Analysis of rescattering effects in 3π final states

Dominik Stamen

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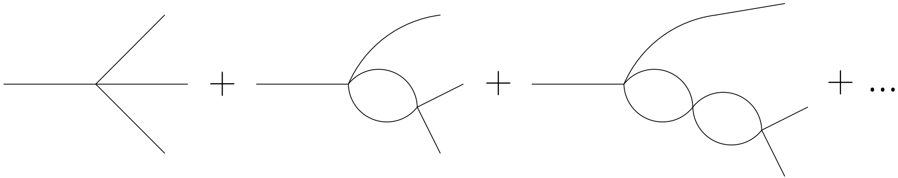
[DS, T. Isken, B. Kubis, M. Mikhasenko, M. Niehus]

Motivation

- different decay processes that decay into $\rho\pi$ final states
- effects of the ρ described by Omnès function [Omnès; 1958]



Motivation

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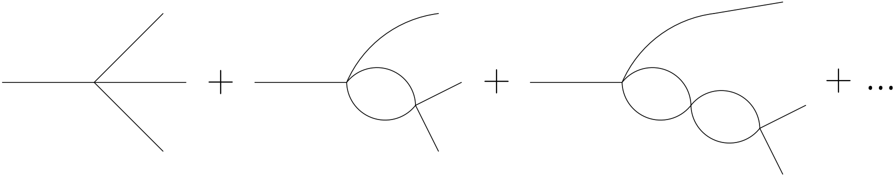
- effects of the ρ described by Omnès function [Omnès; 1958]
- include full rescattering effects
- Khuri–Treiman equations [Khuri, Treiman; 1960]



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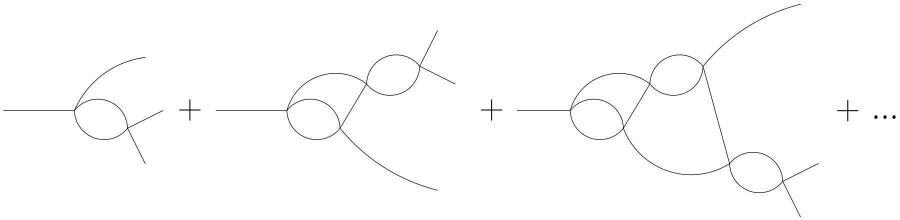


- in which process at what energy are rescattering effects observable?

Khuri–Treiman equations

- unitarity condition

$$f_1(s) = \mathcal{F}(s) + \widehat{\mathcal{F}}(s)$$
$$\text{disc}\mathcal{F}(s) = 2i\mathcal{F}(s)\sin\delta(s)e^{-i\delta(s)}\theta(s - 4M_\pi^2)$$

- homogeneous solution with $\widehat{\mathcal{F}}(s) = 0$ gives Omnès function

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

Khuri–Treiman equations

- unitarity condition

$$f_1(s) = \mathcal{F}(s) + \widehat{\mathcal{F}}(s)$$

$$\mathrm{disc}\mathcal{F}(s) = 2i\left(\mathcal{F}(s) + \widehat{\mathcal{F}}(s)\right)\sin\delta(s)e^{-i\delta(s)}\theta(s-4M_\pi^2)$$

- inhomogeneous solution leads to KT-equations

$$\mathcal{F}(s) = \Omega(s)\left(P_n(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\sin\delta(s')\widehat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right)$$

Decays

- four different processes
- angular-momentum quantum numbers of decaying particle
- $J^{PC} = 0^{-+}, 1^{--}, 1^{-+}, 2^{++}$
- restrictive analysis with only one subtraction
- only consider P-waves
- only elastic unitarity
- not include processes that involve S-waves (1^{++})

Reconstruction theorems

- derive reconstruction theorems using fixed-t dispersion relations
- 1^{--}
  - isoscalar vector meson (ω, ϕ)
  - symmetric in Mandelstam variables

$$\mathcal{M}(s,t,u) = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_3^{\beta}\mathcal{F}(s,t,u)$$
$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- 1^{-+}
  - exotic states
  - isovector meson ($\pi_1(1400)$, $\pi_1(1600)$)

$$\mathcal{M}^{ijkl}(s,t,u) = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_3^{\beta}\mathcal{H}^{ijkl}(s,t,u)$$
$$\mathcal{H}^{ijkl}(s,t,u) = \delta^{ij}\delta^{kl}\mathcal{H}(s,t,u) + \delta^{ik}\delta^{jl}\mathcal{H}(t,u,s) + \delta^{il}\delta^{jk}\mathcal{H}(u,s,t)$$
$$\mathcal{H}(s,t,u) = \mathcal{H}(t) - \mathcal{H}(u)$$

