Resonant processes via lattice QCD an overview

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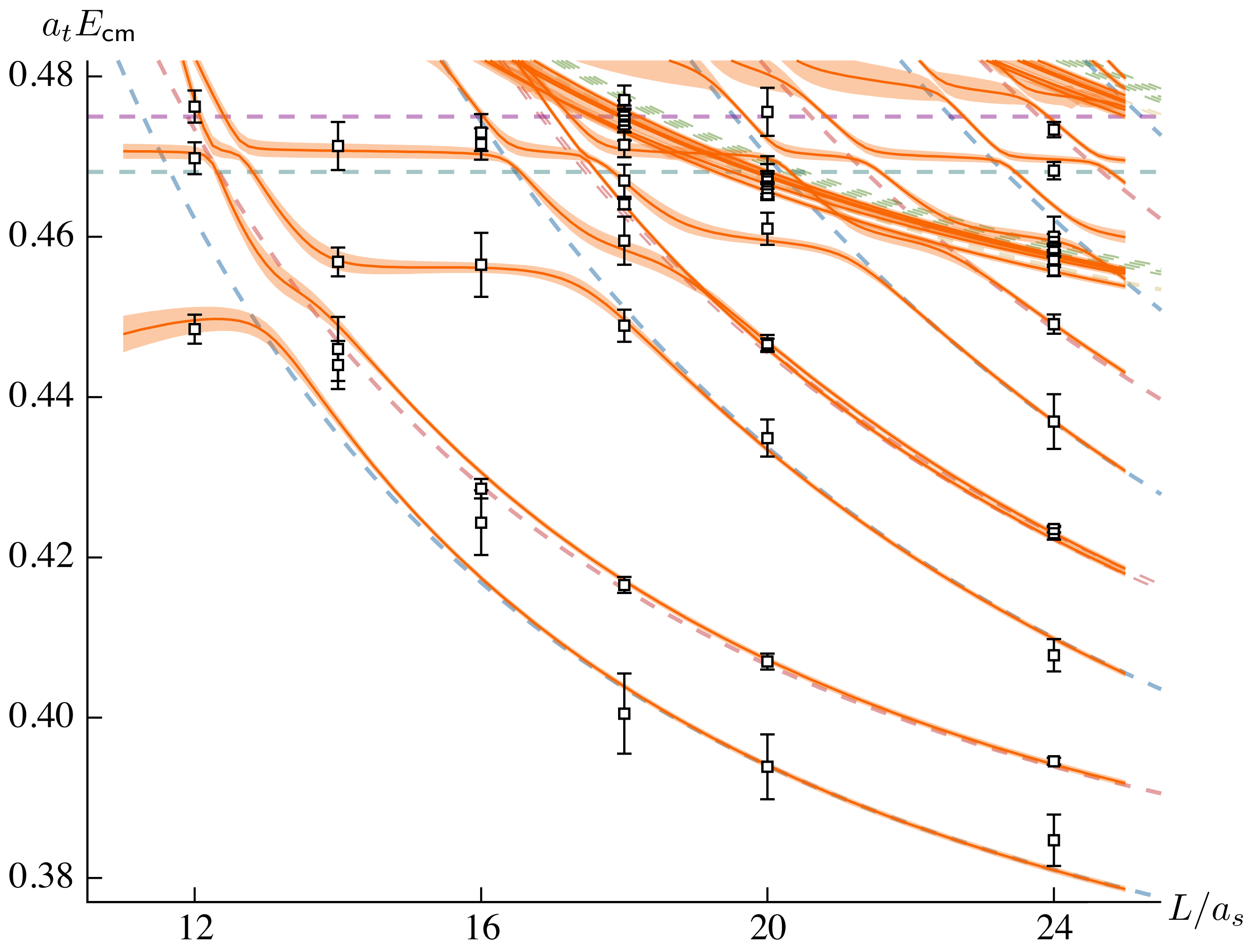
hadspec

Modern-day spectroscopy rooted in QCD

$$\sum_f \overline{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f - \frac{1}{4}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

+ electroweak sector



Woss, Dudek,
Edwards, Thomas, Wilson (2020)

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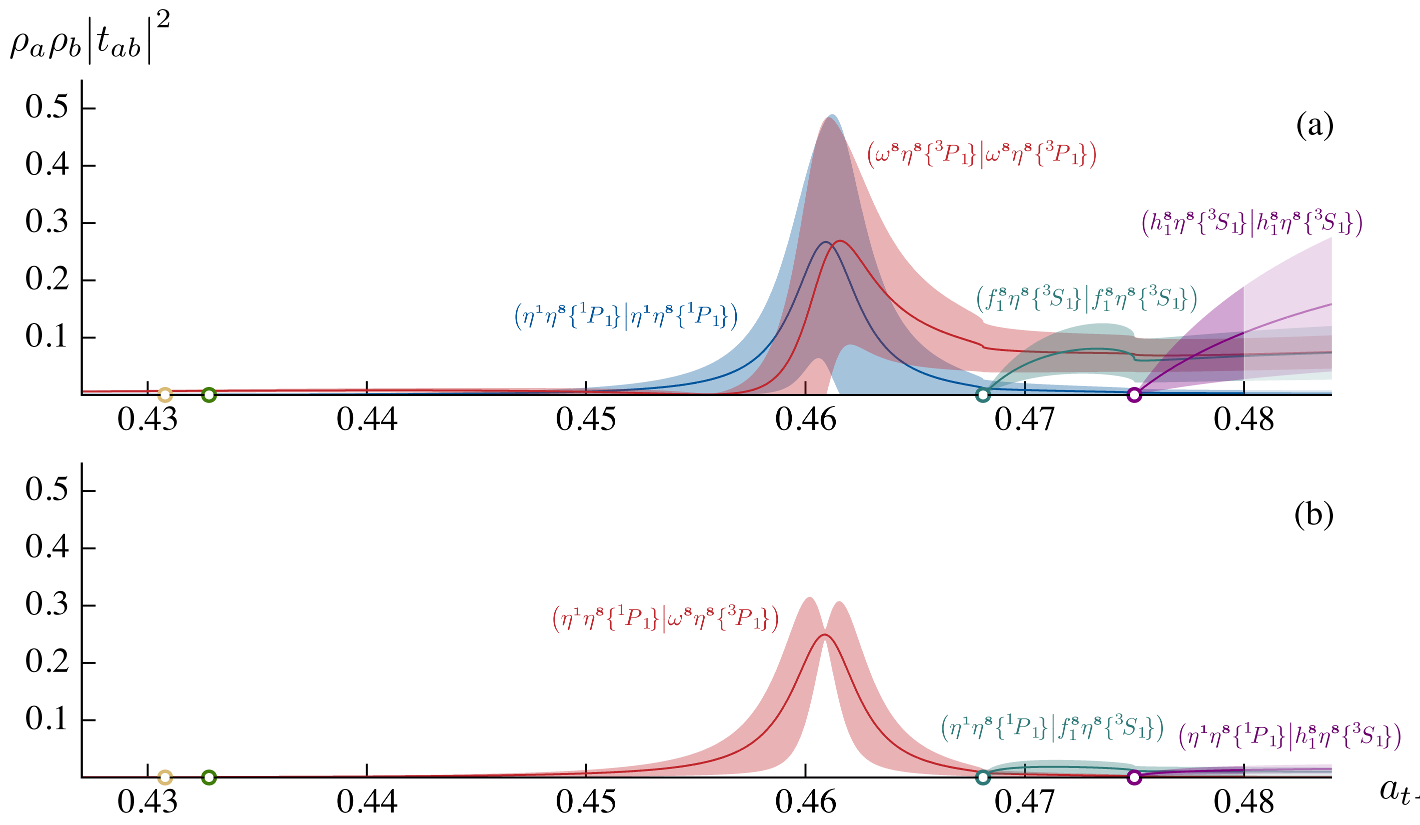

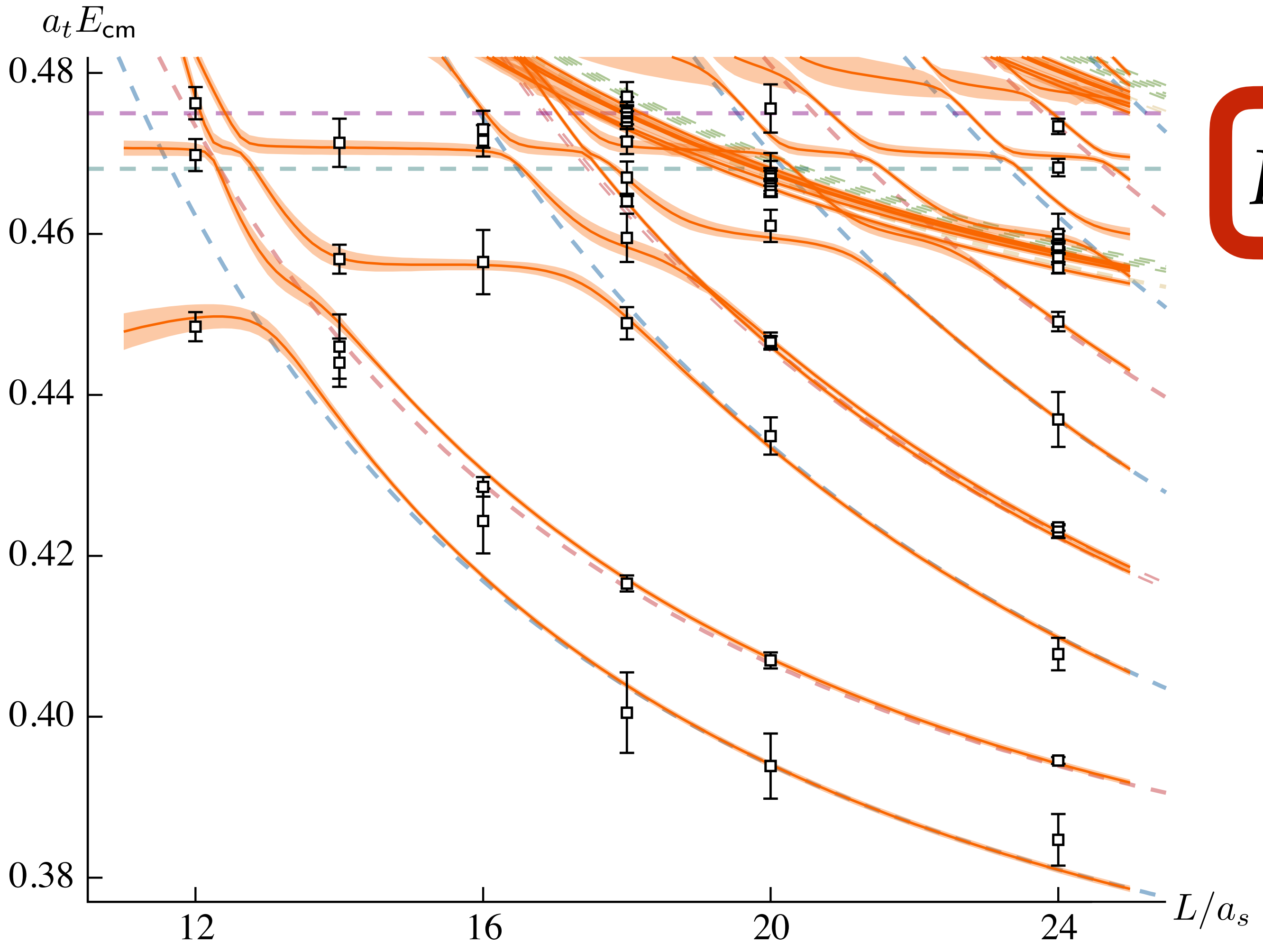

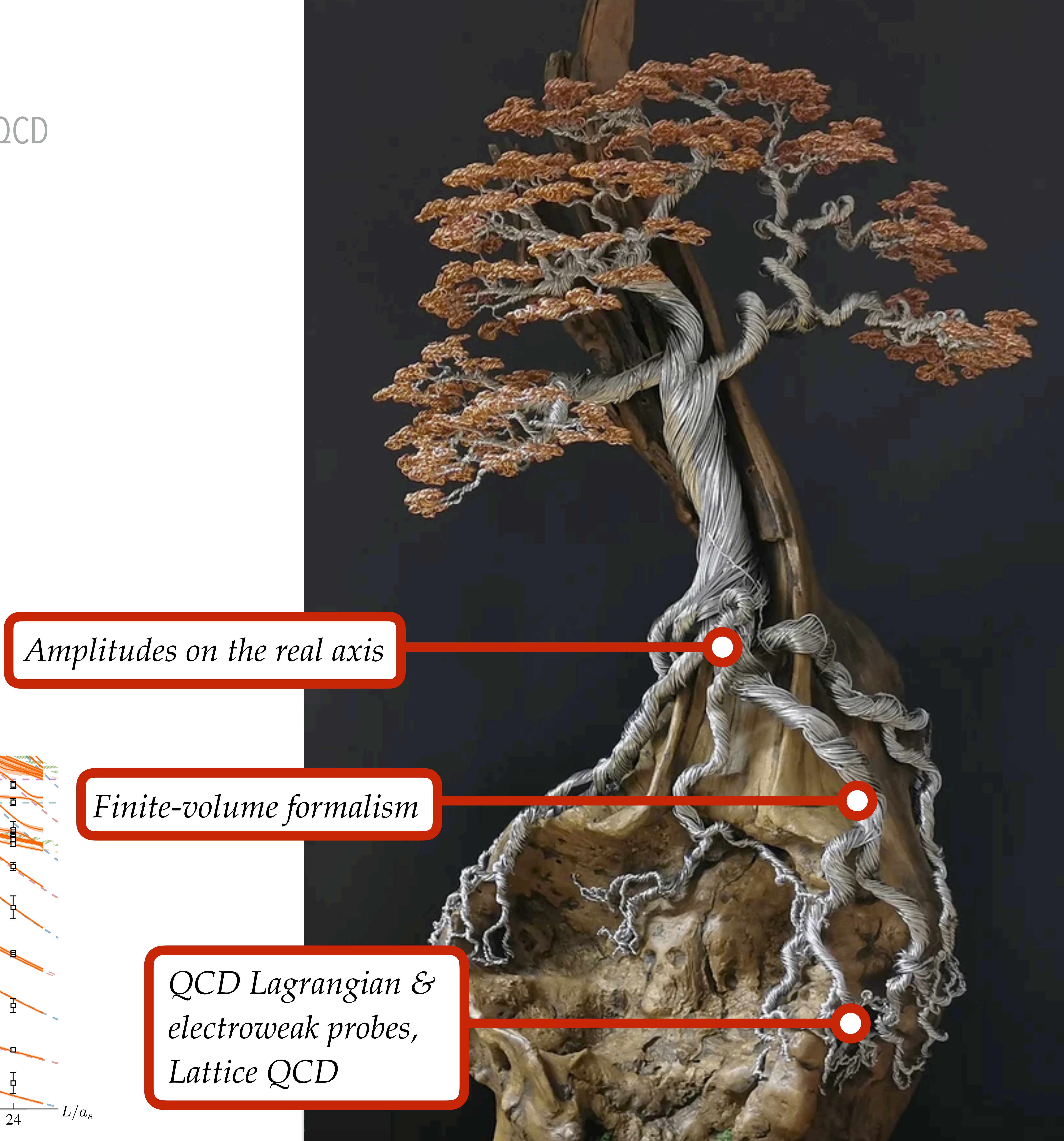

Woss, Dudek, Edwards, Thomas, Wilson (2020)

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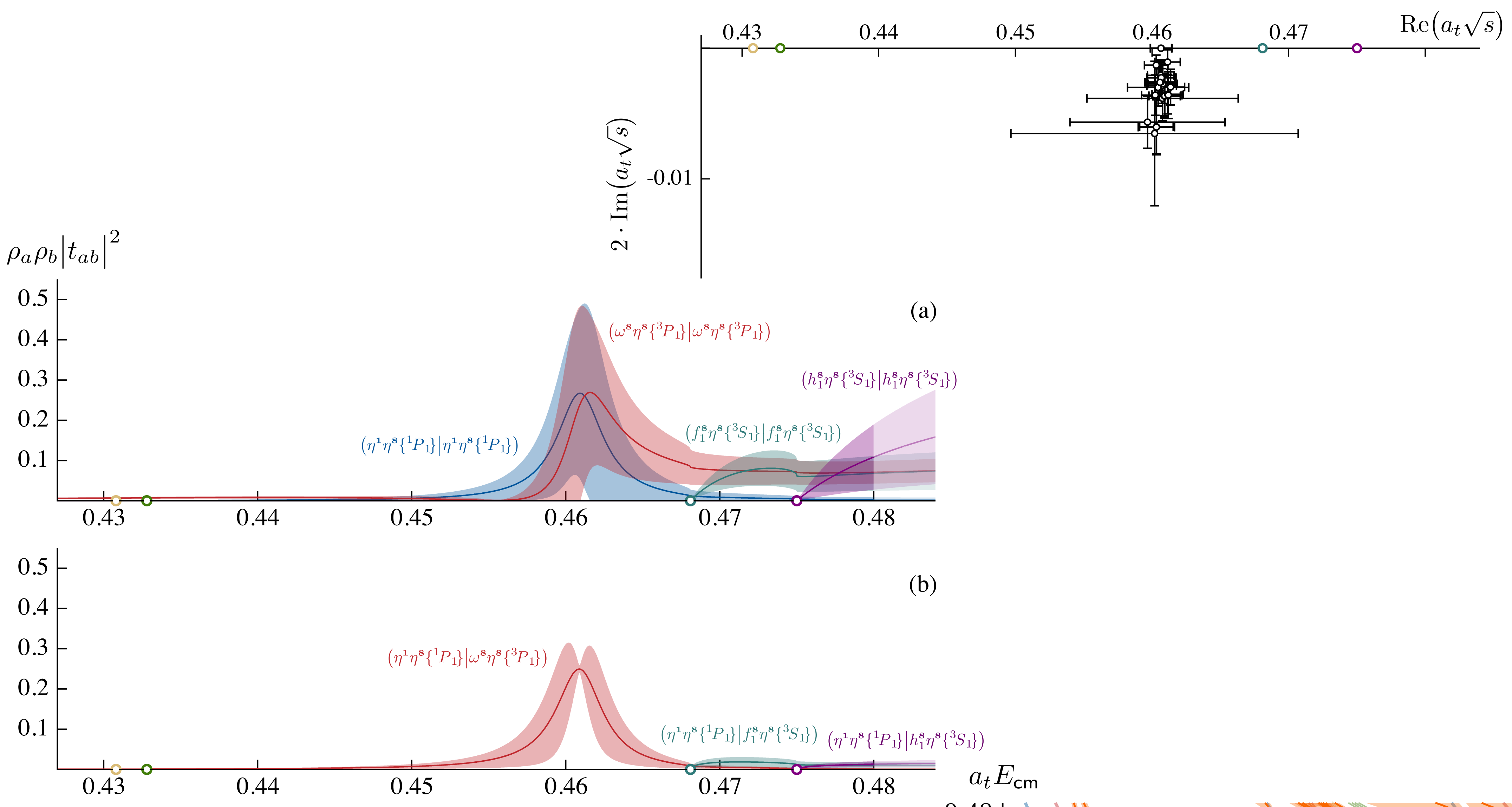

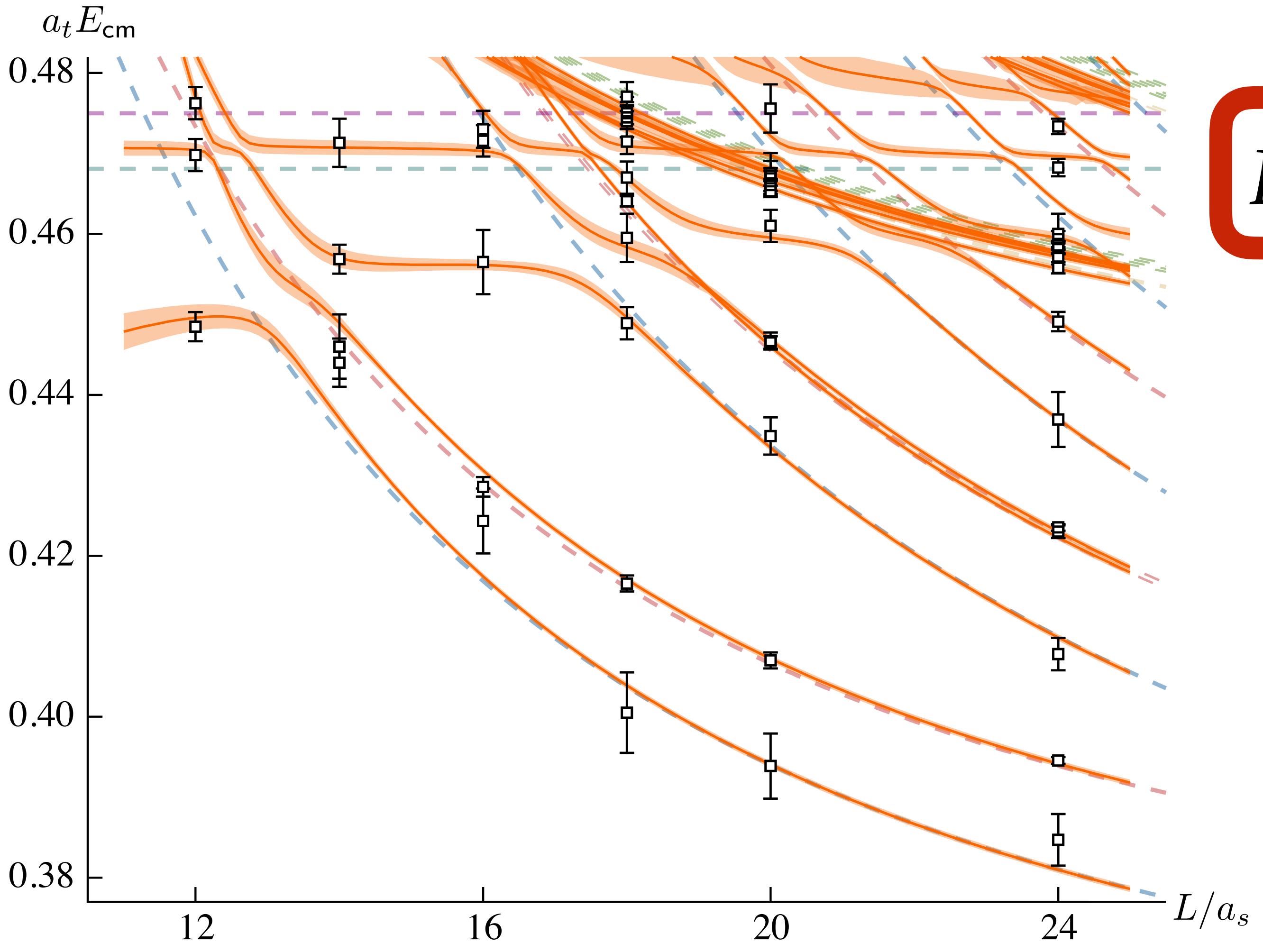

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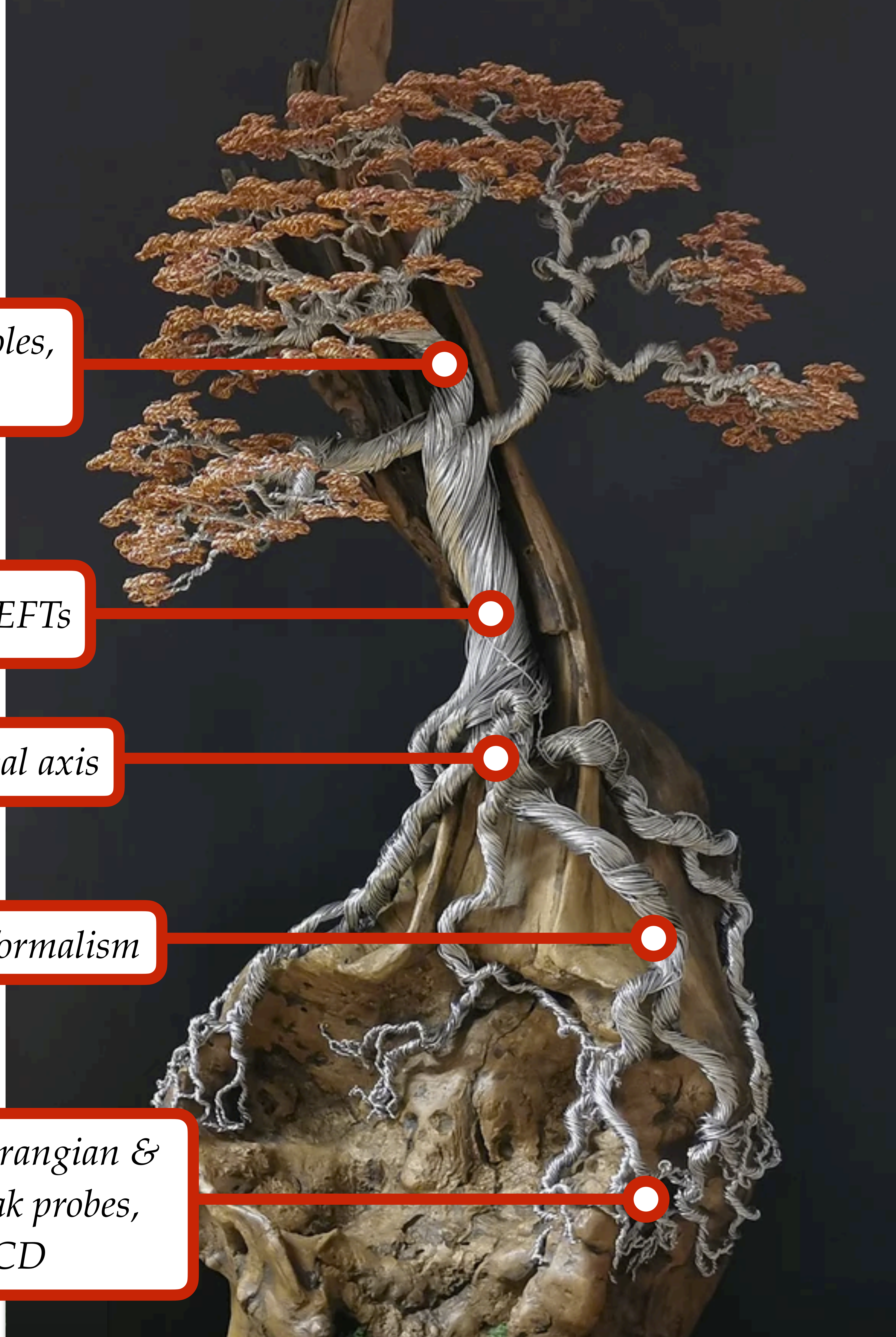


