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# On the semileptonic production of the $\rho$ resonance from B-mesons

4th Workshop on Future Directions in Spectroscopy Analysis (FDSA2022)
JLab, Newport News, VA
November 2022

in collaboration with:
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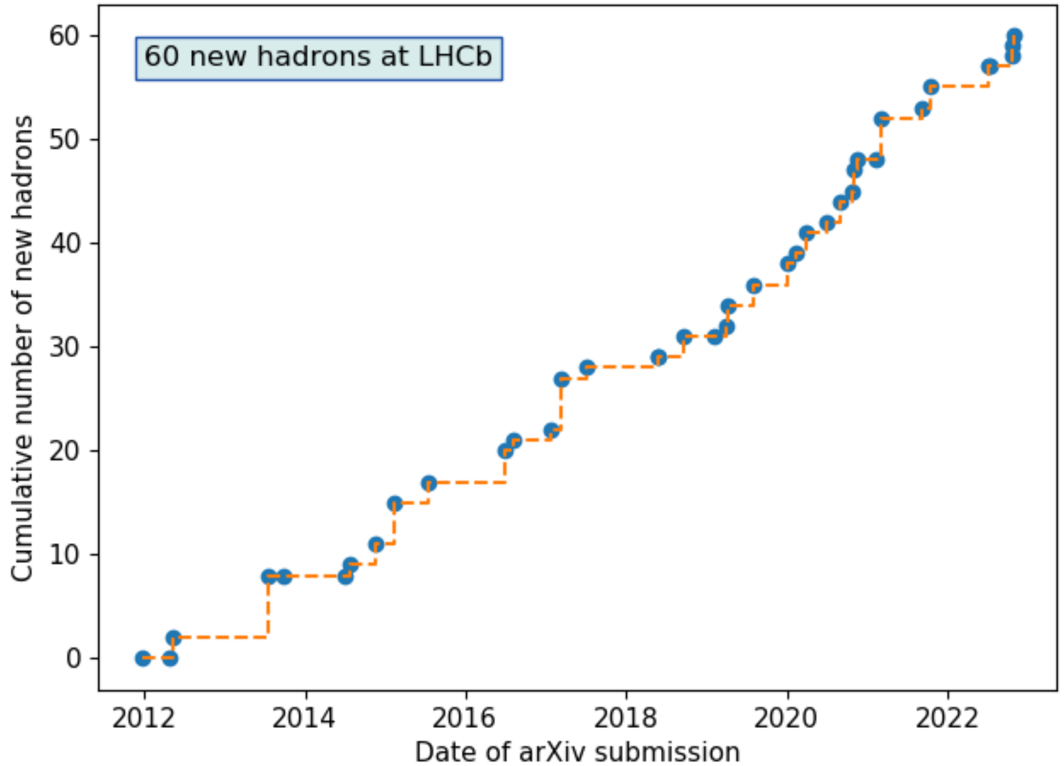
Gumaro Rendon, John W. Negele,
Constantia Alexandrou, Andrew Pochinsky

# QCD and the hadron zoo

(a very long list on PDG)

https://www.nikhef.nl/~pkoppenb/particles.html



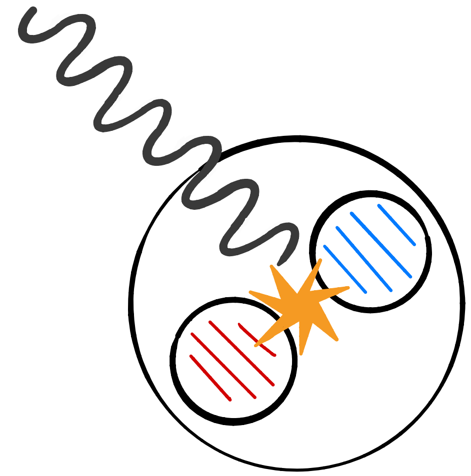

## New hadrons

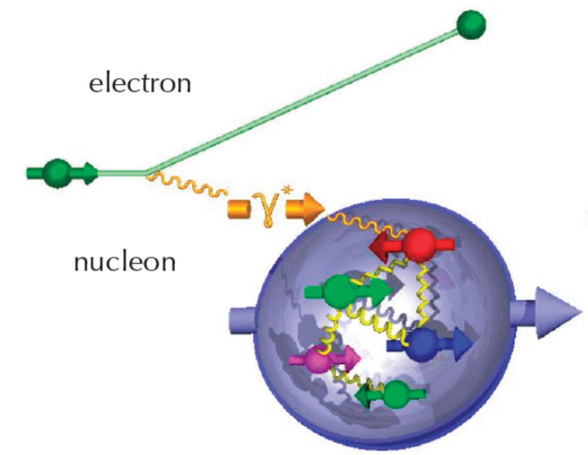
| Counter | Experiment | Particle | Mass [MeV] | Quarks | Date | Reference | Note |
|---|---|---|---|---|---|---|---|
| 1. | ATLAS | $\chi_b(3P)$ | $10530 \pm 10$ | $b\bar{b}$ | 21 Dec 2011 | Phys. Rev. Lett. 108 (2012) 152001 | Later resolved into $\chi_{b1}(3P)$ and $\chi_{b2}(3P)$ |
| 2. | CMS | $\Xi_b(5945)^0$ | $5945 \pm 3$ | $bsu$ | 26 Apr 2012 | Phys. Rev. Lett. 108 (2012) 252002 | |
| 3. | LHCb | $\Lambda_b(5920)^0$ | $5919.8 \pm 0.7$ | $bud$ | 15 May 2012 | Phys. Rev. Lett. 109 (2012) 172003 | |
| 4. | LHCb | $\Lambda_b(5912)^0$ | $5912.0 \pm 0.7$ | $bud$ | 15 May 2012 | Phys. Rev. Lett. 109 (2012) 172003 | |
| 5-6. | LHCb | $D_J^*(3000)^{+,0}$ | $3008 \pm 4$ | $c\bar{q}$ | 17 Jul 2013 | JHEP 09 (2013) 145 | Mass of $+$ state fixed in fit |
| 7. | LHCb | $D_J(3000)^0$ | $2972 \pm 9$ | $c\bar{u}$ | 17 Jul 2013 | JHEP 09 (2013) 145 | |
| 8. | LHCb | $D_J^*(2760)^+$ | $2772 \pm 4$ | $c\bar{d}$ | 17 Jul 2013 | JHEP 09 (2013) 145 | |
| 9. | LHCb | $D_J(2740)^0$ | $2737 \pm 12$ | $c\bar{u}$ | 17 Jul 2013 | JHEP 09 (2013) 145 | |
| 10. | LHCb | $D_J(2580)^0$ | $2580 \pm 6$ | $c\bar{u}$ | 17 Jul 2013 | JHEP 09 (2013) 145 | |

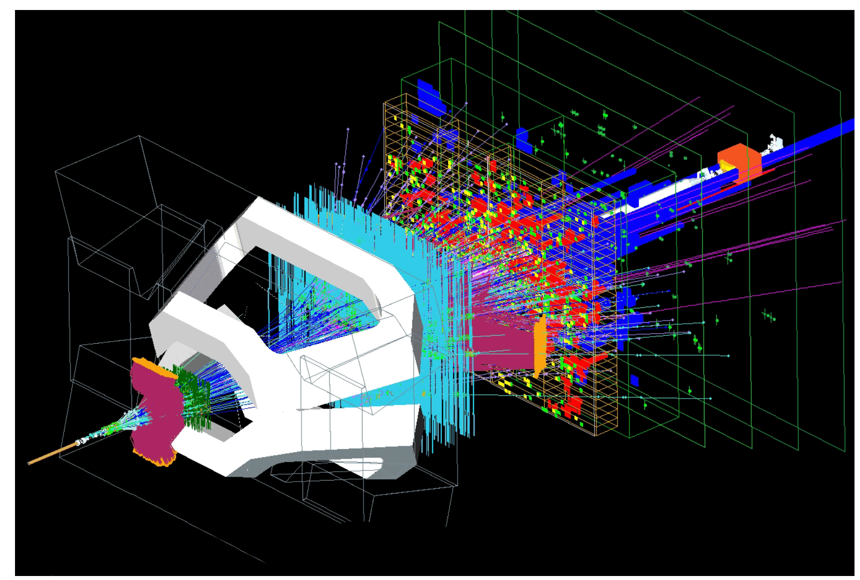
hadrons

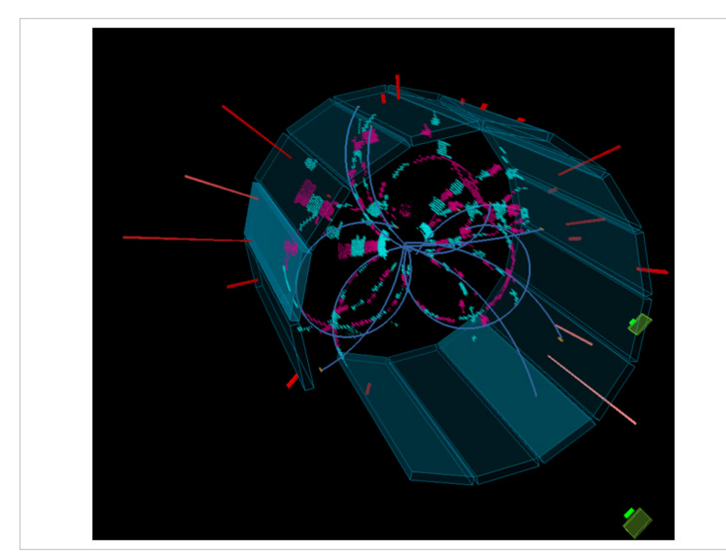
# how are they made?

- shining a light on a nucleus (JLab-glueX)
- hitting nuclei with electrons (JLab)
- smashing proton bunches (LHCb)
- hitting electrons with positrons (Belle, BES)
- ...and others





