Molecular states with charm and strangeness

R. Molina, F. Gil, T. Branz, L. R. Dai and E. Oset

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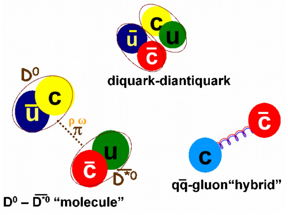
Intro

Many exotics have been discovered ...

- $D_{s0}(2317)$, $D_{s1}(2460)$, $X(3872)$ close to $D^{(*)}K$, $D^*\bar{D}$, BABAR, CLEO, BELLE (2003)
- $Z_c(3900)$, BESIII, 2013
  close to $D\bar{D}^*$, $c\bar{q}q\bar{c}$ $(q = u, d)$
- $Z_{cs}(3985)$, BESIII, 2021
  close to $\bar{D}_s^* D/\bar{D}_s D^*$, $c\bar{q}s\bar{c}$
- $X_0(2866), X_1(2900)$ now $T_{cs}(2900)$, LHCb, 2020
  close to $D^*\bar{K}^*$, $c\bar{q}s\bar{q}$
- $T_{cc}(3875)$, LHCb, 2021
  close to DD^*, $c\bar{q}c\bar{q}$
- $T_{c\bar{s}}(2900)$, LHCb, 2022
  close to D^*K^*, $c\bar{s}q\bar{q}$



$\Longrightarrow$ Do not fit into $q\bar{q}$ basic mesons of the quark model predictions

Are the meson-meson molecules? tetraquarks? interaction?

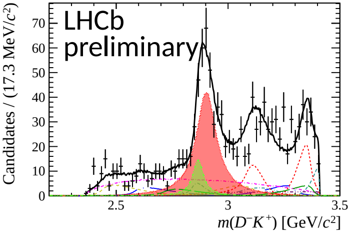
New flavor exotic tetraquark ($C = -1, S = 1, I_3 = 0$)

LHCb (2020)

Two states $J^P = 0^+, 1^-$ decaying to $\bar{D}K$. First clear example of an heavy-flavor exotic tetraquark, $\sim \bar{c}\bar{s}ud$. Now $T_{cs}(2900)$

$$X_0(2866) : M = 2866 \pm 7 \quad \text{and} \quad \Gamma = 57.2 \pm 12.9\,\text{MeV},$$

$$X_1(2900) : M = 2904 \pm 5 \quad \text{and} \quad \Gamma = 110.3 \pm 11.5\,\text{MeV}.$$

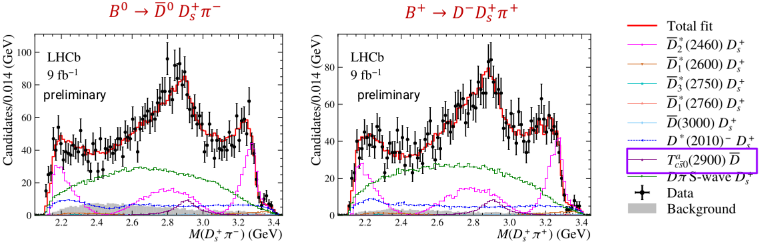


R. Aaij et al. (LHCb Collaboration), PRL125(2020), PRD102(2020)

New exotic tetraquark seen in $D_s^+\pi^+$ $(C = 1, S = 1, I = 1)$

LHCb (2022)

One state decaying $T_{c\bar{s}}(2900)$ decaying to $D_s^+\pi^-$ and $D_s^+\pi^+$ has been observed.



- The analysis favors $J^P = 0^+$
- Mass, $m = 2908 \pm 11 \pm 20$ MeV
- Width, $\Gamma = 136 \pm 23 \pm 11$ MeV

LHCb-PAPER-2022-026

Note that,

D^*K^* th.: 2903 MeV

$D_s^*\rho$ th.: 2890 MeV

Flavour exotic states

- 2010. Prediction of several flavour exotic states

PHYSICAL REVIEW D **82,** 014010 (2010)

New interpretation for the $D^*_{s2}(2573)$ and the prediction of novel exotic charmed mesons

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(Received 4 May 2010; published 21 July 2010)

In this manuscript we study the vector-vector interaction within the hidden-gauge formalism in a coupled channel unitary approach. In the sector $C = 1$, $S = 1$, $J = 2$ we get a pole in the T matrix around 2572 MeV that we identify with the $D^*_{s2}(2573)$, coupling strongly to the $D^*K^*(D^*_s\phi(\omega))$ channels. In addition we obtain resonances in other exotic sectors which have not been studied before such as $C = 1$, $S = -1$, $C = 2$, $S = 0$ and $C = 2$, $S = 1$. These "flavor-exotic" states are interpreted as $D^*\bar{K}^*$, D^*D^*, and $D^*_sD^*$ molecular states but have not been observed yet. In total we obtain nine states with different spin, isospin, charm, and strangeness of non-$C = 0$, $S = 0$ and $C = 1$, $S = 0$ character, which have been reported before.

