

Proton spectral function from the $Ti(e,e'p)$ cross sections

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for the E12-14-012 experiment**

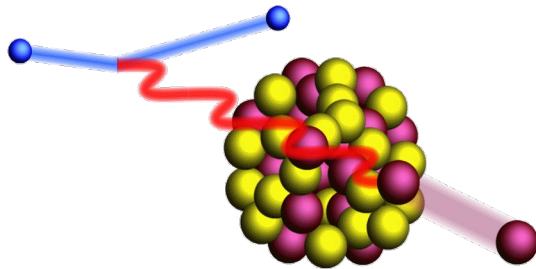
JLab Hall A Summer Collaboration Meeting, June 16–17, 2022

E12-14-012: (e, e') and $(e, e'p)$ on Ar and Ti

Aim: Obtaining the experimental input indispensable to construct the argon spectral function, thus paving the way for a reliable estimate of the neutrino cross sections in DUNE. In addition, stimulating a number of theoretical developments, such as the description of final-state interactions.

[Benhar *et al.*, arXiv:1406.4080]

$$E_e = 2.222 \text{ GeV}$$

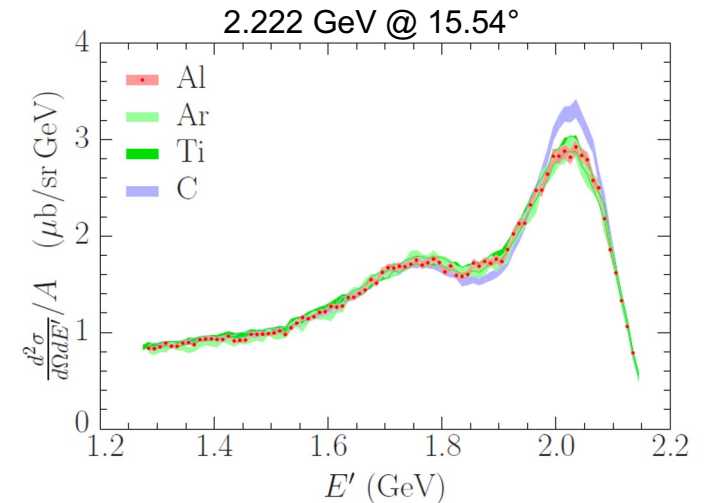


	E'_e (GeV)	θ_e (deg)	$ \mathbf{p}' $ (MeV)	$\theta_{p'}$ (deg)	$ \mathbf{q} $ (MeV)	p_m (MeV)	E_m (MeV)
kin1	1.777	21.5	915	-50.0	865	50	73
kin2	1.716	20.0	1030	-44.0	846	184	50
kin3	1.799	17.5	915	-47.0	741	174	50
kin4	1.799	15.5	915	-44.5	685	230	50
kin5	1.716	15.5	1030	-39.0	730	300	50

