PbF_2 Calorimeter Resolution Simulation

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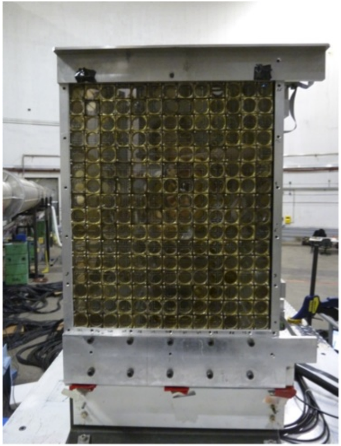
Outline

Calorimeter Properties



13x16 grid of 3x3x18 cm^3 Lead Fluoride (PbF_2) crystals

- Purely Cherenkov medium
- $\rho = 7.77$ g/cm^3
- $X_0 = 0.93$ cm
- $n = 1.8$

Motivation

What is the major contributor to the resolution?

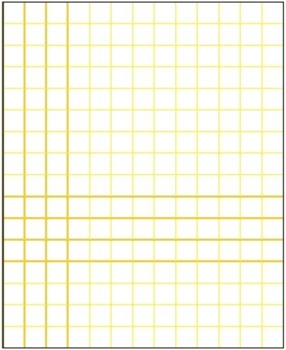
- $\frac{\sigma}{E} \approx a \oplus \frac{b}{\sqrt{E}}$

What is the origin of the calorimeter response line shape?

What is the total light output? (collection efficiency)

- 1-2 photons per MeV of incident particle

Simulation



Simple model (no wrappings, PMTs, etc.)

10k events of e^- for 1-8 GeV beam

2 values extracted:

- Visible energy
- Total path length

Visible Energy

Perfect world: all incident energy dumped into calorimeter and converted into signal

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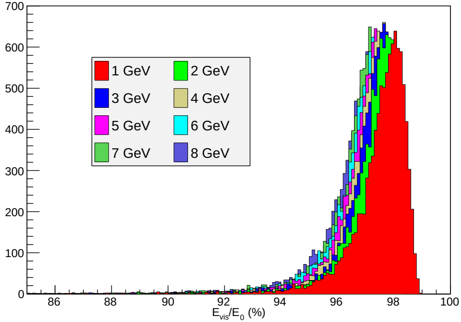
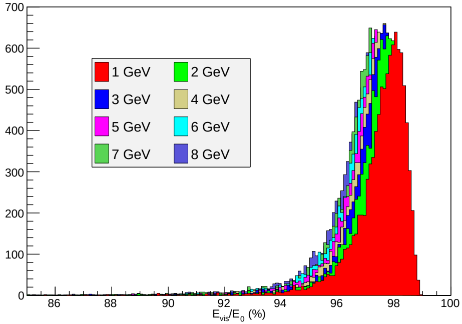


Visible Energy

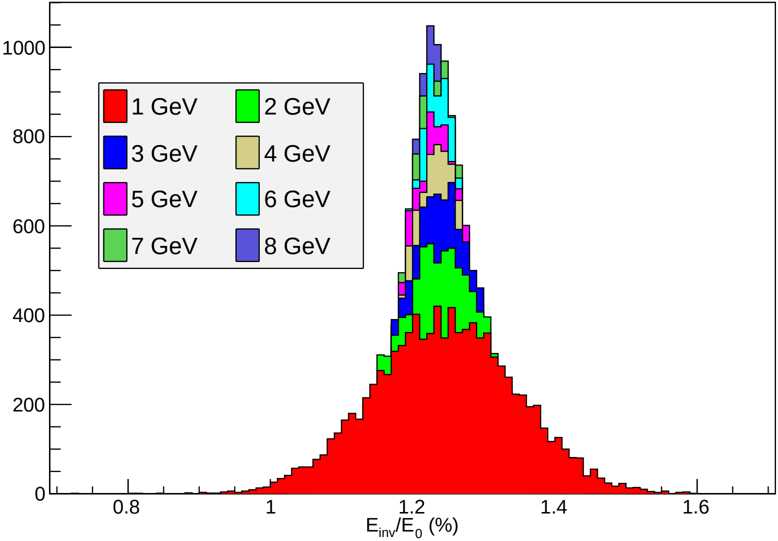
Perfect world: all incident energy dumped into calorimeter and converted into signal

