
The Generalized Polarizabilities of the Proton

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On behalf of E12-15-001 Collaboration

 Jefferson Lab



HALL A/C COLLABORATION MEETING 06/16/2022

Content

- **Theoretical Background**
- **VCS Experiment E12-15-001**
- **Analysis work**
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 - **VCS Analysis**
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Polarizabilities

Polarizability:

- A fundamental characteristic of the proton
- Characterizes the nucleon dynamical response to an external electromagnetic field

N BARYONS
(S = 0, I = 1/2)
p, N⁺ = uud; n, N⁰ = udd

p

$$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$

Mass $m = 1.00727646688 \pm 0.00000000009$ u

Mass $m = 938.272081 \pm 0.000006$ MeV [a]

$|m_p - m_{\bar{p}}|/m_p < 7 \times 10^{-10}$, CL = 90% [b]

$|\frac{q_p}{m_p}|/(\frac{q_p}{m_p}) = 1.00000000000 \pm 0.00000000007$

$|q_p + q_{\bar{p}}|/e < 7 \times 10^{-10}$, CL = 90% [b]

$|q_p + q_e|/e < 1 \times 10^{-21}$ [c]

Magnetic moment $\mu = 2.7928473446 \pm 0.00000000008 \mu_N$

$(\mu_p + \mu_{\bar{p}}) / \mu_p = (0.3 \pm 0.8) \times 10^{-6}$

Electric dipole moment $d < 0.021 \times 10^{-23}$ ecm

Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}$ fm³

Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}$ fm³ (S = 1.2)

Charge radius, μp Lamb shift = 0.84087 ± 0.00039 fm [d]

Charge radius, $e p$ CODATA value = 0.8751 ± 0.0061 fm [d]

Magnetic radius = 0.78 ± 0.04 fm [e]

Mean life $\tau > 2.1 \times 10^{29}$ years, CL = 90% [f] ($p \rightarrow$ invisible mode)

Mean life $\tau > 10^{31}$ to 10^{33} years [f] (mode dependent)

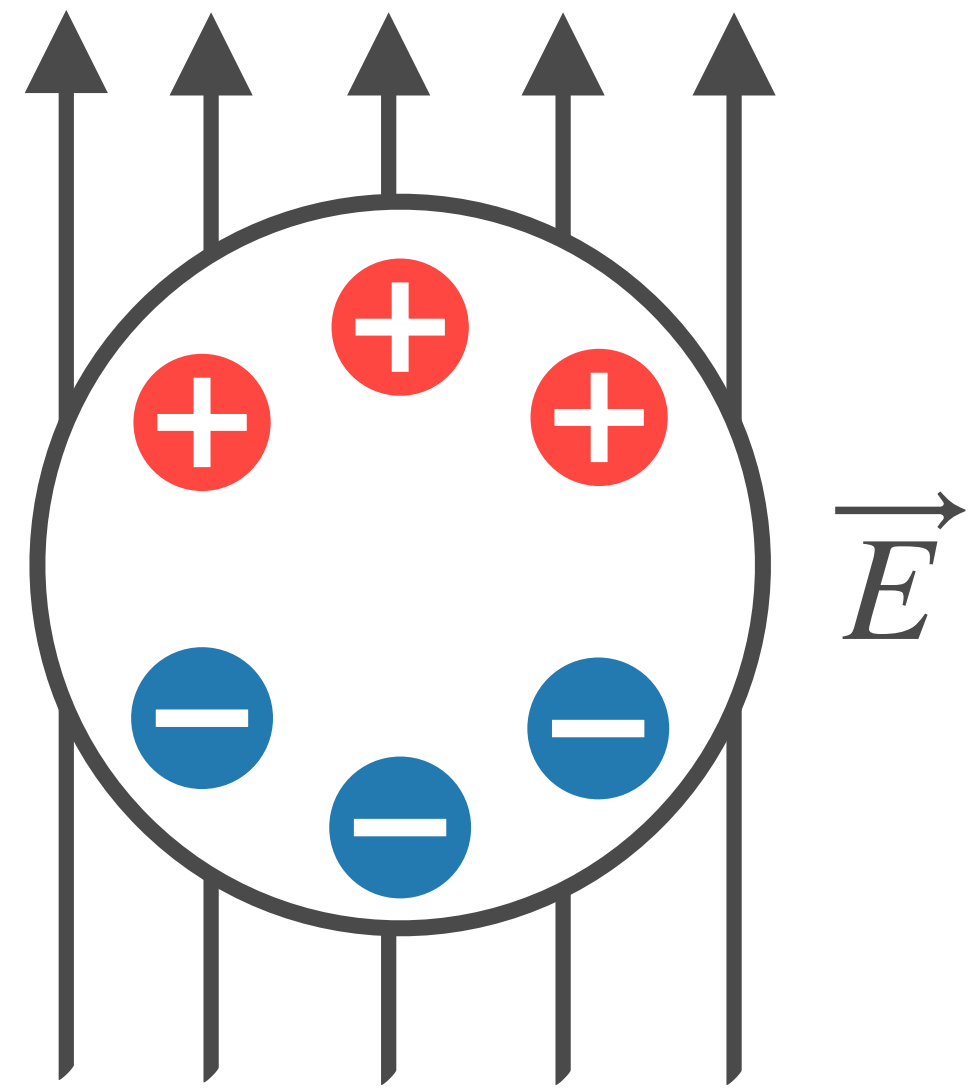
Generalized Polarizabilities (GPs):

- Generalization at finite Q^2 of the polarizability
- Access by Virtual Compton Scattering (VCS)
- 2 scalar and 4 vector GPs
- Fourier transform can map out the spatial distribution density of the polarization induced by an EM field



Scalar GP at the four-momentum transferred squared $Q^2=0$ (RCS limit)

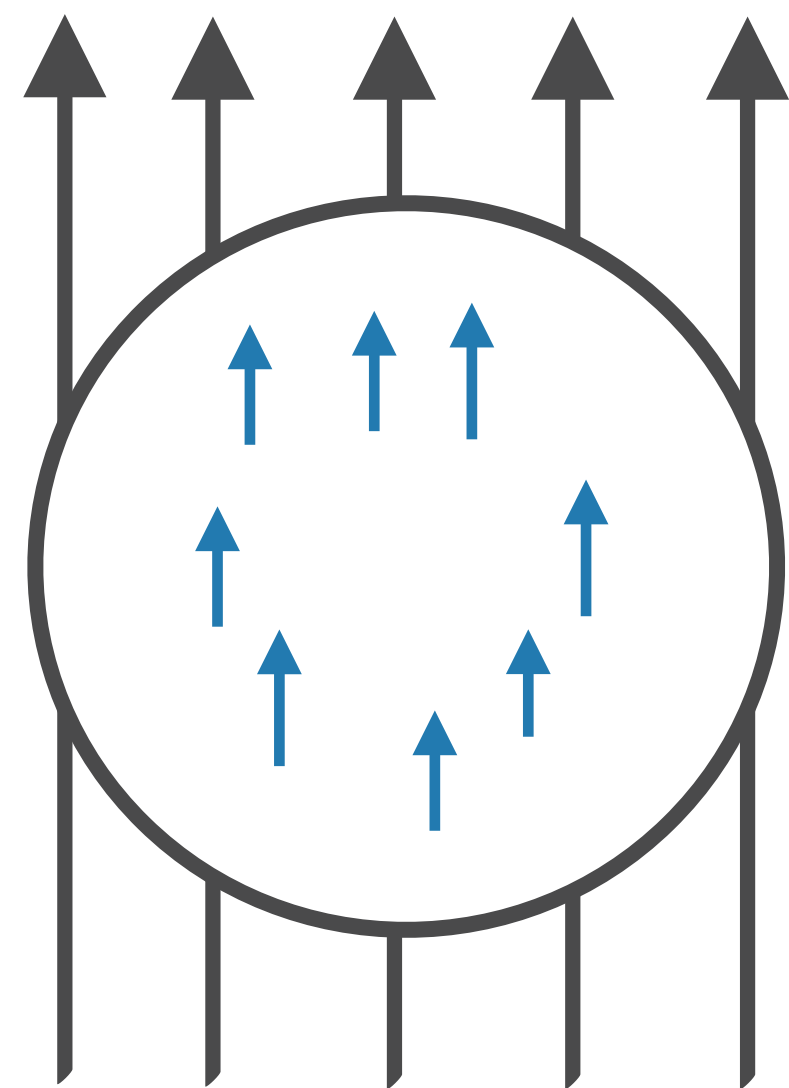
Generalized Polarizabilities



$$\vec{p} = \alpha_E \vec{E}$$

Electric Polarizability

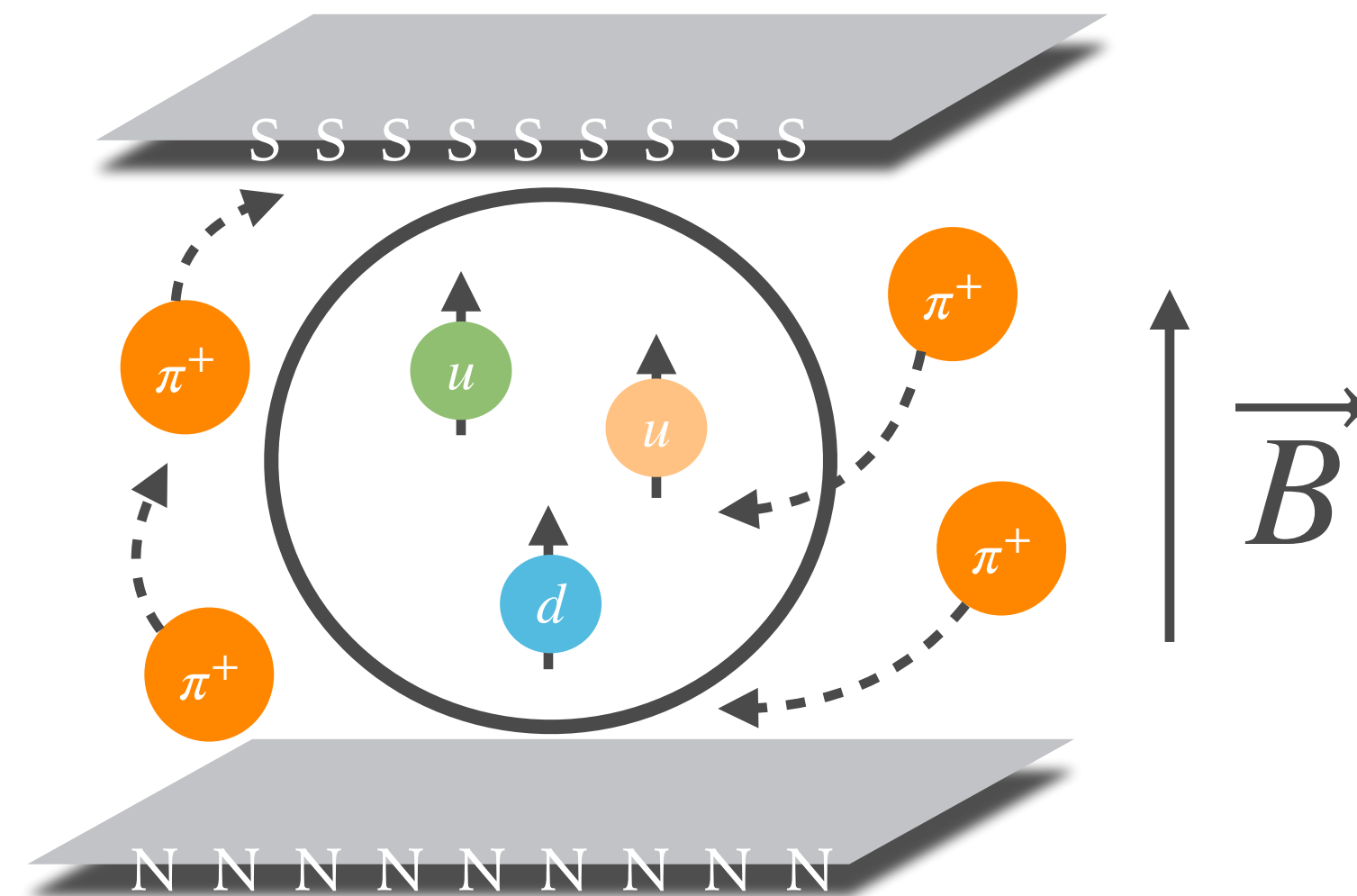
- Electric polarizability α_E reflects the **rigidity** of proton