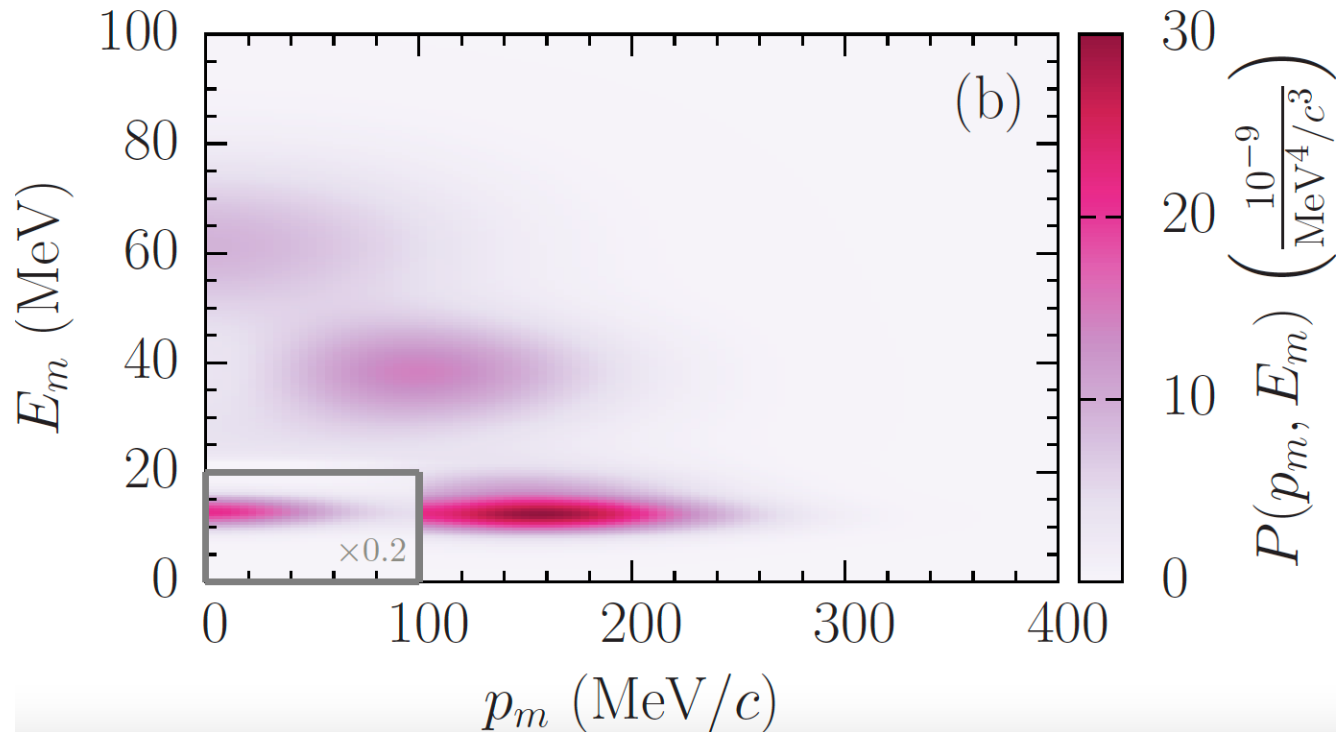
Exploratory analysis of the full dataset

Publications

- ❖ Inclusive cross sections for C and Ti, [Dai *et al.*, PRC 98, 014617 (2018)]
- ❖ Inclusive cross section for Ar, [Dai *et al.*, PRC 99, 054608 (2019)]
- ❖ Inclusive cross section for Al-7075, Ar, C and Ti of all (e, e') data [Murphy *et al.*, PRC 100, 054606 (2019)]
- ❖ Exclusive Ar & Ti cross sections for a single kinematics, $p_m \sim 50\text{--}60$ MeV, $E_m \sim 50\text{--}70$ MeV [Gu *et al.*, PRC 103, 034604 (2021)]
- ❖ Exclusive Ar cross sections for all kinematics, $p_m \sim 50\text{--}350$ MeV/c, $E_m \sim 10\text{--}70$ MeV [Jiang *et al.*, PRD 105, 112002 (2022)]



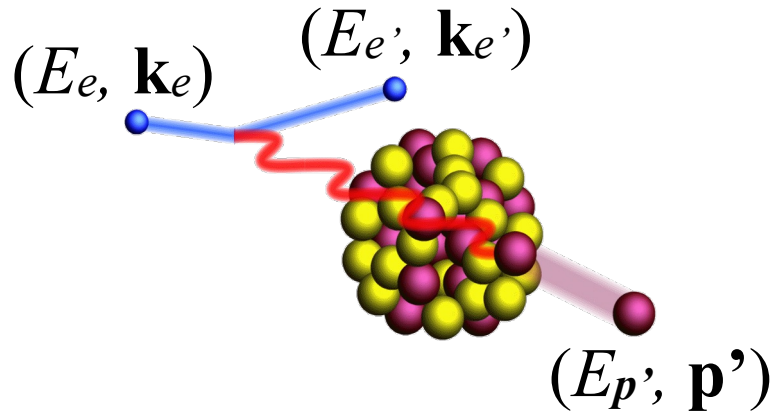
This analysis: extraction of the spectral function



Universal property of the nucleus, independent of the interaction.

Missing momentum \mathbf{p}_m and missing energy E_m

Without final-state interactions,



$$E_e + M - \underline{E_m} = E_{e'} + E_{p'}$$

known

$$\mathbf{k}_e + \underline{\mathbf{p}_m} = \mathbf{k}_{e'} + \mathbf{p}'$$

$E_m - E_{\text{thr}}$ is the excitation energy

$p_m \equiv |\mathbf{p}_m|$ is the initial proton momentum

(e,e'p) cross section

elementary cross section

nuclear transparency

$$\frac{d^6 \sigma_{\text{IA}}}{d\Omega_{e'} dE_{e'} d\Omega_{p'} dE_{p'}} \propto \sigma_{ep} S(p_m, E_m) T(E_{p'})$$

