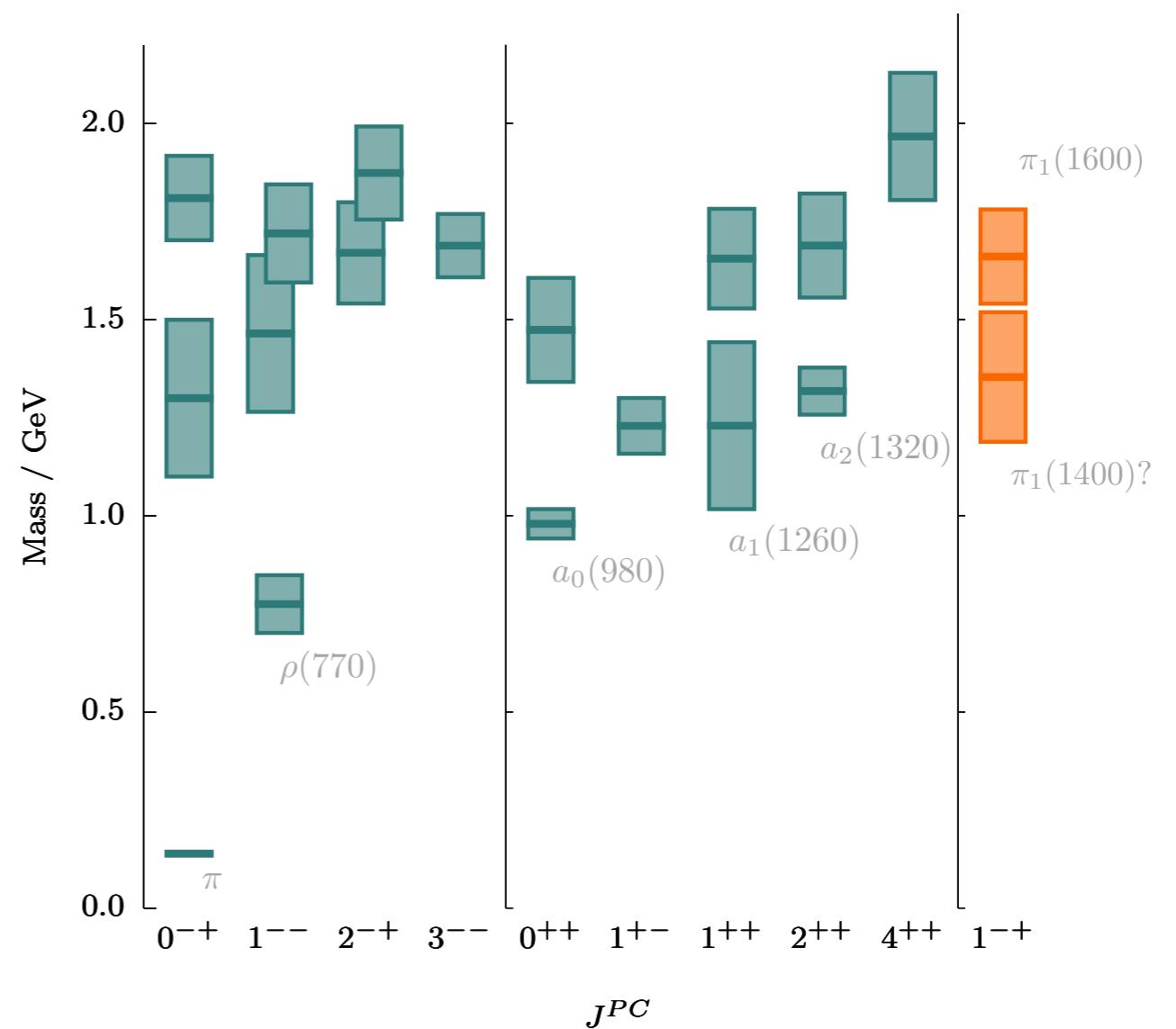


The Hadron Spectrum

How to connect QCD to the hadron spectrum?

- Need to understand how to quantify what the hadrons are in nature
- Need to find non-perturbative approach to access these hadrons *rigorously*

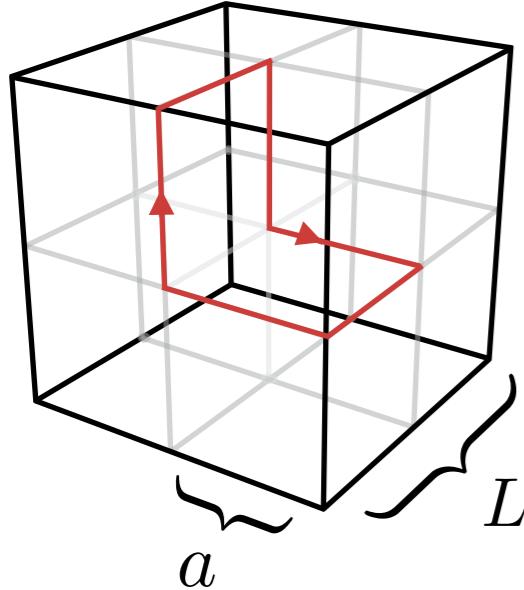
$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (i \not{D} - m_f) \psi_f - \frac{1}{2} \text{tr} (\mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu})$$



Few-Body Physics from QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



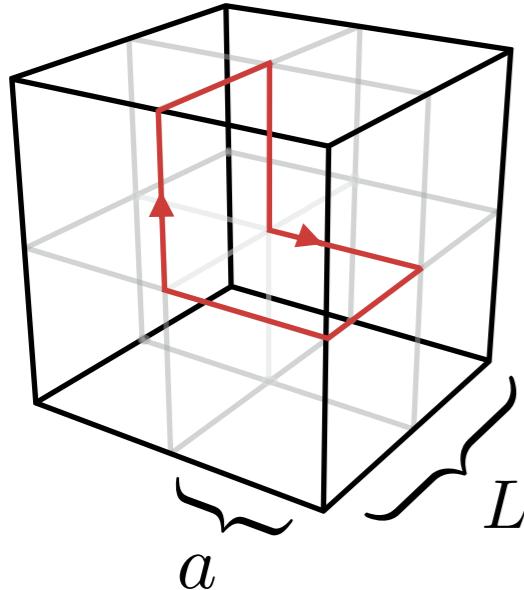
$$Z_{\text{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu e^{iS_{\text{QCD}}(\psi, \bar{\psi}, A_\mu)}$$

- *Euclidean spacetime, $t \rightarrow -i\tau$*
- *Finite volume, L*
- *Discrete spacetime, a*
- *Heavier than physical quark mass, $m > m_{\text{phys.}}$*

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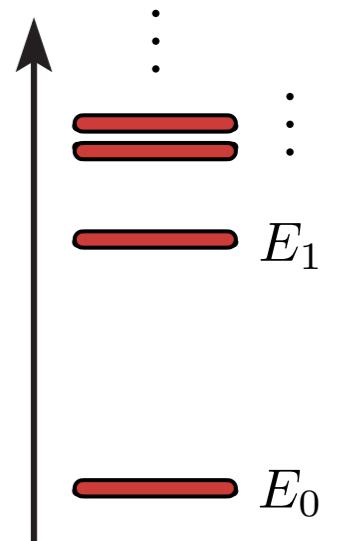


$$Z_{\text{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu e^{iS_{\text{QCD}}(\psi, \bar{\psi}, A_\mu)}$$

- Euclidean spacetime, $t \rightarrow -i\tau$
- Finite volume, L
- Discrete spacetime, a
- Heavier than physical quark mass, m

Correlation functions yield discrete spectrum

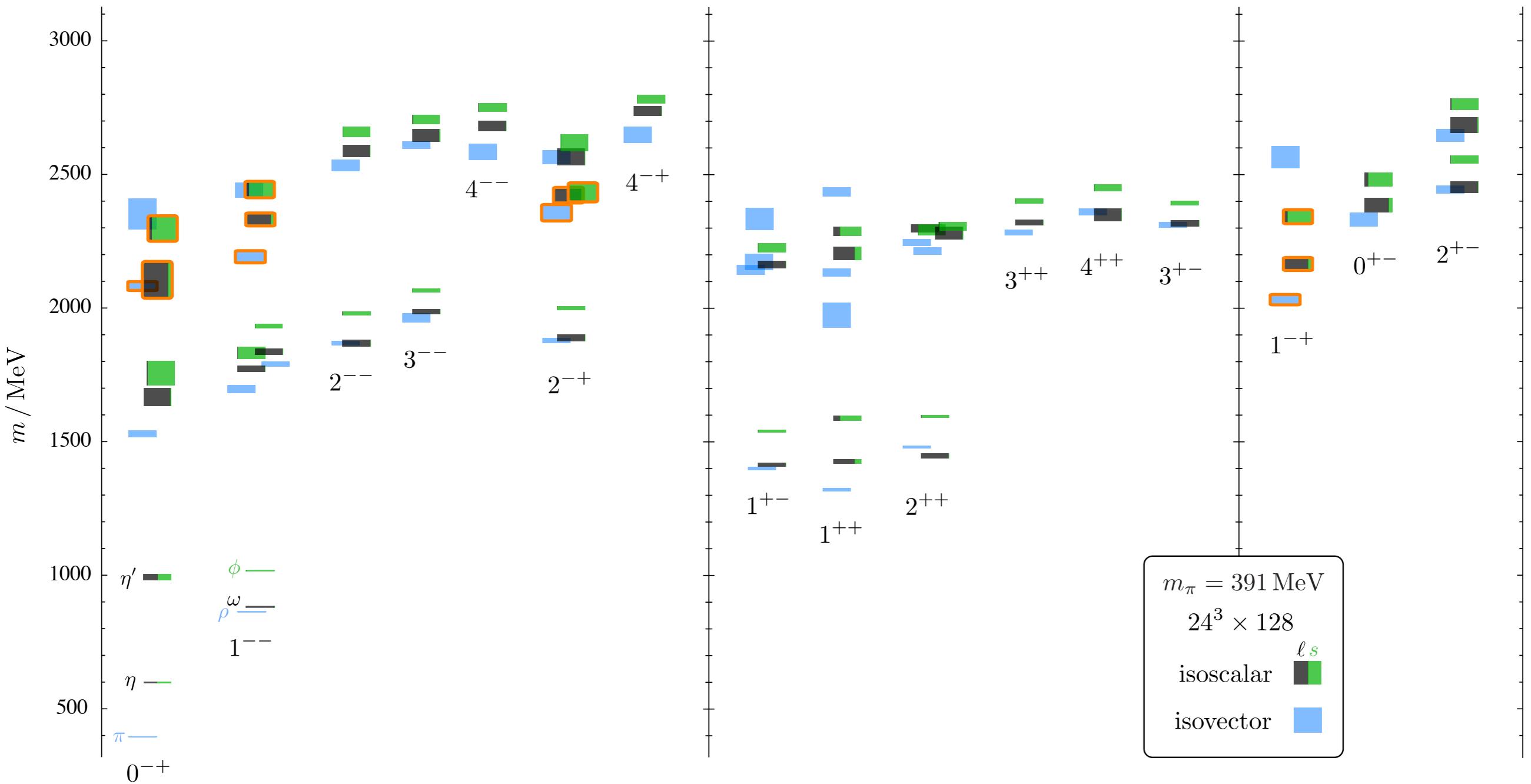
$$\langle \mathcal{O}(\tau) \mathcal{O}^\dagger(0) \rangle = \sum_{\mathfrak{n}} |\langle 0 | \mathcal{O} | \mathfrak{n} \rangle|^2 e^{-E_{\mathfrak{n}} \tau}$$



Few-Body Physics from QCD

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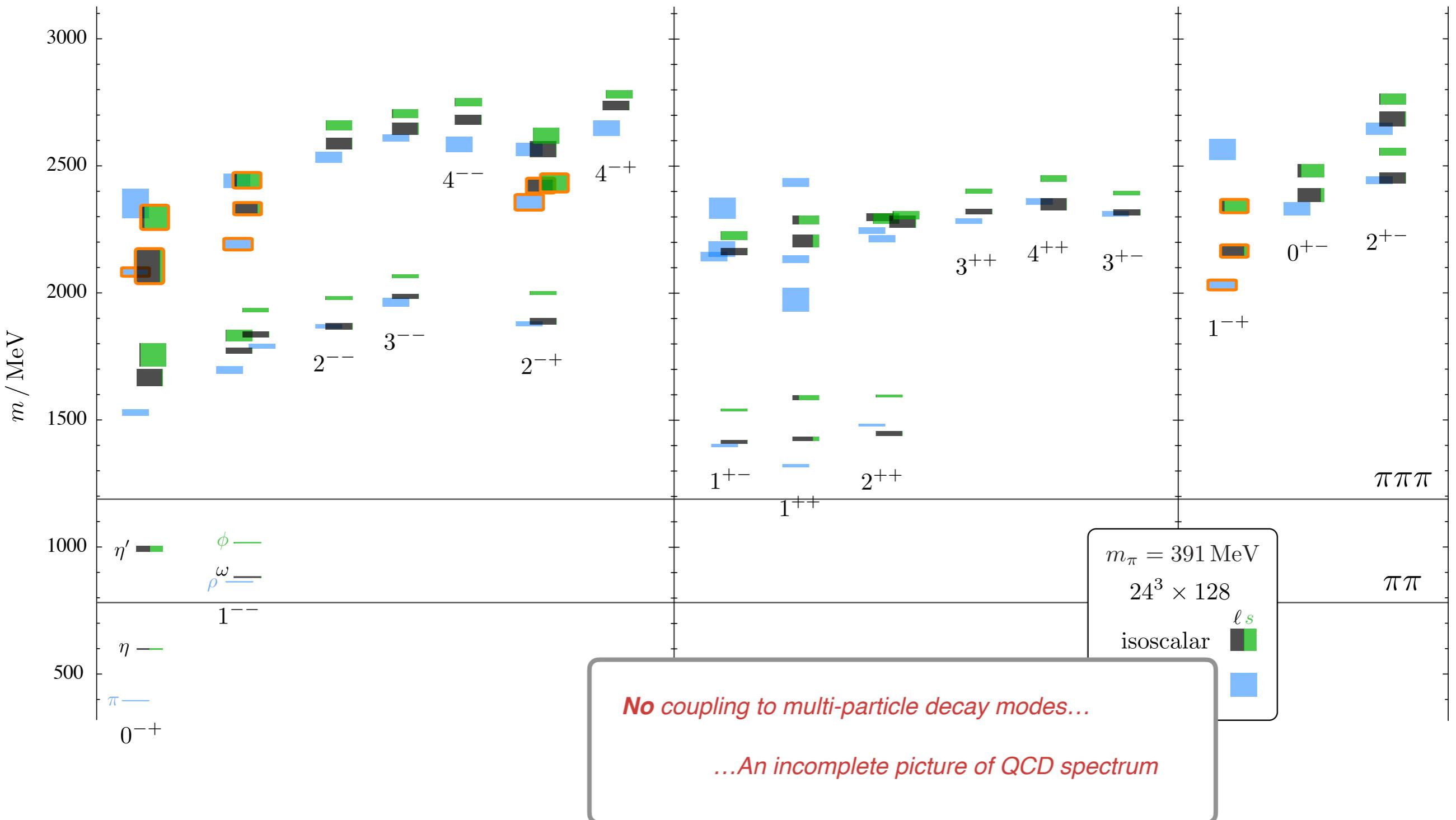
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Few-Body Physics from QCD

