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DVCS Radiative Corrections

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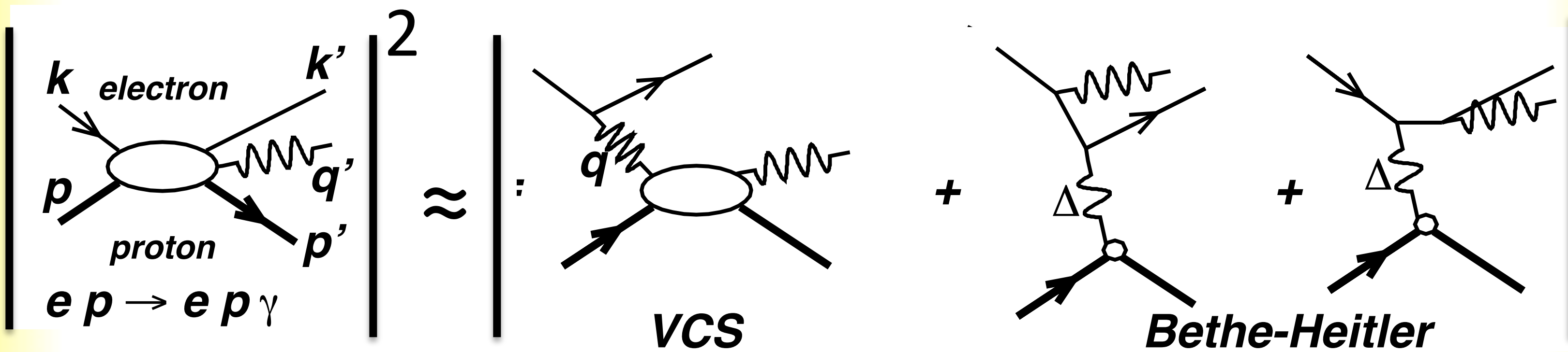
https://hallaweb.jlab.org/dvcslog/DVCS2/235
(and update to come)

M. Vanderhaeghen, et al., Phys Rev C **62,** 025501 (2000)
I.Akushevich and A. Ilyich, Phys Rev D **85**, 053008 (2012)
A. Afanasev, I. Akushevich, et al, Phys Rev D **66** (2002) (EXCLURAD)

The Born Approximation

- $d\sigma(e,e')$ =



- BUT:
  - An accelerating charge **always** radiates
  - The probability of scattering without additional radiation is ***0***.
- Radiative corrections **not** small
  $\sim \alpha \ln(Q^2/m_e^2) \sim 20\%$

