Finite volume corrections for form factors of two hadrons

Felipe Ortega-Gama

William & Mary

felortga@jlab.org

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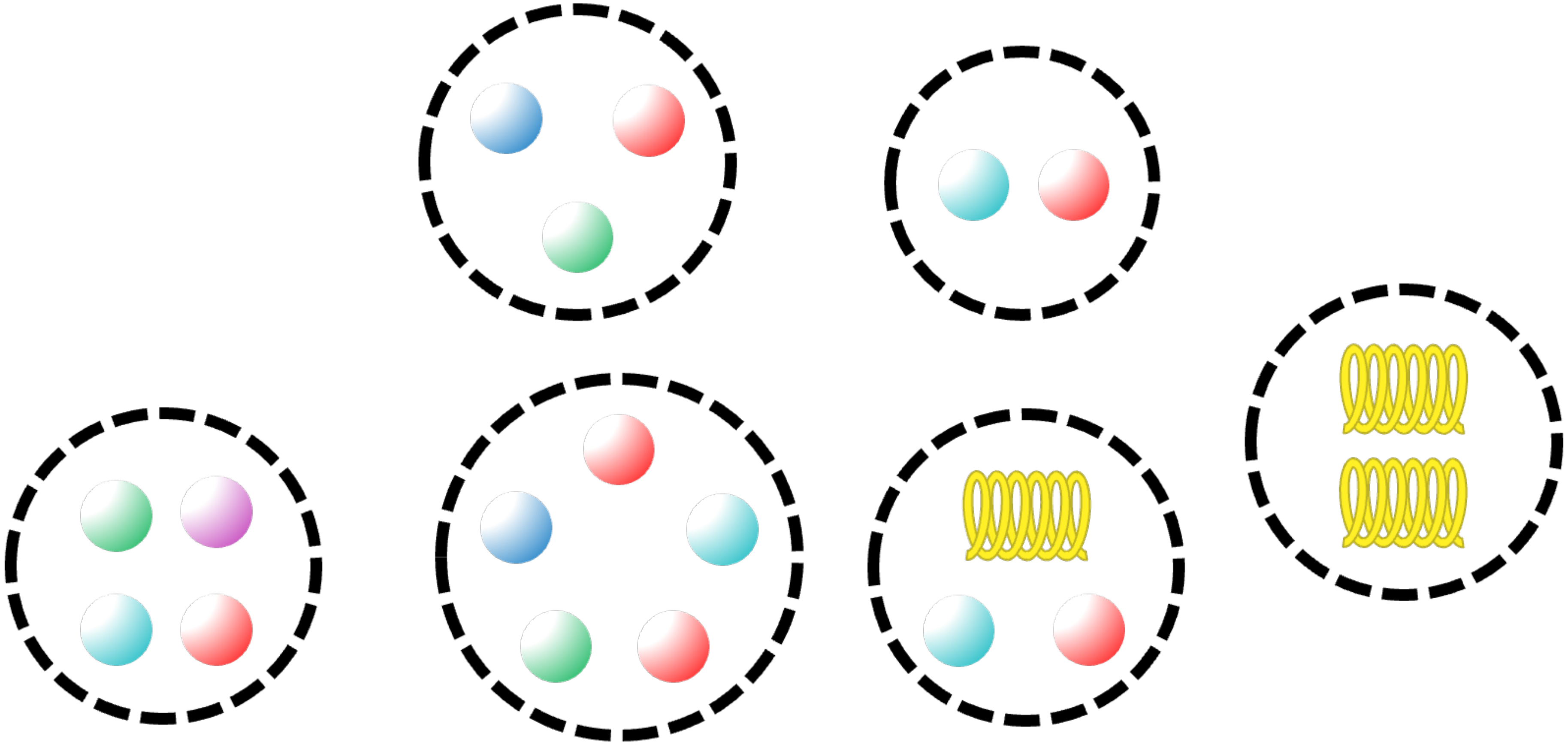
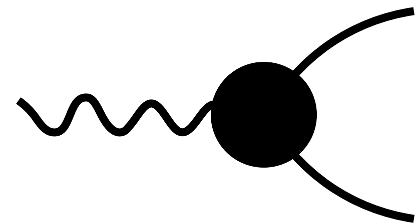
Characterization of hadrons



- Spin - parity identification, e.g. 1^{-+} exotic channels
- Masses
- Couplings to hadronic decay channels
- *Couplings to electroweak currents*

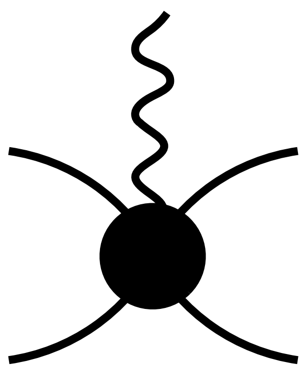
Infinite Volume transition amplitudes from lattice QCD

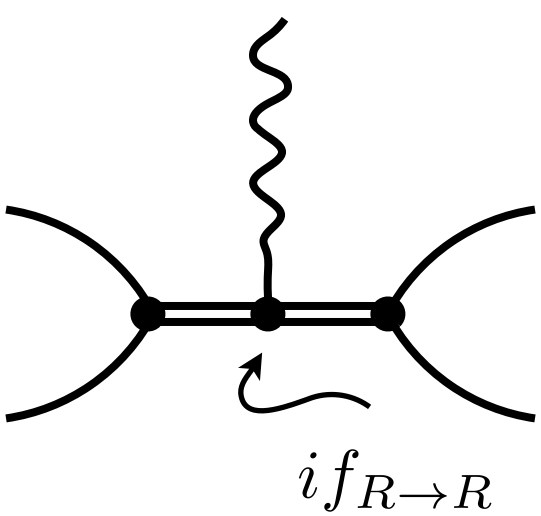
$\mathcal{J} \to 2$



[arXiv:1105.1892] Meyer, [arXiv:0003023] Lellouch, Lüscher

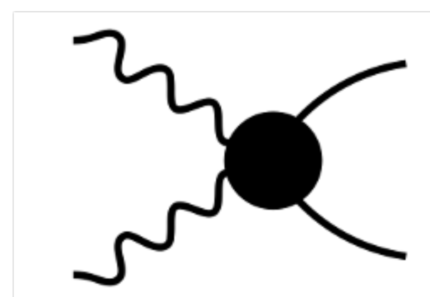
$2 + \mathcal{J} \to 2$

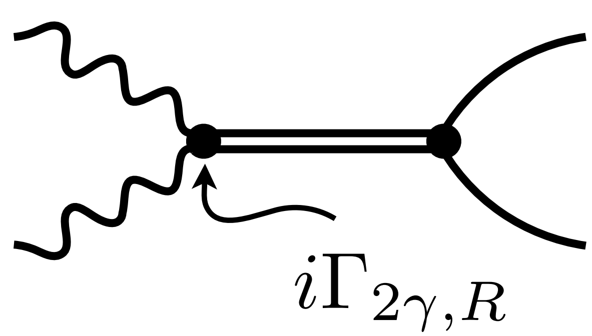




[arXiv:1812.10504] FO, etal

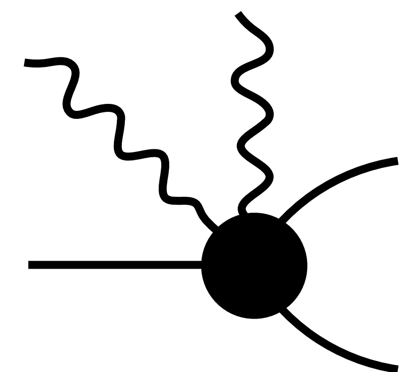
$\mathcal{J} + \mathcal{J} \to 2$

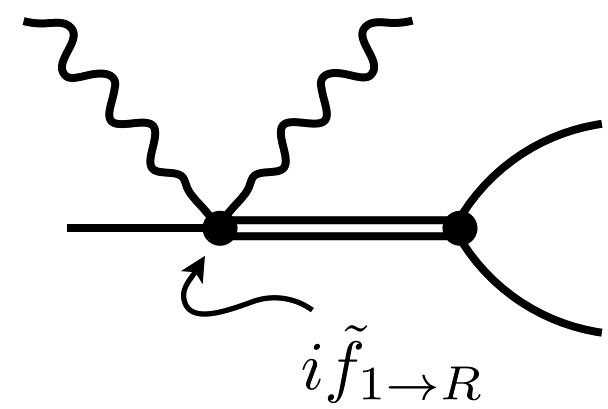




[arXiv:2210.08051] Briceño, etal.

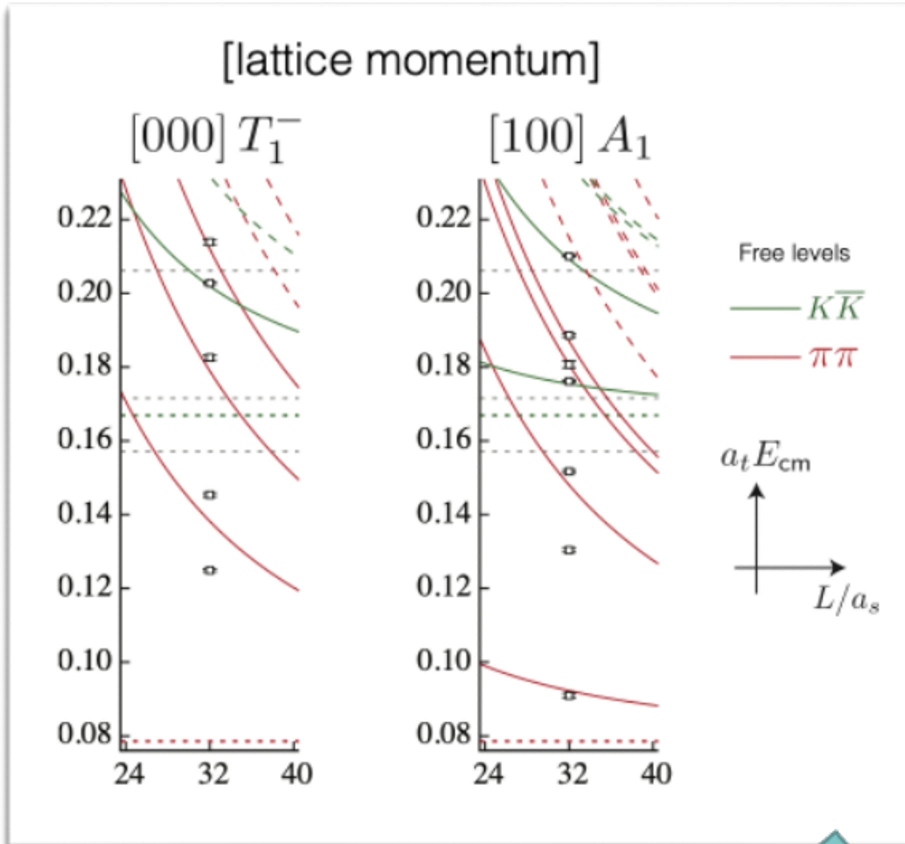
$1 + \mathcal{J} \to 2 + \mathcal{J}$

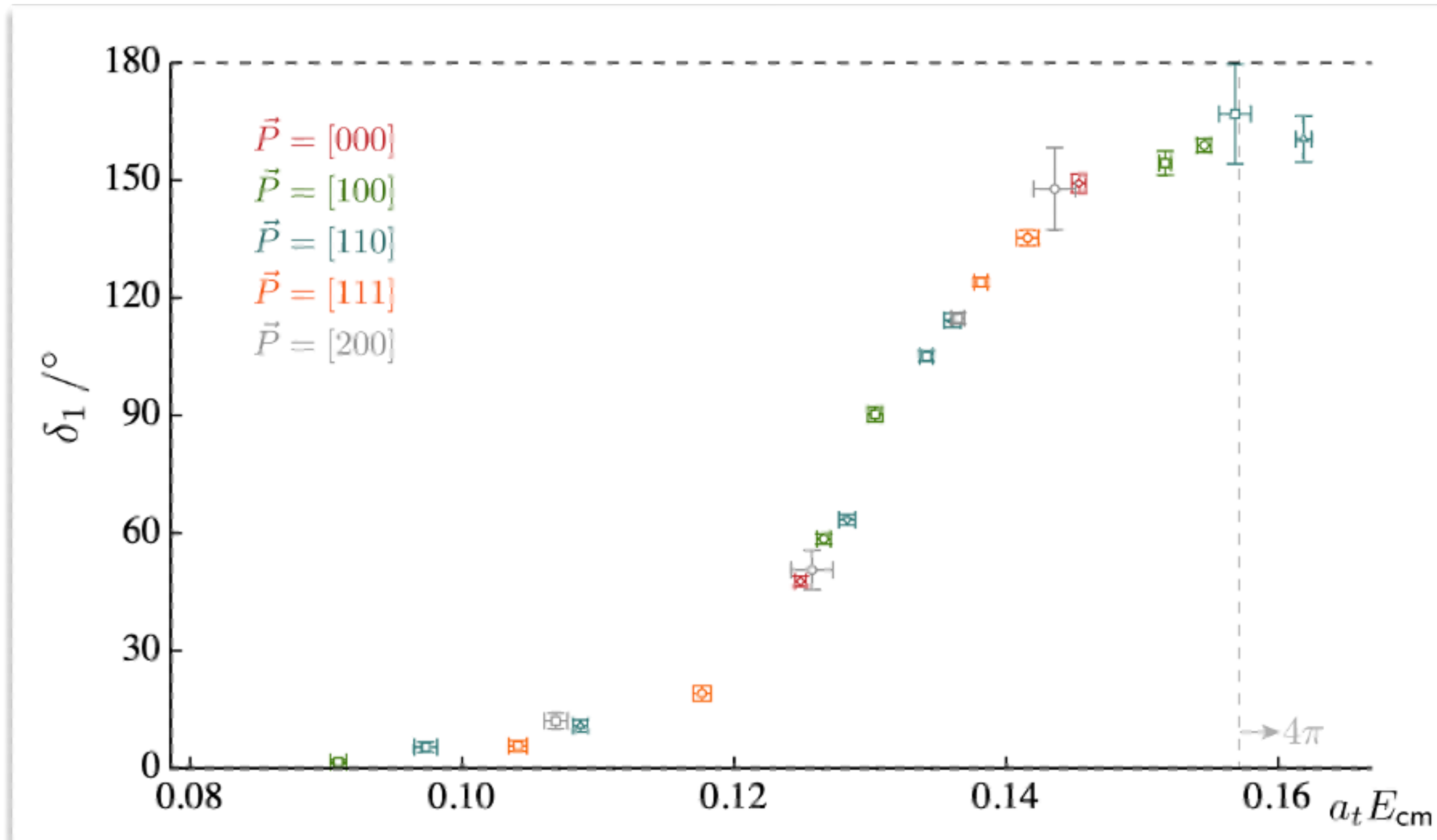




[arXiv:2202.02284] FO, etal.