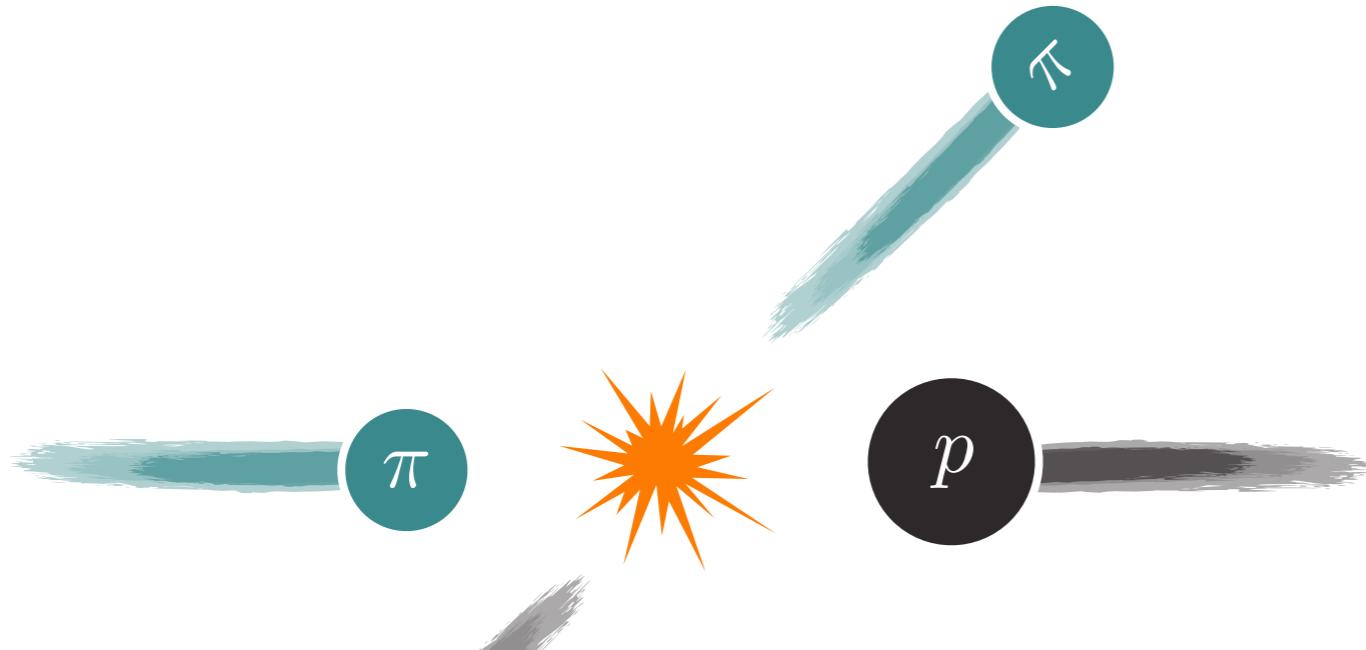
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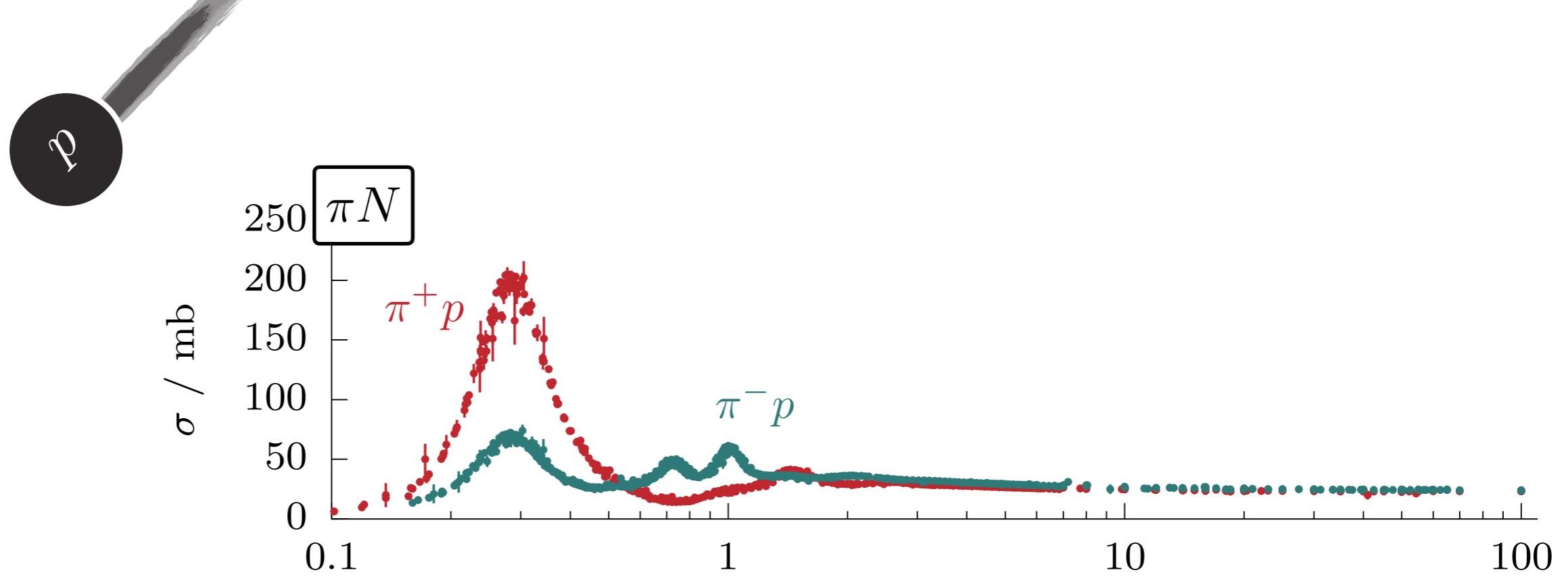


Few-Body Physics from QCD

All our knowledge of the hadrons comes from *scattering experiments*



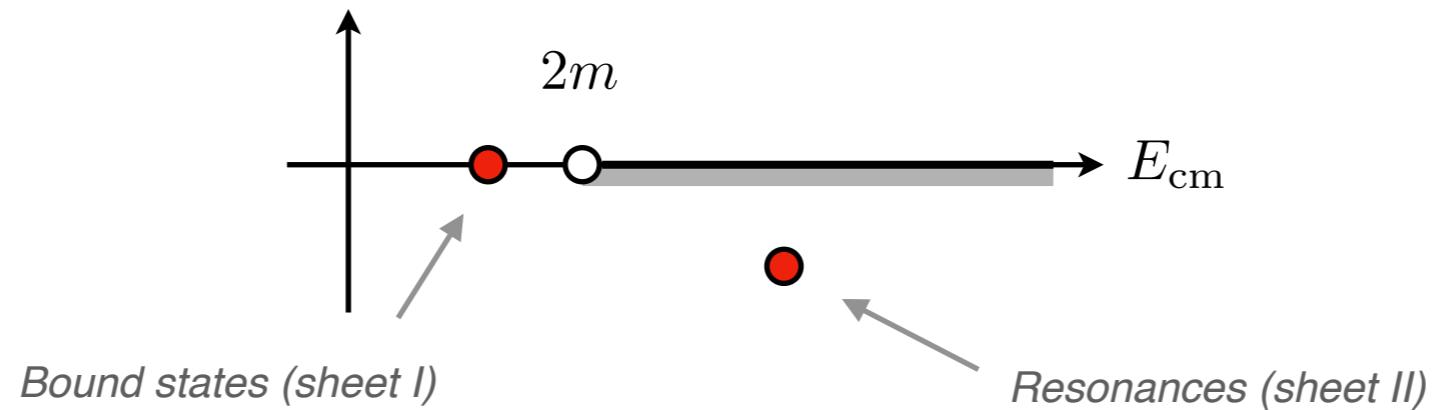
How to quantify such a process
with Lattice QCD?



Scattering Theory & QCD Spectrum

Resonances & Bound states are pole singularities of scattering amplitudes

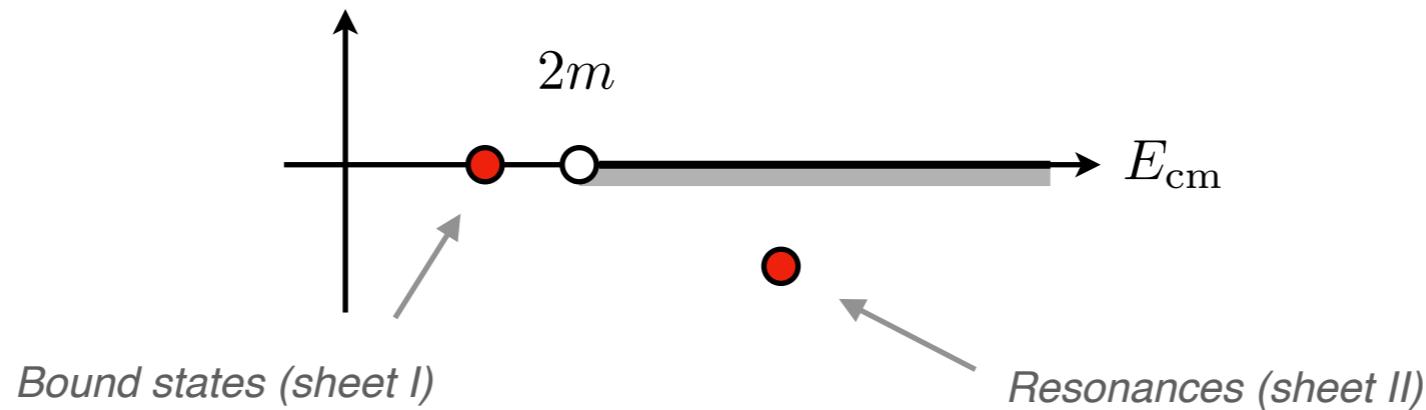
$$\mathcal{M}_2 = \mathcal{K}_2 \frac{1}{1 - i\rho \mathcal{K}_2}$$



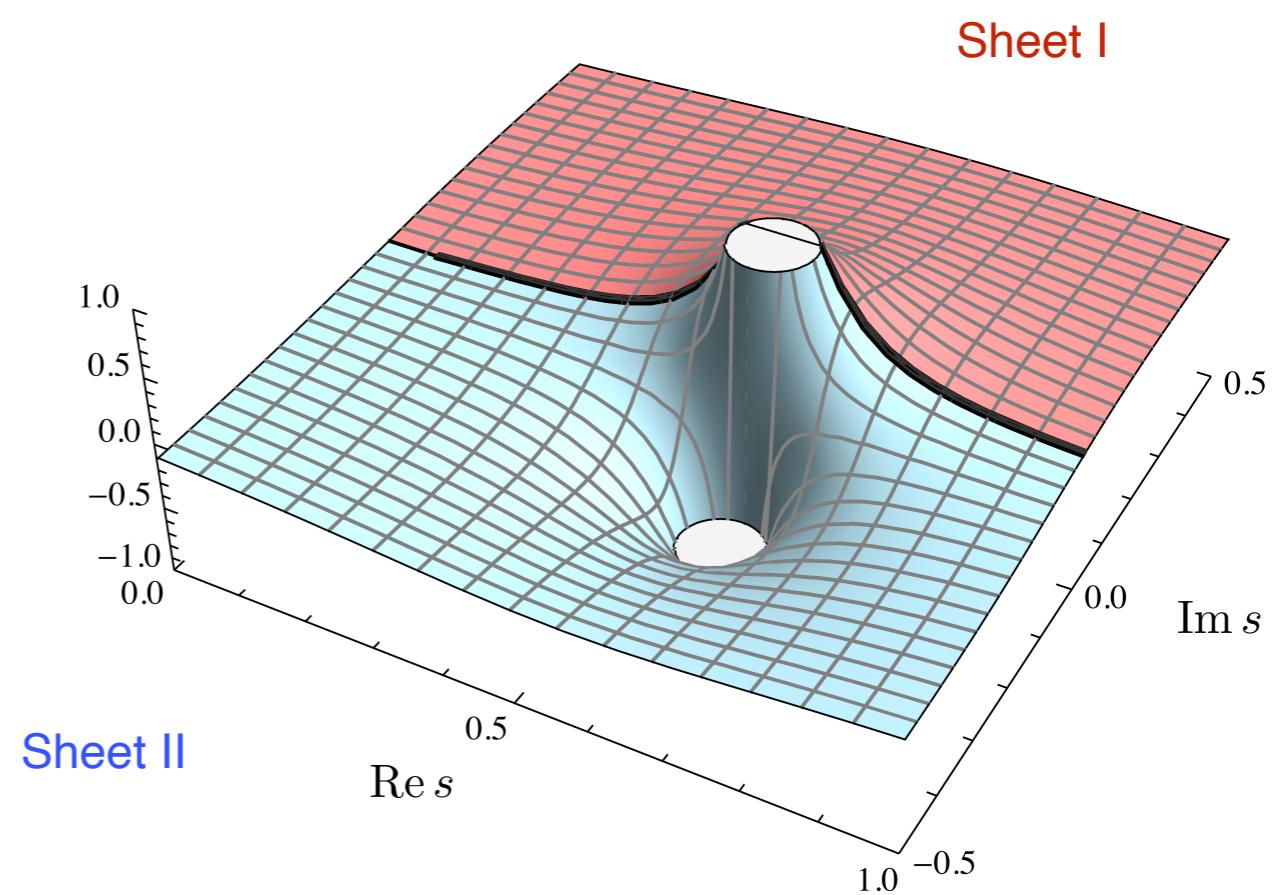
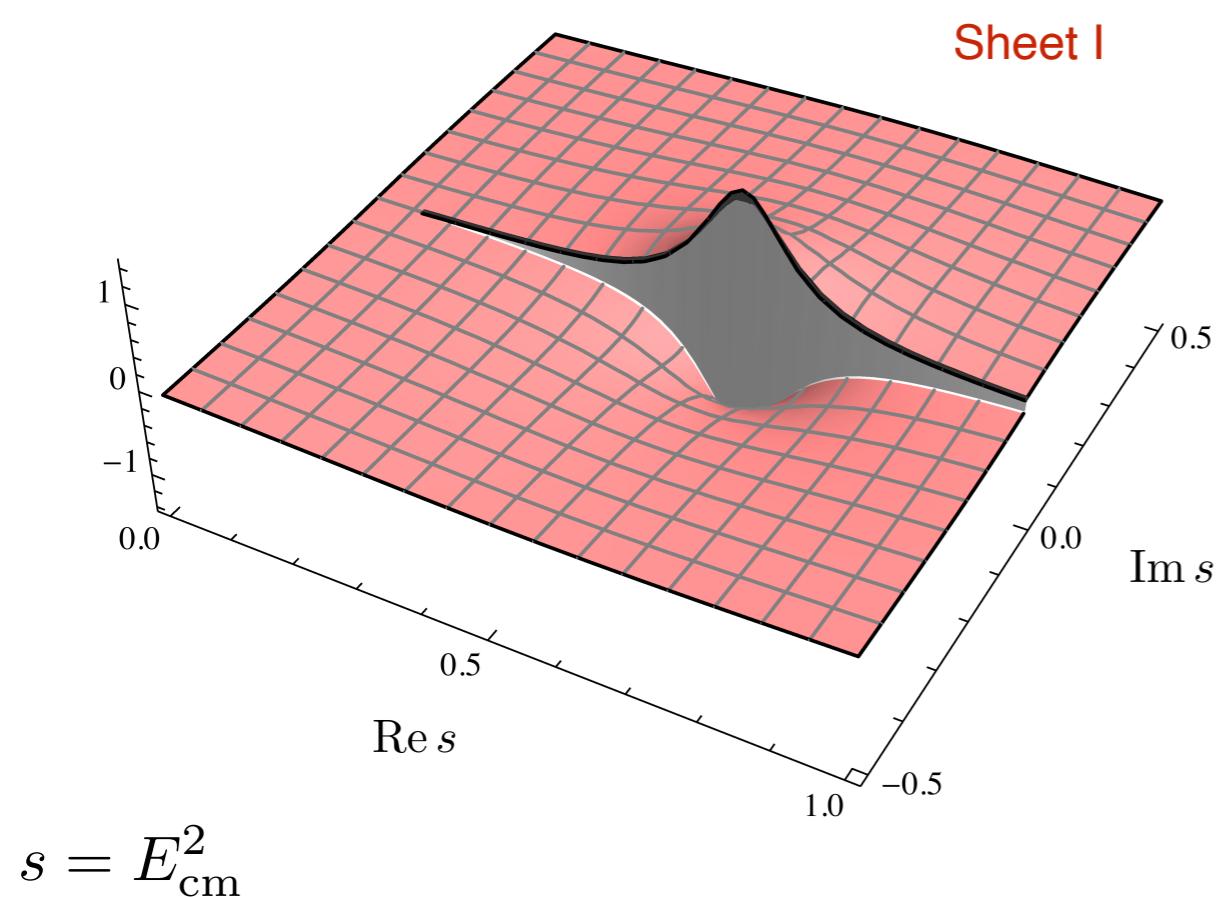
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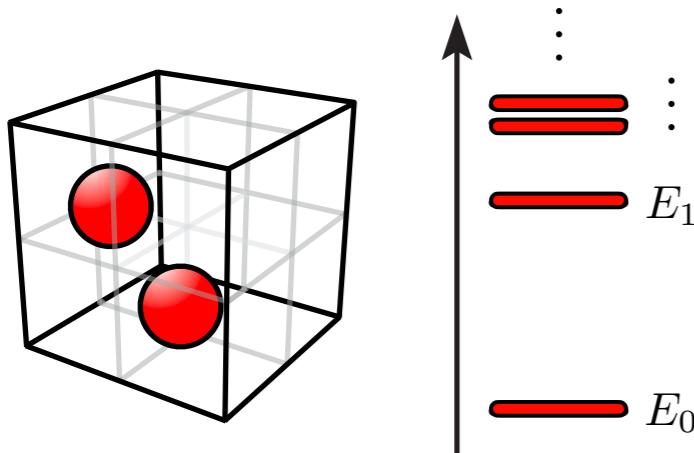


e.g., Narrow resonance



Connecting Scattering Physics to QCD

Employing scattering theory and EFTs to all-orders
connects lattice QCD spectra to scattering observables