Magnetic Polarizability

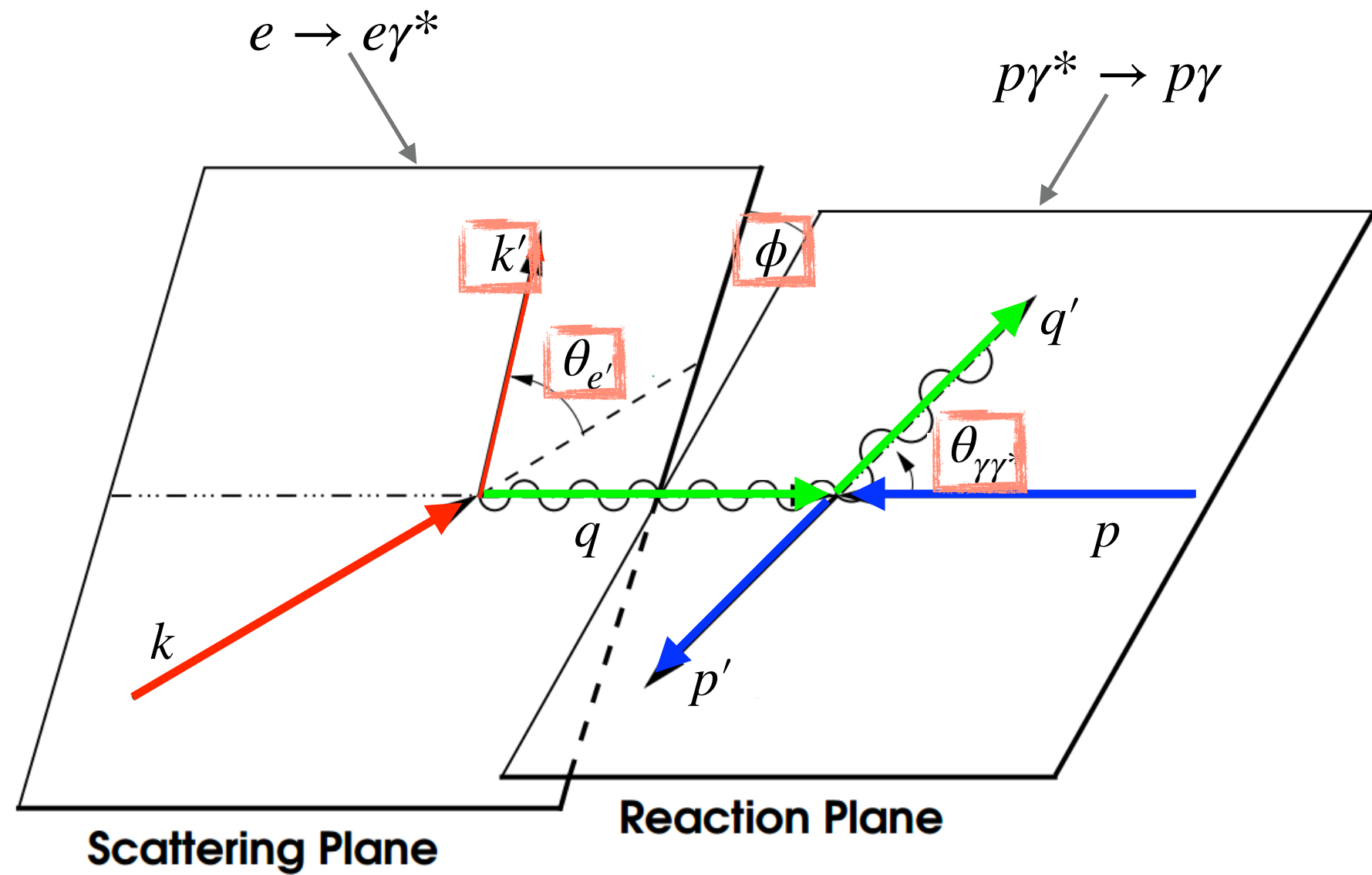
$$\vec{m} = \beta_M \vec{B}$$

- **Paramagnetic:** >0 , quarks align along magnetic field;
- **Diamagnetic:** <0 , pion cloud induced magnetic field in opposite direction
- Partially cancels each other, makes β_M value small



Reaction & Amplitudes

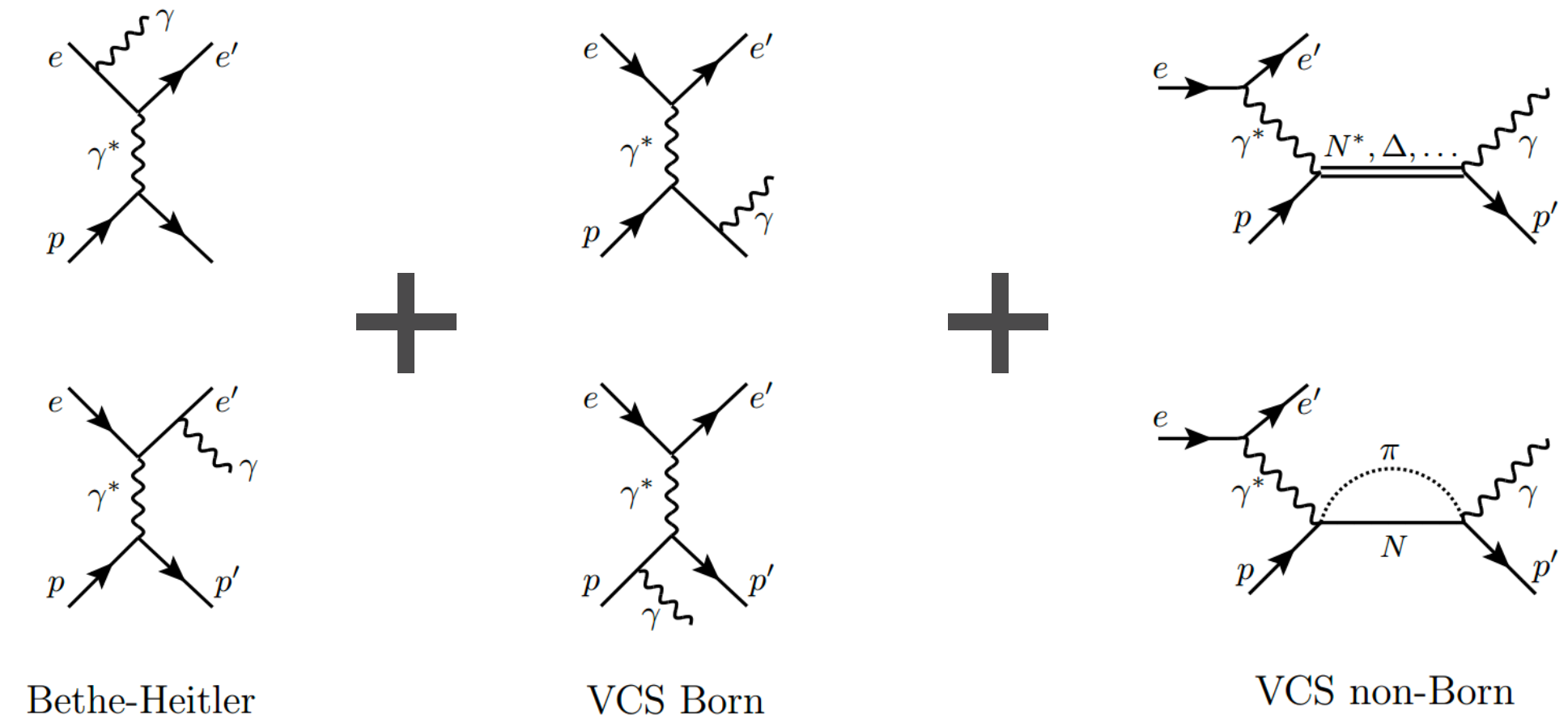
k -incoming electron q -virtual photon p -initial proton
 k' -scattered electron q' -real photon p' -final proton



Kinematics of $ep \rightarrow e\gamma$

$$VCS \text{ cross-section} = d^5 \sigma / (dk'_{lab} d\Omega'_{elab} d\Omega_{p_{cm}})$$

VCS process \rightarrow photon electro-production reaction



Calculable with nucleon
form factors G_E, G_M

$$\alpha_{E1}(Q^2) = -\frac{e^2}{4\pi} \cdot \sqrt{\frac{3}{2}} \cdot P^{(L1,L1)0}(Q^2)$$

$$\beta_{M1}(Q^2) = -\frac{e^2}{4\pi} \cdot \sqrt{\frac{3}{8}} \cdot P^{(M1,M1)0}(Q^2)$$