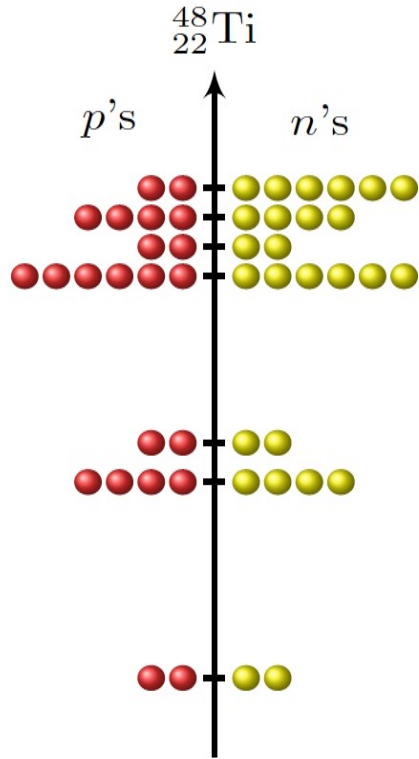
spectral function

The diagram illustrates the components of the (e,e'p) cross section. The central equation is $\frac{d^6 \sigma_{\text{IA}}}{d\Omega_{e'} dE_{e'} d\Omega_{p'} dE_{p'}} \propto \sigma_{ep} S(p_m, E_m) T(E_{p'})$. Three text labels are connected to this equation by blue arrows: 'elementary cross section' points to σ_{ep} , 'nuclear transparency' points to $T(E_{p'})$, and 'spectral function' points to $S(p_m, E_m)$.

Analysis procedure

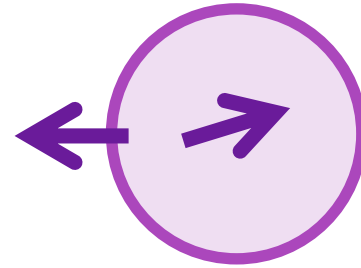
- 1) Extract of the $(e, e'p)$ cross section
- 2) Using σ_{cc1} of de Forest and nuclear transparency, obtain the reduced cross sections as a function of (a) p_m and (b) E_m .
- 3) Find the parameters of the spectral function (*i.e.*, spectroscopic factors) from the fits to the reduced cross sections as a function of p_m .
- 4) Using the priors from Step 3), find the parameters of the spectral function (*i.e.*, spectroscopic factors, peak positions, distribution widths) from the fits to the reduced cross sections as a function of E_m . Correct for transparency.

Test spectral function: 80% mean-field + 20% *NN* correlations



Independent-particle shell model

+



Convolution model of the correlated spectral function

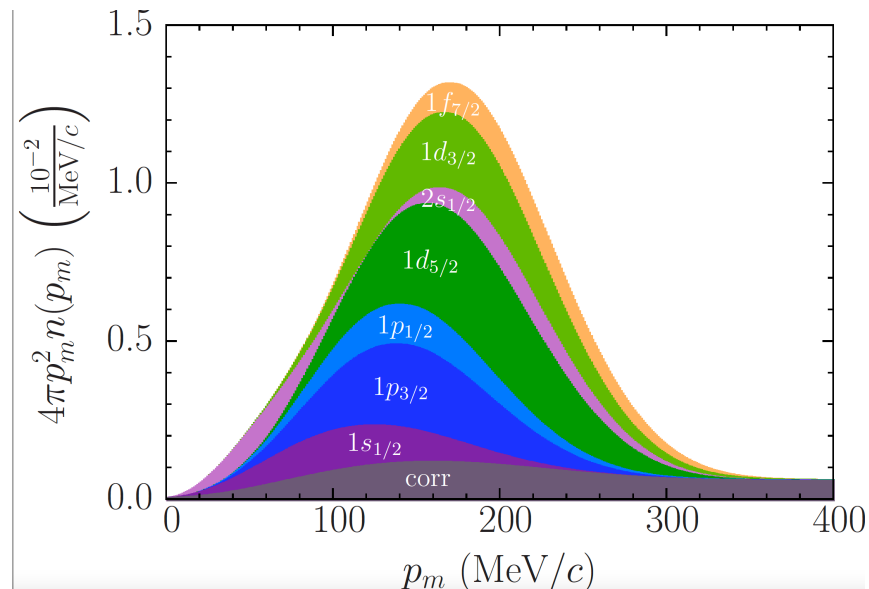
Mean-field part of the spectral function