Virtual Radiative Corrections

Vertex, Self-Energy, Vacuum-Polarization corrections to BH

Vertex, Vacuum-Polarization corrections to VCS

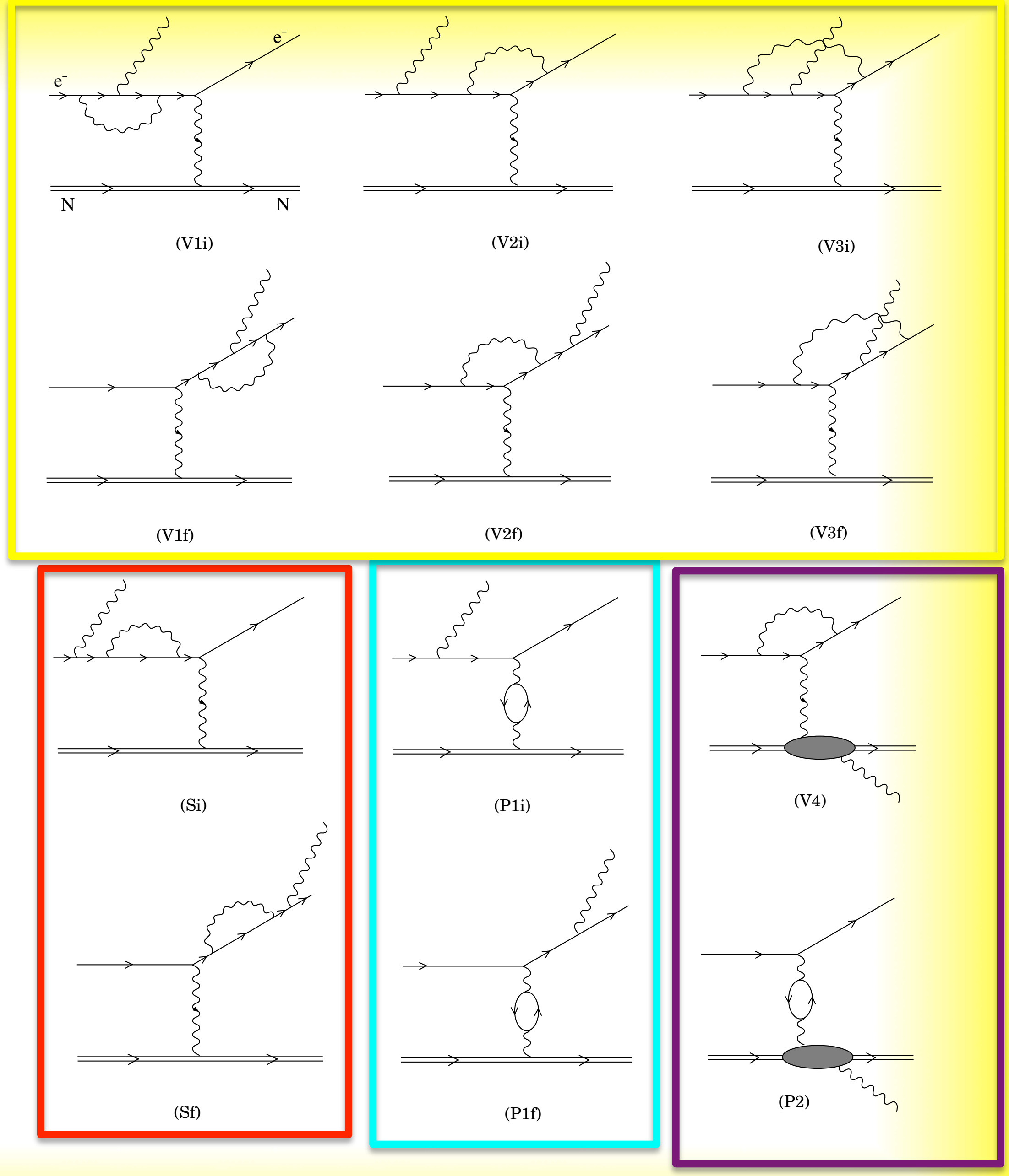


FIG. 2. First order virtual photon radiative corrections to the $ep \to ep\gamma$ reaction.