Real world: 3 contributions to signal fluctuations

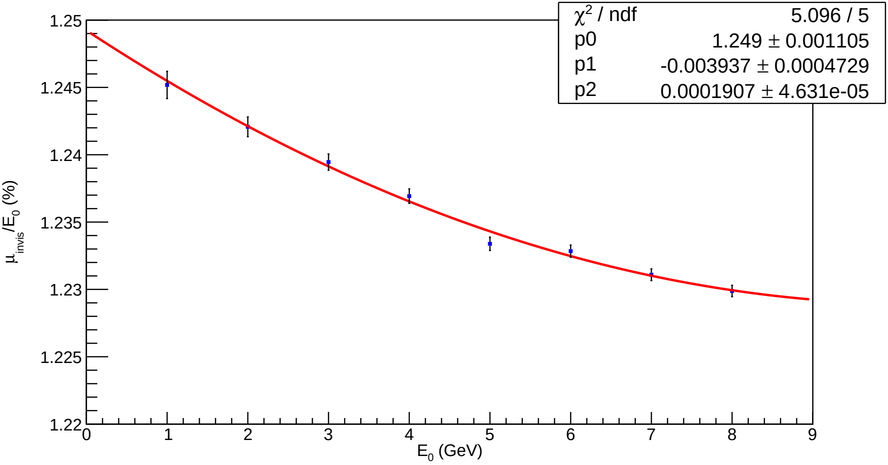
- Non-radiating particles (invisible)
- Escaping particles
- Fluctuations in light detection



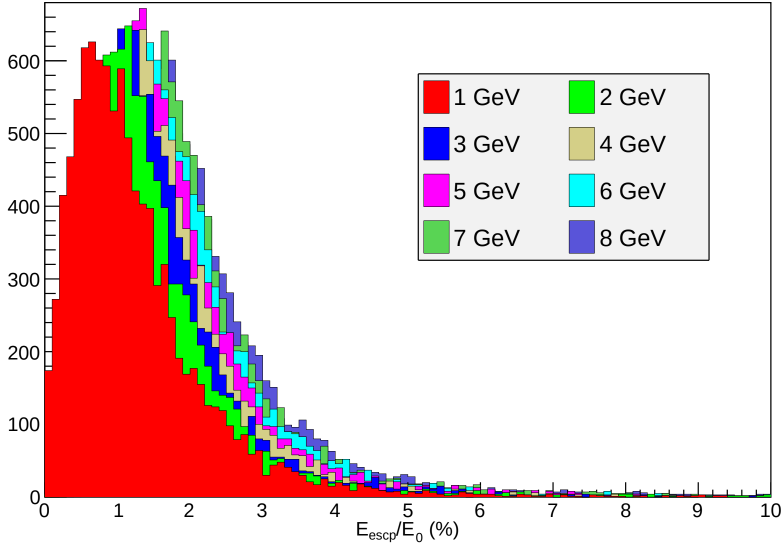
Invisible Energy



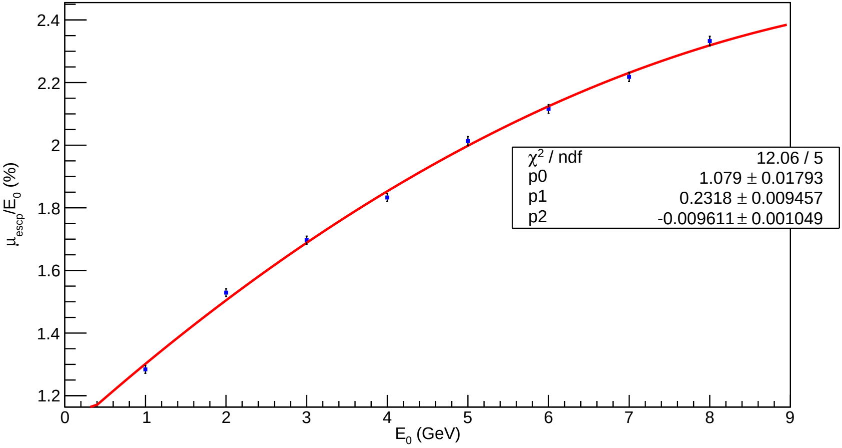
Invisible Energy



Escaping Energy



Escaping Energy



Light Output

Used Frank-Tamm formula to find rough estimate of number of Cherenkov photons:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}x\mathrm{d}\lambda} = \frac{2\pi\alpha Z^2}{\lambda^2}\sin^2\theta_c$$