spectroscopic factor energy distribution

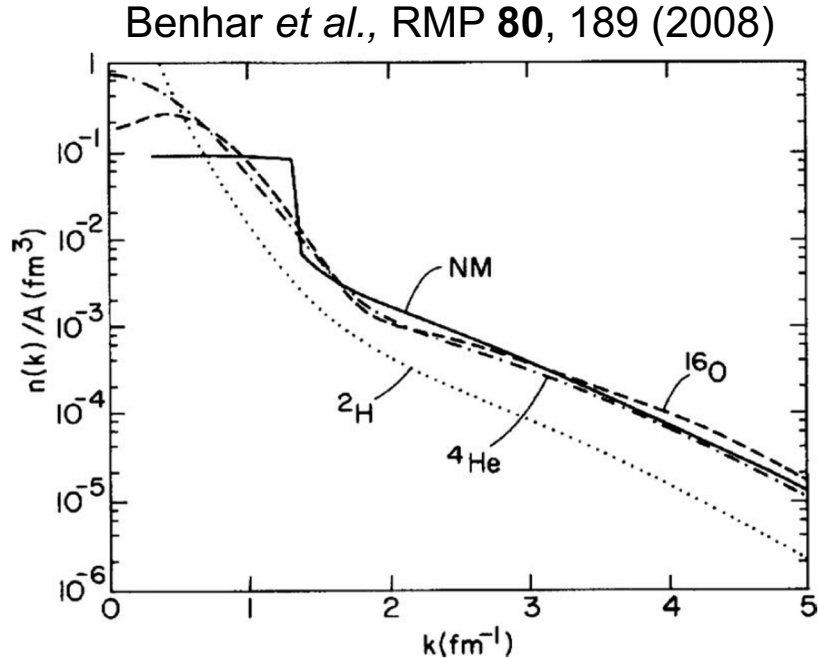
$$P_{\text{MF}}(p_m, E_m) = \sum_{\alpha} S_{\alpha} |\phi_{\alpha}(p_m)|^2 f_{\alpha}(E_m)$$

wave function in momentum space

Relativistic MF calculations by C. Giusti



Correlated part of the spectral function



Ciofi degli Atti and Simula, PRC **53**, 1689 (1996)

- Correlated nucleons form quasi-deuteron pairs, with the relative momentum distributed as in deuteron.
- NN pairs undergo CM motion (Gaussian distrib.)
- Excitation energy of the $(A - 1)$ -nucleons is their kinetic energy plus the pn knockout threshold

ρ_m fit results

Spectroscopic factor normalized by $2j+1$, no transparency correction

1f72 - Spectroscopic factor(2) = 0.78 +/- 0.28

1d32 - Spectroscopic factor(4) = 2.60 +/- 0.22

2s12 - Spectroscopic factor(2) = 1.94 +/- 0.09

1d52 - Spectroscopic factor(6) = 2.34 +/- 1.15

1p12 - Spectroscopic factor(2) = 2.73 +/- 0.05

1p32 - Spectroscopic factor(4) = 5.46 +/- 0.01

1s12 - Spectroscopic factor(2) = 2.10 +/- 0.1-

Correlated part - Spectroscopic factor = 1.64 +/- 0.17

Chi2/ndof = 1.42

Ndof = 683

In the ρ_m fit, only deeply bound states are sensitive to the correlated spectral function.