DOI: 10.1103/PhysRevD.82.014010 PACS numbers: 14.40.Rt, 12.40.Vv, 13.75.Lb, 14.40.Lb

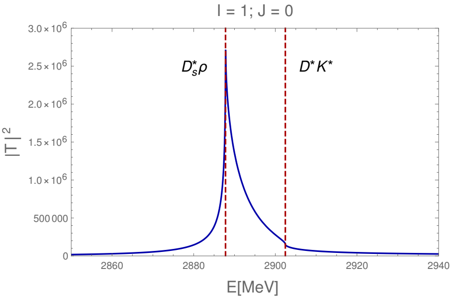
- Free parameter fixed with $D_{s2}(2573)$; couples to D^*K^*, $c\bar{q}q\bar{s}$
- Flavour exotic states with $I = 0$, $J^P = \{0, 1, 2\}^+$ coupling to $D^*\bar{K}^*$ are predicted, $c\bar{q}s\bar{q}$
- Doubly charm states, $I = 0$; $J^P = 1^+$, close to D^*D^* are predicted, $c\bar{q}c\bar{q}$, and $I = 1/2$; $J^P = 1^+$, close to $D^*D^*_s$ $c\bar{q}c\bar{s}$

Exotic states

Phys. Rev. D 82 (2010), Molina, Branz, Oset

3.5 $C = 1; S = 1; I = 1$

In this sector the potential is attractive for the $D^*K^* \to D_s^*\rho$ reaction. For $J = 0$ and 1 this potential is around $-7\,g^2$ whereas it is by a factor of two bigger $-13\,g^2$ for $J = 2$ (see Table 14). In fact, we only obtain a pole for $J = 2$. For $J = 0$ and 1 we only observe a cusp in the $D_s^*\rho$ threshold. In Table 5 we show the pole position and couplings to the different channels. Both channels, D^*K^* and $D_s^*\rho$, are equally important as one can deduce from the corresponding couplings. The broad width of the ρ meson has to be taken into account



$\alpha = -1.6$

ρ width not included

$D^*K^* \to DK$ considered

Cusp around $D_s^*\rho$, D^*K^* th.

separated only by 14 MeV

Flavour exotic states

Molina,Branz,Oset, PRD82(2010)

C, S	Channels	$I[J^P]$	$\sqrt{s}$	$\Gamma_A(\Lambda = 1400)$	$\Gamma_B(\Lambda = 1200)$	State	$\sqrt{s}_{\rm exp}$	$\Gamma_{\rm exp}$
$1, -1$	$D^* \bar{K}^*$	$0[0^+]$	2848	23	59	$X_0(2866)$ or $T_{cs}(2900)$	2866	57
		$0[1^+]$	2839	3	3			
		$0[2^+]$	2733	11	36			
$1, 1$	$D^* K^*$, $D_s^* \omega$	$0[0^+]$	2683	20	71			
	$D_s^* \phi$	$0[1^+]$	2707	4×10^{-3}	4×10^{-3}			
		$0[2^+]$	2572	7	23	$D_{s2}(2573)$	2572	20
$1, 1$	$D^* K^*$, $D_s^* \rho$	$1[0^+]$	Cusp structure around $D_s^* \rho$, $D^* K^*$			new $T_{c\bar{s}}(2900)$	2908	136
$1, 1$		$1[1^+]$	Cusp structure around $D_s^* \rho$, $D^* K^*$					
$1, 1$		$1[2^+]$	2786	8	11			
$2, 0$	$D^* D^*$	$0[1^+]$	3969	0	0			
$2, 1$	$D^* D_s^*$	$1/2[1^+]$	4101	0	0			

Table 1: Summary of the nine states obtained. The width is given for the model A, Γ_A, and B, Γ_B. All the quantities here are in MeV. Repulsion in $C = 0, S = 1, I = 1/2$; $C = 1, S = -1, I = 1$; $C = 1, S = 2, I = 1/2$; $C = 2, S = 0, I = 1$ and $C = 2, S = 2, I = 0$ is found.

Form factors in the $D^* D\pi$ vertex; Model A: $F_1(q^2) = \frac{\Lambda_b^2 - m_\pi^2}{\Lambda_b^2 - q^2}$, Titov, Kampfer EPJA7, PRC65 with $\Lambda_b = 1.4, 1.5$ GeV and $g = M_\rho / 2 f_\pi$. Model B: $F_2(q^2) = e^{q^2/\Lambda^2}$ Navarra, Nielsen, Bracco PRD65 (2002), $\Lambda = 1, 1.2$ GeV and $g_D = g^{\rm exp}_{D^* D\pi} = 8.95$ (experimental value). Subtraction constant $\alpha = -1.6$.

Many studies appeared after these discoveries ...