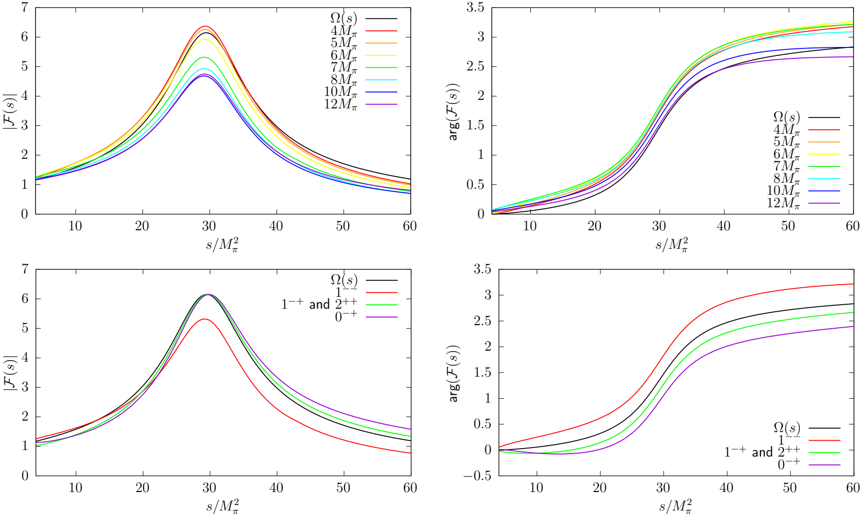
Reconstruction theorems

- 0^{-+}
  - related to exotic 0^{--}
  - C-violating $I = 0$ ($\eta^{(\prime)}$)
  - antisymmetric in Mandelstam variables

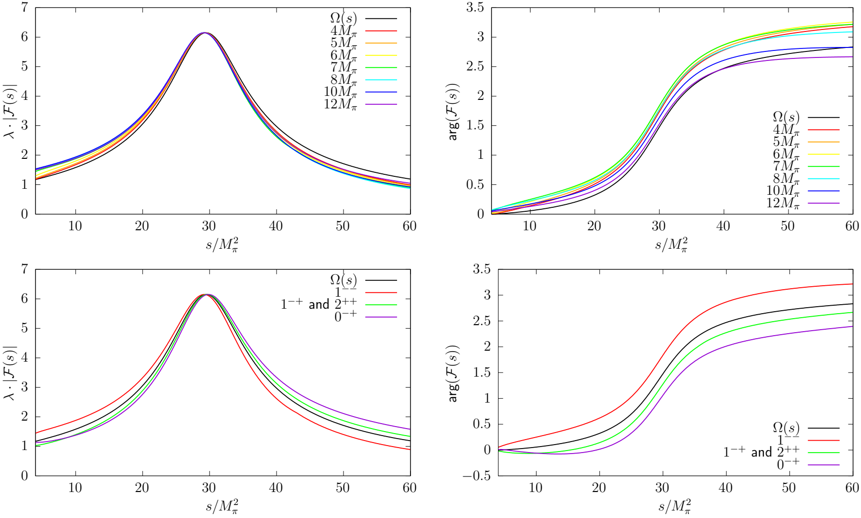
$$\mathcal{M}(s,t,u) = (t-u)\mathcal{G}(s) + (u-s)\mathcal{G}(t) + (s-t)\mathcal{G}(u)$$

- 2^{++}
  - tensor meson ($a_2(1320)$)
  - main decay channel $\rho\pi$
  - more complicated (helicity) structure
  - only P-waves leads to 1^{-+} RT

Basis Functions



Basis Functions



Log-likelihood differences

- KT equations set as truth
- to which extent do Omnès functions reproduce these?
- probability density function

$$f(s,t) = \frac{|\mathcal{M}(s,t)|^2}{\int_D |\mathcal{M}(s,t)|^2 \mathrm{d}s\mathrm{d}t}$$

- draw (s_i, t_i) from f^{KT}
- likelihood function

$$L(\mathbb{D}, f) = \prod_{i=1}^{N} f(s_i, t_i)$$

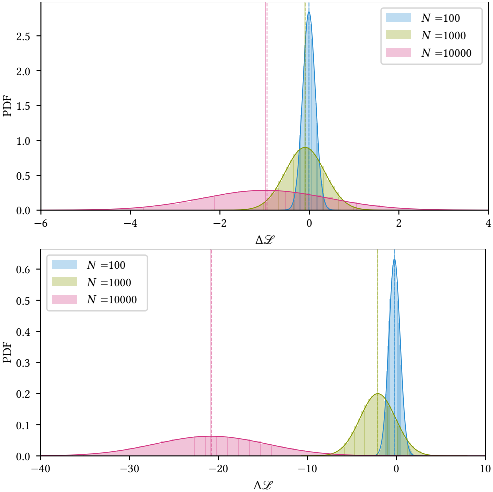
- log-likelihood difference

$$\Delta\mathcal{L}(\mathbb{D}) = \ln(L(\mathbb{D}, f^{\text{Omnès}})) - \ln(L(\mathbb{D}, f^{\text{KT}}))$$