Electric & Magnetic
Scaler GP

LEX & DR Formalism

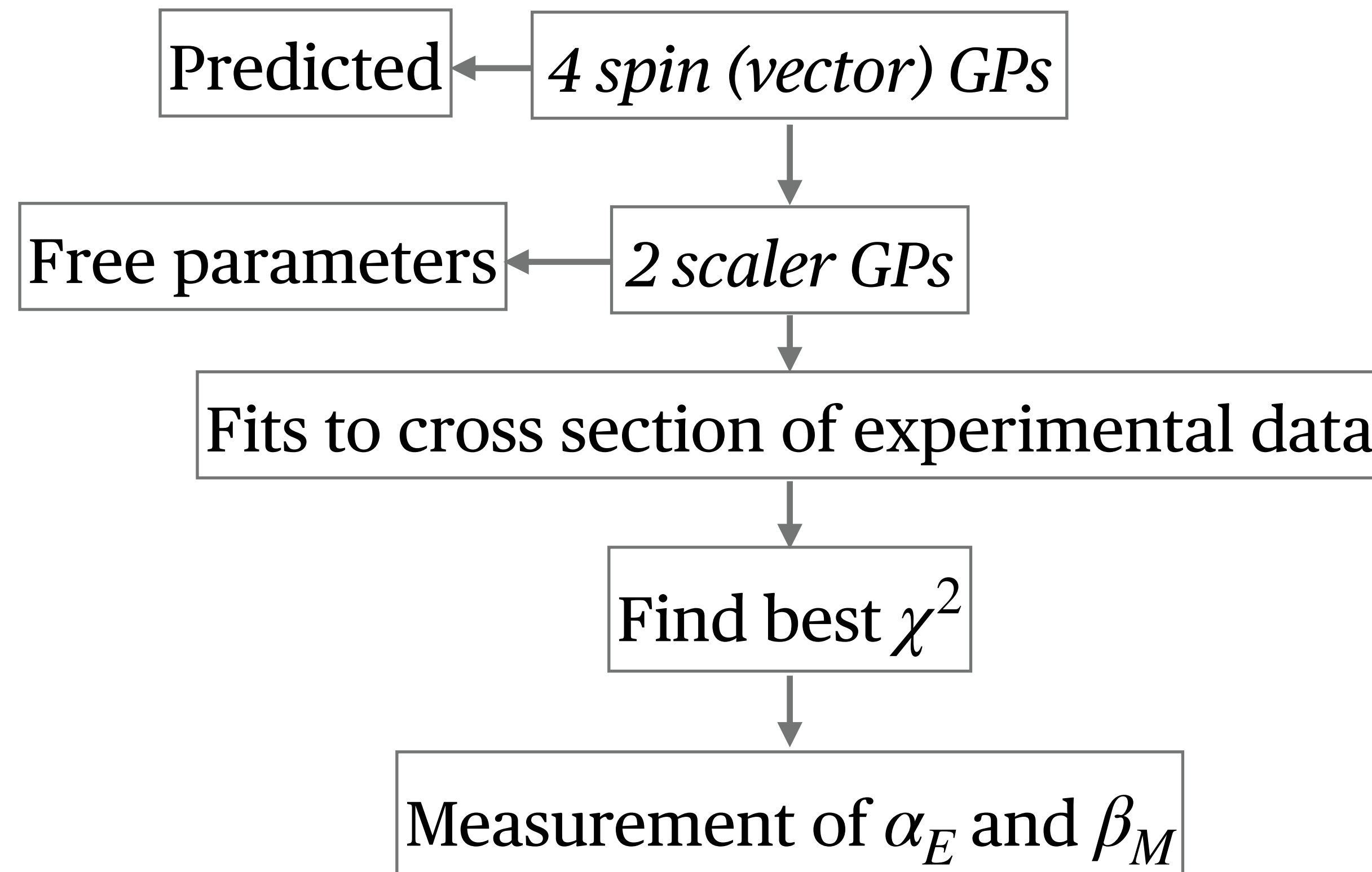
•LEX - Low Energy Expansion

Below pion threshold



•DR - Dispersion Relation Formalism

Below & Above pion threshold



“The effect of the GPs in the photon electroproduction cross-section roughly scales with the outgoing photon energy.”

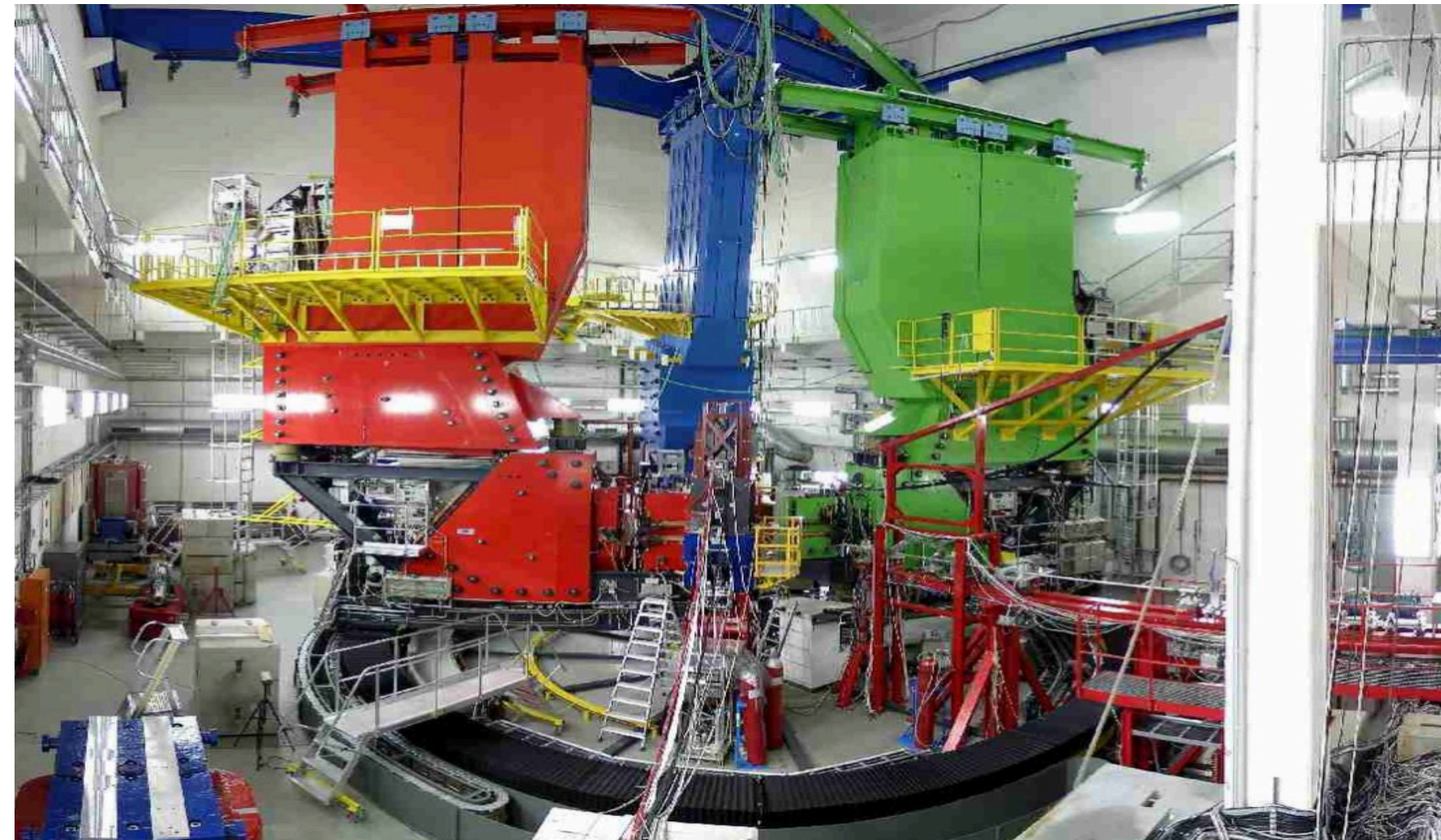
Phys. Rev C 86, 015210 (2012)

Phys. Rev Lett. 93, 122001 (2004)