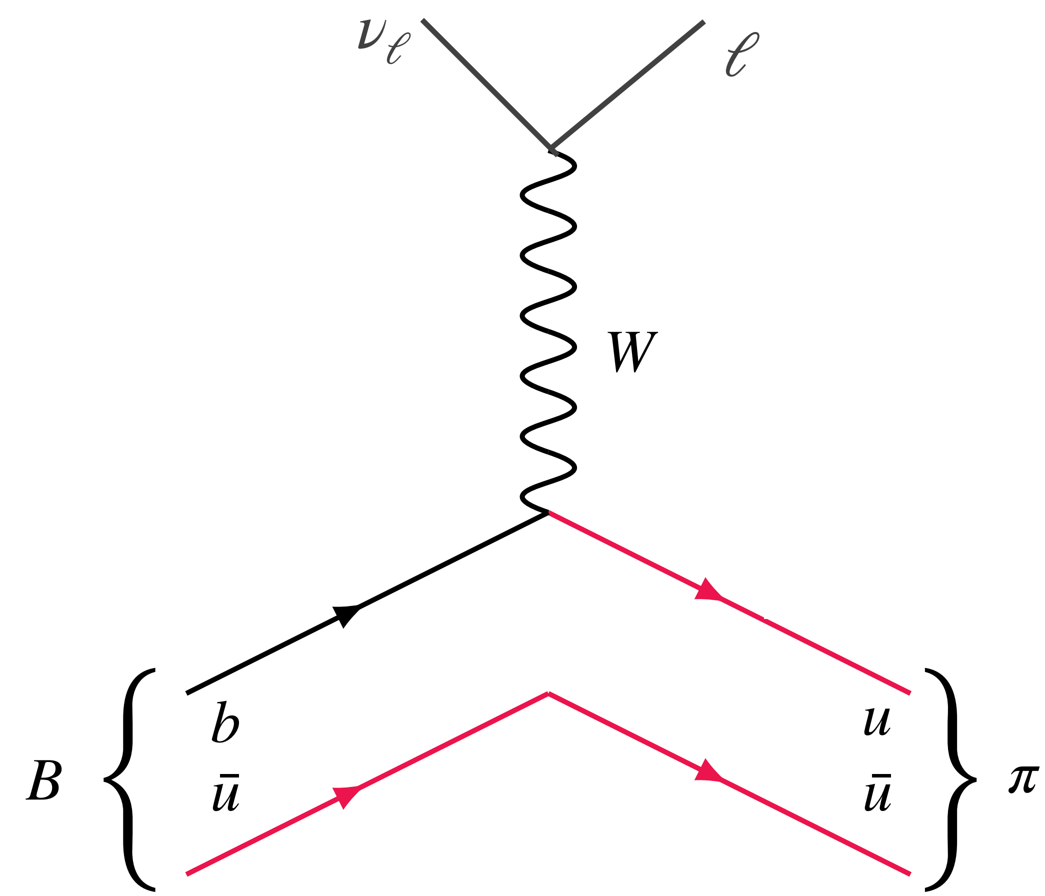



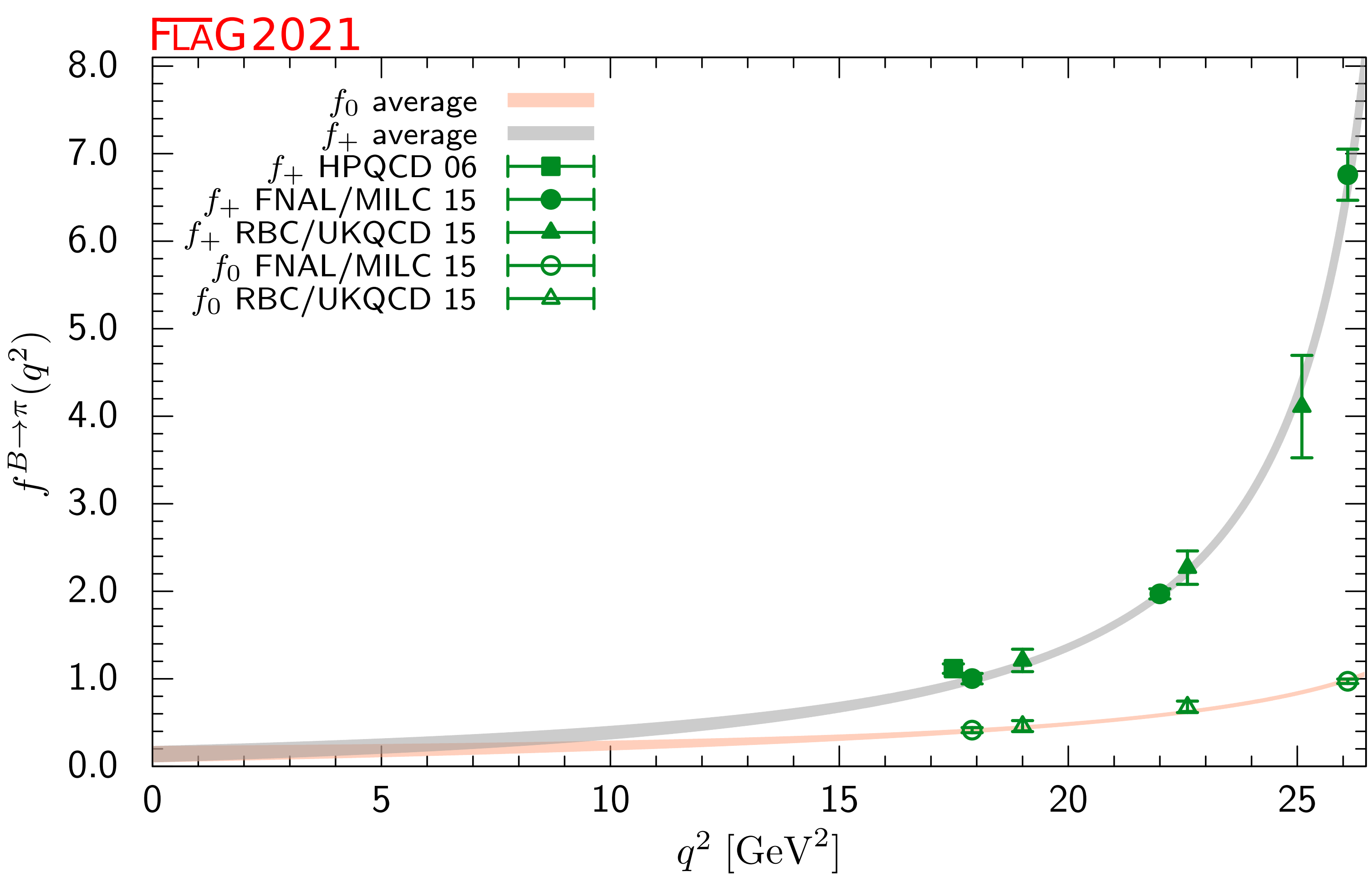


# example: $B \to \pi \ell \nu$

$$\langle \pi | J^{\mu}_{V-A} | B \rangle = f_+(q^2)\left[p_B^{\mu} + p_{\pi}^{\mu} + \frac{M_B^2 - m_\pi^2}{q^2} q^{\mu}\right]$$

$$+ f_0(q^2)\frac{M_B^2 - m_\pi^2}{q^2} q^{\mu}$$

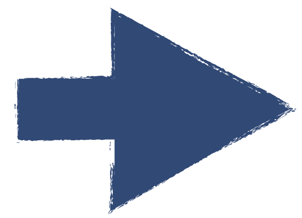


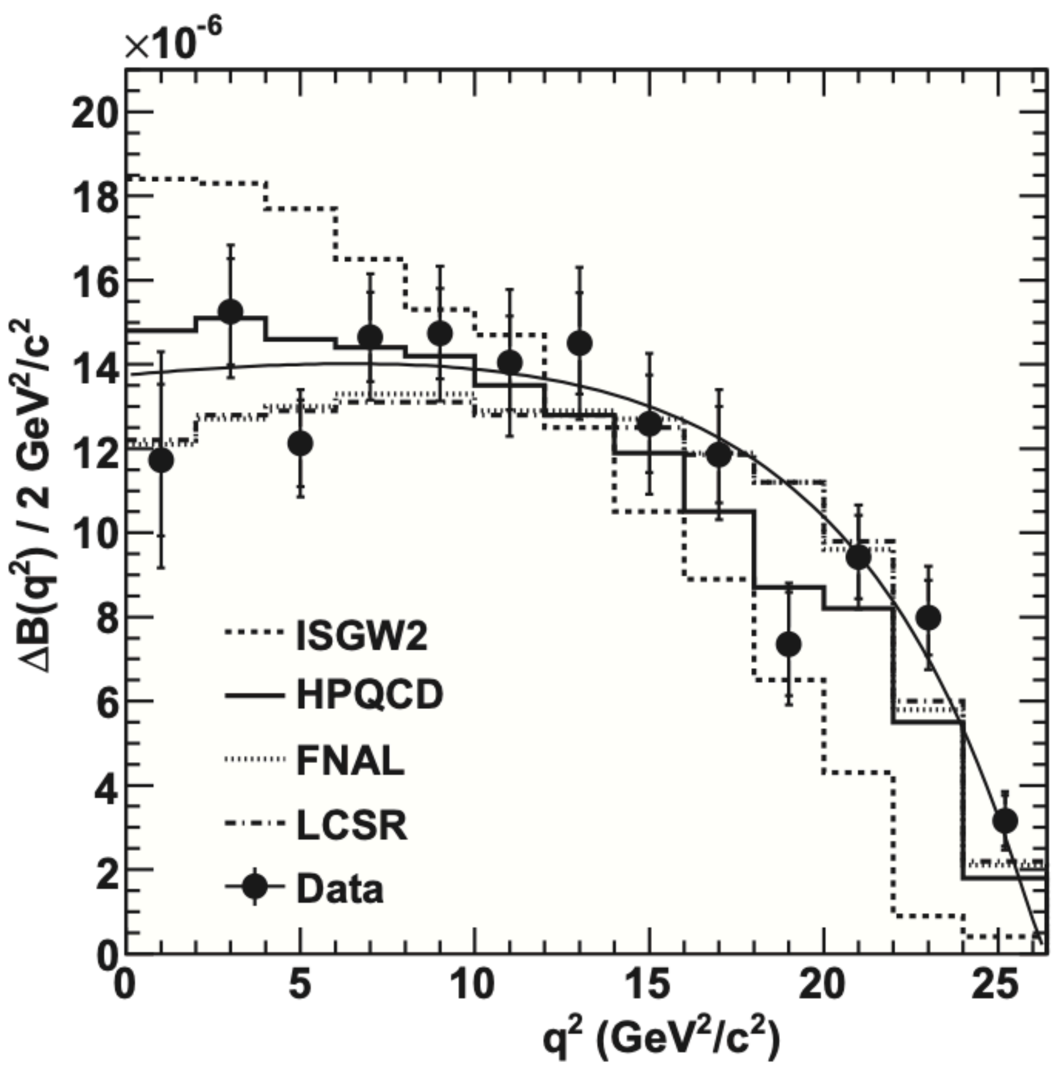

$$\frac{d\mathbf{B}}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3 q^2} \vec{p}_\pi^{\,3} |f_+(q^2)|^2$$

# example: $B \to \pi \ell \nu$

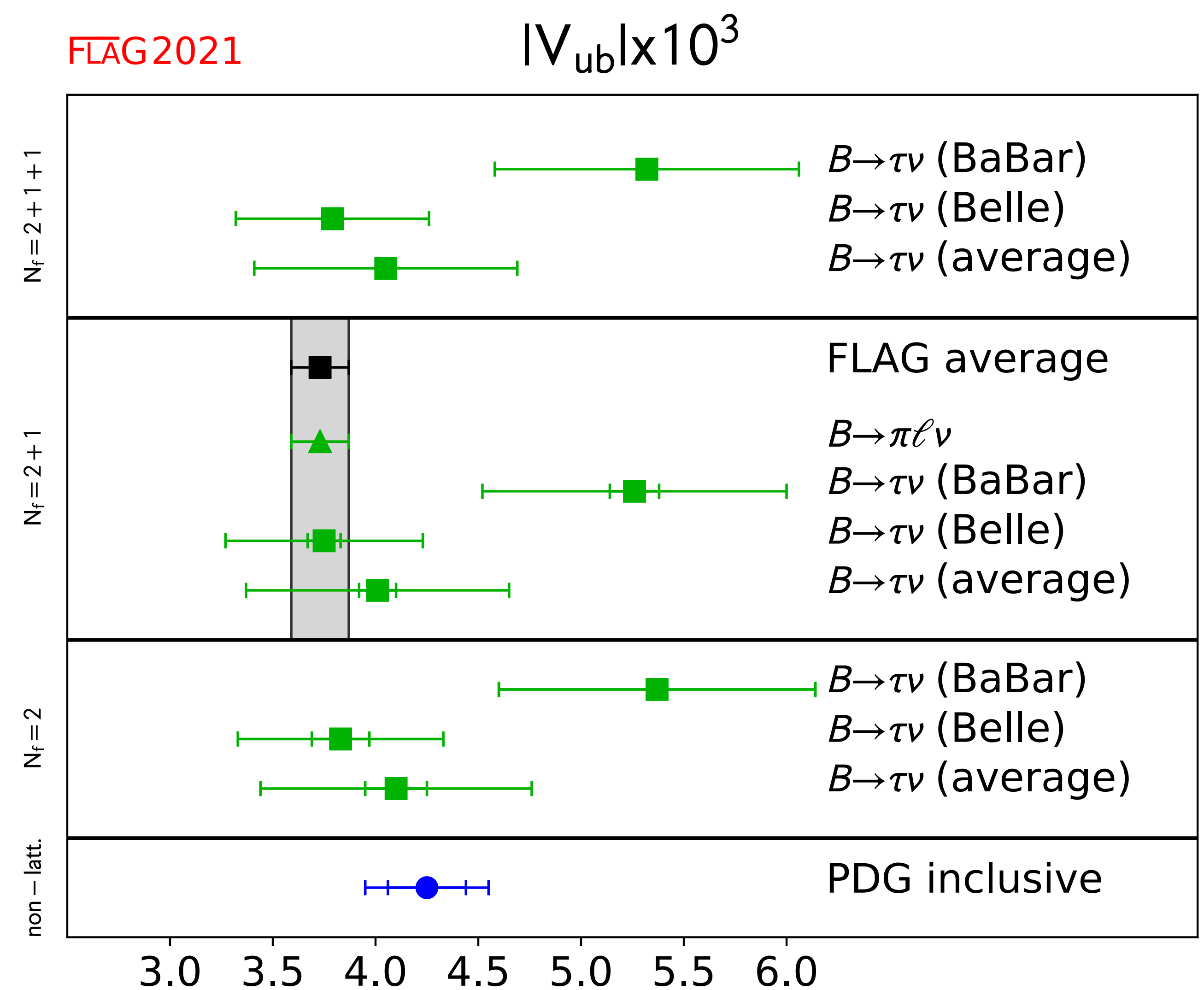
production well known (exp and theory)



determine SM parameters




PTEP12 - Belle

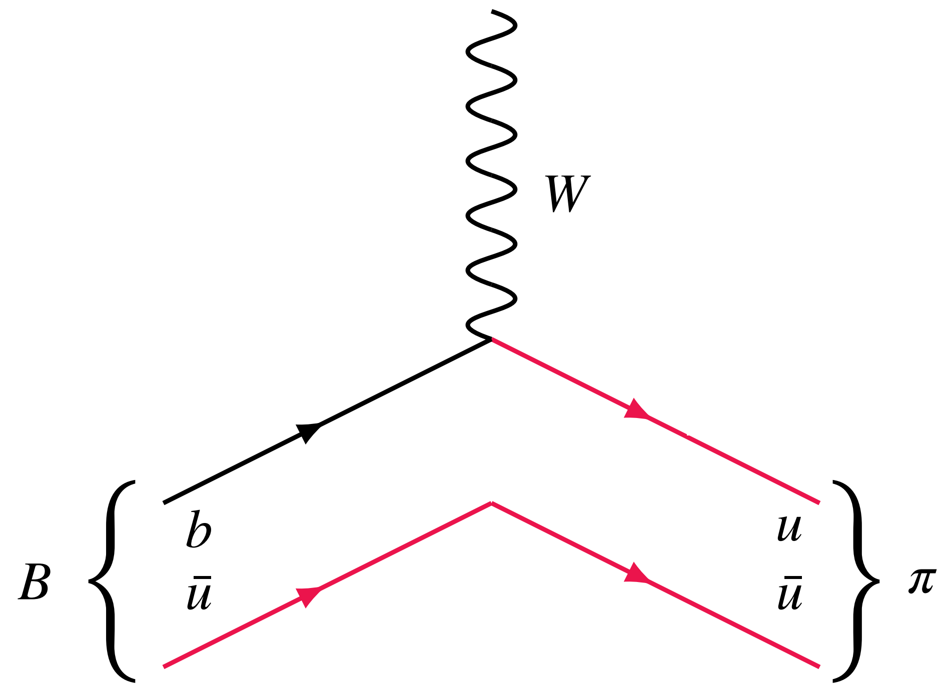


# can we do the same with resonances?

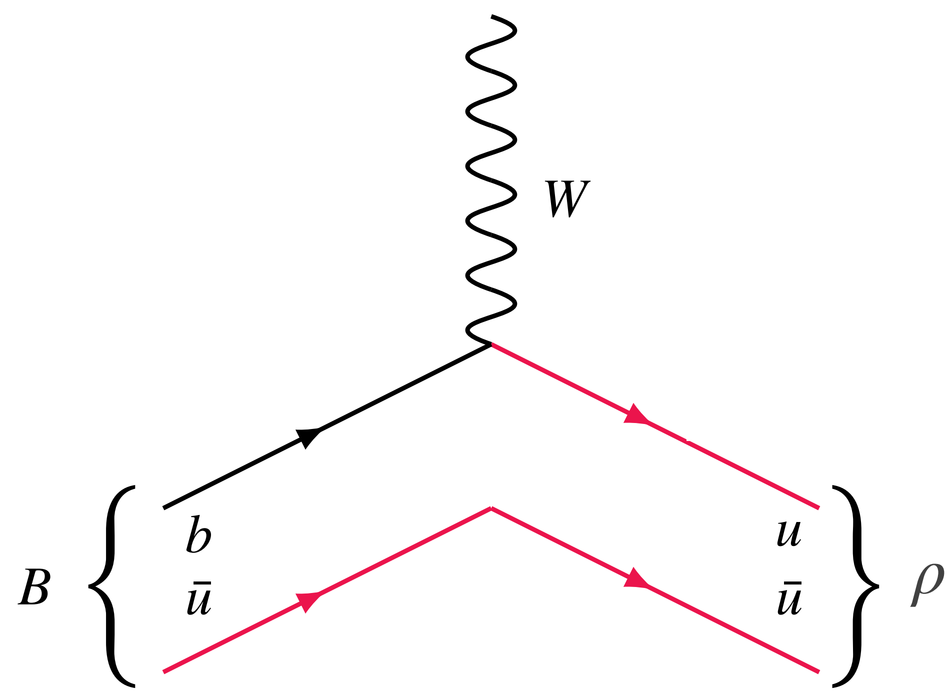
- theoretical treatment of resonances? ✓
- theoretical treatment of production? (✓)
- experimental treatment of resonances? (✓)
- experimental measurements of production? ✓