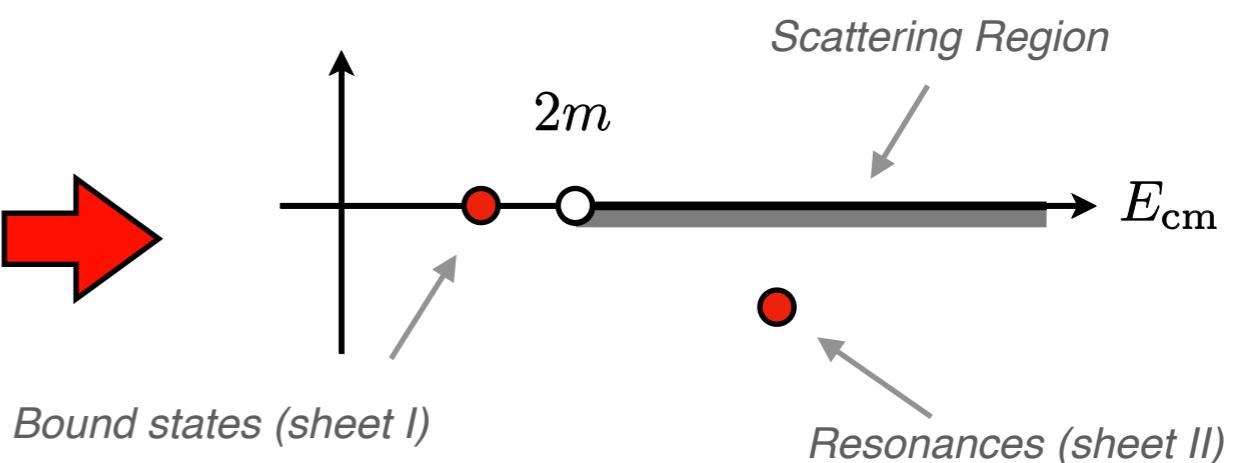


$$\det(1 + \mathcal{K}_2 F_L)_{E=E_n} = 0$$

$$\mathcal{M}_2 = \mathcal{K}_2 \frac{1}{1 - i\rho \mathcal{K}_2}$$

M. Lüscher
Commun.Math.Phys. **105**, 153 (1986)
Nucl.Phys. **B354**, 531 (1991)

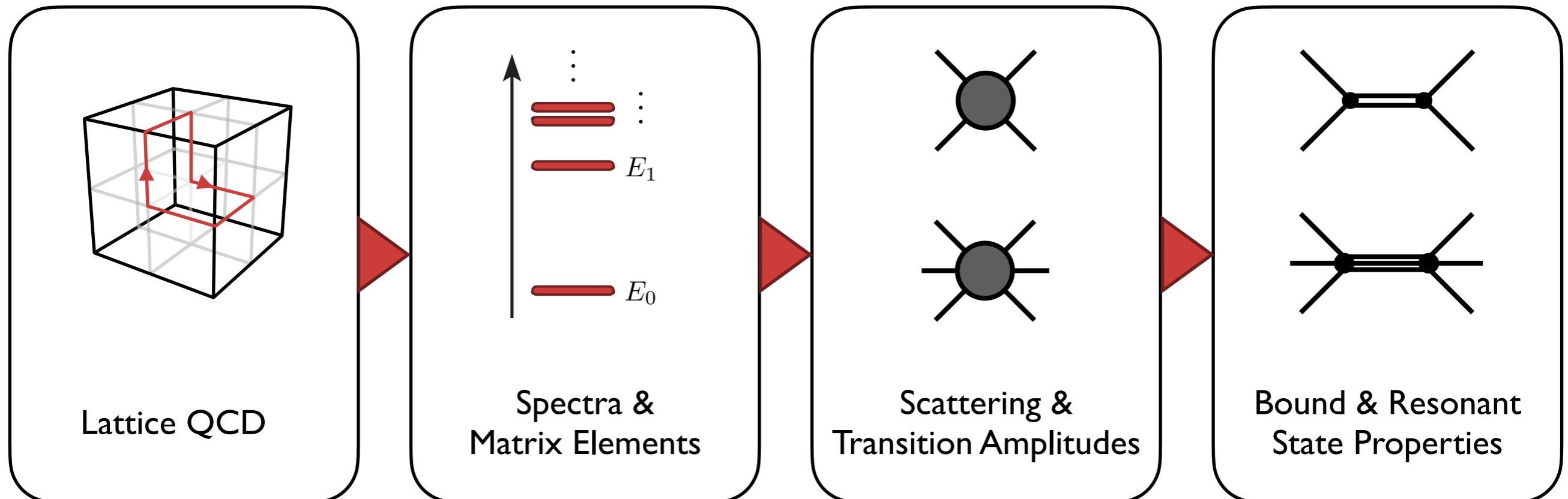
Many others...



Few-Body Physics from QCD

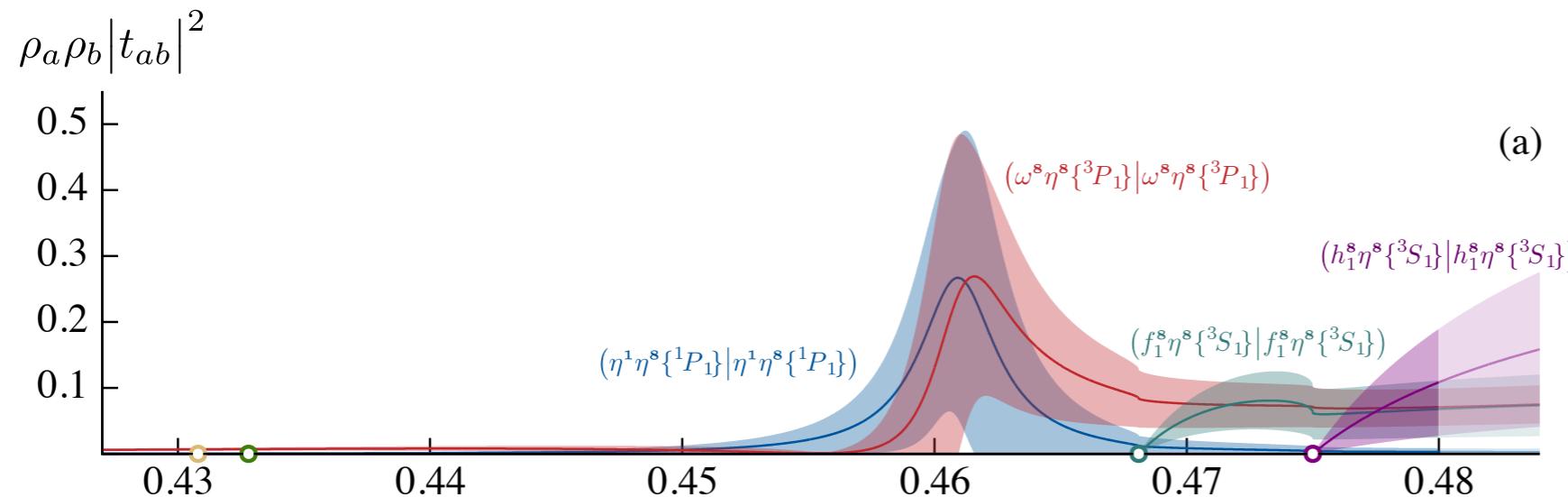
Path to few-body physics from QCD

- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD, Scattering Theory, & Effective Field Theory*



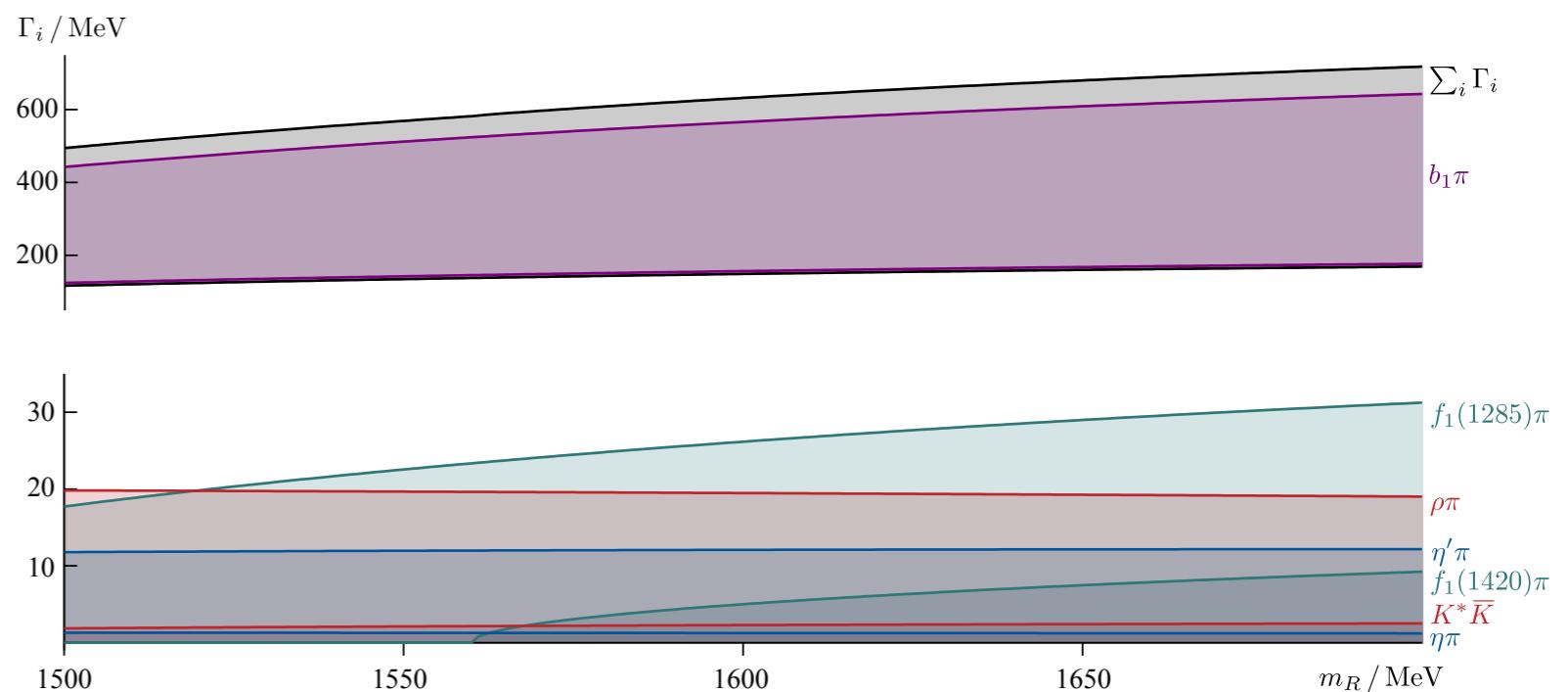
Connecting Scattering Physics to QCD

First determination of hybrid candidate, $J^{PC} = 1^{-+}$, $m_\pi \sim 700$ MeV



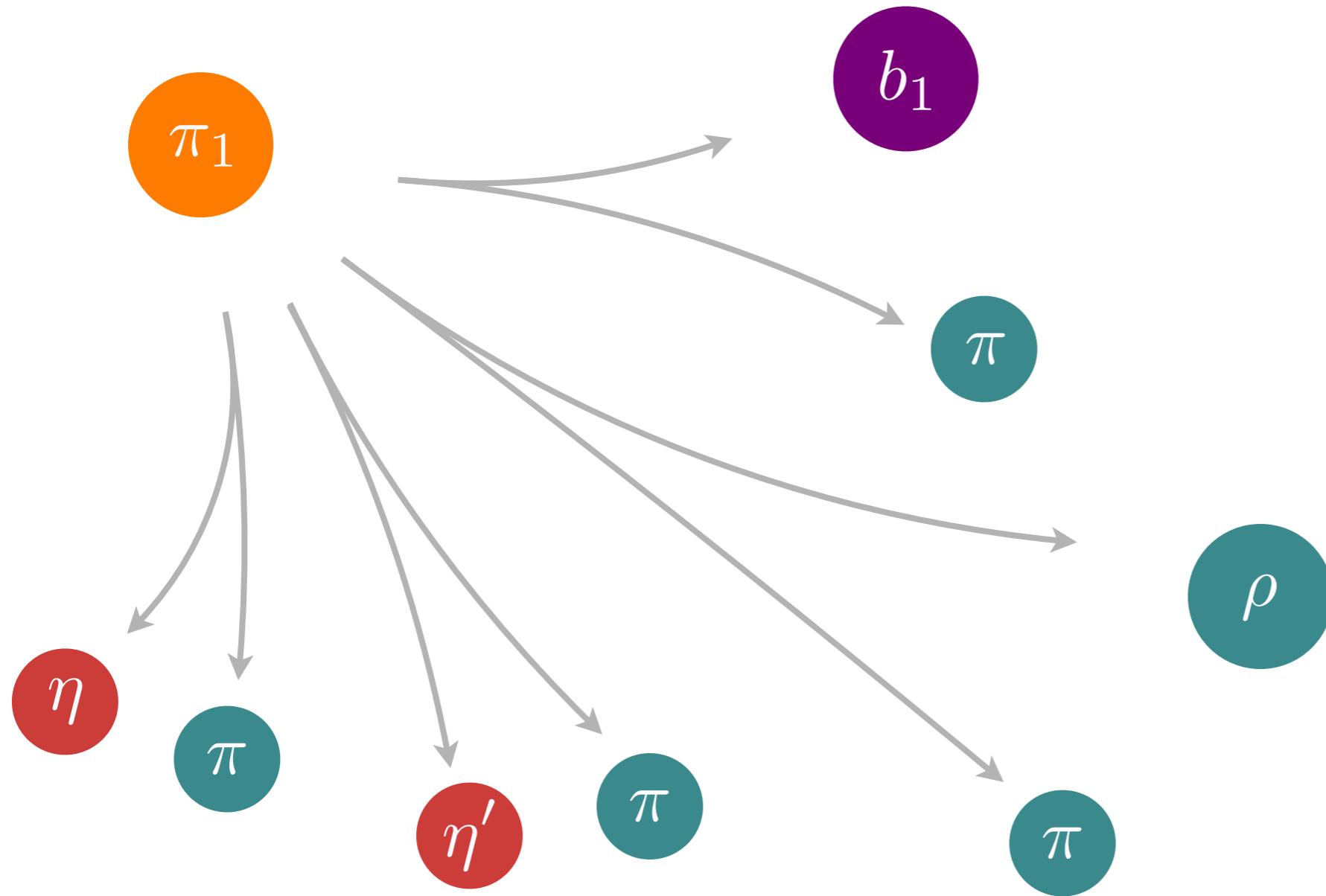
A.J. Woss et al. [HadSpec]
Phys. Rev. D103, 054502 (2021)

had spec



Connecting Scattering Physics to QCD

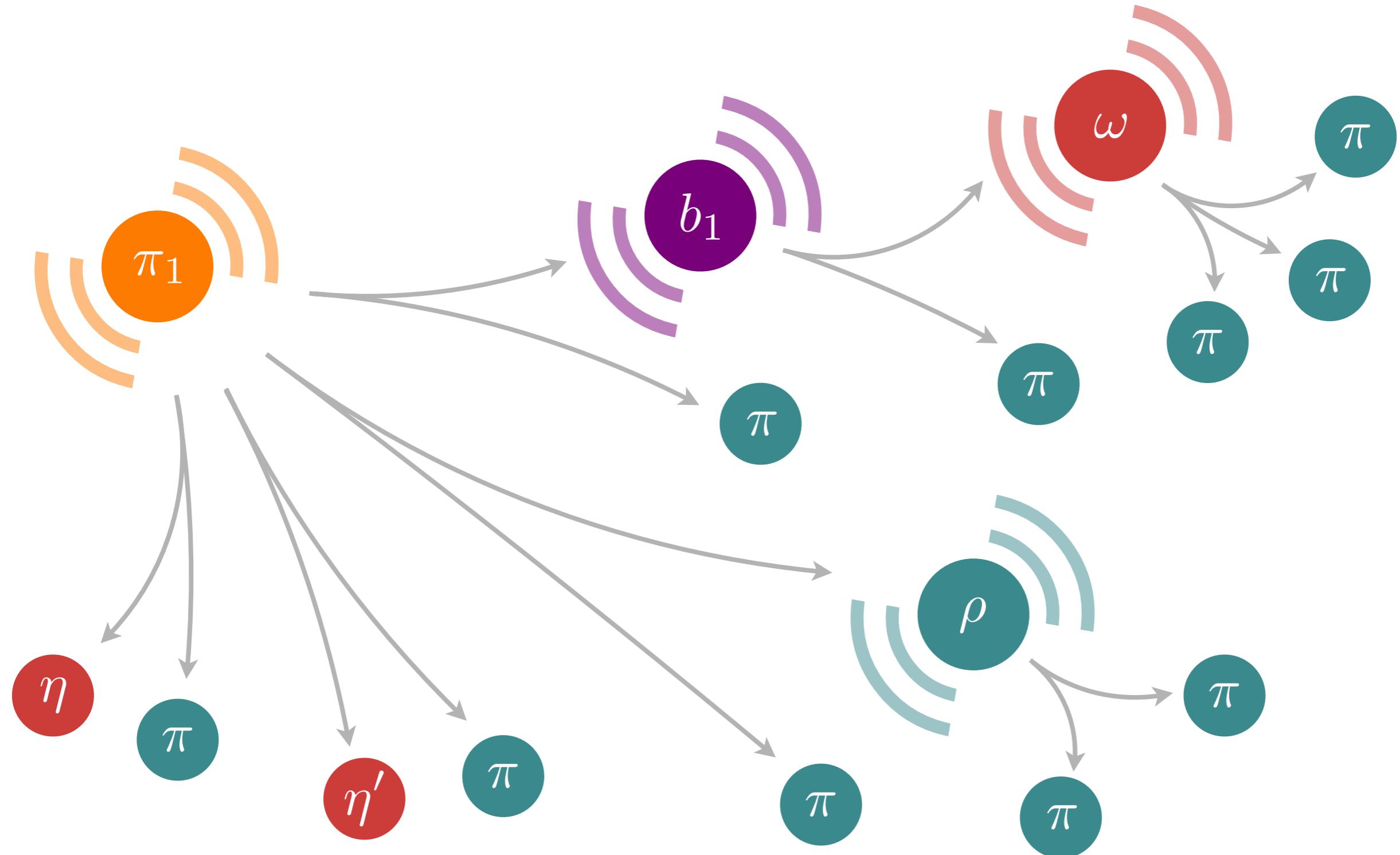
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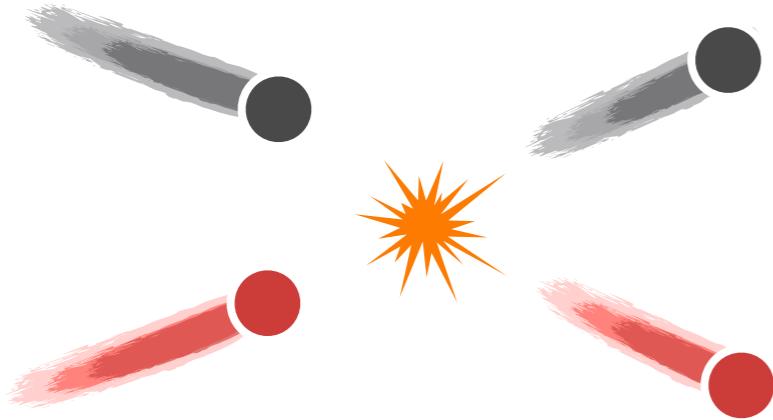
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At physical point, 3, 4,.. body decays