- $\Delta\mathcal{L} < 0$ corresponds to Omnès rejected

Log-likelihood differences

- generate B datasets $\mathbb{D}_b$, $b = 1, \ldots, B$ of size N
- compute $\Delta\mathcal{L}$ on each set
- normal distribution



$M = M_\omega$

$M = M_\phi$

Log-likelihood differences

- use Kullback-Leibler divergence/variance [Kullback, Leibler; 1951]

$$d_{\mathsf{KL}} = \int_D f^{\mathsf{KT}}(s,t) \ln\left(\frac{f^{\mathsf{KT}}(s,t)}{f^{\mathsf{Omnès}}(s,t)}\right) \mathrm{d}s\mathrm{d}t$$

$$\nu_{\mathsf{KL}} = \int_D f^{\mathsf{KT}}(s,t) \ln\left(\frac{f^{\mathsf{KT}}(s,t)}{f^{\mathsf{Omnès}}(s,t)}\right)^2 \mathrm{d}s\mathrm{d}t - d_{\mathsf{KL}}^2$$

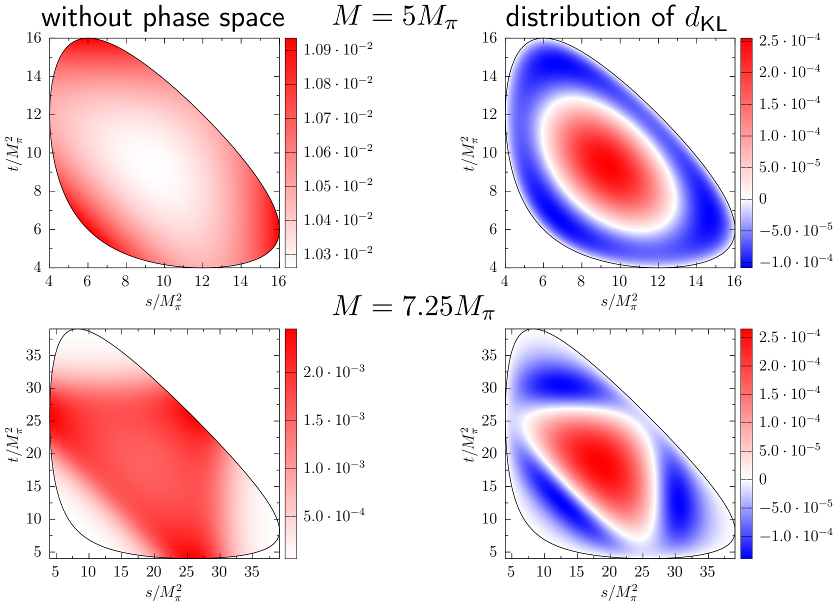
- only use pdfs from here
- probability of $\Delta\mathcal{L} > 0$ via cumulative distribution function

$$q(N) = 1 - \mathcal{N}(0, \mu(N), \sigma(N)) \quad \mu(N) = -N d_{\mathsf{KL}} \quad \sigma(N) = \sqrt{N \nu_{\mathsf{KL}}}$$

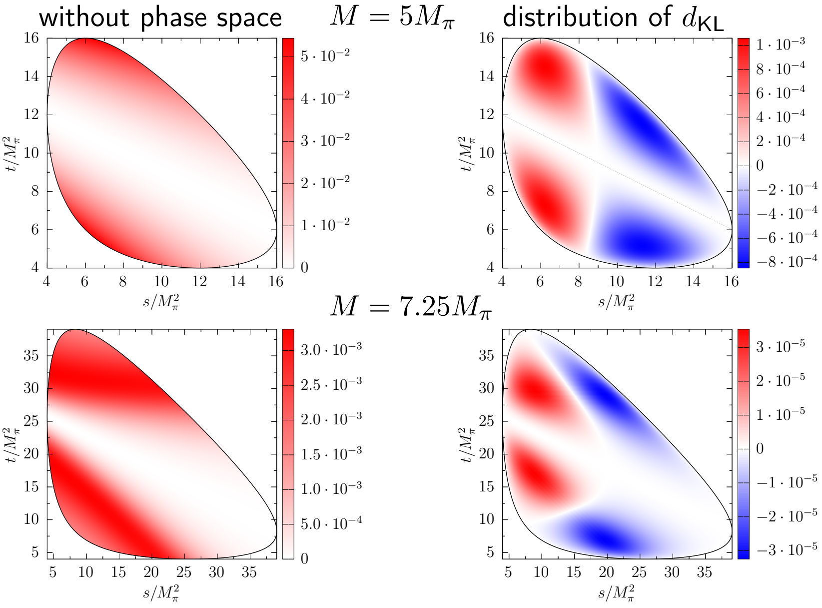
- number of events

$$N(q) = 2\nu_{\mathsf{KL}} \left(\frac{\mathsf{erf}^{-1}(1-2q)}{d_{\mathsf{KL}}}\right)^2$$