ρ_m fit results

Spectroscopic factor normalized by $2j+1$, with transparency correction

1f72 - Spectroscopic factor(2) = 0.78 +/- 1.25

1d32 - Spectroscopic factor(4) = 1.15 +/- 0.09

Sum = 19.18 +/- 1.60

2s12 - Spectroscopic factor(2) = 1.96 +/- 0.09

1d52 - Spectroscopic factor(6) = 2.34 +/- 0.96

1p12 - Spectroscopic factor(2) = 2.73 +/- 0.23

1p32 - Spectroscopic factor(4) = 5.24 +/- 0.06

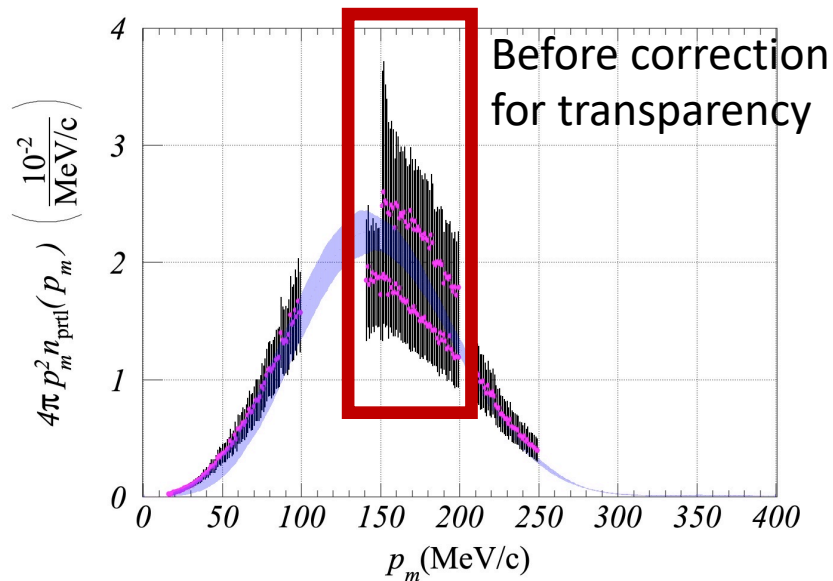
1s12 - Spectroscopic factor(2) = 2.08 +/- 0.10

Corr - Spectroscopic factor = 0.66 +/- 0.17

Transparency corr. for kin2 = 0.75 +/- 0.02

Chi2/ndof = 1.42, Ndof = 683

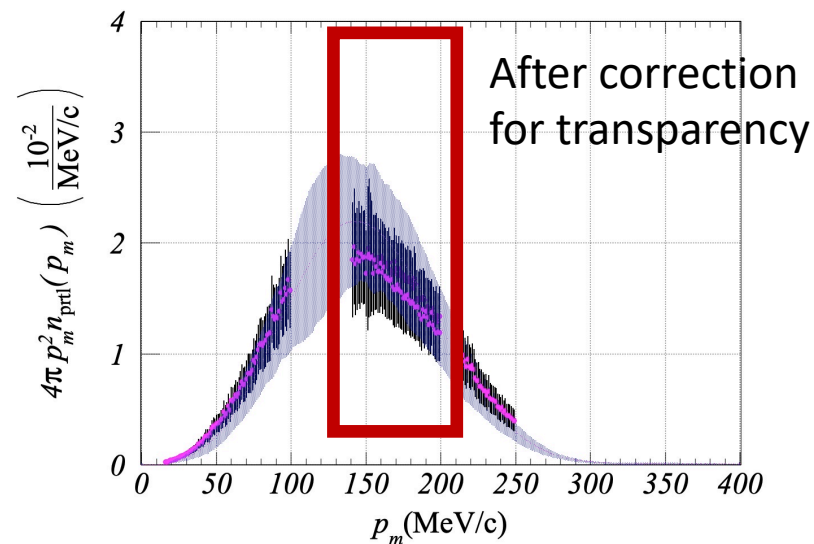
In the ρ_m fit, only deeply bound states are sensitive to the correlated spectral function.



Kinematic 2 has a proton momentum of 1030 MeV/c

Kinematic 3 has a proton momentum of 915 MeV/c

Kin2 data should be corrected for the change in nuclear transparency



E_m fit results

Orbital Strength and Errors

1f72 (2)	1.43	+/-	0.28
1d32 (4)	2.94	+/-	0.50
2s12 (2)	1.89	+/-	0.11
1d52 (6)	2.60	+/-	0.18
1p12 (2)	2.68	+/-	0.06
1p32 (4)	3.53	+/-	0.16
1s12 (2)	2.20	+/-	0.13