Scattering in the finite volume





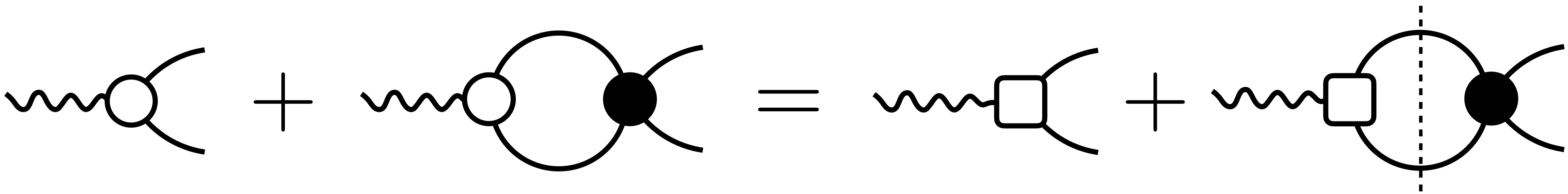
$$\det\left(F^{-1}(E_n, L) + \mathcal{M}(E_n)\right) = 0$$

Finite volume correlation function poles in momentum space

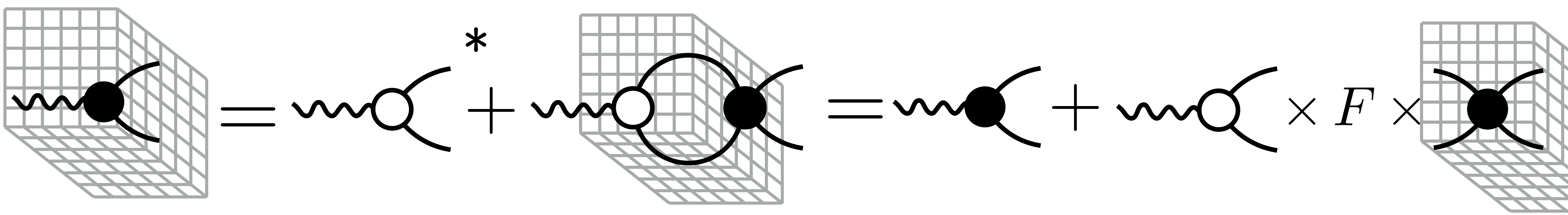
Known geometric function

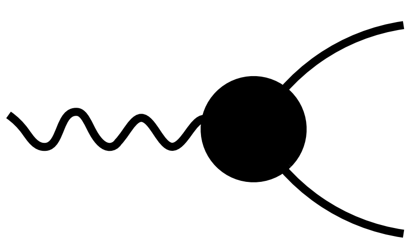
Finite volume corrections

Infinite volume: Watson's theorem. $\mathcal{H}(s) = \mathcal{A}_{02}(s)\,\mathcal{M}(s)$

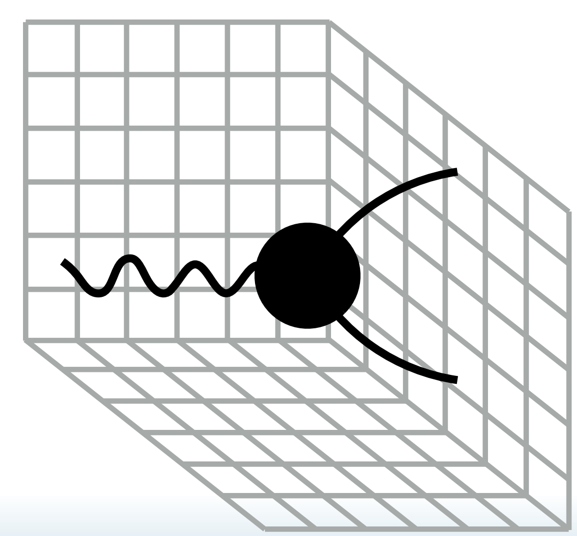


Finite volume: Lellouch-Lüscher factor.



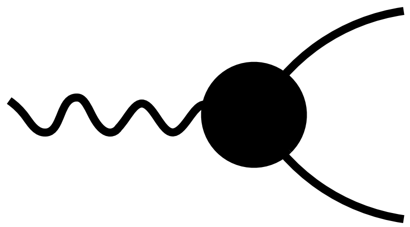


$$\times \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + \mathcal{M}} =$$

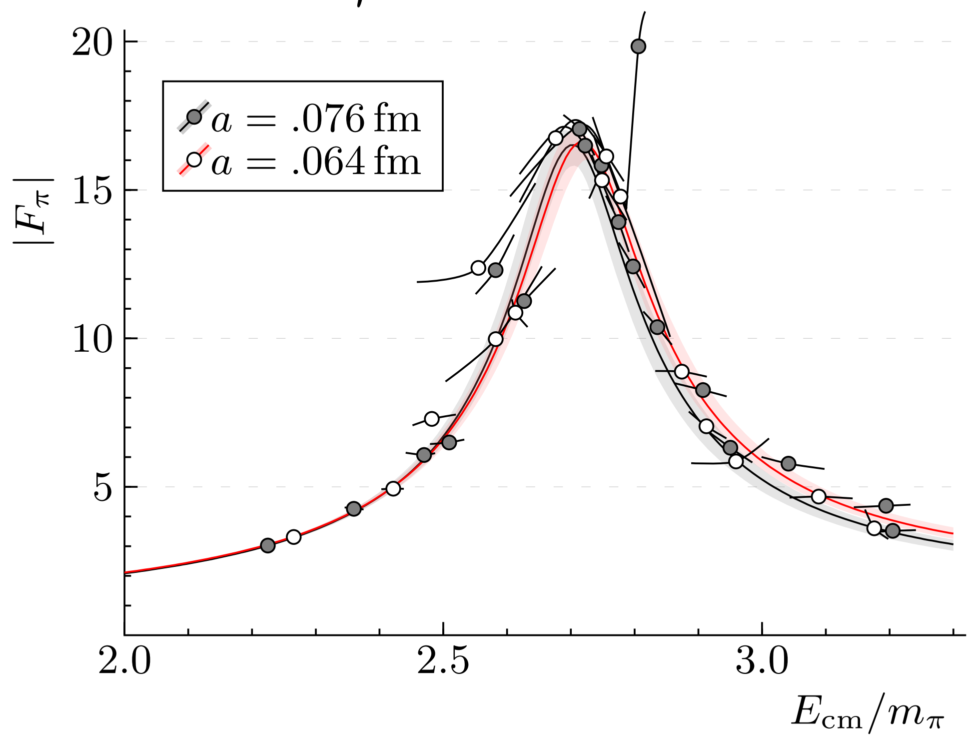


*Up to exponentially small corrections

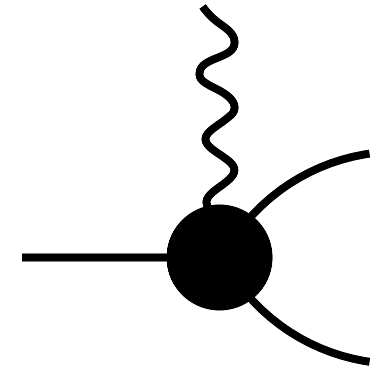
Infinite volume extractions



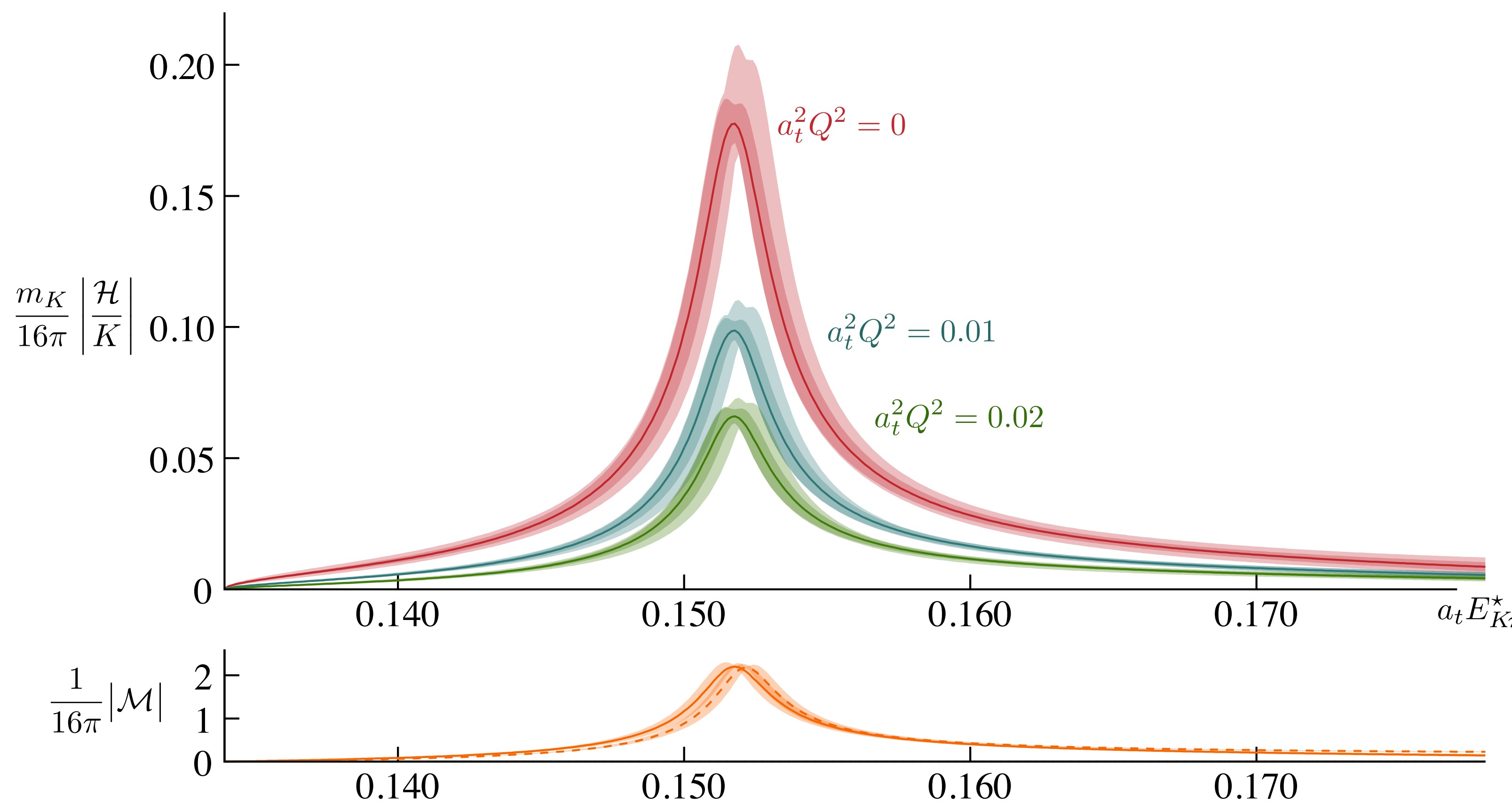
$\gamma \to \pi\pi$



[arXiv:1808.05007] Andersen, Bulava, etal.



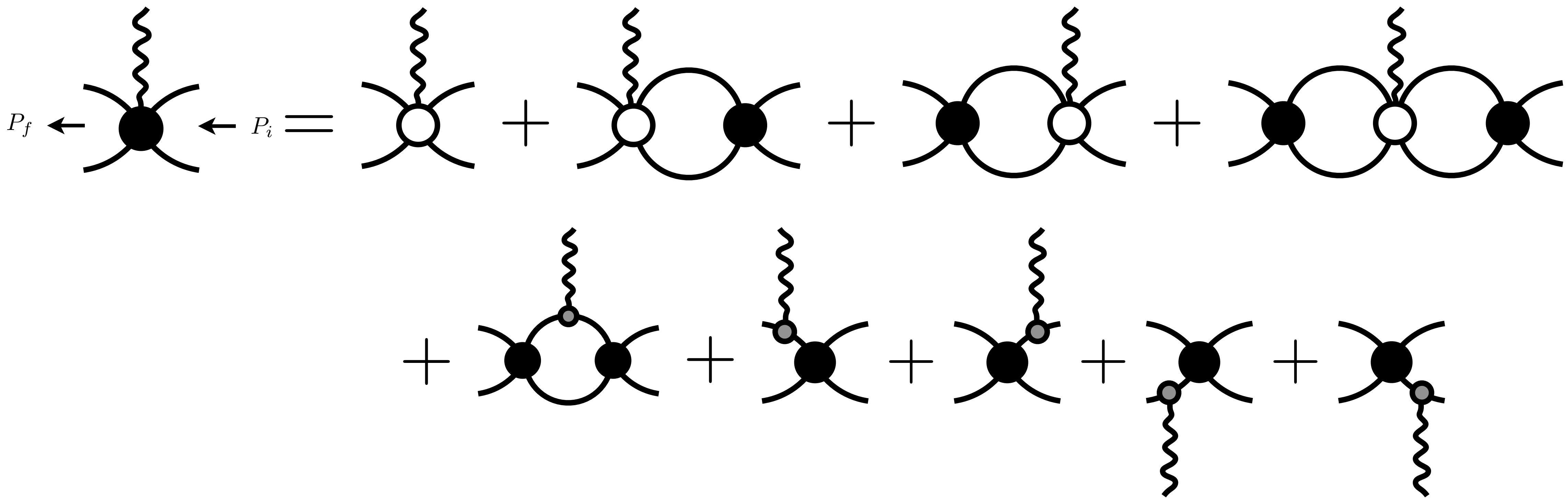
$K + \gamma \to K\pi$



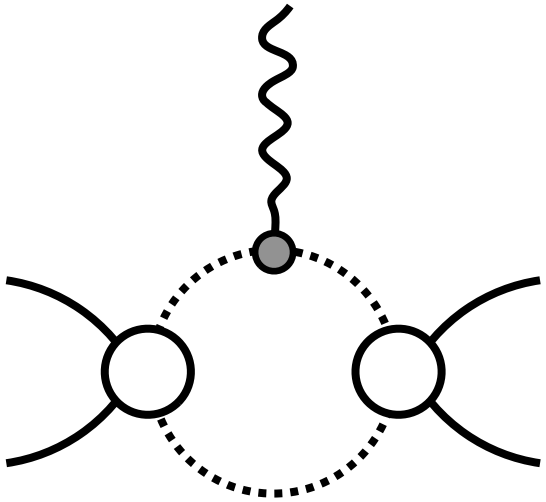
$$m_\pi \approx 280 \text{ MeV}$$

[arXiv:2208.13755] Radhakrishnan, Dudek, etal.