Dalitz plots for 1^{--}

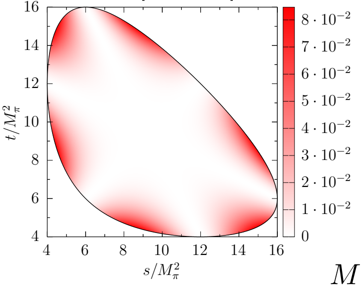


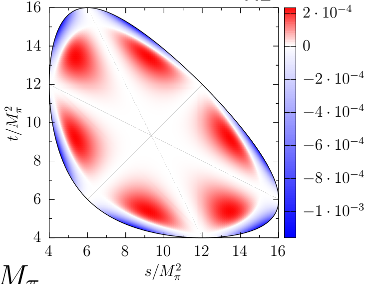
Dalitz plots for 1^{-+}



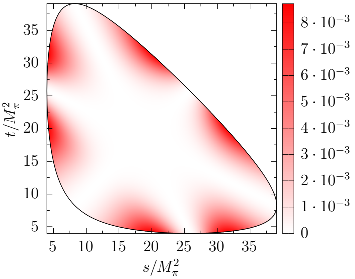
Dalitz plots for 0^{-+}

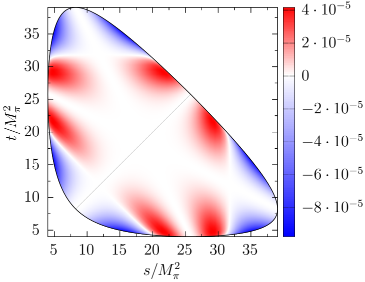
without phase space $M = 5M_\pi$ distribution of d_{KL}





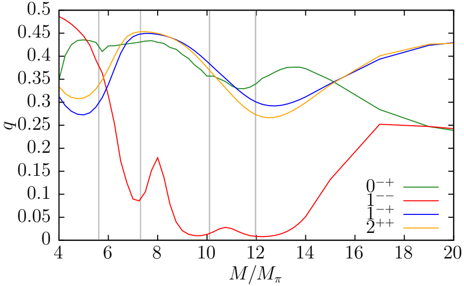
$M = 7.25M_\pi$



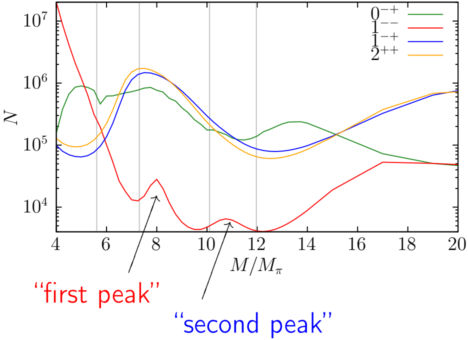


Results

$N = 1000$



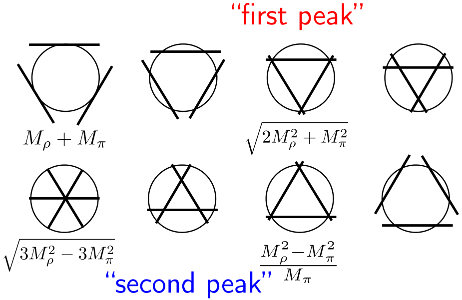
5σ



- 1^{-+} and 2^{++} almost overlap
- kinematic factors play minor role
- form dominated by reconstruction theorem
- rise for limit $M \to \infty$, inelastic effects
- two peaks in 1^{--}

Results

- diagrammatic Dalitz plots for different decay masses



- difference decreases when ρ bands cross in Dalitz plot
- approx. $\sqrt{2M_\rho^2 + M_\pi^2}$
- second peak also due to kinematic effect

Conclusion

- compare Omnès to KT solutions
- in 0^{-+}, 1^{-+} and 2^{++} many events ($\mathcal{O}(10^5) - \mathcal{O}(10^6)$) are needed for all decay masses
- for 1^{--} strong dependence on decay mass
- at ω mass: small difference
- at ϕ mass: rescattering effects easy to observe