Challenges for three-body dynamics

Increased number of kinematic variables



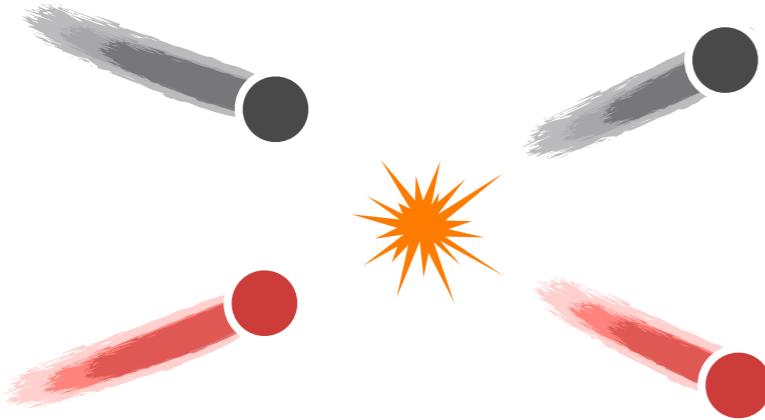
12 momentum components

- 10 Poincaré symmetries
- = **2 degrees of freedom**

$$(E, \theta) \leftrightarrow (s, t)$$

Challenges for three-body dynamics

Increased number of kinematic variables



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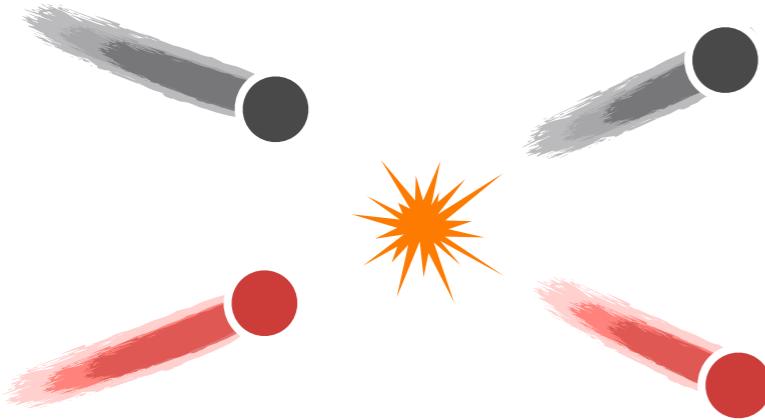
$$s = E^2$$



$$q = \frac{1}{2} \sqrt{s - 4m^2}$$

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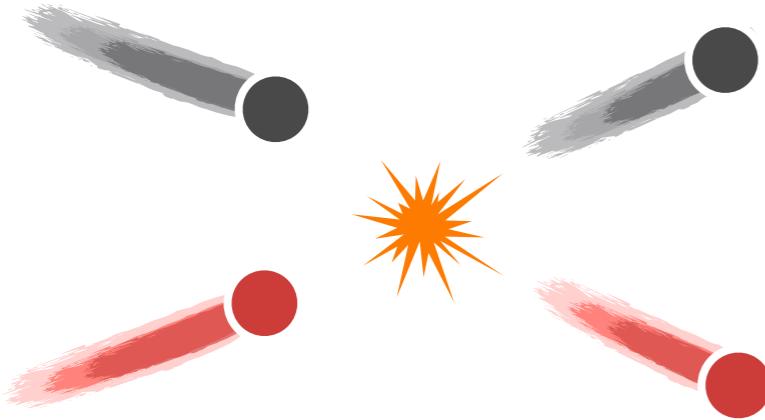
$$s = E^2$$

A diagram illustrating the relationship between the variables s and θ . It features two horizontal teal arrows pointing in opposite directions from a central point. A red arrow points upwards and to the right from the same central point, labeled with the Greek letter θ .

$$q = \frac{1}{2} \sqrt{s - 4m^2}$$

Challenges for three-body dynamics

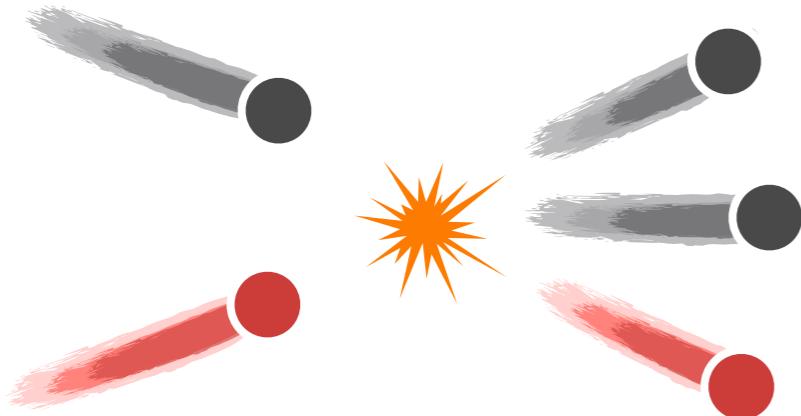
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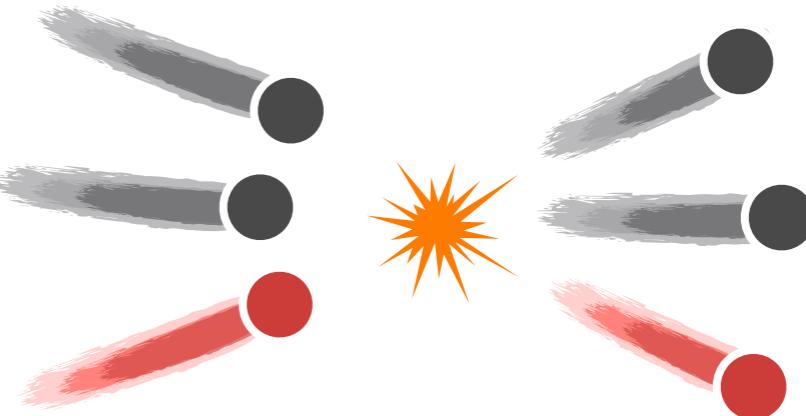
- 10 Poincaré symmetries
- = **2 degrees of freedom**

$$(E, \theta) \leftrightarrow (s, t)$$



15 momentum components

- 10 Poincaré symmetries
- = **5 degrees of freedom**



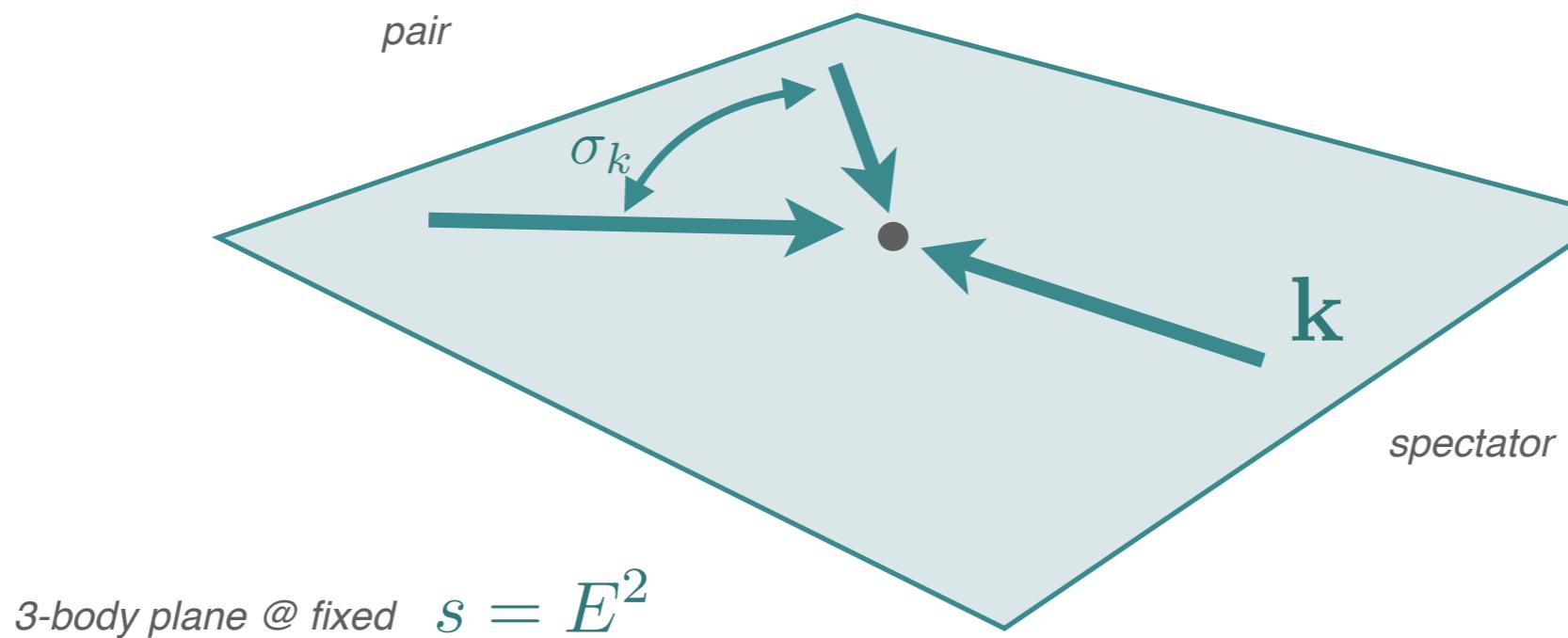
18 momentum components

- 10 Poincaré symmetries
- = **8 degrees of freedom**

Challenges for three-body dynamics

Increased number of kinematic variables

- Convenient to organize into ***pair*** and ***spectators***



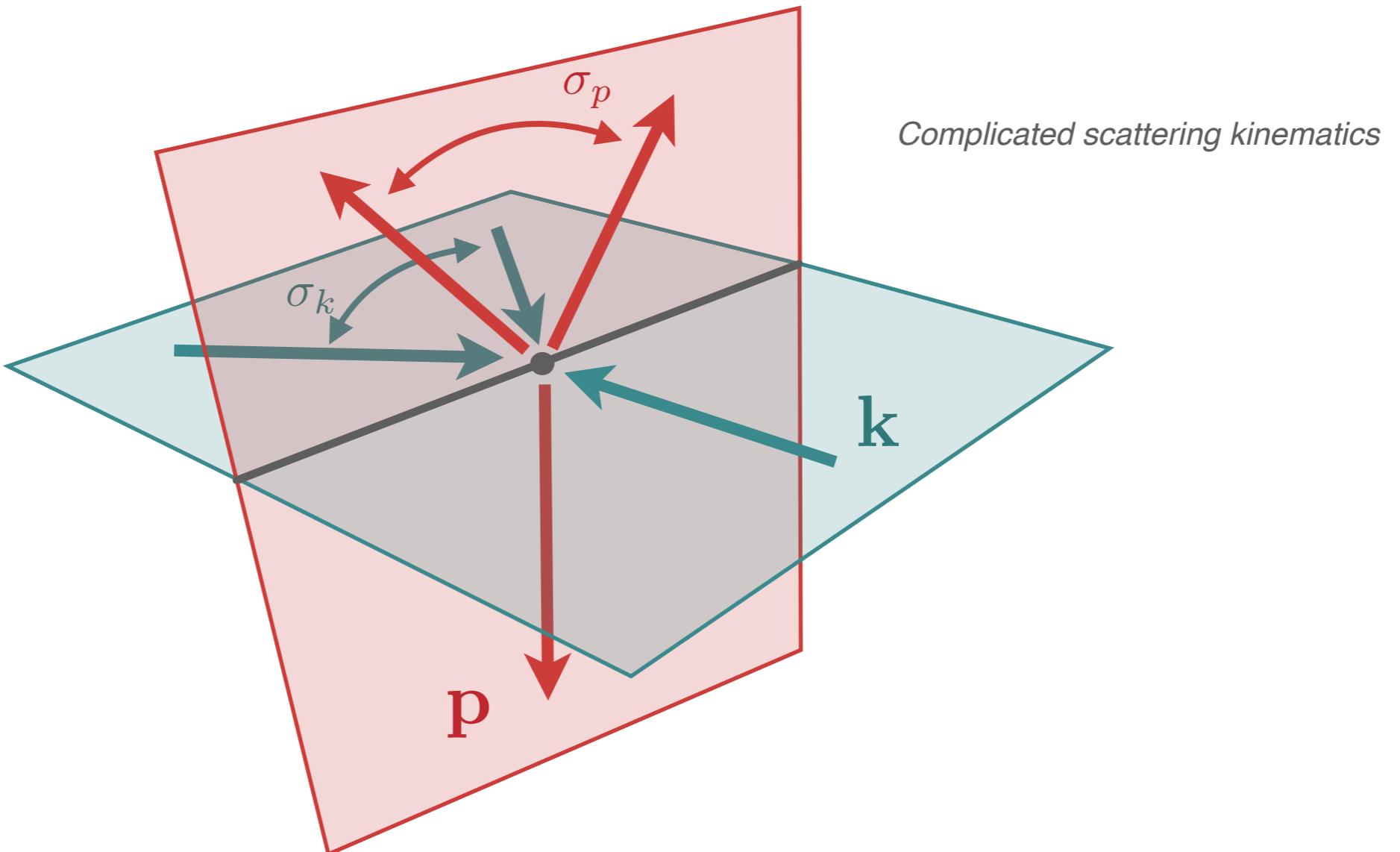
$$3m \leq \sqrt{s} < \infty$$

$$2m \leq \sqrt{\sigma_k} < \sqrt{s} - m$$

Challenges for three-body dynamics

Increased number of kinematic variables

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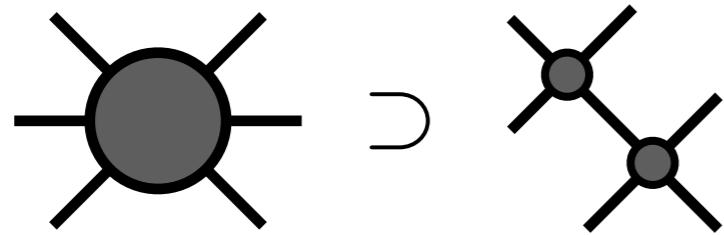
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Challenges for three-body dynamics

Rescattering effects

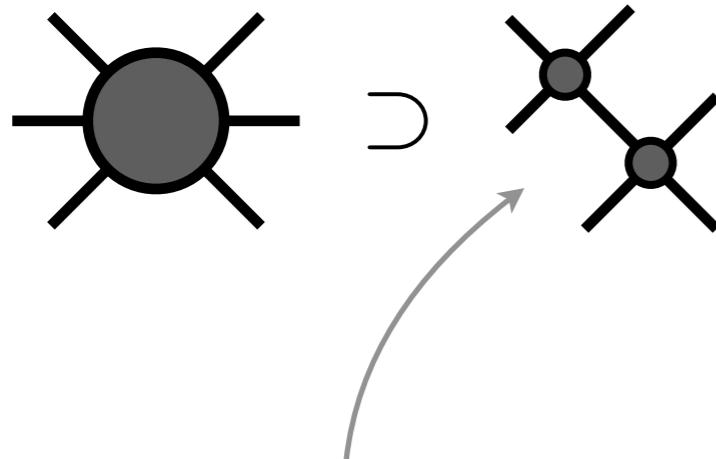
- On-shell exchanges induce kinematic singularities