Real Radiative Corrections

BH

VCS

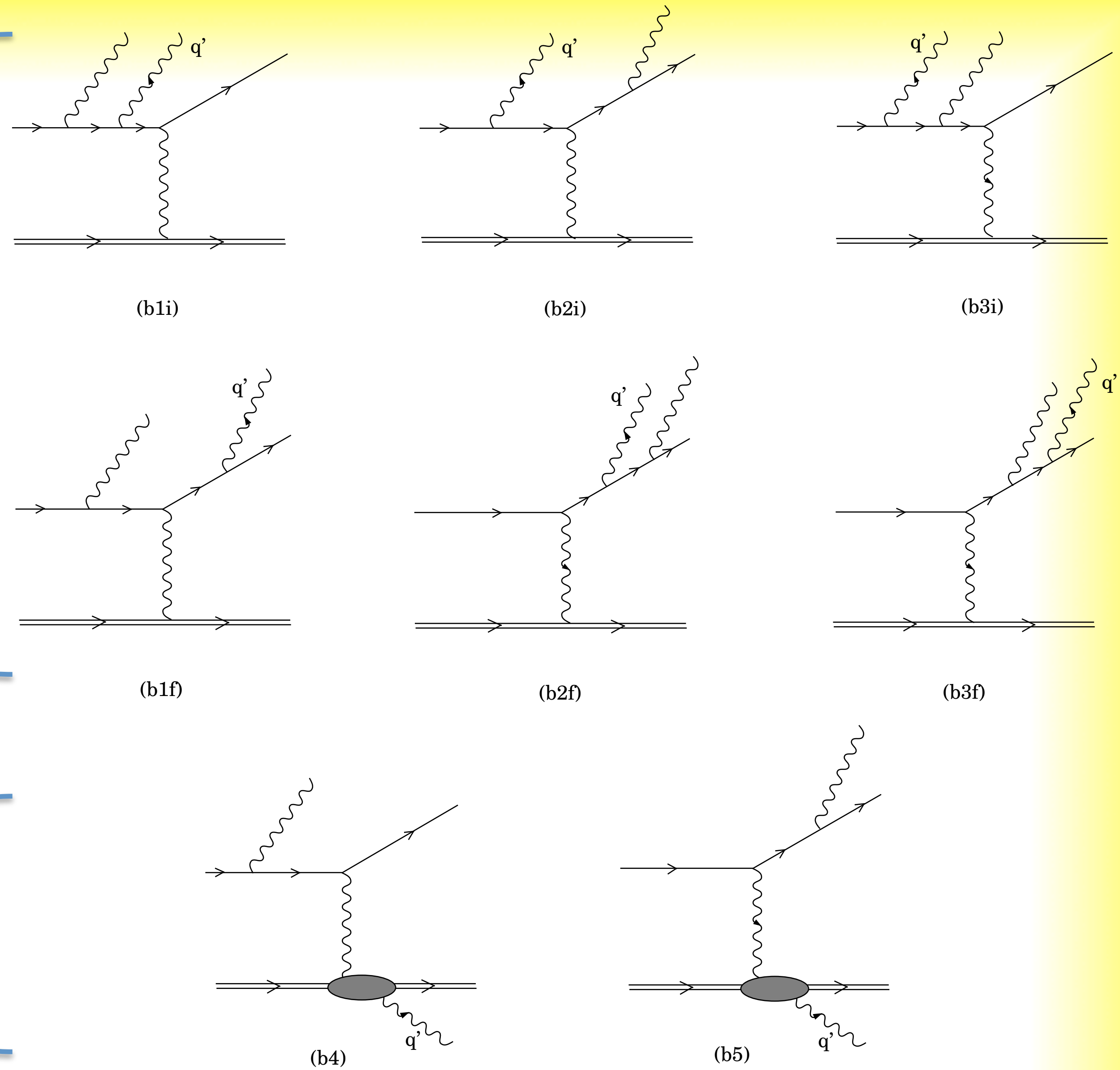


FIG. 3. First order soft photon emission contributions to the $ep \to ep\gamma$ reaction.

UV Divergences of Virtual Corrections

- *e.g.* vertex graph on BH initial state radiation *(V1i):*

$$M_{v1i} \sim \int d^4 l \frac{\gamma^{\alpha}\left(l\cdot\gamma\right)\left(\varepsilon\cdot\gamma\right)\gamma_{\alpha}}{l^2\left(l^2-2l\cdot k\right)\left(l^2-2l\cdot k-2k\cdot q\right)} \xrightarrow[l^2\to\infty]{} \int \frac{dl^2}{l^2}$$

- Q.E.D. is renormalizable! Infinities are tamable.
  - To each order in perturbation theory, redefine Fields, Masses, and Couplings
  - $A^{\mu}_{\text{Bare}} = Z_3^{1/2} A^{\mu}_{\text{Physical}}$ $\quad\psi_{\text{Bare}} = Z_2^{1/2}\psi_{\text{Physical}}$
    $m_{\text{Bare}} = Z_m\, m$ $\quad e_{\text{Bare}} = Z_g\, e_{\text{Physical}}$
  - This cancels all UV divergences, but creates IR-divergences

IR Divergence of Real Radiative Correction

- *e.g.* VCS Bremsstrahlung graphs

$$M_{b4}+M_{b5}\sim\bar{u}(k')\left[\gamma^{\mu}\frac{1}{(k-l)\cdot\gamma-m}\left(\gamma\cdot\varepsilon_f^*(l)\right)+\left(\gamma\cdot\varepsilon_f^*(l)\right)\frac{1}{(k'+l)\cdot\gamma-m}\gamma^{\mu}\right]u(k)\left[\frac{-1}{(k-l-k')^2}H_{\mu\nu}\varepsilon_f^*(q')\right]$$

$$\sim\bar{u}(k')\gamma^{\mu}u(k)\left[\frac{2k\cdot\varepsilon_f^*(l)}{-2k\cdot l-m^2}+\frac{2k'\cdot\varepsilon_f^*(l)}{2k'\cdot l-m^2}\right]\left[\frac{-1}{(k-l-k')^2}H_{\mu\nu}\varepsilon_f^*(q')\right]+\ldots$$

- Must always integrate cross section over photon energies dl less than the experimental resolution:

$$\int\frac{d^3l}{l}\left|M_{b4}+M_{b5}\right|^2\sim d\sigma^{\text{No Radiation}}\left[\int_0^{\Delta E}\frac{dl}{l}\right](\ldots)$$

  - IR-divergence, but cancels order-by-order with IR-divergences of renormalized Virtual corrections.