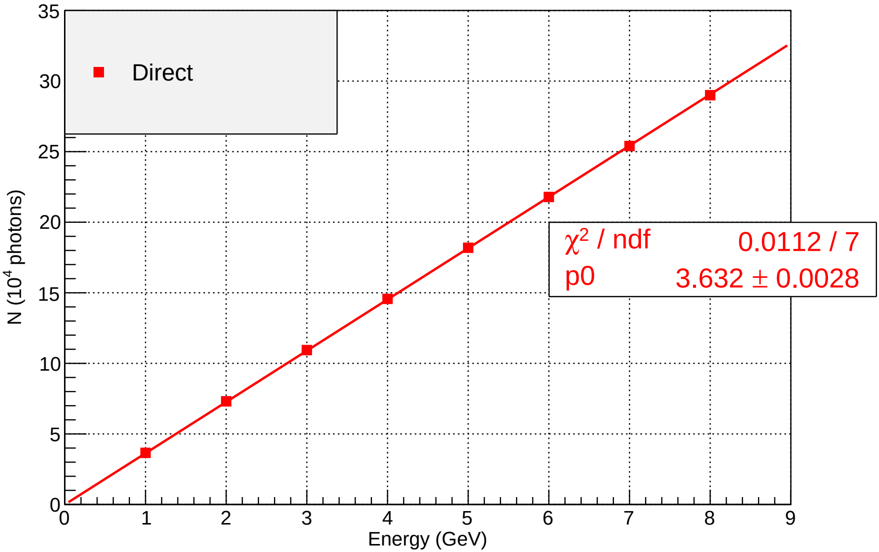
Solving for photons per unit length:

$$\frac{N}{L} \approx 400\frac{\text{photons}}{\text{cm}}$$

Light Output

Two methods used to find the total path length of radiating particles:

- Directly, using Geant4 to sum path lengths
- Indirectly, using Bethe-Bloch equation and mean deposited energy


Direct
χ^2 / ndf 0.0112 / 7
p0 3.632 ± 0.0028
N (10^4 photons)
Energy (GeV)

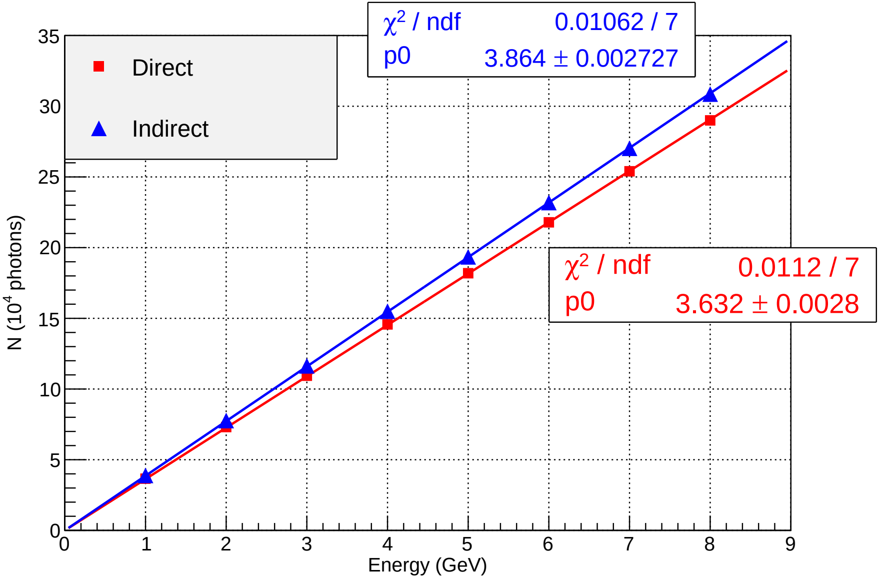
Indirect Path Length

The stopping power of a compound can be approximated with the following sum and table

$$\frac{1}{\rho}\frac{\mathrm{d}E}{\mathrm{d}x} = \sum_i \frac{\omega_i}{\rho_i}\frac{\mathrm{d}E}{\mathrm{d}x}\bigg|_i$$