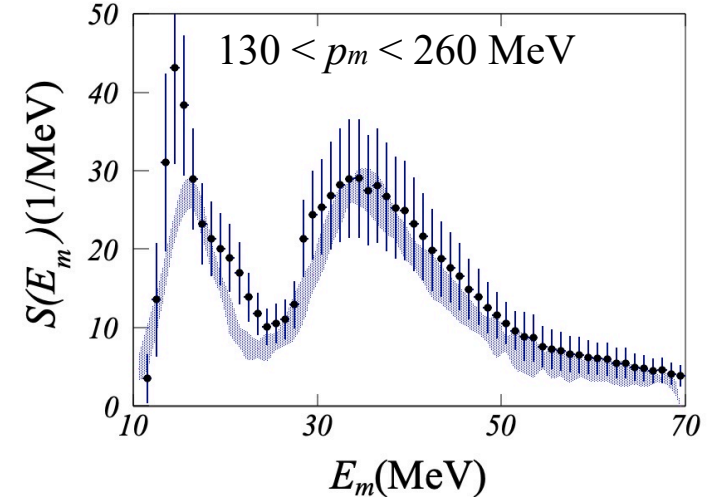
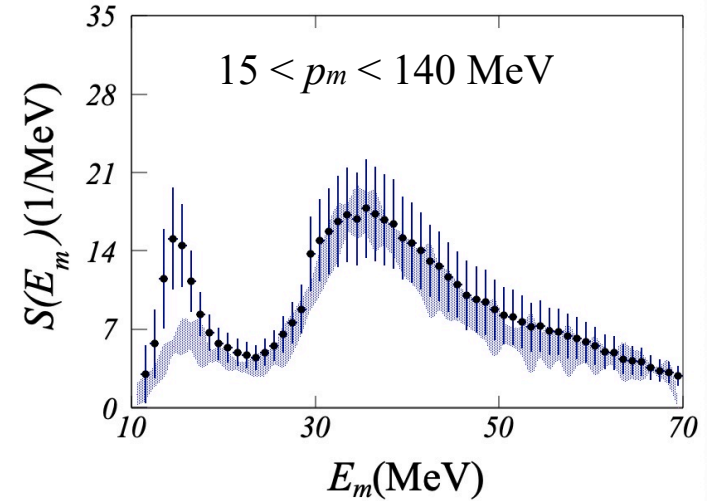
Sum=21.10 +/- 0.81

Corr 0.87+/-0.08

Chi2/ndof = 1.25

Ndof = 125

Valence level – 9.11 MeV



E_m fit results

Orbital Mean and Errors (all in MeV)

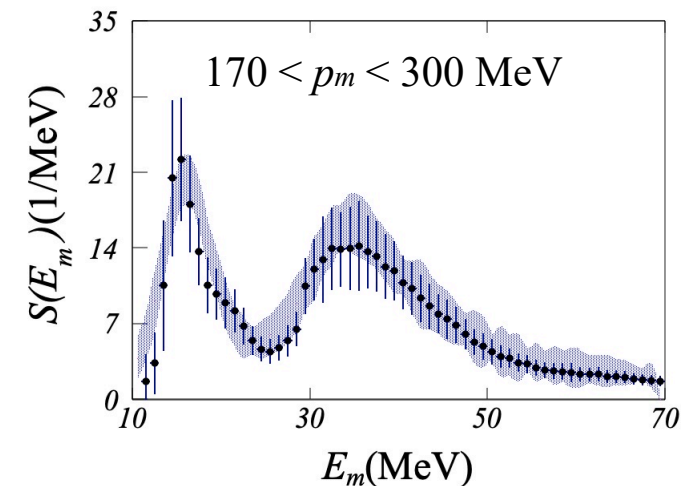
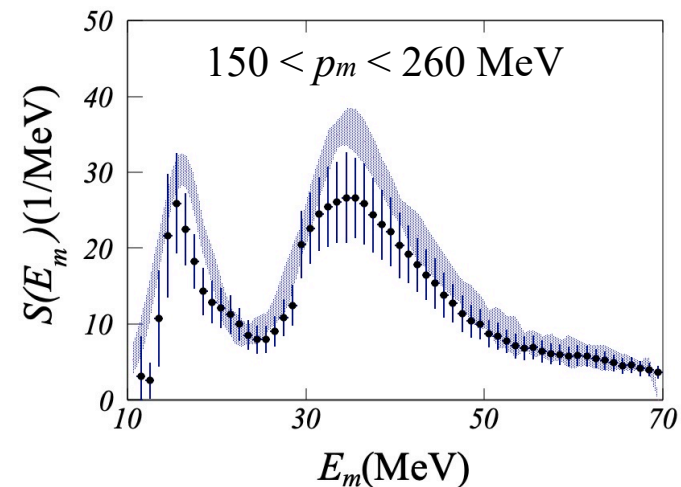
1f72	11.32	+/- 1.65	g.s.
1d32	12.39	+/- 0.40	1.07 MeV
2s12	12.80	+/- 0.95	1.48 MeV
1d52	16.12	+/- 0.29	4.80 MeV
1p12	33.68	+/- 0.37	22.36 MeV
1p32	39.85	+/- 0.86	28.53 MeV
1s12	56.07	+/- 2.69	44.75 MeV