Challenges for three-body dynamics

Rescattering effects

- On-shell exchanges induce kinematic singularities



Intermediate particle can go on mass-shell

$$G^J \sim \mathcal{K}_G^J + \frac{1}{2pk} Q_J(z)$$

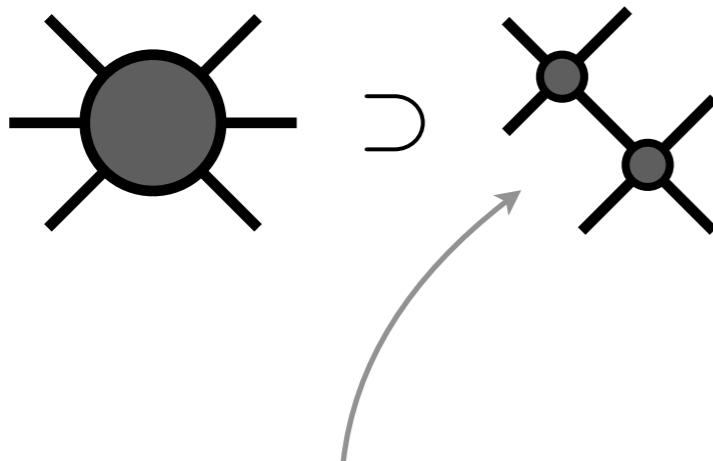
Short-distance contribution

$$\text{Logarithmic singularity, } Q_0(z) = \frac{1}{2} \log \left(\frac{z+1}{z-1} \right)$$

Challenges for three-body dynamics

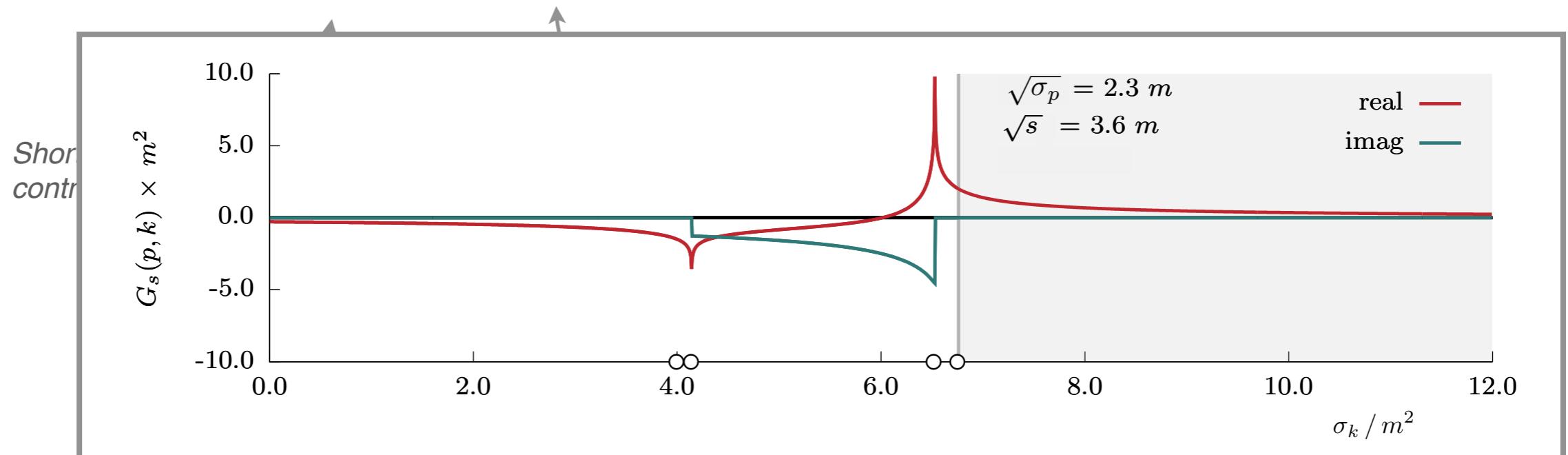
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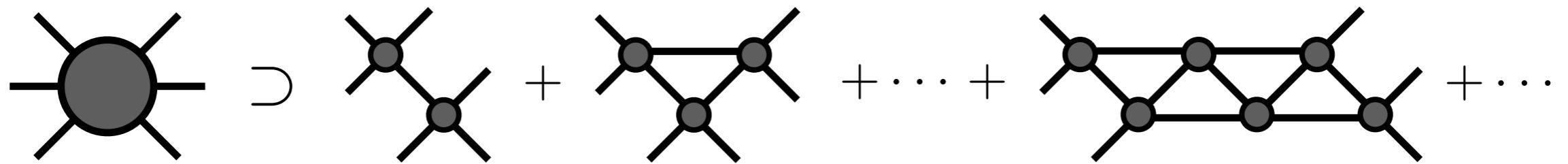
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Challenges for three-body dynamics

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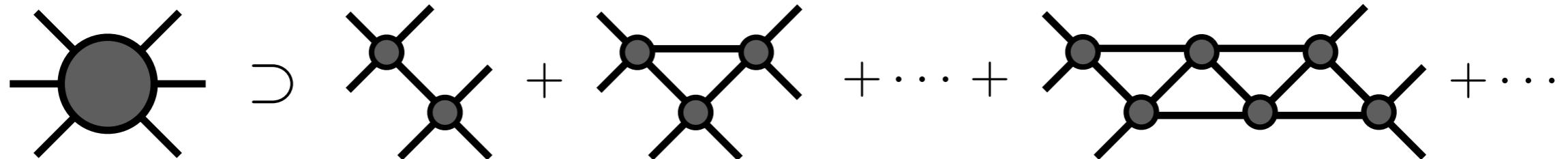
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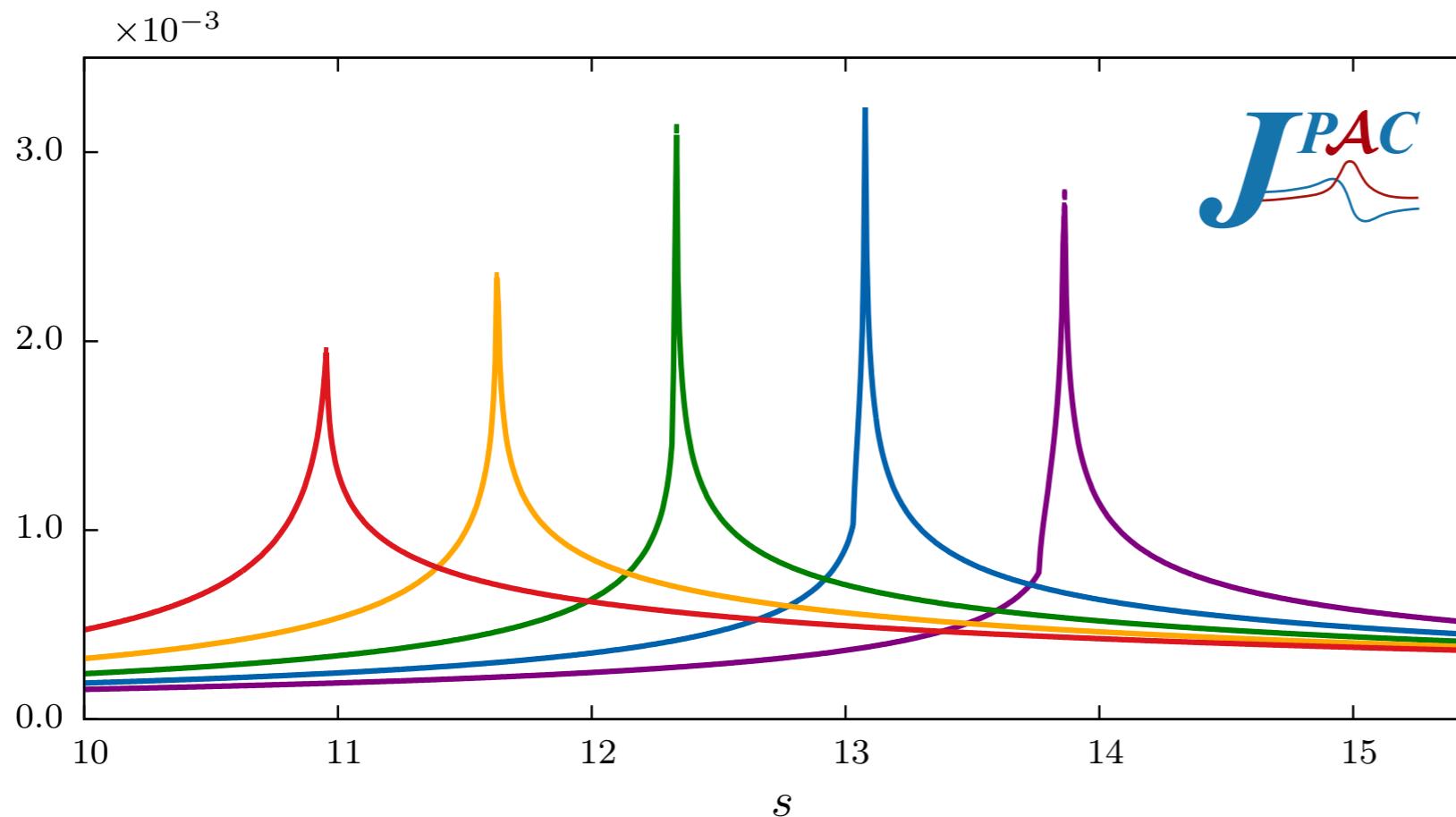
Challenges for three-body dynamics

Rescattering effects

- On-shell exchanges induce kinematic singularities
- Ultimately, requires understanding to not confuse with dynamical enhancements



e.g. Triangle singularities

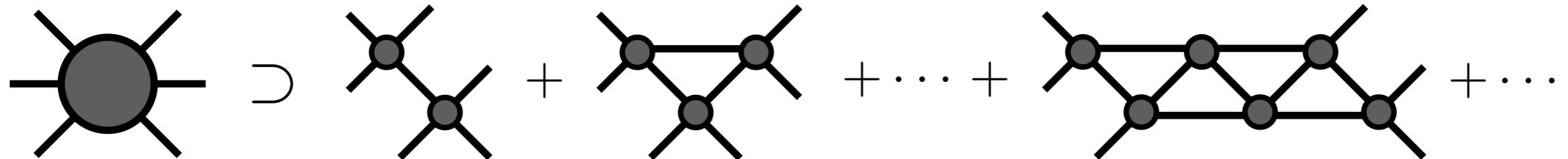


AJ et al. [JPAC]
Eur. Phys. J. C 79, no. 1, 56 (2019)

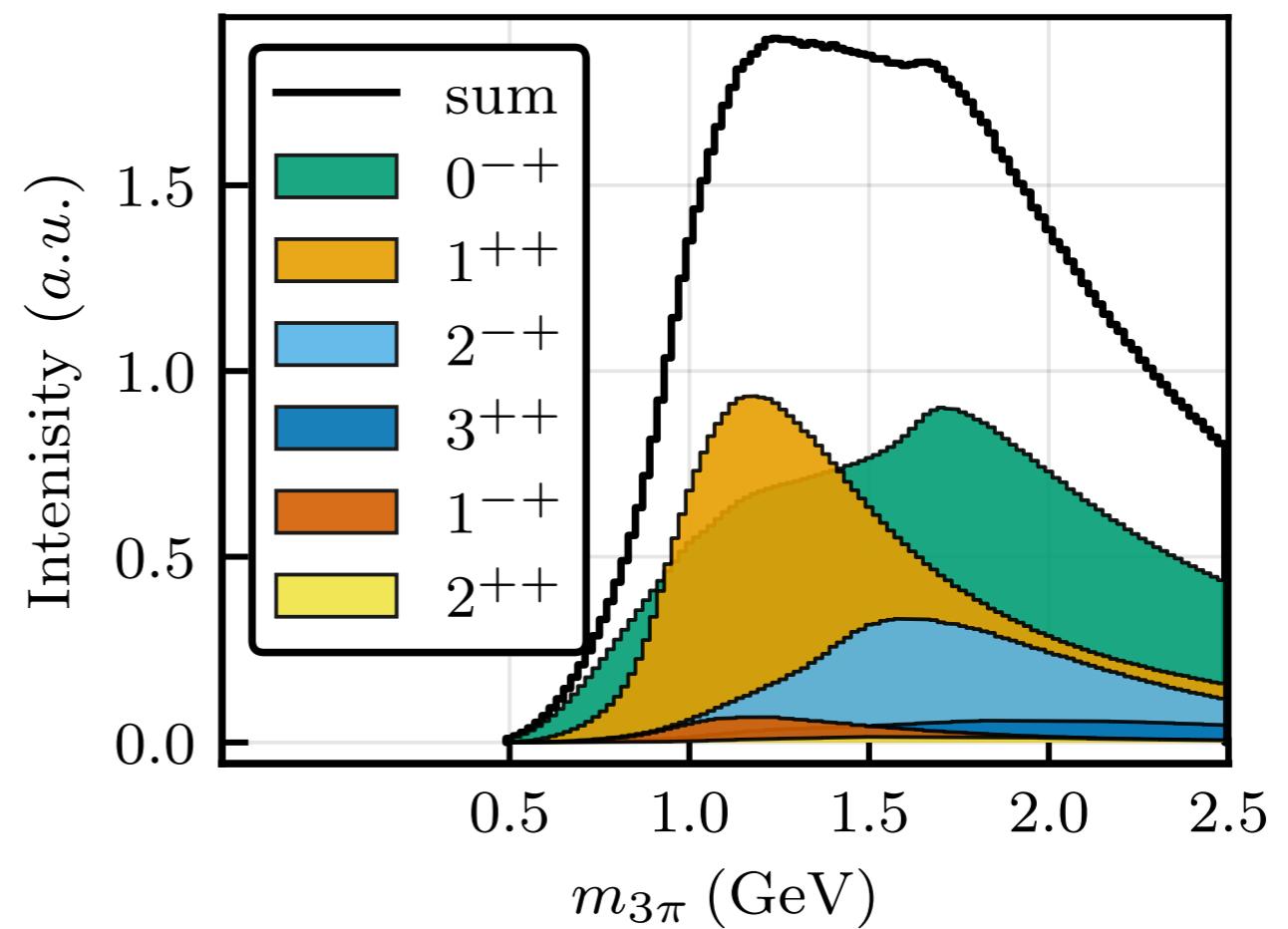
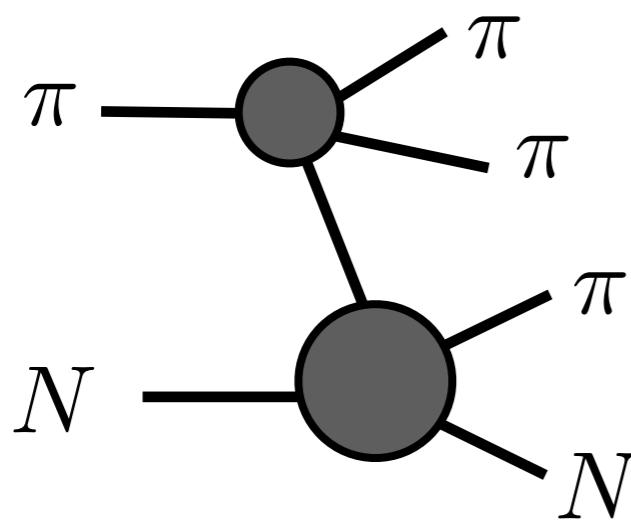
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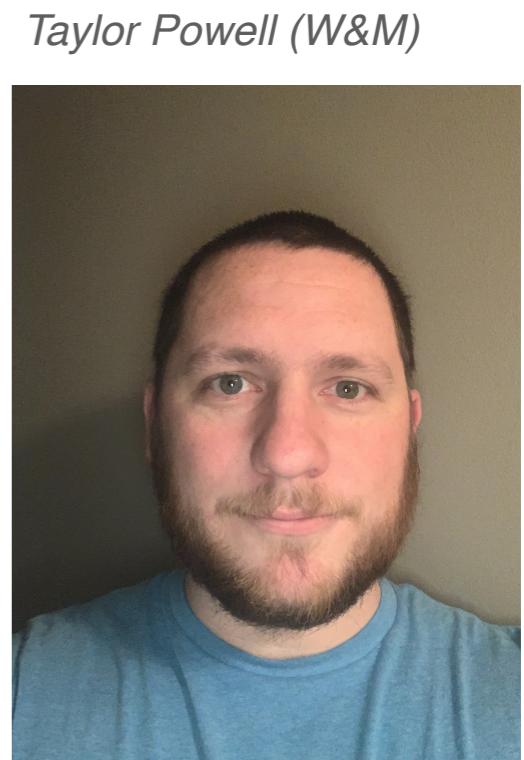
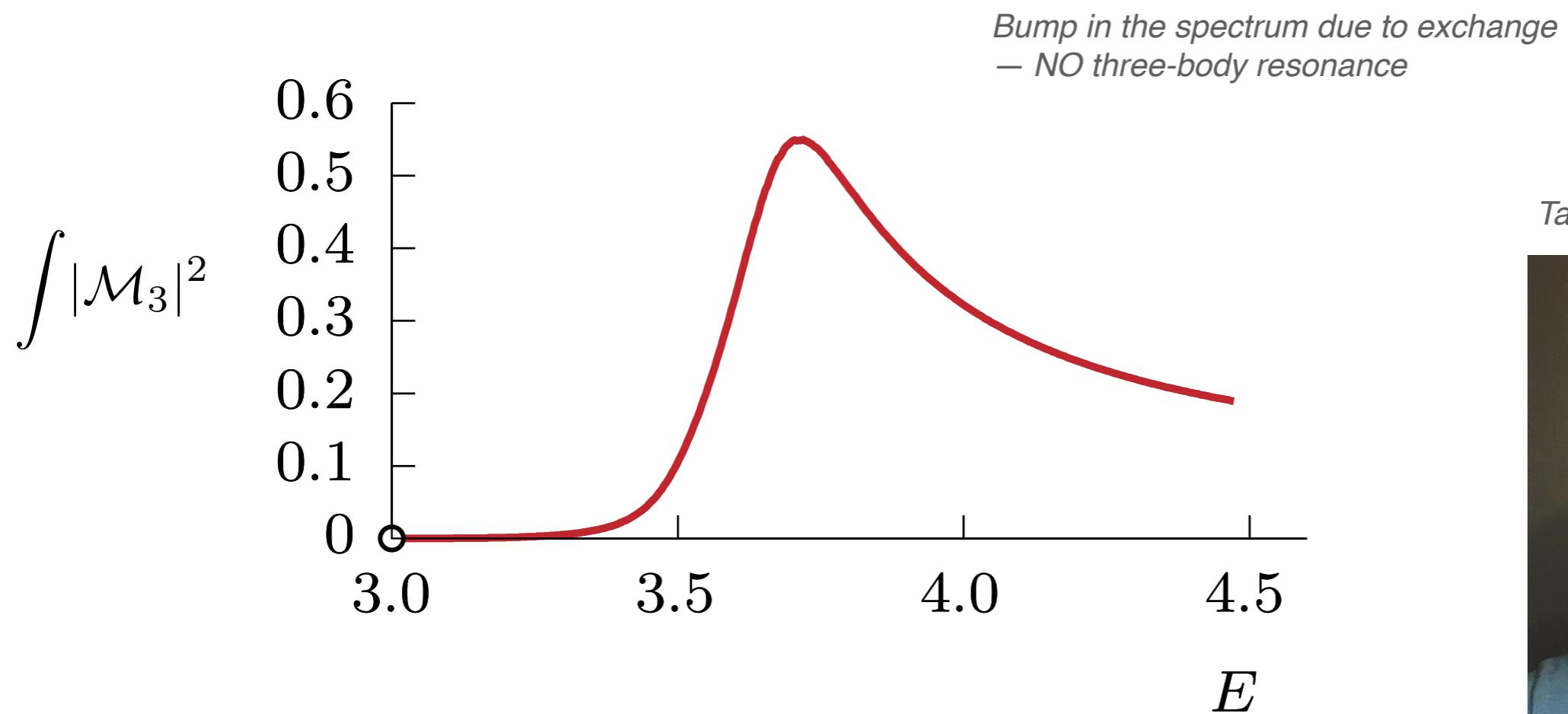
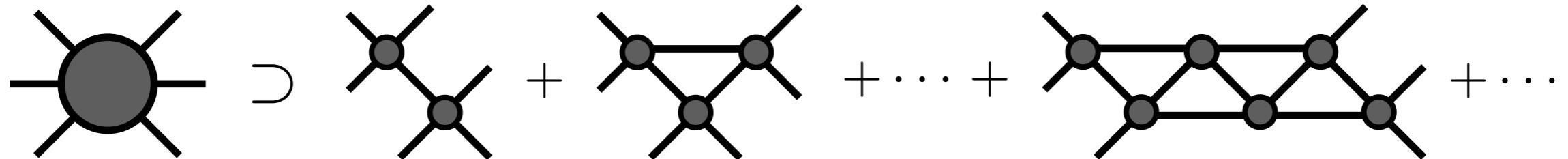
e.g., Deck Effect