# why $B \to \rho \ell \bar{\nu}$ ?

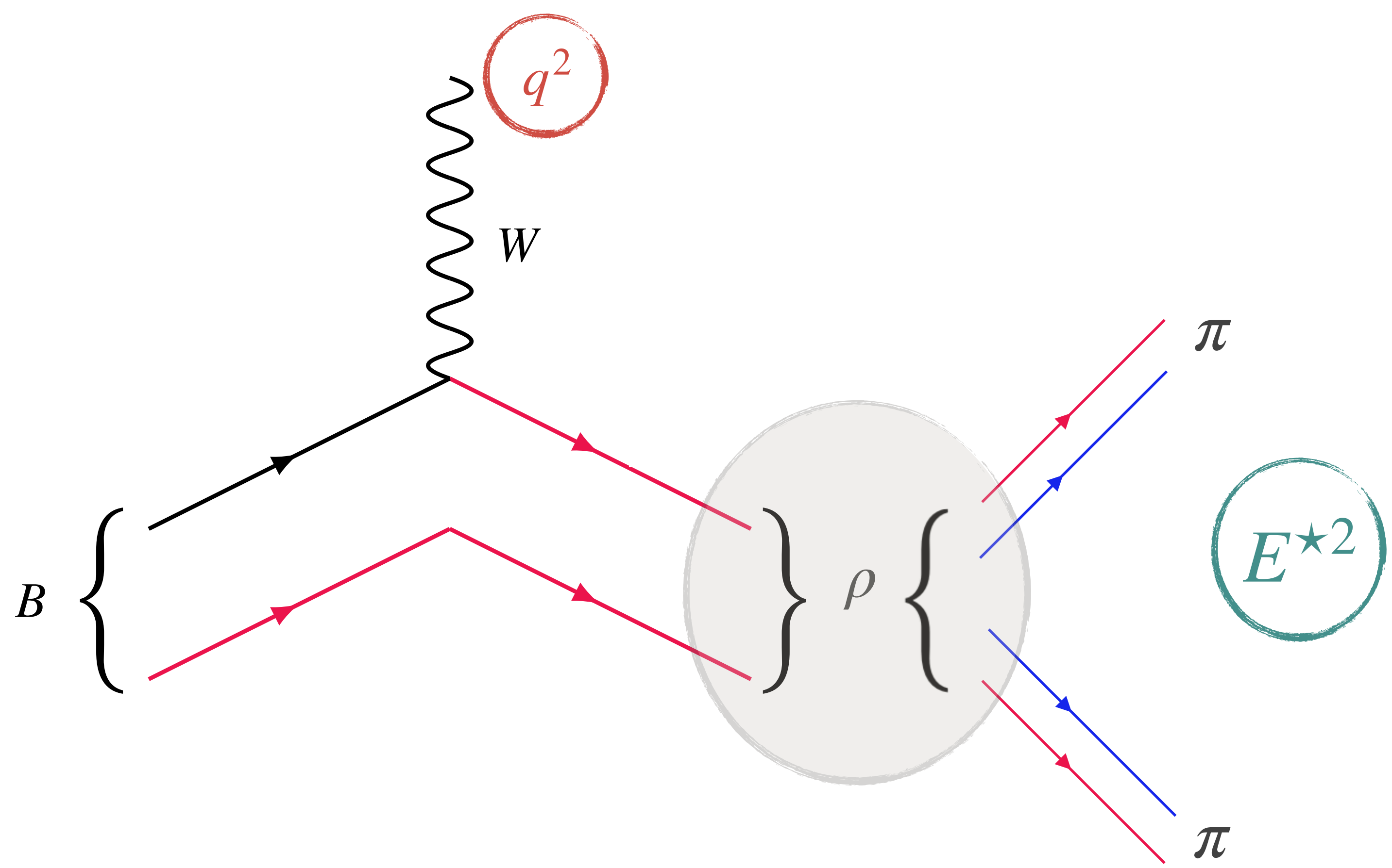
- $B \to \pi \ell \nu$
  - $f_+, f_0$
  - established

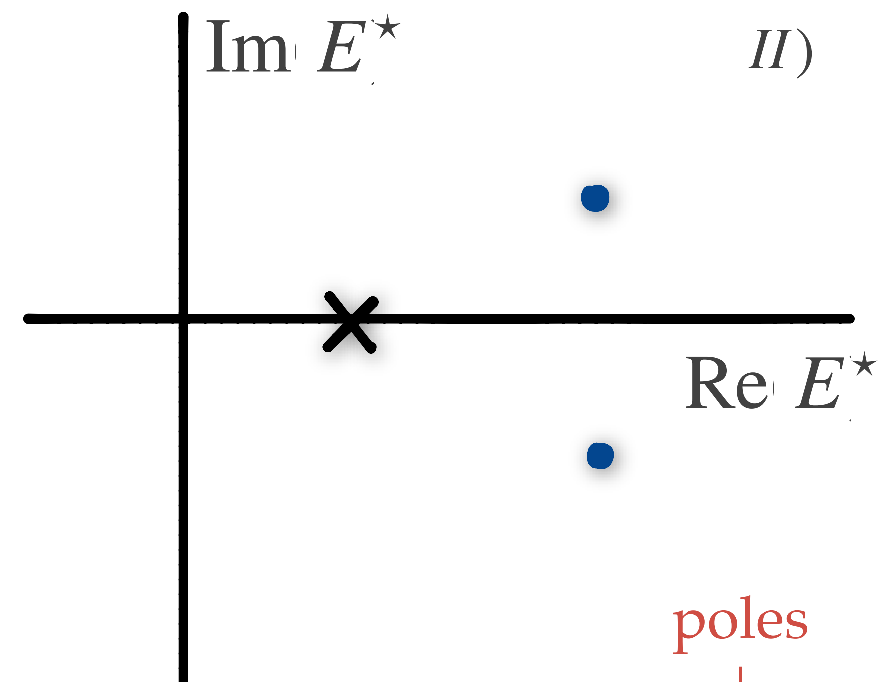



- $B \to \rho \ell \nu$
  - $V, A_0, A_1, A_2$
  - new



# $B \to \rho \ell \nu$



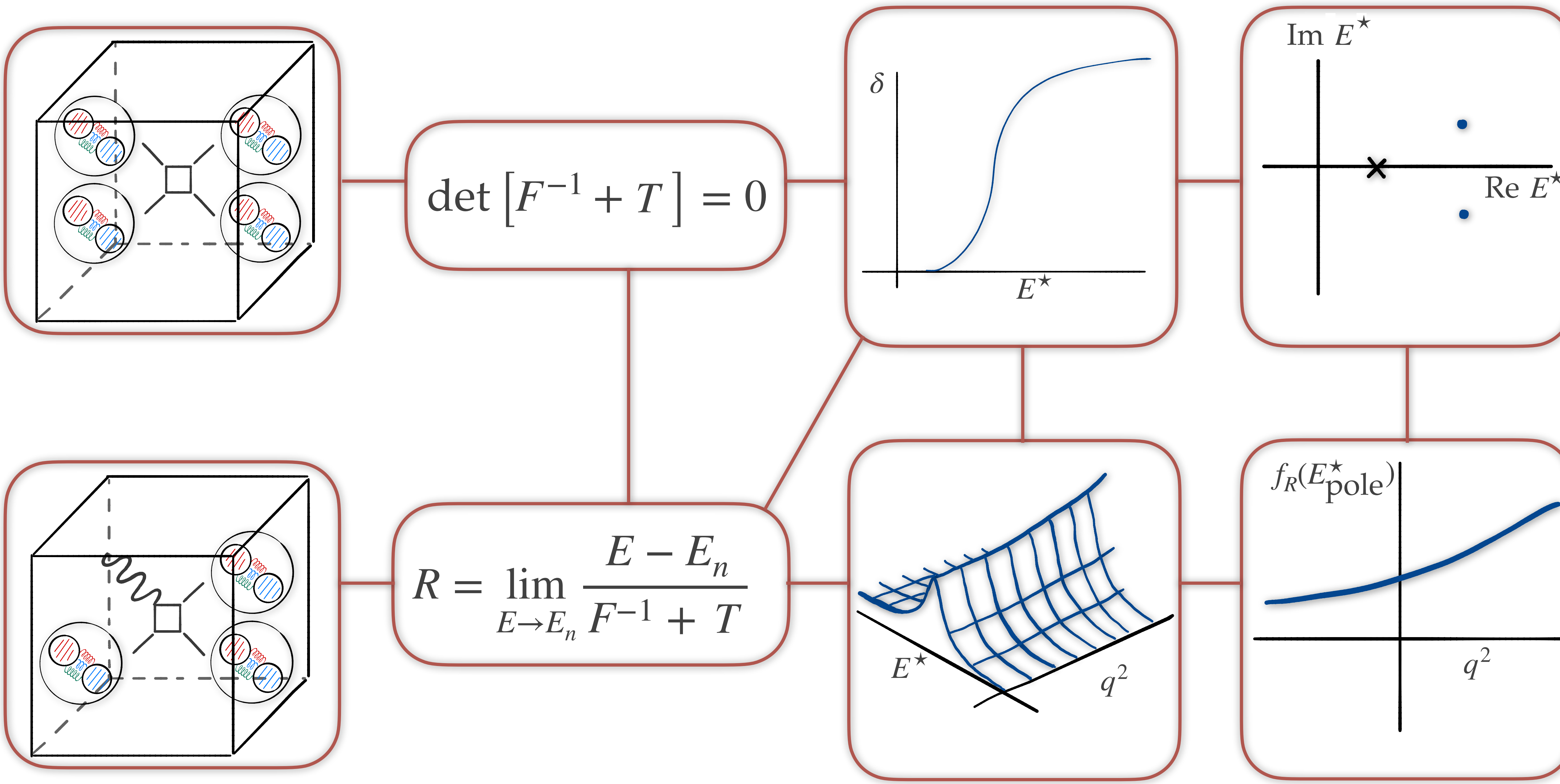

poles

smooth

poles

$$\mathscr{H}^{\mu}_{1,m_\ell}(q^2, E^{\star 2}) = \mathscr{A}^{\mu}_{1,m_\ell}(q^2, E^{\star 2}) \frac{T(E^{\star 2})}{k}$$

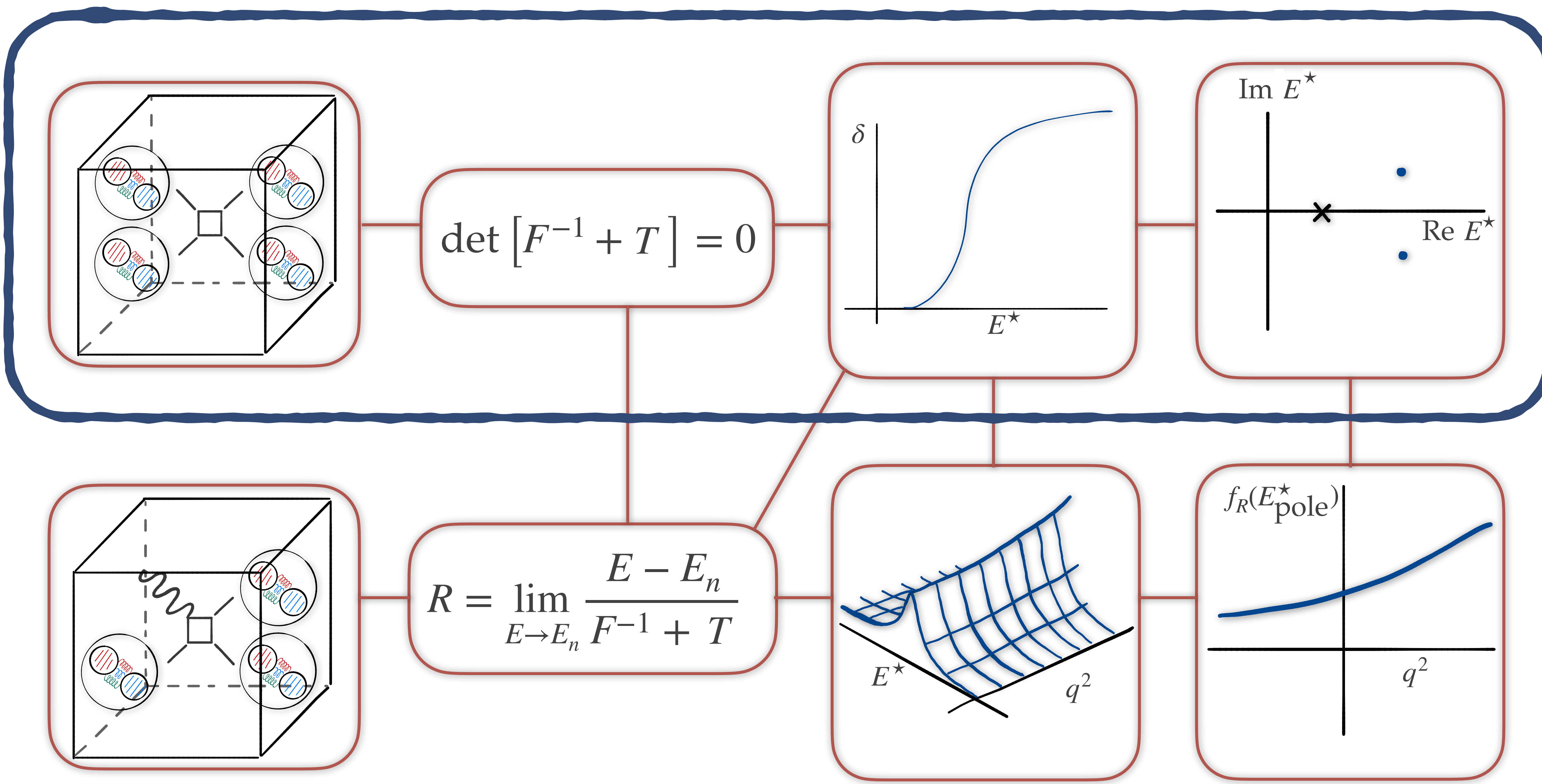
# $B \to \rho \ell \nu$ on the lattice



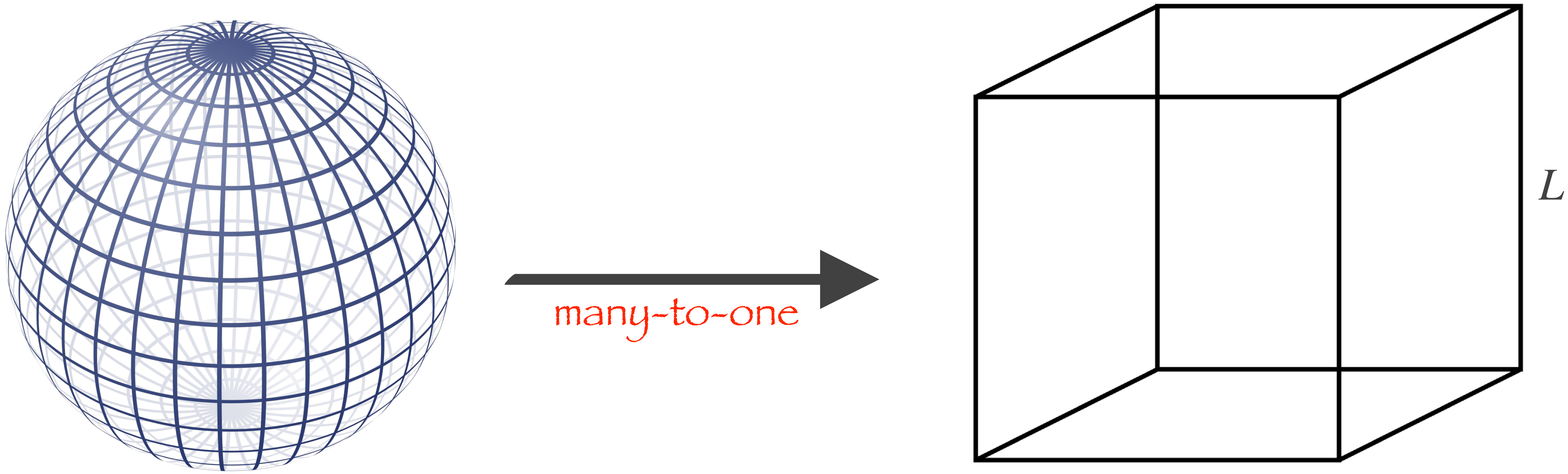

"Briceno-Hansen-Walker-Loud way of doing it"