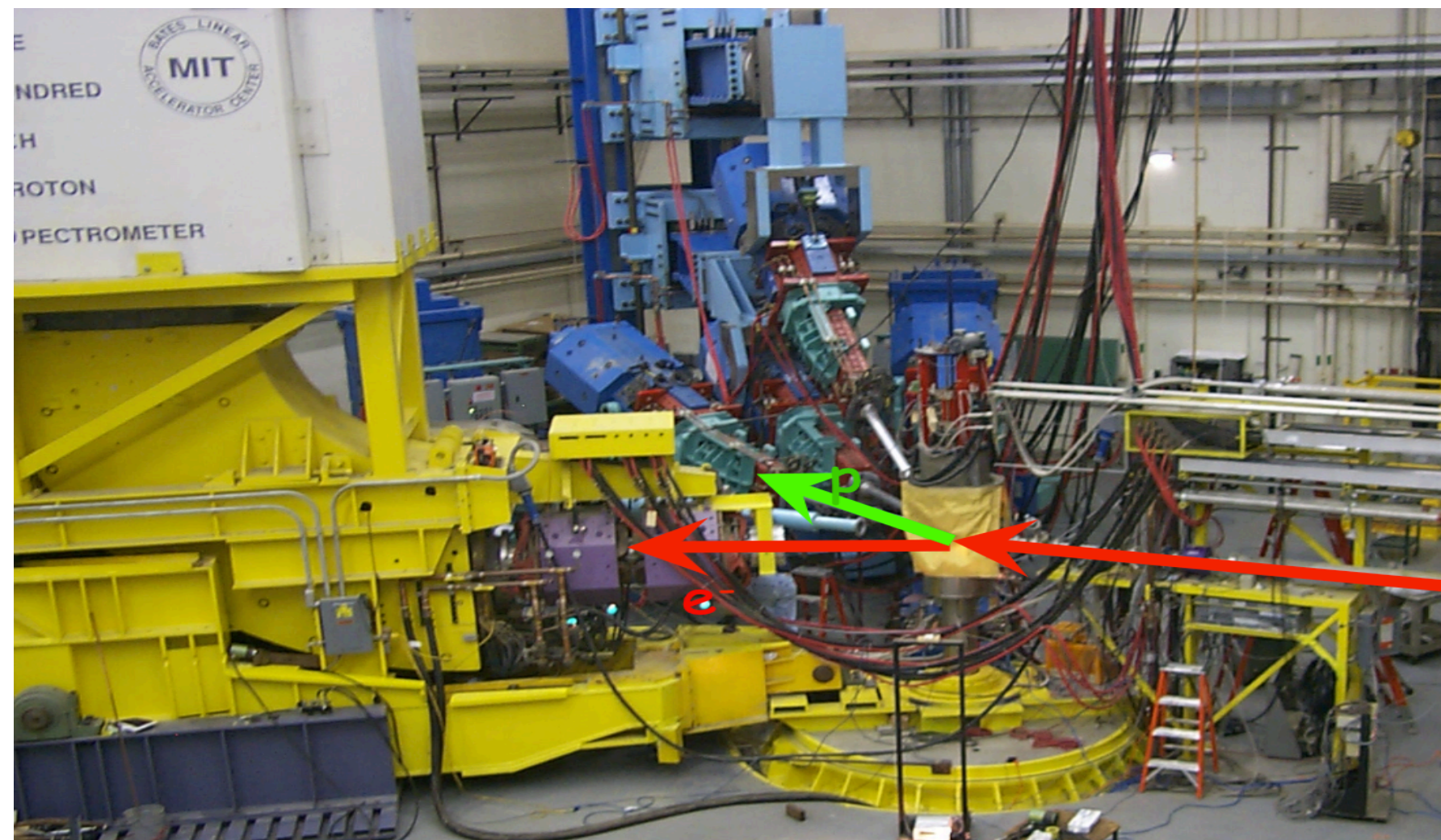
High sensitivity to the GPs

World Data & Motivation

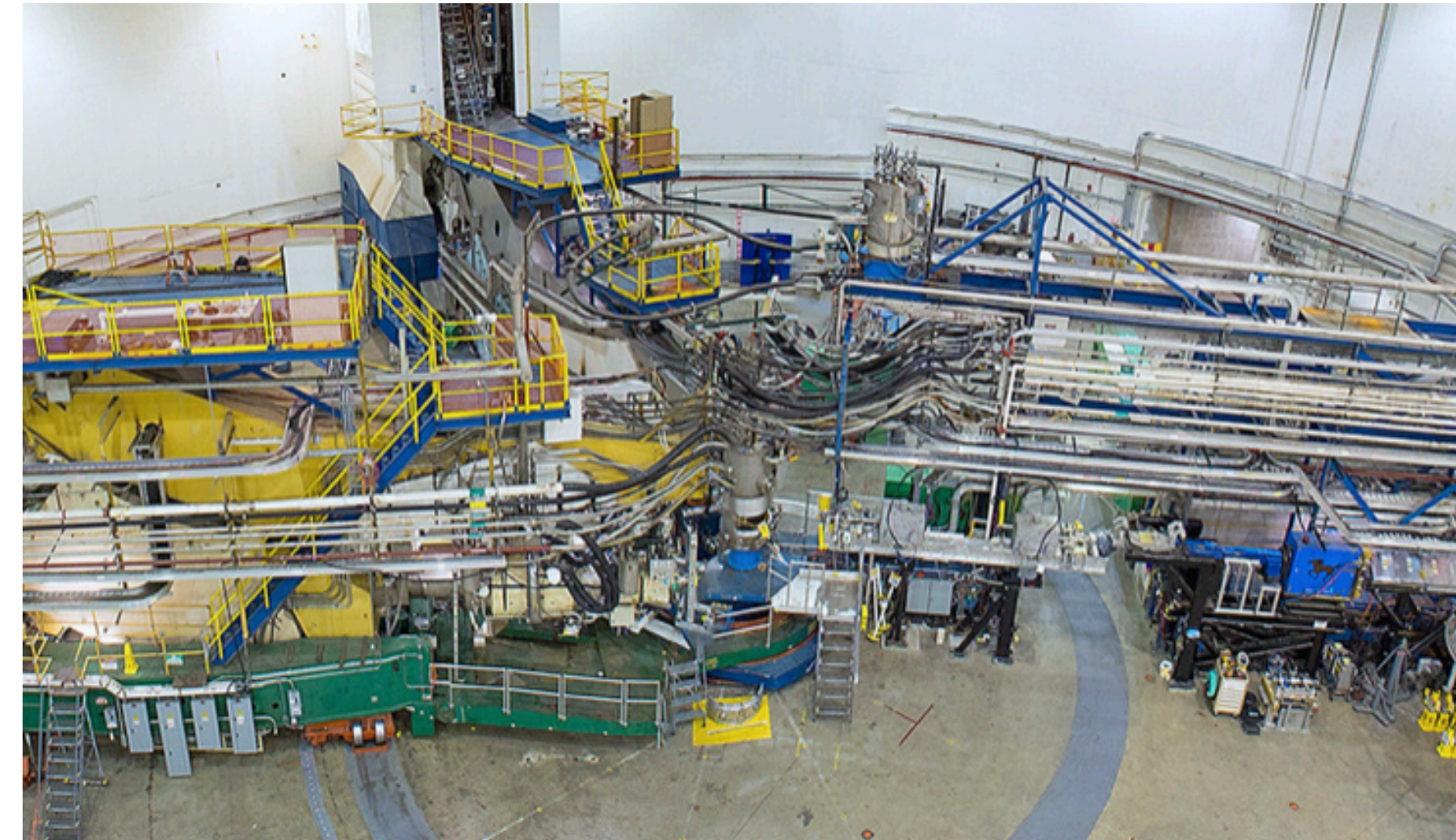
MAMI at $Q^2 = 0.1$ & 0.2 & 0.33 & 0.45 GeV^2



MIT-Bates at $Q^2 = 0.057$ GeV^2



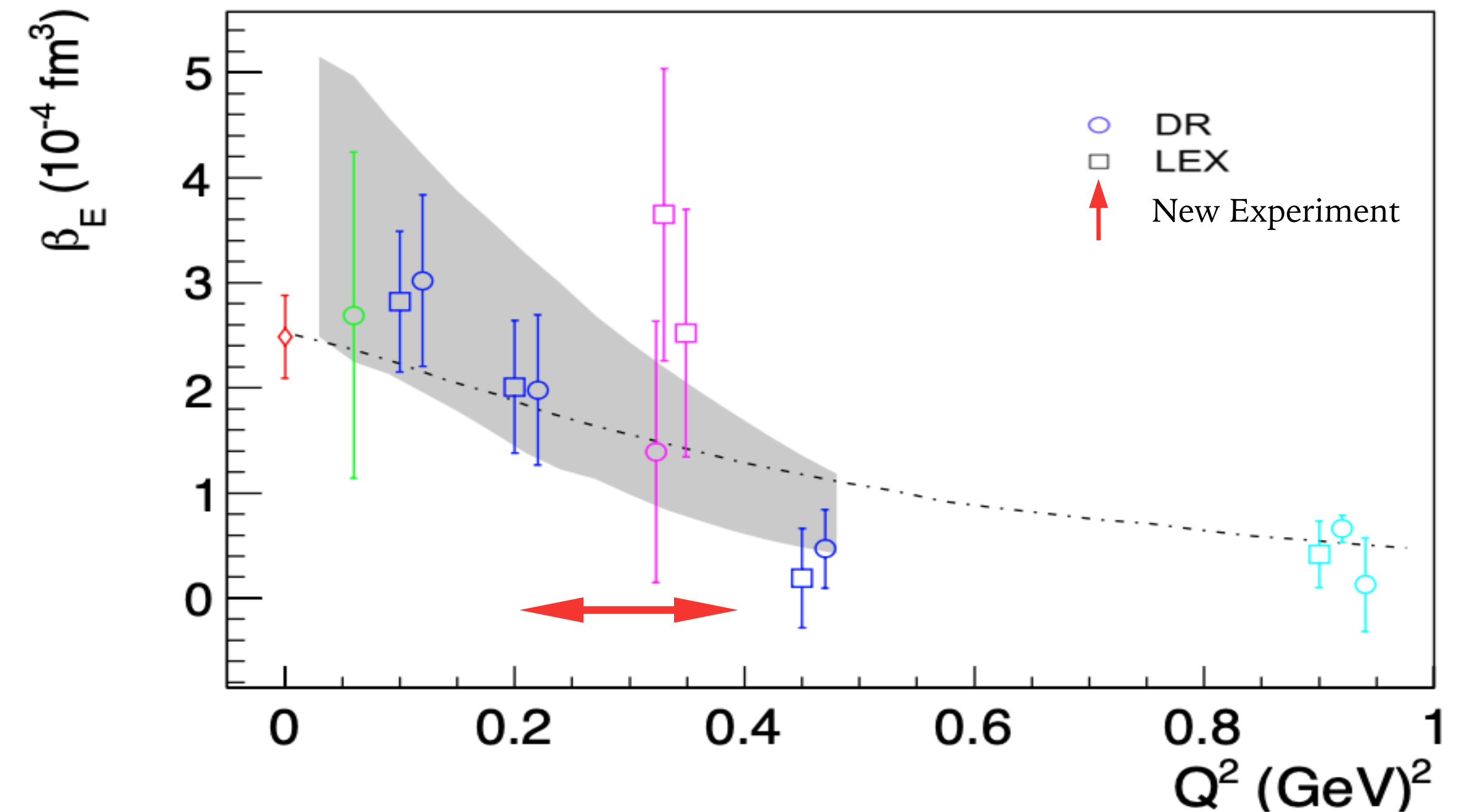
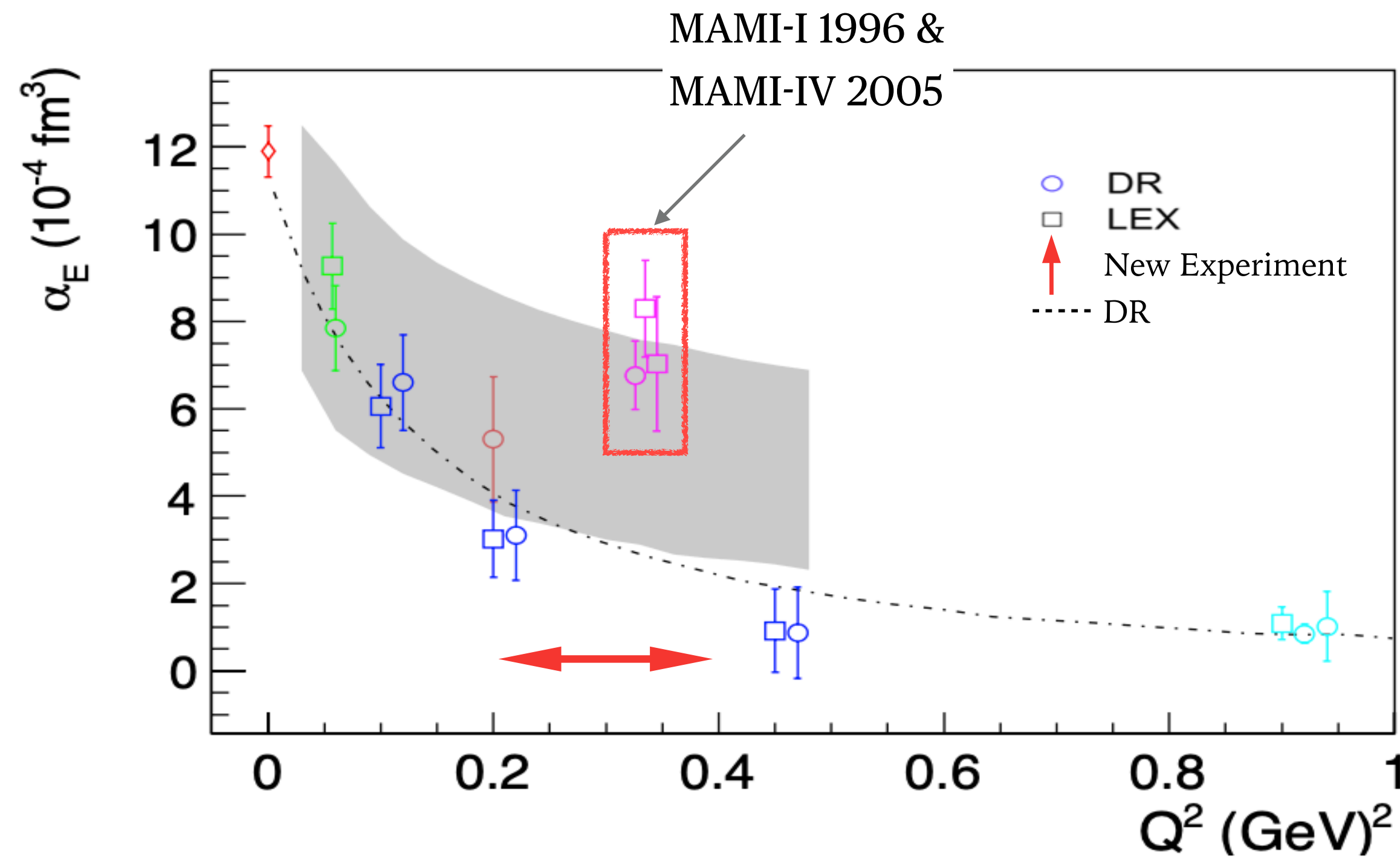
JLAB Hall A at $Q^2 = 0.92$ & 1.76 GeV^2