Analytic continuation: poles, widths, form factors,…

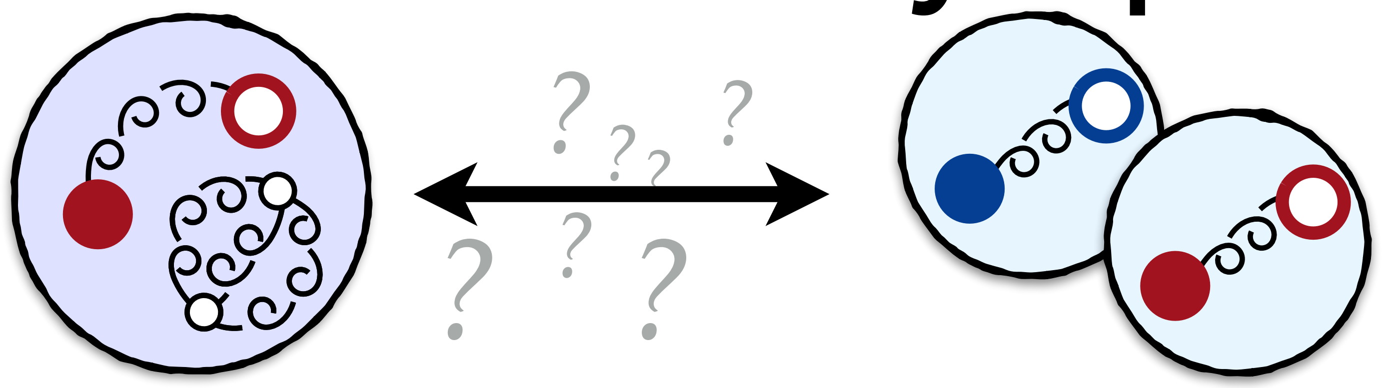
Scattering theory & EFTs

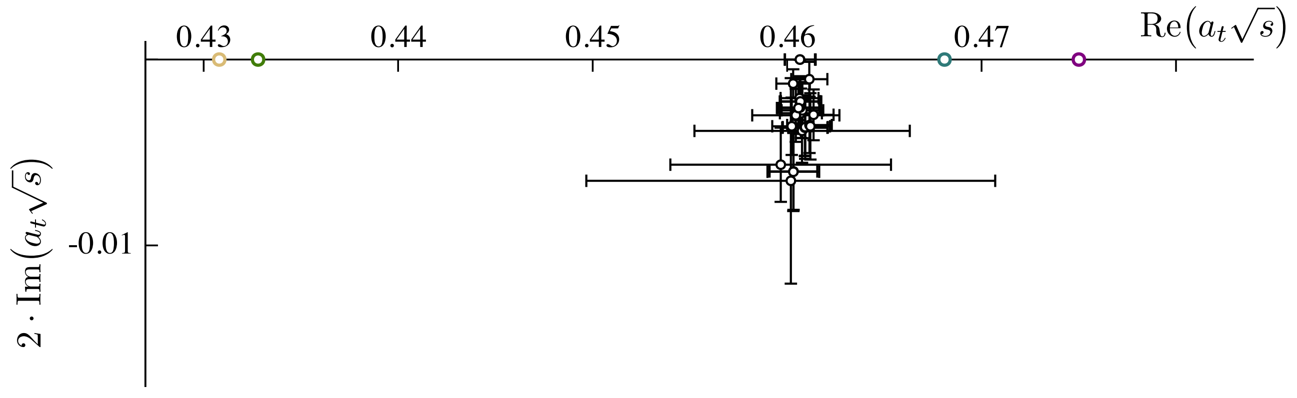
Amplitudes on the real axis

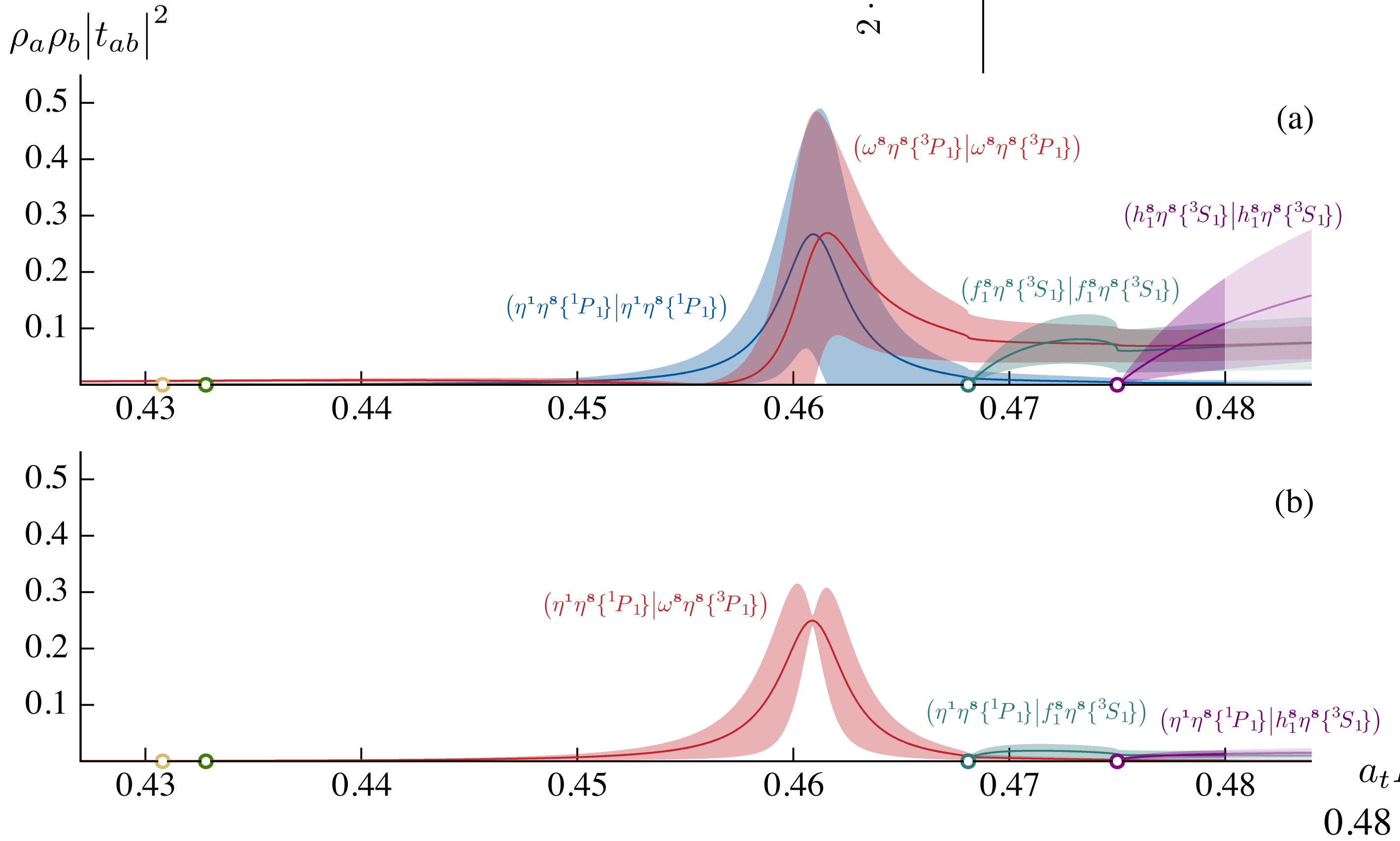
Finite-volume formalism

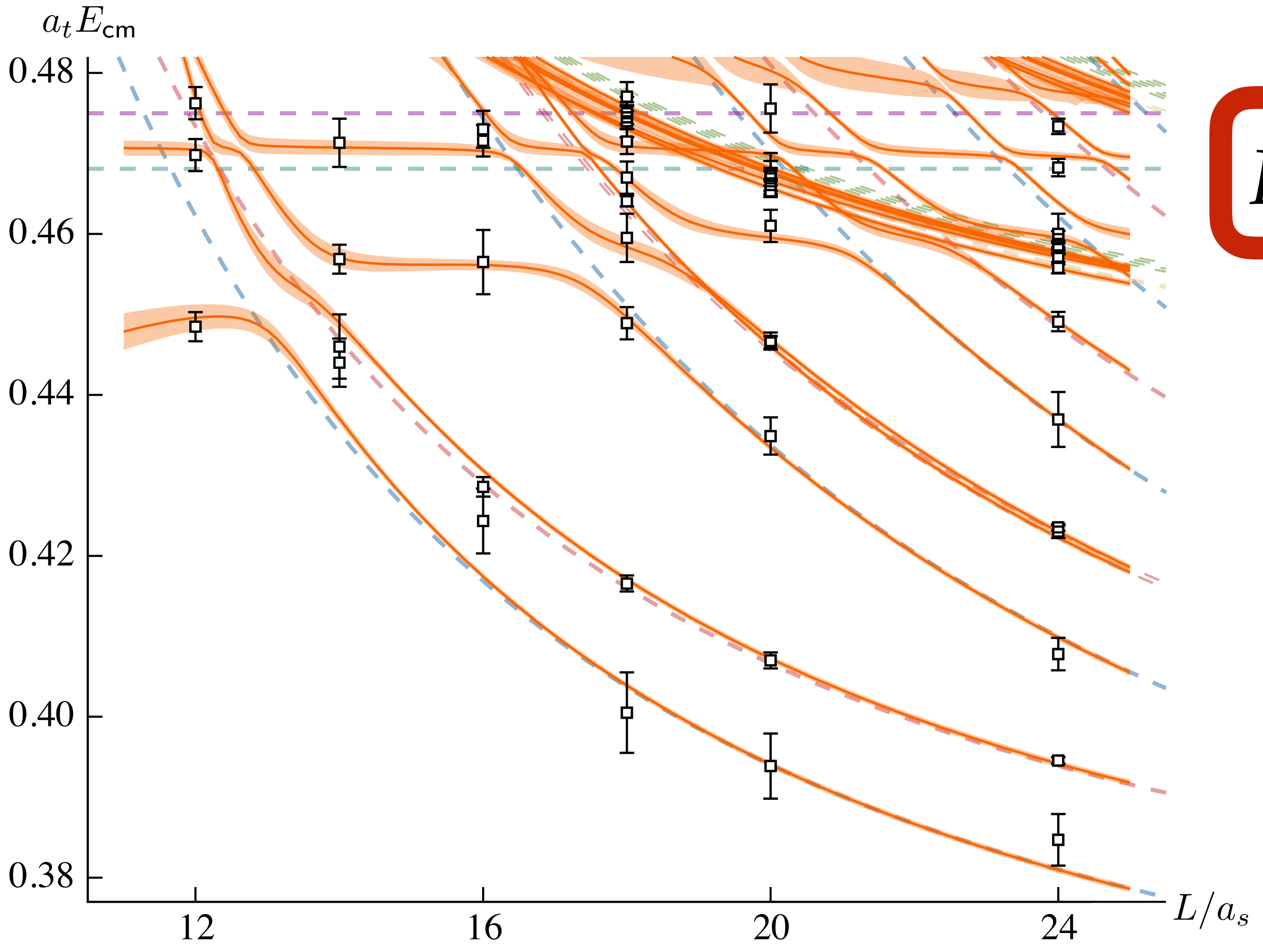
QCD Lagrangian & electroweak probes, Lattice QCD

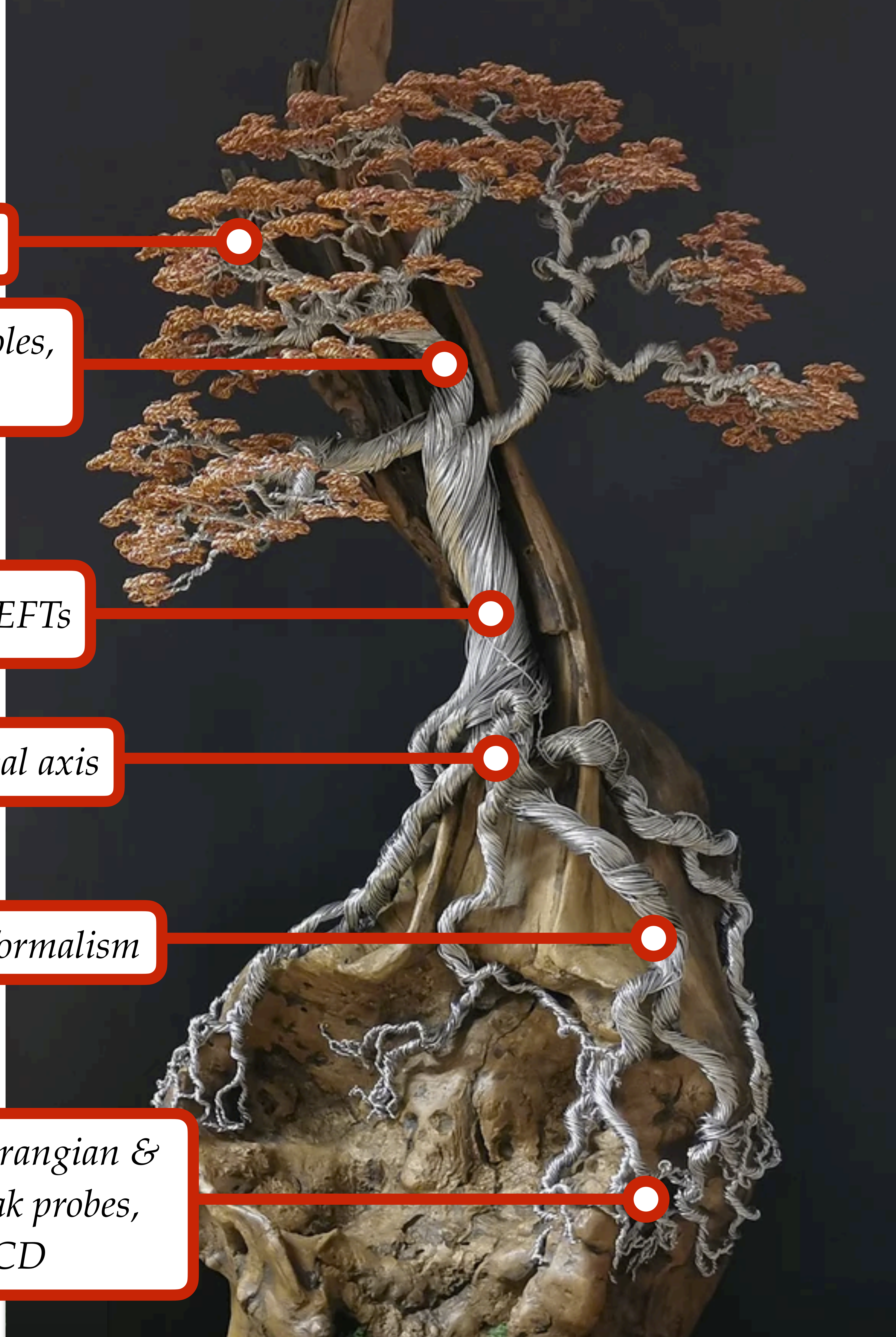
Modern-day spectroscopy rooted in QCD











Nature of states, patterns in the spectrum, …

Analytic continuation: poles, widths, form factors,…

Scattering theory & EFTs

Amplitudes on the real axis

Finite-volume formalism

QCD Lagrangian & electroweak probes, Lattice QCD

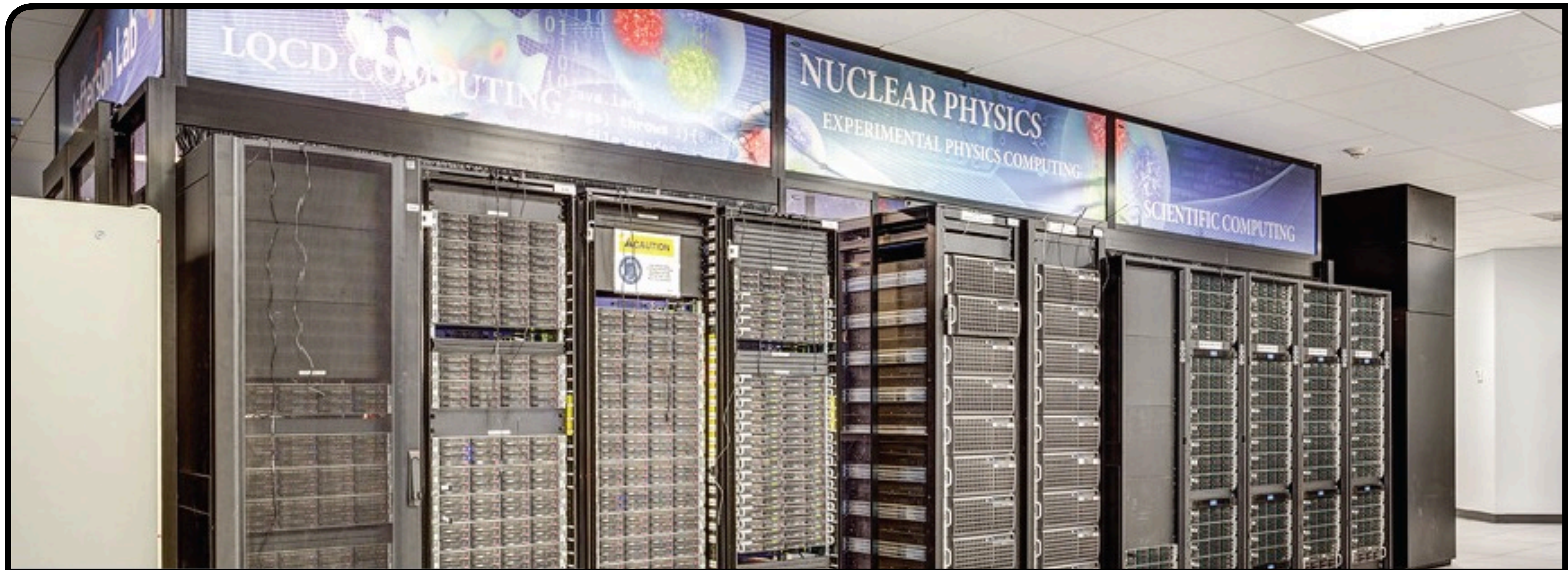
Woss, Dudek, Edwards, Thomas, Wilson (2020)

Lattice QCD

hadspec

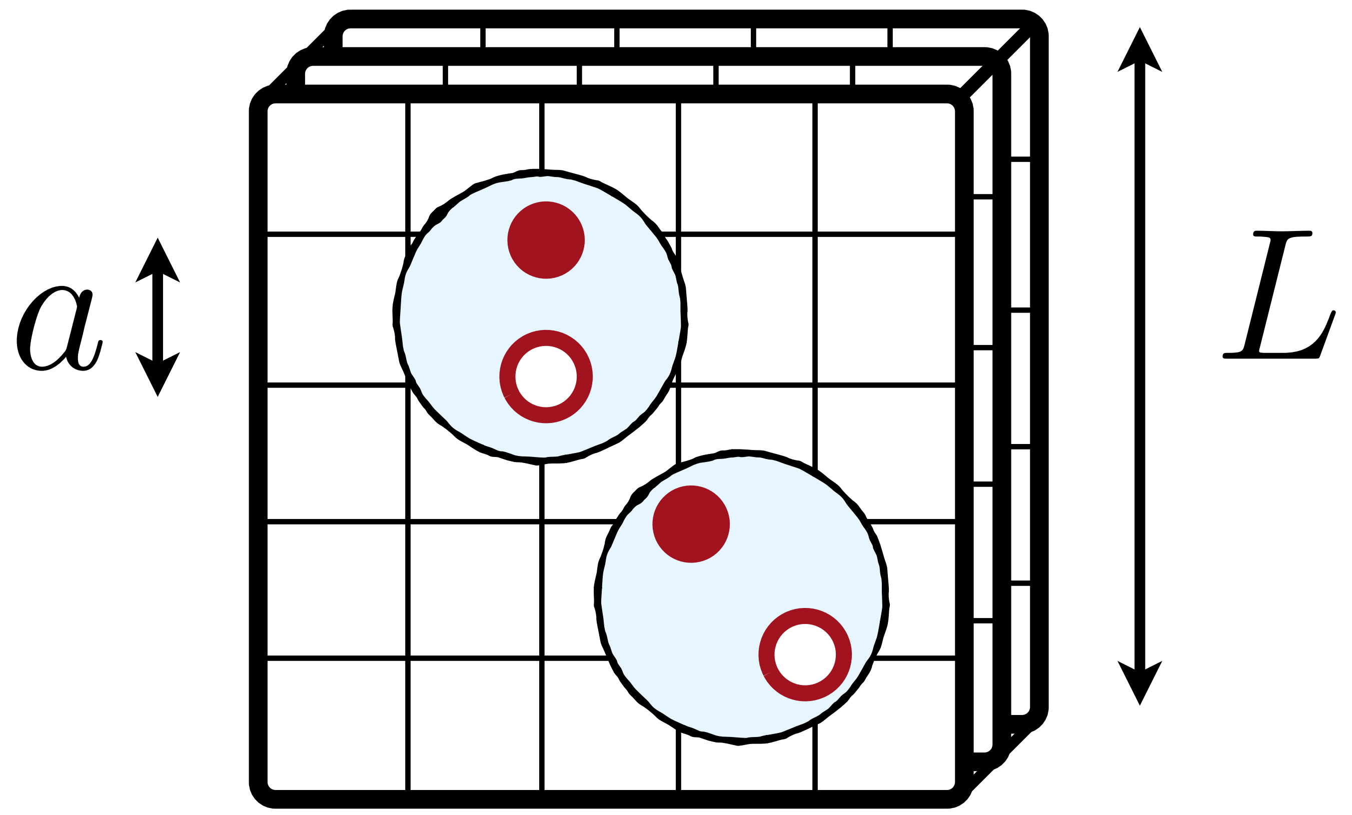
- ☑ Only systematically improvable tool (available) for QCD,
  - ☐ Confirm experimental observations, most states in PDG, …
  - ☐ Guide experimental searches, exotic states,…
  - ☐ Compliment experimental efforts. resonant form factors and structure.

- ☑ Large collaborations with broad set of expertise, akin to experiments
  - ☐ Theoretical formalism,
  - ☐ Algorithmic developments,
  - ☐ Production runs,
  - ☐ Data analysis.



Lattice QCD in a nut shell

- lattice spacing: $a \sim 0.03 - 0.12$ fm
- finite volume: $L \sim 6 - 12$ fm



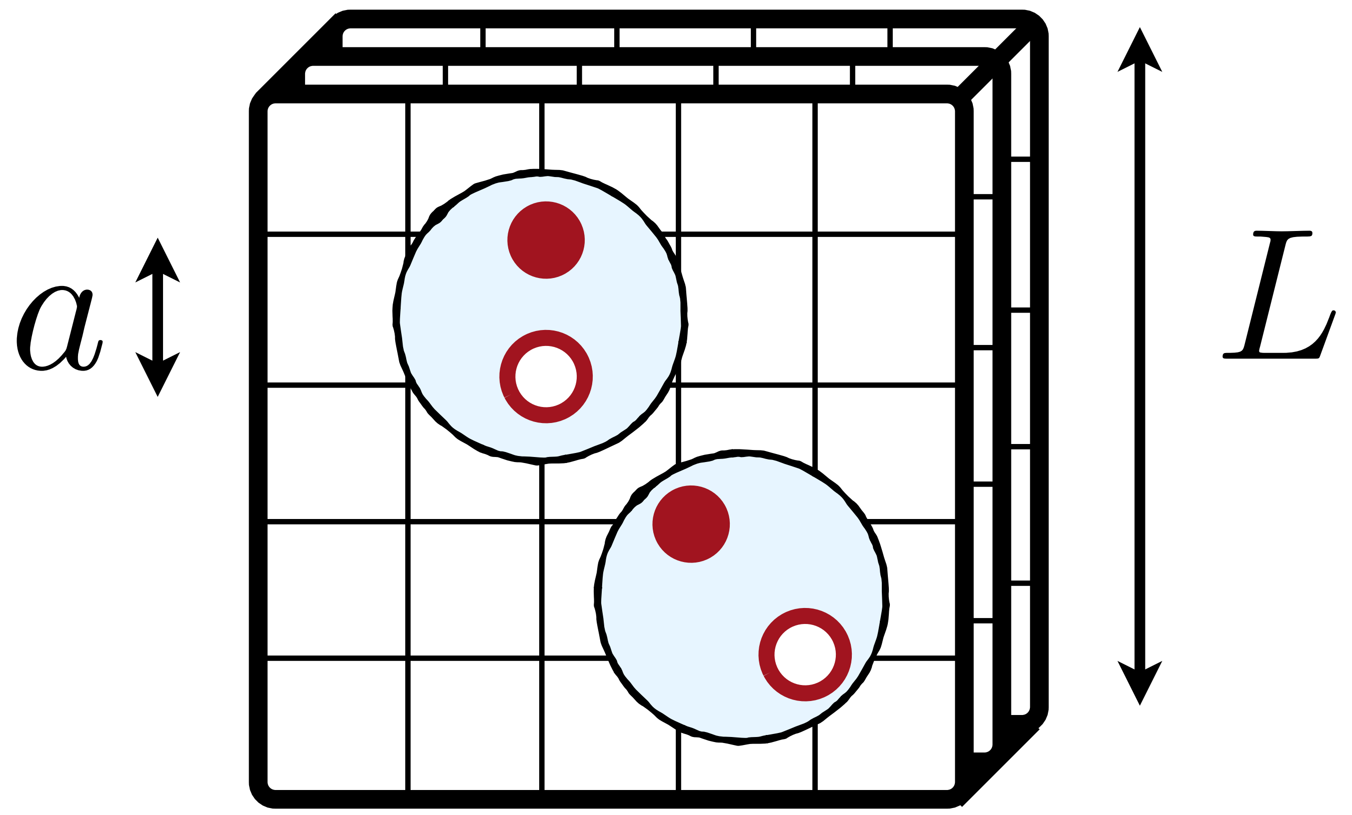
$$D_\mu = \Big(\qquad \Big) \updownarrow (L/a)^3 \times (T/a)$$

Lattice QCD in a nut shell

- lattice spacing: $a \sim 0.03 - 0.12$ fm
- finite volume: $L \sim 6 - 12$ fm
- quark masses
- Euclidean spacetime: $t_M \to -it_E$
- Importance sampling

QFT in real time : $Z = \int \mathscr{D}\varphi(x)\ e^{iS[\varphi]}$

QFT in imaginary time : $Z_E = \int \mathscr{D}\varphi(x)\ e^{-S_E[\varphi]}$



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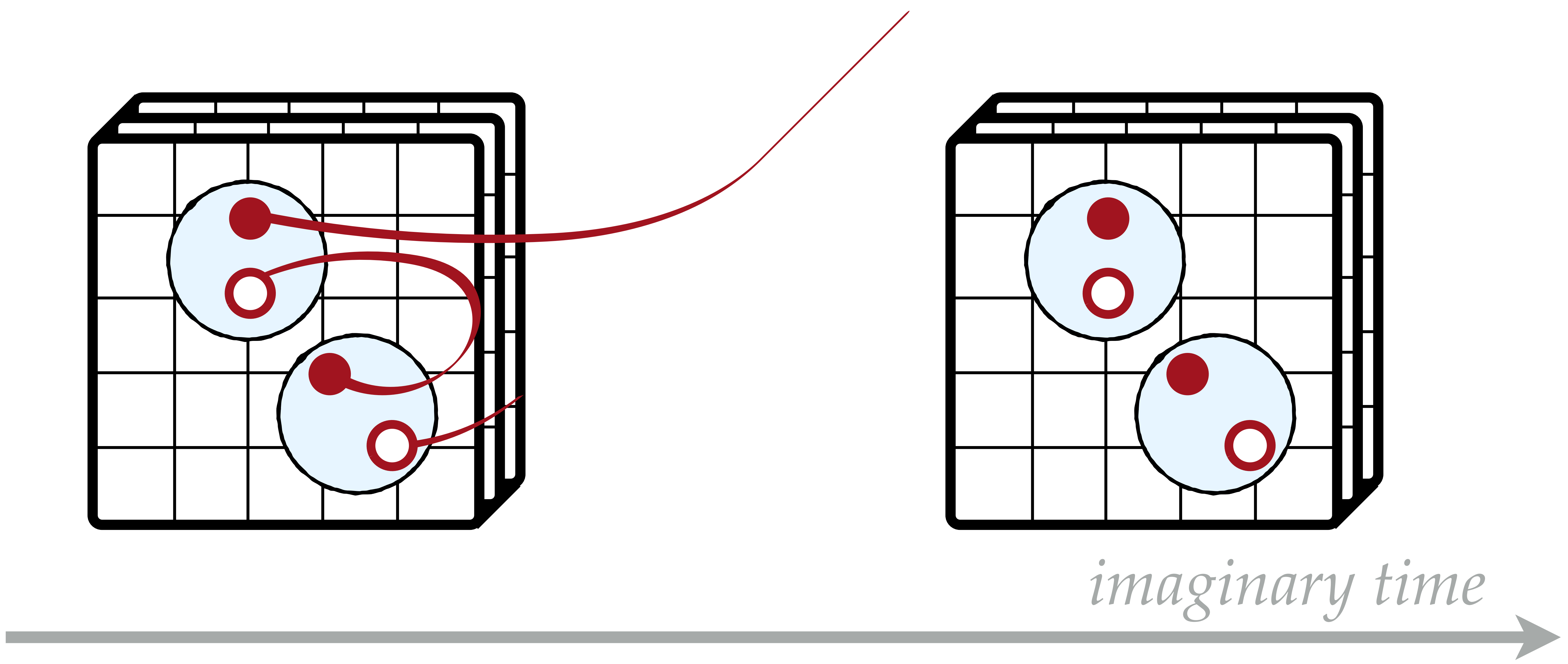
- Importance sampling
- Correlation functions, …

$$C^{2pt.}(t_E) \equiv \langle 0|\mathcal{O}(t_E)\mathcal{O}^\dagger(0)|0\rangle = \sum_n c_n e^{-E_n t_E}$$