Challenges for three-body dynamics

Rescattering effects

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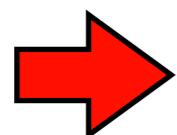
Challenges for three-body dynamics

On-shell scattering governed by integral equations

Unitarity condition

$$\text{Disc} \quad \text{Diagram} = \quad \text{Diagram} + \quad \text{Diagram}$$

$\sim \rho$ $\sim \text{Disc } \mathcal{G}$



On-shell scattering equation

$$\mathcal{M}_3 = \mathcal{K}_3 + \int \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_3 + \iint \mathcal{K}_3 \cdot \mathcal{G} \cdot \mathcal{M}_3$$

\mathcal{K}_3 Unknown!
Obtained from Lattice QCD

M. Mai, B. Hu, M. Döring, A. Pilloni, and A. Szczepaniak
Eur. Phys. J. A **53**, 177 (2017)

AJ et al. [JPAC]
Eur. Phys. J. C **79**, no. 1, 56 (2019)

AJ et al. [JPAC]
Phys. Rev. D **100**, 034508 (2019)

AJ, arxiv:2208.10587

$$\mathcal{G} = \quad \text{Diagram}$$

cf. two-body case:

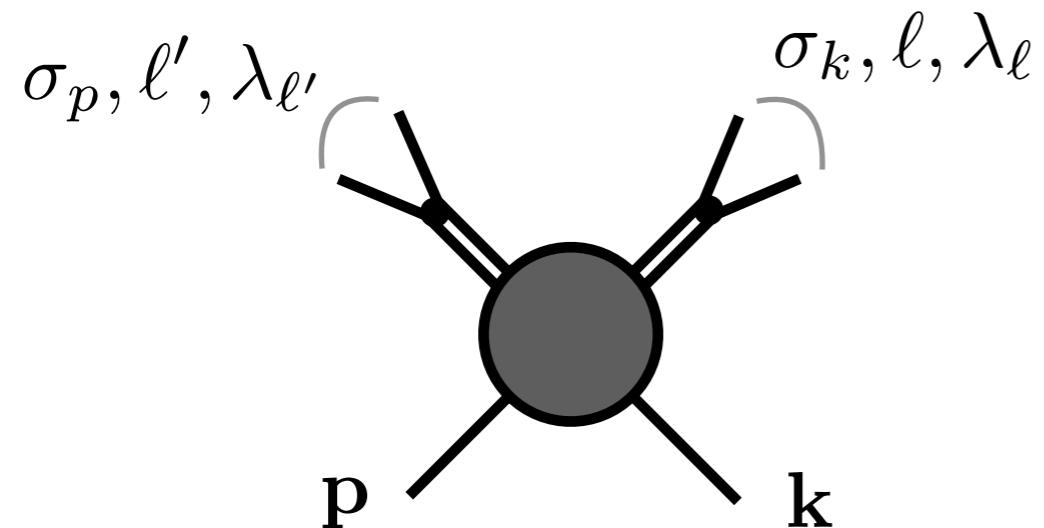
$$\mathcal{M}_2 = \mathcal{K}_2 + \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_2$$

Challenges for three-body dynamics

On-shell scattering governed by integral equations

- Practical implementation – numerical inversion
- Practical systematic – truncation of partial waves (cf. isobar model)

$$\mathcal{M}_3 = \mathcal{K}_3 + \int \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_3 + \iint \mathcal{K}_3 \cdot \mathcal{G} \cdot \mathcal{M}_3$$



two-body case:

$$\text{Im } \mathcal{M}_2^J \sim |\mathcal{M}_2^J|^2$$

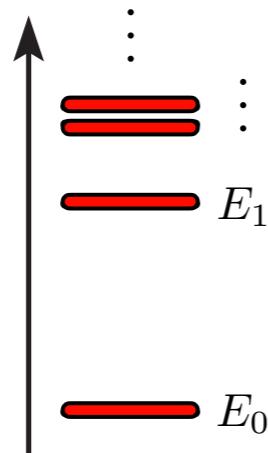
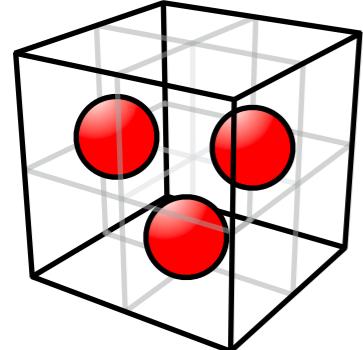
Three-body case:

$$\text{Im}[\mathcal{M}_3^J]_{\ell'\ell} \sim \sum_{\ell''} \int [\mathcal{M}_3^{J*}]_{\ell'\ell''} [\mathcal{M}_3^J]_{\ell''\ell} + \dots$$

Three-Body Dynamics

Connecting to finite-volume spectra

Finite-volume quantization condition



$$\rightarrow \det\left(1 + \mathcal{K}_3 (\mathcal{F}_L + \mathcal{G}_L)\right)_{E=E_n} = 0$$

$$\rightarrow \mathcal{M}_3 = \mathcal{K}_3 + \int \mathcal{K}_2 \cdot i\rho \cdot \mathcal{M}_3 + \iint \mathcal{K}_3 \cdot \mathcal{G} \cdot \mathcal{M}_3$$

M. Hansen and S. Sharpe
Phys. Rev. D **90**, 116003 (2014), Phys. Rev. D **95**, 034501 (2017)

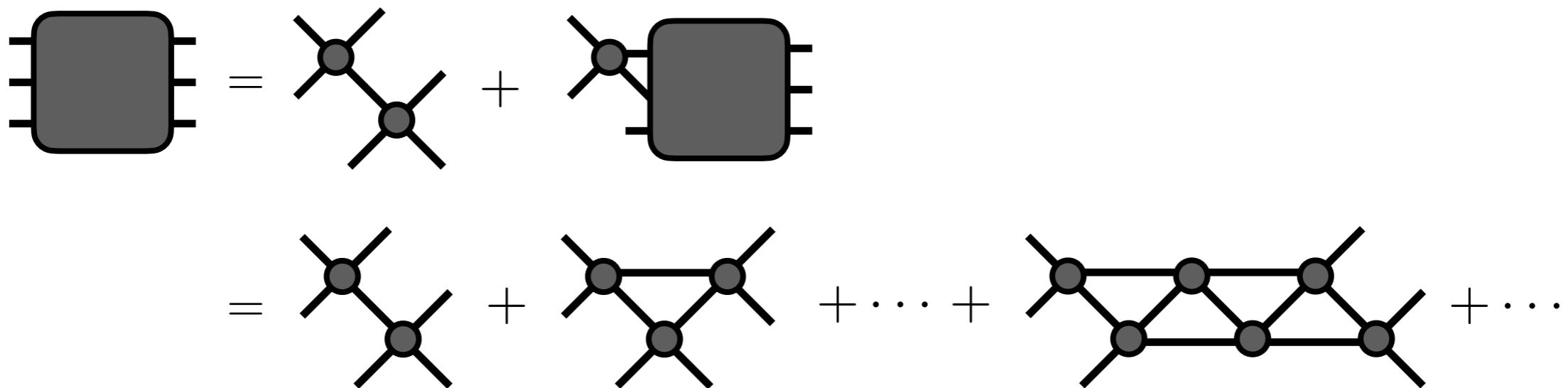
M. Mai and M. Döring
Eur. Phys. J. A **53**, 240 (2017), Phys. Rev. Lett. **122**, 062503 (2019)

Three-Body Dynamics

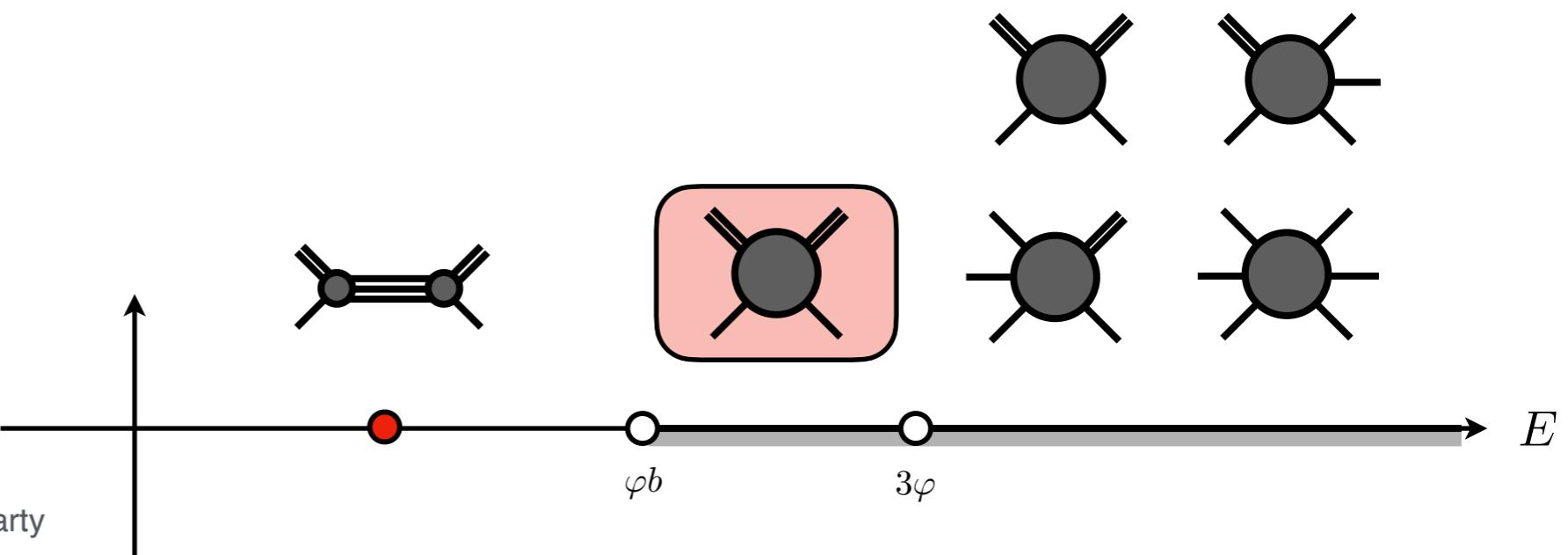
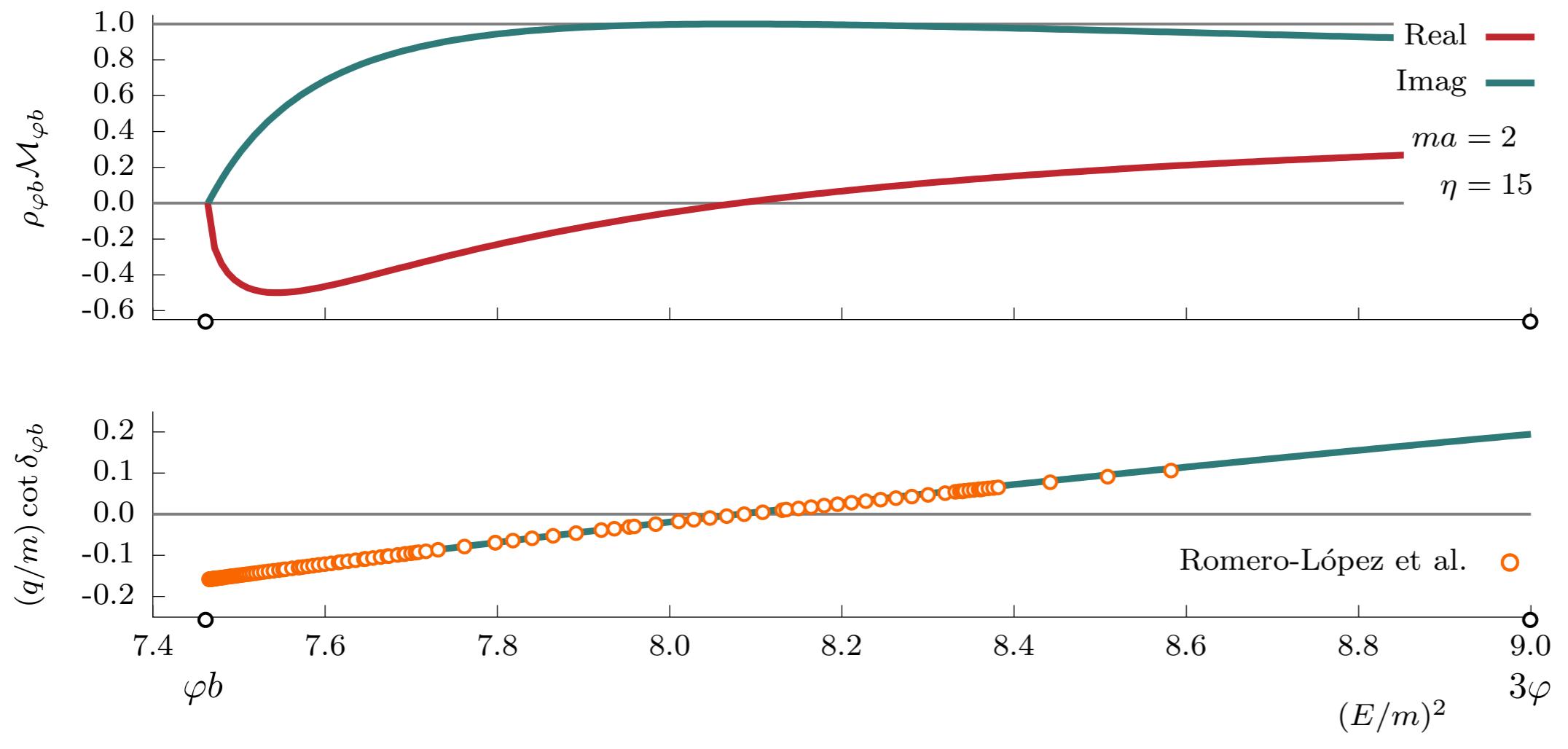
Examine toy-model – $3\varphi \rightarrow 3\varphi$