Integral of Soft Bremsstrahlung

- M. Vanderhaeghen *et al*:
  - Neglect non-factorized part of real radiative correction?
  - Integrate $d^3 \boldsymbol{l}$ in frame `*S*´: $\mathbf{P'} + \boldsymbol{l} = \mathbf{q} + \mathbf{P} - \mathbf{q'} = \boldsymbol{0}$

$$
\begin{aligned}
d\sigma\Big|_{\text{Real Soft}} &= d\sigma\Big|_{\text{`Born}'} \left[1 + \alpha \int^{\Delta M_X^2} \frac{d^3 l}{(2\pi)^2 l} \left(\frac{k'^{\mu}}{k' \cdot l} - \frac{k^{\mu}}{k \cdot l} \right) \left(\frac{k'_{\mu}}{k' \cdot l} - \frac{k_{\mu}}{k \cdot l} \right) \right] \\
&= d\sigma\Big|_{\text{`Born}'} \left[1 + \delta_R(\Delta M_X^2) + \text{divergent integral cancelled by virtual terms} \right]
\end{aligned}
$$

Finite part of Real Radiative Correction

$$\delta_R = \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_S)^2}{\tilde{E}_e\tilde{E}'_e}\right)\left[\ln\frac{Q^2}{m_e^2}-1\right]-\frac{1}{2}\ln^2\frac{\tilde{E}_e}{\tilde{E}'_e}+\frac{1}{2}\ln^2\frac{Q^2}{m_e^2}-\frac{\pi^2}{3}+\mathrm{DiLog}\left(\cos^2\frac{\tilde{\theta}_e}{2}\right)\right\}$$

- with

$$\Delta E_S = \frac{M_X^2 - M^2}{\sqrt{M_X^2}}$$

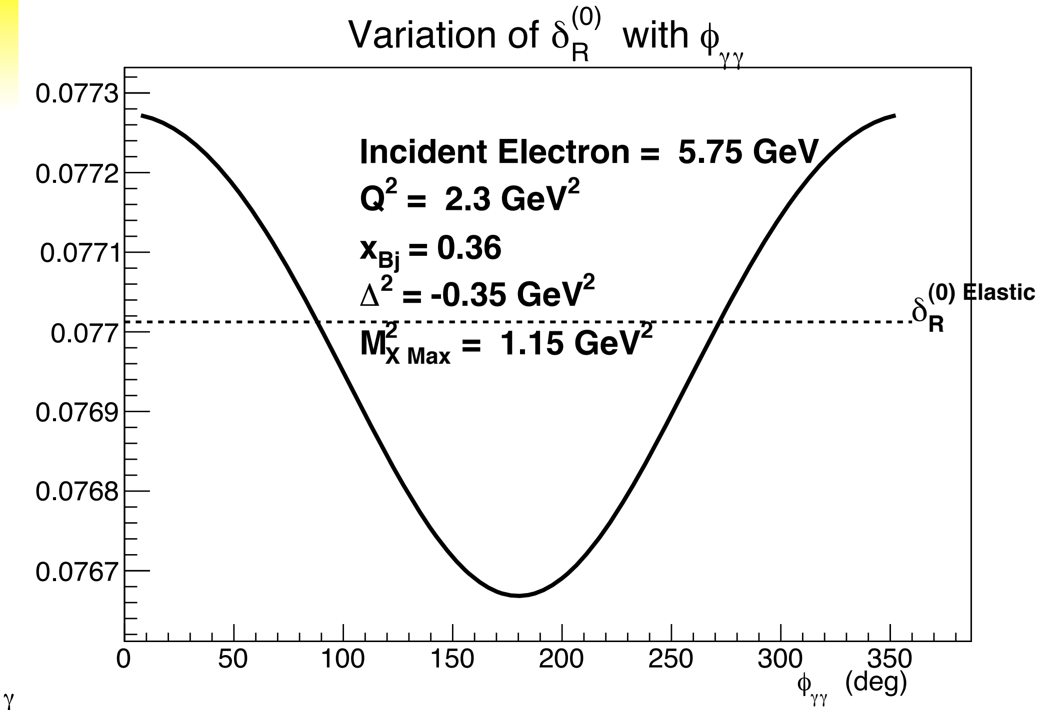
$$\tilde{E}_e \approx \frac{k\cdot(P+q-q')}{\sqrt{M_X^2}} = \frac{M}{\sqrt{M_X^2}}\left(E_e^{\mathrm{Lab}} - \frac{Q^2}{2M} - \frac{k\cdot q'}{M}\right)$$