- He, Wang, Zhu, EPJC80, 1026 (2020), Karliner, Rosner, PRD102(2020), $X_0(2866)$, compact tetraquark
- X. H. Liu, Yan et al., EPJC80(2020), $X_0(2866)$, Triangle Singularity
- M. Z. Liu, Xie, Geng, PRD102(2020), $X_0(2866)$, $D^*\bar{K}^*$ molecule (one-boson ex.), $X_1(2900)$ cannot be, Qi, Wang et al. EPJC81(2021), X_1 is a $\bar{D}_1K$ molecule (ρ, ω ex.)
- Ying-Hui Ge, X.H. Liu and H. W. Kei, 2207.09900, the $T_{c\bar{s}}$ could be a TS from the $\chi_{c1}D^*K^*$ loop. However, the TS peak around the $D_s^*\rho$ threshold from the $D^{**}D_s^*\rho$ loop cannot explain the $T_{c\bar{s}}(2900)$
- Du, Baru, Dong, Filin, Nieves, F. K. Guo, T_{cc}, PRD105, (2022), 3-body dynamics, $D^0D^0\pi^+$, contact+OPE, DD^* molecule
- Albaladejo, T_{cc} from DD^*, can have $I = 0$ or 1
- Feijoo, Liang, Oset, PRD104(2021), T_{cc} as DD^*, has $I = 0$, decay width to $D^0D^0\pi^+ \sim 43\ \mathrm{MeV}$
- Padmanath, Prelovsek, virtual s-wave bound state for $m_\pi = 280$ MeV of DD^* in LatticeQCD ...

The Local Hidden Gauge Approach

The hidden gauge formalism Bando,Kugo,Yamawaki

Lagrangian

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}_{III} \tag{1}$$

$$\mathcal{L}^{(2)} = \frac{1}{4} f^2 \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \tag{2}$$

$$\mathcal{L}_{III} = -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle + \frac{1}{2} M_V^2 \langle [V_\mu - \frac{i}{g} \Gamma_\mu]^2 \rangle$$

$$D_\mu U = \partial_\mu U - ieQA_\mu U + ieUQA_\mu, \qquad U = e^{i\sqrt{2}P/f} \tag{3}$$

$$\mathcal{L}_{V\gamma} = -M_V^2 \frac{e}{g} A_\mu \langle V^\mu Q \rangle$$

$$\mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial_\mu P] \rangle; \quad g = M_V / 2f$$

$$\widetilde{\mathcal{L}}^{(2)} = \frac{1}{12 f^2} \langle [P, \partial_\mu P]^2 + M P^4 \rangle. \tag{4}$$

Local Hidden Gauge Approach
Vector-vector scattering

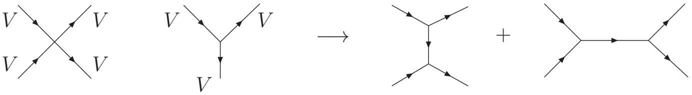
$$\boxed{\mathcal{L}_{III} = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle} \longrightarrow \boxed{\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle}$$

$$\boxed{\mathcal{L}_{III}^{(c)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle}$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu]$$

$$g = \frac{M_V}{2f}$$

$$V_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi \end{pmatrix}_\mu$$



$a)$ $b)$ $c)$ $d)$

Local Hidden Gauge Approach

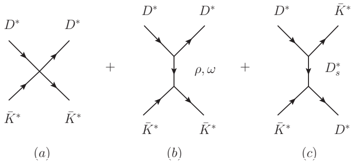


Figure 1: The $D^* \bar{K}^* \to D^* \bar{K}^*$ interaction at the tree level; (a) contact term; (b) exchange of light vectors; (c) exchange of a heavy vector.

Approximation

$$\epsilon_1^\mu = (0,1,0,0) \qquad\qquad k^\mu = (k^0, 0, 0, |\vec{k}|) \qquad\qquad \epsilon_1^\mu = (0,1,0,0)$$
$$\epsilon_2^\mu = (0,0,1,0) \qquad\qquad \vec{k}/m \simeq 0,\ k_j^\mu \epsilon_\mu^{(l)} \simeq 0 \qquad\qquad \epsilon_2^\mu = (0,0,1,0)$$
$$\epsilon_3^\mu = (|\vec{k}|, 0, 0, k^0)/m \qquad\qquad \epsilon_3^\mu = (0,0,0,1)$$

$$\mathcal{L}_{III}^{(3V)} = ig\langle(\partial_\mu V_\nu - \partial_\nu V_\mu)V^\mu V^\nu\rangle = ig\langle[V_\mu, \partial_\nu V_\mu]V^\nu\rangle$$

Spin projectors

$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu; \quad \mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu - \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu)$$
$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu + \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu) - \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu\}\ .$$

The $X_0(2866)$ or $T_{cs}(2900)$

Local Hidden Gauge Approach

Potential V: contact + vector-meson exchange (ρ, ω)

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-15.9g^2$

Table 2: Tree level amplitudes for $D^*\bar{K}^*$ in $I=0$. Last column: $(V_{\rm th.})$.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.7g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D^*_s}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$15.7g^2$

Table 3: Tree level amplitudes for $D^*\bar{K}^*$ in $I=1$. Last column: $(V_{\rm th.})$.

The interaction is attractive for $I=0$ and repulsive for $I=1$.