- Assume exchange dominance – **No short-range three-body forces**
- Scalar system – $J = 0$
- Two-hadron pair forms bound state – $2\varphi \rightarrow b$

Toy model version of $3N \rightarrow 3N$ with $2N \rightarrow d$



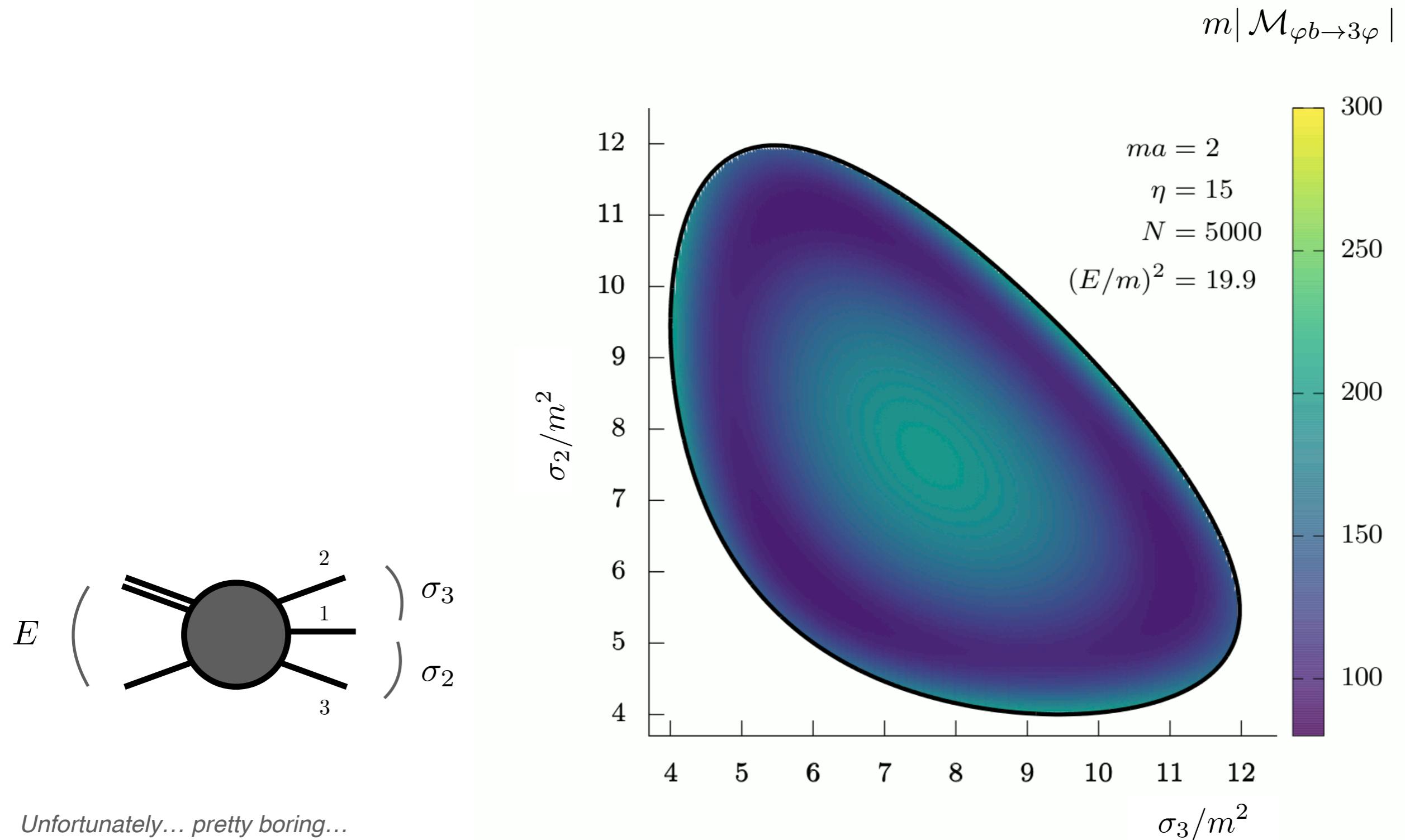
Three-Body Dynamics



Three-Body Dynamics

Methodology not limited to below 3-body threshold

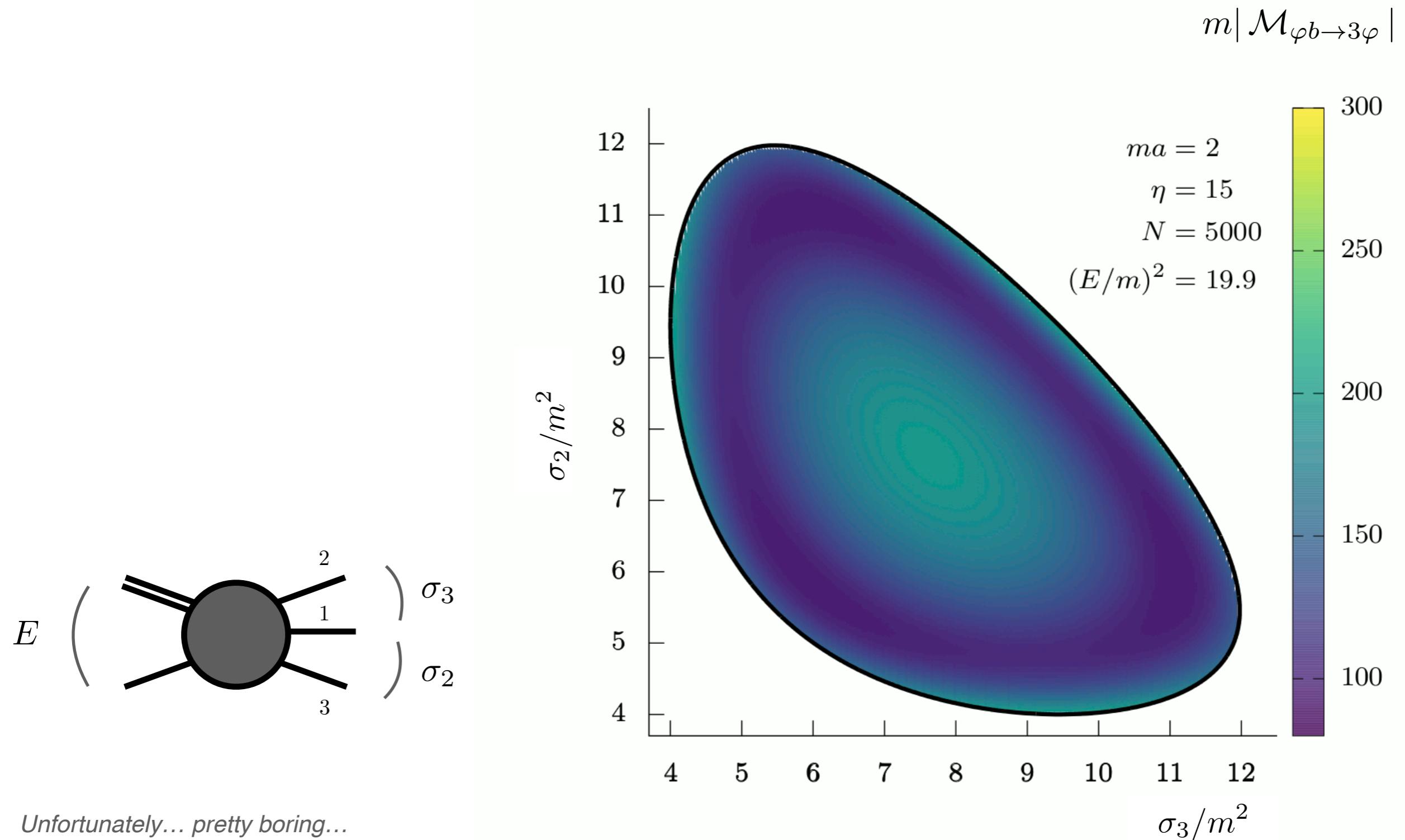
- Allows for calculation of breakup / recombination amplitude



Three-Body Dynamics

Methodology not limited to below 3-body threshold

- Allows for calculation of breakup / recombination amplitude

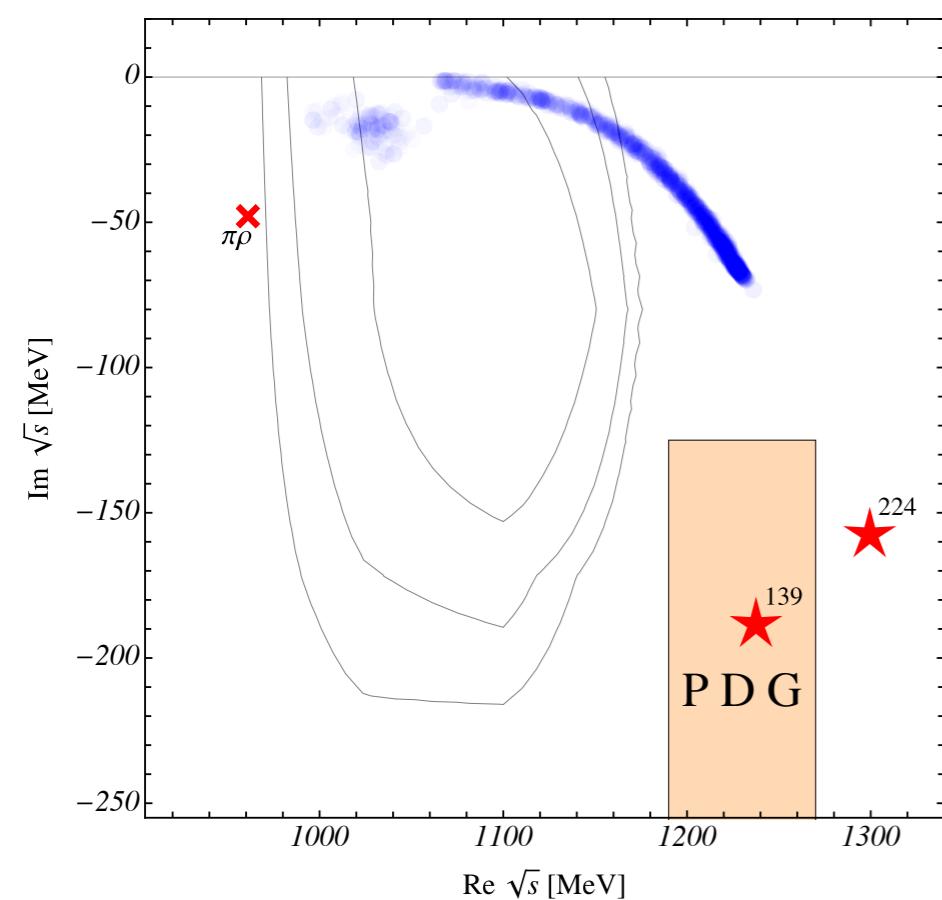


Applications

First applications appearing in literature

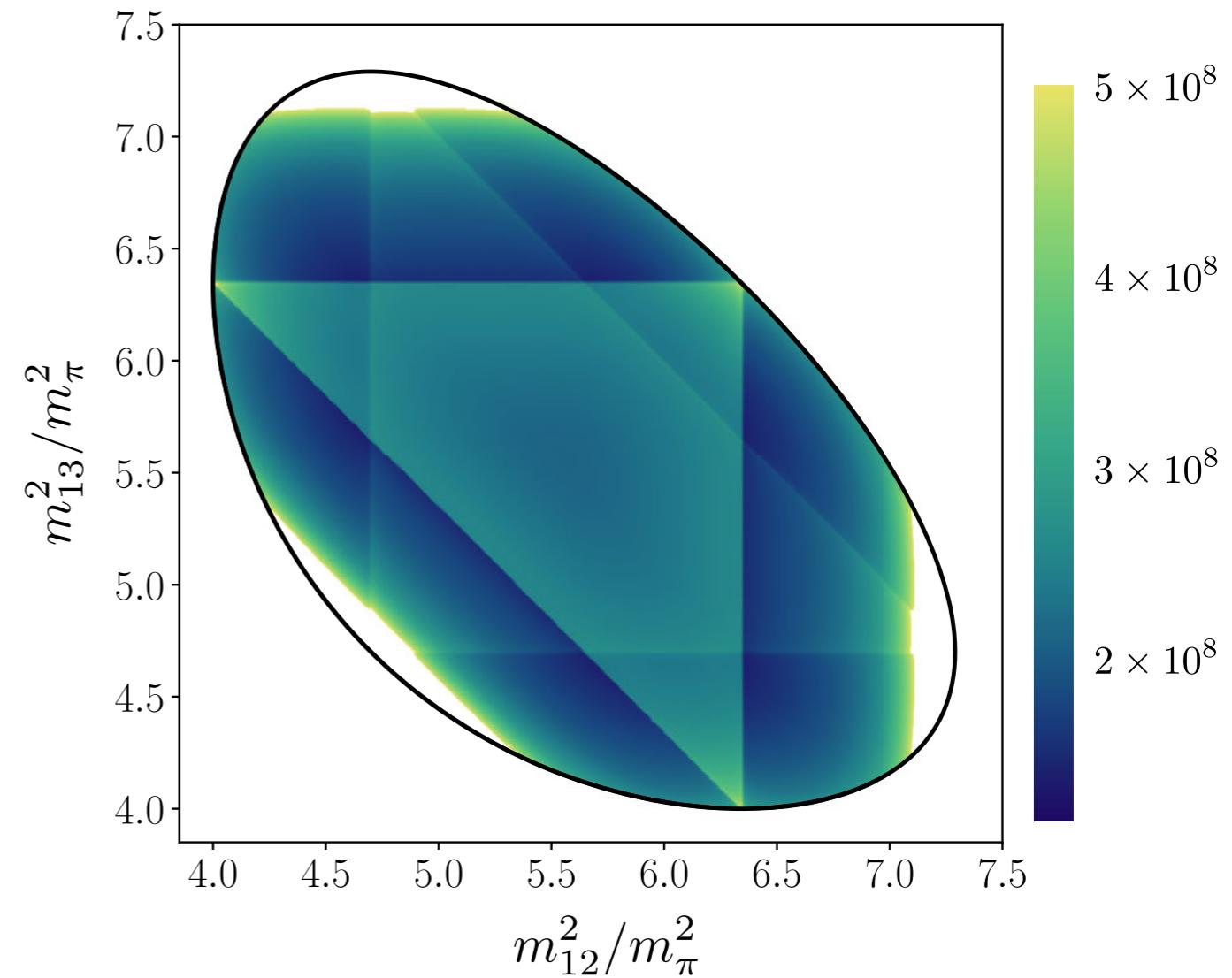
- $3\pi^+, 3K^+, \pi^+\pi^+K^+, \dots$

Resonant a_1 !