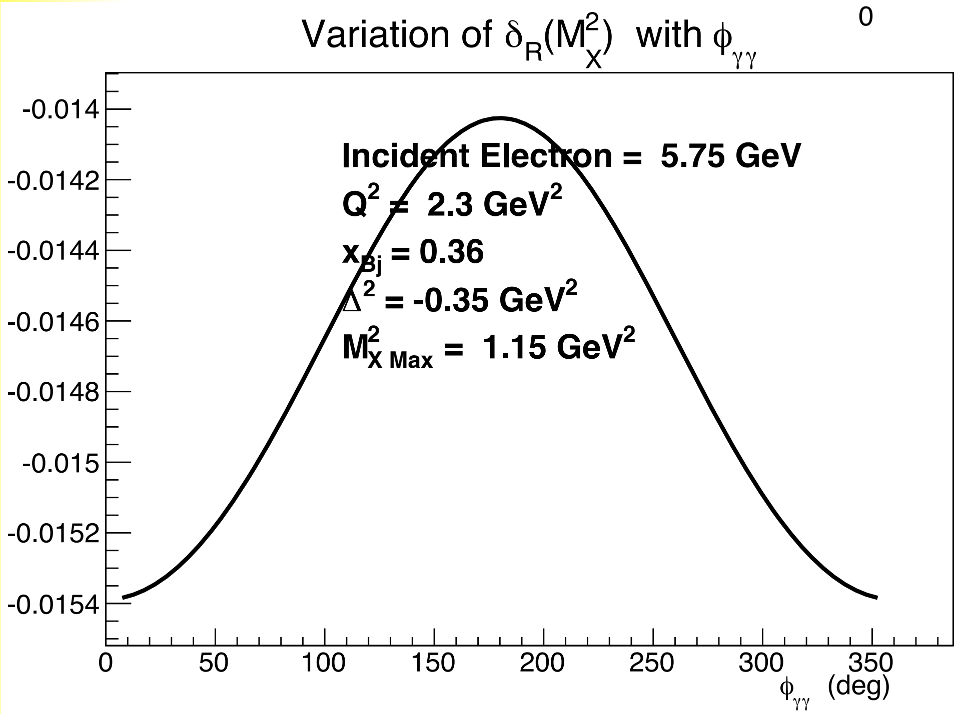
$$\tilde{E}_e \approx \frac{k'\cdot(P+q-q')}{\sqrt{M_X^2}} = \frac{M}{\sqrt{M_X^2}}\left(E_e^{'\mathrm{Lab}} + \frac{Q^2}{2M} - \frac{k'\cdot q'}{M}\right)$$

$$\sin^2\frac{\tilde{\theta}_e}{2} = \left[\frac{\left(E_eE'_e\right)^{\mathrm{Lab}}}{\left(\tilde{E}_e\tilde{E}_e\right)_S}\right]\sin^2\frac{\theta_e^{\mathrm{Lab}}}{2}$$

- All variables are external:
- At detector, not vertex
- Soft approx.: $Q^2_{\text{vertex}} \approx Q^2_{\text{detected}}$

$\phi_{\gamma\gamma}$ dependence of δ_R





E00-110
Kin-3
Variations
$< \pm 0.005$

Exponentiation

$$(1+\delta_R) \Rightarrow e^{+\delta_R(M_X^2)} = \left(\frac{\Delta E_S}{\tilde{E}_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]} \left(\frac{\Delta E_S}{\tilde{E}'_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]} \; e^{\delta_R^{(0)}}$$

- "Exact" (soft-radiation) result
  - No peaking approximation
- $\delta_R{}^{(0)}$ depends weakly on $M_X{}^2$ cutoff,
  - set $M_X{}^2 \rightarrow M^2$ in definition of $\delta_R{}^{(0)}$

$$\delta_R^{(0)} = \frac{\alpha}{\pi}\left\{-\frac{1}{2}\ln^2\frac{\tilde{E}_e}{\tilde{E}'_e} + \frac{1}{2}\ln^2\frac{Q^2}{m_e^2} - \frac{\pi^2}{3} + \mathrm{DiLog}\left(\cos^2\frac{\tilde{\theta}_e}{2}\right)\right\}$$

- Consistency requires exponentiation of virtual radiative corrections also.