Briceno, Hansen, Walker-Loud 1406.5965

# $B \to \rho \ell \nu$ on the lattice



# $\rho$ with lattice QCD

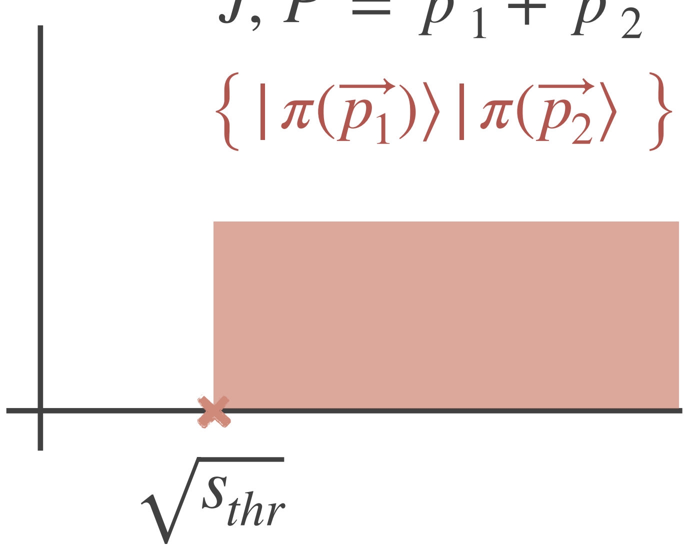



infinite volume:

- $O(3)$ symmetry
- infinite irreps ($J^P$)

$$J, \vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\{\, |\pi(\vec{p_1})\rangle |\pi(\vec{p_2})\rangle \,\}$$



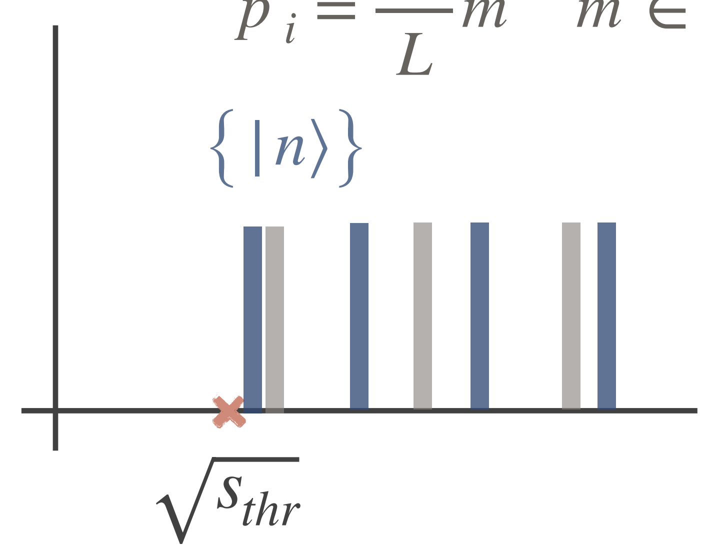

finite volume:

- discrete symmetries, $\Lambda$

$$L, \Lambda, \vec{P} = \vec{p}_1 + \vec{p}_2$$

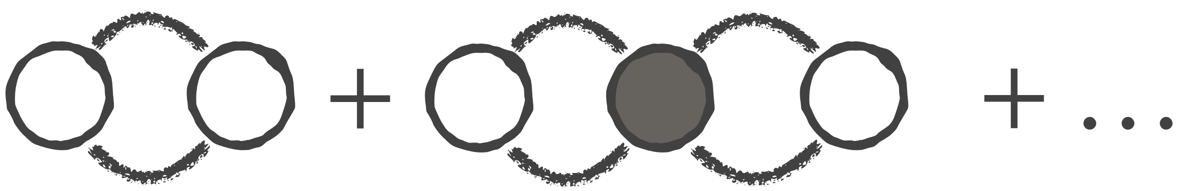
$$\vec{p}_i = \frac{2\pi}{L}\vec{m} \quad \vec{m} \in Z^3$$

$$\{\, |n\rangle \,\}$$

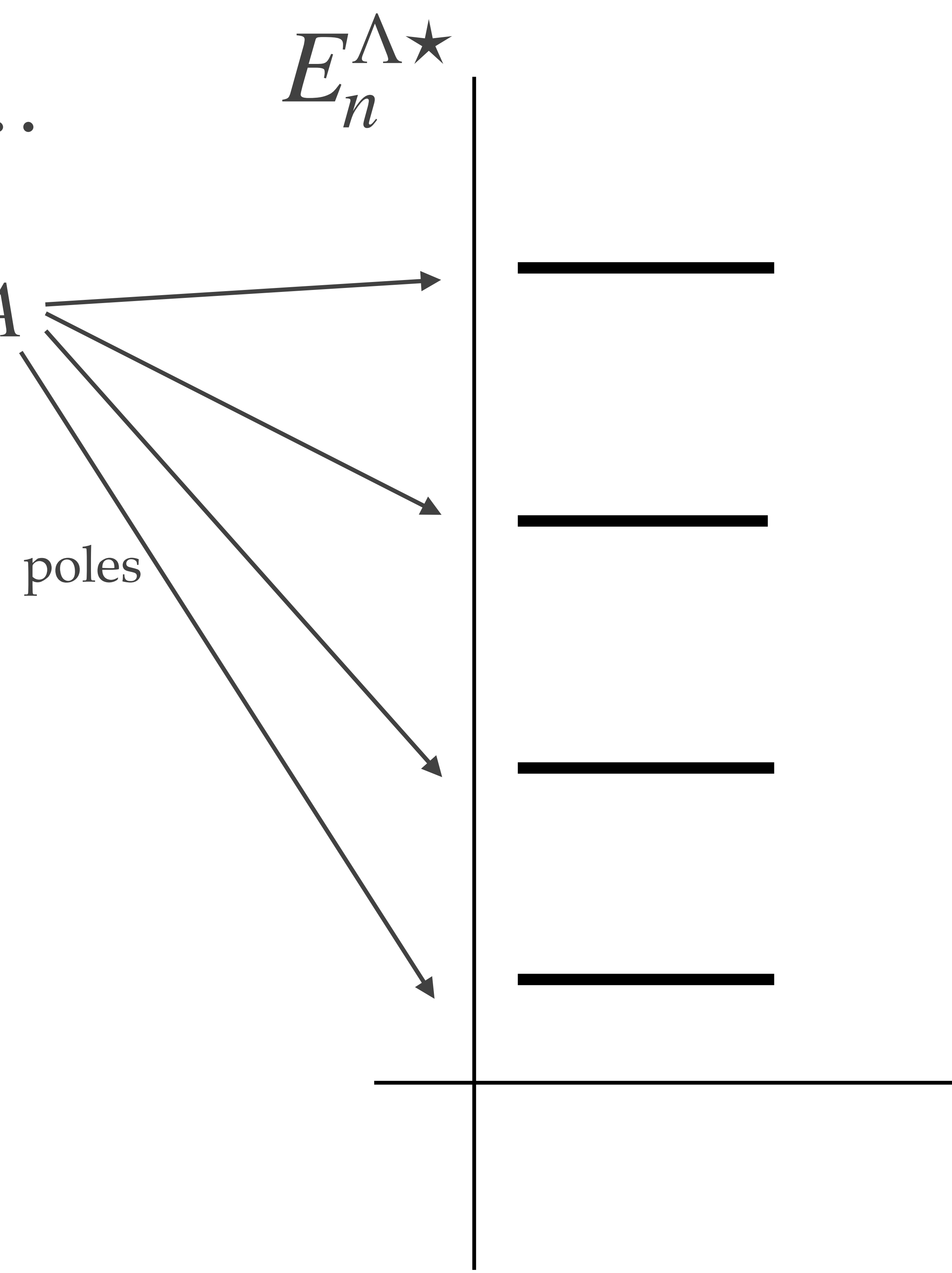


# $\rho$ with lattice QCD

$C_L^{(2)} =$



$$C_L^{(2)} = C_\infty^{(2)} - A' \frac{1}{F^{-1}(E^\star) + T(E^\star)} A$$



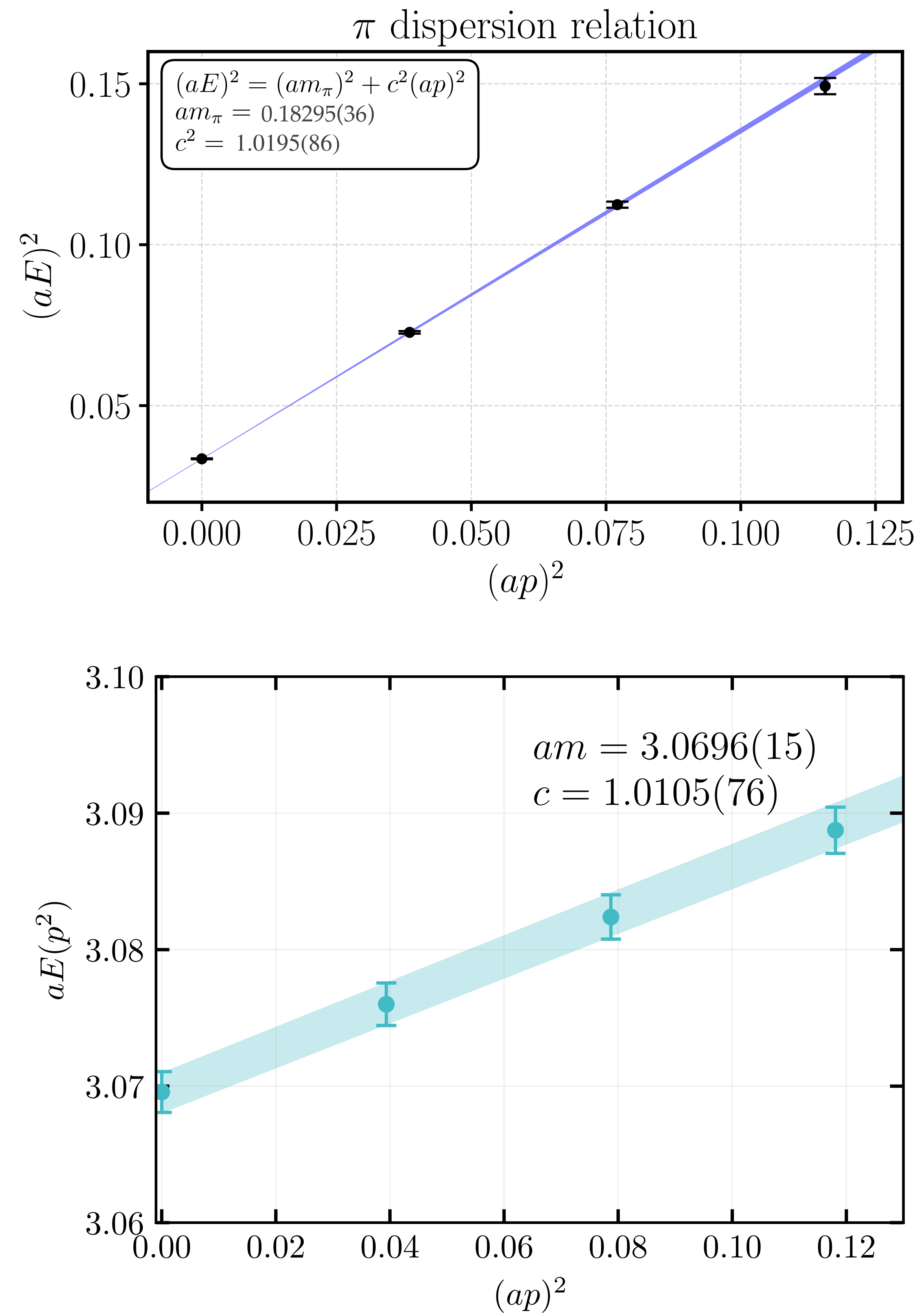

discrete spectrum where:

$$\det\left[F^{-1}(E^\star) + T(E^\star)\right] = 0$$

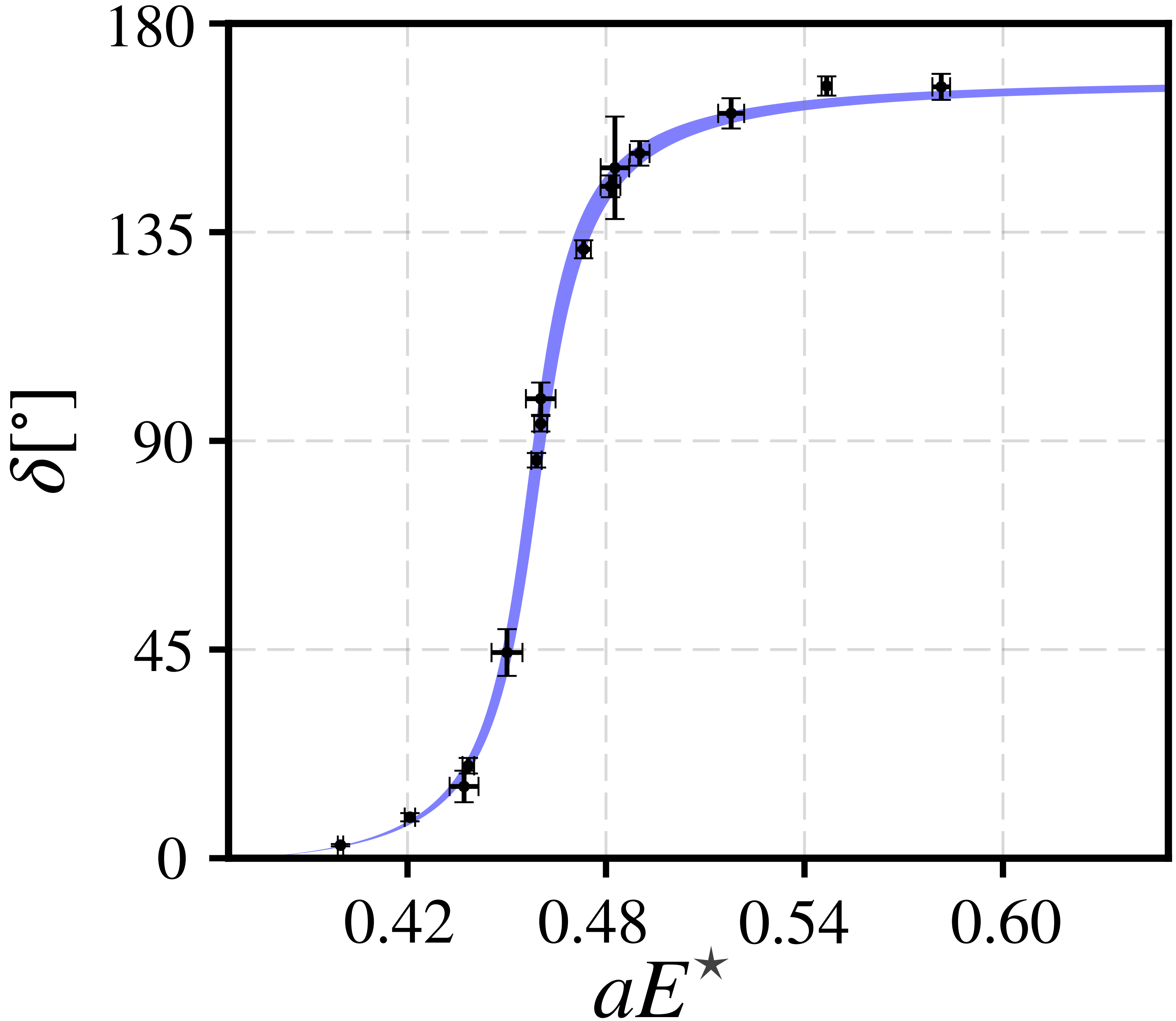

Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
Briceno 1401.3312
Woss, Wilson, Dudek 2001.08474
Briceno, Dudek, Young 1706.06223
[and many more]

# ensemble

- $N_f = 2 + 1$
- $32^3 \times 96$
- clover-Wilson light q
- RHQ heavy q
- $m_\pi \approx 320$ MeV
- $m_B = 5319.8(2.6)$ MeV
- $L \approx 3.6$ fm
- $a \approx 0.11$ fm
- 1039 configs



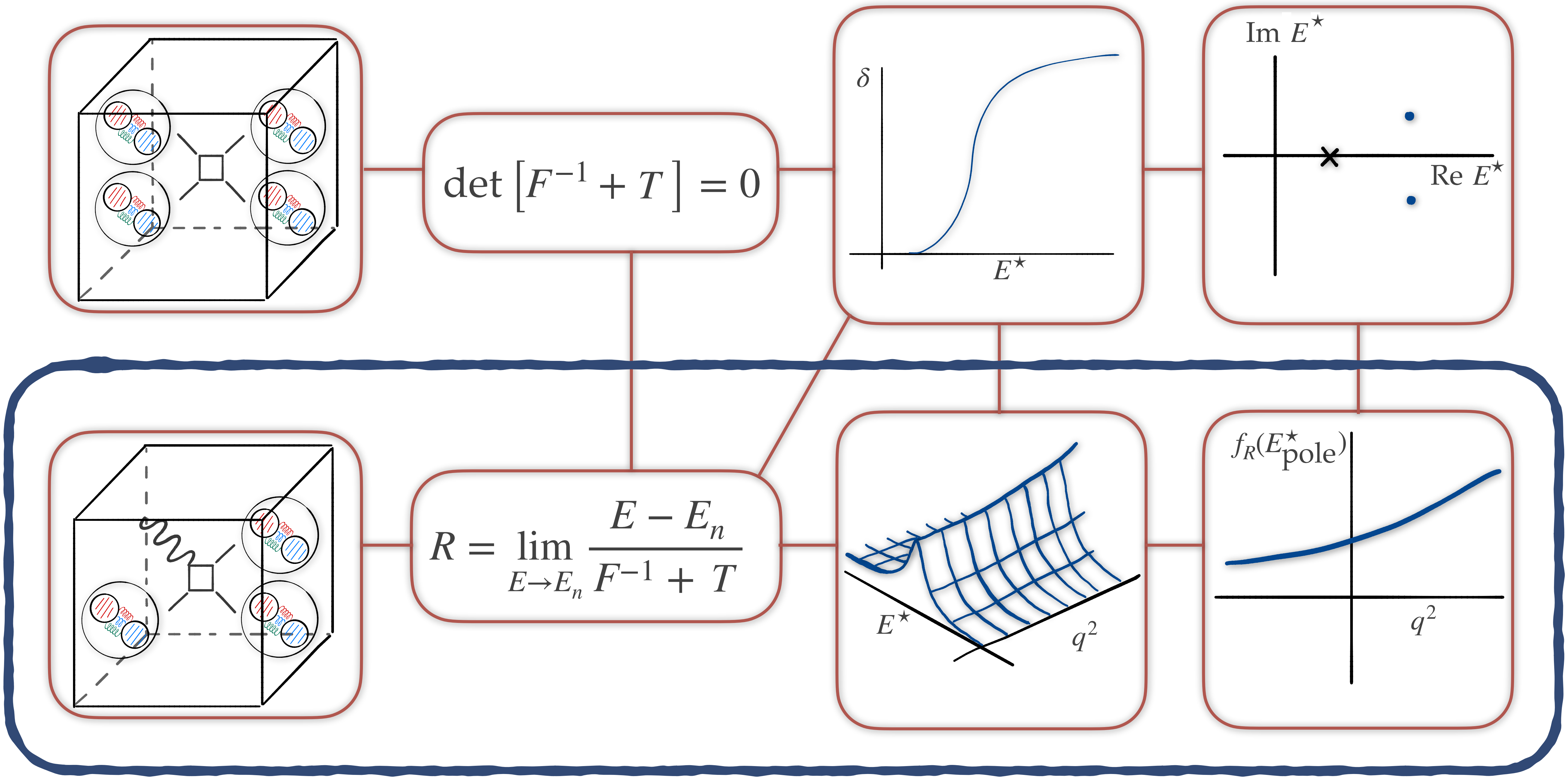
# $\rho$ from lattice QCD



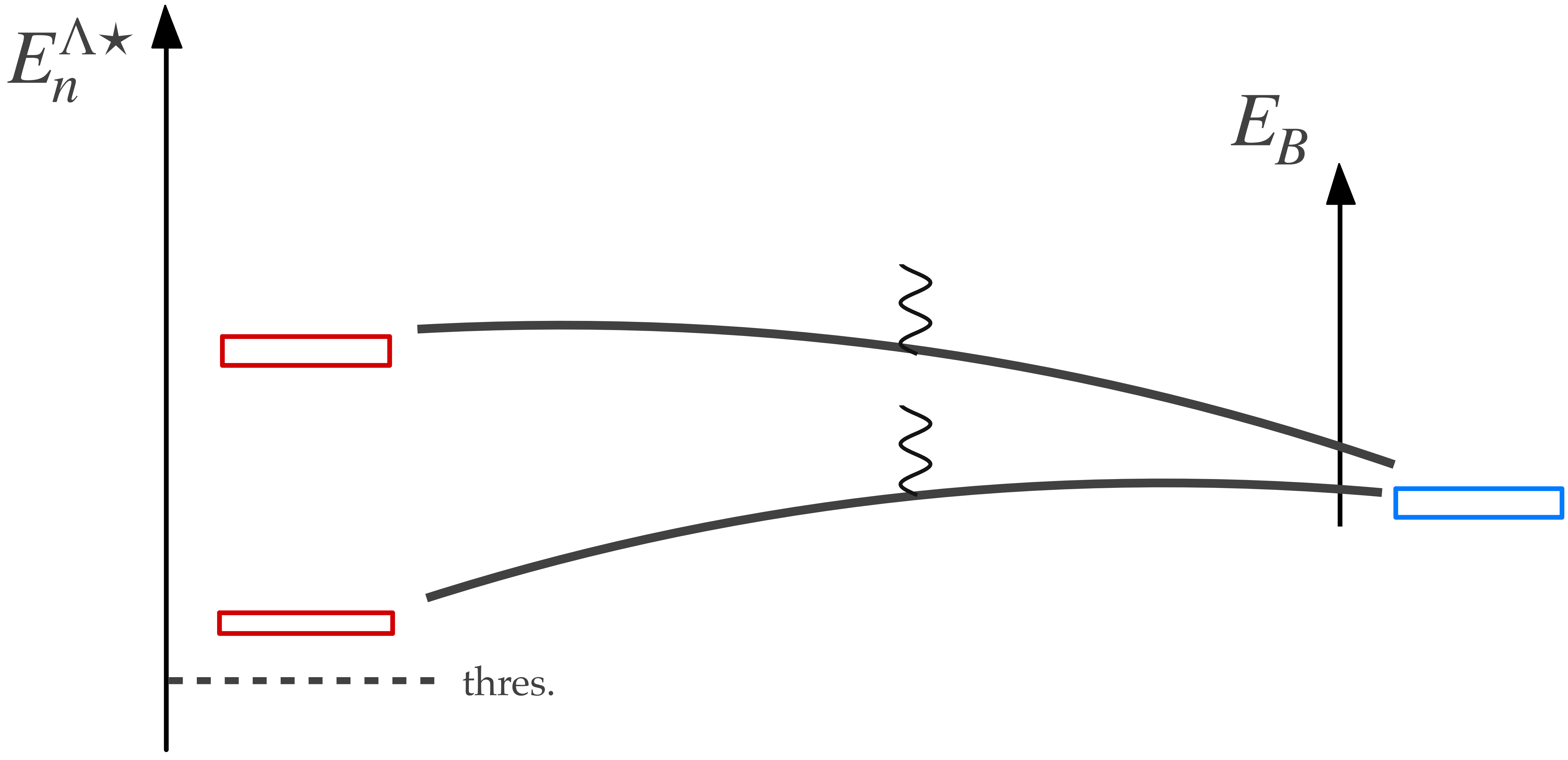

$$T(E^\star) = \frac{1}{\rho}\frac{1}{\cot\delta - i}$$

1704.05439

# $B \to \rho \ell \nu$ on the lattice

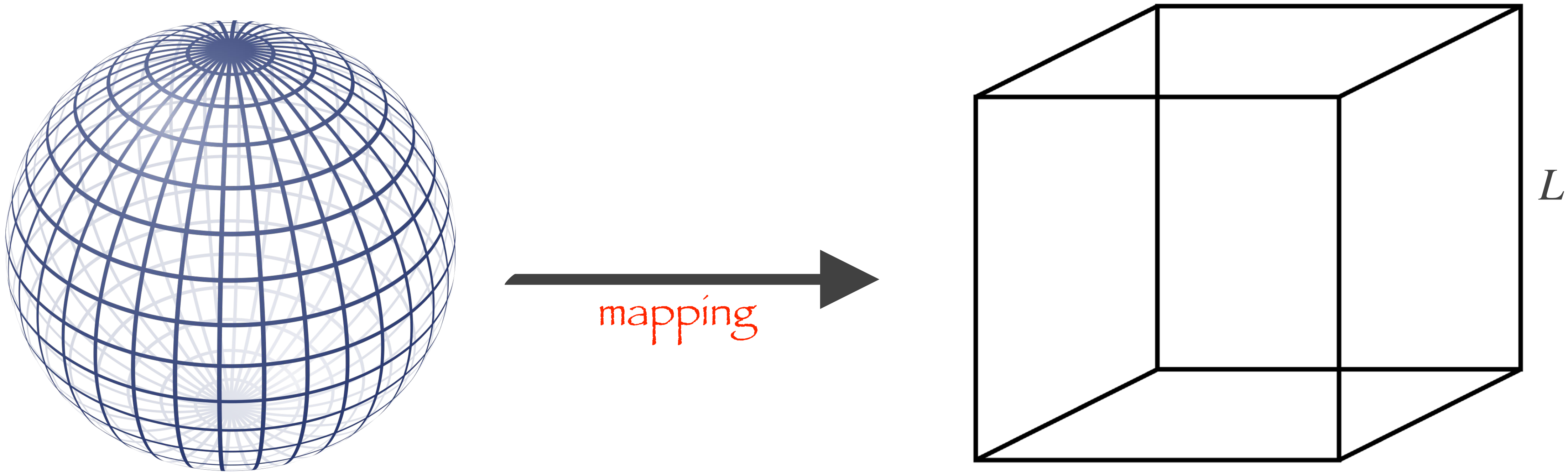


# transitions on the lattice




$$\langle n, E_n^{\Lambda\star} \,|\, V_\mu(\overrightarrow{q}) \,|\, E_B \rangle$$