Local Hidden Gauge Approach

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1+p_3).(p_2+p_4)$	$0.11g^2$
0	$D^*K^* \to D_s^*\rho$	$4g^2$	$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K*}^2}$	$-6.8g^2$
0	$D_s^*\rho \to D_s^*\rho$	0	0	0
1	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1+p_3).(p_2+p_4)$	$0.11g^2$
1	$D^*K^* \to D_s^*\rho$	0	$\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K*}^2}$	$-6.6g^2$
1	$D_s^*\rho \to D_s^*\rho$	0	0	0
2	$D^*K^* \to D^*K^*$	0	$\frac{g^2}{2}(\frac{1}{m_\rho^2} - \frac{1}{m_\omega^2})(p_1+p_3).(p_2+p_4)$	$0.11g^2$
2	$D^*K^* \to D_s^*\rho$	$-2g^2$	$-\frac{g^2(p_1+p_4)(p_2+p_3)}{m_{D*}^2} - \frac{g^2(p_1+p_3).(p_2+p_4)}{m_{K*}^2}$	$-12.8g^2$
2	$D_s^*\rho \to D_s^*\rho$	0	0	0

Table 4: Tree level amplitudes for $D^*K^*, D_s^*\rho$ in $I=1$. Last column: $(V_{\rm th.})$ for $C=1$, $S=1$ and $I=1$.

The interaction is attractive for both $I=0$ and $I=1$, favoring a $J^+ = 2^+$ state. (see PRD82 (2010) Molina, Branz, Oset, for $I=0$)

New flavor exotic tetraquark ($C = 1, S = -1$)

Two-meson loop function

$$G_i(s) = \frac{1}{16\pi^2}\left(\alpha + \mathrm{Log}\frac{M_1^2}{\mu^2} + \frac{M_2^2 - M_1^2 + s}{2s}\mathrm{Log}\frac{M_2^2}{M_1^2}\right.$$
$$\left. + \frac{p}{\sqrt{s}}\left(\mathrm{Log}\frac{s - M_2^2 + M_1^2 + 2p\sqrt{s}}{-s + M_2^2 - M_1^2 + 2p\sqrt{s}} + \mathrm{Log}\frac{s + M_2^2 - M_1^2 + 2p\sqrt{s}}{-s - M_2^2 + M_1^2 + 2p\sqrt{s}}\right)\right),$$

Bethe-Salpeter

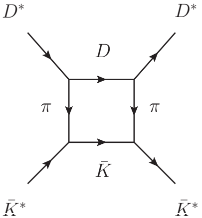
$$\boxed{T = [\hat{1} - VG]^{-1}V}$$

The states with $J^P = \{0,2\}^+$ decay into $D\bar{K}$

$$\mathcal{L}_{VPP} = -ig\langle[P, \partial_\mu P]V^\mu\rangle$$

$F(q) = e^{((p_1^0 - q^0)^2 - \vec{q}^{\,2})/\Lambda^2}$ Navarra, PRD65(2002)

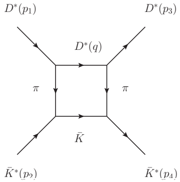
with $q_0 = (s + m_D^2 - m_K^2)/2\sqrt{s}$.



New flavor exotic tetraquark ($C = 1, S = -1$)

Recent work: Molina, Oset PLB811 2020, $\alpha = -1.474$, $\Lambda = 1300$.
Evaluation of the decay width of the $J^P = 1^+$ state

$$\mathcal{L} = \frac{iG'}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \delta_\mu V_\nu \delta_\alpha V_\beta P \rangle$$
$$\mathcal{L}_{VPP} = -ig \langle [P, \partial_\mu P] V^\mu \rangle$$



Amplitude:

$$-it = \frac{9}{2}(G' g m_{D*})^2 \int \frac{d^4 q}{(2\pi)^4} \epsilon^{ijk} \epsilon^{i'j'k'} \left(\frac{1}{(p_1 - q)^2 - m_\pi^2 + i\epsilon} \right)^2 \frac{1}{q^2 - m_{D*}^2 + i\epsilon} \frac{1}{(p_1 + p_2 - q)^2 - m_K^2 + i\epsilon}$$
$$\times \epsilon^{j(1)} \epsilon^{m(2)} \epsilon^{k(3')} q^i q^m \epsilon^{j'(1)} \epsilon^{m'(4)} \epsilon^{k'(3')} q^{i'} q^{m'} F^4(q) \quad (5)$$

$$t = \frac{9}{2}(G' g m_{D*})^2 \int \frac{d^3 q}{(2\pi)^3} (\delta_{ii'} \delta_{jj'} - \delta_{ij'} \delta_{i'j}) \left(\frac{1}{(p_1 - q)^2 - m_\pi^2} \right)^2 \frac{1}{2\omega^*(q)} \frac{1}{2\omega(q)} \frac{1}{\sqrt{s} - \omega^*(q) - \omega(q) + i\epsilon}$$
$$\times \epsilon^{j(1)} \epsilon^{m(2)} \epsilon^{j'(3)} \epsilon^{m'(4)} q^i q^m q^{i'} q^{m'} F^4(q) \quad (6)$$

with $\omega^*(q) = \sqrt{m_{D*}^2 + \vec{q}^{\,2}}$, $\omega(q) = \sqrt{m_K^2 + \vec{q}^{\,2}}$, $p_1^0 = m_{D*}$, $q^0 = \omega^*(q)$. We use $\mathrm{Im}\, \frac{1}{x + i\epsilon} = -i\pi\delta(x)$.