Peaking Approximation

- ~~Generate soft photon in 4π phase space~~
- Generate soft photons only parallel to **k** or **k'**

$$\left(\frac{\Delta E_S}{\tilde{E}_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\left(\frac{\Delta E_S}{\tilde{E}'_e}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]} =? \left(\frac{\Delta E_{\mathrm{Inc}}}{E_{e,\mathrm{Inc}}}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}\left(\frac{\Delta E_{\mathrm{Scatt}}}{E_{e,\mathrm{Scatt}}}\right)^{\frac{\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2}-1\right]}$$

- Differential distribution $I(E,\Delta E,\delta)d(\Delta E)=\delta\left(\frac{\Delta E}{E}\right)^{\delta}\frac{d\Delta E}{E}$
  - Generate uniform deviate r in [0,1]
    $\Delta E = E\, r^{1/\delta}$

Analytic Approximation to Virtual Radiative Corrections

- These formulae do not include self-energy and proton radiation terms included in Vanderhaeghen *et al* code. 2-4% discrepancy from full calculations
- Independent of structure of VCS amplitude
- VCS ≠ BH

$$\boxed{\frac{1}{2}\delta_{\text{vertex}}^{\text{VCS}} = \frac{\alpha}{4\pi}\left\{-\left[\ln\frac{Q^2}{m_e^2}\right]^2 + \frac{\pi^2}{3} - 4 + 3\ln\frac{Q^2}{m_e^2}\right\} = \frac{1}{2}\delta_{\text{vertex}}^{\text{Elastic}}}$$

$$\boxed{\frac{1}{2}\delta_{\text{vertex}}^{\text{BH}} = \frac{\alpha}{4\pi}\left\{-\left[\ln\frac{Q^2}{m_e^2}\right]^2 + \frac{\pi^2}{3} - 6 - 2\ln\frac{Q^2}{m_e^2}\right\}}$$

Vacuum Polarization

- VCS:

$$\Pi(Q^2) = \frac{\alpha}{3\pi}\left\{\ln\frac{Q^2}{m^2} - \frac{5}{3}\right\}$$

- BH

$$\Pi(-t) = \frac{\alpha}{3\pi}\left\{\ln\frac{-t}{m^2} - \frac{5}{3}\right\}$$

Correction to Cross section does not factorize (P. Guichon version of code)