# $\rho$ with lattice QCD



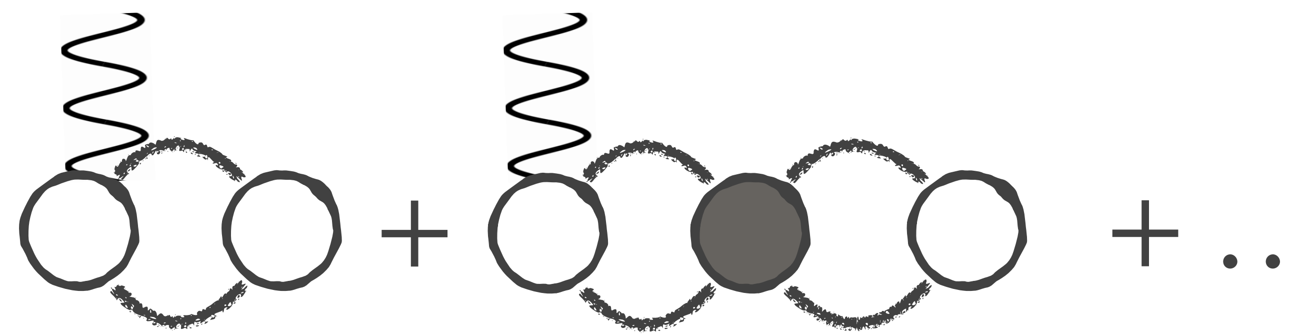

$\{ |\pi(\overrightarrow{p_1})\rangle |\pi(\overrightarrow{p_2})\rangle \}$

$\{|n\rangle\}$

- one particle normalization
  $\langle \pi, p | \pi, p' \rangle = 2E_\pi (2\pi)^3 \delta^3(\overrightarrow{p} - \overrightarrow{p}')$

- one particle normalization
- normalization due to strong interaction

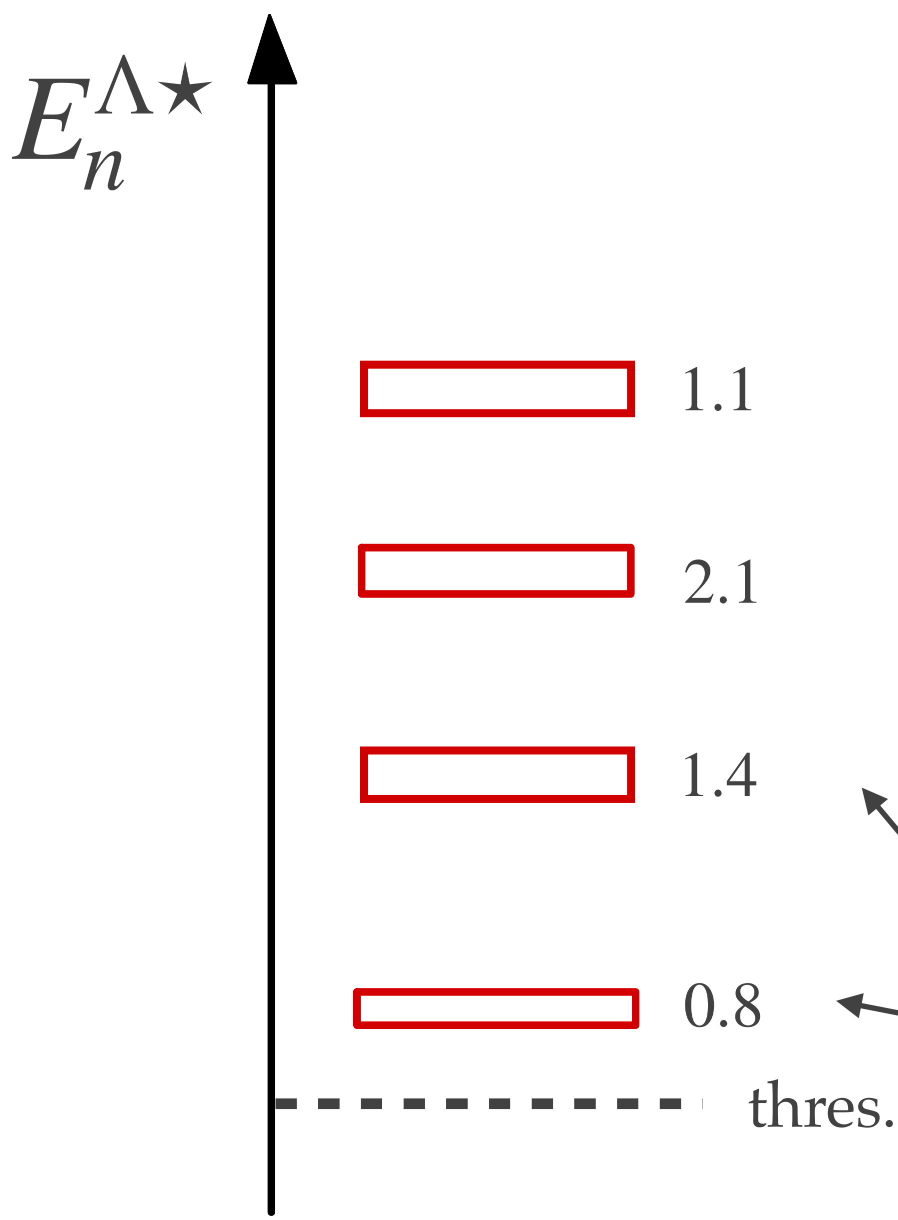
# the finite volume

$C_L^{(3)} =$ 

$$C_L^{(3)} = C_\infty^{(3)} - A'RA$$

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

$$R = \lim_{E\to E_n} \frac{E-E_n}{F^{-1}+T}$$

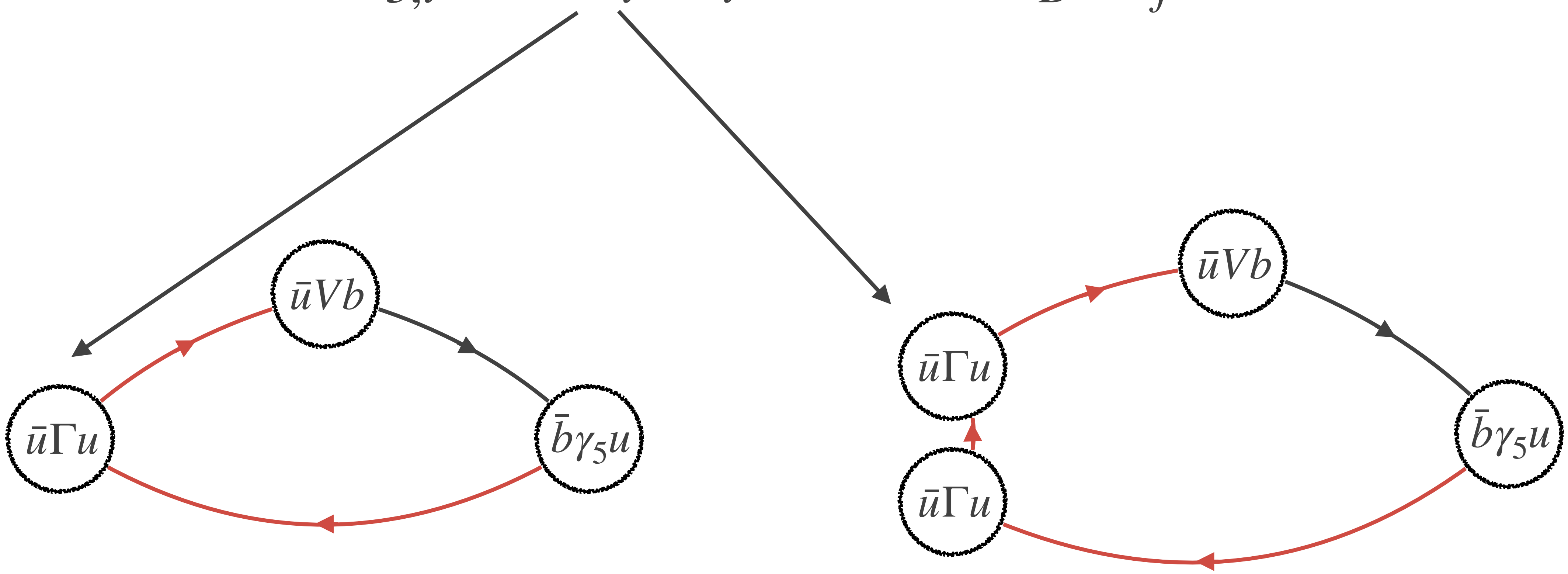



normalization of finite-volume states

$$|E_n^{\Lambda\star}\rangle_L \sim \sqrt{R}\,|\pi\pi(E^\star = E_n^{\Lambda\star})\rangle_\infty$$

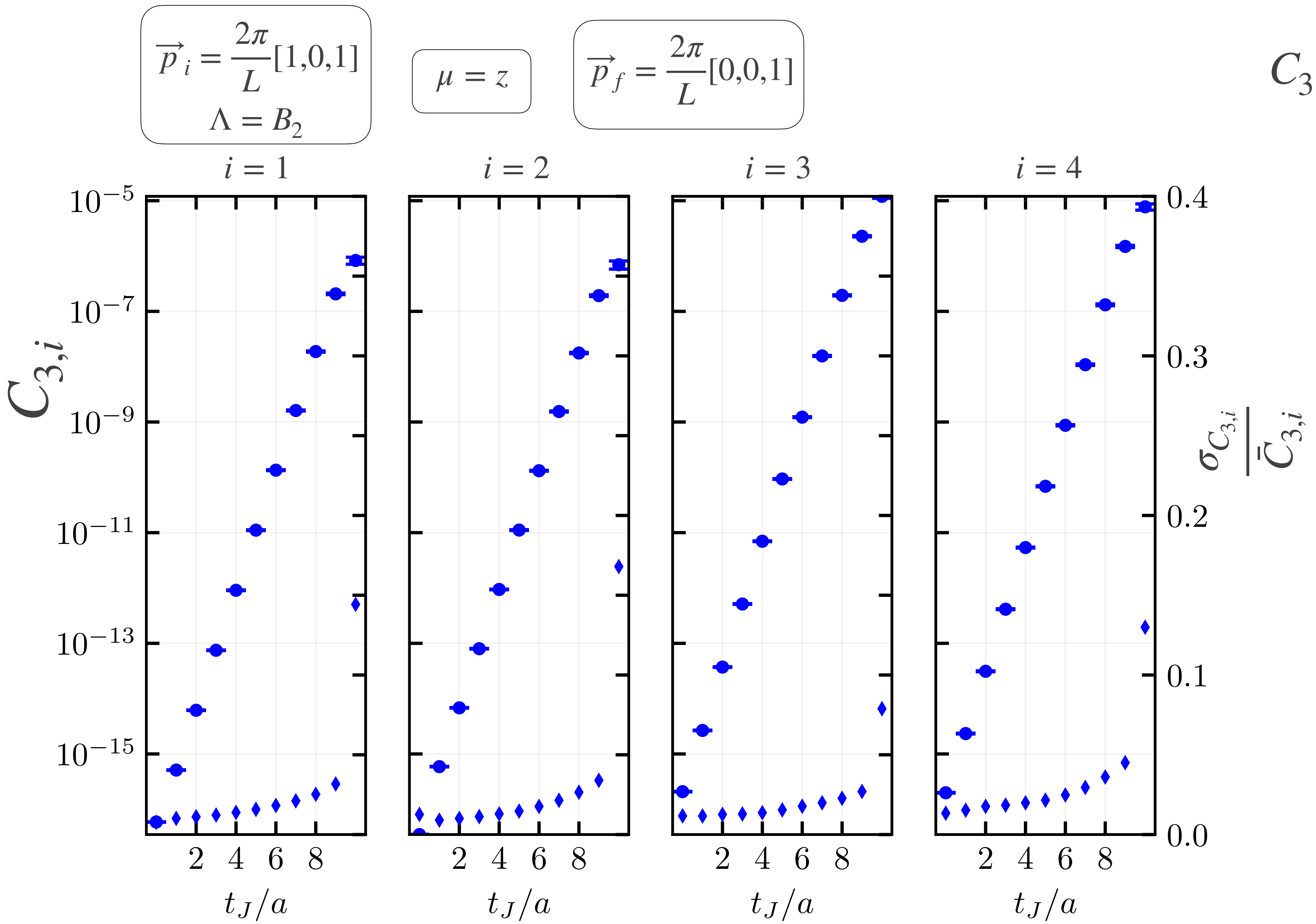
# the calculation

$$C_{3,i} = \langle\ O_i(\overrightarrow{p}_i, \Lambda)\ \ V^\mu\ \ O_B(\overrightarrow{p}_f)\ \rangle$$