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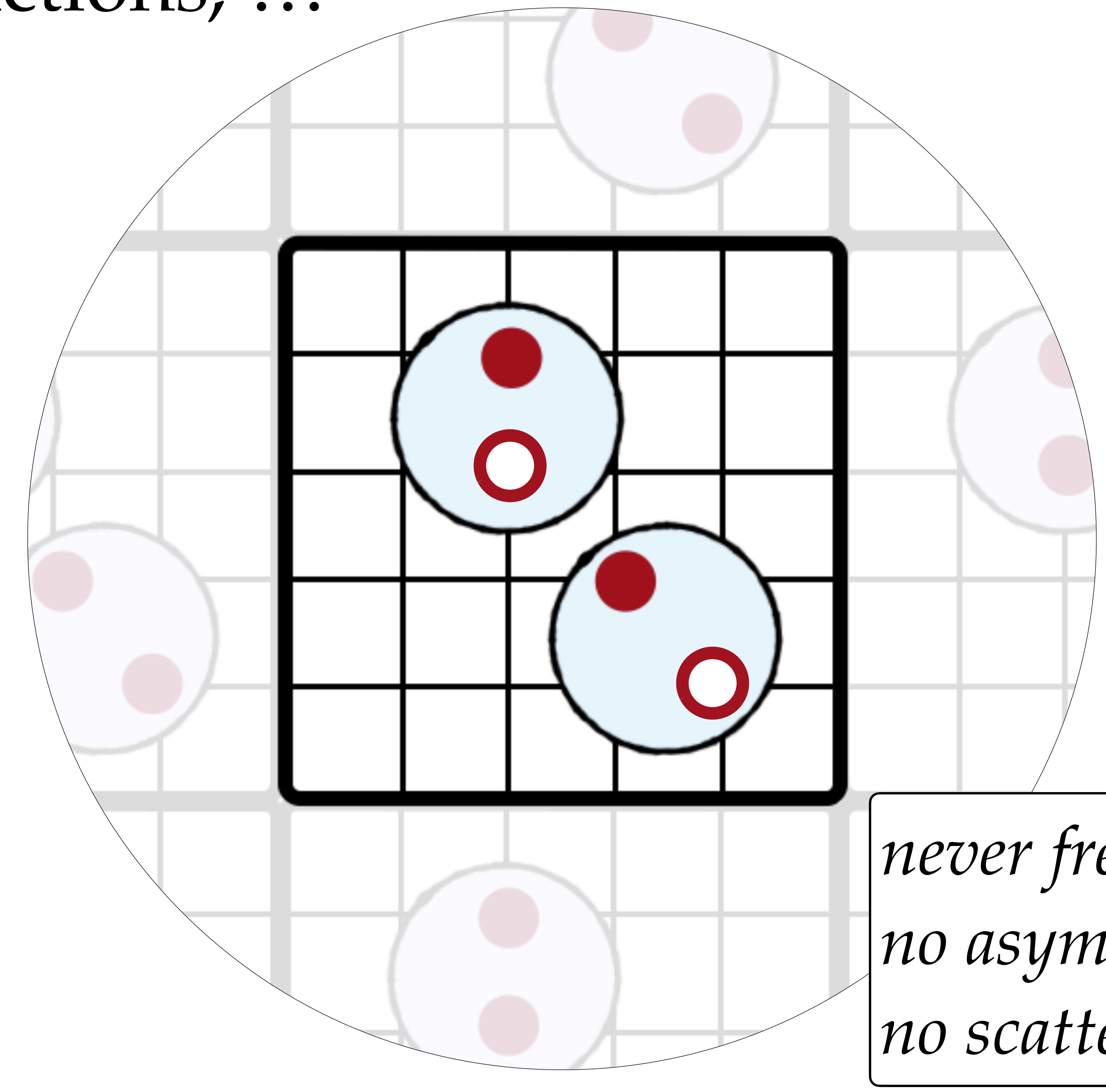
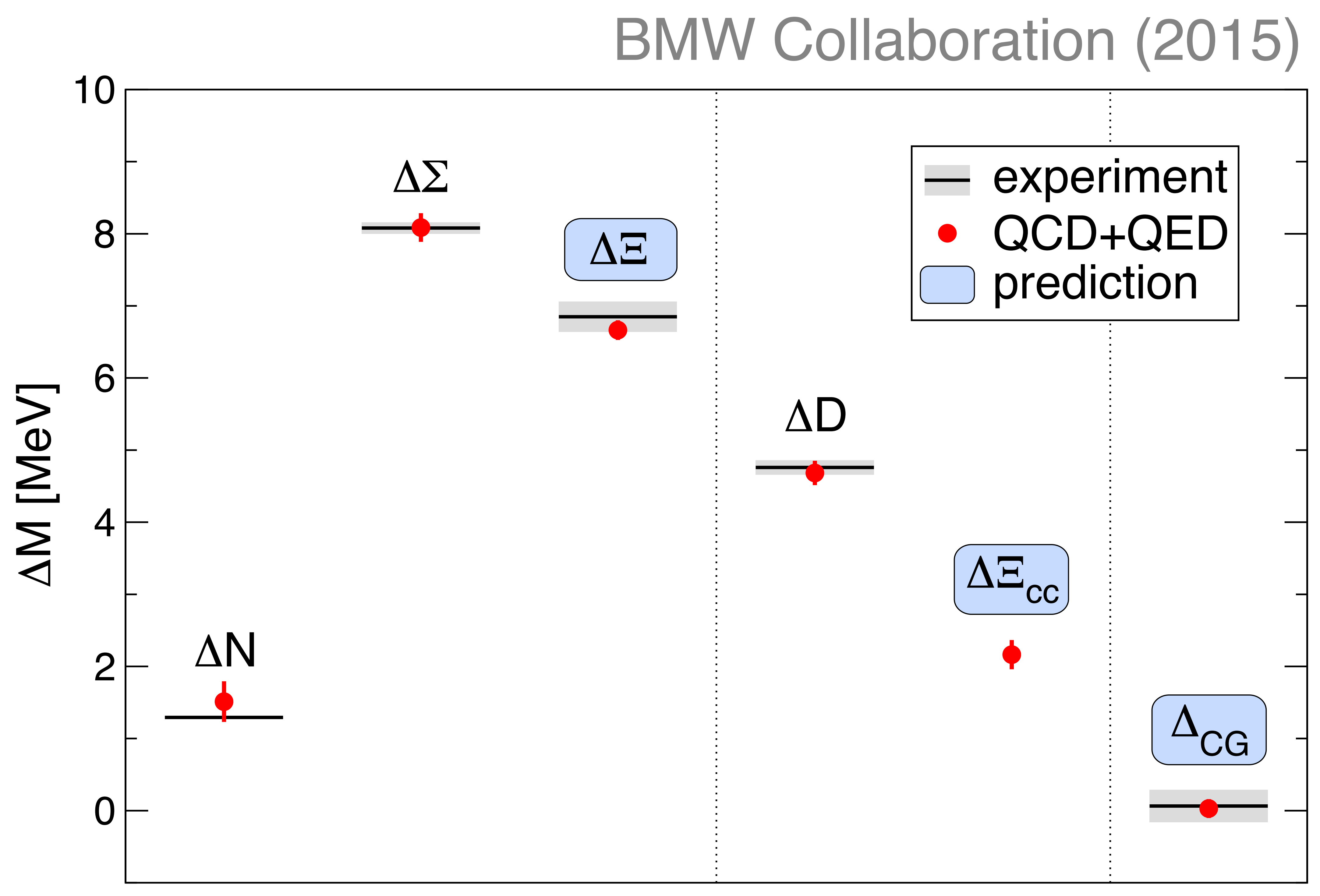
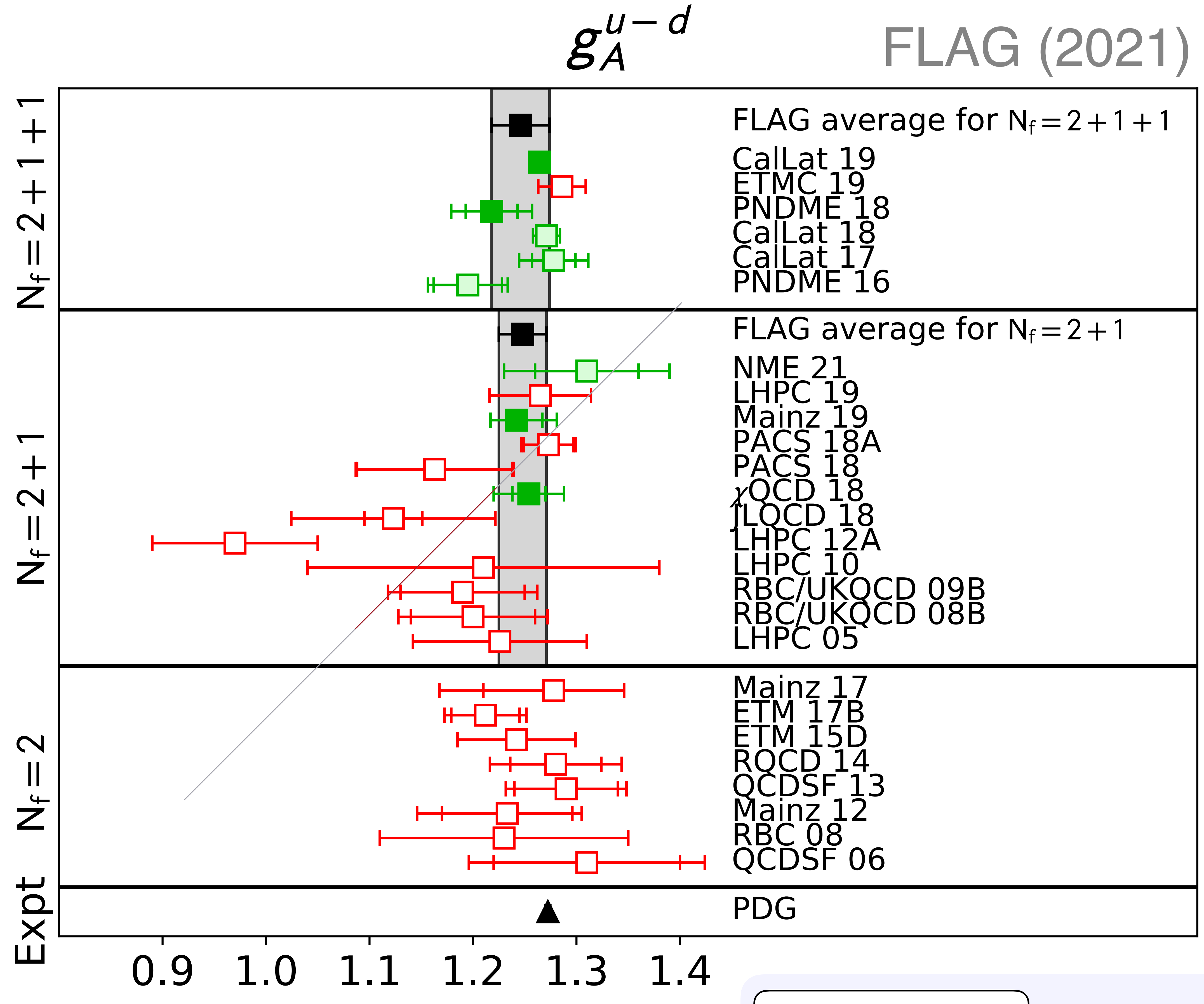
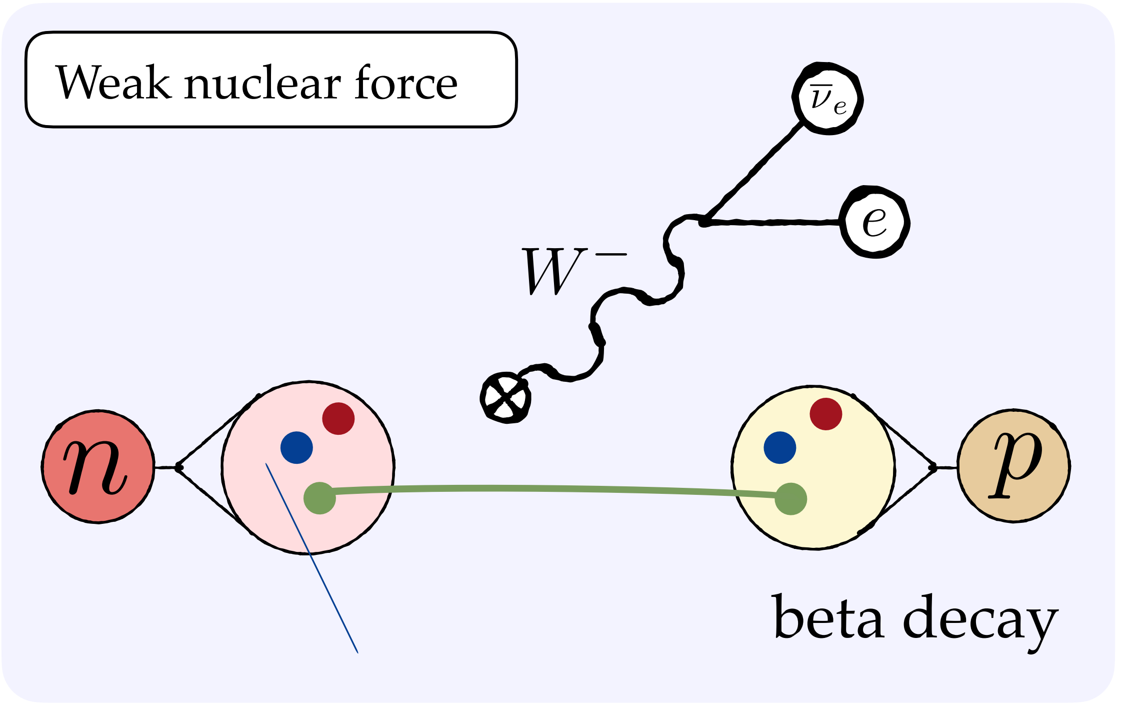


never free
no asymptotic states
no scattering

"Simple quantities" from lattice QCD

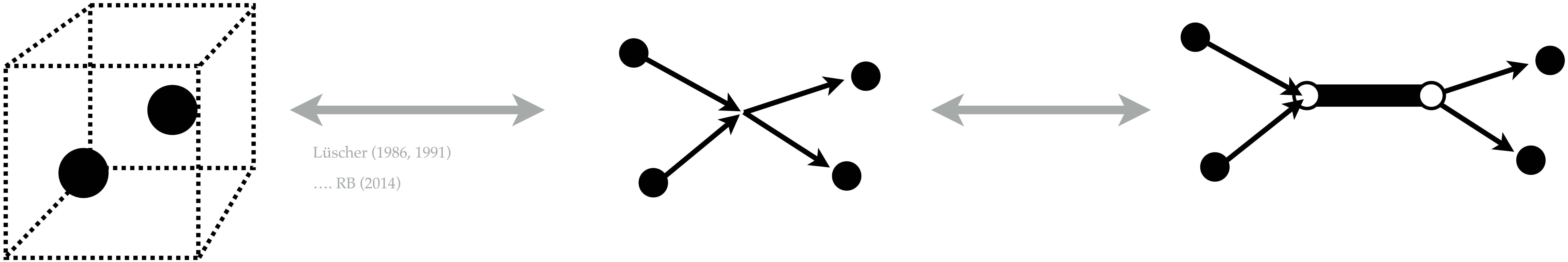






Coleman-Glashow mass difference $\Delta_{CG} = \Delta M_N - \Delta M_\Sigma + \Delta M_\Xi$

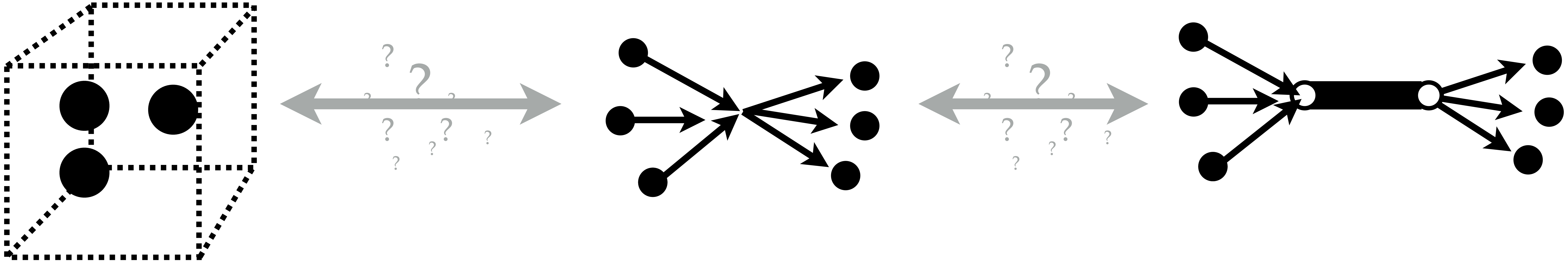
Purely hadronic amplitudes



finite-volume
spectroscopy

infinite-volume
scattering amplitudes

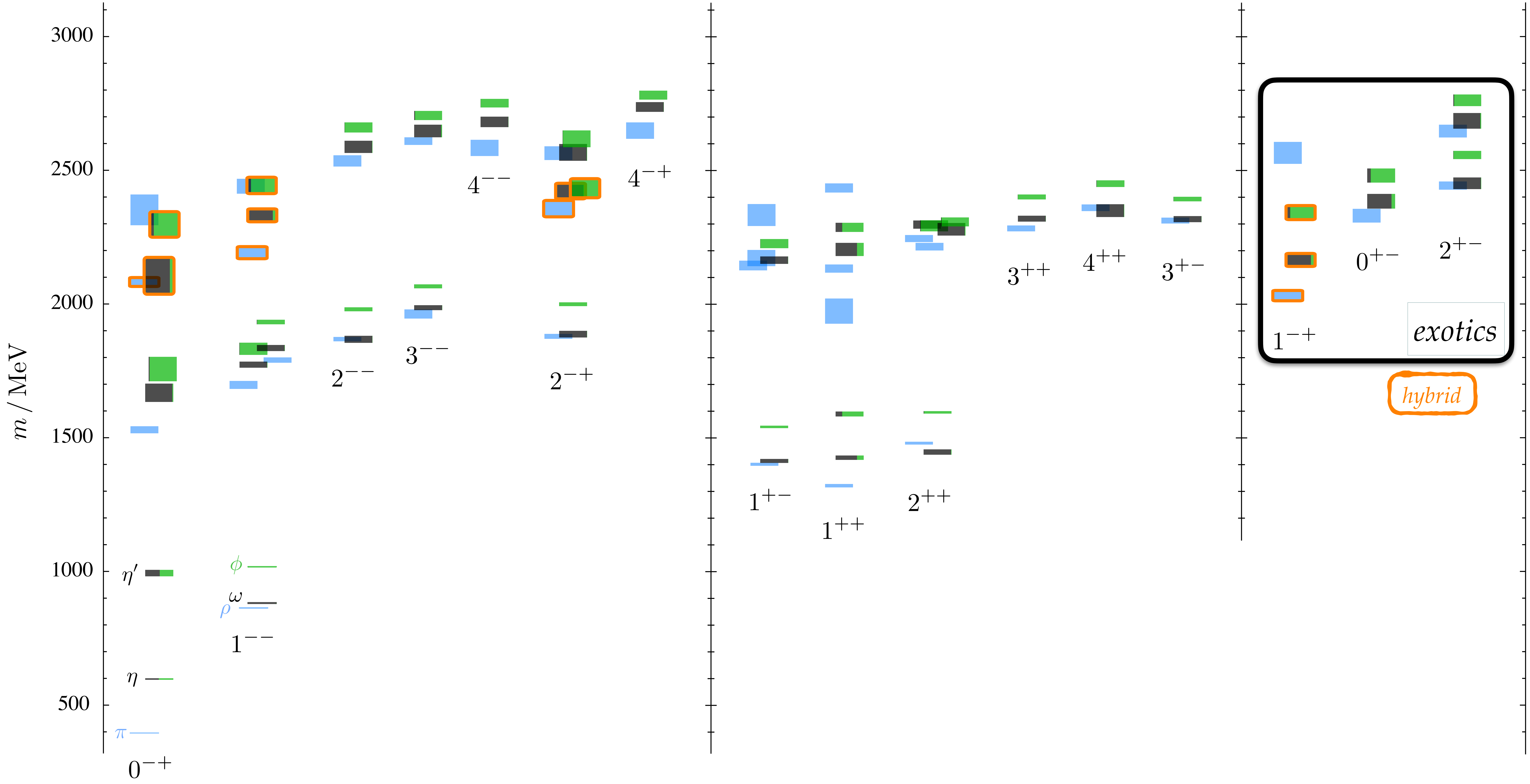
bound state and
resonance poles



Jackura [Tuesday Morning]

Narrow-width approximation

Using a large number [10-30] of local ops, $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\,q$



m_π=391 MeV

Dudek, Edwards, Guo, Thomas (2013) RB

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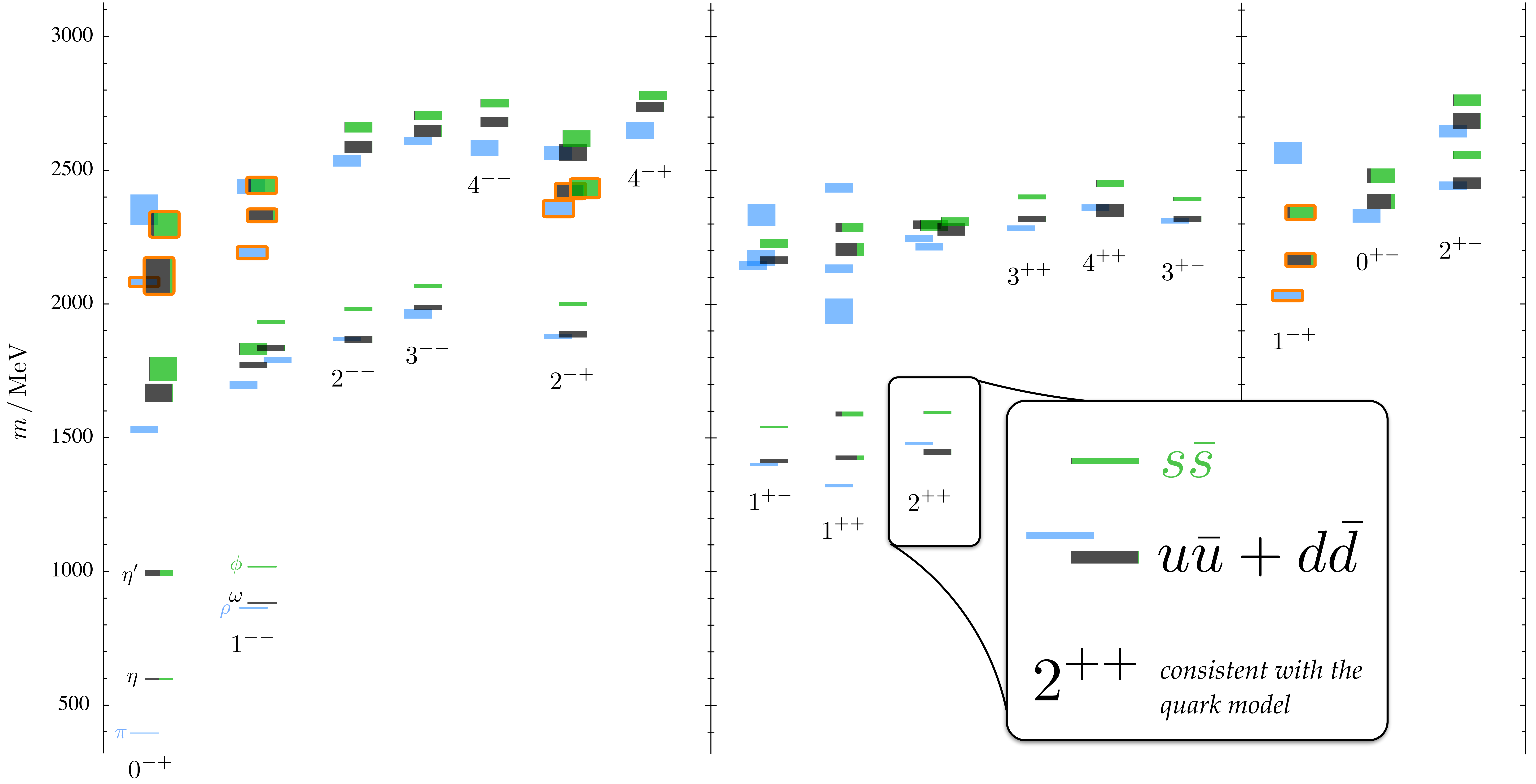

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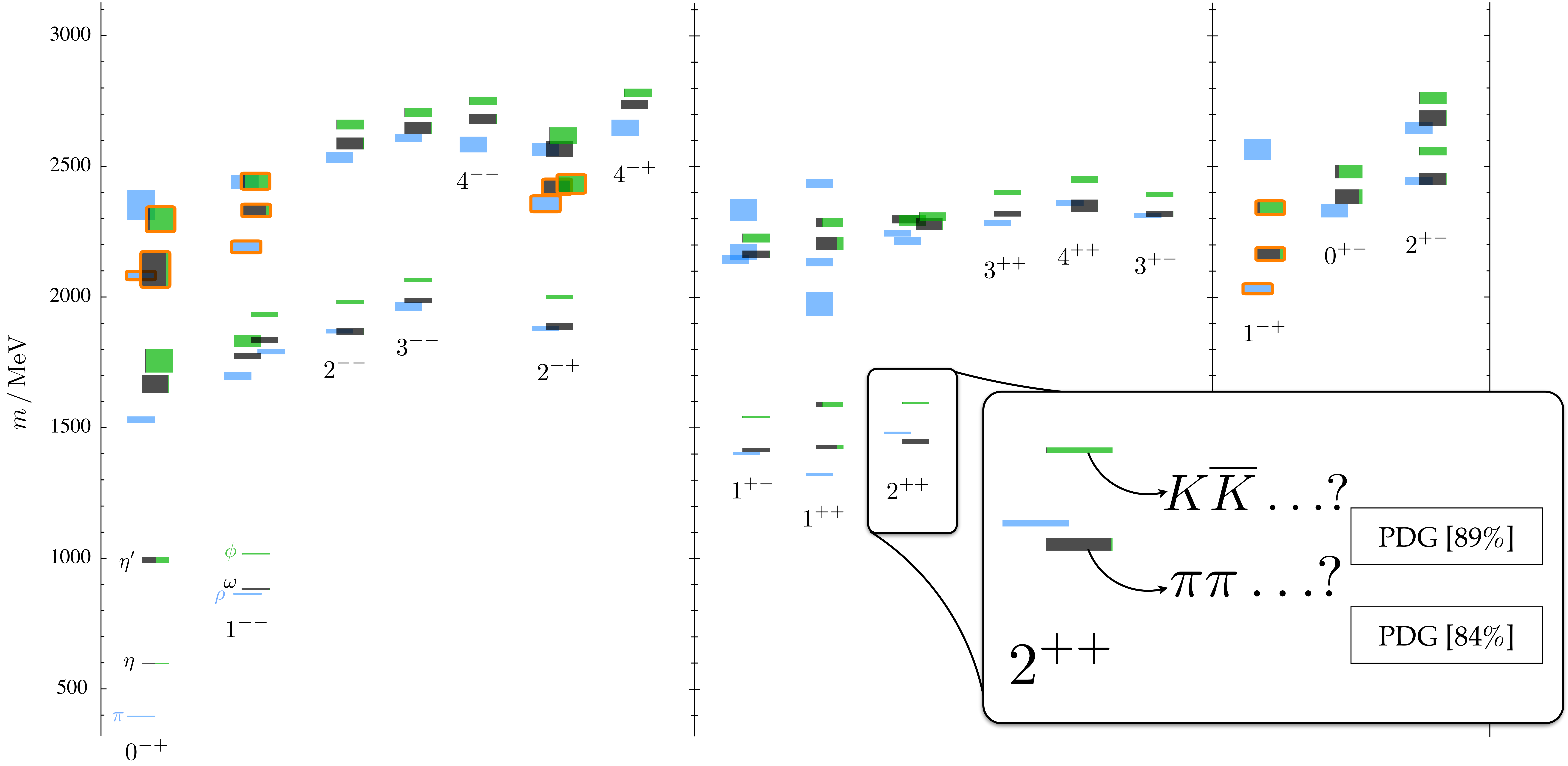

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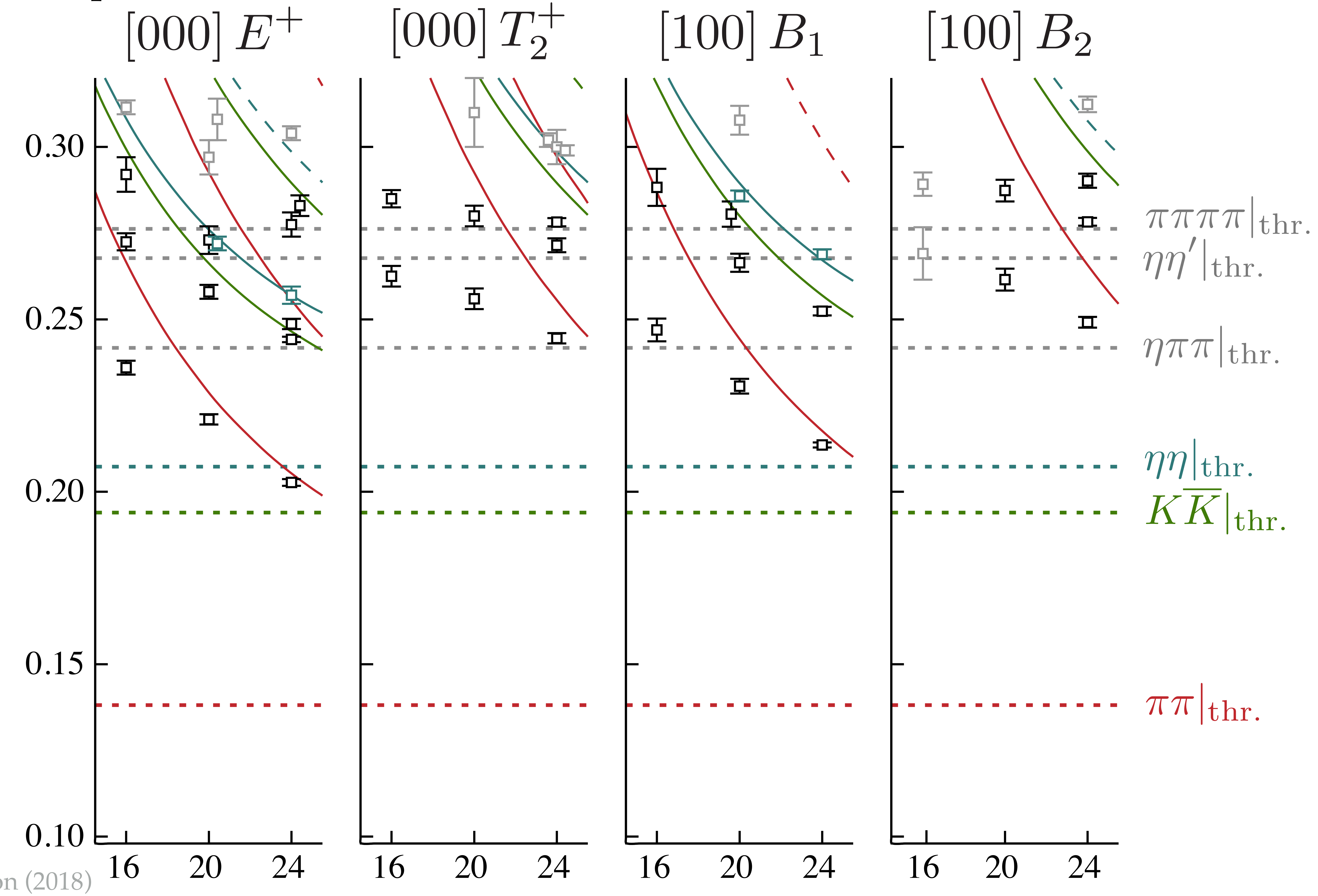
Fuller Spectrum: Isoscalar 2^{++}

Enhancing basis of operators to include $\{\pi\pi, K\overline{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



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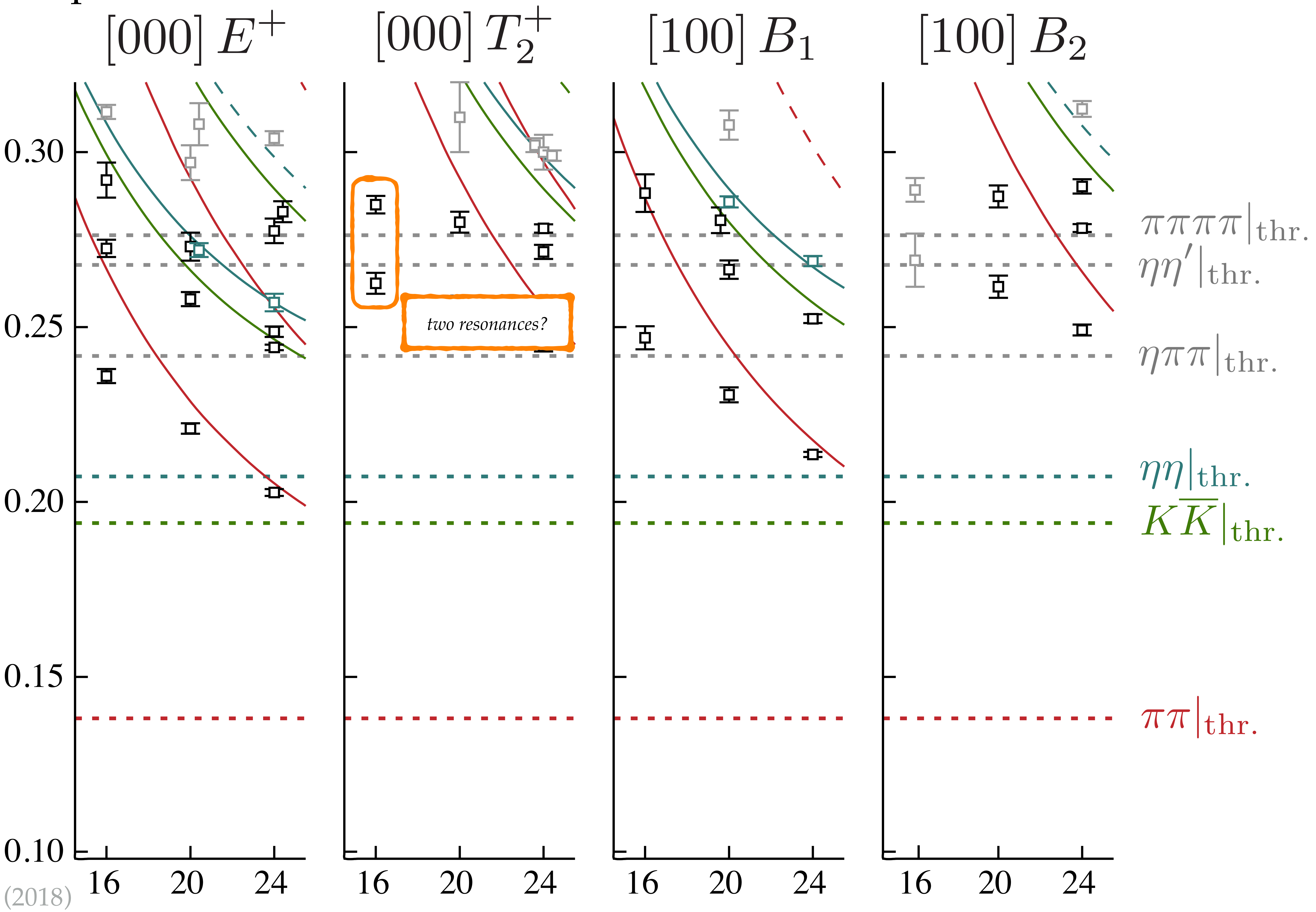