JLAB Hall C at $Q^2 = 0.33$ GeV^2



World Data & Motivation

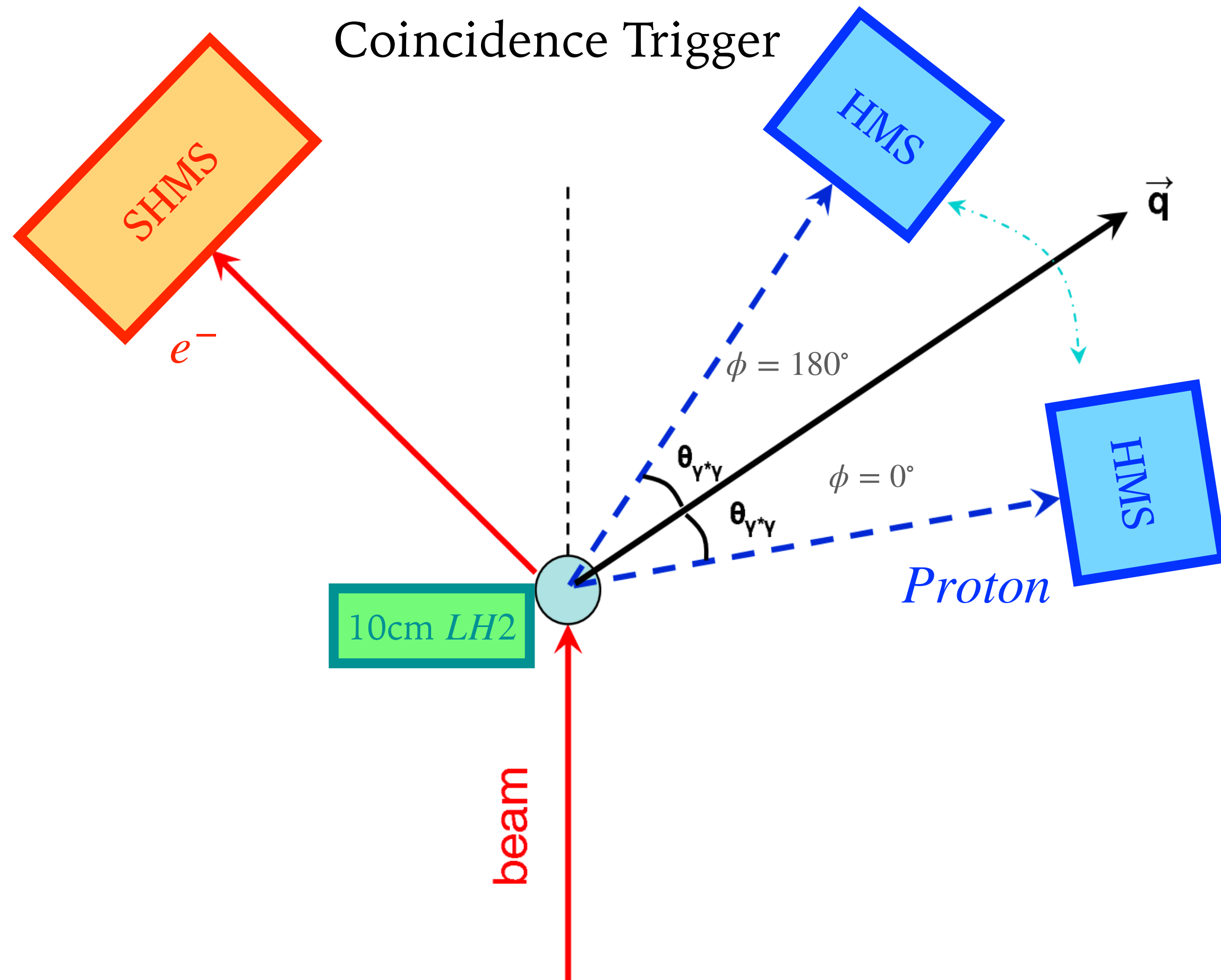


- Initial theoretical models predicted smooth fall off of α_E
 - data at $Q^2 = 0.33$ implies a non-trivial structure
- New experiment can:
 - Address puzzling α_E enhancement
 - Reduce error by 2

- Small values, $1/3 \sim 1/4$ of α_E
- Large uncertainties
- New experiment can:
 - Improve precision
 - Explore para-& dia-magnetic mechanism inside nucleon

J. Roche, et al., Phys. Rev. Lett. 85 (2000) 708-711; P. Janssens, et al., Eur. Phys. J. A37 (2008) 1-8; G. Laveissiere, et al., Phys. Rev. Lett. 93 (2004) 122001; H. Fonvieille, et al., Phys. Rev. C86 (2012) 015210; P. Bourgeois, et al., Phys. Rev. Lett. 97 (2006) 212001; Eur.Phys.J.A55(2019)no. 10,182; Phy.Rev.Lett. 123(2019)no.19,192302; Phys.Rev.C 103, 025205(2021) *Figure Credit: Hamza Atac

JLab E12-15-001 Experiment



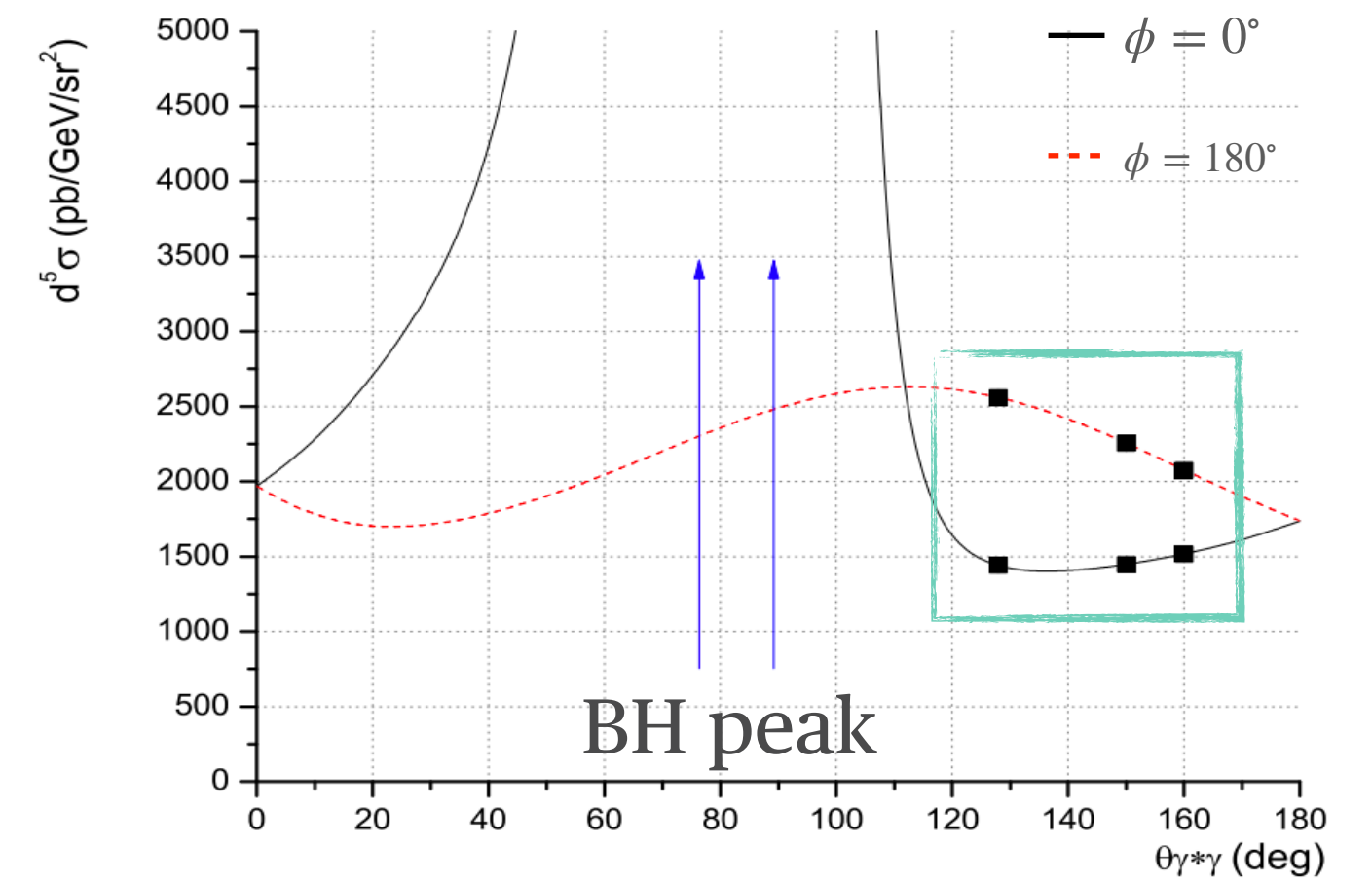
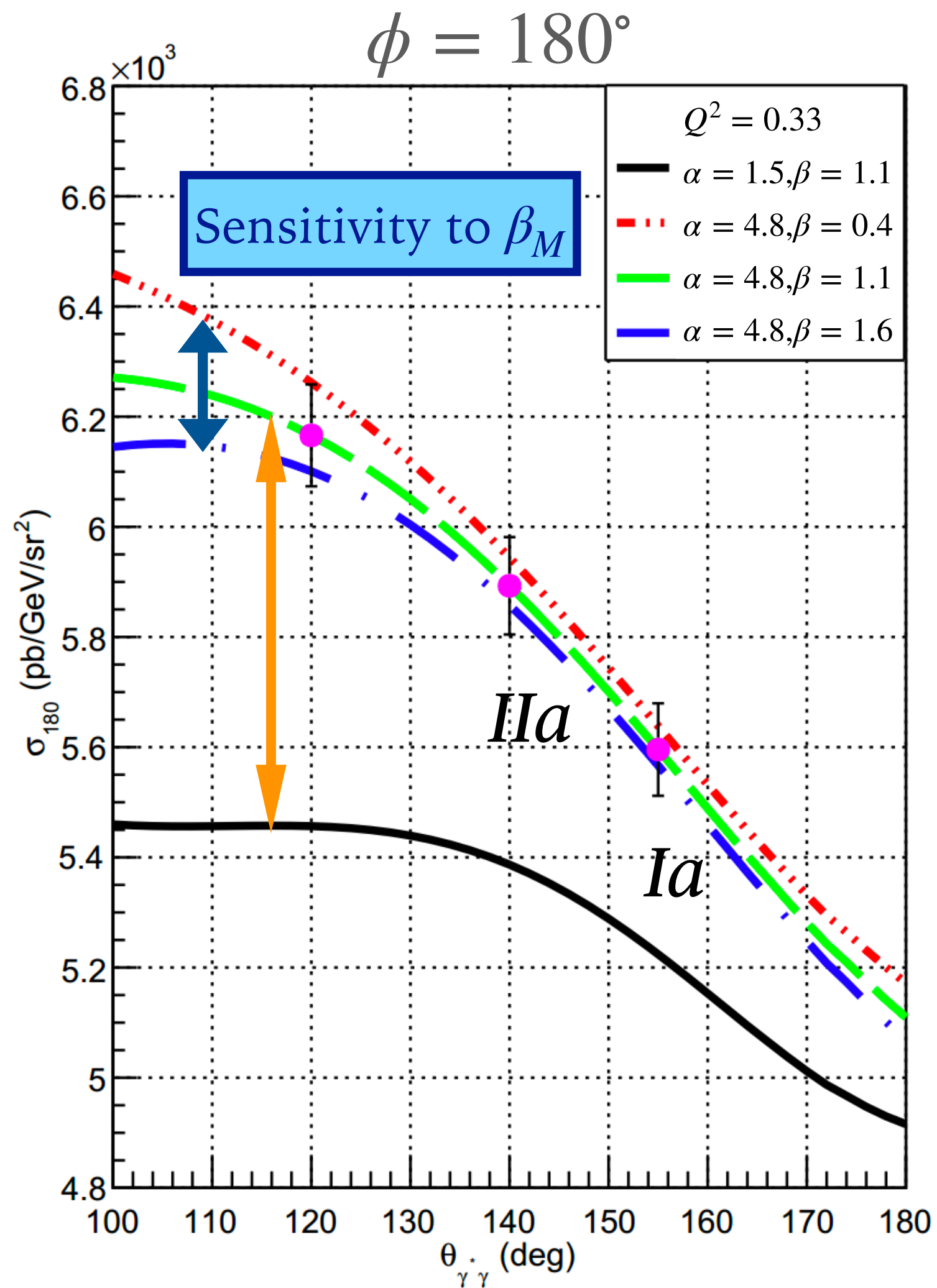
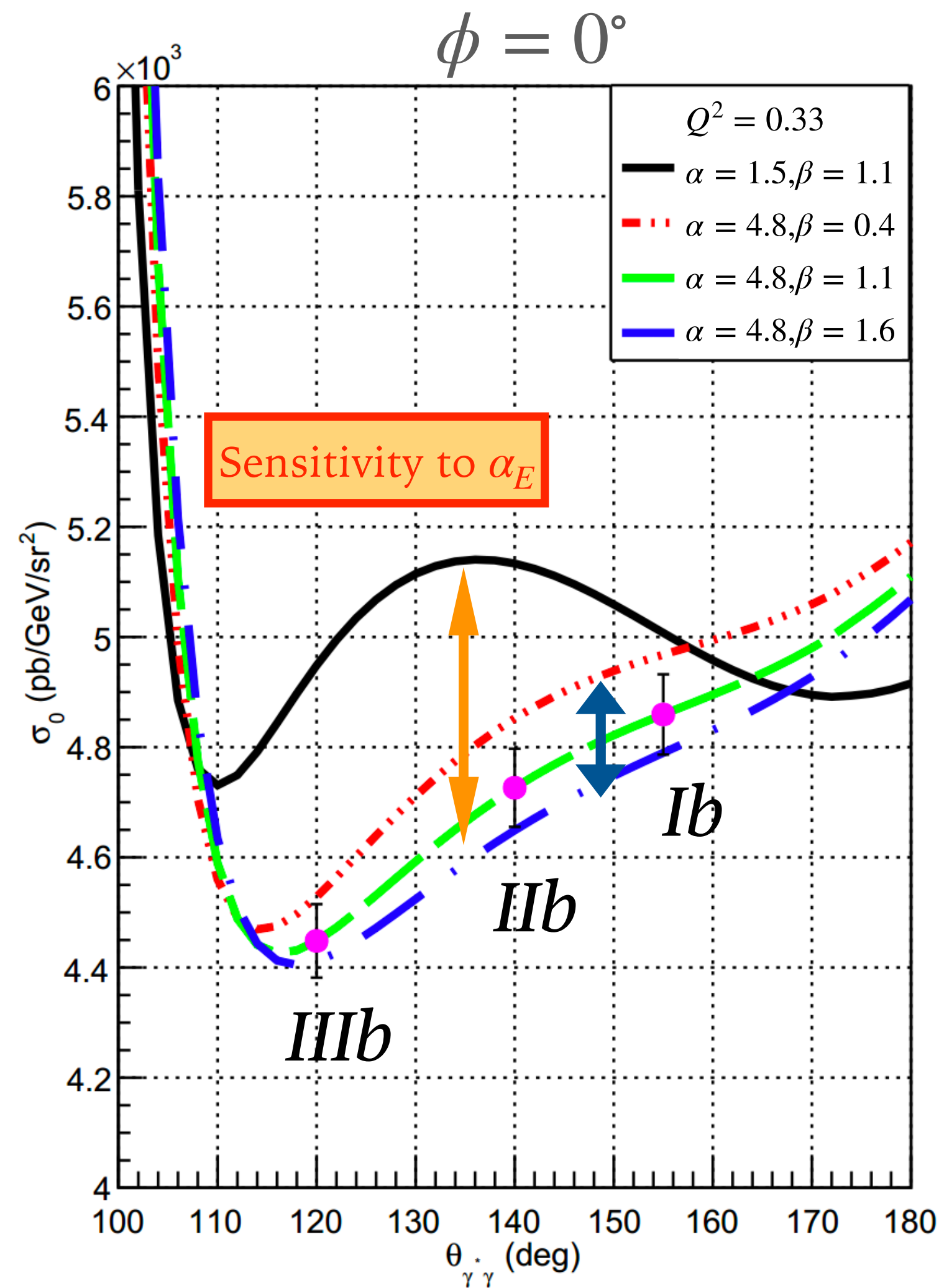
- Summer 2019: July 20 - August 5
- Beam $E = 4.56 GeV$
- $Q^2 = 0.33 GeV^2$, $W = 1.232 GeV$