Taking now into account that,

$$\int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) q^i q^m q^{i'} q^{m'} = \frac{1}{15} \int \frac{d^3q}{(2\pi)^3} f(\vec{q}^{\,2}) \vec{q}^{\,4} (\delta_{im}\delta_{i'm'} + \delta_{ii'}\delta_{mm'} + \delta_{im'}\delta_{m'i}) \ ,$$

one obtains,

$$4\epsilon^{j(1)}\epsilon^{m(2)}\epsilon^{j(3)}\epsilon^{m(4)} - \epsilon^{j(1)}\epsilon^{j(2)}\epsilon^{m(3)}\epsilon^{m(4)} - \epsilon^{j(1)}\epsilon^{m(2)}\epsilon^{m(3)}\epsilon^{j(4)} \ ,$$

which is a combination of the spin projectors, $5\mathcal{P}^{(1)} + 3\mathcal{P}^{(2)}$, zero component for $J = 0$ (violates parity). The imaginary part for $J = 1$ is,

$$\boxed{\mathrm{Im}\,t = -\frac{3}{2}\frac{1}{8\pi}(G' g m_{D^*})^2 q^5 \left(\frac{1}{(m_D^* - \omega^*(q))^2 - \omega^2(q)}\right)^2 \frac{1}{\sqrt{s}} F^4(q)}$$

$$\omega(q) = \sqrt{m_K^2 + \vec{q}^{\,2}}; \omega^*(q) = \sqrt{m_{D^*}^2 + \vec{q}^{\,2}}; q = \frac{\lambda^{1/2}(s, m_{D^*}^2, m_K^2)}{2\sqrt{2}}$$

Decay of the $T_{cs}(2900)$ to $D^*\bar{K}$

$I(J^P)$	$M[\mathrm{MeV}]$	$\Gamma[\mathrm{MeV}]$	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$T_{cs}(2900)$

Table 5: New results including the width of the D^*K channel.

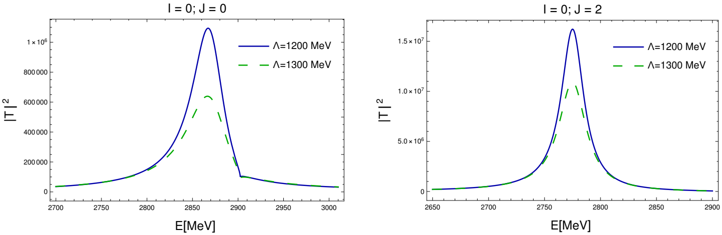


Figure 2: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 2$.

Decay of the $T_{cs}(2900)$ to $D^*\bar{K}$

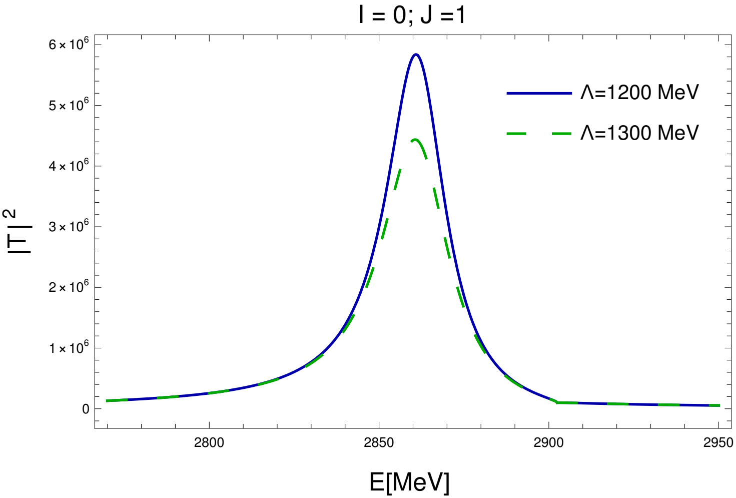


Figure 3: $|T|^2$ for $C = 1, S = -1, I = 0, J = 0$ and $J = 1$.

The $T_{c\bar{s}}(2900)$

$C = 1, S = 1, I = 1$ **The** $T_{c\bar{s}}(2900)$

Two channels: D^*K^*, $D_s^*\rho$. New results, $\alpha = -1.474$ to obtain the $T_{cs}(2900)$ state in $D^*\bar{K}^*$. We also consider both the ρ and K^* width.