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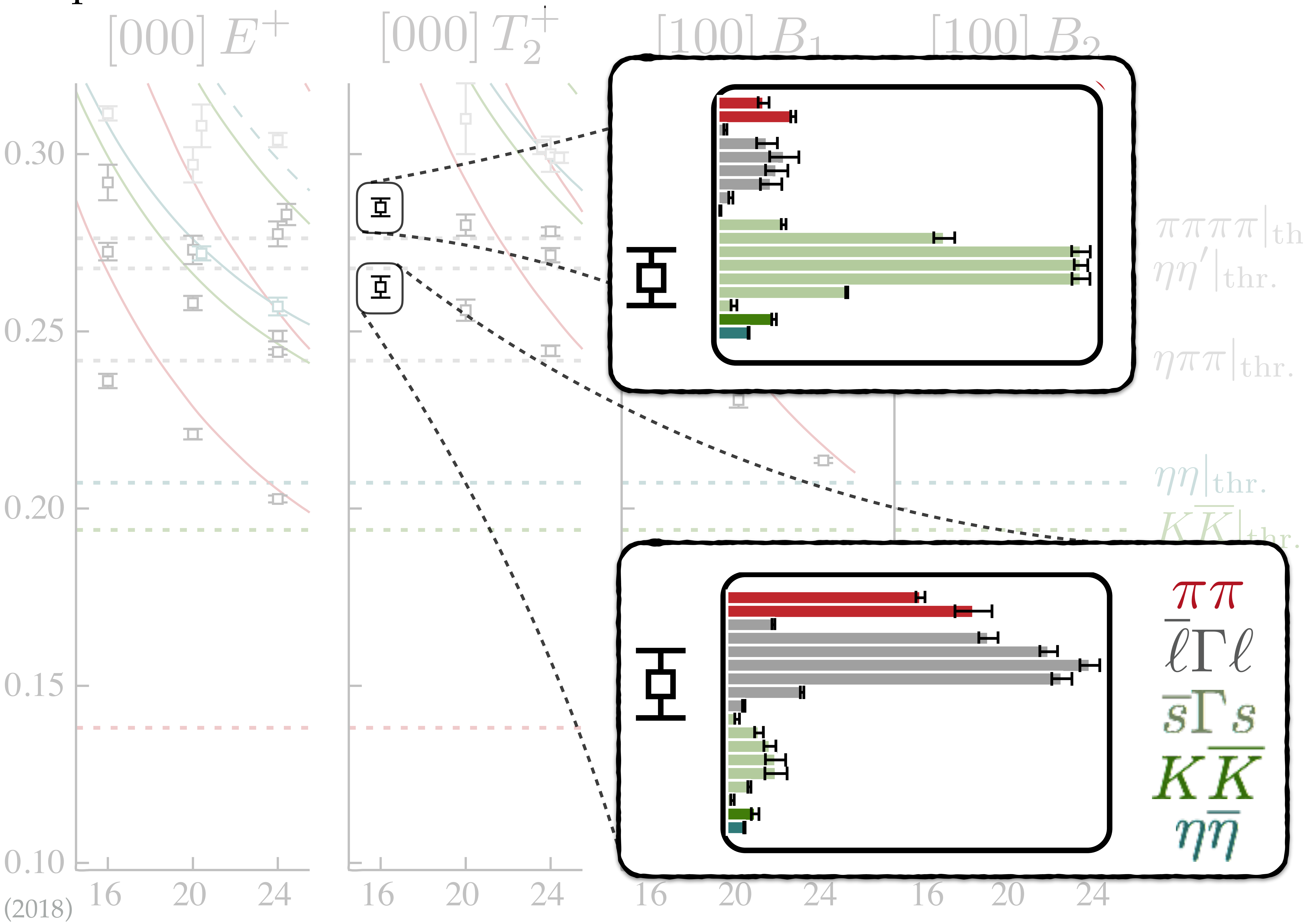

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Fuller Spectrum: Isoscalar 0^{++}

no simple story

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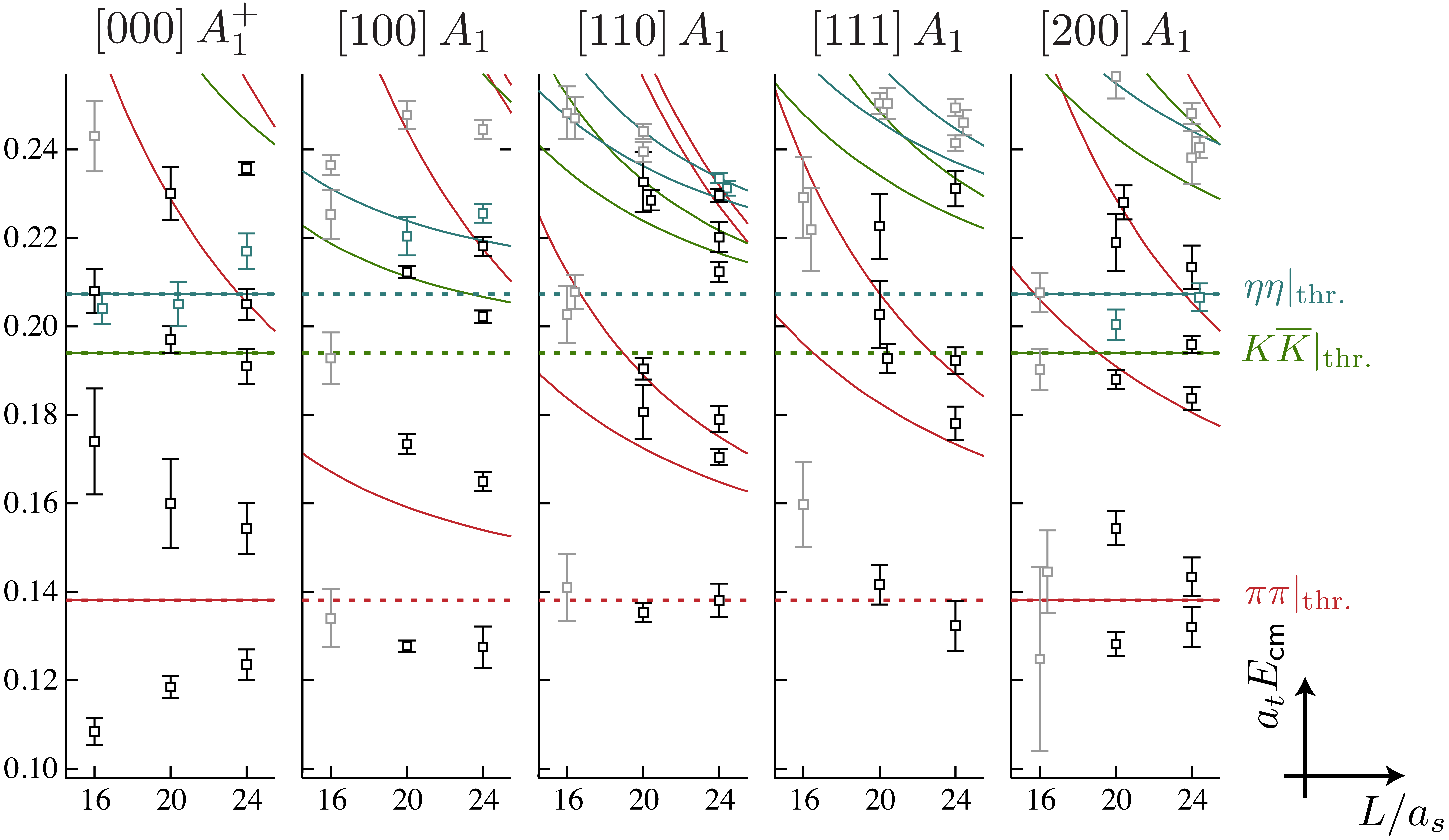

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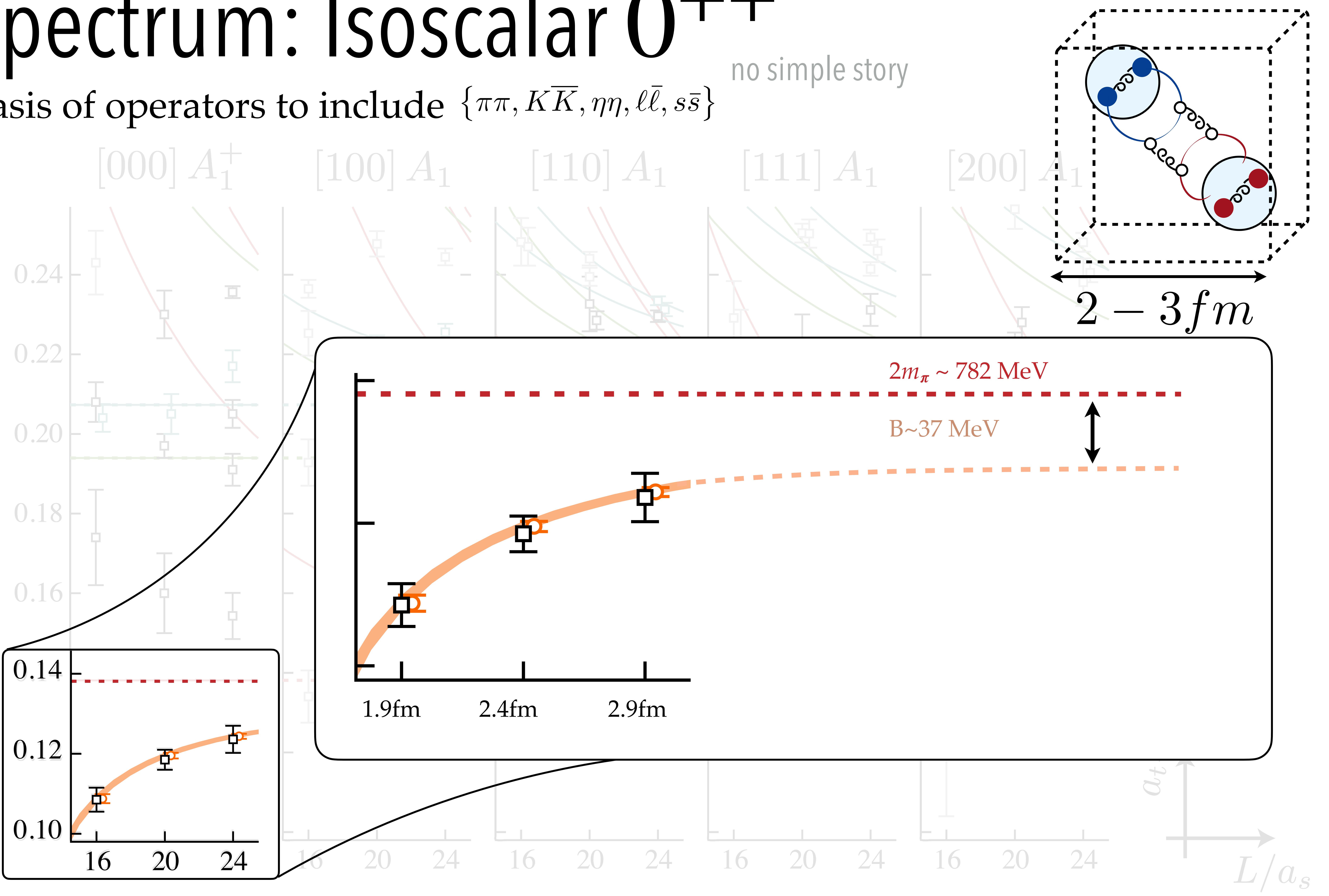

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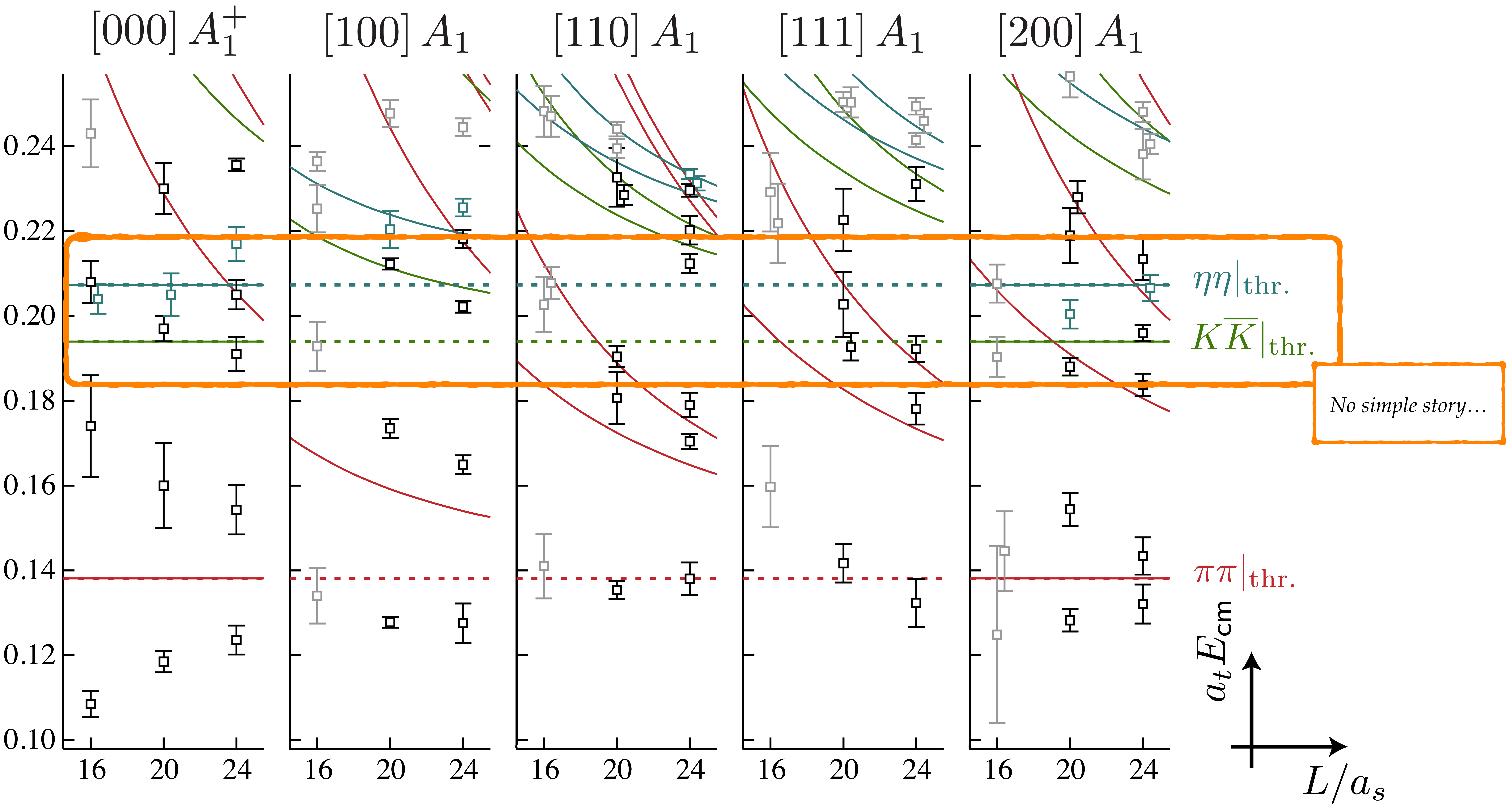
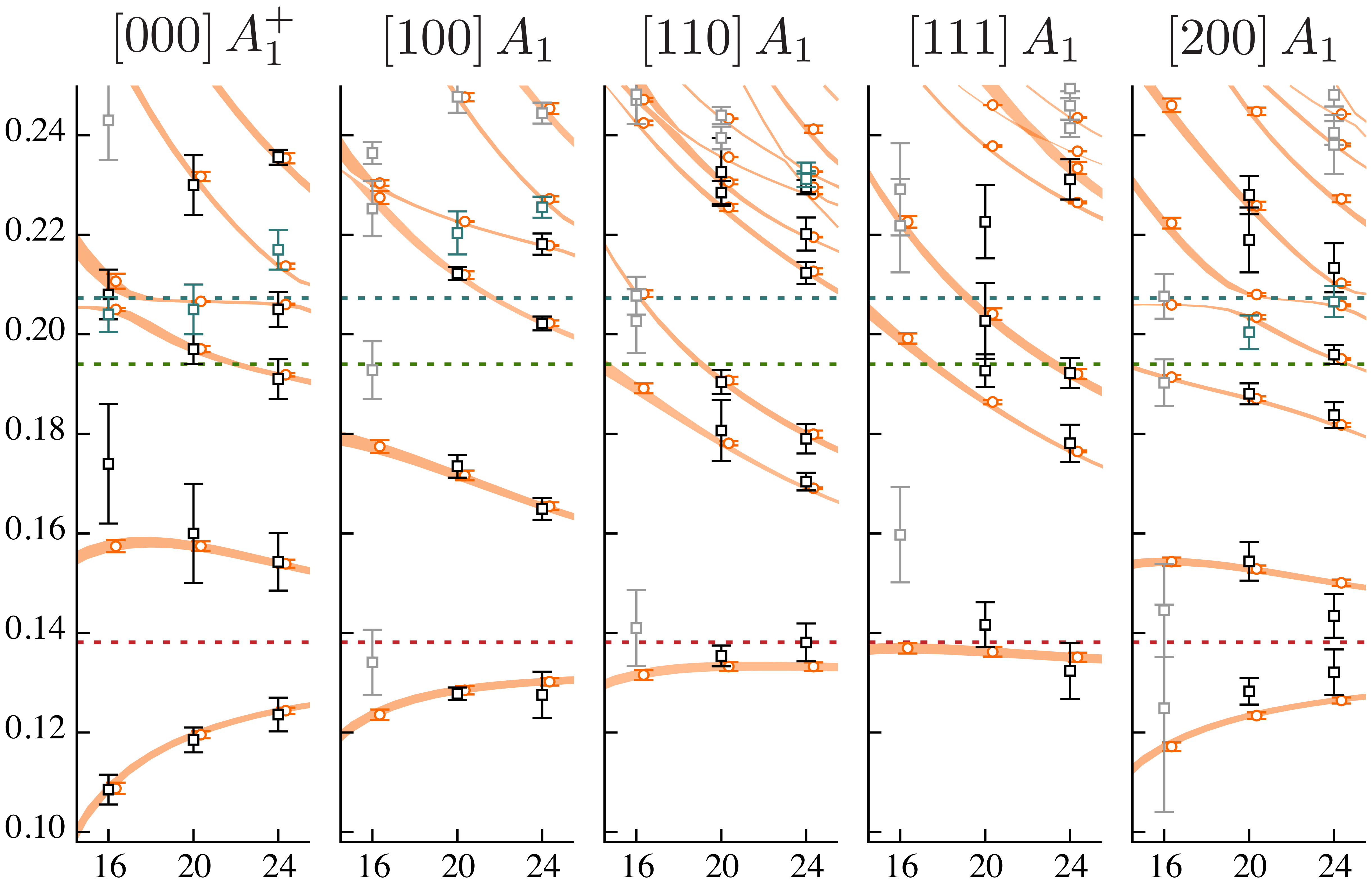


m_π=391 MeV

RB, Dudek, Edwards, Wilson (2018)

Spectrum analysis

- Above $K\overline{K}$-threshold, spectrum satisfies: $\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$
- No one-to-one correspondence,
- Parameterize amplitude and perform global fit.



Ansatz: $\mathbf{K}^{-1}(s) = \begin{pmatrix} a + bs & c + ds & e \\ c + ds & f & g \\ e & g & h \end{pmatrix}$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

Spectrum analysis

- ☑ Above $K\overline{K}$-threshold, spectrum satisfies: $$\det\begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$
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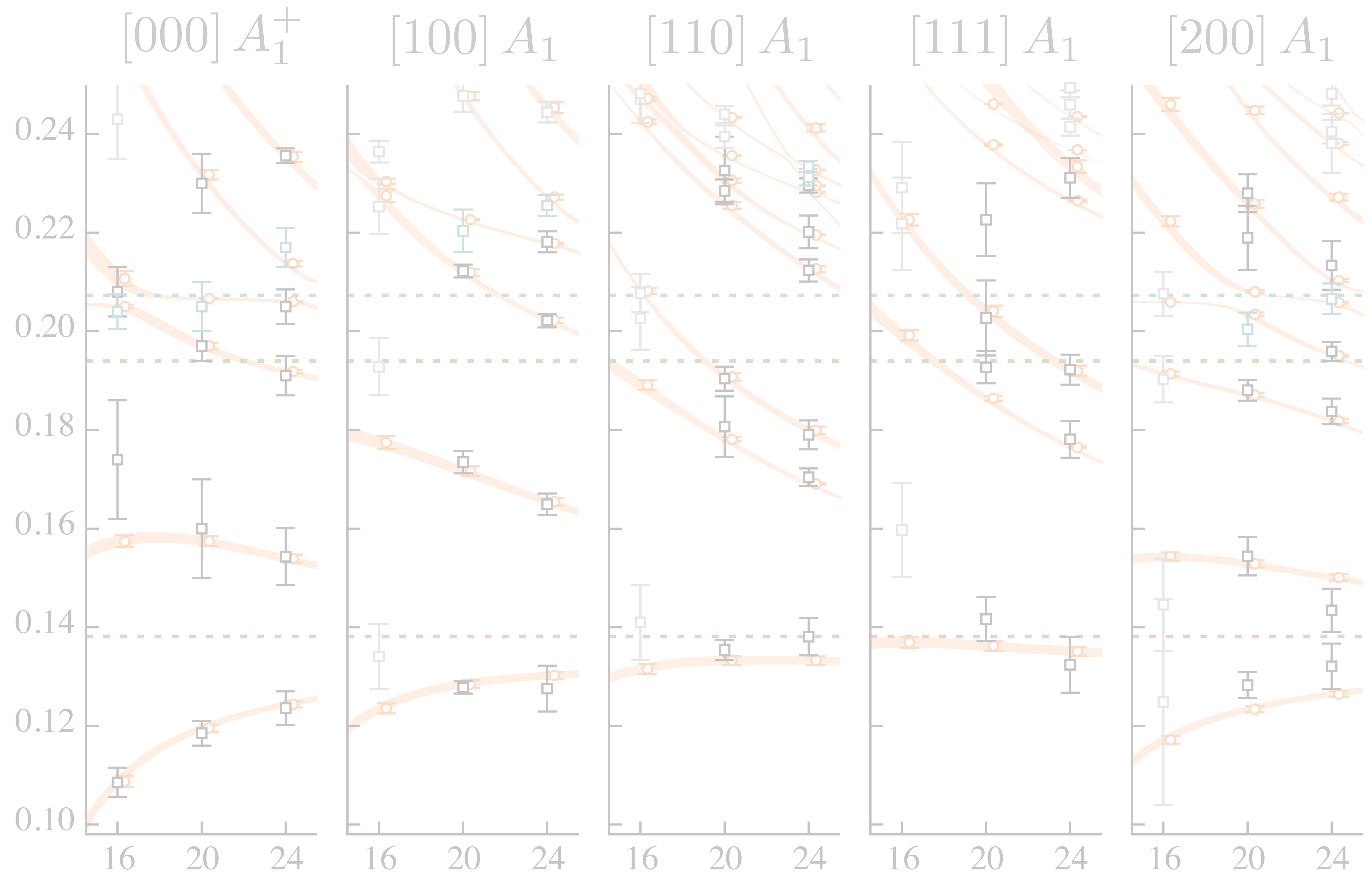
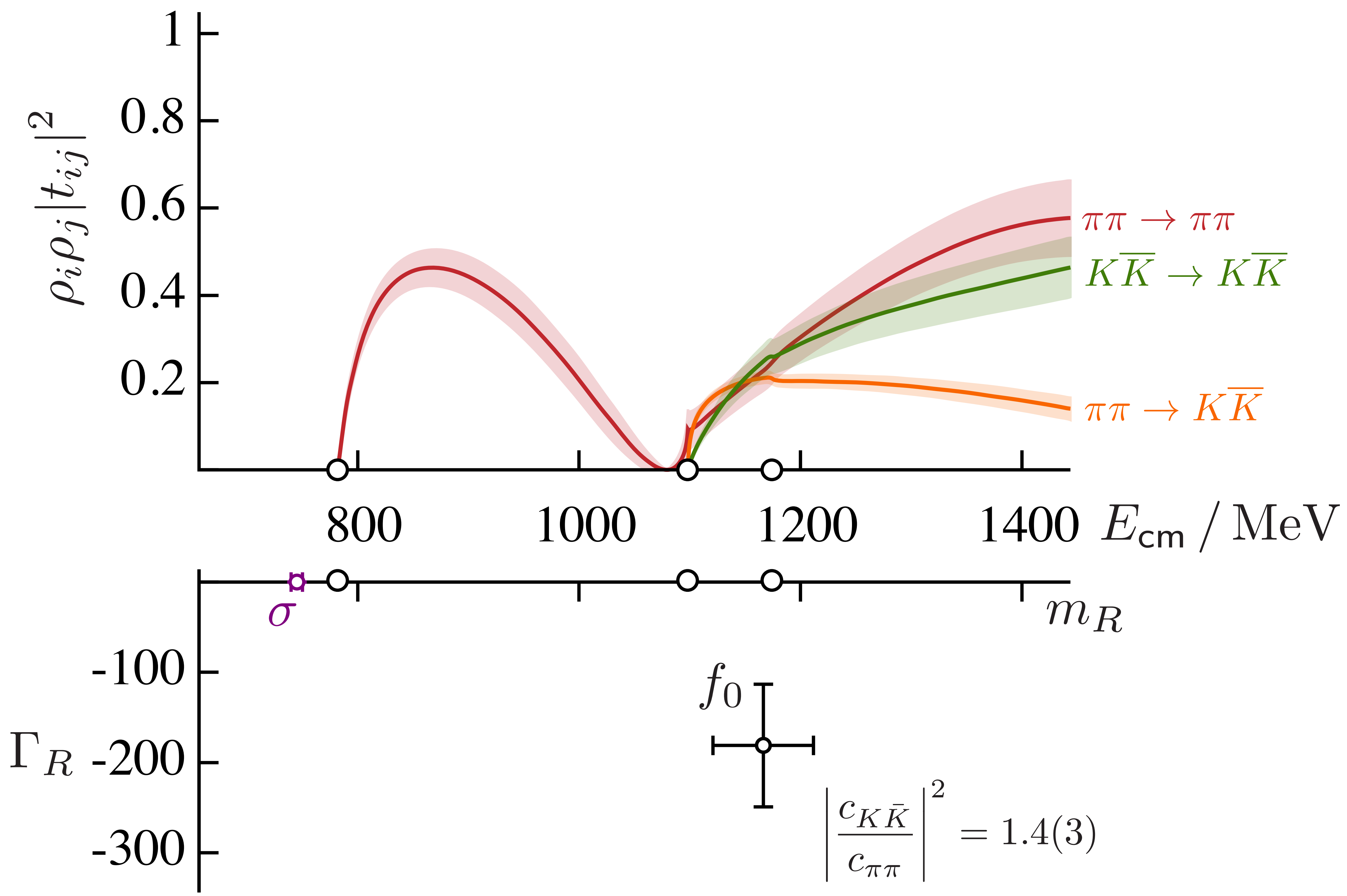
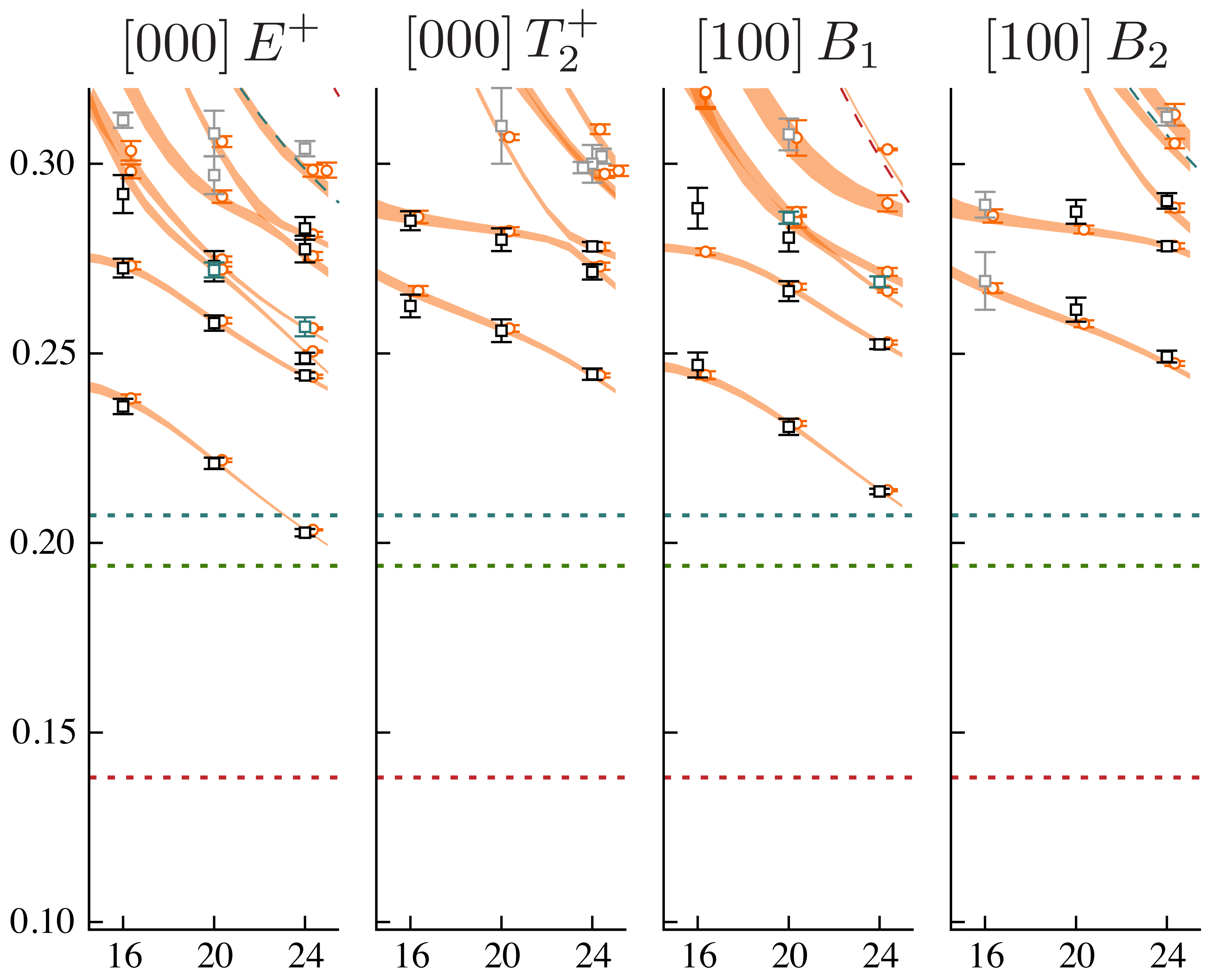
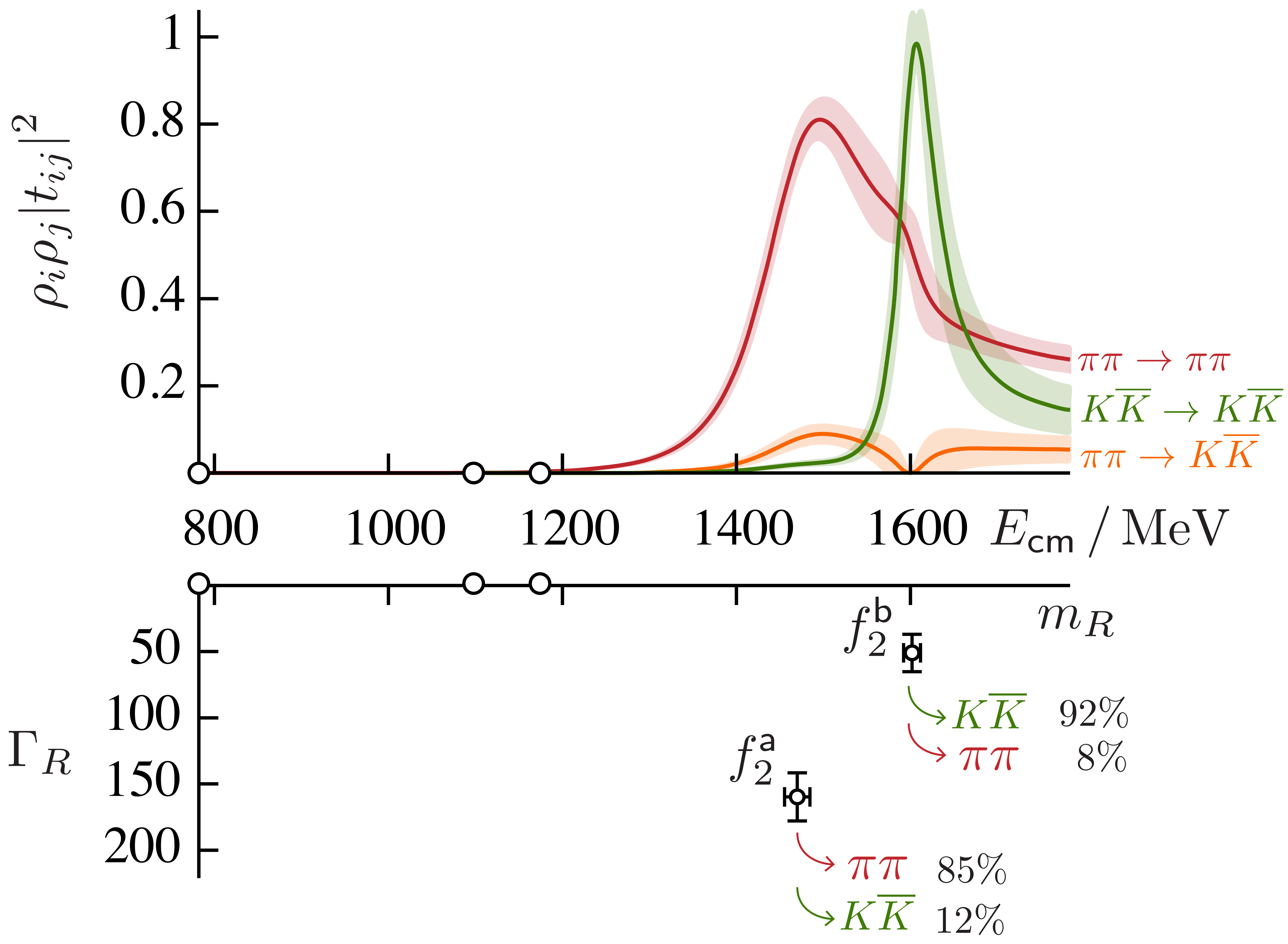