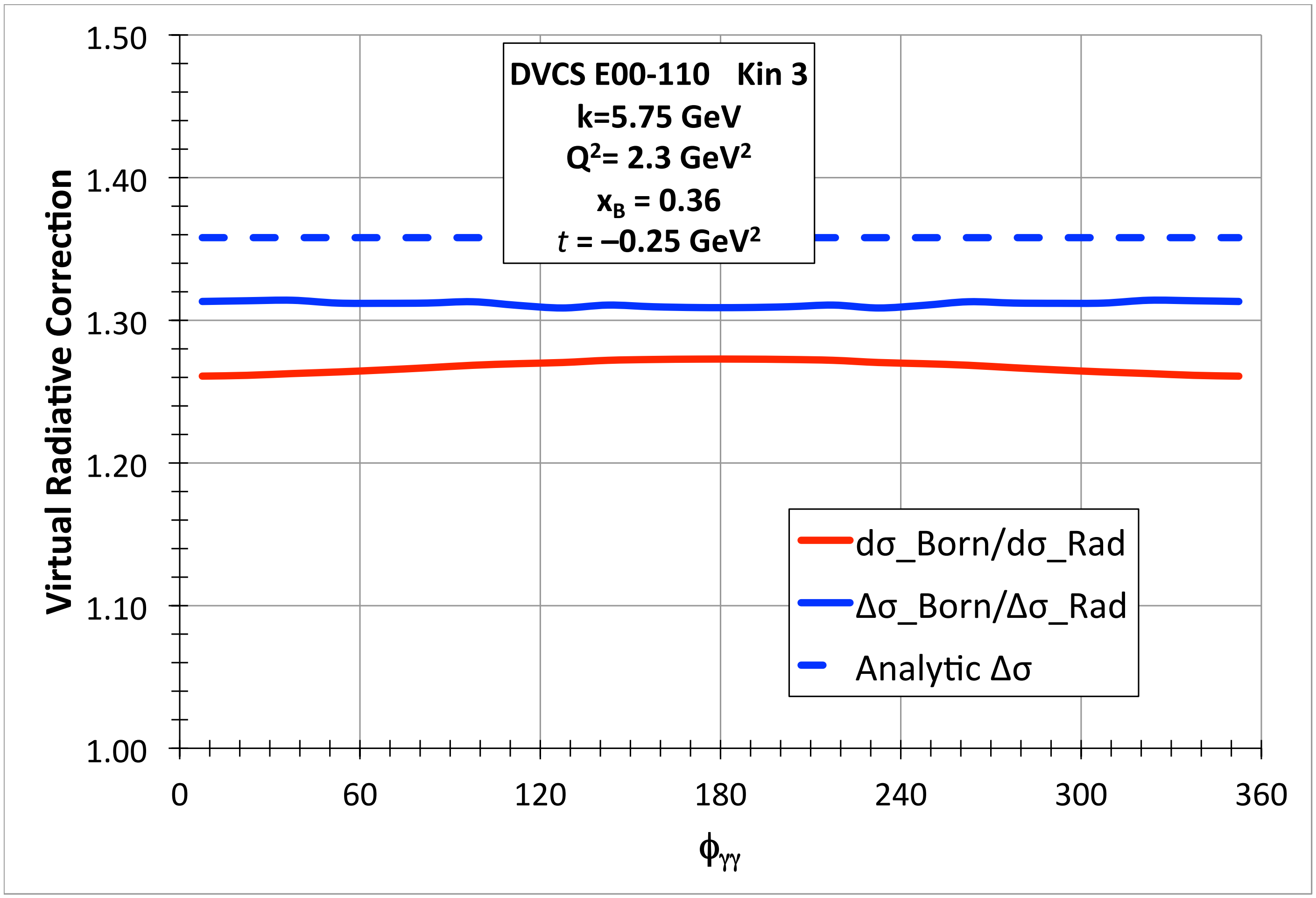
$$d\sigma = d\sigma^{\mathrm{VCS}} + d\sigma^{\mathrm{BH}} + \left[VCS^{\dagger}BH\right]$$

$$\Rightarrow d\sigma^{\mathrm{VCS}}\left[1+\frac{\delta^{\mathrm{VCS}}_{\mathrm{vertex}}/2}{1-\Pi(Q^2)}\right]^2 + d\sigma^{\mathrm{BH}}\left[1+\frac{\delta^{\mathrm{BH}}_{\mathrm{vertex}}/2}{1-\Pi(-t)}\right]^2 + \left[VCS^{\dagger}BH\right]\left[1+\frac{\delta^{\mathrm{VCS}}_{\mathrm{vertex}}/2}{1-\Pi(Q^2)}\right]\left[1+\frac{\delta^{\mathrm{BH}}_{\mathrm{vertex}}/2}{1-\Pi(-t)}\right]$$

- Each term, VCS, BH, Interference,
  - different radiative corrections.
- This form differs slightly from the usual correction factor of $(1+\delta_{\mathrm{vertex}}+2\Pi)$

Sample Virtual Radiative Corrections

BH + DVCS (factorized GPD model)



Asymmetries have a ~5% radiative correction

Questions

for M. Vanderhaeghen and P. Guichon, and for us

- Confirm, VCS Rad-corrections are model independent, but different from BH
- How to exponentiate virtual corrections?
  - Publication says $$\frac{e^{\delta(\text{vertex+self energy})}}{(1-\Pi)^2}$$
  - Try to extract δ(vertex + self-energy) from code
- How to publish cross sections?
  - Calculate helicity dependent cross section assuming pure $VCS^{\dagger}BH$ interference
  - Calculate average correction to unpolarized cross section
    - Publish correction
    - Theorists/GPD-fitters must undo our correction, and then apply separate corrections to $|DVCS|^2$, $|BH|^2$ and Interference terms.