Convolution due to the vector meson mass distribution

$$\tilde{G}(s) = \frac{1}{N}\int_{(M_1-4\Gamma_1)^2}^{(M_1+4\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi}\right)\mathcal{I}m\frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} G(s, \tilde{m}_1^2, M_2^2) \ ,$$

with

$$N = \int_{(M_1-4\Gamma_1)^2}^{(M_1+4\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi}\right)\mathcal{I}m\frac{1}{\tilde{m}_1^2 - M_1^2 + i\Gamma(\tilde{m})\tilde{m}_1} \ , \tag{7}$$

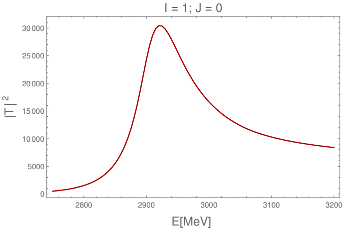
where M_1 and Γ_1 are the nominal mass and width of the vector meson.

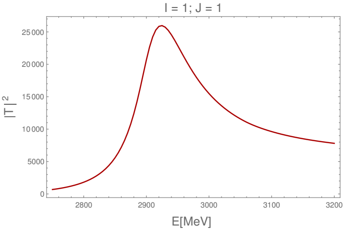
$$\tilde{\Gamma}(\tilde{m}) = \Gamma_0 \frac{q_{\rm off}^3}{q_{\rm on}^3}\Theta(\tilde{m} - m_1 - m_2) \tag{8}$$

with

$$q_{\rm off} = \frac{\lambda^{1/2}(\tilde{m}^2, m_1^2, m_2^2)}{2\tilde{m}}, \quad q_{\rm on} = \frac{\lambda^{1/2}(M_1^2, m_1^2, m_2^2)}{2M_1} \ . \tag{9}$$

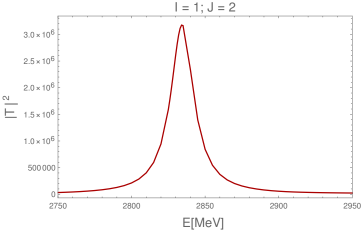
$C = 1, S = 1, I = 1$ **The** $T_{c\bar{s}}(2900)$





$(1,0):\ \ m = 2920$ MeV, $\Gamma = 130$ MeV $\quad (1,1):\ \ m = 2922$ MeV, $\Gamma = 145$ MeV

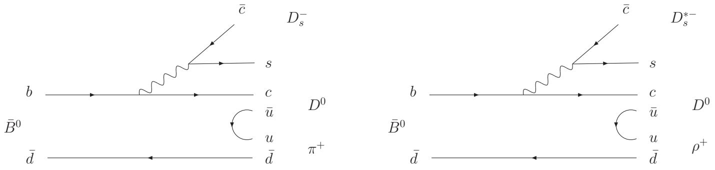
Exp. $m = 2908 \pm 11 \pm 20$ MeV, $\Gamma = 136 \pm 23 \pm 11$ MeV



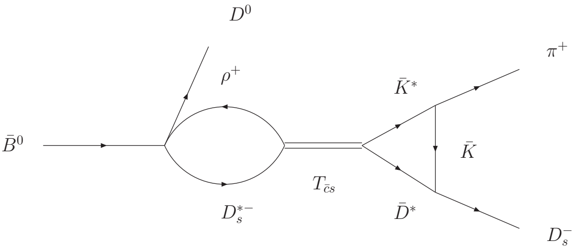
$(I,J) = (1,2):\ \ m = 2835$ MeV, $\Gamma = 20$ MeV

Production of the $T_{\bar{c}s}(2900)$

$\bar{B}^0 \to D_s^- D^0 \pi^+$ in B decays



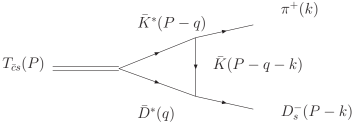
The $T_{\bar{c}s}(2900)$ can be produced by means of external emission



Production of the $T_{\bar{c}s}(2900)$ in B decays

$$\boxed{T(E) = aG(E)_{D_s^*\rho}t_{D_s^*\rho\to\bar{D}^*\bar{K}^*}(E)t_L(E) + b} \tag{10}$$

where $E = M_{inv}(\pi^+D_s^-)$, and a, b are constants adjusted to reproduce the experimental data. b stands for the background. t_L is the amplitude for the triangle loop.