Tensors: the f_2's

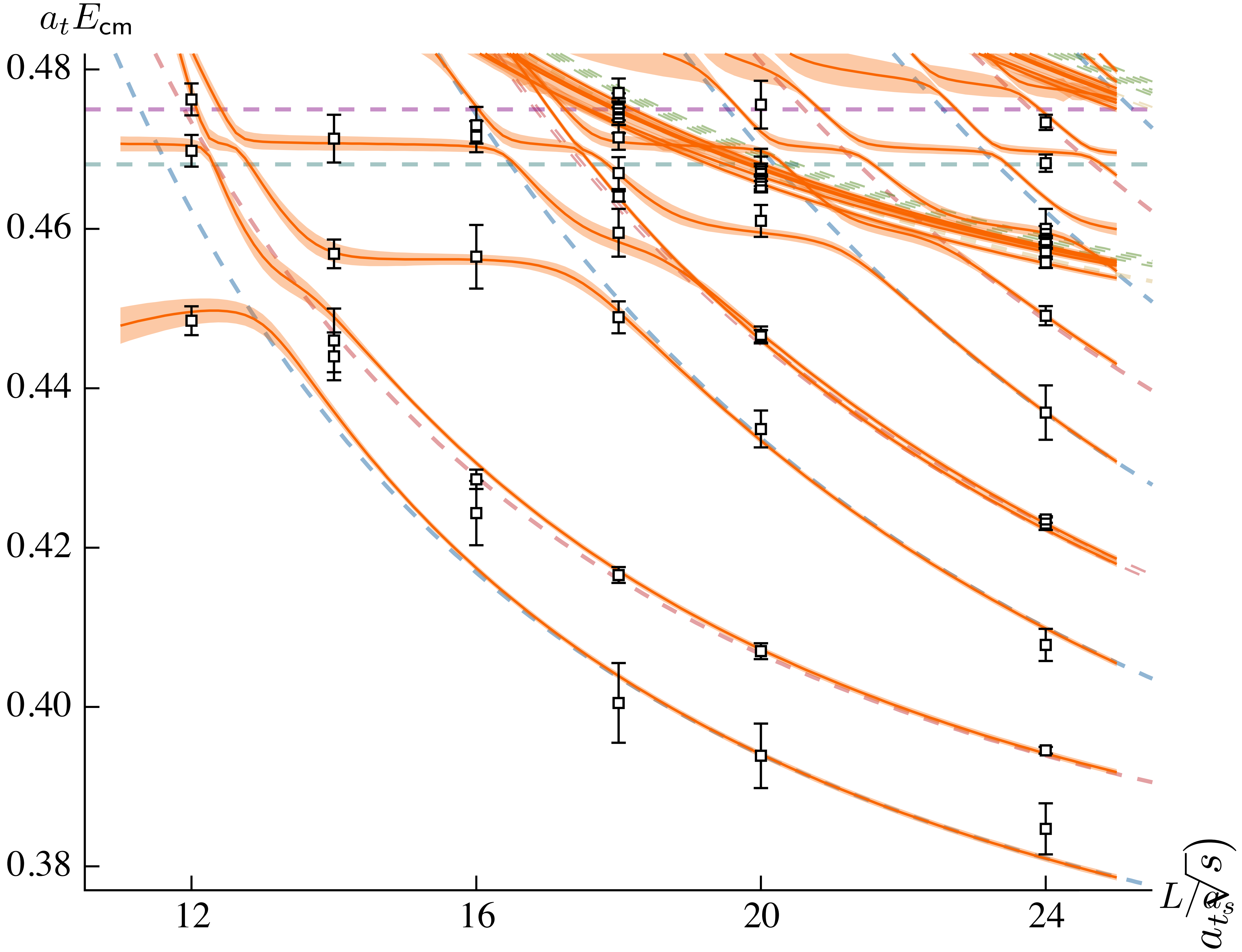


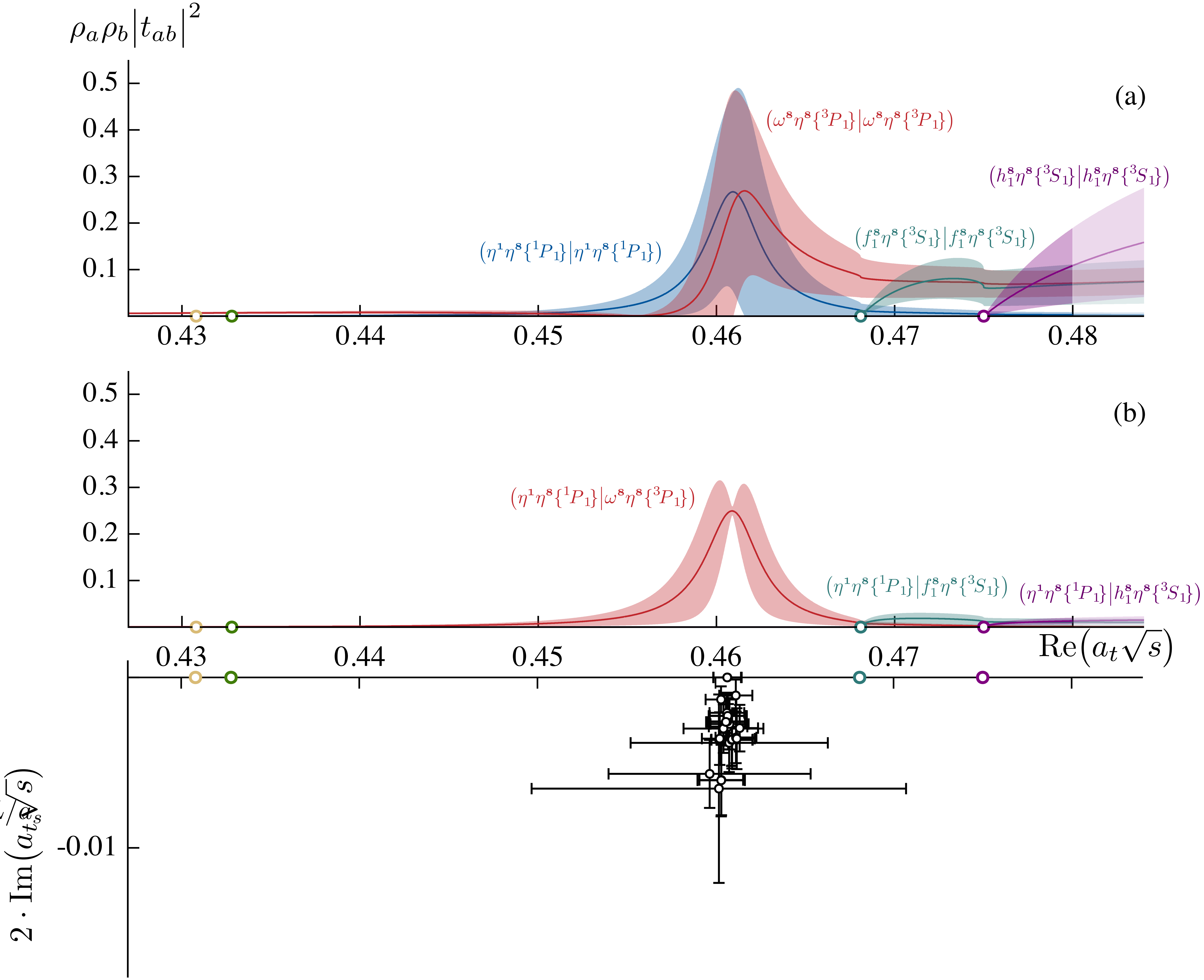
$$\det[F^{-1}(P,L)+\mathcal{M}(P)]=0$$



π_1 channel

$(m_\pi \sim 700\,\text{MeV})$



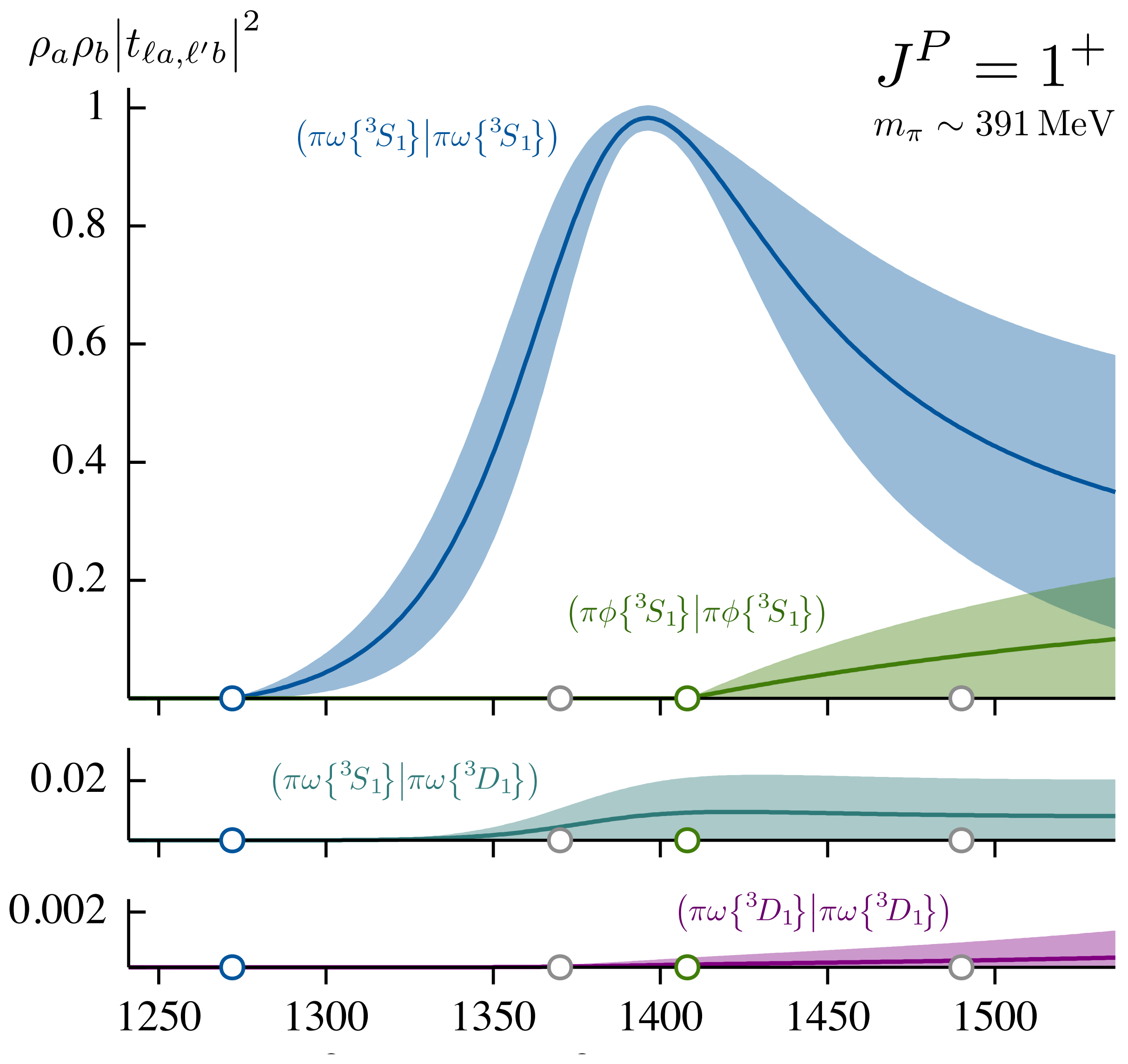


~~RB~~

Woss, Dudek, Edwards, Thomas, Wilson (2020)

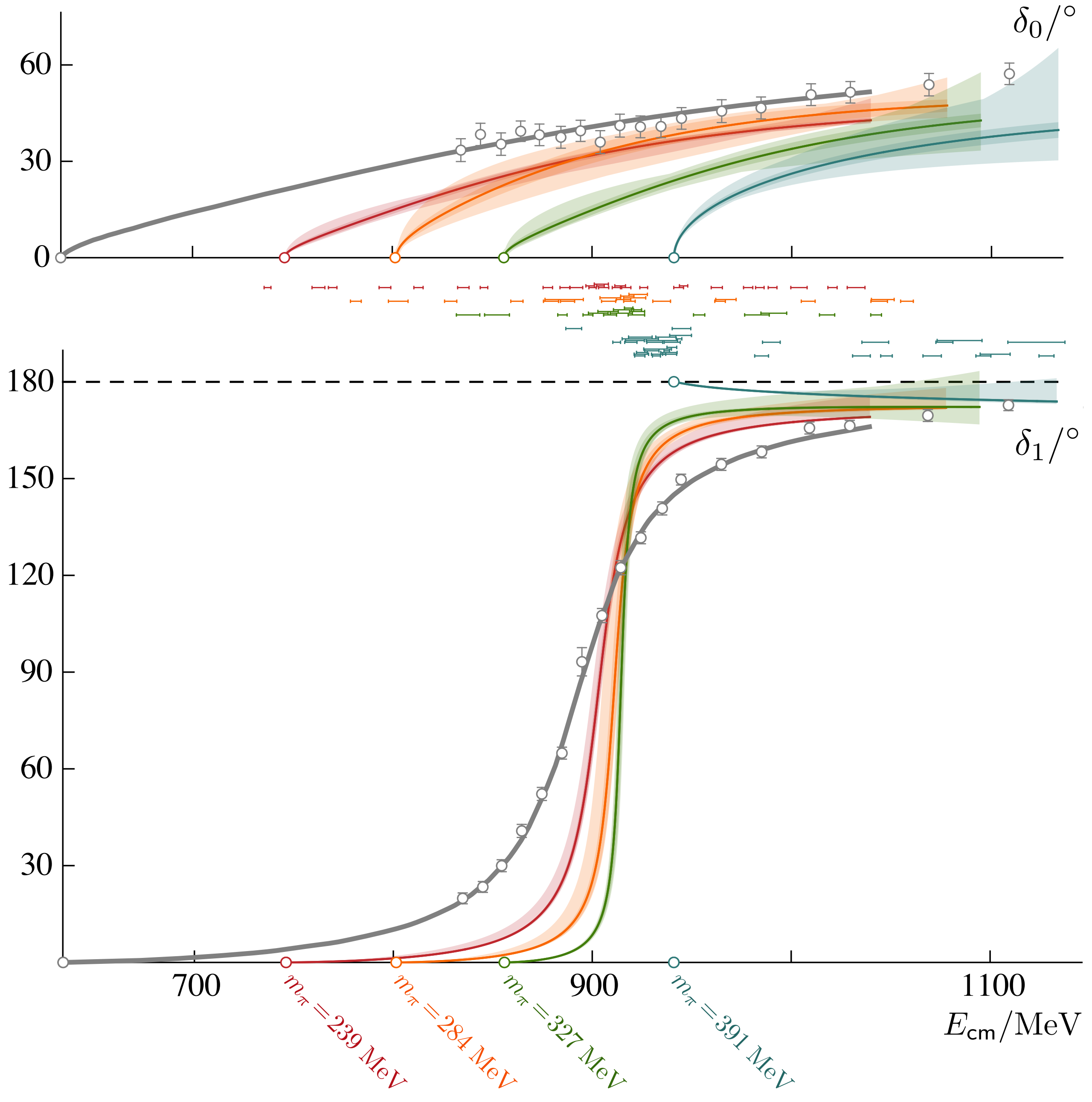
$$b_1\pi \gg [f_1(1285)\pi \sim \rho\pi \sim \pi\eta \sim \pi\eta' \sim K^*K \sim f_1(1420)\pi]$$