Infinite volume analytic structure: $\mathcal{W}$

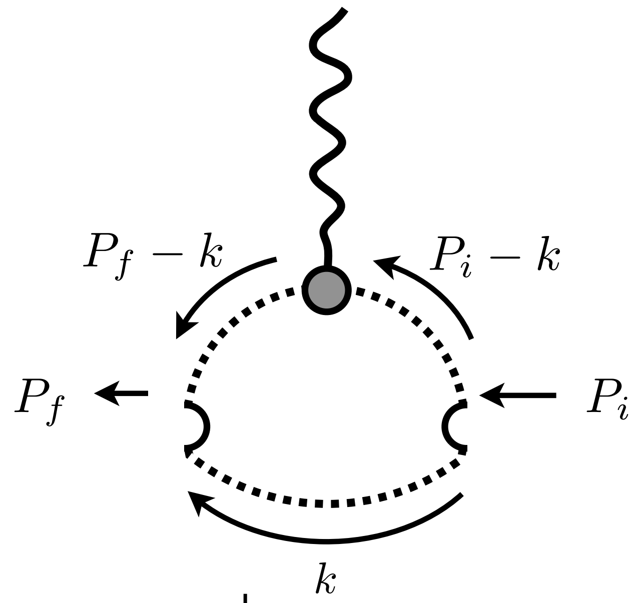


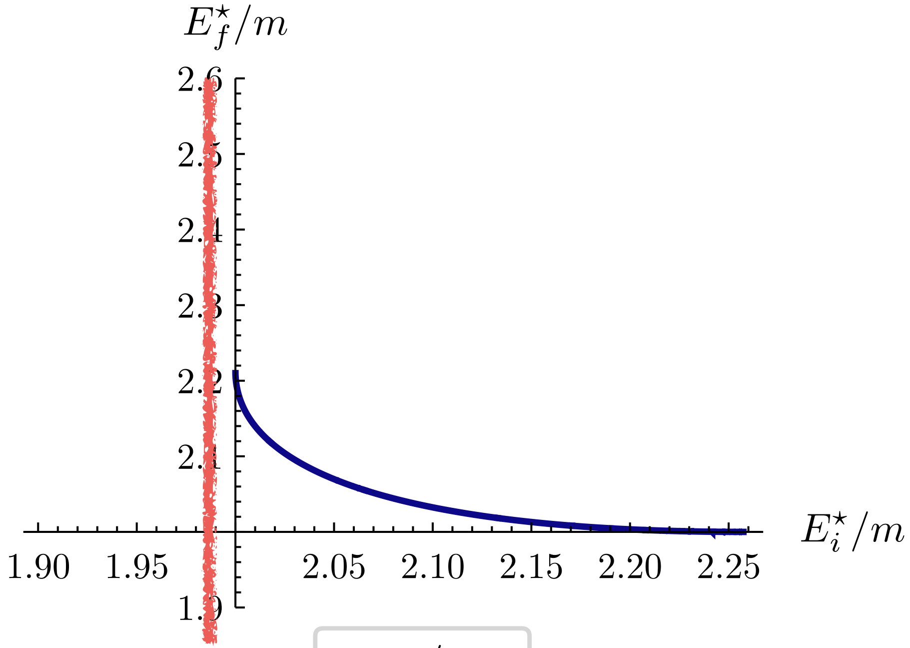
A singularity structure in s channel loops not present in the two-body scattering amplitude

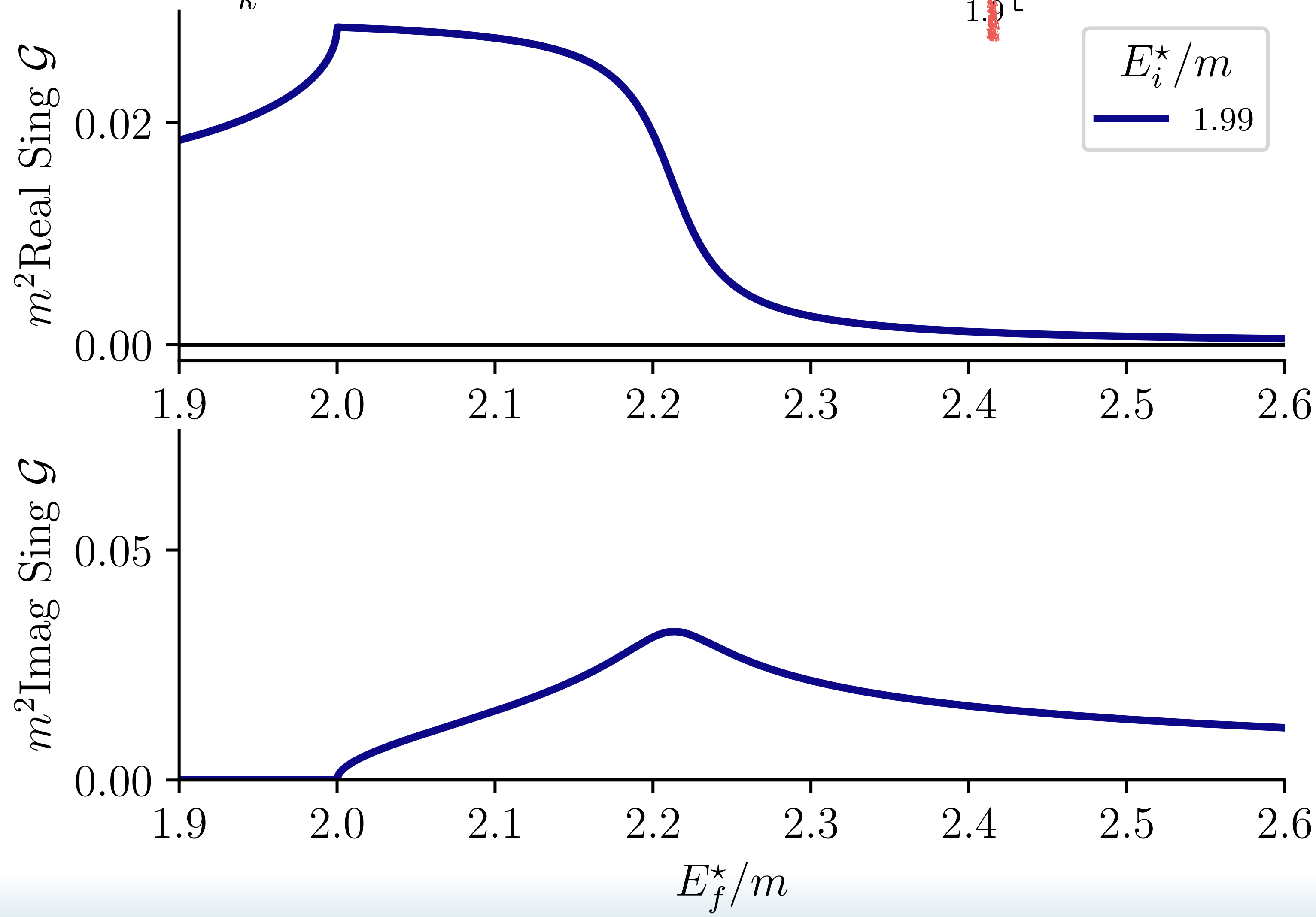


Infinite volume analytic structure: $\mathcal{W}$

$$f \cdot \mathcal{G} =$$







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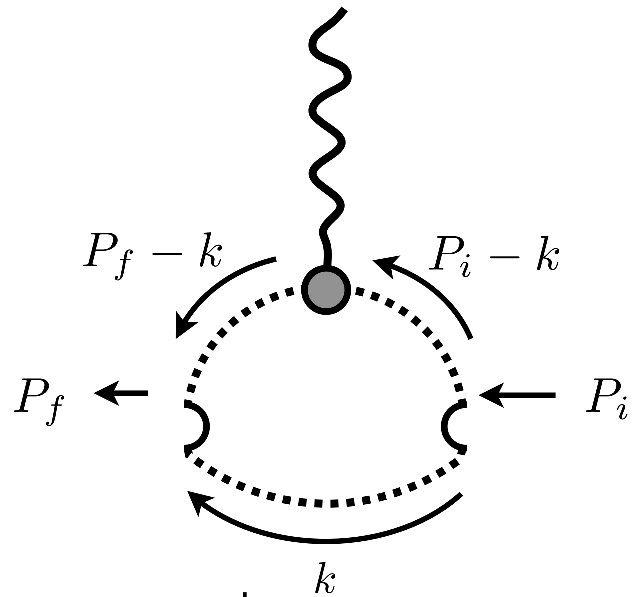
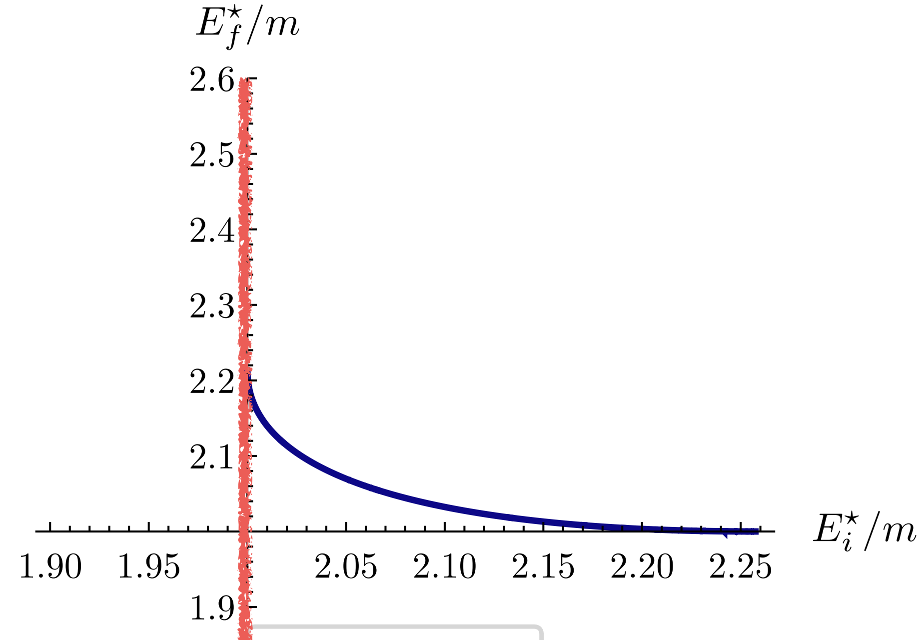
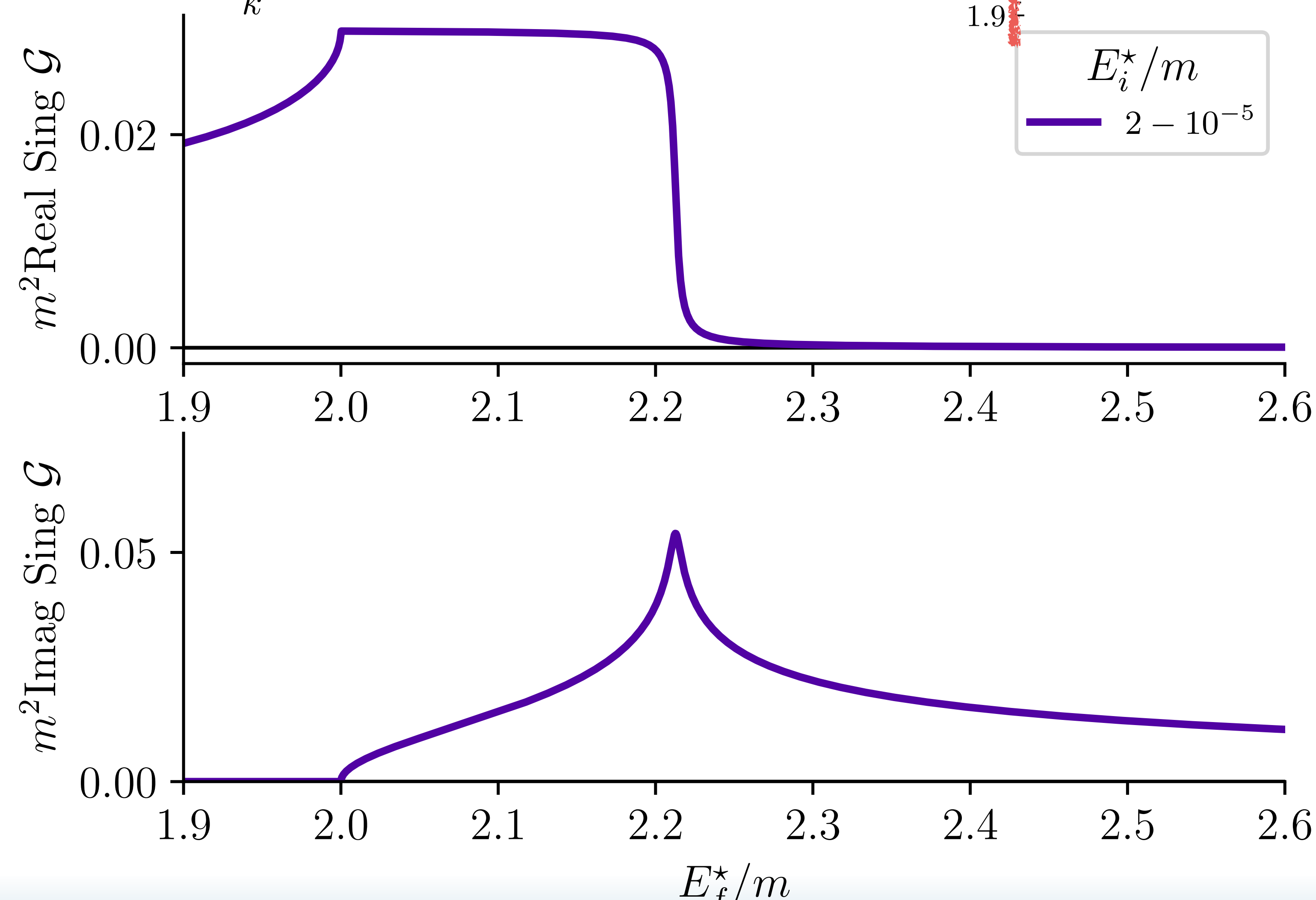
$f \cdot \mathcal{G} =$ 