M. Mai et al. [GWQCD]
Phys. Rev. Lett. **127** (2021) 22, 222001

See M. Döring @ 1:30pm

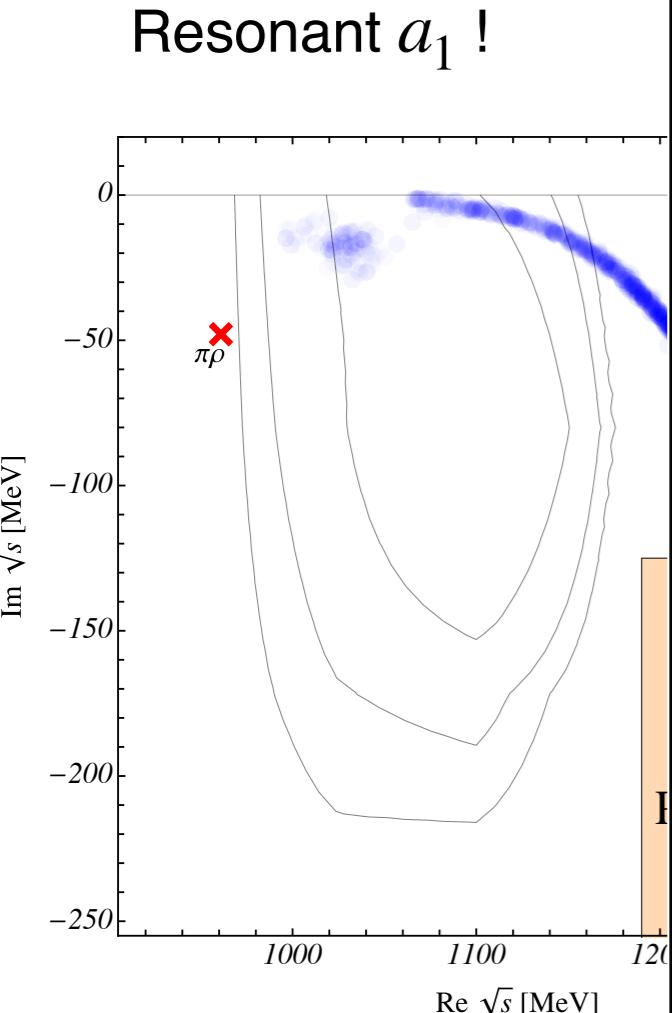


M. Hansen et al. [HadSpec]
Phys. Rev. Lett. **126**, (2021) 012001

Applications

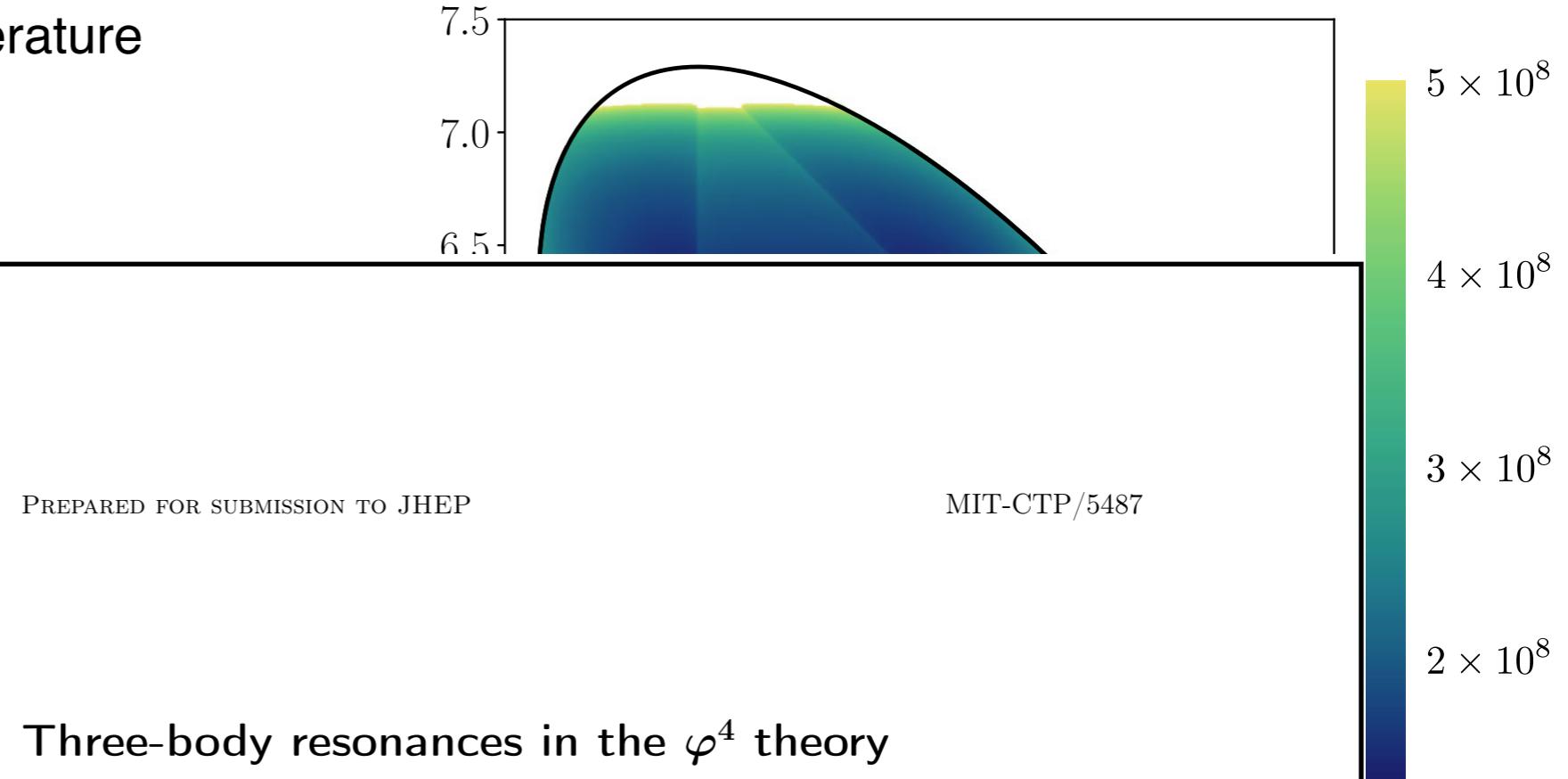
First applications appearing in literature

- $3\pi^+, 3K^+, \pi^+\pi^+K^+, \dots$



M. Mai et al. [GWQCD]
Phys. Rev. Lett. **127** (2021) 22, 222

05605v1 [hep-lat] 10 Nov 2022



PREPARED FOR SUBMISSION TO JHEP

MIT-CTP/5487

Three-body resonances in the φ^4 theory

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^dTbilisi State University, 0186 Tbilisi, Georgia

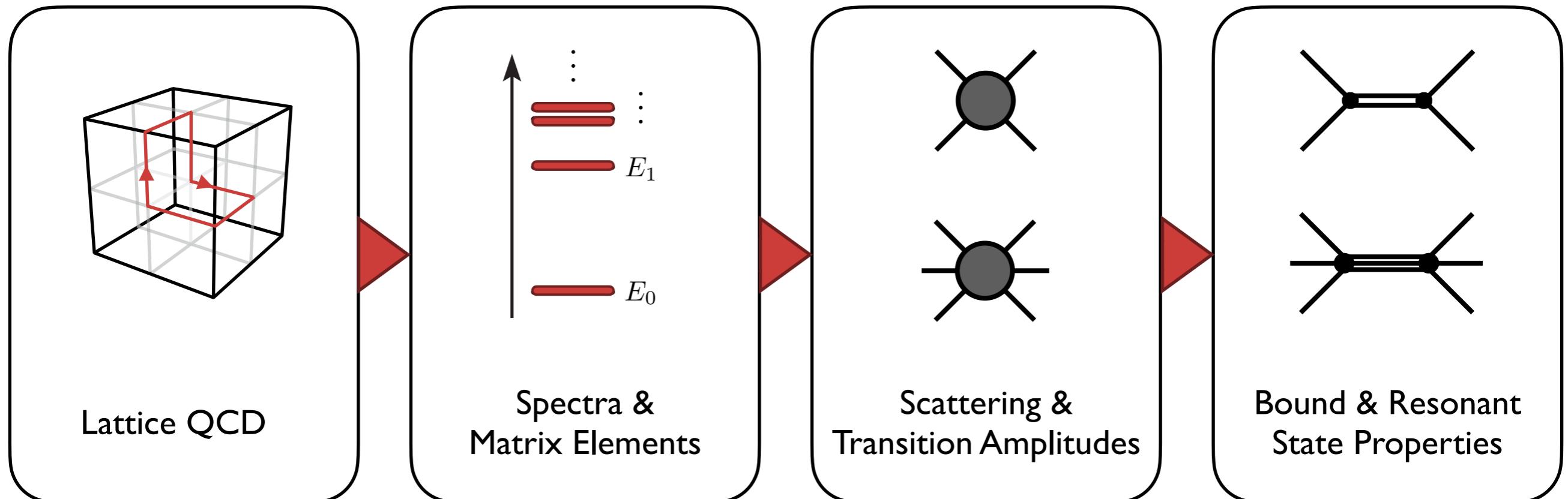
E-mail: garofalo@hiskp.uni-bonn.de

ABSTRACT: We study the properties of three-body resonances using a lattice complex scalar φ^4 theory with two scalars, with parameters chosen such that one heavy particle can decay into three light ones. We determine the two- and three-body spectra for several lattice volumes using variational techniques, and then analyze them with two versions of the three-particle finite-volume formalism: the Relativistic Field Theory approach and the Finite-Volume Unitarity approach. We find that both methods provide an equivalent description of the energy levels, and we are able to fit the spectra using simple parametrizations of the energy levels. These findings are also confirmed by a direct comparison with the results obtained by the Finite-Volume Unitarity approach for the two-body case.

Few-Body Physics from QCD

Path to few-body physics from QCD

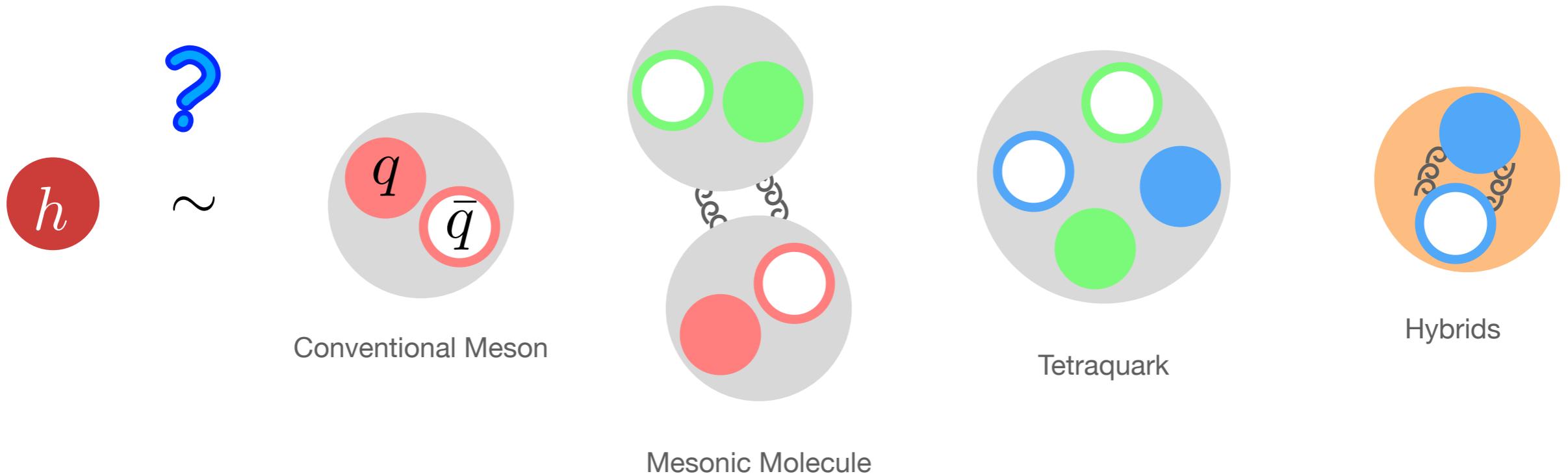
- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD, Scattering Theory, & Effective Field Theory*



Hadronic Structure & Electroweak Probes

Can we get more than the spectrum?

- Want to know the substructure of the excited hadrons



See L. Leskovec @ 11:35

See F. Ortega-Gama @ 2:40 Wednesday

Summary

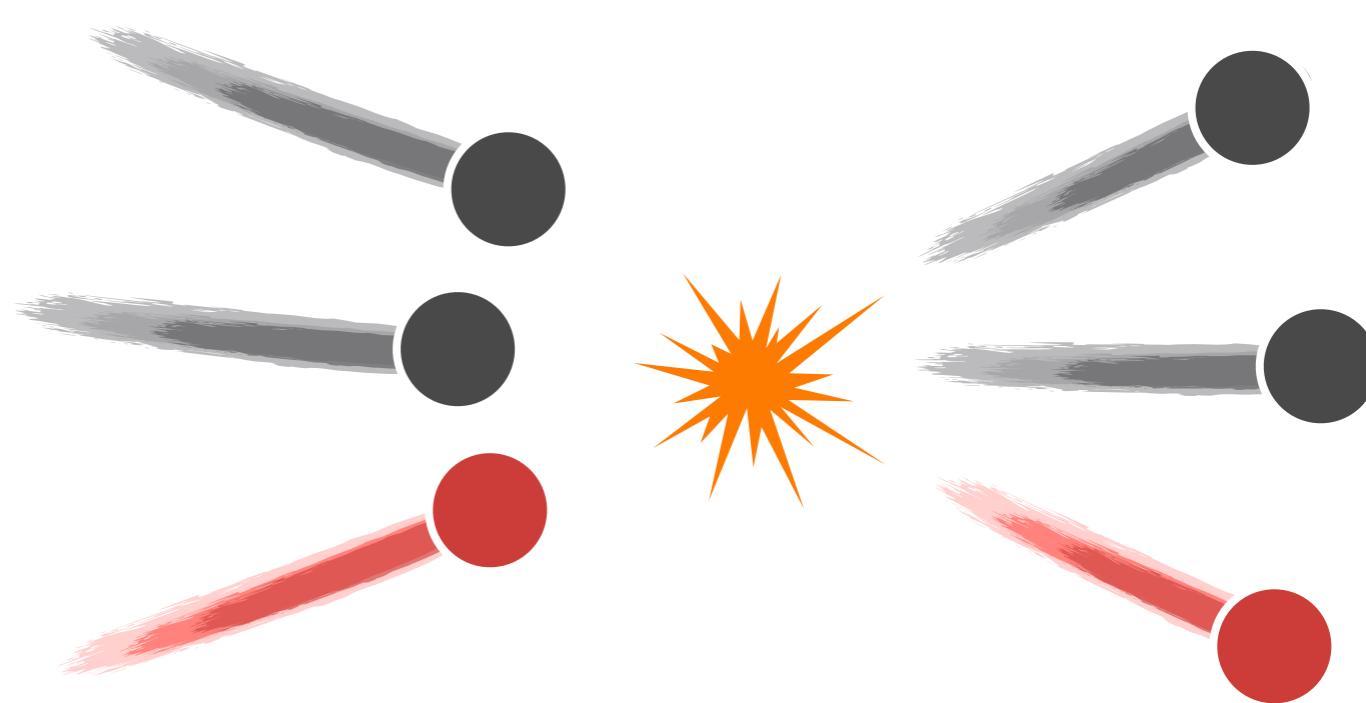
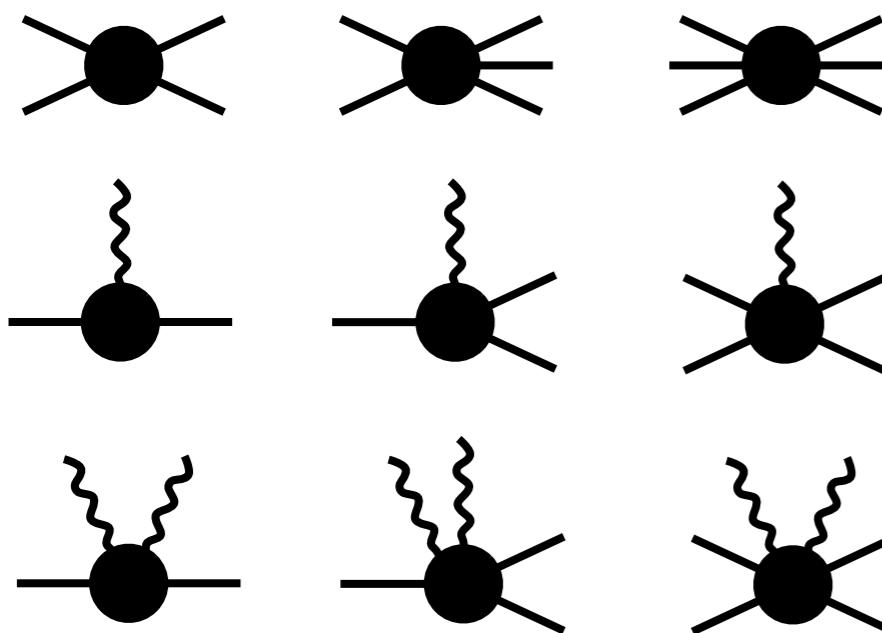
Few-body interactions play a key role in many outstanding problems in nuclear & hadron physics

Lattice QCD, EFTs, & Scattering theory combined provide useful tools to extract physics from QCD

- Rapid development in formalisms relating lattice QCD observables to amplitudes
- Scattering phenomenology is advancing in tandem

Latest developments in three-body scattering & two-body matrix elements

- First applications appearing in literature
- Can address increasingly complicated processes



Much more to come!