Corr 24.2 MeV

Chi2/ndof = 1.25

Ndof = 125

Valence level – 9.11 MeV,



E_m fit results - priors

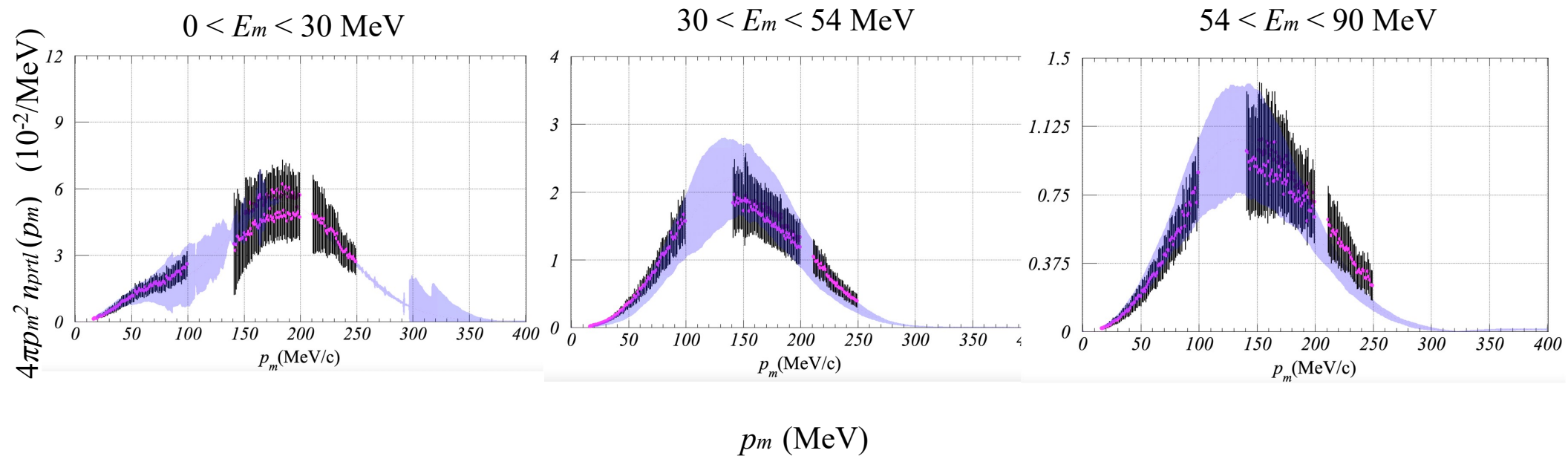
Priors included in the E_m fit as
penalty terms to the Chi^2

1f72	11.31	+/-	0.2	MeV
1d32	12.10	+/-	0.5	MeV
2s12	12.70	+/-	0.5	MeV
1d52	15.36	+/-	0.9	MeV