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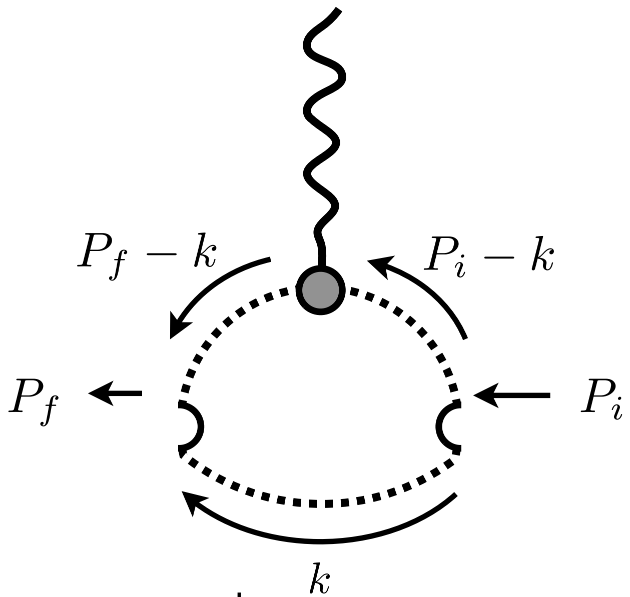
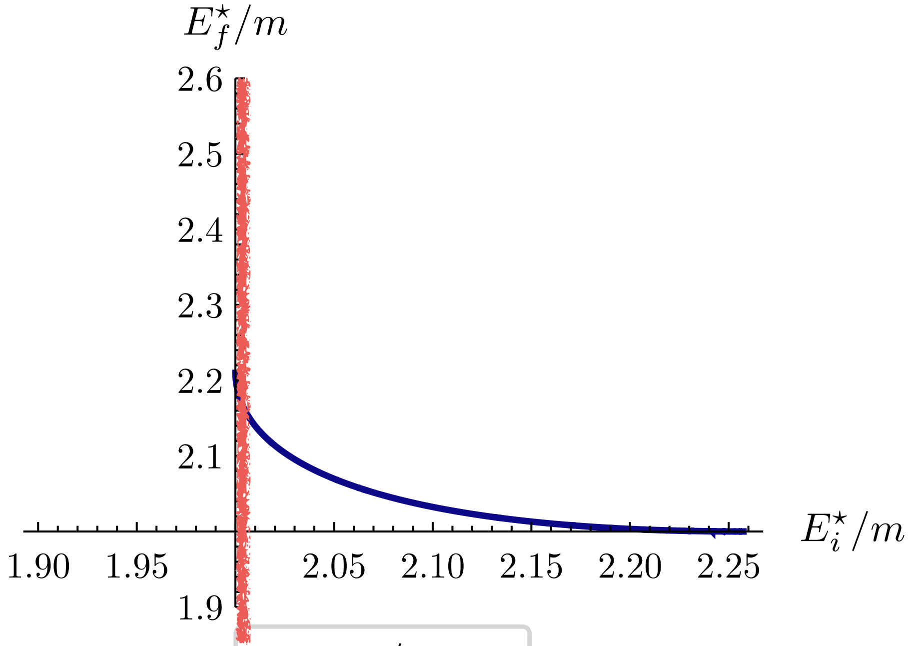
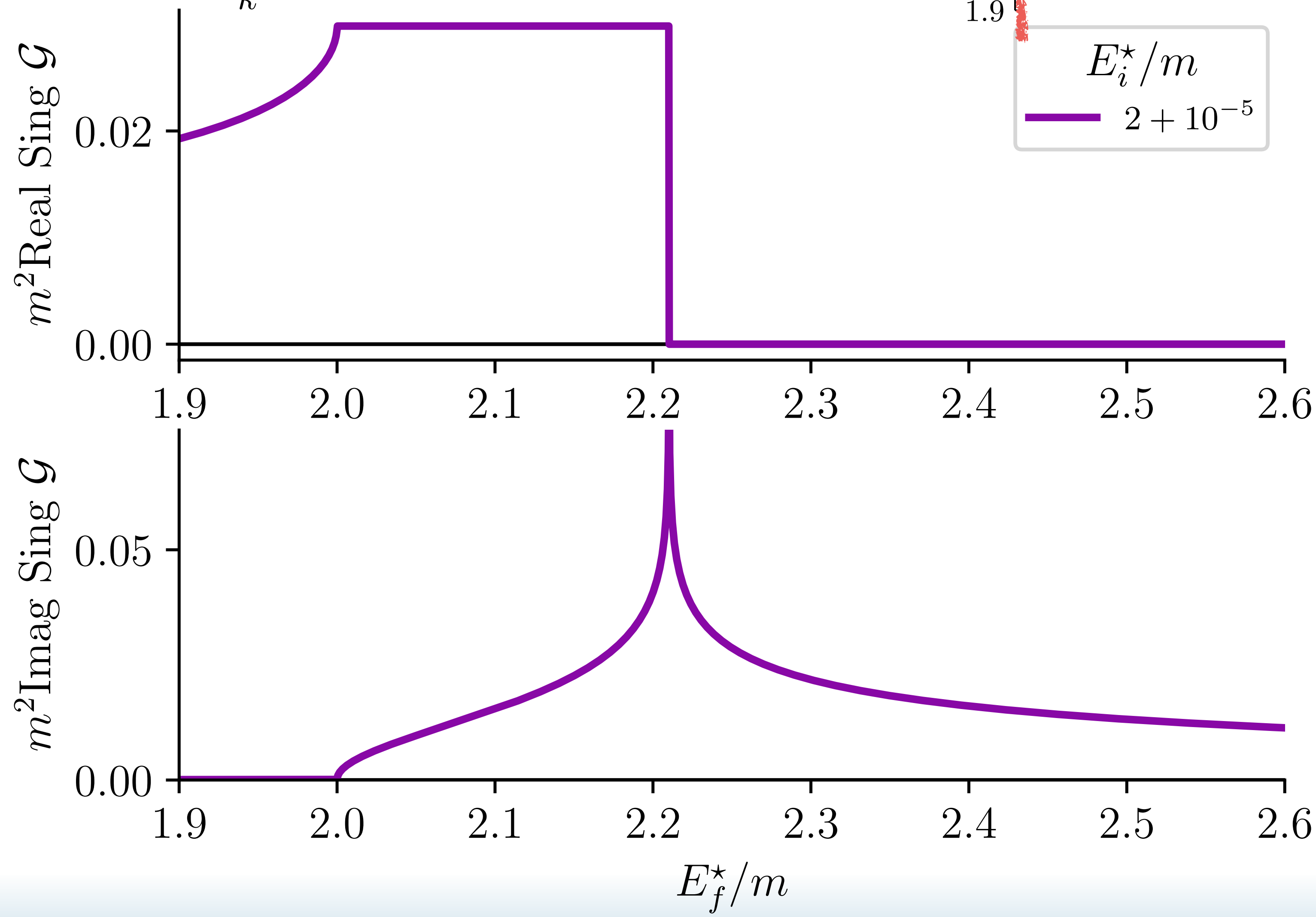
$f \cdot \mathcal{G} =$ 





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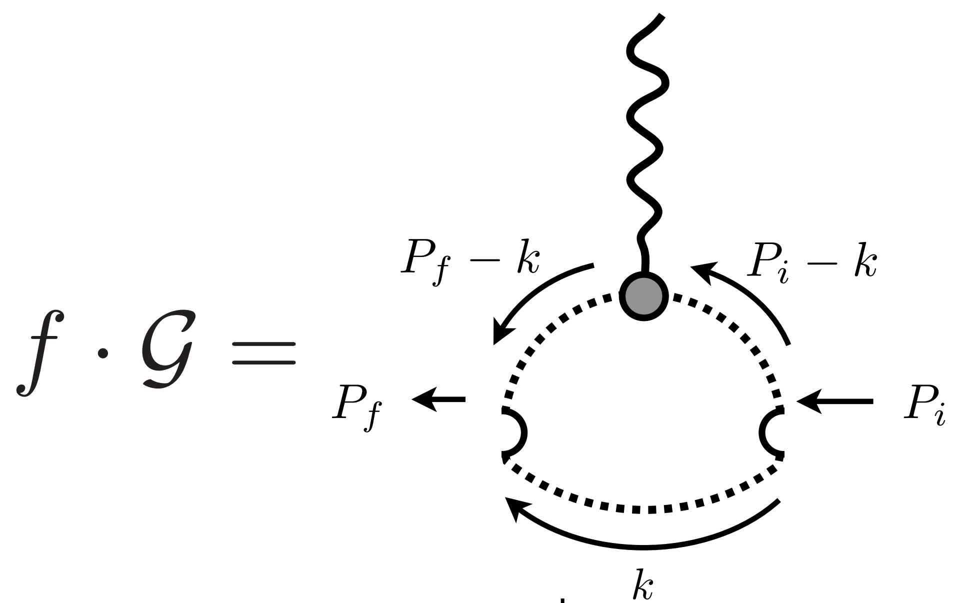
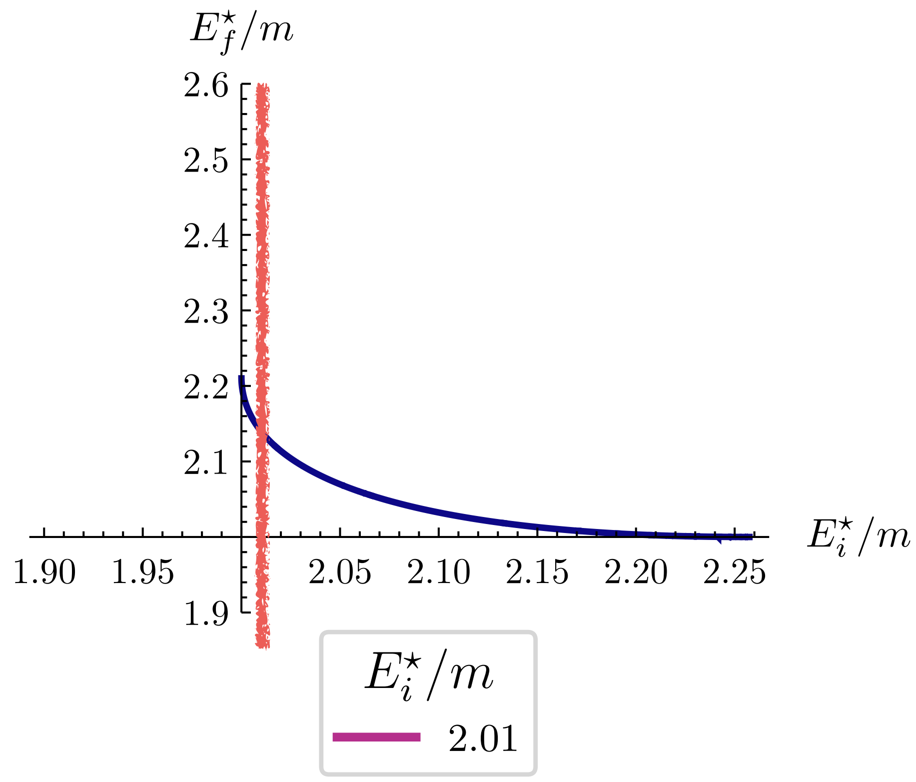
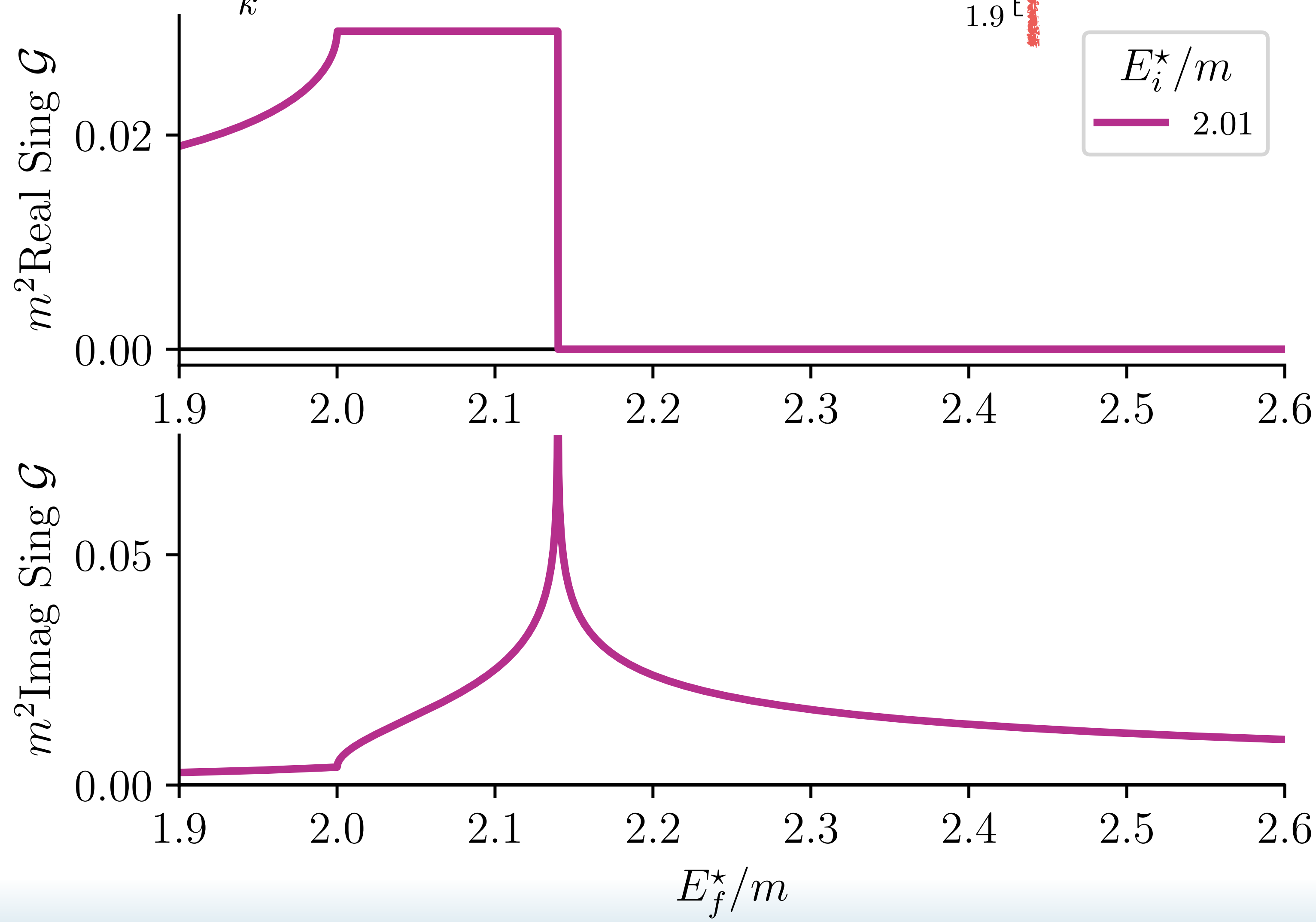
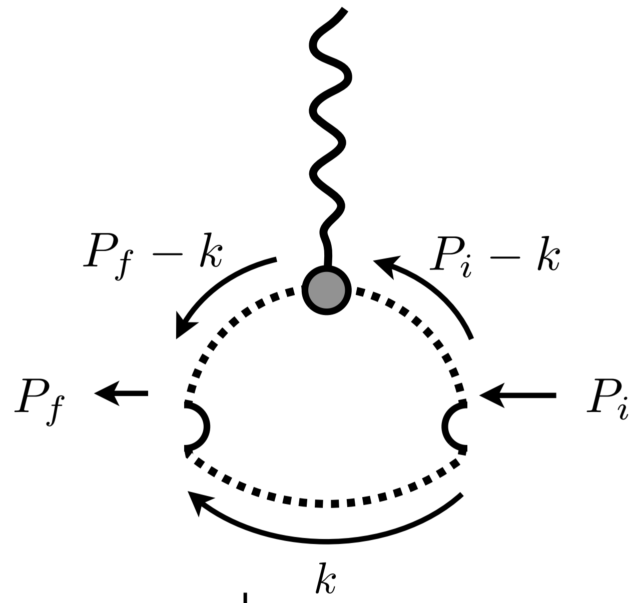