Kinematics	θ_e°	$P_e(GeV/c)$	θ_p°	$P_p(GeV/c)$
Kin 1a	7.69	4.034	37.33	0.893
Kin 1b	7.69	4.034	51.40	0.893
Kin 2a	7.69	4.034	33.53	0.863
Kin 2b	7.69	4.034	55.22	0.863
Kin 3b	7.69	4.034	60.74	0.795

- In-plane azimuthal asymmetries to suppress systematic uncertainties:

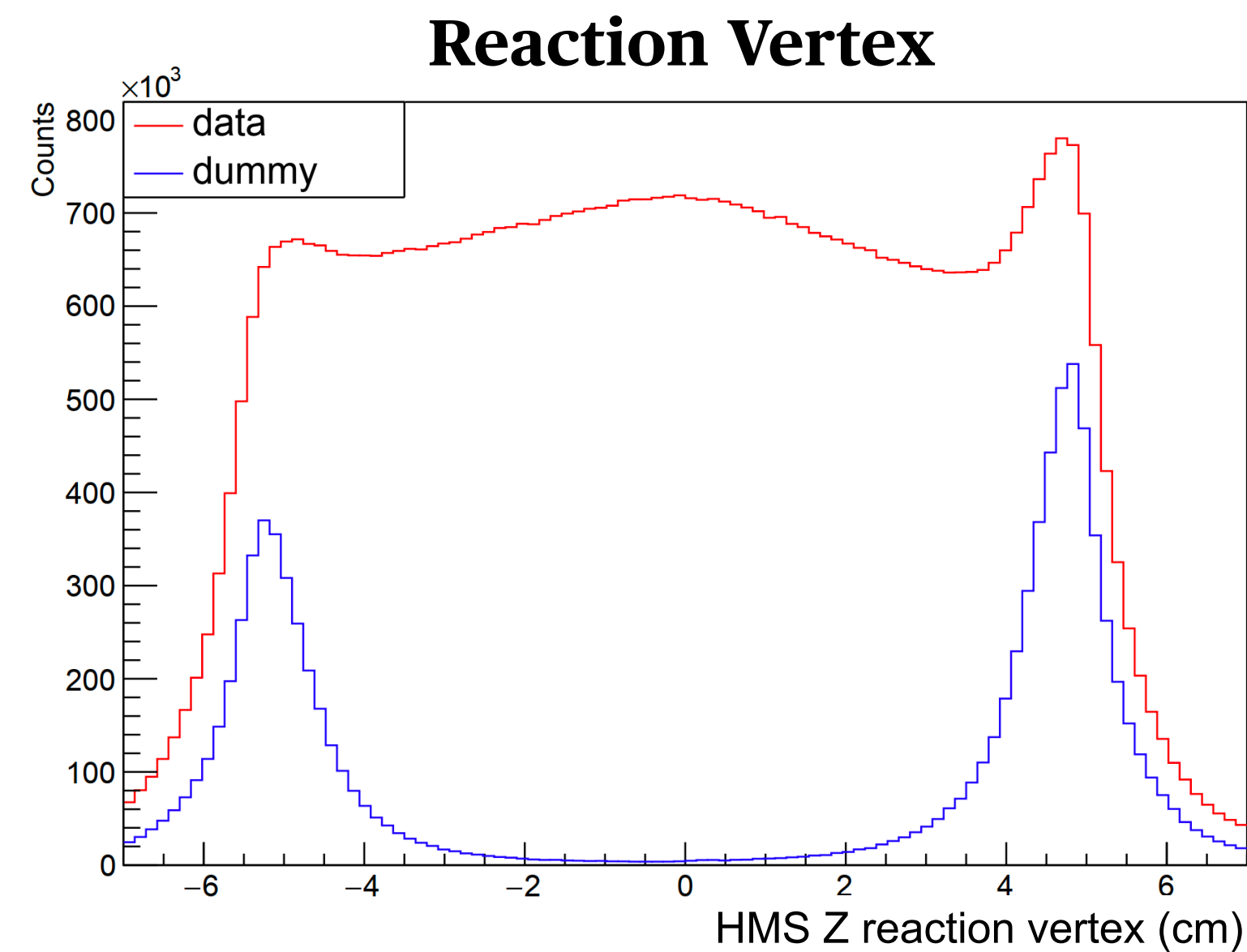
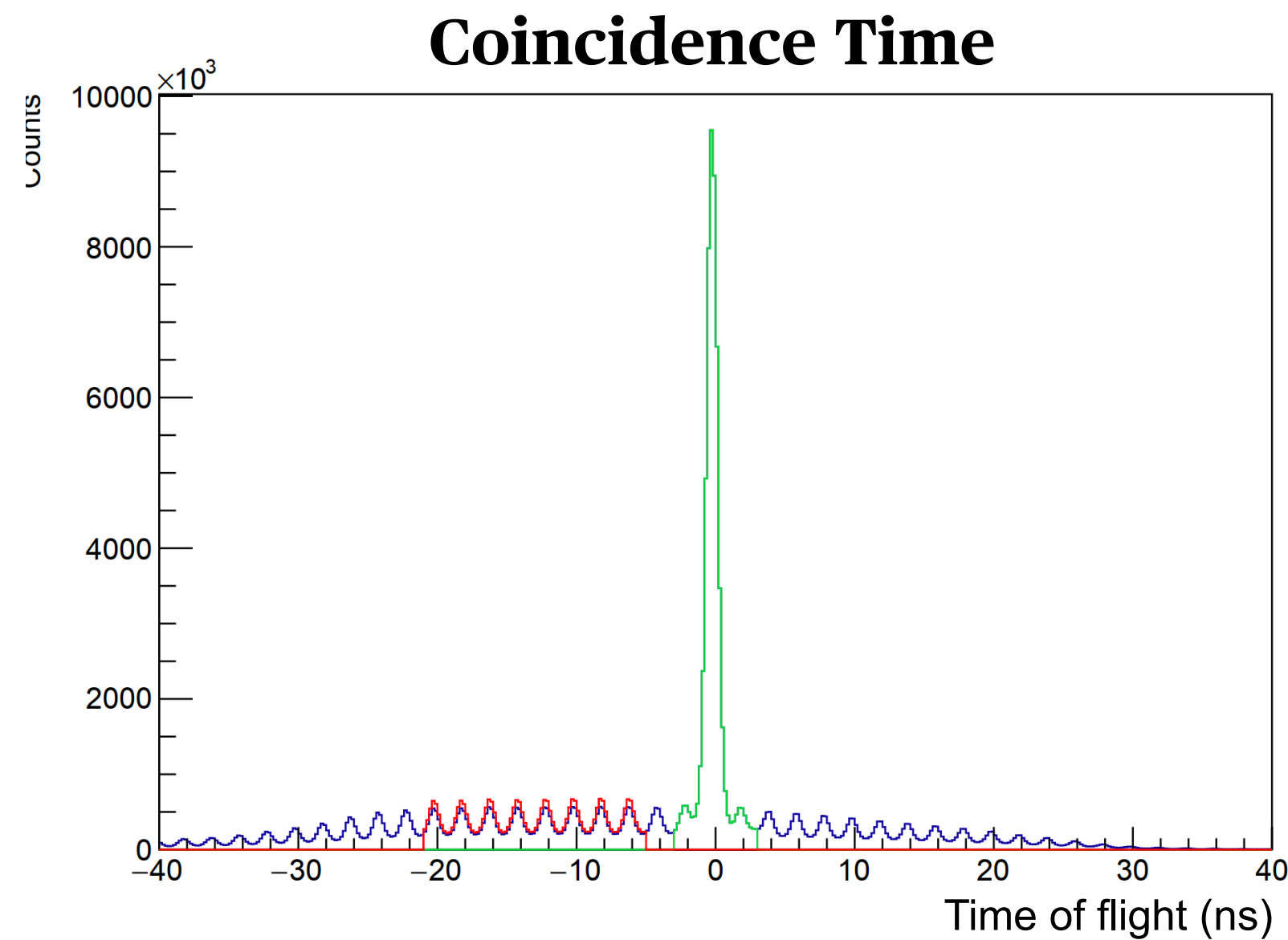
$$\frac{d\sigma_{\phi=180^\circ} - d\sigma_{\phi=0^\circ}}{d\sigma_{\phi=180^\circ} + d\sigma_{\phi=0^\circ}}$$