Many other calculations

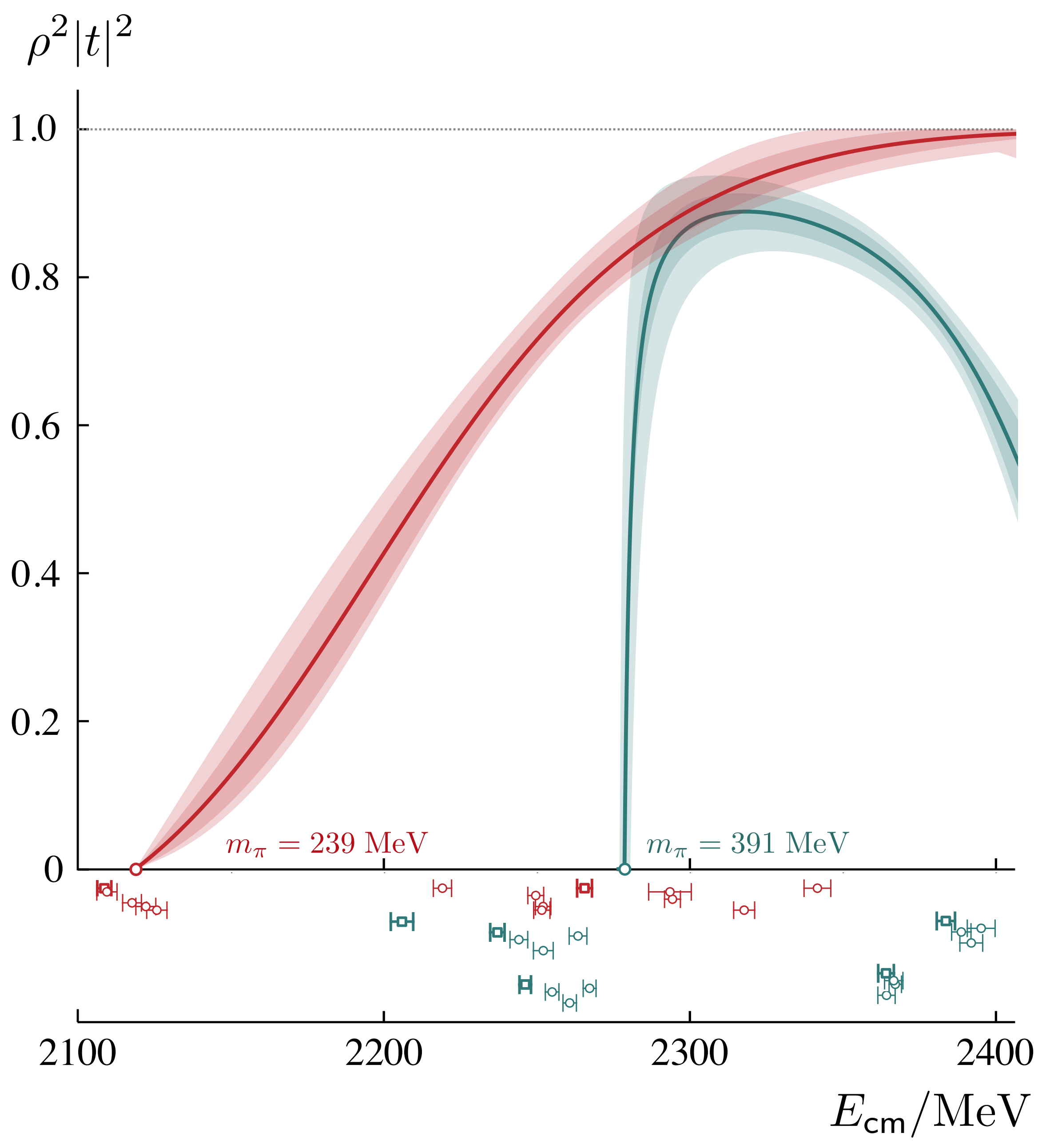


πω-πφ and the b_1

~~RB~~



Kπ in the κ and K⋆

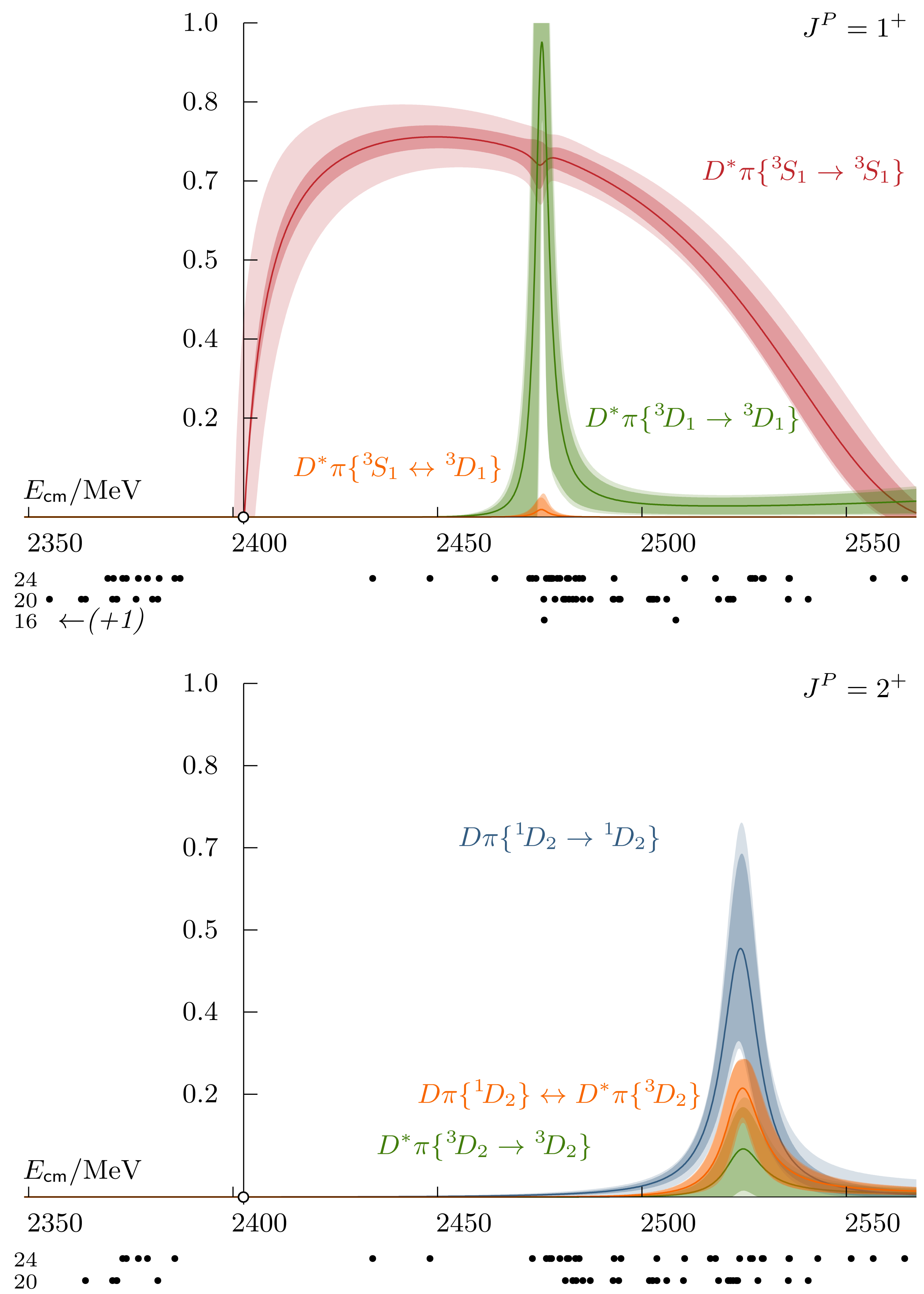


Dπ and the $D_0^\star$

~~RB~~

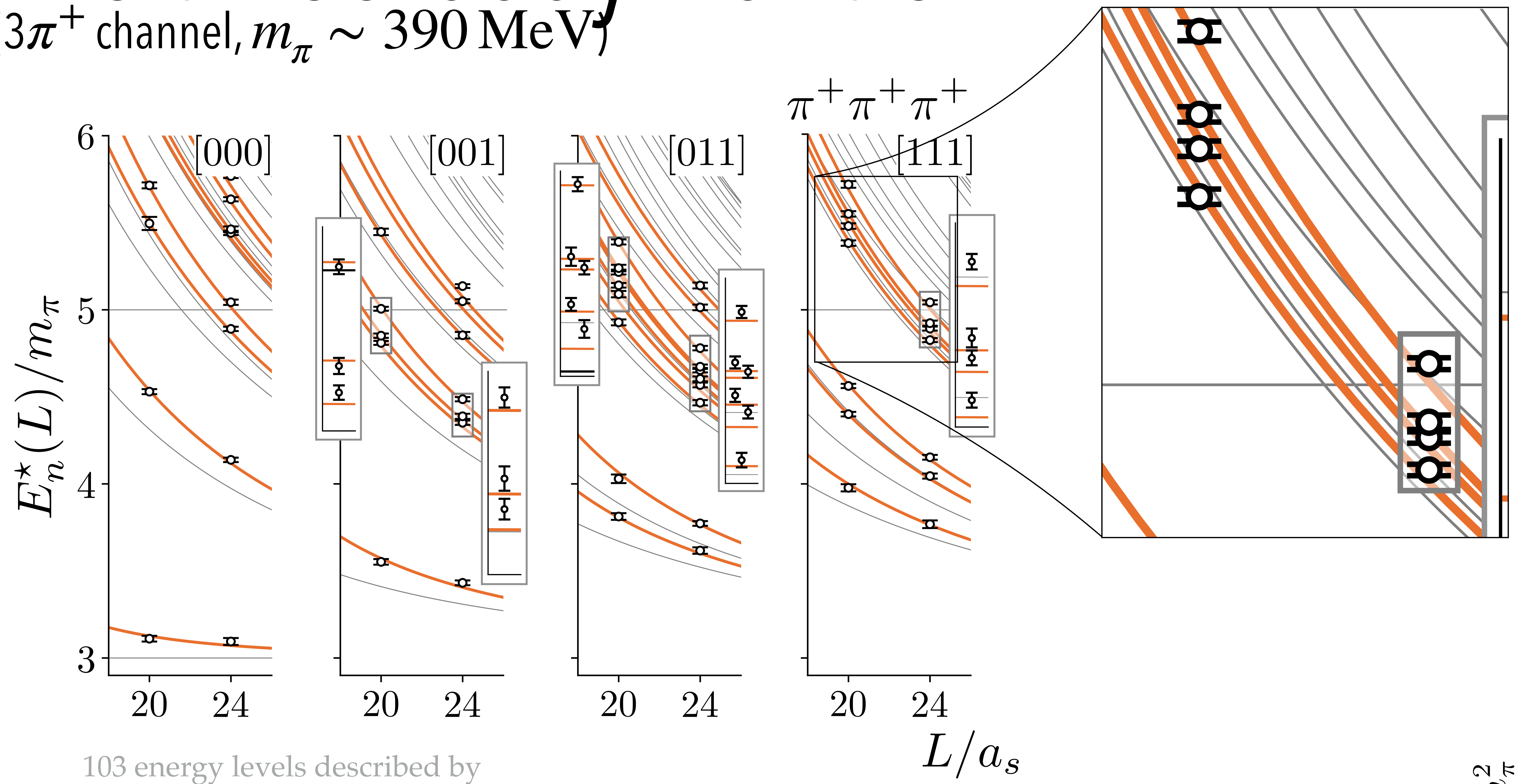
~~RB~~

$D^\star\pi$ & the D_1 and the D_2

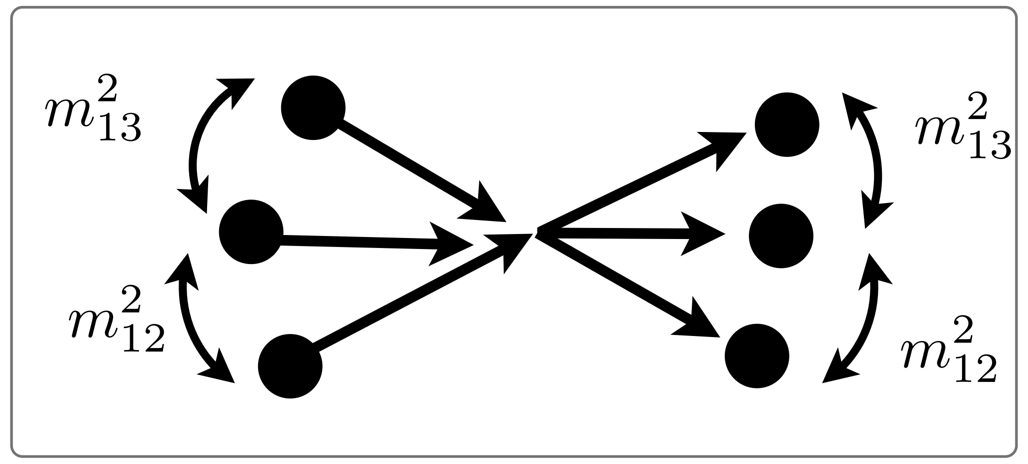


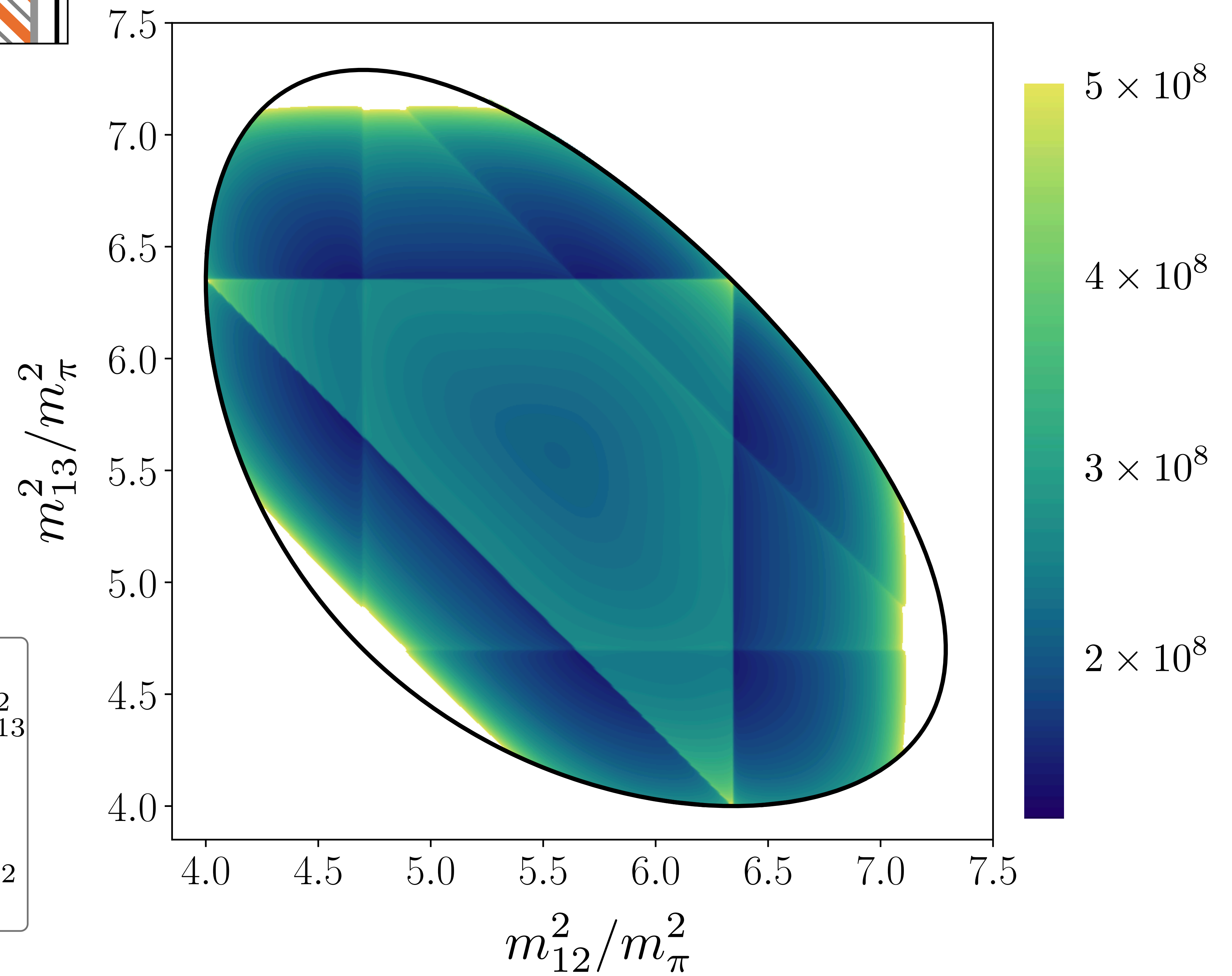
The three-body frontier

($3\pi^+$ channel, $m_\pi \sim 390\,\mathrm{MeV}$)



103 energy levels described by three numbers: m_π, $a_{\pi\pi}$, $\mathscr{K}_{3,\mathrm{iso}}$

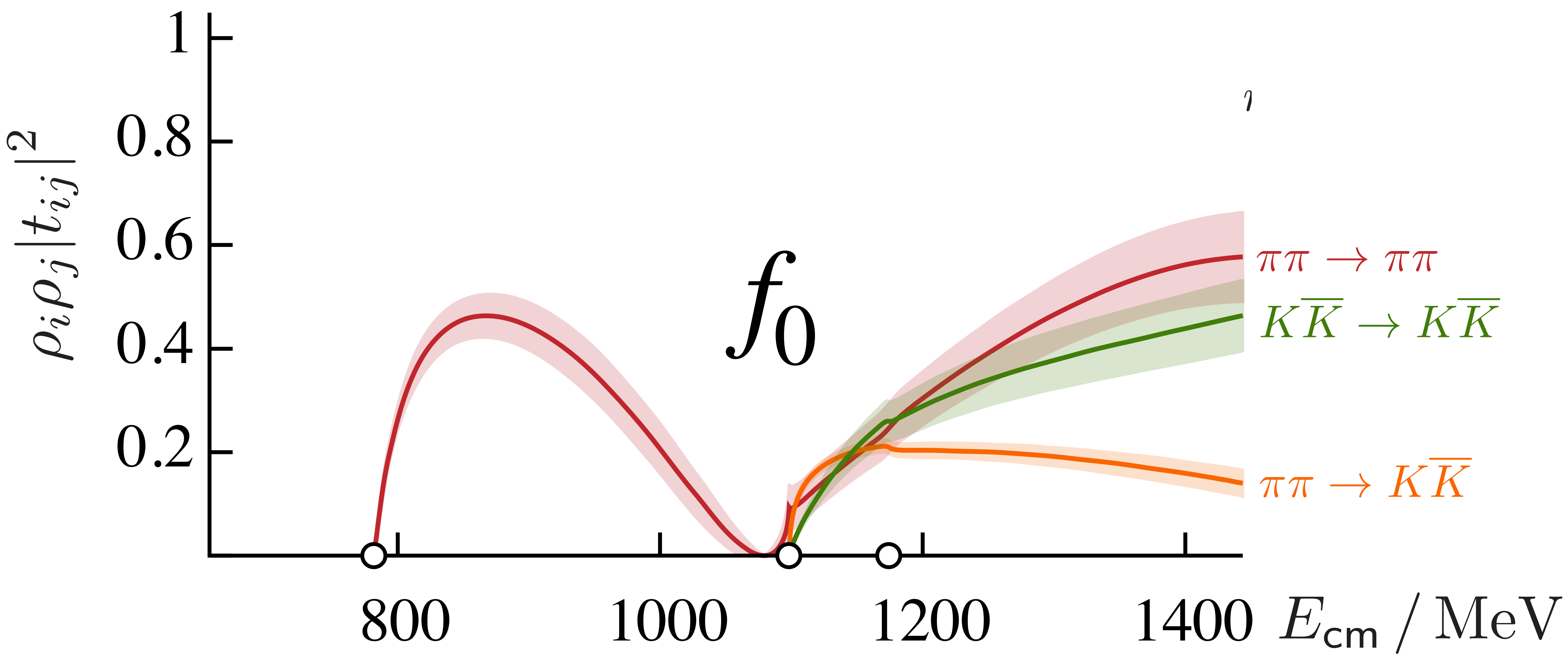


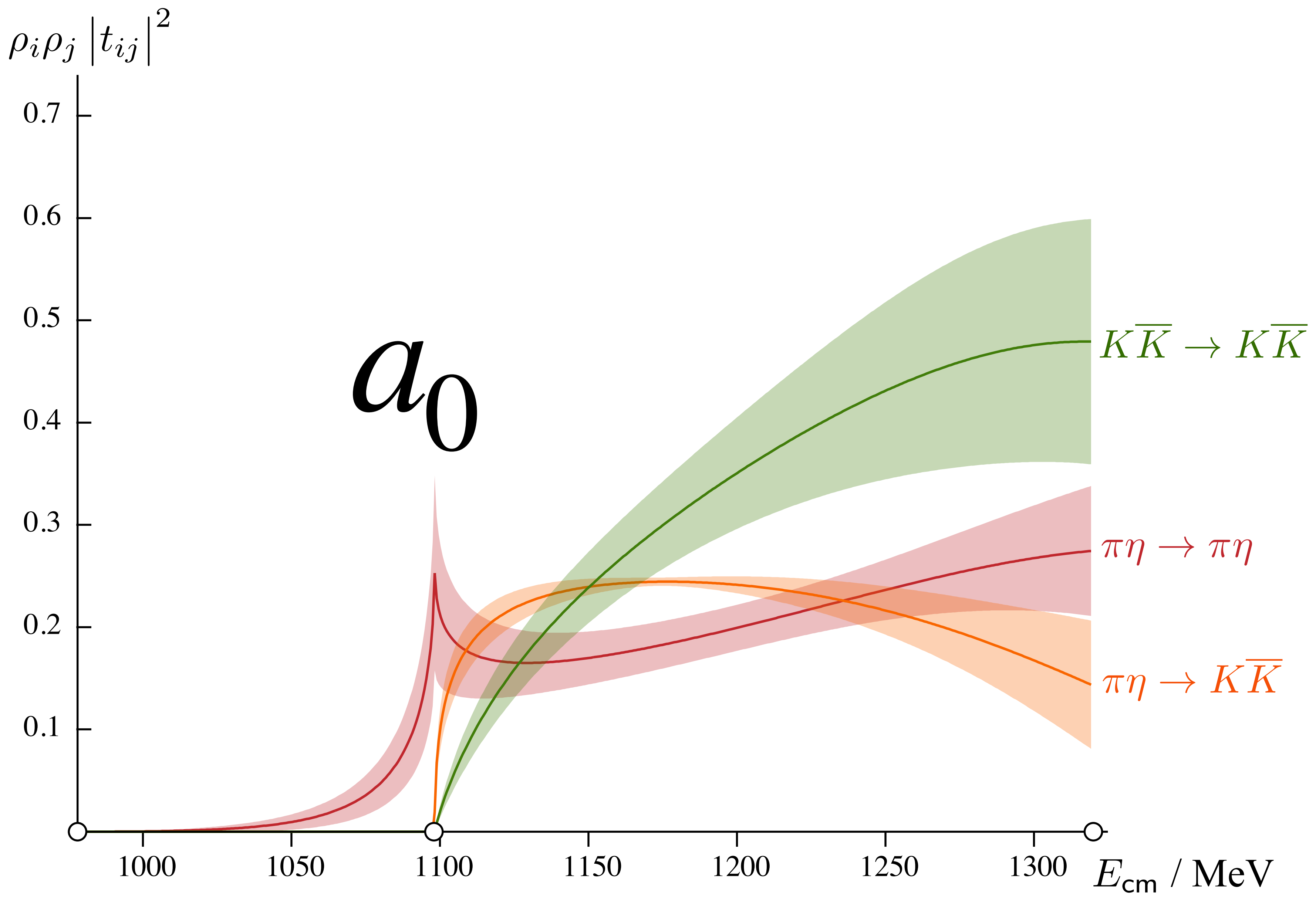


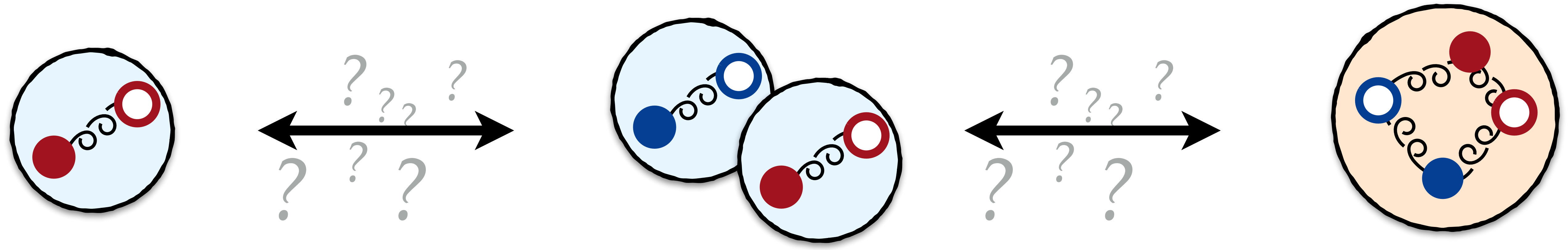
m_π=391 MeV

Hansen, RB, Edwards, Thomas, & Wilson (2020)

The spectrum just isn't enough!

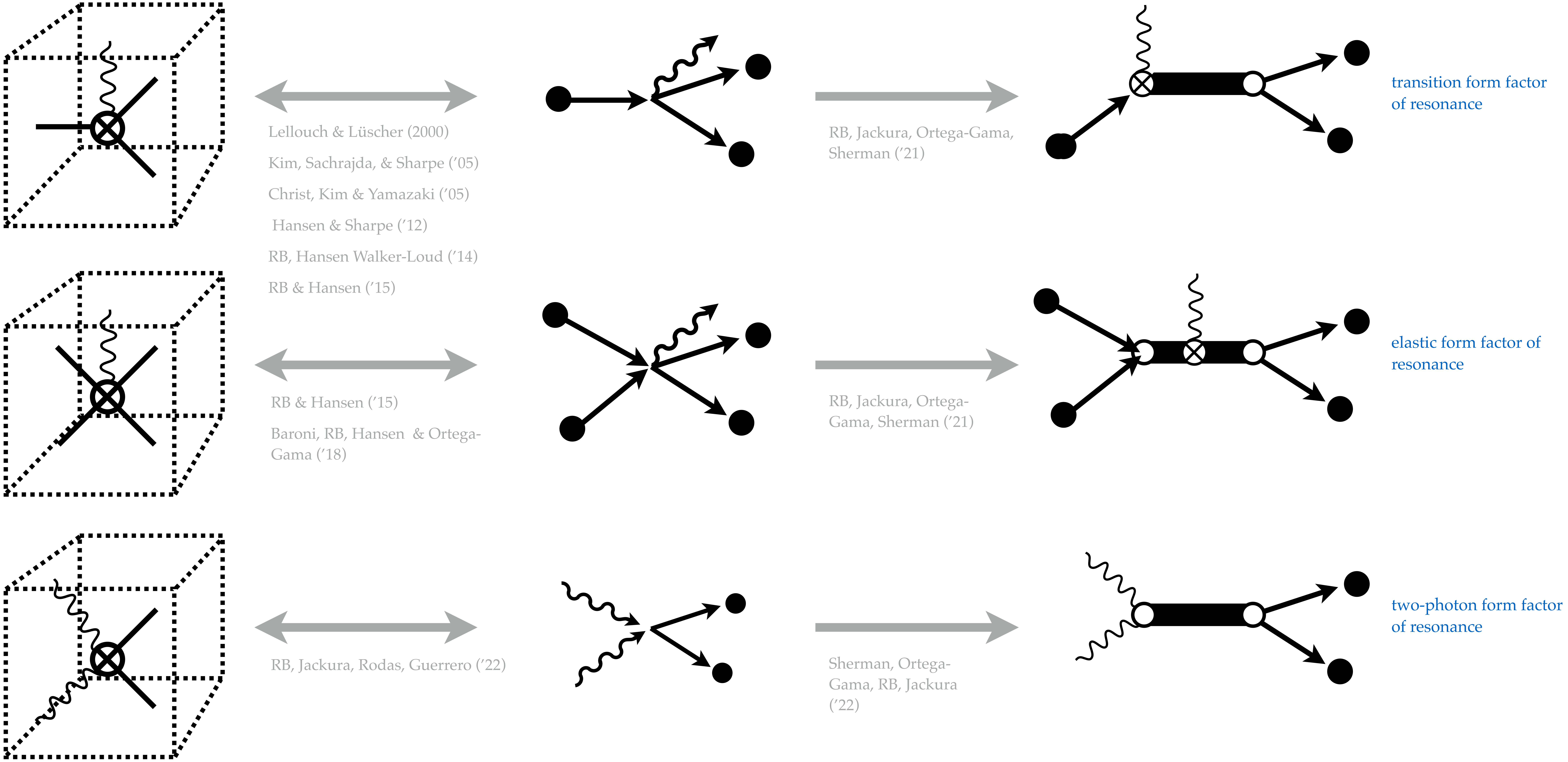






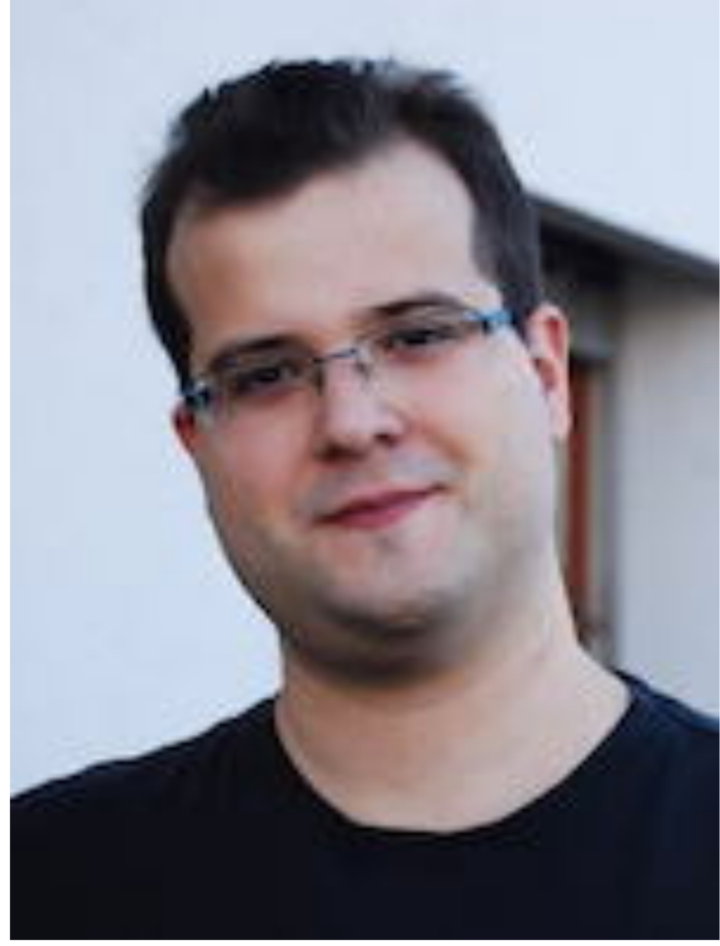
RB, Dudek, Edwards, Wilson (2016)
Dudek, Edwards, Wilson (2016)

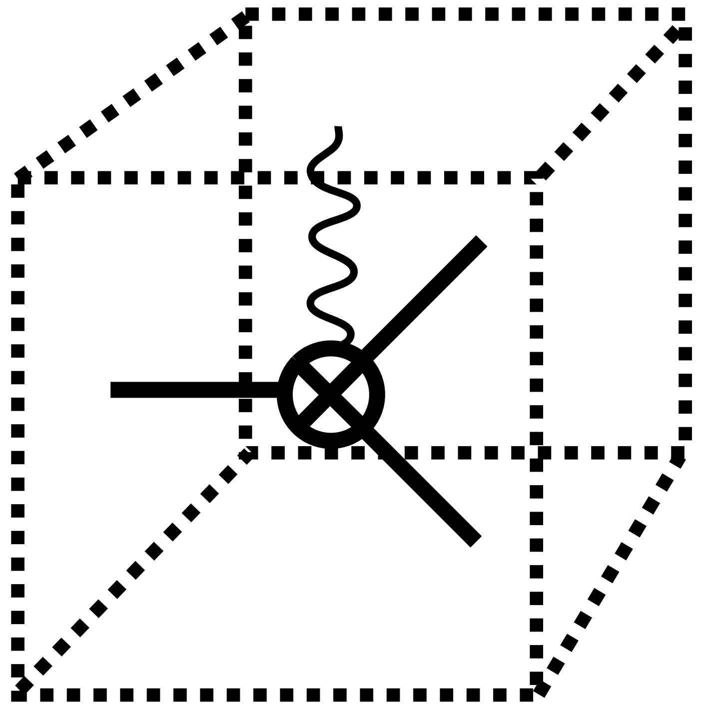
Two-body Matrix Elements



Two-body Matrix Elements

Leskovec [Tuesday morning]





Lellouch & Lüscher (2000)

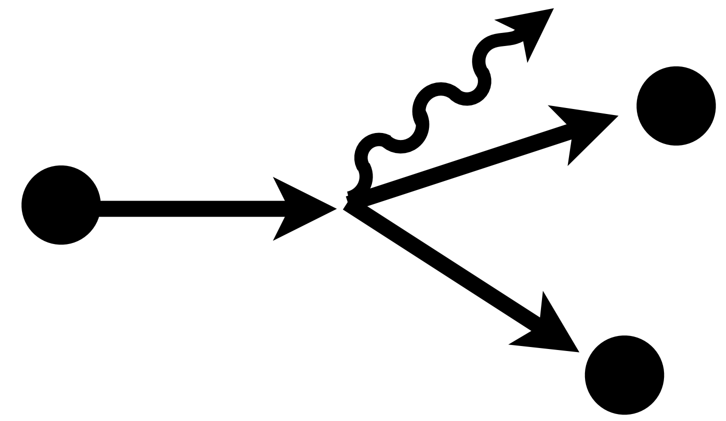
Kim, Sachrajda, & Sharpe ('05)

Christ, Kim & Yamazaki ('05)

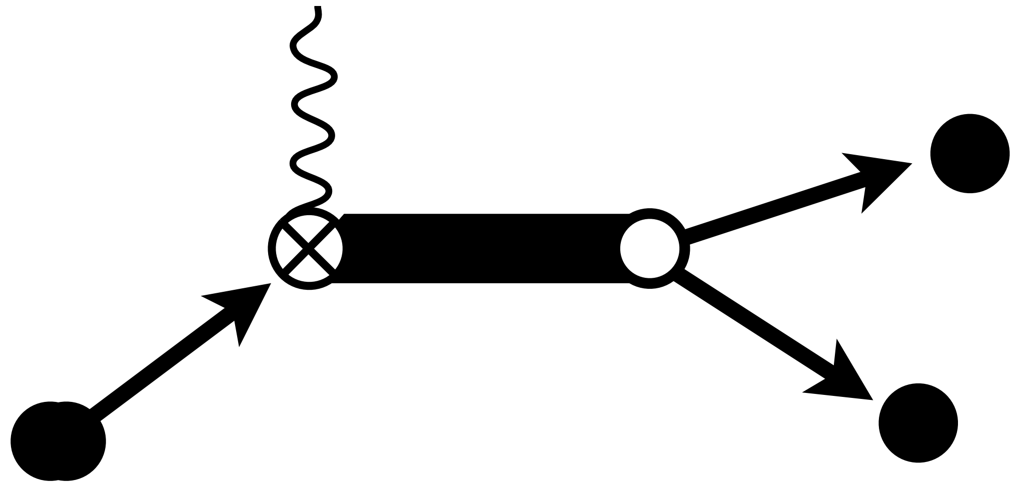
Hansen & Sharpe ('12)

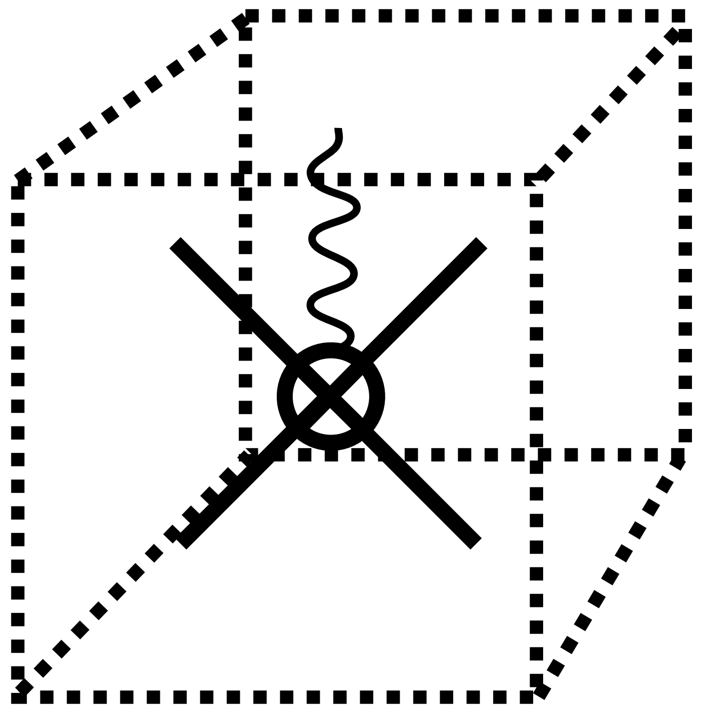
RB, Hansen Walker-Loud ('14)

RB & Hansen ('15)



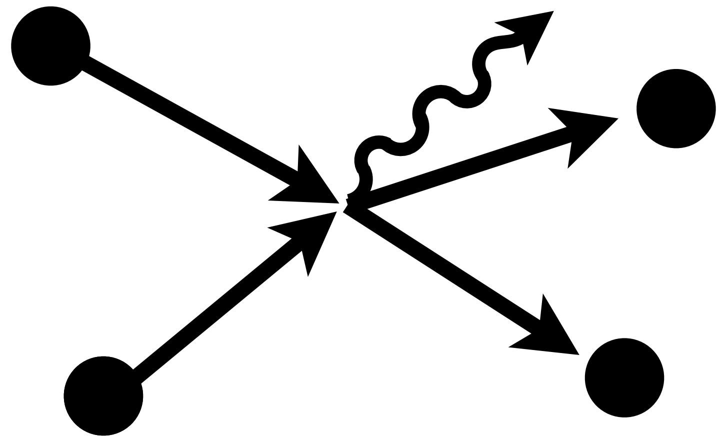
RB, Jackura, Ortega-Gama, Sherman ('21)