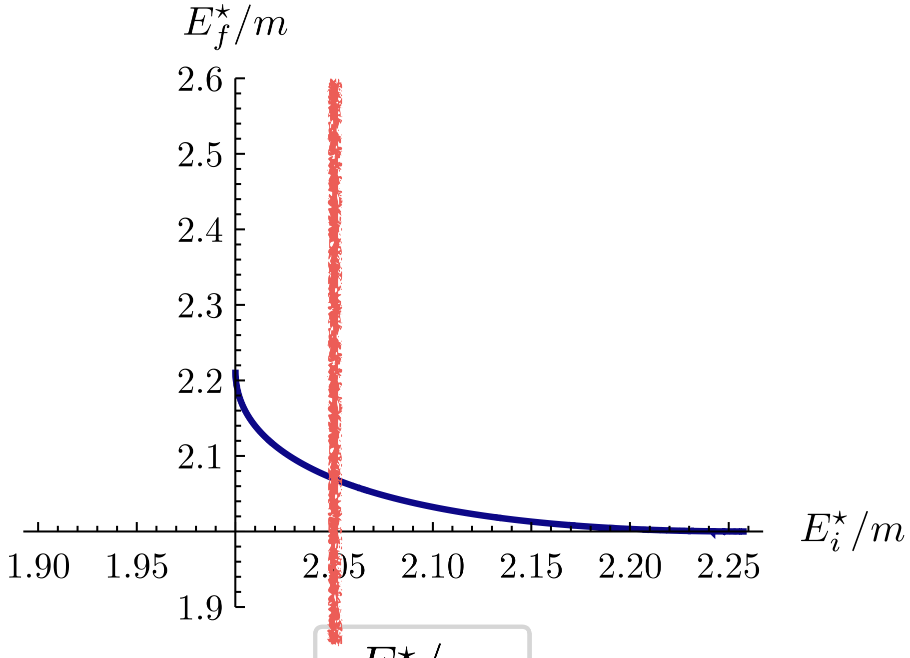


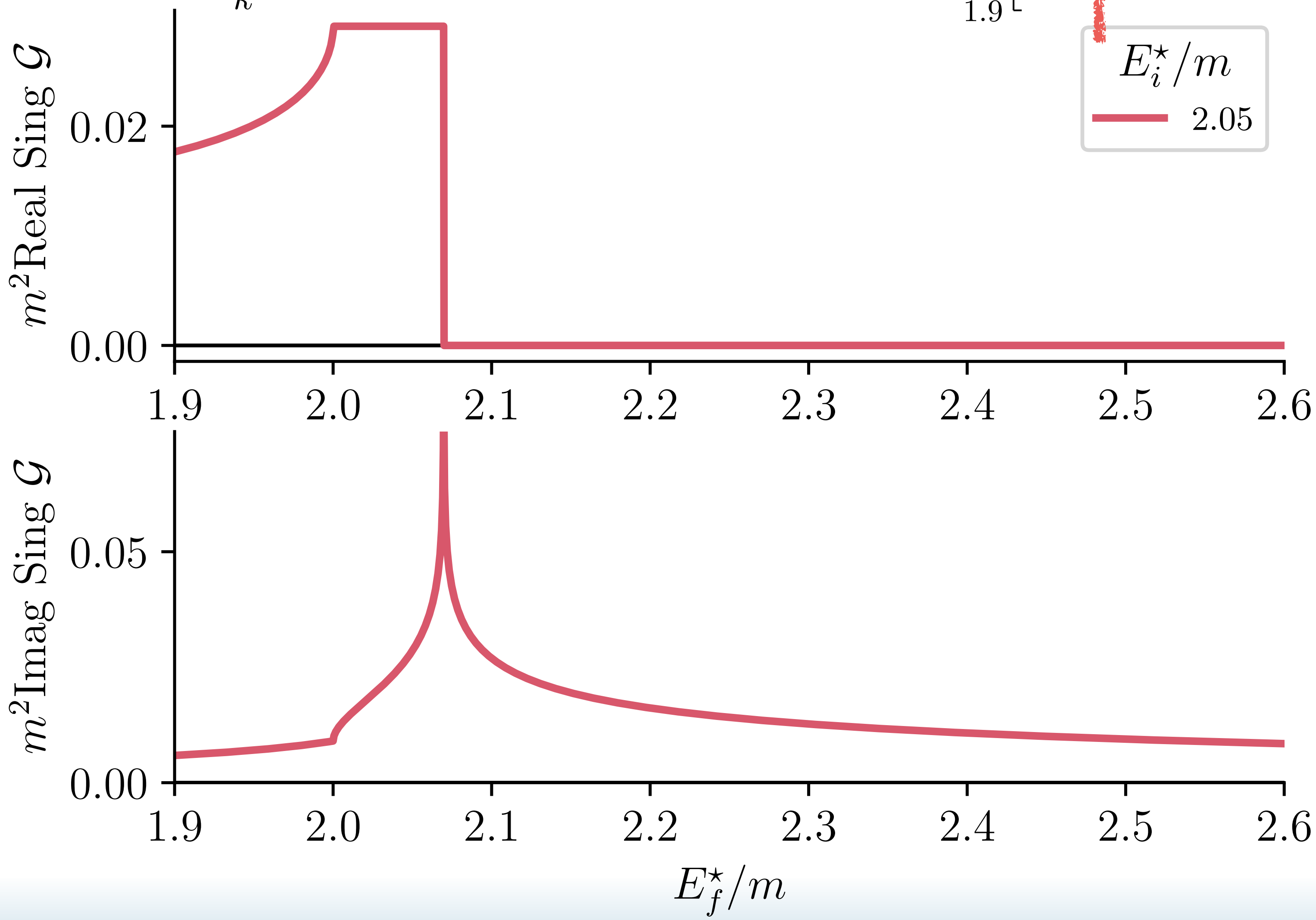


Infinite volume analytic structure: $\mathcal{W}$

$f \cdot \mathcal{G} =$







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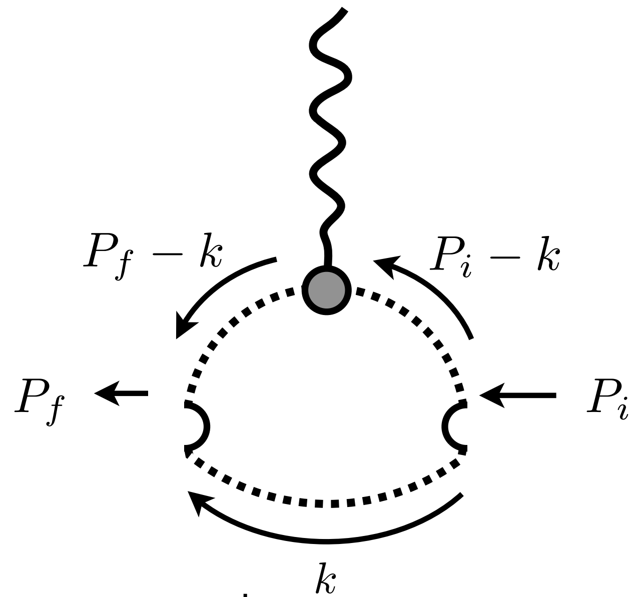
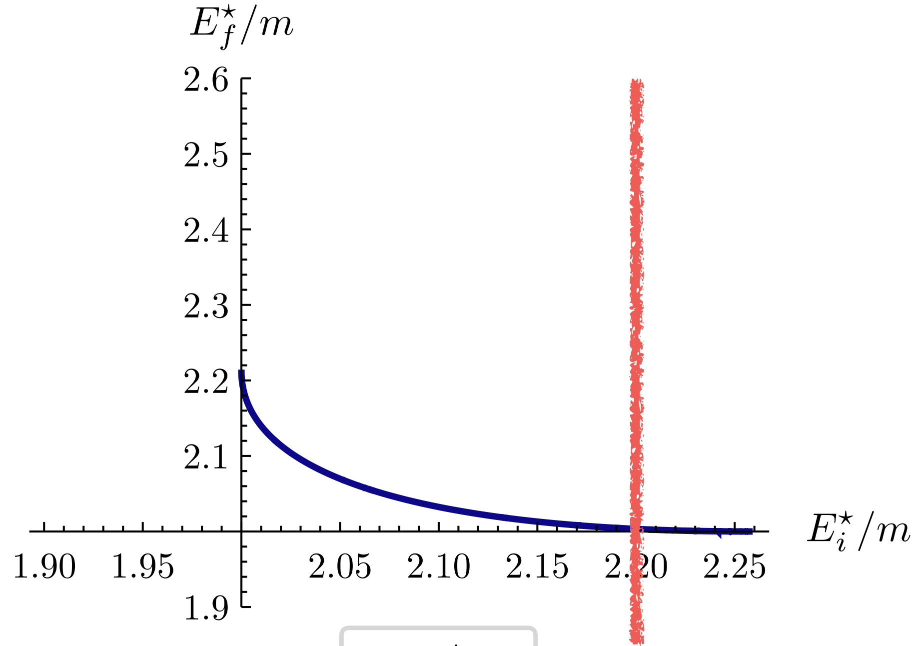
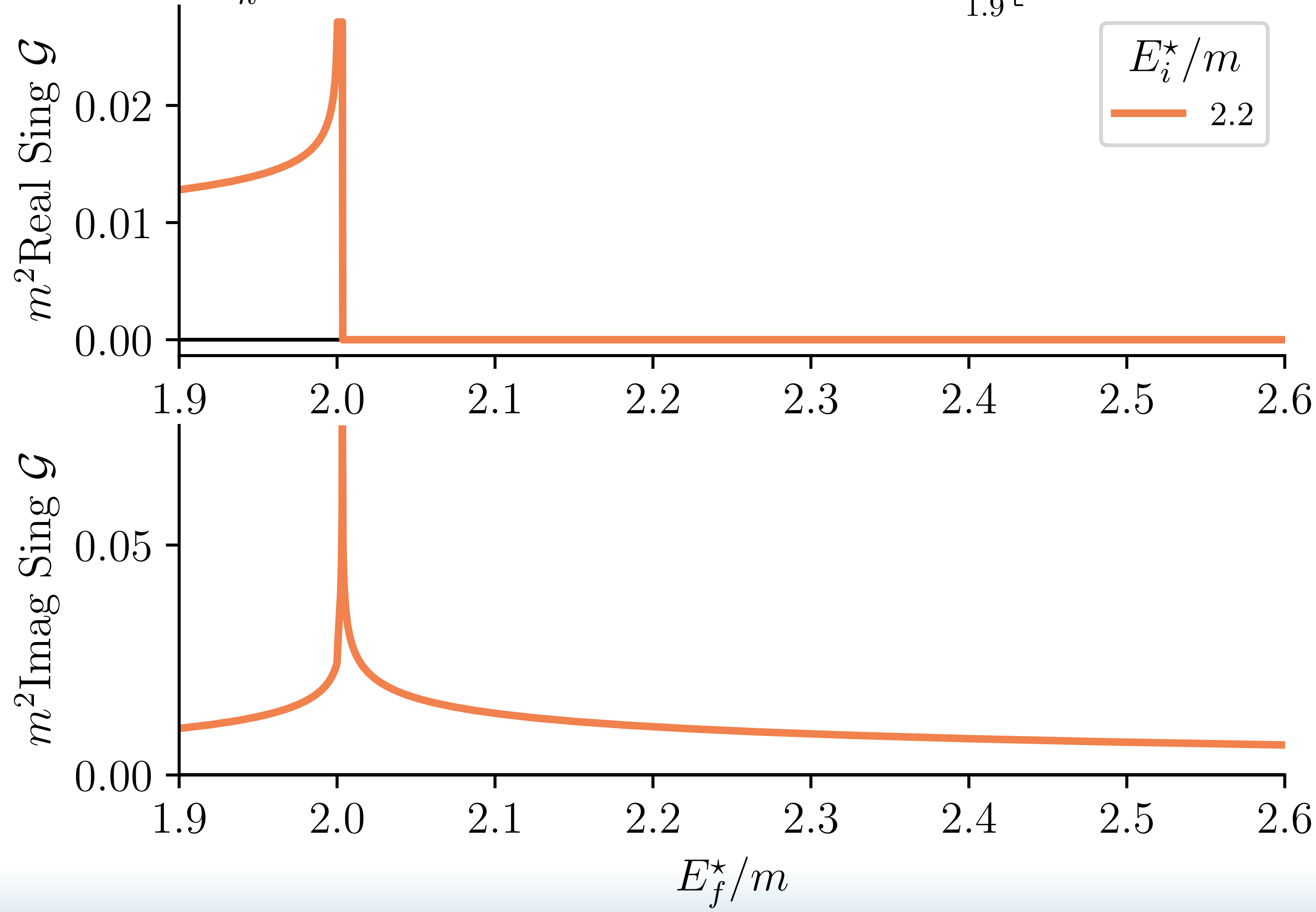
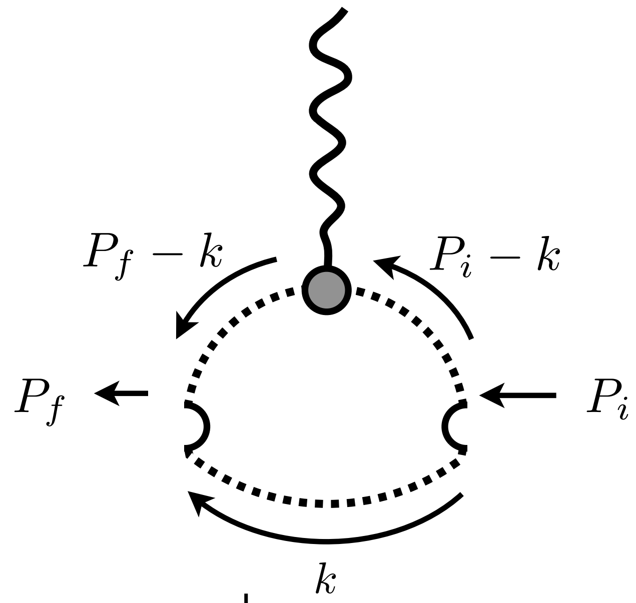
$f \cdot \mathcal{G} =$ 