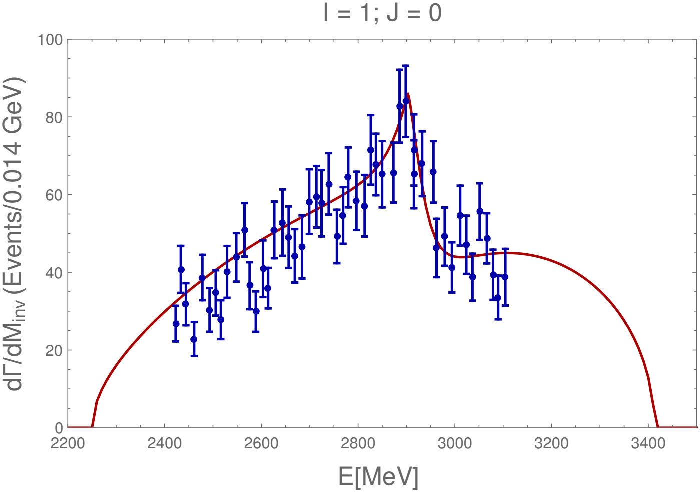


$$t_L = -g^2\int\frac{d^3q}{(2\pi)^3}(2\vec{k}+\vec{q})^2F(\vec{k}+\vec{q})\frac{1}{2\omega_{K^*}(q)}\frac{1}{2\omega_{D^*}(q)}\frac{1}{2\omega_K(\vec{q}+\vec{k})}$$
$$\times\left\{\frac{1}{P^0-\omega_{D^*}(q)-\omega_{K^*}(q)+i\epsilon}\frac{1}{P^0-\omega_{D^*}-k^0-\omega_K(\vec{q}+\vec{k})+i\epsilon}\right.$$
$$\left.-\frac{1}{P^0-\omega_{K^*}(q)-\omega_{D^*}(q)+i\epsilon}\frac{1}{\omega_{K^*}(q)+\omega(\vec{k}+\vec{q})-k^0-i\epsilon}\right\} \tag{11}$$

Production of the $T_{\bar{c}s}(2900)$ in B decays

$\bar{B}^0 \to D_s^- D^0 \pi^+$

$$\frac{d\Gamma}{dM_{Inv}} = \frac{1}{(2\pi)^3}\frac{1}{4M_B^2} p_D \tilde{p}_\pi |T|^2$$



Analysis of LQCD data on DK scattering and the $D_{s0}(2317)$

Quark mass dependence of the $D(D^*)$ mesons

Heavy Hadron Chiral Perturbation Theory (HHχPT)

E. Jenkins, NPB412 (1994)

$$\frac{1}{4}(D+3D*) = m_H + \alpha_a - \sum_{X=\pi,K,\eta} \beta_a^{(X)} \frac{M_X^3}{16\pi f^2} + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\alpha_a\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + c_a$$

$$(D* - D) = \Delta + \sum_{X=\pi,K,\eta} \left(\gamma_a^{(X)} - \lambda_a^{(X)}\Delta\right) \frac{M_X^2}{16\pi^2 f^2} \log\left(M_X^2/\mu^2\right) + \delta c_a$$

$\mu = 770$ MeV; $g^2 = 0.55$ MeV (Decay of the D^* meson)

$$\left.\begin{aligned} \frac{1}{4}(D+3D*) &= m_H + f(\sigma, a, b, c, d) \\ (D* - D) &= \Delta + g(\Delta^{(\sigma)}, \Delta^{(a)}) \end{aligned}\right\}$$

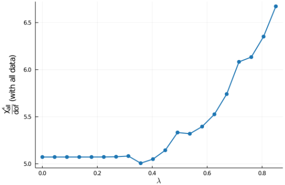
9 parameters, but different collaborations/scale settings, $7 + 2 \times 7 = 21$ parameters, ~ 80 data points

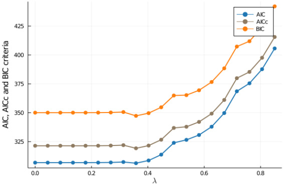
ETMC, PACS, HSC, CLS, RQCD, S.Prelovsek, MILC

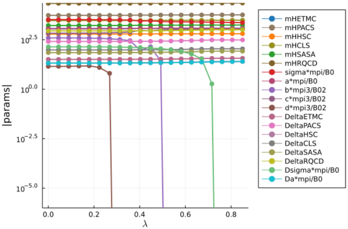
Quark mass dependence of the $D(D^*)$ mesons

LASSO + information criteria;

$$\chi_P^2 = \chi^2 + \lambda \sum_i^n |p_i|; \quad \text{Data} = \text{Training}\,(70\%) + \text{Test}\,(30\%) \qquad (12)$$