Element	Mass Fraction (ω)	$\frac{1}{\rho}\frac{dE}{dx}$
Lead	0.845	1.122
Fluorine	0.155	1.676

$$\frac{\mathrm{d}E}{\mathrm{d}x} \approx 9.36\frac{MeV}{cm} \rightarrow L \approx \frac{E_\mu/MeV}{9.36}\ cm$$



Direct
Indirect
χ2 / ndf 0.01062 / 7
p0 3.864 ± 0.002727
χ2 / ndf 0.0112 / 7
p0 3.632 ± 0.0028
N (10^4 photons)
Energy (GeV)
0
5
10
15
20
25
30
35
0
1
2
3
4
5
6
7
8
9

Light Output

6% agreement, and N is linear with incident energy.

Should expect ~ 37 photons per MeV of incident energy.

Past observations suggest only one to two photoelectrons detected per MeV $\rightarrow$ collection/detection efficiency between 2.5% and 5.5%

Fluctuations in Light Detection

Signal will have an approximate Poisson distribution centered around average $\mu \equiv$ mean number of detected Cherenkov photons

As μ becomes large, signal will be more Gaussian, with $\sigma \sim \sqrt{\mu}$

So statistical contribution to resolution can be written as

$$\frac{\sigma_{Poisson}}{E} \propto \frac{1}{\sqrt{E}}$$

Resolution

Total resolution expected to be of the form

$$\frac{\sigma_{\mathit{Tot}}}{E} = a \oplus \frac{b}{\sqrt{E/GeV}}$$

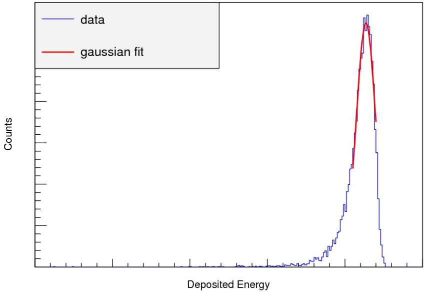
Already shown contribution from Poisson statistics is

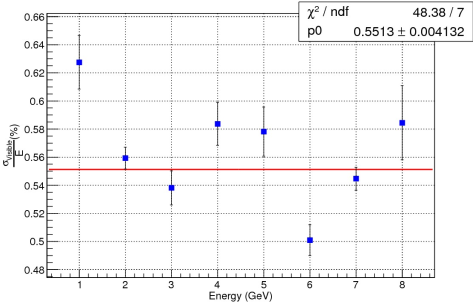
$$\frac{\sigma_{\mathit{Poisson}}}{E} = \frac{b}{\sqrt{E/GeV}}$$

$$\rightarrow \frac{\sigma_{\mathit{Visible}}}{E} = a$$

Resolution

High-energy spectrum of visible energy fitted with Gaussian





$$\frac{\sigma_{Visible}}{E} \approx 0.55\%$$

Resolution

Using the assumption that only one photoelectron is detected per MeV, then the total resolution looks like

$$\frac{\sigma_{Tot}}{E} \approx 0.55\% \oplus \frac{3\%}{\sqrt{E/GeV}}$$

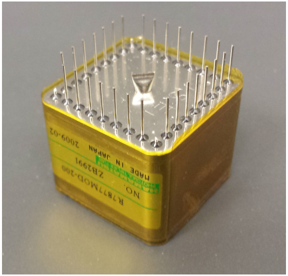
NB: the contribution from $\frac{\sigma_{Visible}}{E}$ will grow with defects in physical calorimeter (*e.g.* cracks, dead material, and calibration fluctuations)

High Quantum Efficiency PMTs

Proposal during commissioning: Swap 16 high QE PMTs and measure resolution in elastic $H(e, e', p)$

Will be swapped back out for calibration

Expecting double the light yield (at a possible sacrifice of lifetime)



Summary

- Line shape determined by escaping and non-radiating particles
- Escaping partiles biggest contributor to long tail
- Poisson fluctuations dominate resolution for incident energy $E < 25$ GeV
- Better light collection and QE can significantly improve resolution