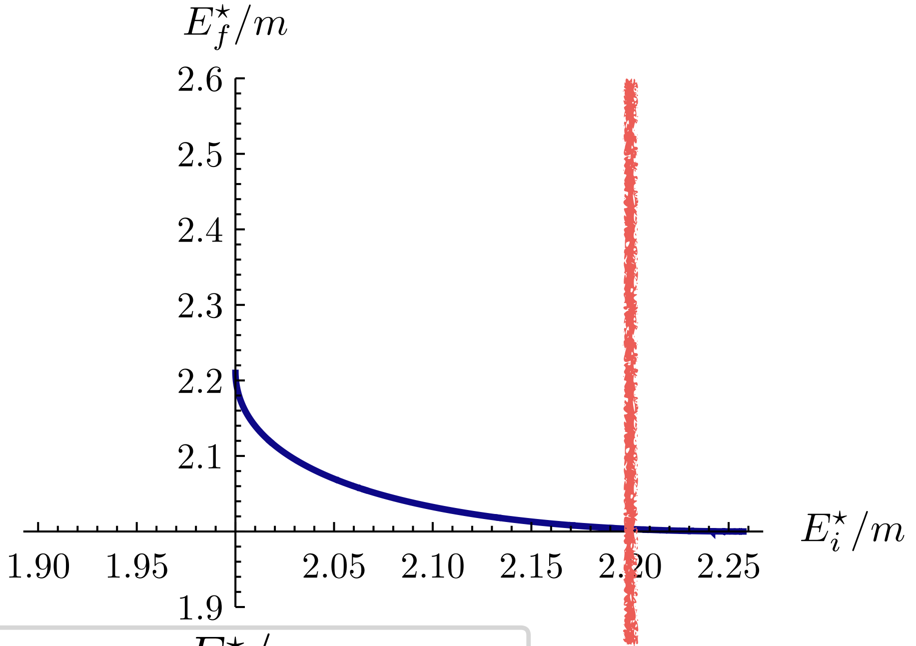


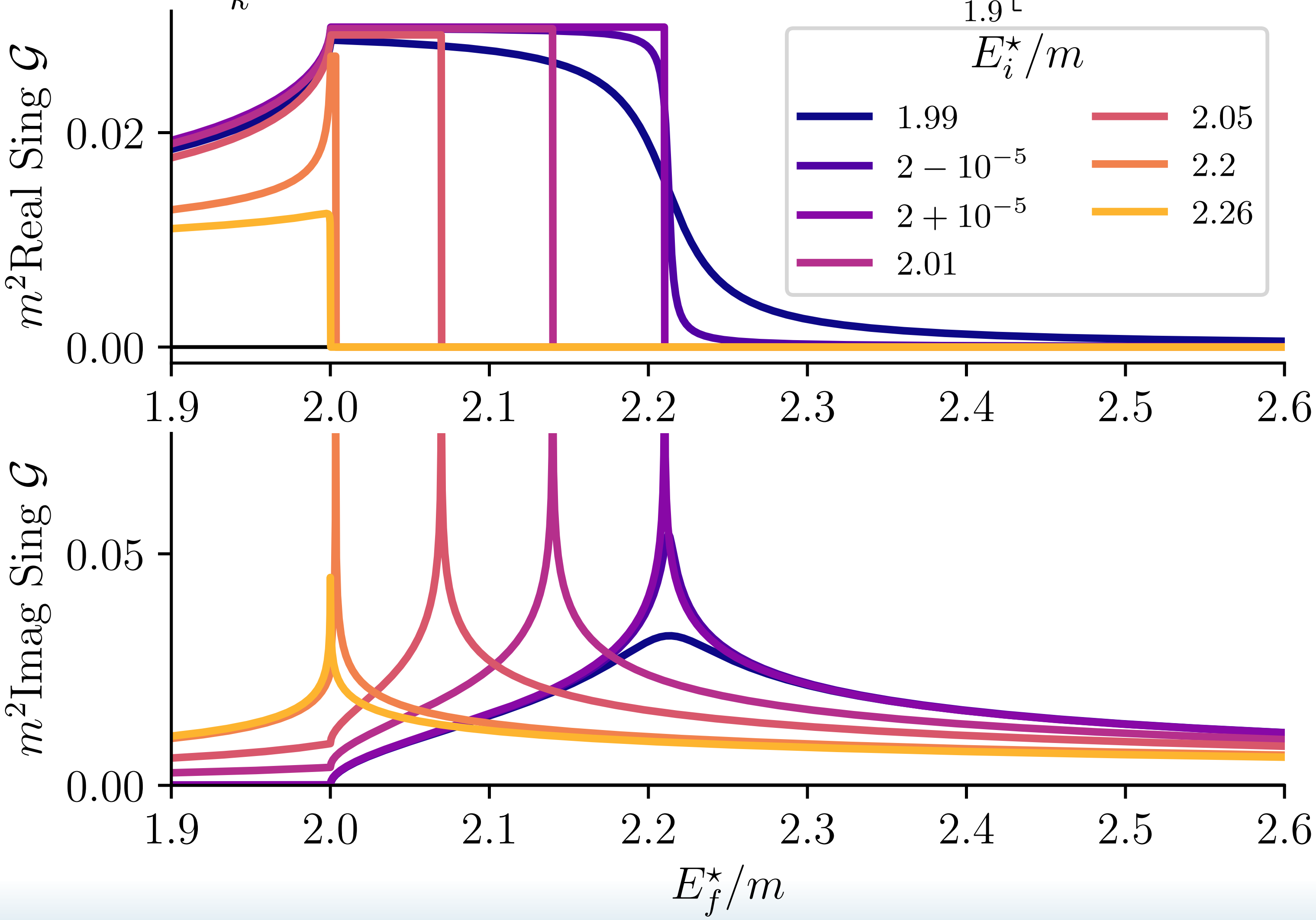


Infinite volume analytic structure: $\mathcal{W}$

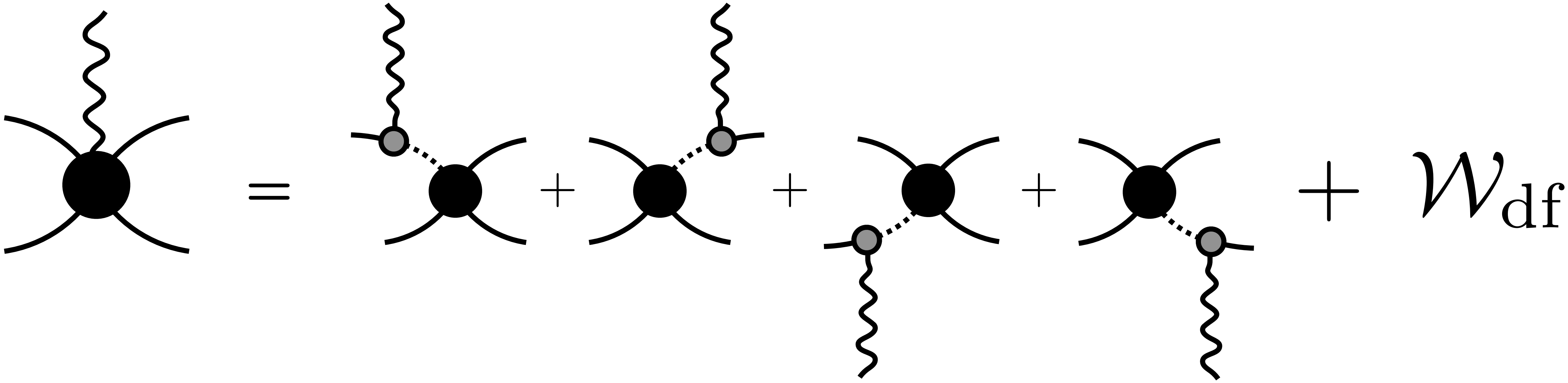
$$f \cdot \mathcal{G} =$$



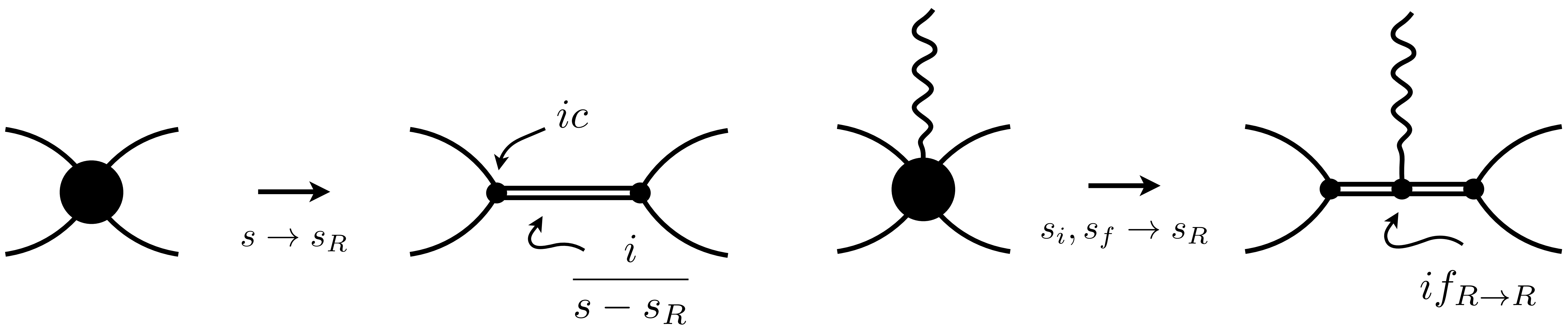




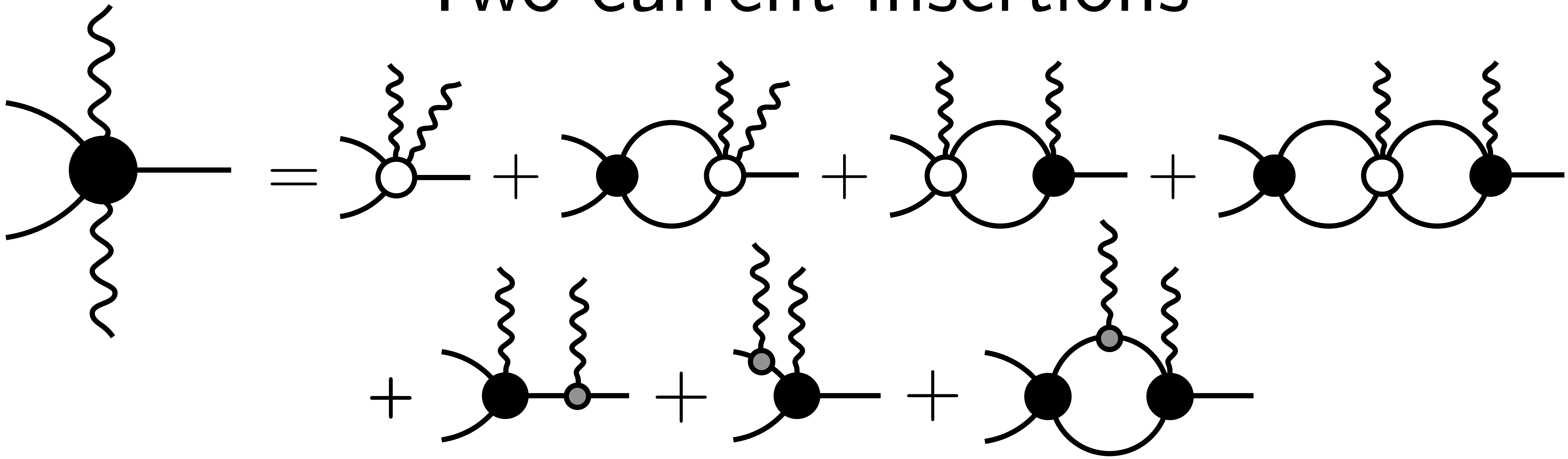
Resonance form factors

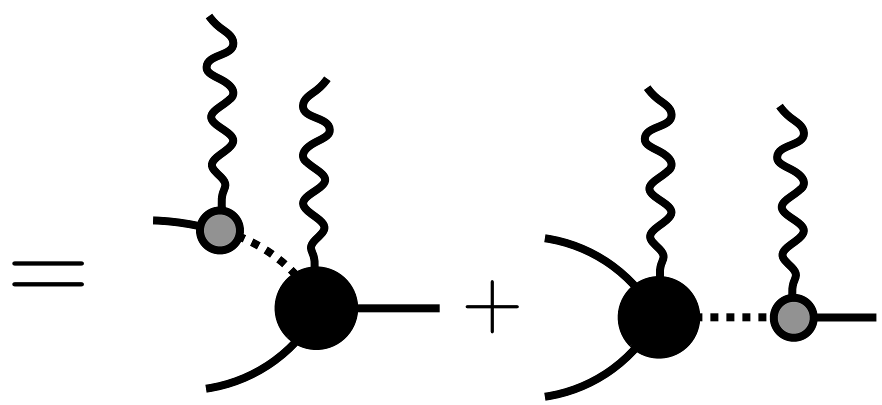


$$\mathcal{W}_{\rm df} = \mathcal{M}(\mathcal{A}_{22} + f \cdot \mathcal{G})\mathcal{M}$$



Two current insertions





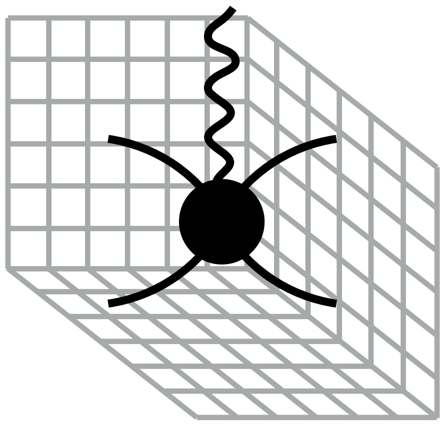
$+\mathcal{M}\,\mathcal{B}_{21} + \mathcal{W}_{\mathrm{df}}\,\mathcal{A}_{21}$

Extracted from the respective subprocess

Finite volume two-hadron transitions

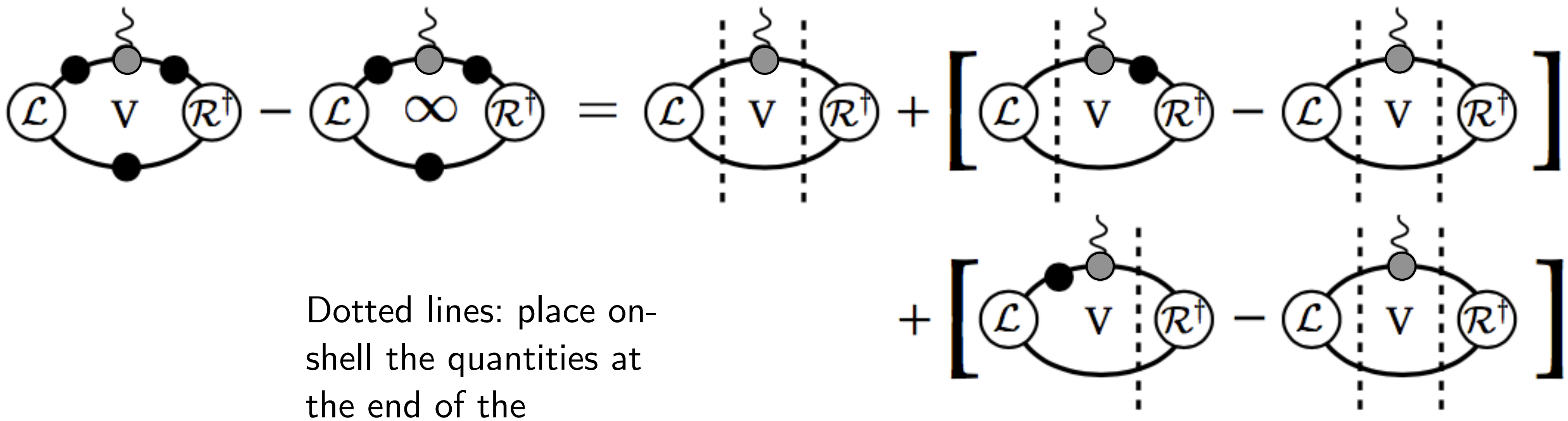
$$\left\langle \mathcal{O}_{\pi\pi}(\tau_2)\mathcal{J}(\tau_1)\mathcal{O}^\dagger_{\pi\pi}(0)\right\rangle_L \sim \sum_{n,m} \langle E_n|\mathcal{J}(0)|E_m\rangle_L \, e^{-E_n(\tau_2-\tau_1)-E_m\tau_1}$$

$$\lim_{E\to E_n} \frac{E-E_n}{F^{-1}+\mathcal{M}} \times \underbrace{(\mathcal{W}_{\rm df}+\mathcal{M}[f\cdot G]\mathcal{M})}_{\mathcal{W}_L} \times \underbrace{\lim_{E\to E_n} \frac{E-E_n}{F^{-1}+\mathcal{M}}}_{\mathcal{R}} =$$



$$|\langle E_n|\mathcal{J}(0)|E_m\rangle_L|^2 = \mathrm{tr}\big(\mathcal{R}(E_n)\mathcal{W}_L(E_n,E_m,Q^2)\mathcal{R}(E_m)\mathcal{W}_L(E_m,E_n,Q^2)\big)$$

Finite volume corrections of the triangle diagram

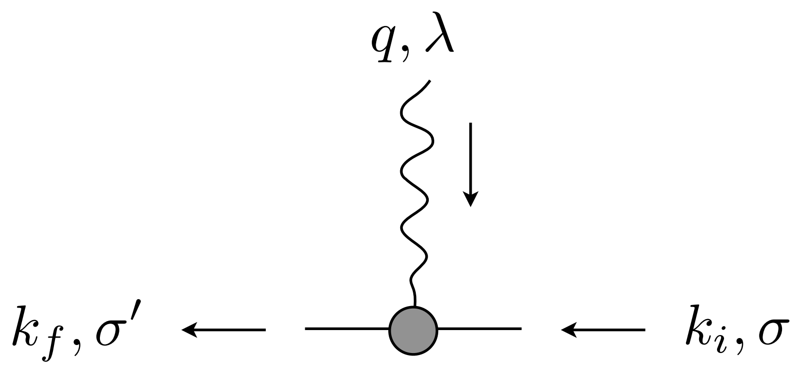


$+\mathcal{O}(e^{-mL})$

Dotted lines: place on-shell the quantities at the end of the propagator.