S0 1d32-1d52 3.7 +/- 2.0 MeV

S0 1p12-1p32 6.1 +/- 1.0 MeV

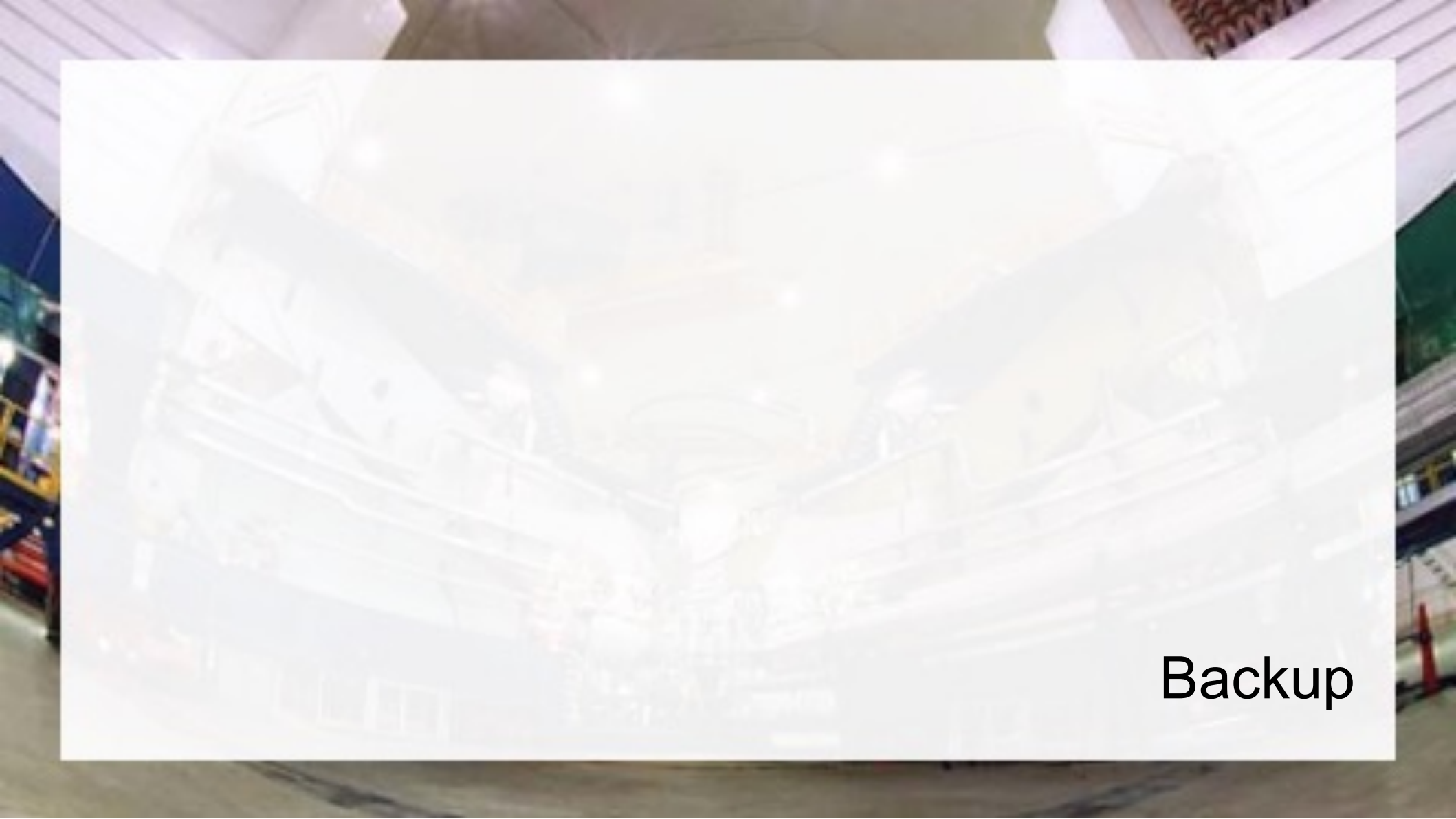
P_m fit results



Data from different kinematics are consistent within uncertainties.

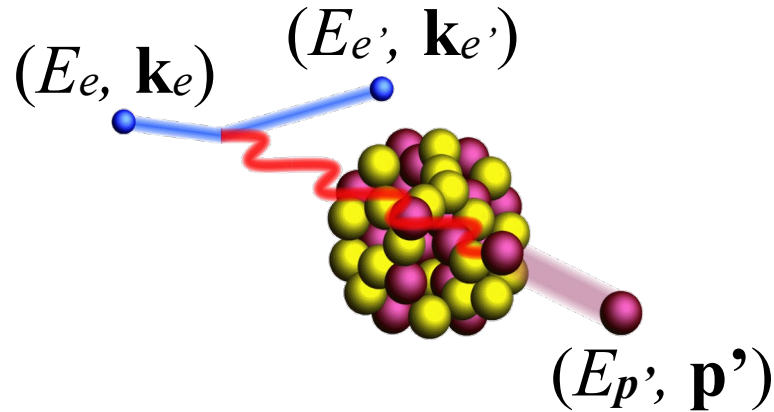
Summary

- ❖ The first, exploratory analysis of the full dataset for Ti, results still preliminary.
- ❖ Reasonable parametrization of the spectral function of ^{48}Ti is found.
- ❖ Comparison with past results is underway.
- ❖ Separation of individual contributions requires improved analysis. Numerous theoretical developments are necessary.
- ❖ Need to work on using this measurement for computing the Ar-n spectral function
- ❖ Paper is in preparation
- ❖ Investigate the transparency measurement and correction



Backup

Missing momentum \mathbf{p}_m and missing energy E_m



$$E_e + M_A = E_e' + E_{p'} + \underline{E_{A-1}^*}$$

known

determined

$$\mathbf{k}_e + 0 = \mathbf{k}_e' + \mathbf{p}' + \underline{\mathbf{p}_{A-1}}$$

In the absence of final state interactions

– $\mathbf{p}_{A-1} = \mathbf{p}_m$ initial proton momentum; $p_m \equiv |\mathbf{p}_m|$

$$E_{A-1}^* = \sqrt{(M_A - M + E_m)^2 + \mathbf{p}_m^2}, \text{ with excitation energy } E_m - E_{\text{thr}}$$