# the raw 3-point functions

$$C_{3,i} = < O_i(\vec{p}_i, \Lambda)\ V^\mu\ O_B(\vec{p}_f)\rangle$$




$$O_1 = \bar{u}\Gamma_{B_1}u\,[\vec{p}_i]$$

$$O_2 = \bar{u}\gamma_t\Gamma_{B_1}u\,[\vec{p}_i]$$

$$O_3 = \pi^+(\vec{p}_{i,1})\pi^-(\vec{p}_{i,2})\,|_{\vec{p}_i}$$

$$O_4 = \pi^+(\vec{p}_{i,1})\pi^-(\vec{p}_{i,2})\,|_{\vec{p}_i}$$

# state projection

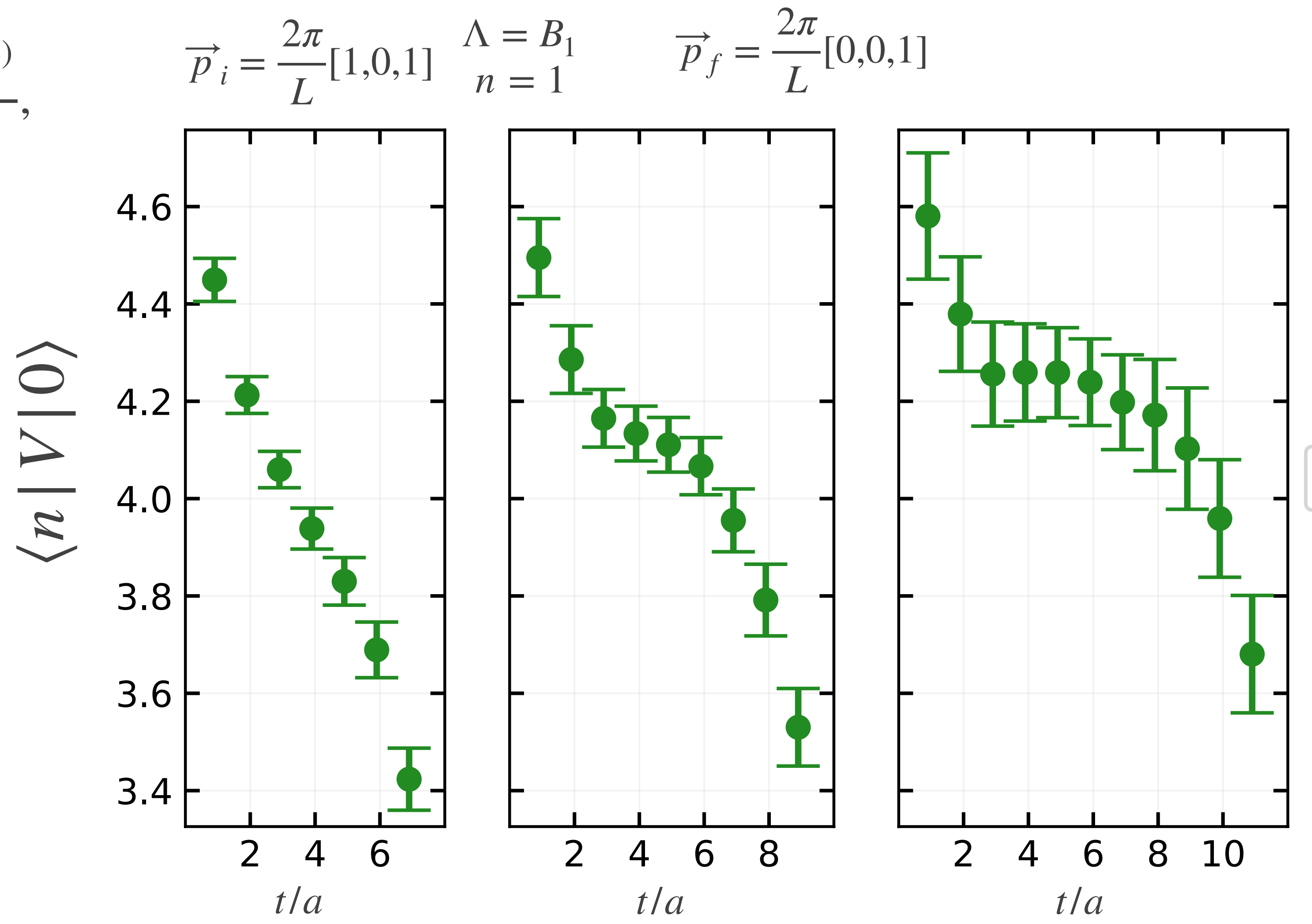
$$C_{3,i} = \sum_{m\in B}\sum_{n\in[\pi\pi]} \langle 0|O_i|n\rangle\langle n|V|m\rangle\langle m|O_B|0\rangle \frac{e^{-E_n(t_f-t)}e^{-E^B_m(t-t_i)}}{2E_n 2E^B_m},$$

$$\boxed{C_3^n = u_i^n C_{3,i}}$$

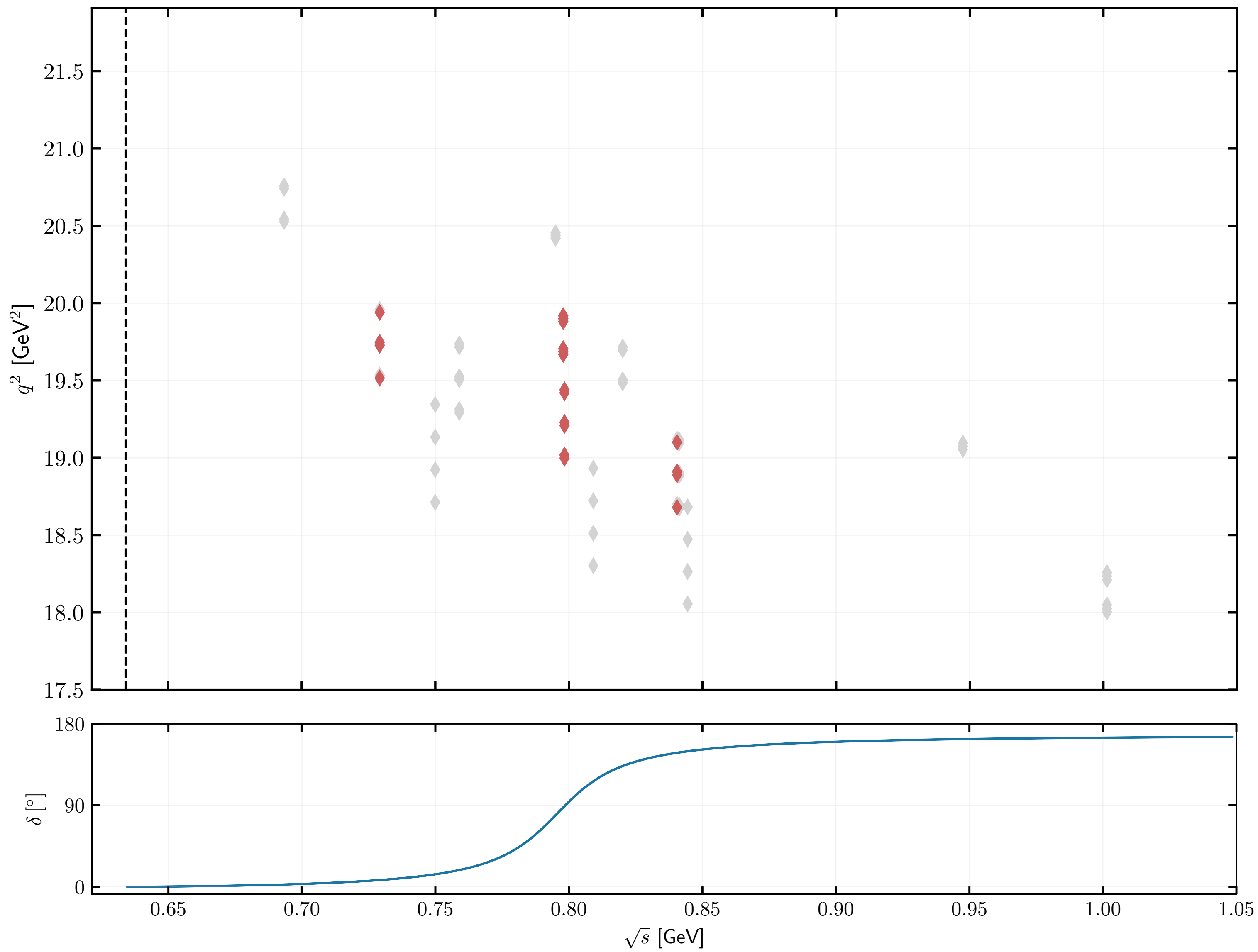
weights from $\pi\pi$ GEVP

$$C_3^n = \langle n|V|0\rangle\langle 0|O_B|0\rangle \frac{e^{-E_n(t_f-t)}e^{-E^B_0(t-t_i)}}{2E_n 2E^B_0}$$

+excited state cont.



# matrix element set




1704.05439

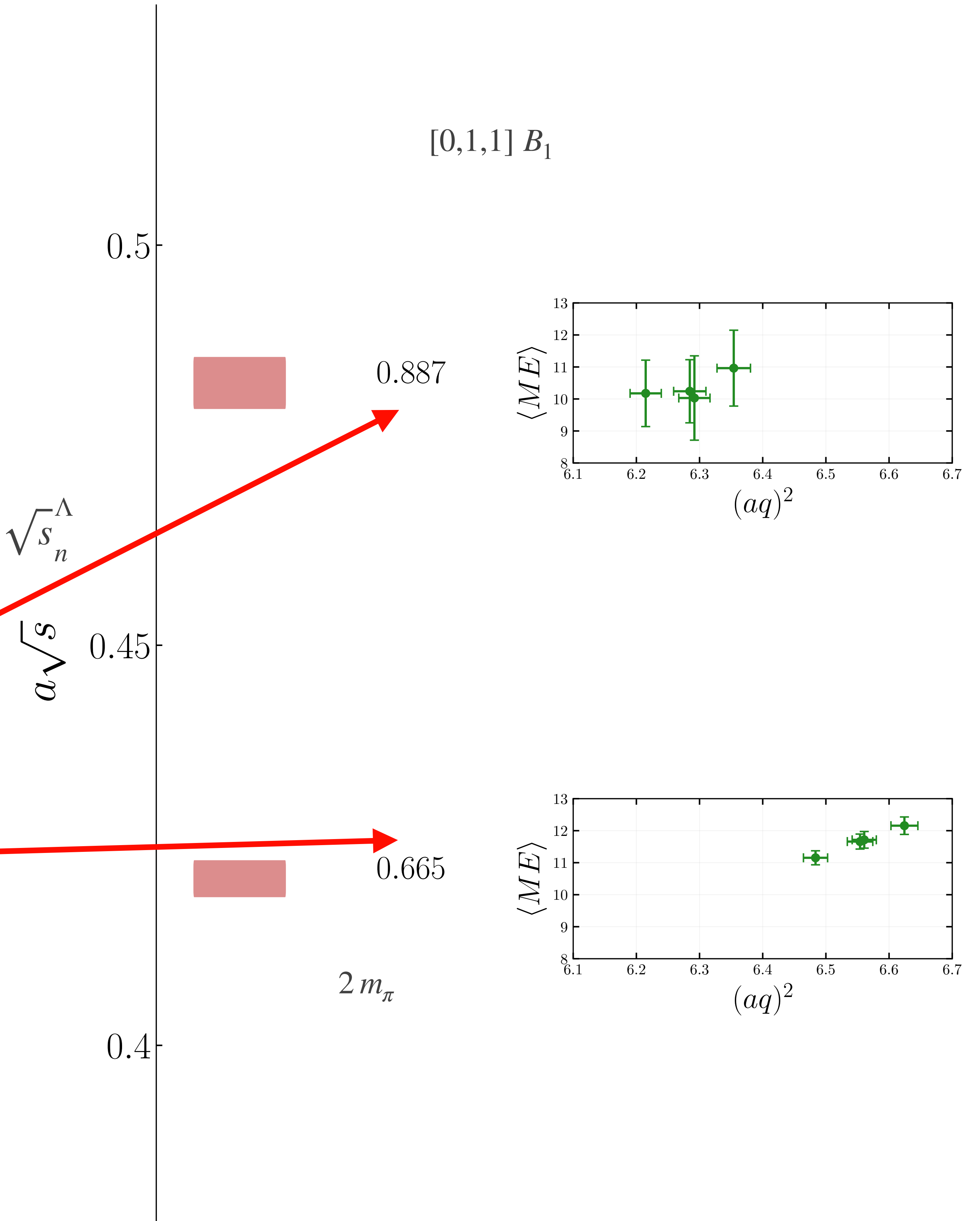
# matrix element set

normalization of
finite-volume states

$$|\sqrt{s_n}^{\Lambda}\rangle_L \sim \sqrt{R_n}\,|\pi\pi(\sqrt{s} = \sqrt{s_n}^{\Lambda})\rangle_\infty$$

$$R_n = 2E_n \lim_{E \to E_n^\Lambda} \frac{E - E_n^\Lambda}{F^{-1} + \mathscr{M}} = \frac{2\sqrt{s_n}^{\Lambda}}{\mu_0^{\star\,\prime}} \mathbf{w}_0 \mathbf{w}_0^T \Big|_{\sqrt{s_n}^{\Lambda}}$$

$$|\langle ME \rangle_L| = \frac{1}{\sqrt{2E_B}\sqrt{2E_n^\Lambda}} \sqrt{\frac{2E_n^\star}{-\mu_0^{\star\,\prime}}}\, w_0^T \cdot V$$

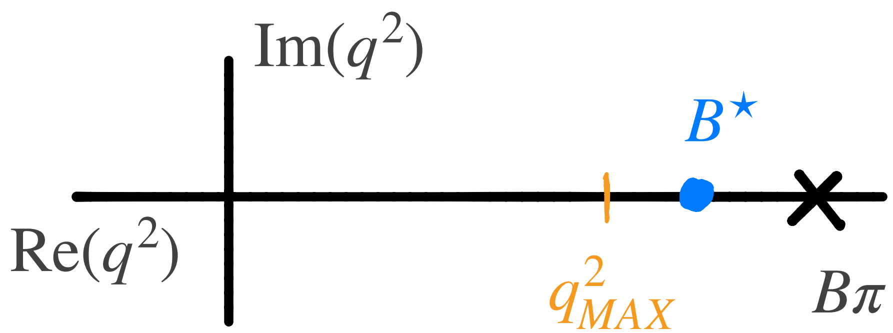


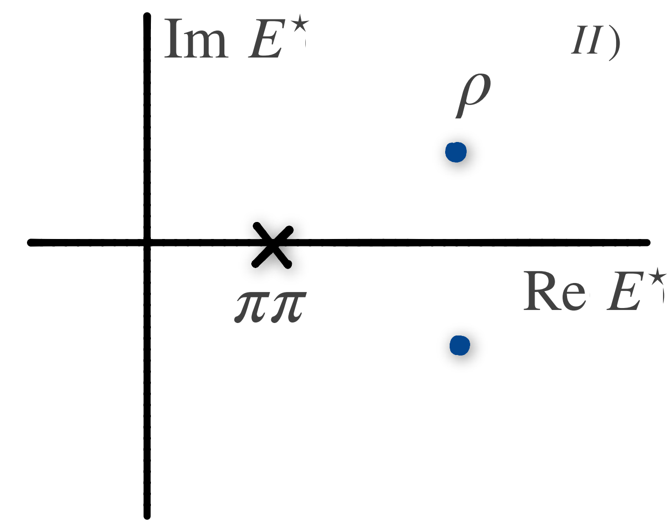

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

# $B \to \rho(\to \pi\pi)\ell\bar{\nu}$ vector transition amplitude

$$\mathcal{H}^{\mu}_{1,m_\ell} = \langle \pi\pi(\varepsilon(m_\ell), p_f) \,|\, \bar{q}\gamma^\mu b \,|\, B(p_i) \rangle = \frac{2iV(q^2, E^\star)}{m_B + 2m_\pi} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu *} p_i^\alpha p_f^\beta$$