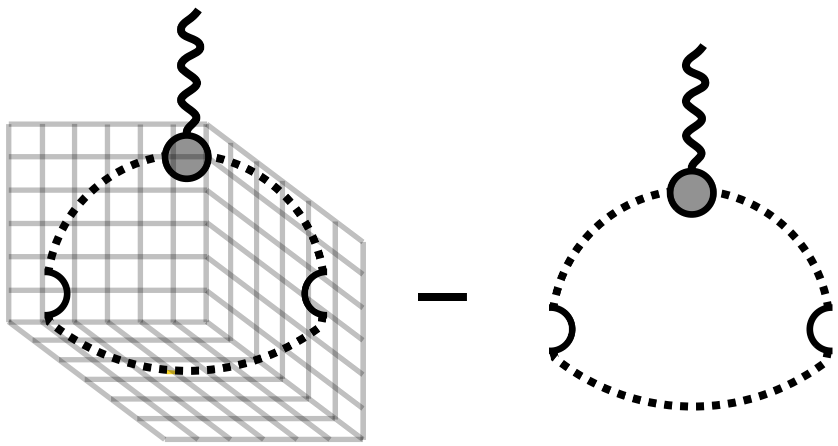
Prescription is needed for on shell expansion

$w^{\lambda}_{\sigma,\sigma'} =$



$$w^{\lambda}_{\sigma,\sigma'}(Q^2, k_f^2, k_i^2) = w^{\lambda}_{\sigma,\sigma'}(Q^2) + \delta w^{\lambda}_{\sigma,\sigma'}(Q^2, k_i^2) + w^{\lambda}_{\sigma,\sigma'}(Q^2, k_f^2)\delta + \delta w^{\lambda}_{\sigma,\sigma'}(Q^2, k_f^2, k_i^2)\delta\,.$$

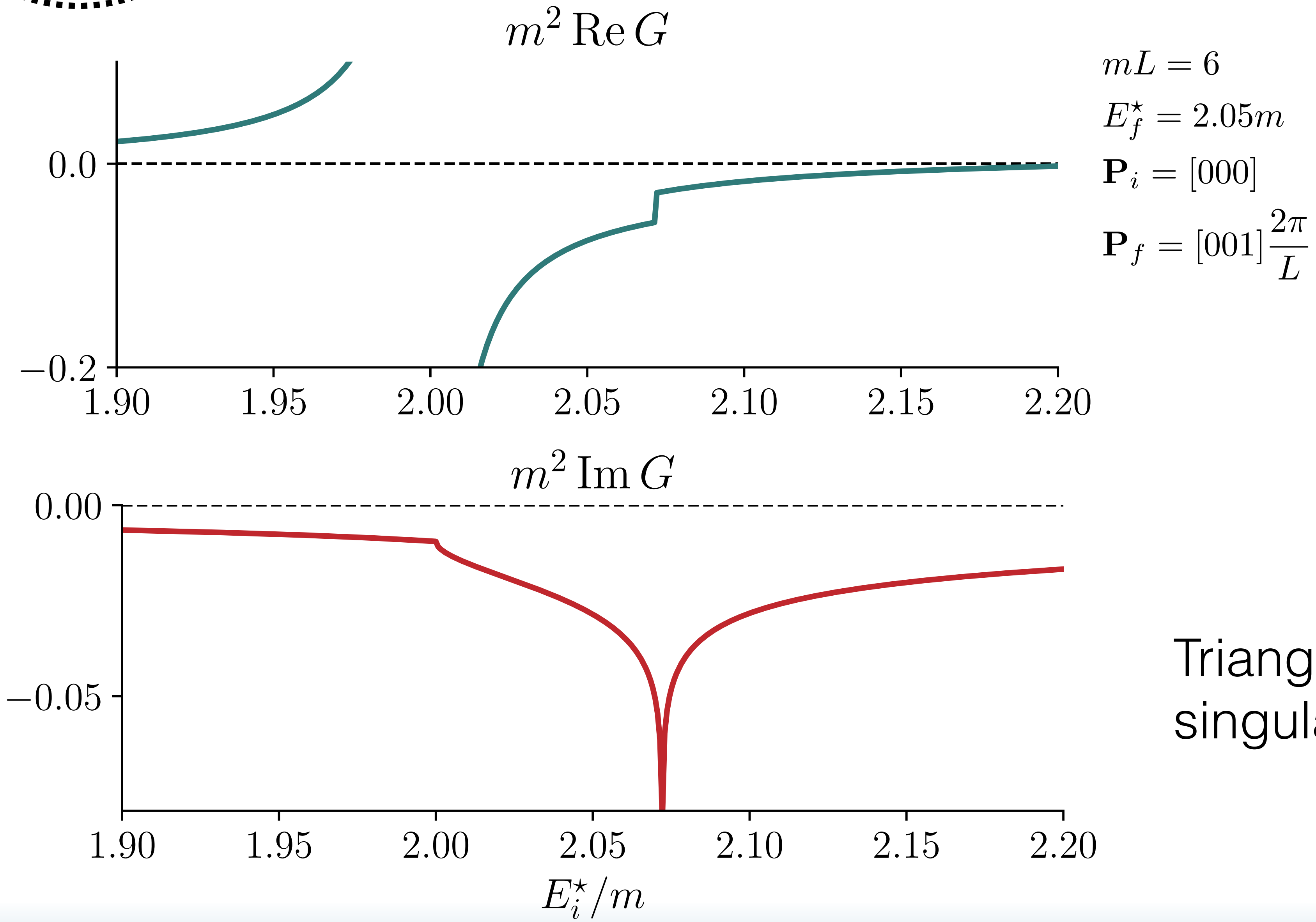
Finite volume corrections of the triangle diagram: G



$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f-k)^2-m^2+i\epsilon} f(Q^2) \frac{i}{(P_i-k)^2-m^2+i\epsilon} = f(Q^2)\cdot G(P_f,P_i)$$

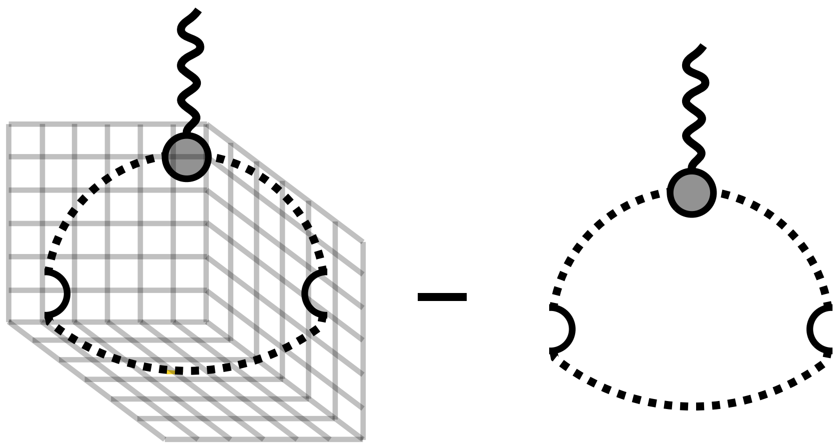
Poles at FV free energies.

Threshold singularity.



Triangle singularity.

Finite volume corrections of the triangle diagram: G



$$\sim \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \right] \frac{i}{(P_f - k)^2 - m^2 + i\epsilon} f(Q^2) \frac{i}{(P_i - k)^2 - m^2 + i\epsilon} = f(Q^2) \cdot G(P_f, P_i)$$

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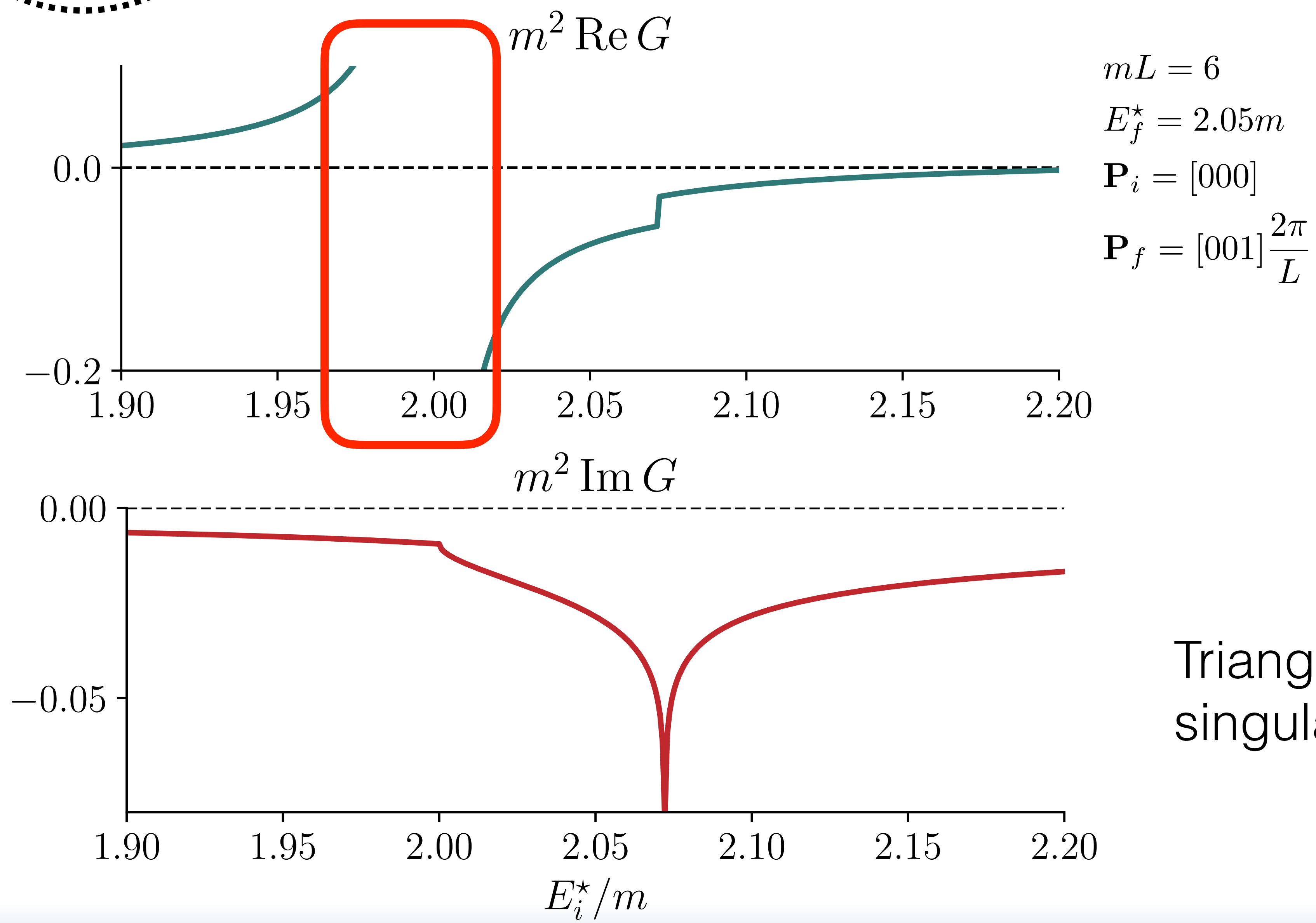

Threshold singularity.

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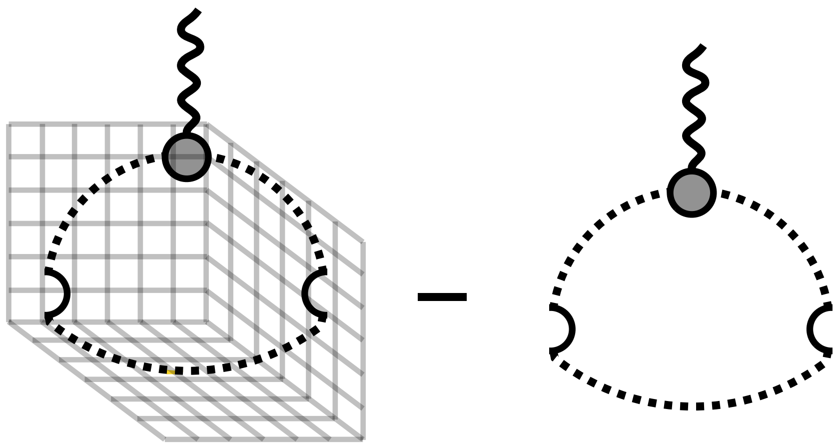
Finite volume corrections of the triangle diagram: G



$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f-k)^2-m^2+i\epsilon} f(Q^2) \frac{i}{(P_i-k)^2-m^2+i\epsilon} = f(Q^2)\cdot G(P_f,P_i)$$

Poles at FV free energies.

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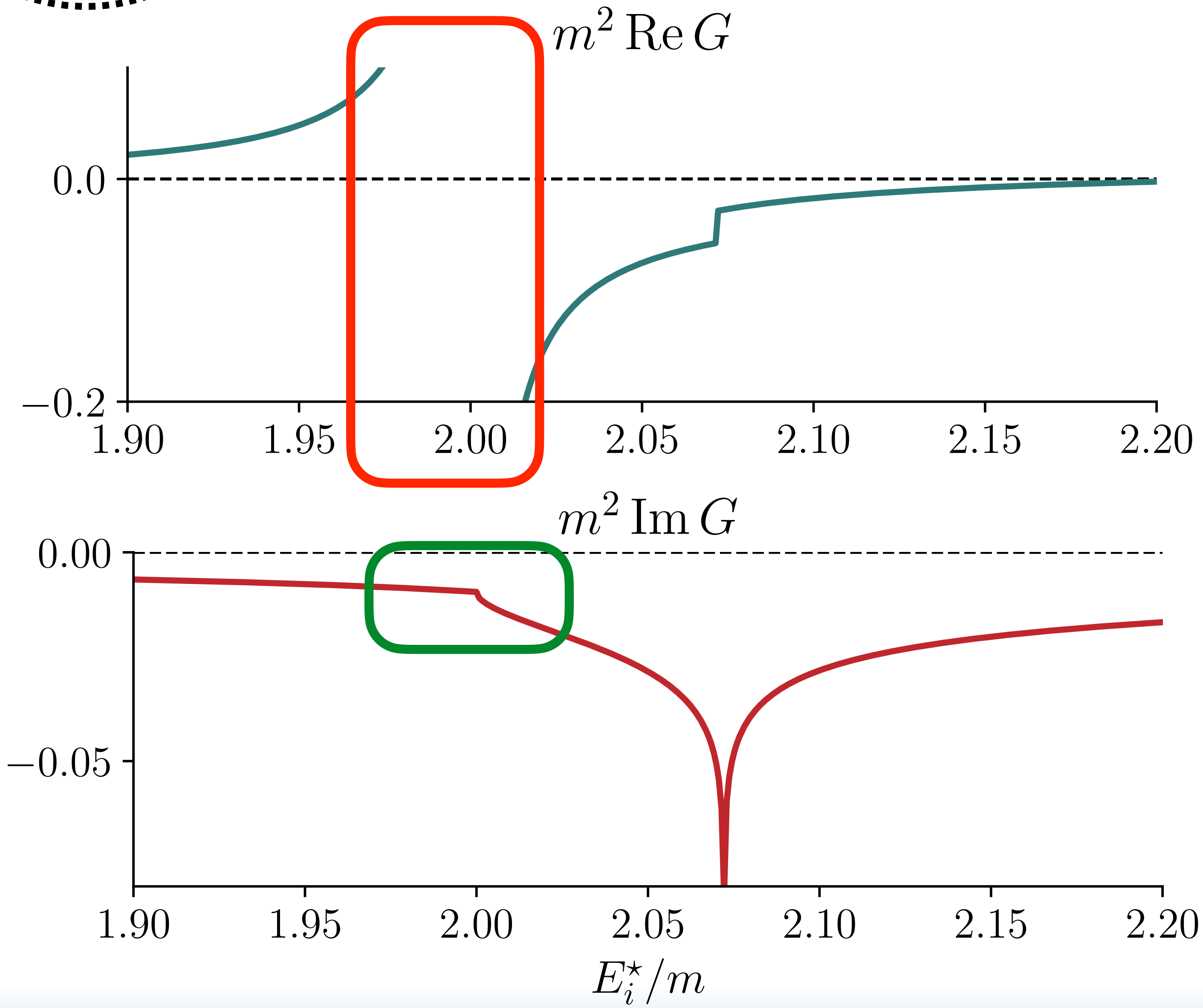