Predicted Measurement



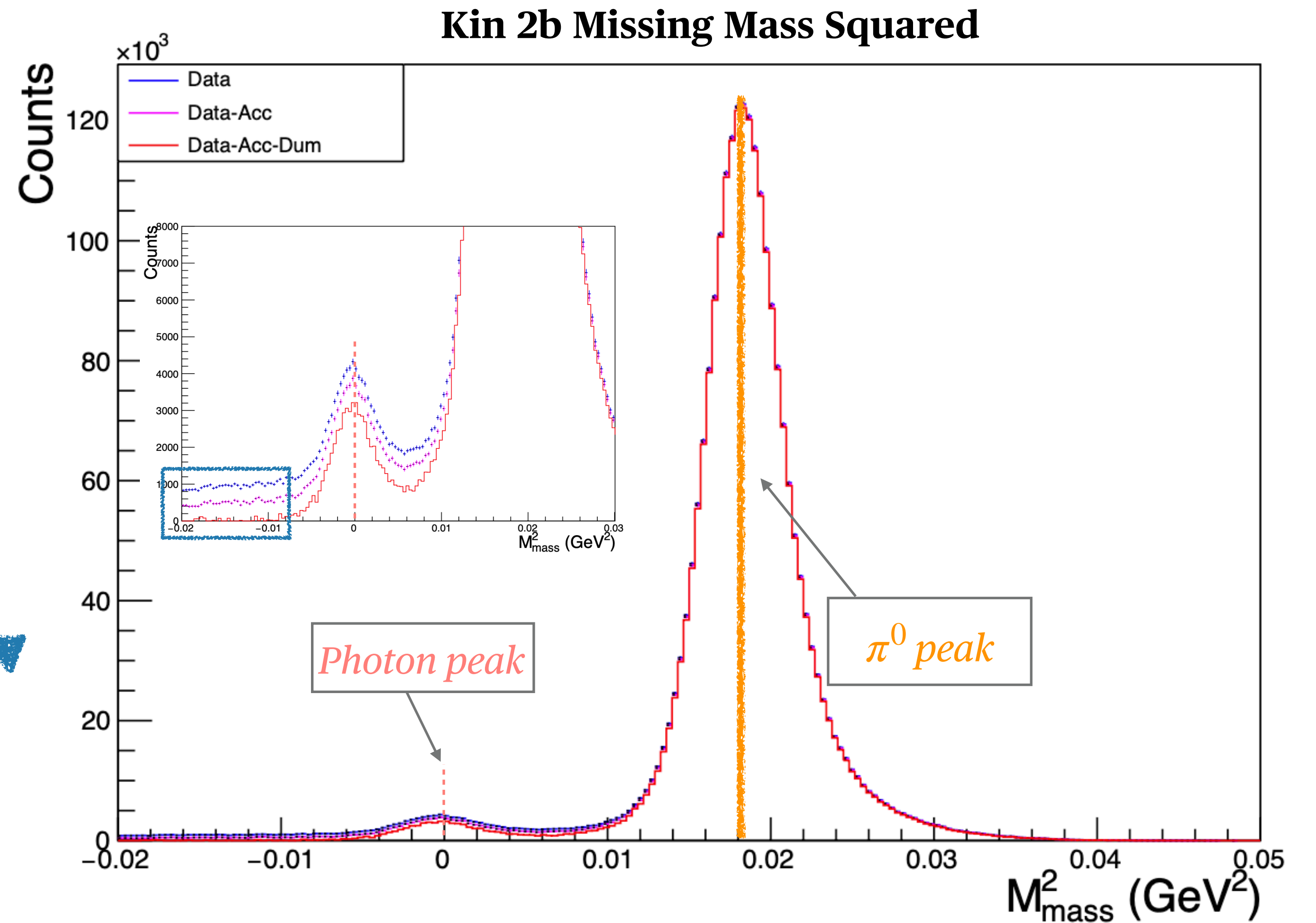
- High enough $\theta_{\gamma\gamma}^*$ to avoid BH peak
- Avoid rapid cross-section variation
- ϵ increases from 0.6 to 0.98 – doubles the sensitivity to GPs

VCS Peak and pi0 Peak



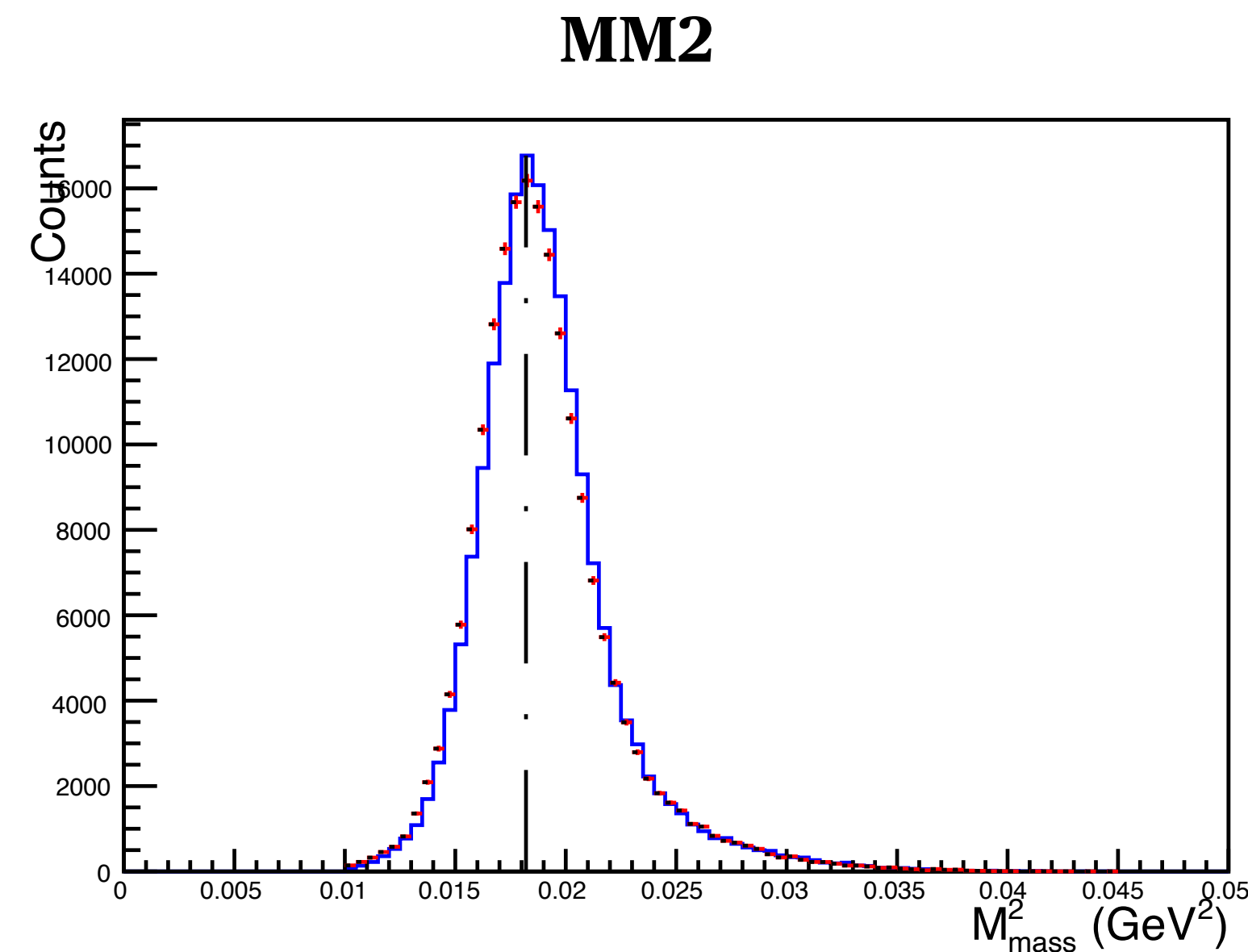
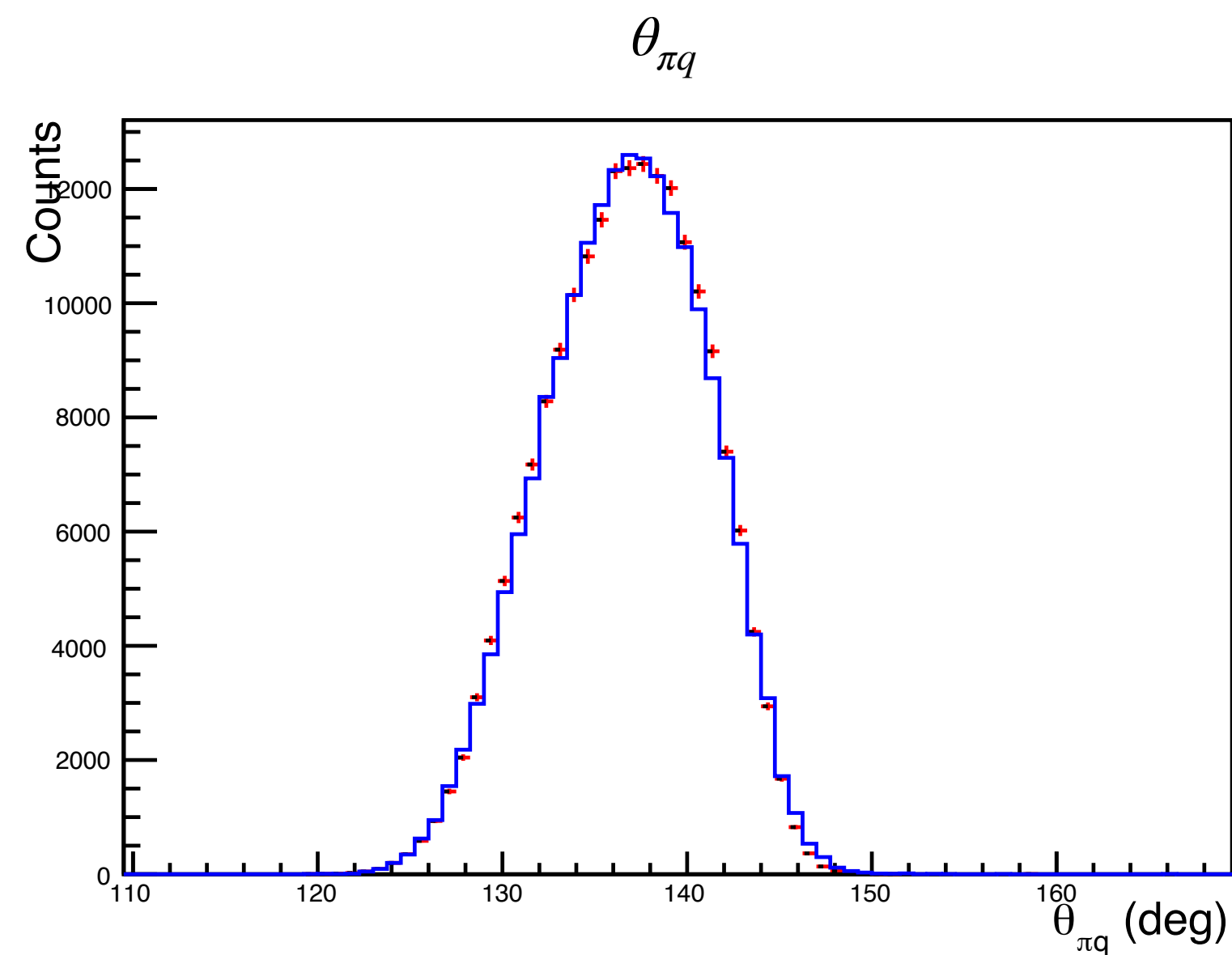
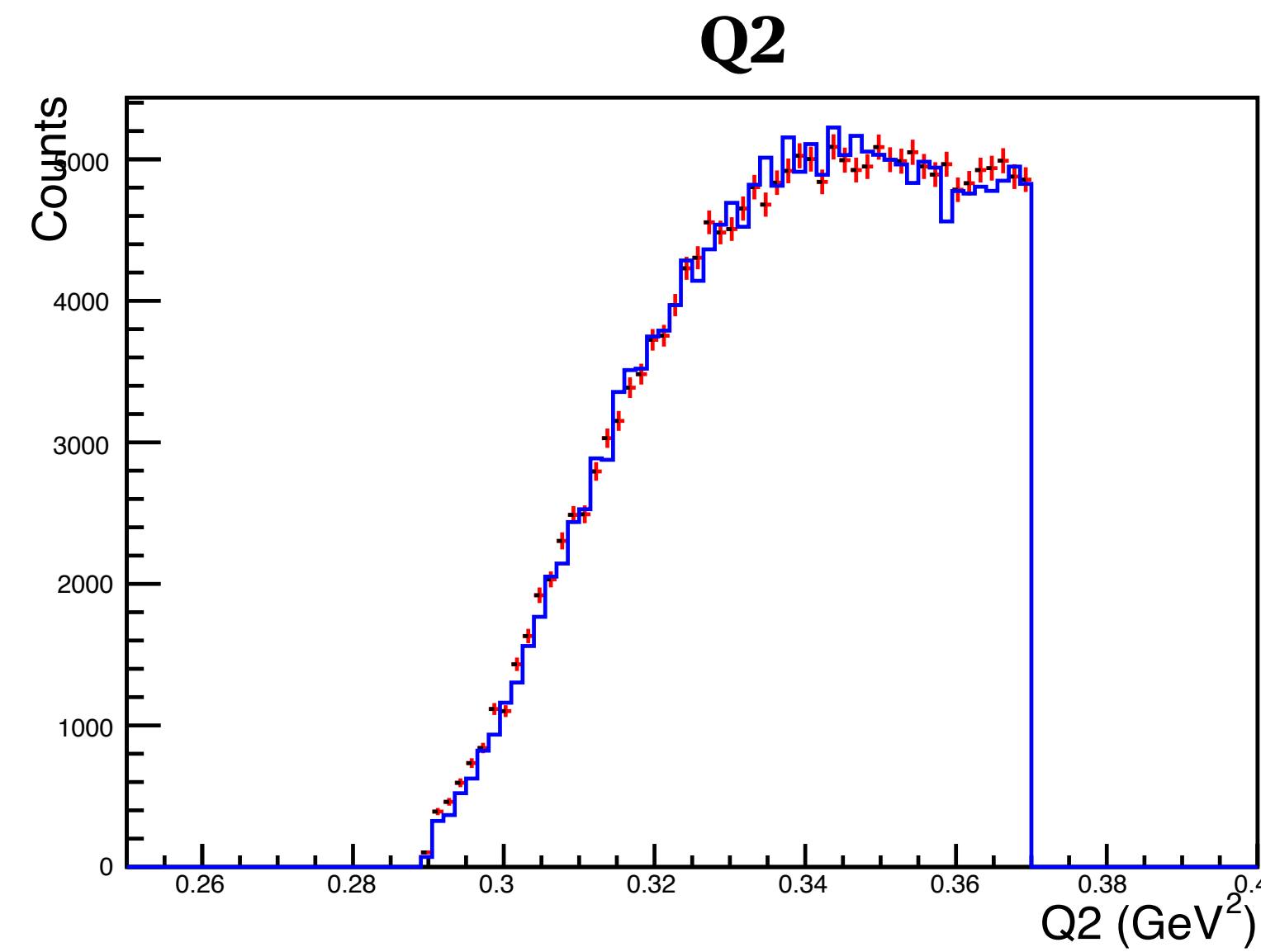
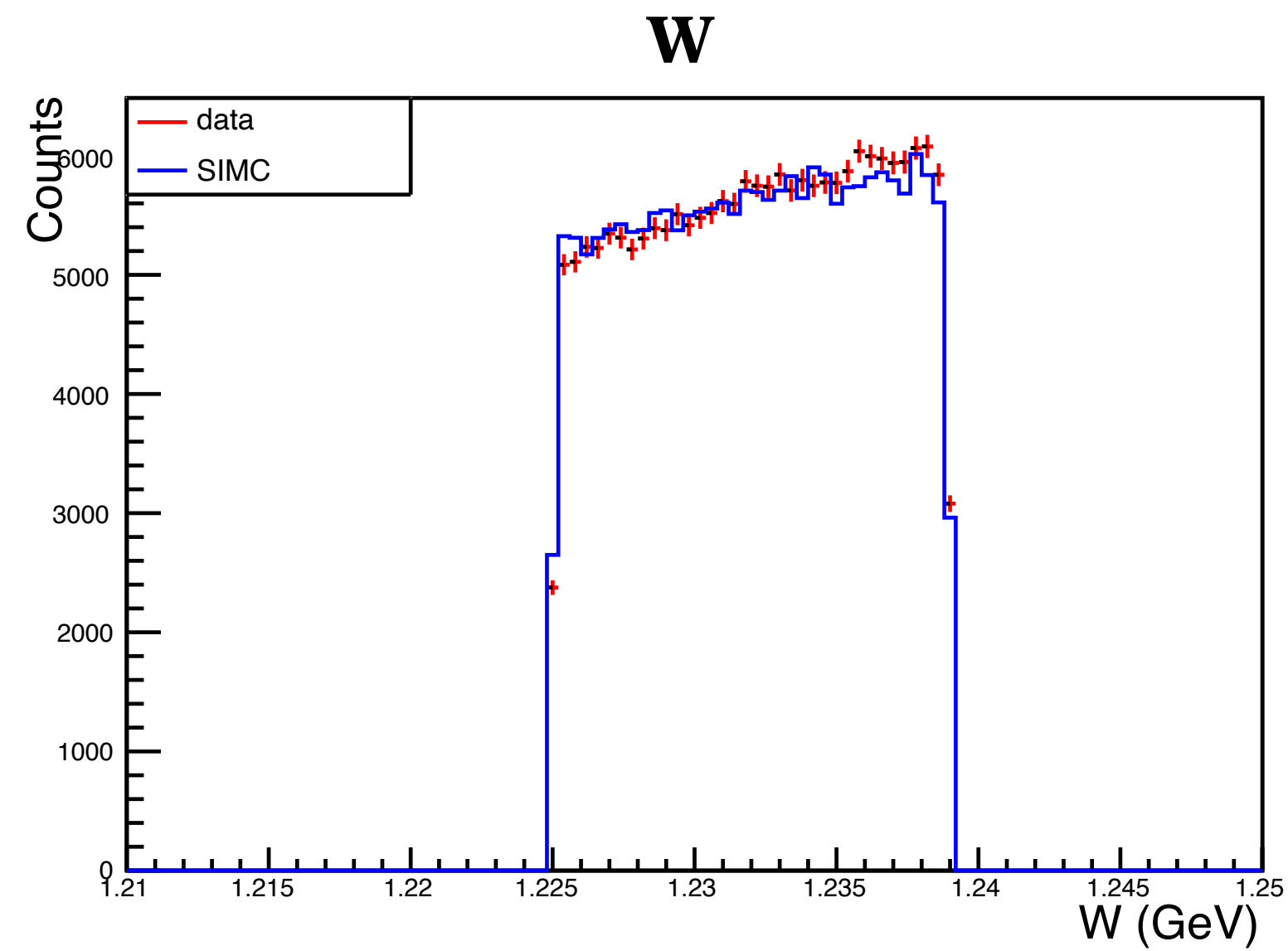
Accidentals subtraction