$$V(q^2, E^\star) = F(q^2, E^\star) \frac{T(E^\star)}{k}$$

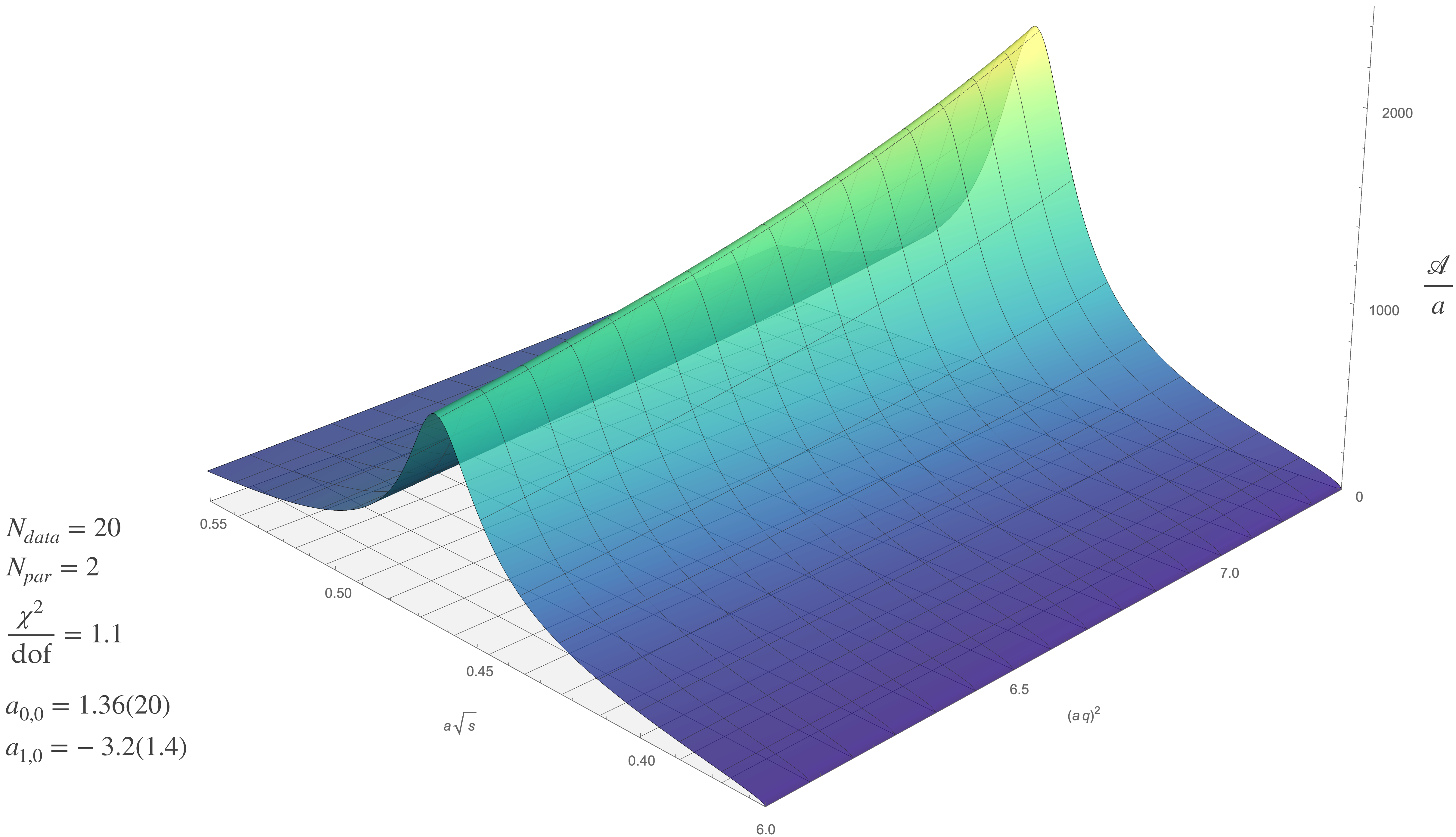


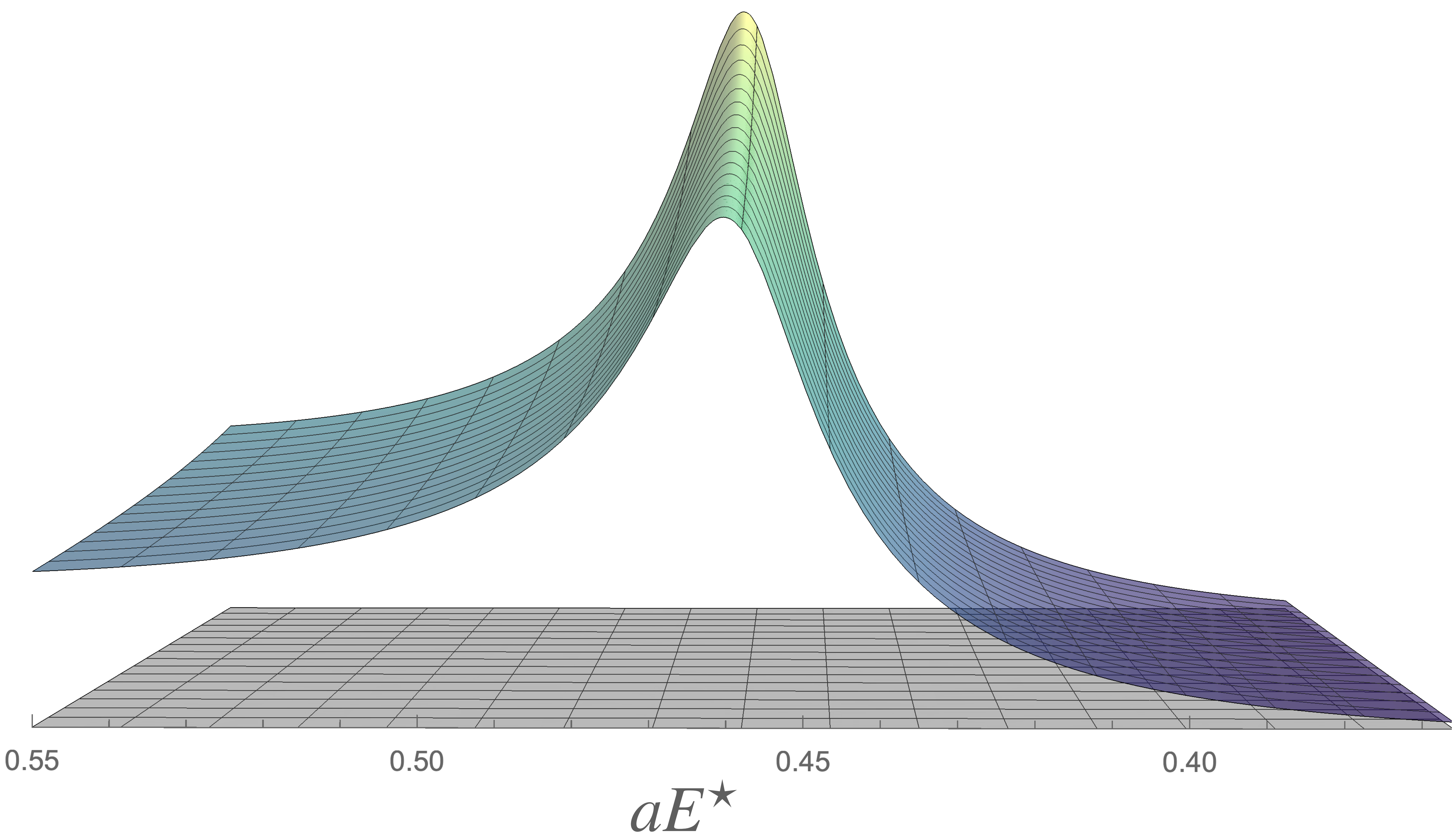

- $F(q^2, E^\star)$:
  - smooth in $E$
  - $q^2$ has poles and thresholds
  - use $z$-expansion

Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357

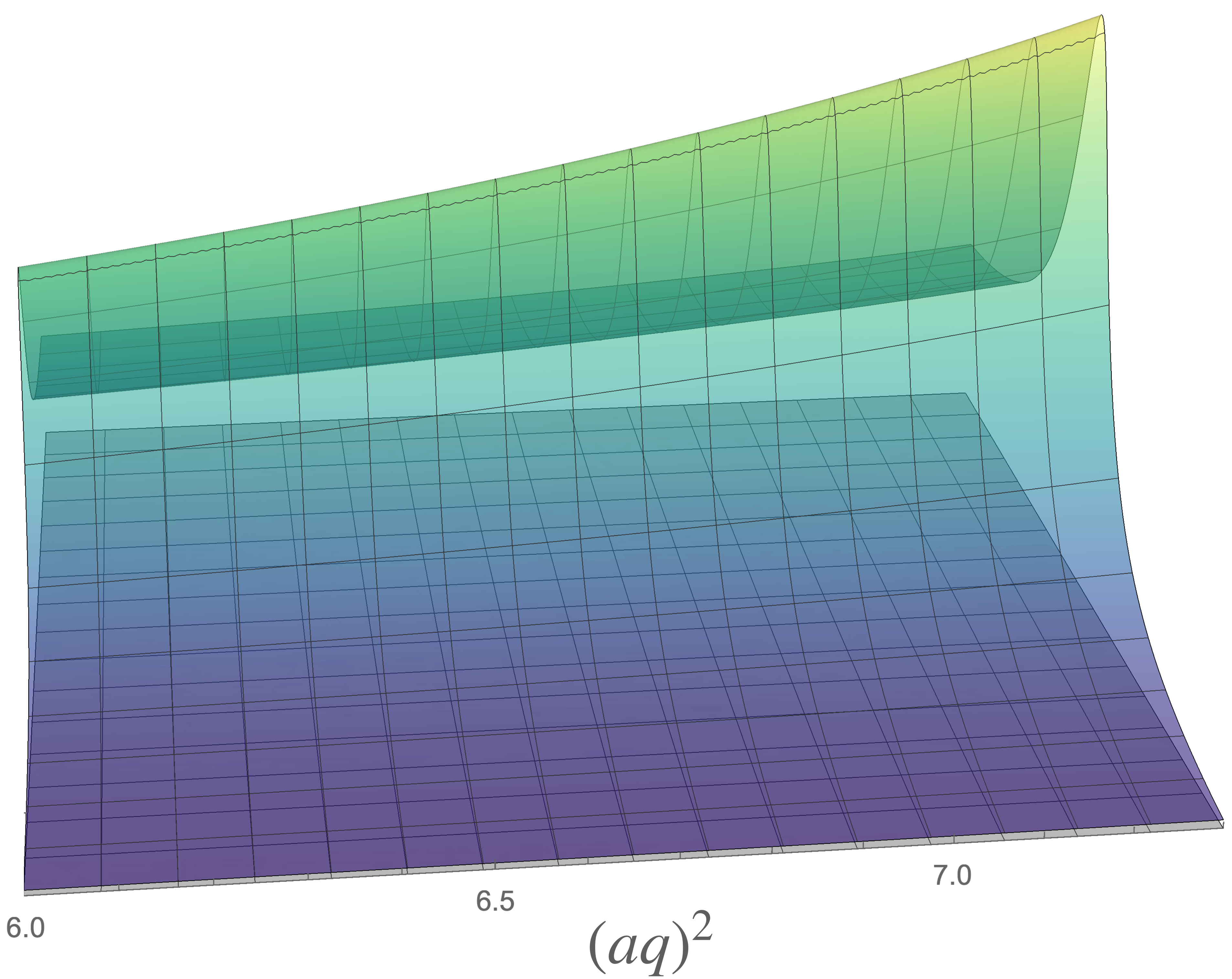
- $T(E^\star)$:
  - $\pi\pi$ threshold
  - $\rho$ pole



$N_{data} = 20$
$N_{par} = 2$
$\frac{\chi^2}{\mathrm{dof}} = 1.1$
$a_{0,0} = 1.36(20)$
$a_{1,0} = -3.2(1.4)$
$a\sqrt{s}$
0.55
0.50
0.45
0.40
6.0
6.5
7.0
$(a\,q)^2$
0
1000
2000
$\frac{\mathcal{A}}{a}$



0.55
0.50
0.45
0.40
$aE^{\star}$


6.0
6.5
7.0
$(aq)^2$

# outlook

- understanding of production mechanism allows us to further constrain SM parameters
- great process to gain confidence
- combining flavor physics with hadronic physics

# Thank you!

https://arxiv.org/pdf/2005.07766.pdf

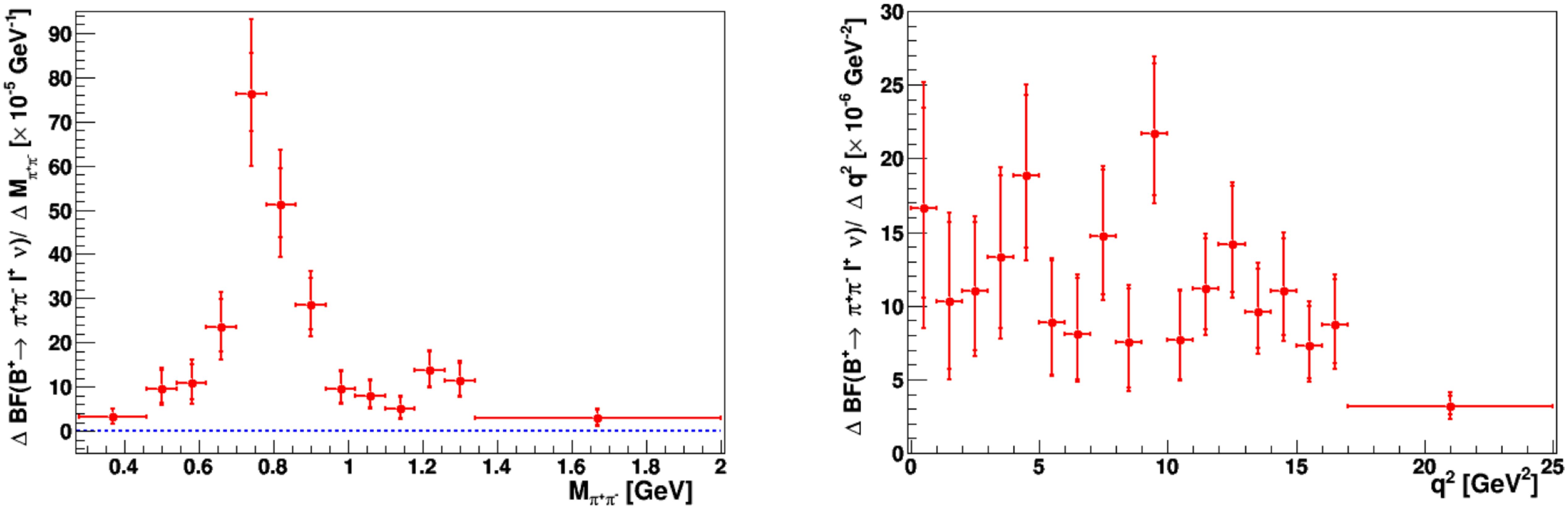



FIG. 5: Partial branching fractions for the decay $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$ in bins of: (left) the $\pi^+\pi^-$ invariant mass according to the results in the 1D($M_{\pi\pi}$) configuration, and (right) the momentum-transfer squared according to the results in the 1D($q^2$) configuration. As there is no upper limit in the $\pi^+\pi^-$ invariant mass, we use a cut-off at 2 GeV for the left figure.