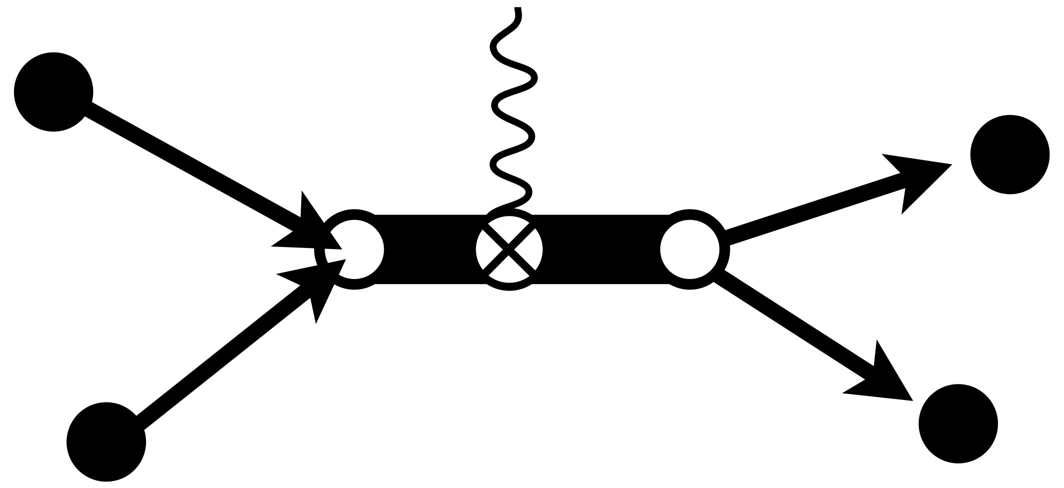


RB & Hansen ('15)

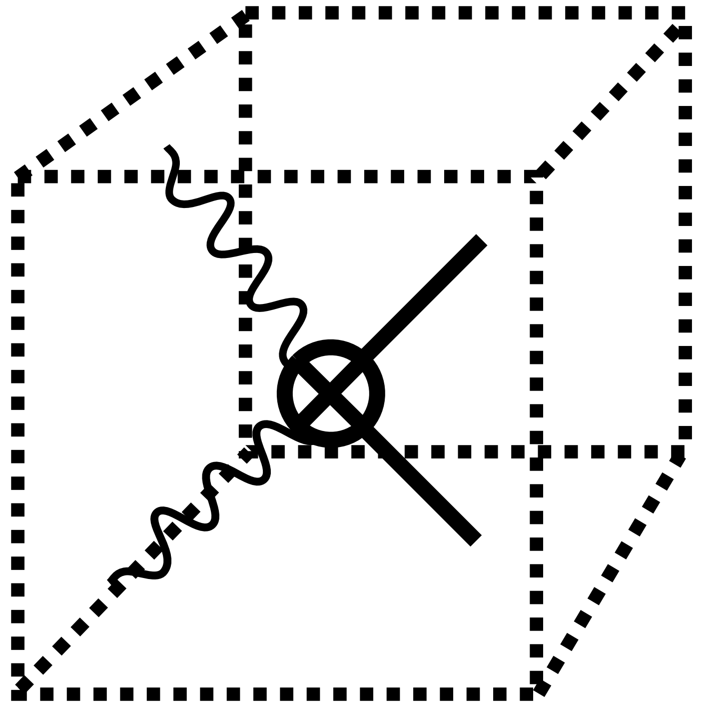
Baroni, RB, Hansen & Ortega-Gama ('18)



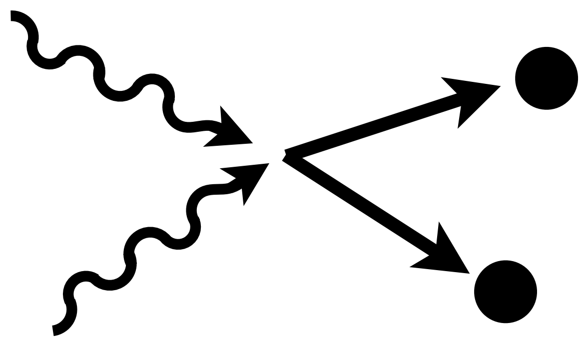
RB, Jackura, Ortega-Gama, Sherman ('21)



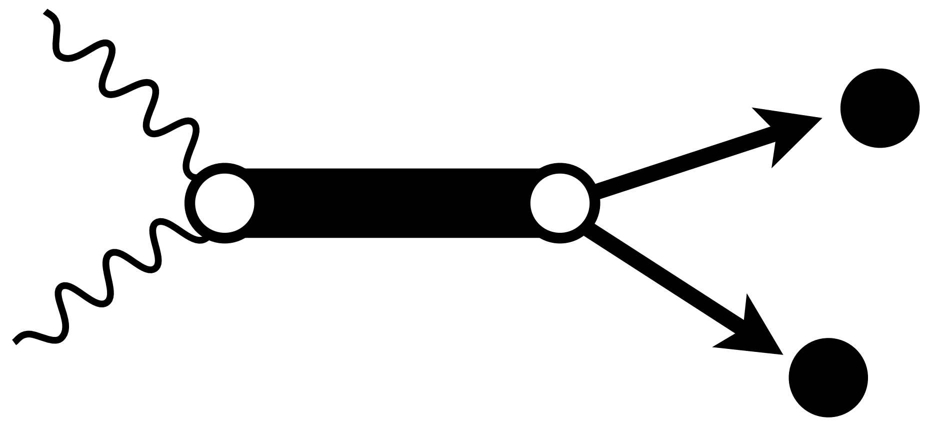
elastic form factor of resonance



RB, Jackura, Rodas, Guerrero ('22)

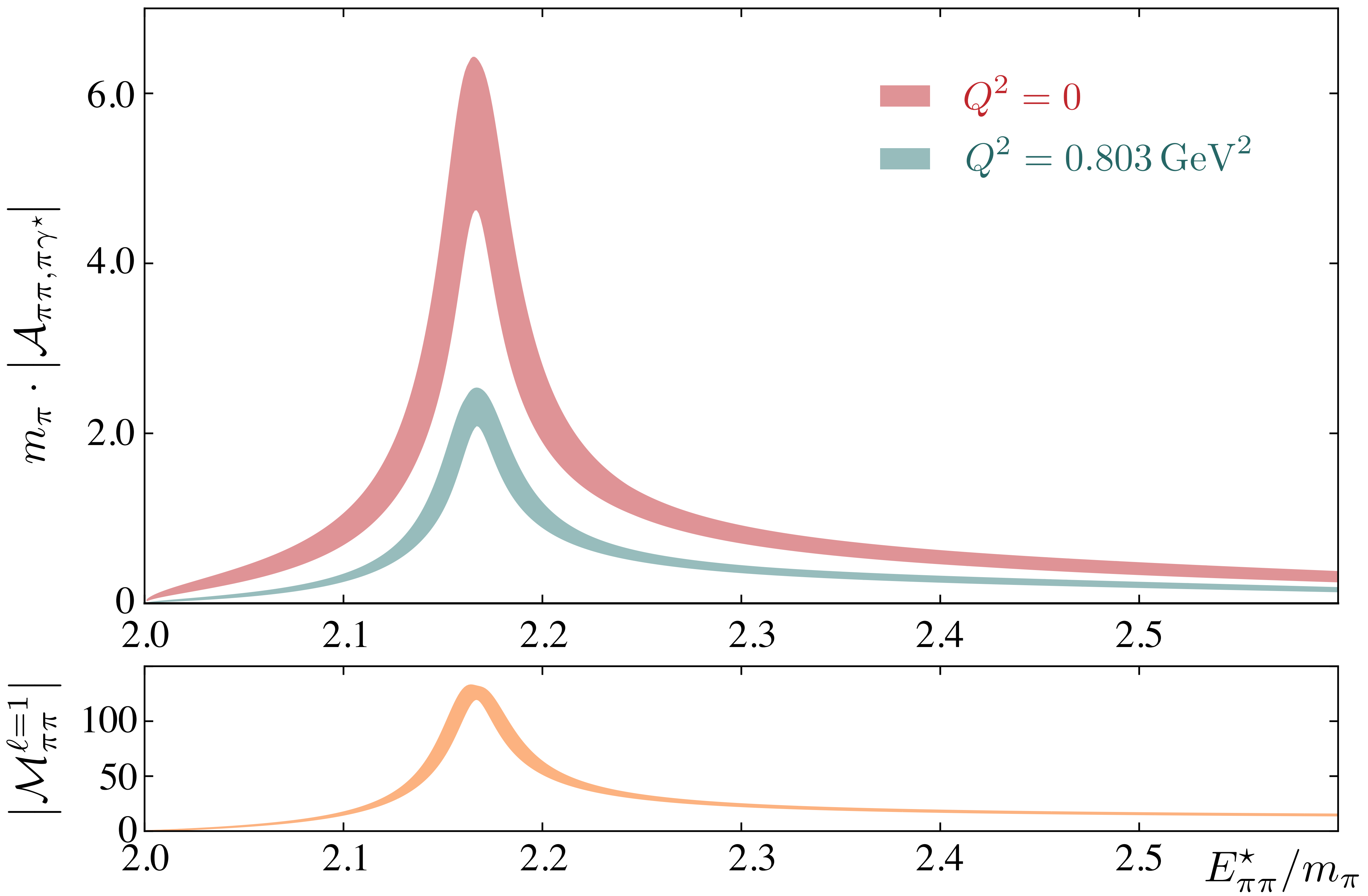


Sherman, Ortega-Gama, RB, Jackura ('22)

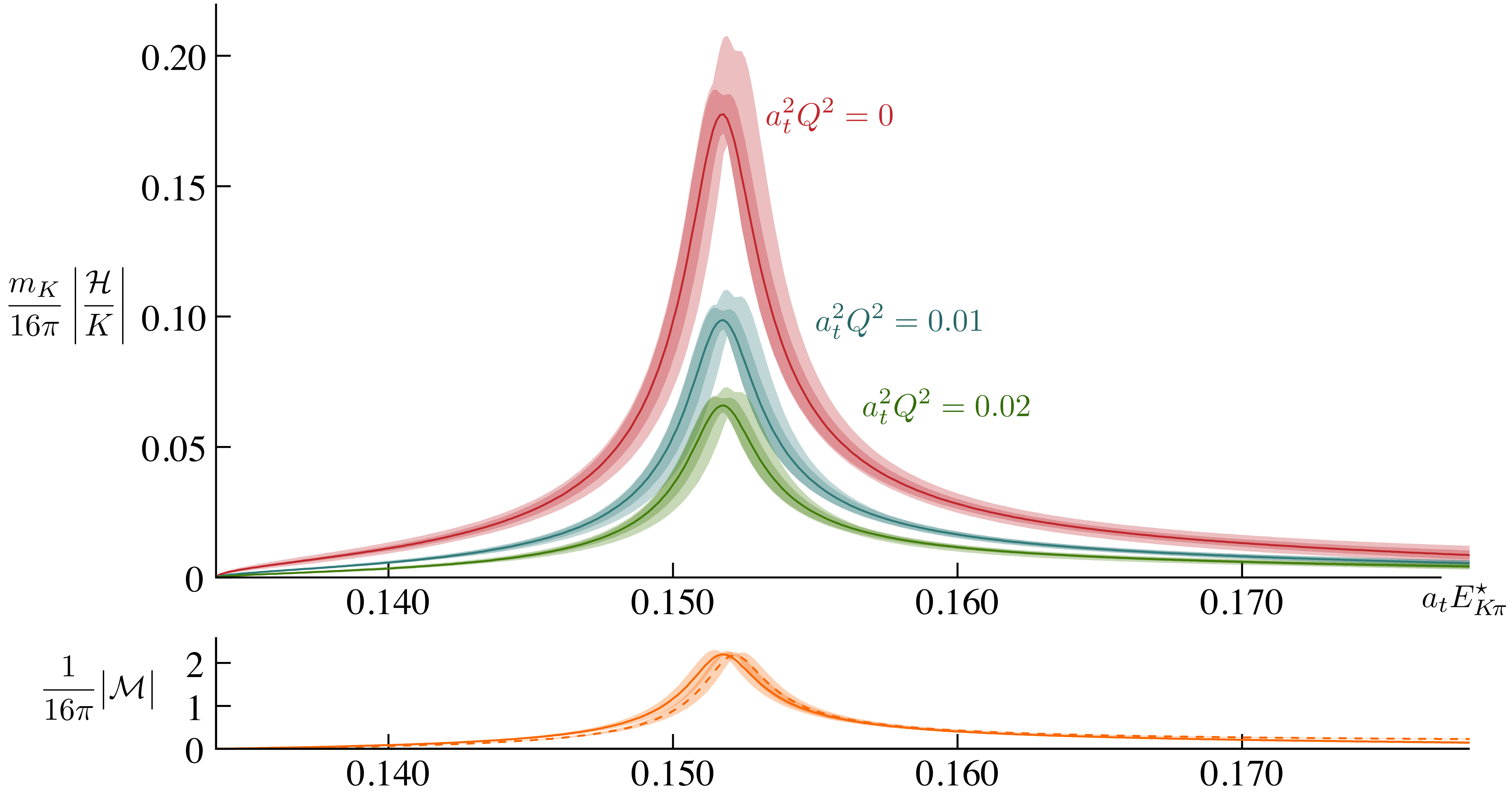


Ortega-Gama [Wednesday afternoon]

$\pi\gamma^\star \to \pi\pi$ m_π=391 MeV

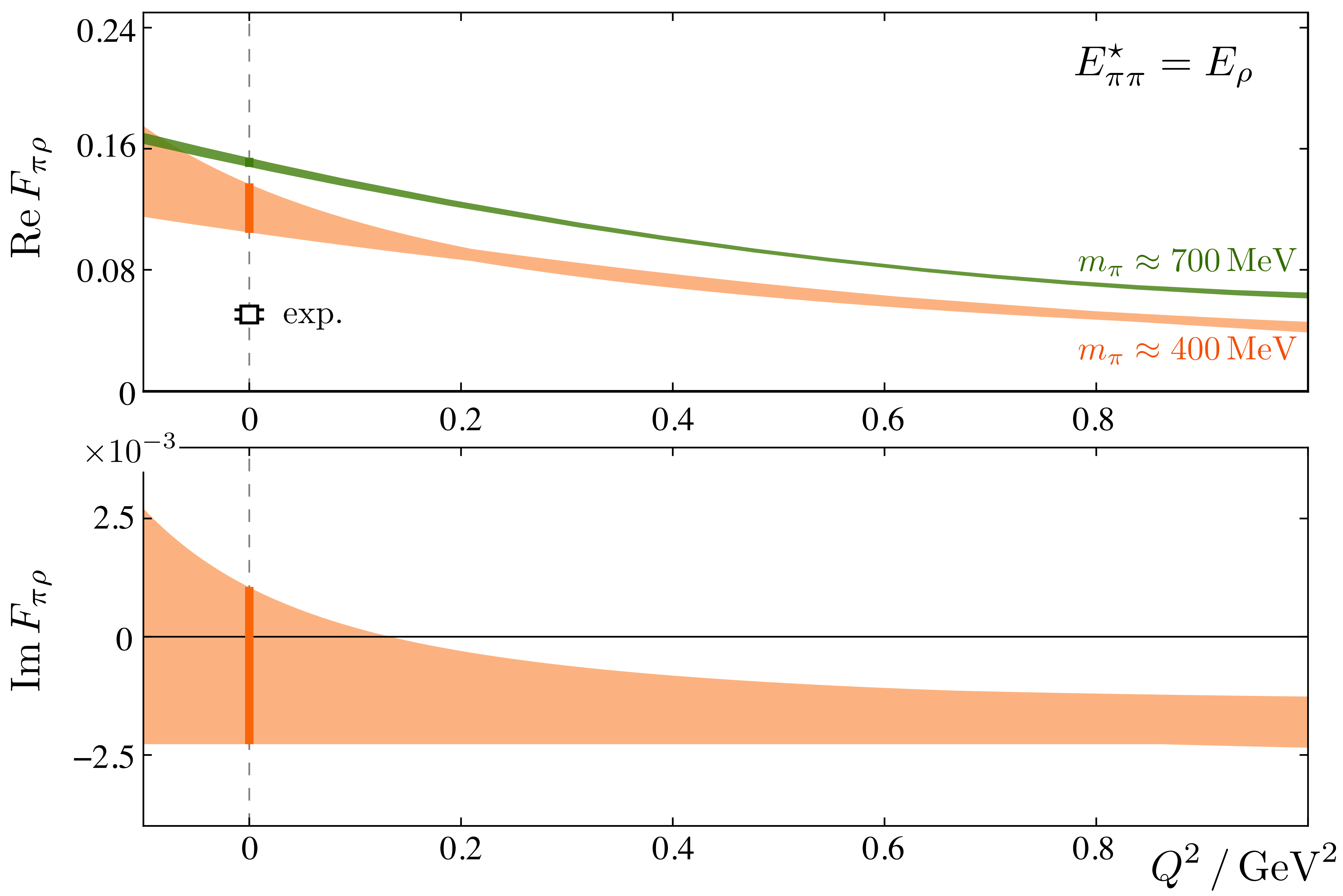


$K\gamma^\star \to K\pi$ m_π=280 MeV

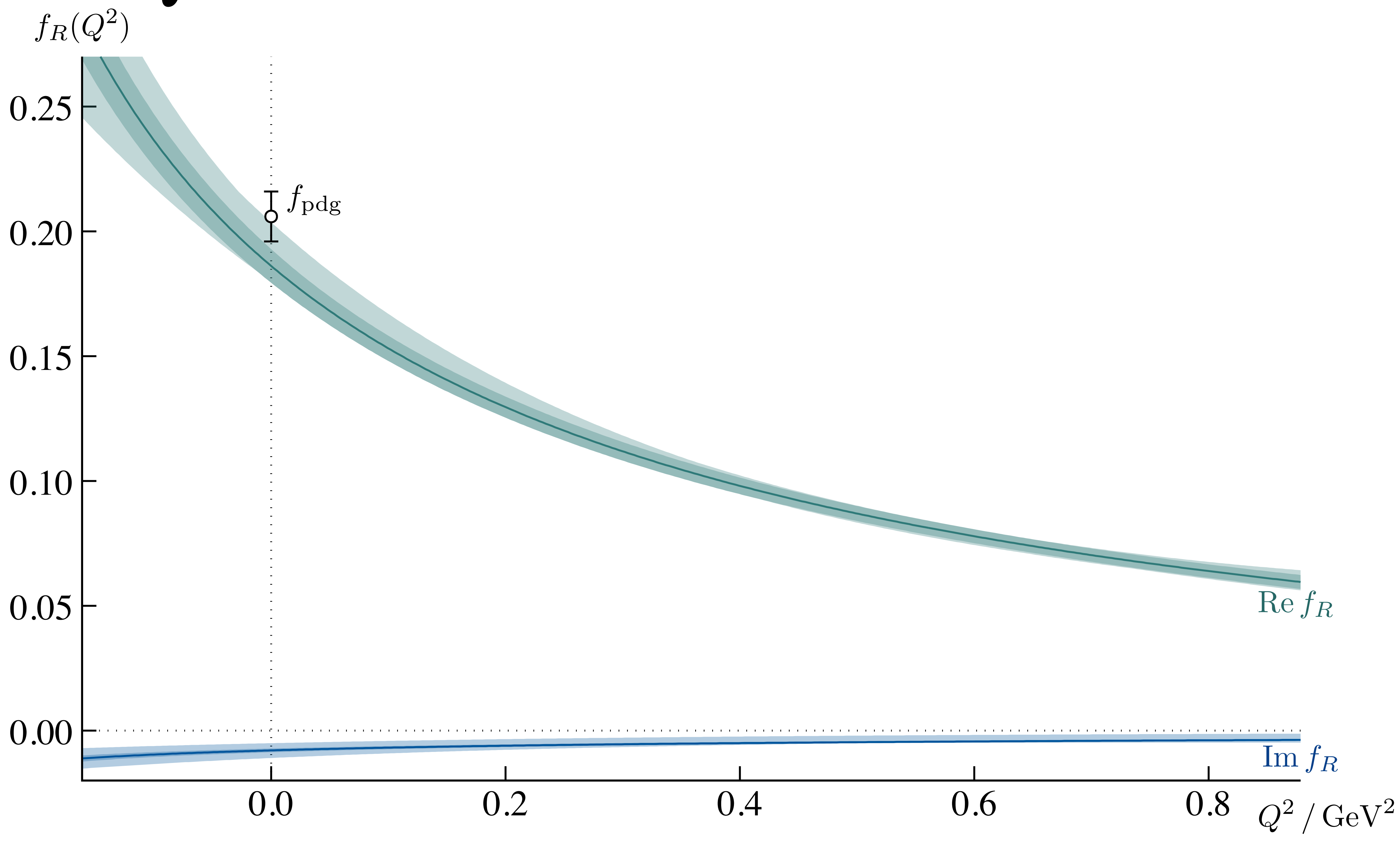


RB, Dudek, Edwards, Schultz, Thomas, Wilson (2016)
Radhakrishnan, Dudek, Edwards (2022) ~~RB~~

$\pi\gamma^\star \to \rho$ m_π=391 MeV

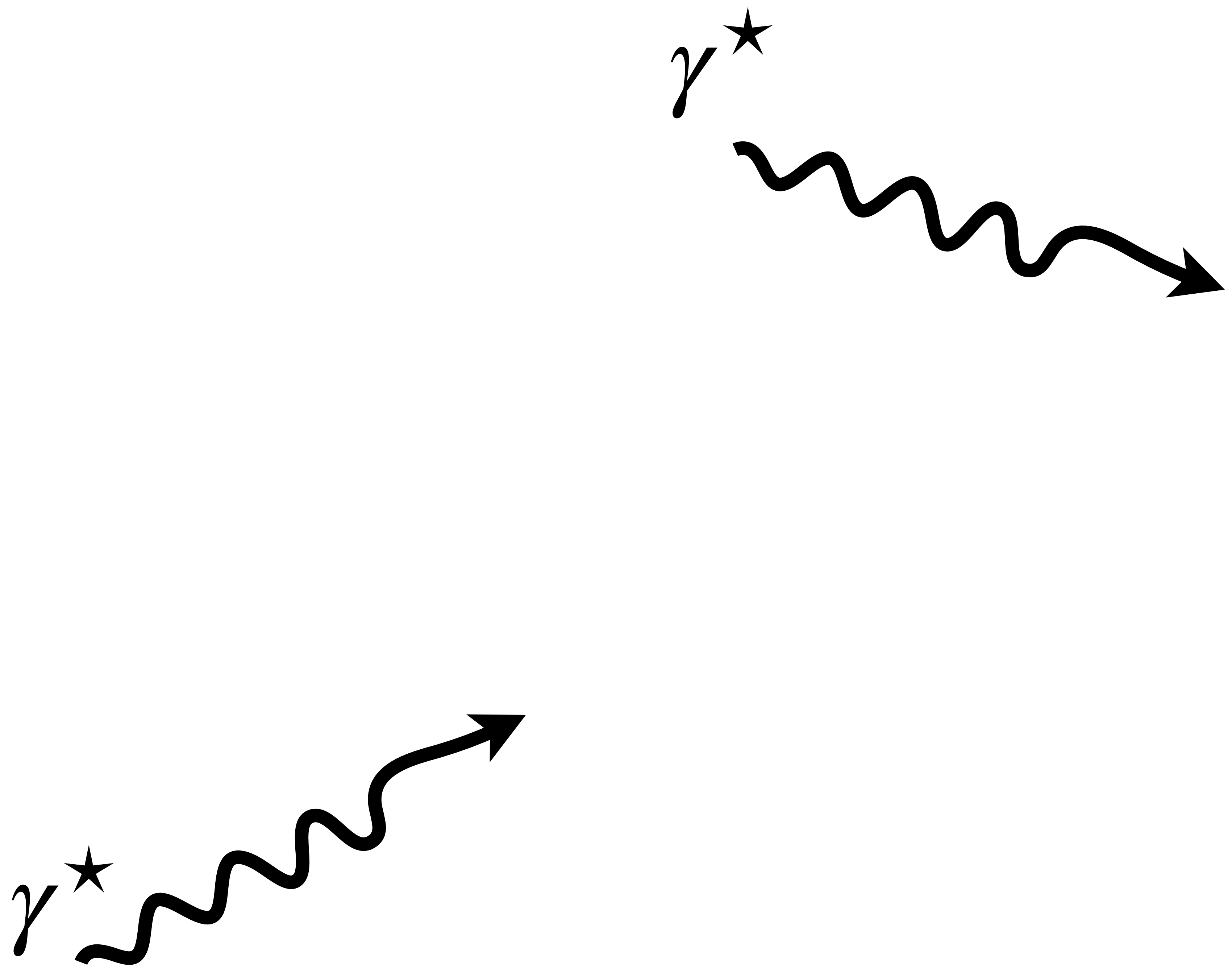


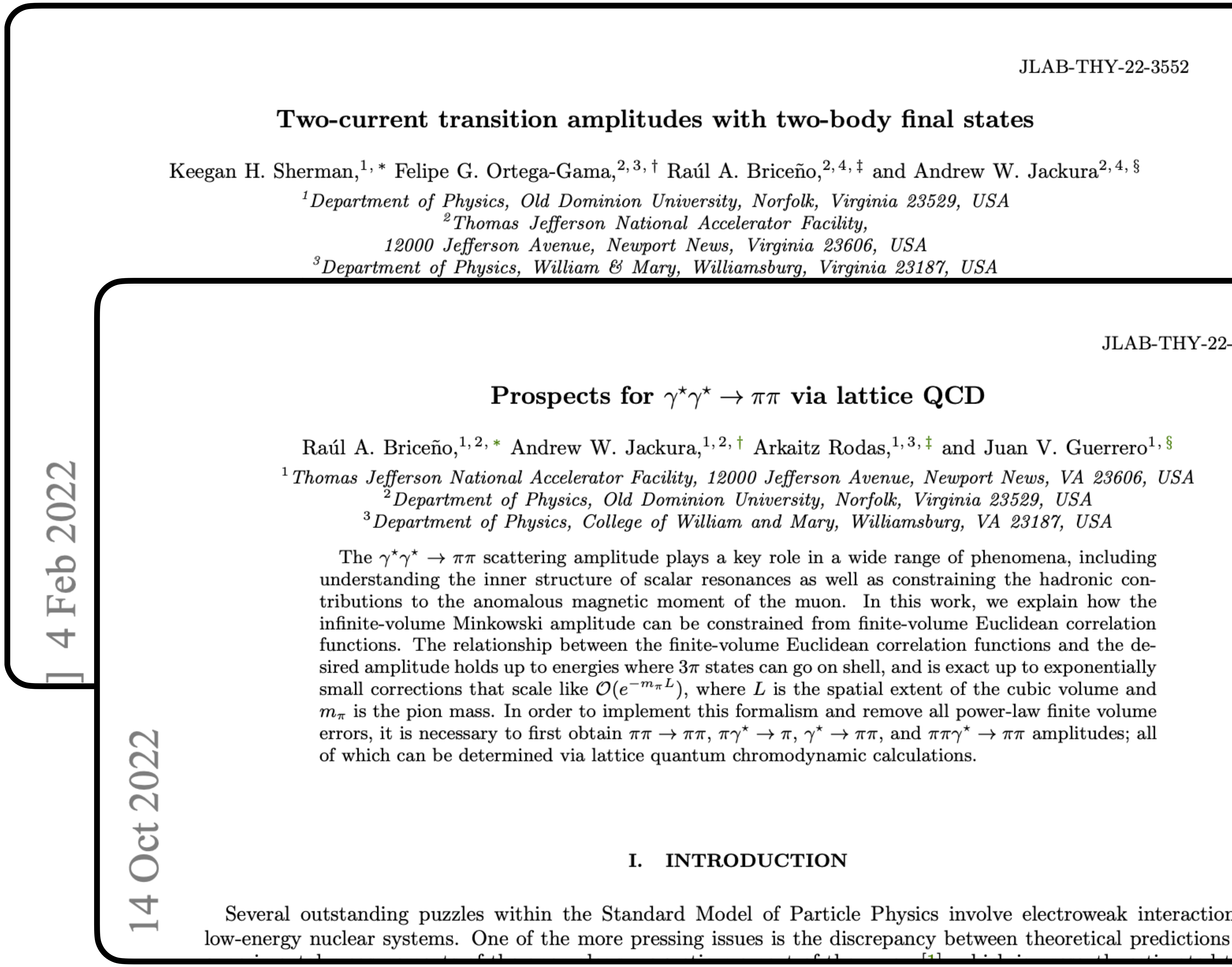
$K\gamma^\star \to K^\star$ m_π=280 MeV



RB, Dudek, Edwards, Schultz, Thomas, Wilson (2016)
Radhakrishnan, Dudek, Edwards (2022) ~~RB~~

$\gamma^\star\gamma^\star \to \pi\pi$ not total pipe dream





JLAB-THY-22-3552

Two-current transition amplitudes with two-body final states

Keegan H. Sherman,[1,*] Felipe G. Ortega-Gama,[2,3,†] Raúl A. Briceño,[2,4,‡] and Andrew W. Jackura[2,4,§]

[1]Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA
[2]Thomas Jefferson National Accelerator Facility,
12000 Jefferson Avenue, Newport News, Virginia 23606, USA
[3]Department of Physics, William & Mary, Williamsburg, Virginia 23187, USA

4 Feb 2022

JLAB-THY-22-

Prospects for $\gamma^\star\gamma^\star \to \pi\pi$ via lattice QCD

Raúl A. Briceño,[1,2,*] Andrew W. Jackura,[1,2,†] Arkaitz Rodas,[1,3,‡] and Juan V. Guerrero[1,§]

[1]Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA
[2]Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA
[3]Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA

The $\gamma^\star\gamma^\star \to \pi\pi$ scattering amplitude plays a key role in a wide range of phenomena, including understanding the inner structure of scalar resonances as well as constraining the hadronic contributions to the anomalous magnetic moment of the muon. In this work, we explain how the infinite-volume Minkowski amplitude can be constrained from finite-volume Euclidean correlation functions. The relationship between the finite-volume Euclidean correlation functions and the desired amplitude holds up to energies where 3π states can go on shell, and is exact up to exponentially small corrections that scale like $\mathcal{O}(e^{-m_\pi L})$, where L is the spatial extent of the cubic volume and m_π is the pion mass. In order to implement this formalism and remove all power-law finite volume errors, it is necessary to first obtain $\pi\pi \to \pi\pi$, $\pi\gamma^\star \to \pi$, $\gamma^\star \to \pi\pi$, and $\pi\pi\gamma^\star \to \pi\pi$ amplitudes; all of which can be determined via lattice quantum chromodynamic calculations.