$mL = 6$

$E_f^\star = 2.05m$

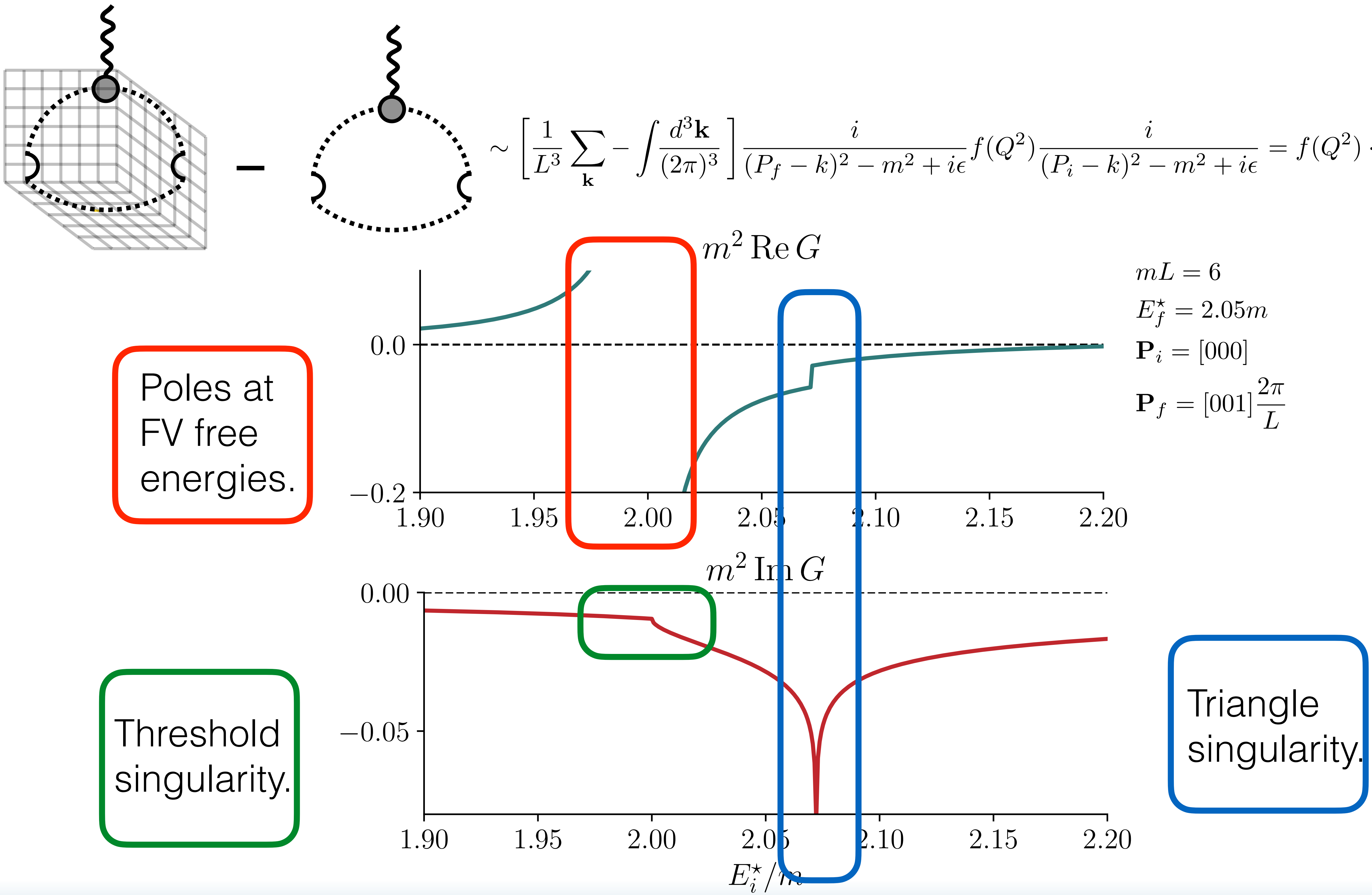
$\mathbf{P}_i = [000]$

$\mathbf{P}_f = [001]\frac{2\pi}{L}$

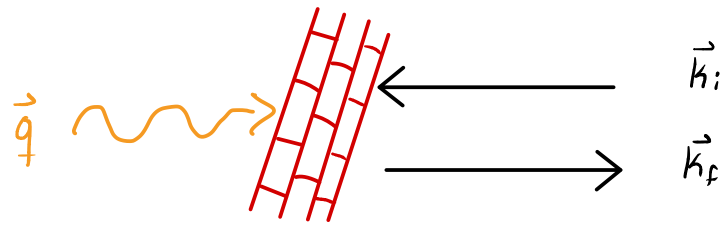
Triangle singularity.

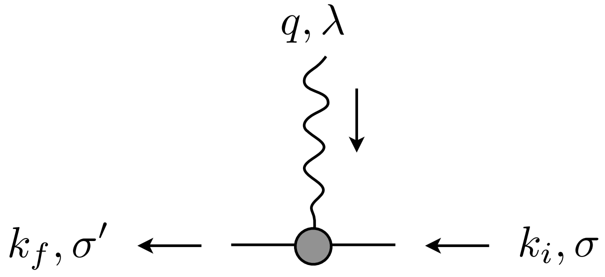
Finite volume corrections of the triangle diagram: G

$$\sim \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{i}{(P_f-k)^2 - m^2 + i\epsilon} f(Q^2) \frac{i}{(P_i-k)^2 - m^2 + i\epsilon} = f(Q^2)\cdot G(P_f, P_i)$$



Brick-Wall frame: one-body FFs





The multipole expansion of currents in the BW frame:

[Durand et al., 1962]

$$w^{\lambda,BW}_{\sigma\sigma'}(k_f, k_i) = 2m_N \sum_J \begin{pmatrix} 1/2 & J & 1/2 \\ \sigma & \lambda & \sigma' \end{pmatrix} M^{\lambda}_J(Q^2)\,,$$

Spinors + EM current: Charge, and magnetic dipole

The relationship between multipoles and Lorentz FFs is well known for arbitrary spin

[Lorce, 2009]

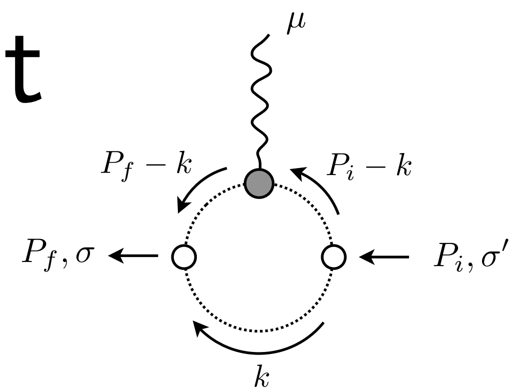
$$F_1(Q^2) = \frac{1}{1+Q^2/(4M^2)}\left(C(Q^2) + \frac{Q^2}{4M^2}M_1(Q^2)\right),$$

$$F_2(Q^2) = \frac{1}{1+Q^2/(4M^2)}\left(-C(Q^2) + M_1(Q^2)\right),$$

Dirac and Pauli FFs

Sachs FFs: multipole charge and magnetic dipole

G with spin 1/2 and an EM current



$$[G \cdot w]^{\mu}_{\sigma;\sigma'}(P_f, P_i, L) = \left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3\mathbf{k}}{(2\pi)^3}\right] \frac{1}{2\omega_k} \frac{1}{(P_f - k)^2 - m^2 + i\epsilon} w^{\mathrm{BW},\mu}_{\sigma\sigma'} \frac{1}{(P_i - k)^2 - m^2 + i\epsilon}$$

- Boost to from BW to lattice frame
- Thomas precession
- $SO(3)$ to cartesian

$$w^{\mathrm{BW},\mu}_{\sigma\sigma'} = [\Lambda_{BW}(-\mathbf{k}/\omega_k)]^{\mu}{}_{\nu} D^{1/2}_{\tau\sigma}(R_T(\hat{k}_f)) 2m \left[\sum_{J,\lambda=0,1} \epsilon^{\nu}_{\lambda} \begin{pmatrix} 1/2 & J & 1/2 \\ \tau & \lambda & \tau' \end{pmatrix} M^{\lambda}_{J}(Q^2)\right] D^{1/2*}_{\tau'\sigma'}(R_T(\hat{k}_i))$$

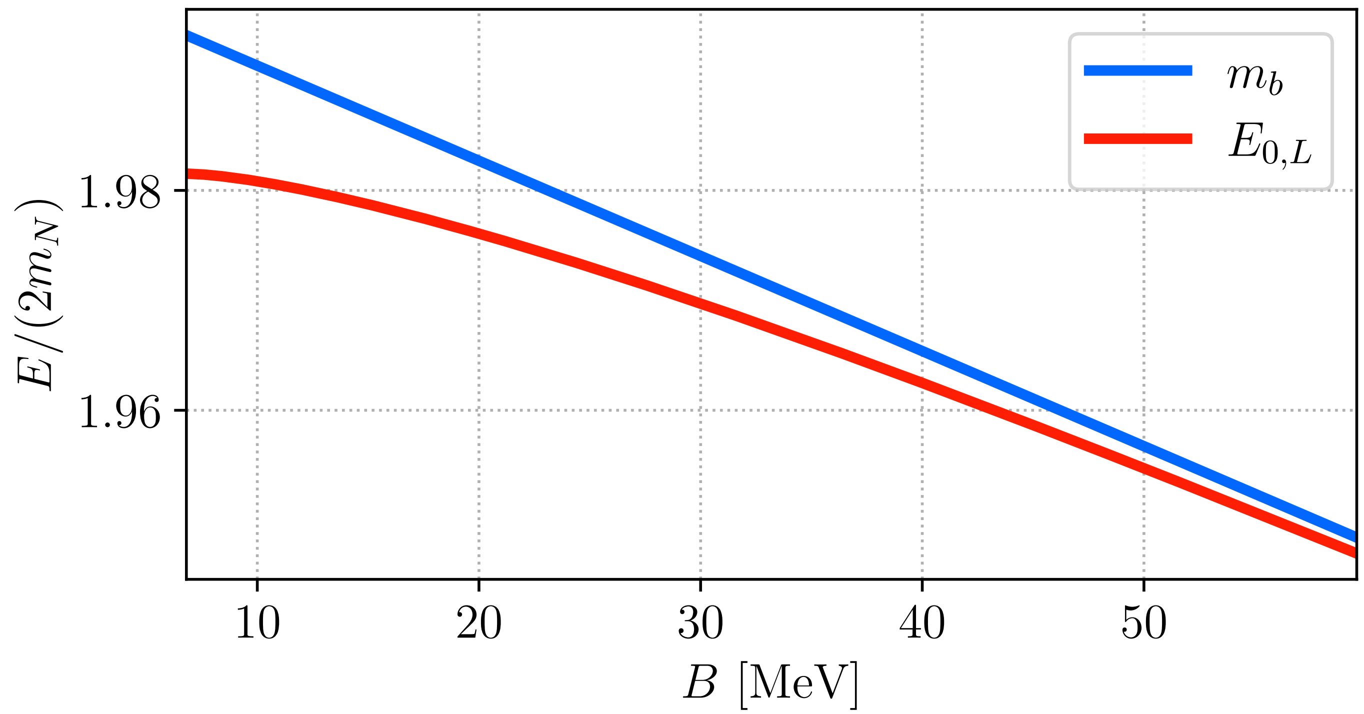
$$\epsilon^{\mu}_{0} = \delta^{\mu}_{0} \qquad M^{0}_{J}(Q^2) = \delta_{J0} C(Q^2)$$

$$\epsilon^{\mu}_{\pm 1} = \epsilon^{\mu}_{\pm 1}(0) \qquad M^{1}_{J}(Q^2) = \delta_{J1} M_1(Q^2)$$

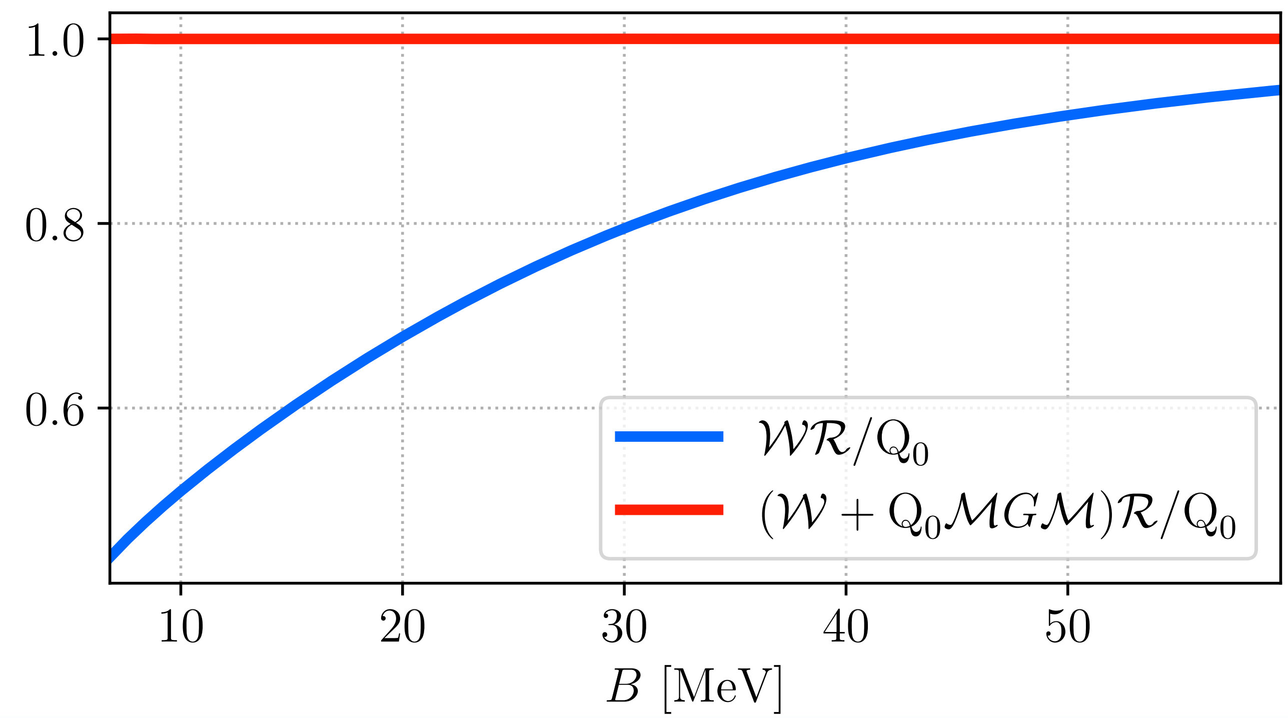
A toy example: the Deuteron electric charge

$m_N L \approx 16$

- Energies: Finite volume effects vanish for deeply bound states.



- Matrix elements: more significant volume dependence from *G even* below threshold.



[Briceño et al., 2019] $\mathcal{W}^0(P_f = P_i = (E, \mathbf{0})) = \mathrm{Q}_0 \dfrac{\partial \mathcal{M}}{\partial E}$

Verifications of the formalism

- Perturbative systems [R. Briceño, M. Hansen, A. Jackura (2020)]

$$L^3 \langle E_0 | \mathcal{J} | E_0 \rangle_L = \sum_{j=0}^{5} \frac{\beta_j}{L^j} + \mathcal{O}\big(1/L^6\big)$$

  - Consistent with Feynman-Hellmann theorem.
  - Matches with the results of a perturbative interaction.

- Ward identities: forward limit [R. Briceño, M. Hansen, A. Jackura (2019)] [A. Jackura, FO (in preparation)]

$$\langle E_n | \hat{\mathrm{Q}} | E_n \rangle_L = \mathrm{tr}\big(\mathcal{R}(E_n)\mathcal{W}_L(E_n, E_n, Q^2)\big)$$

$$\mathcal{R} \propto \mathcal{W}_L^{-1}$$

Summary and outlook

- Lattice QCD offers a great opportunity to measure hadronic and electroweak properties of the QCD spectrum which complement experimental efforts.

- Analytic structure of two-body amplitudes in the infinite volume drives finite volume corrections of the lattice.

- First calculations at heavier-than-physical pion mass lattices have shown the feasibility of this research direction.

- FV corrections can be significant *even* for bound states.