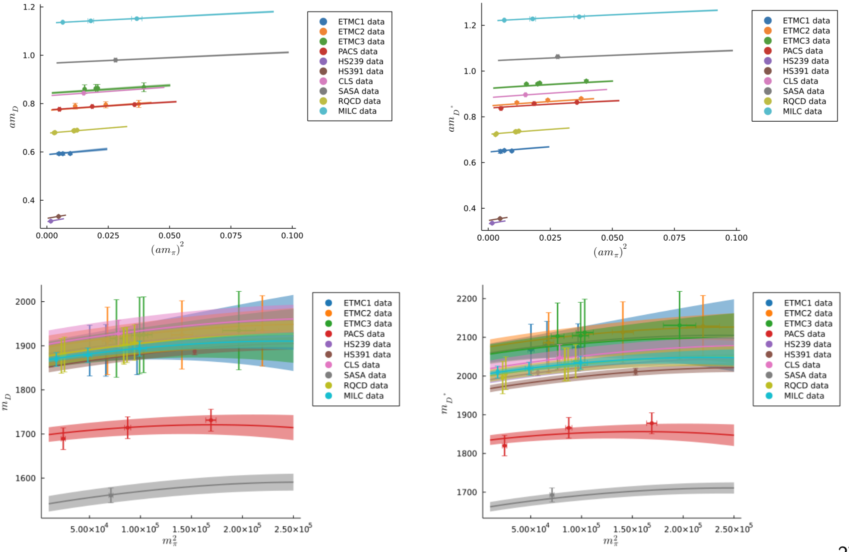




$$\frac{1}{4}(D + 3D^*) = m_H + f(\sigma, a, \cancel{b}, c, \cancel{d})$$

$$(D^* - D) = \Delta + g(\cancel{\Delta^{(\sigma)}}, \Delta^{(a)})$$

Quark mass dependence of the $D(D^*)$ mesons



Quark mass dependence of the $D_{s0}(2317)$ resonance in DK

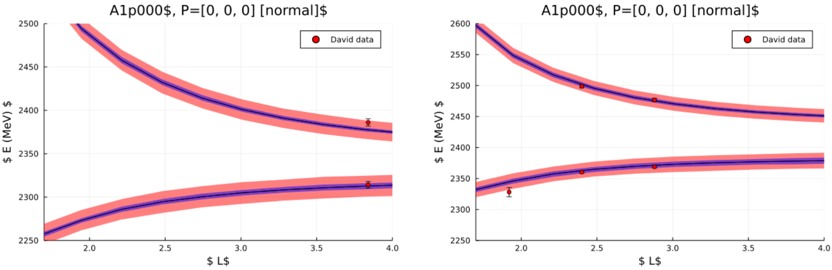
Fit to the energy levels of HSC; JHEP02, 100 (2021)

$$T = V[1 - VG]^{-1} \Longrightarrow \tilde{T} = V[1 - V\tilde{G}]^{-1}; \quad V_{LO} = -(s-u)/2f^2$$

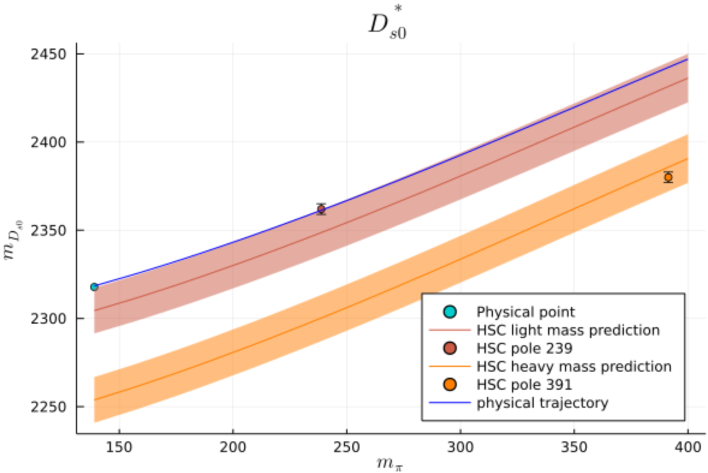
$$\tilde{G}(E) = \frac{1}{L^3} \sum_{\vec{q}_i} I(E, \vec{q}_i) \ , \qquad (13)$$

$$\tilde{G} = G^{DR} + \lim_{q_{max} \to \infty} \left(\frac{1}{L^3} \sum_{q < q_{max}} I(E, \vec{q}) - \int_{q < q_{max}} \frac{d^3 q}{(2\pi^3)} I(E, \vec{q}) \right) \qquad (14)$$

A. Martinez et al. PRD85 (2012)



Quark mass dependence of the $D_{s0}(2317)$ resonance in DK



$m_\pi = 236$ MeV; $a_t^{-1} = 5.667$ GeV; $a_t M_{\eta_c} = 0.2412$, $M_{\eta_c} = 2986$ MeV;
$m_\pi = 391$ MeV; $a_t^{-1} = 6.079$ GeV; $a_t M_{\eta_c} = 0.2735$; $M_{\eta_c} = 2963$ MeV;

Conclusions

Conclusions

- The $X_0(2866)$ or $T_{cs}(2900)$ is compatible with a $D^*\bar{K}^*$ resonance decaying to $D\bar{K}$. Its spin partners can be also searched for in B decays. Proposed reactions to observe the 1^+ state:
  $\bar{B}^0 \to D^{*+}\bar{D}^{*0}K^-$, Phys. Lett. B832 (2022), Dai, Molina, Oset,
  $\bar{B}^0 \to D^{*+}K^-\bar{K}^{*0}$, Phys. Rev. D105 (2022); and the 2^+ state:
  $B^+ \to D^+D^-K^+$, Phys. Lett. B833 (2022), Bayar and Oset.
- The $T_{c\bar{s}}(2900)$ is more likely to be a failed bound state, or cusp structure around the $D^*K^*, D_s^*\rho$ thresholds. The width of the ρ is responsible for most of its width. There should be a bound state for $J^P = 2^+$ with mass of around 2830 MeV and width 20 MeV.
- The study of the pion mass dependence of the $D_{s0}(2317)$ supports the DK molecular picture