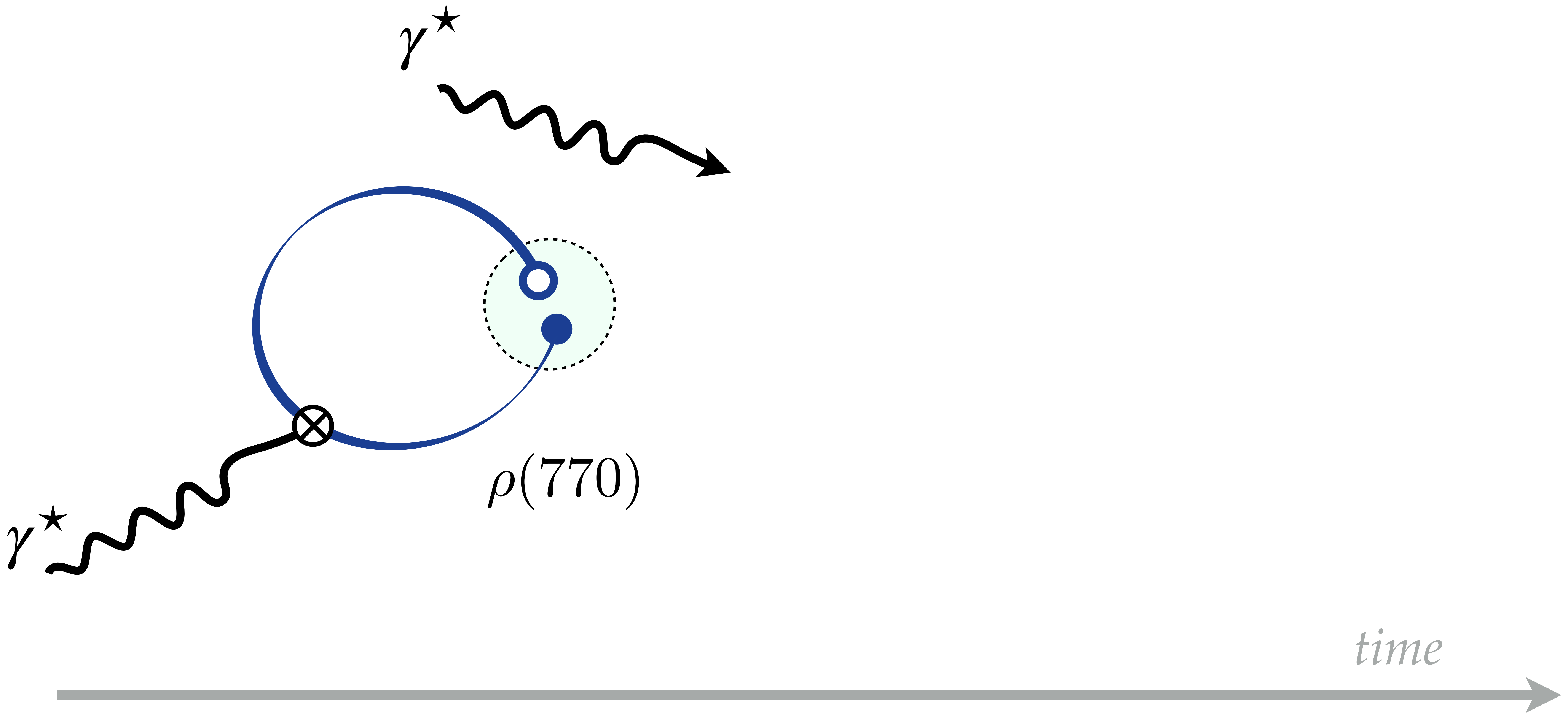
14 Oct 2022

I. INTRODUCTION

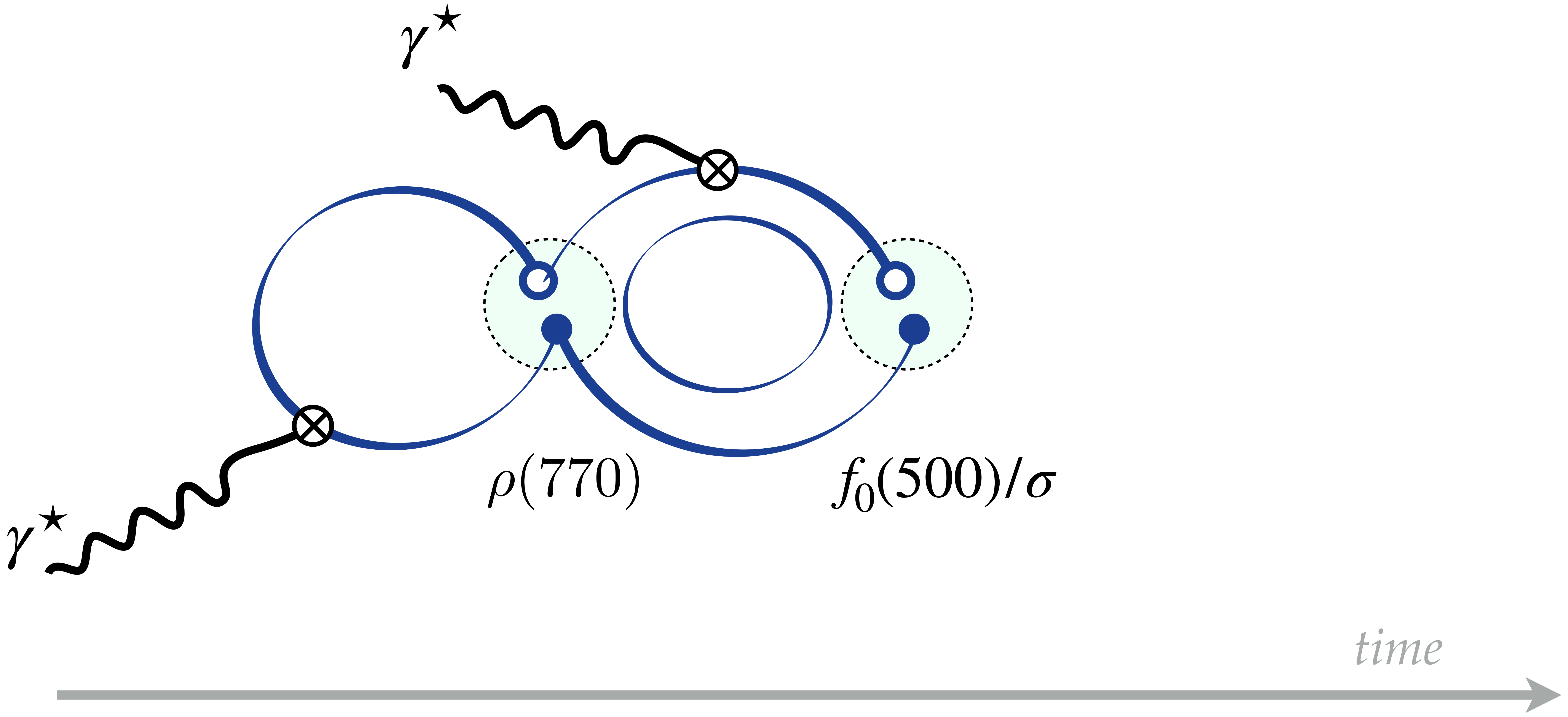
Several outstanding puzzles within the Standard Model of Particle Physics involve electroweak interaction
low-energy nuclear systems. One of the more pressing issues is the discrepancy between theoretical predictions

time

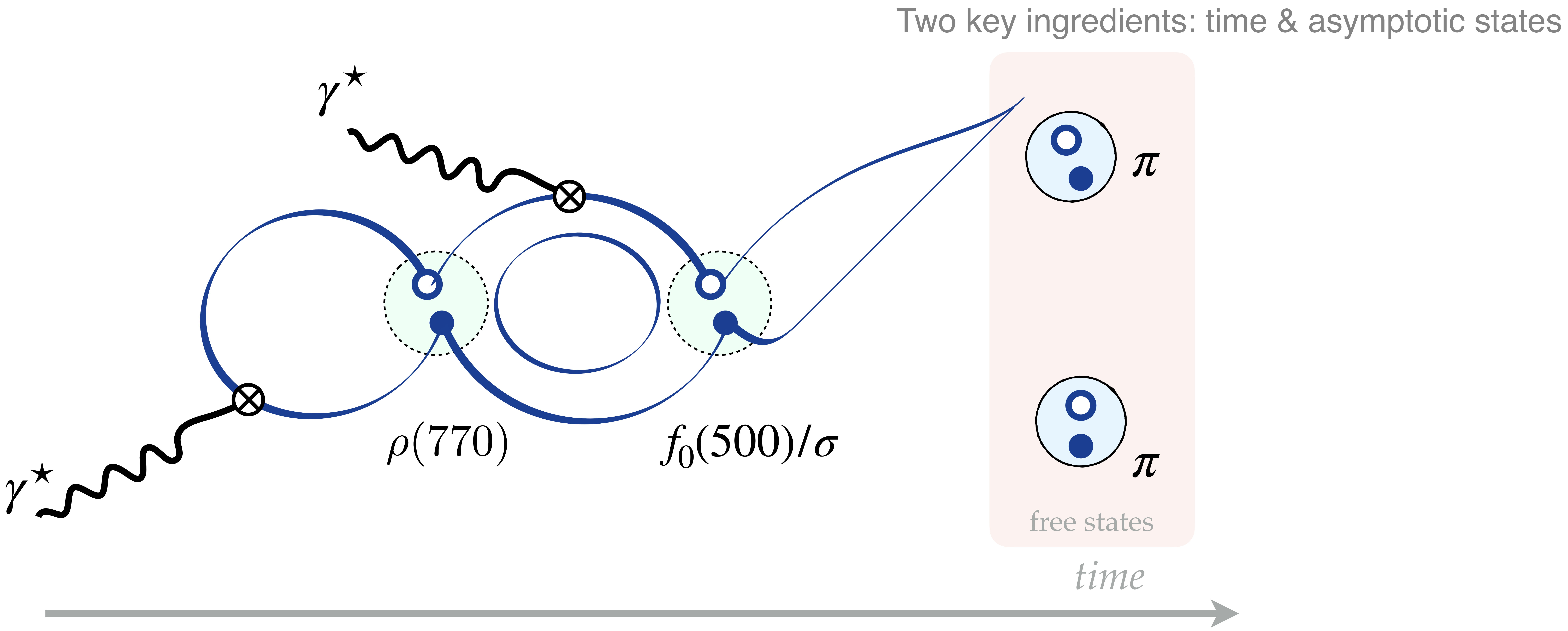
$\gamma^\star \gamma^\star \to \pi\pi$



$\gamma^\star\gamma^\star \to \pi\pi$



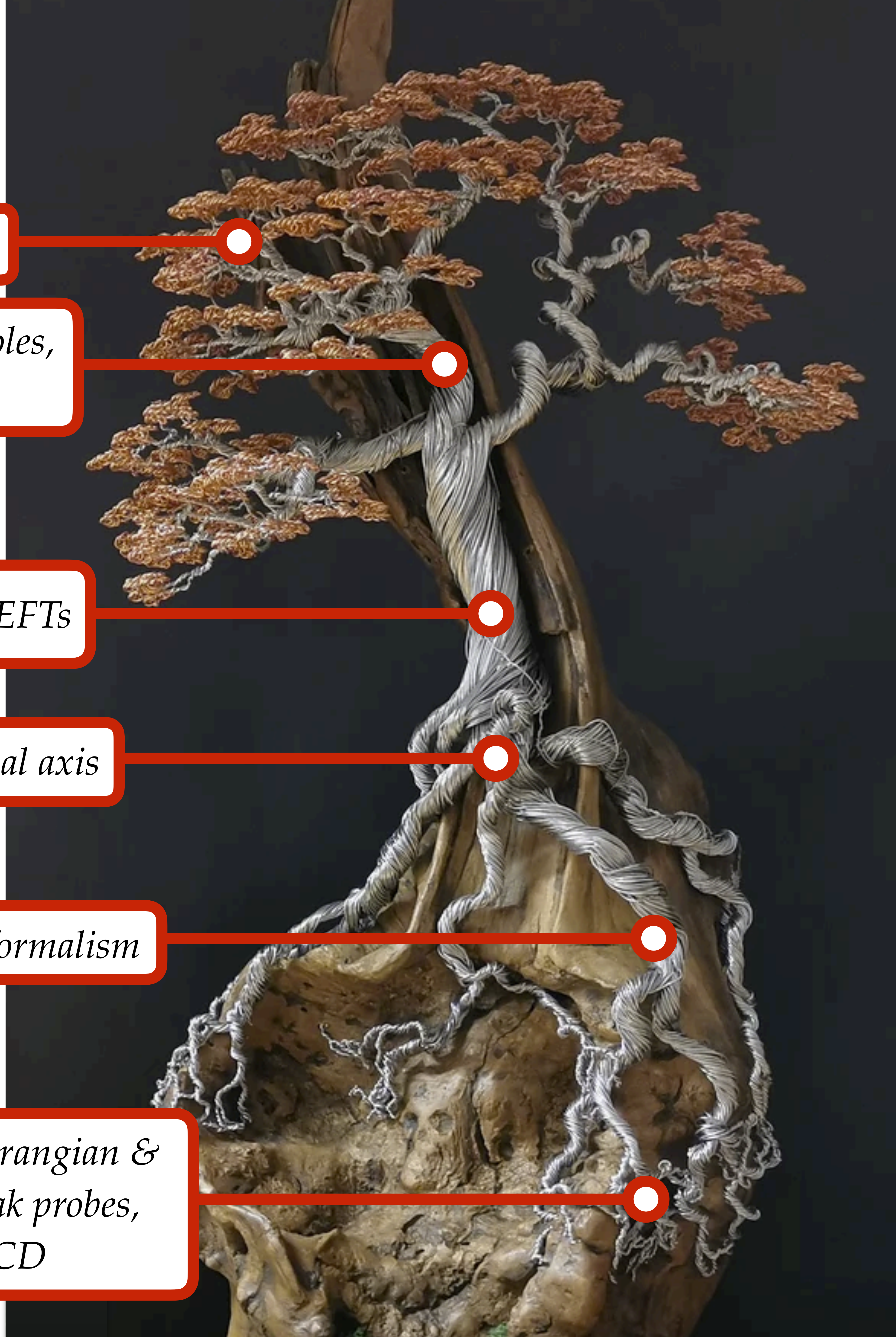
$\gamma^\star\gamma^\star \to \pi\pi$



Modern-day spectroscopy rooted in QCD

Challenging but not impossible

- ☑ Enhancement of operator basis,
  - ☐ Tetraquarks, glueballs, pentraquarks, three-particles, ….
- ☑ Contraction costs,
  - ☐ Cost grows with the number of externals legs…
- ☑ Extensions of finite-volume formalism,
  - ☐ Multi-channel 3-body, spin, etc.
  - ☐ Electroweak processes, …
- ☑ Scattering theory,
  - ☐ "Earnest amplitudes", analytic continuations,…



A review/introduction

slightly out of date

Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

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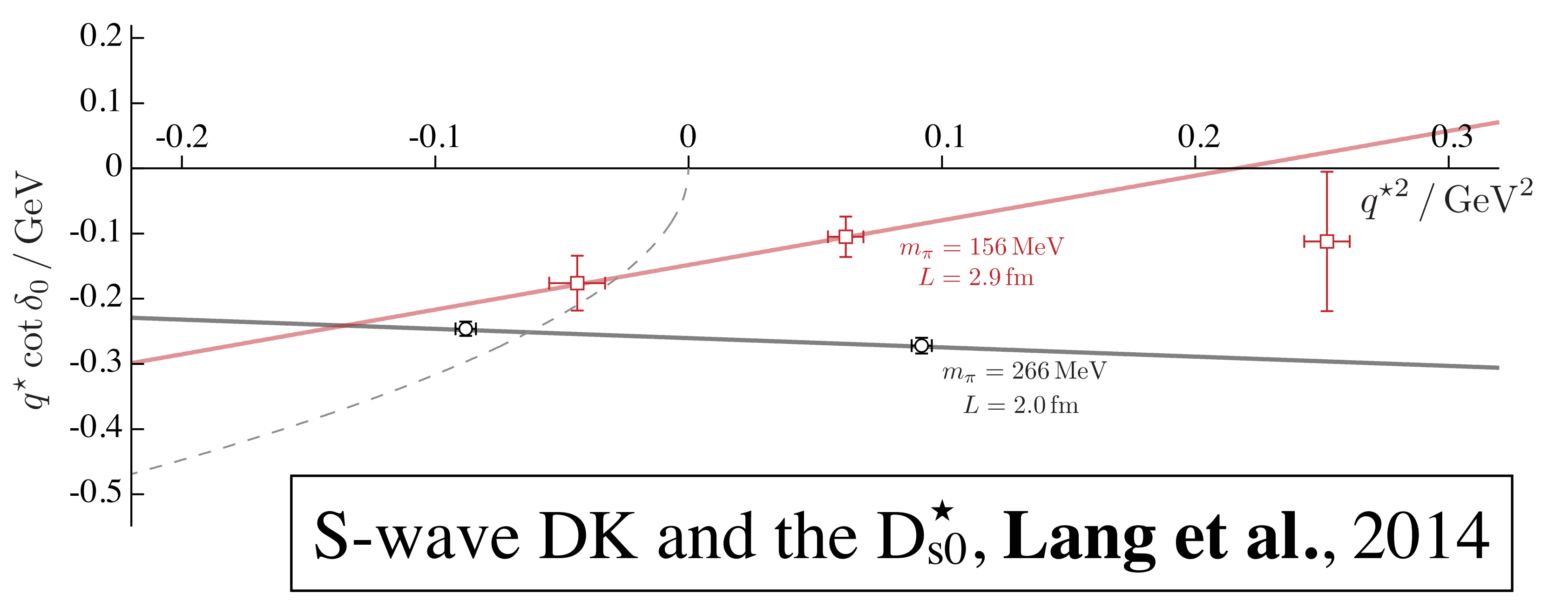
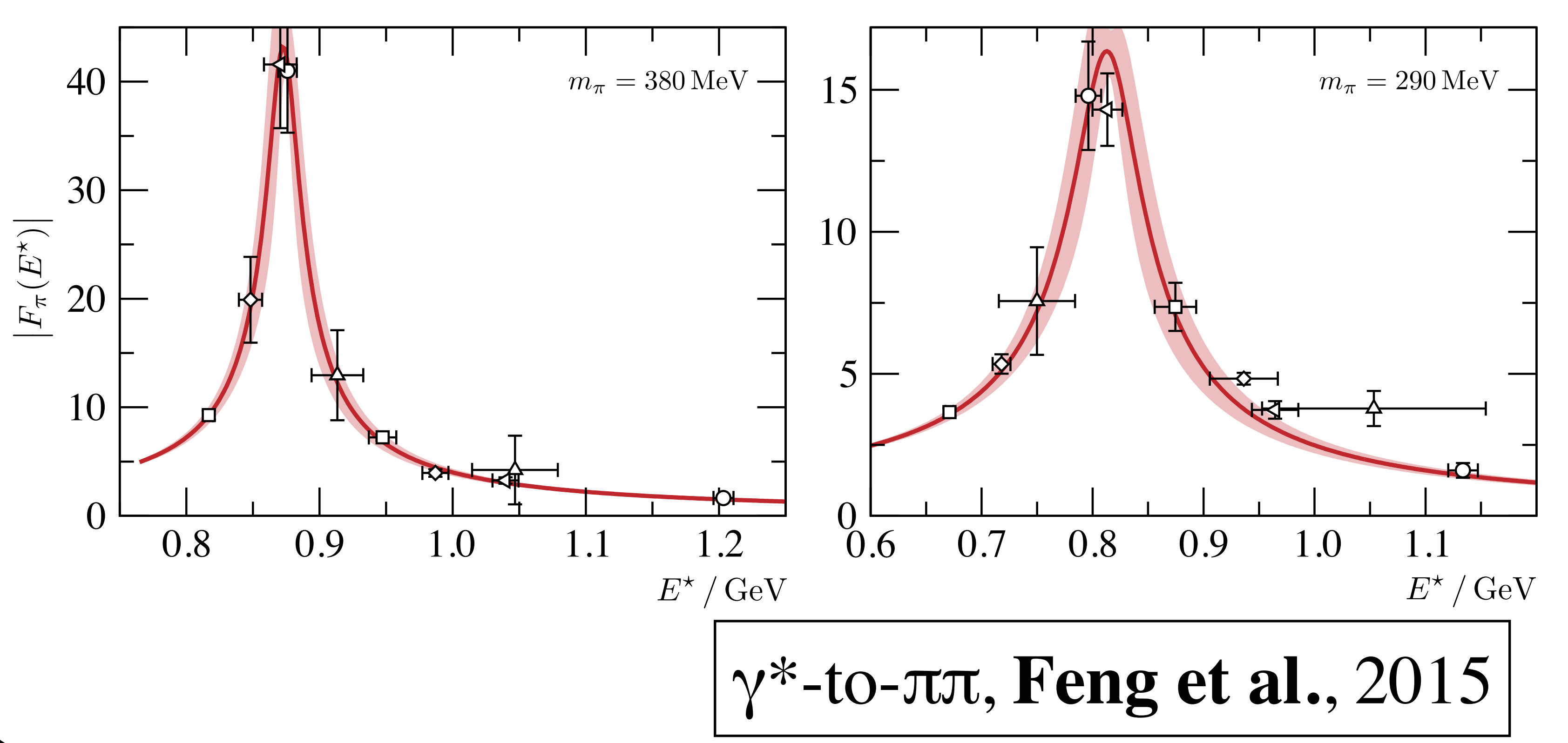
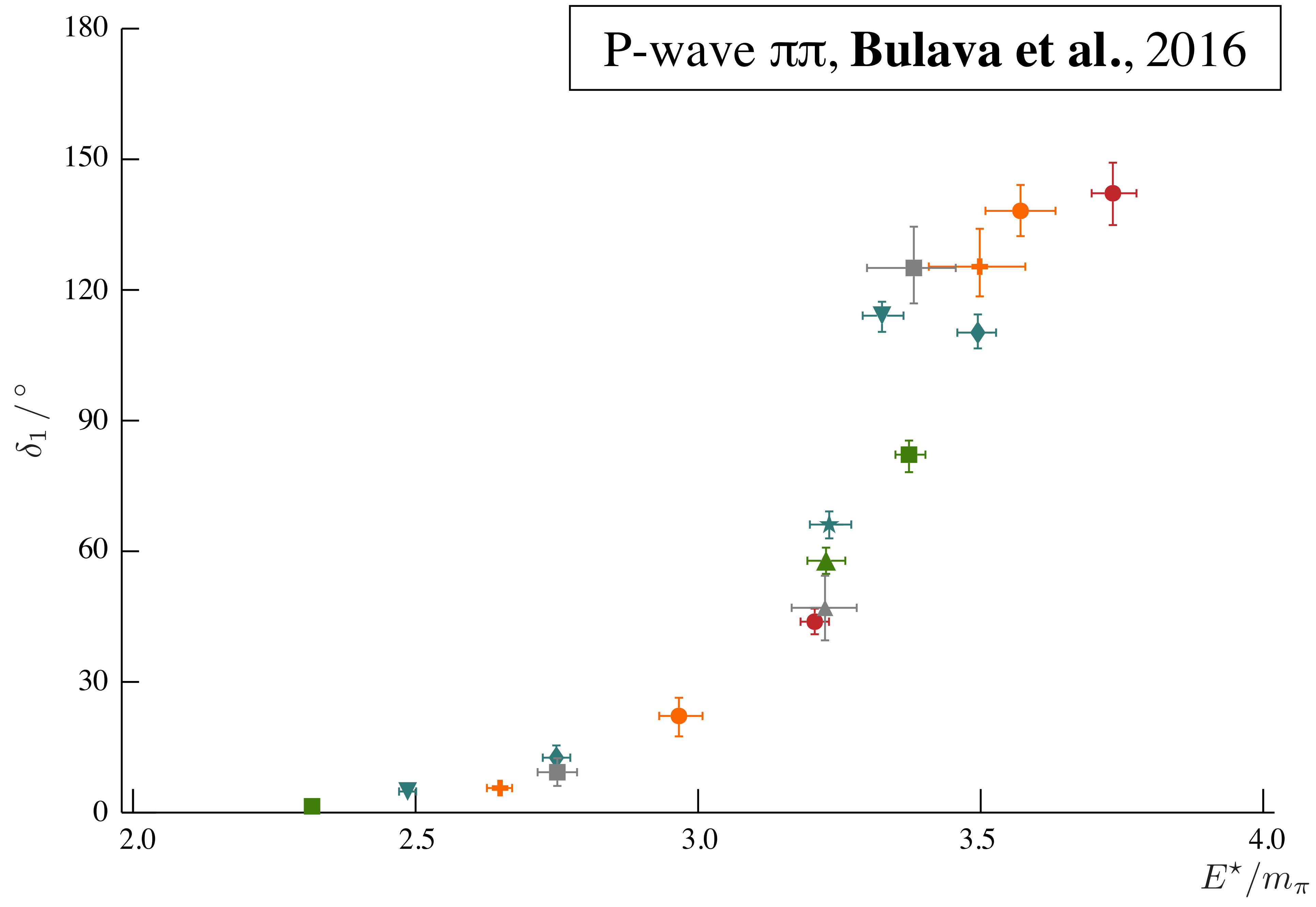


S-wave DK and the $D^\star_{s0}$, **Lang et al.**, 2014



γ*-to-ππ, **Feng et al.**, 2015



P-wave ππ, **Bulava et al.**, 2016

2 minimum requirements

Two "musts" for few-body systems:

☑ Generalized eigenvalue problem (GEVP),

☑ large basis of ops,

$$\mathcal{O}_b \sim \bar{q}\,\Gamma_b\,q, \pi\pi, K\overline{K}, \ldots,$$

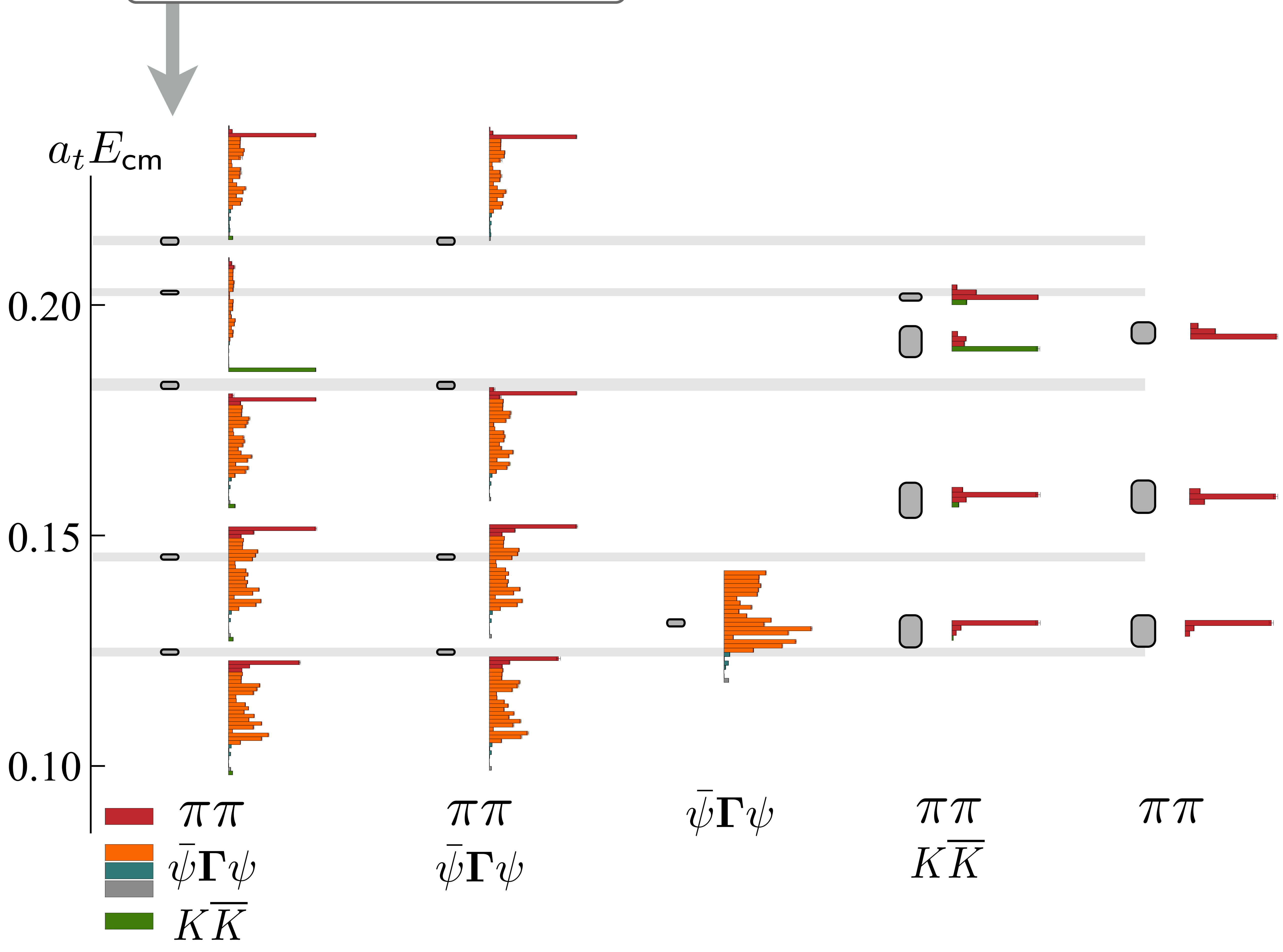
☑ diagonalization,

$$C^{2pt.}_{ab}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}^\dagger_a(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n}Z^*_{a,n}e^{-E_n t}$$

$$C(t)\vec{v}^{(n)}(t,t_0) = \lambda_n(t,t_0)C(t_0)\vec{v}^{(n)}(t,t_0)$$

☑ Finite-volume formalisms.

GEVP with large basic is **necessary** but **not sufficient** to trust spectrum!



Wilson, RB, Dudek, Edwards, & Thomas (2015)