Dummy subtraction



• The missing mass technique is used to identify photons

Pion Preliminary Analysis



Kin2b data vs. MAID

Cuts:

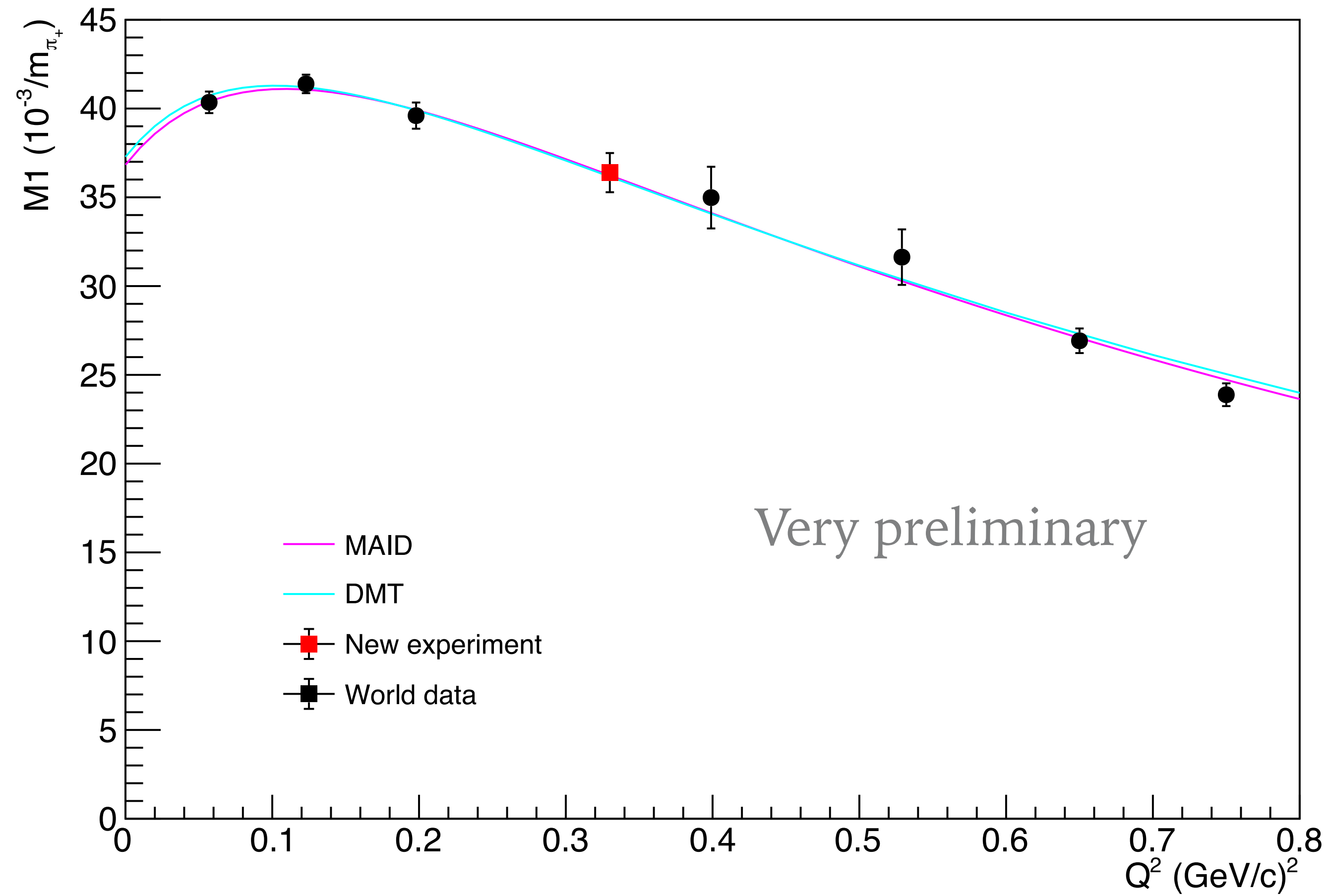
$$0.01 < mm^2 < 0.05 \text{ (GeV}^2\text{)}$$

$$\text{abs}(W - 1.232) < 0.007 \text{ (GeV)}$$

$$\text{abs}(Q2 - 0.33) < 0.04 \text{ (GeV}^2\text{)}$$

$$\text{abs}(\phi_{pq} - \phi_{\text{center}}) < 25 \text{ (deg)}$$

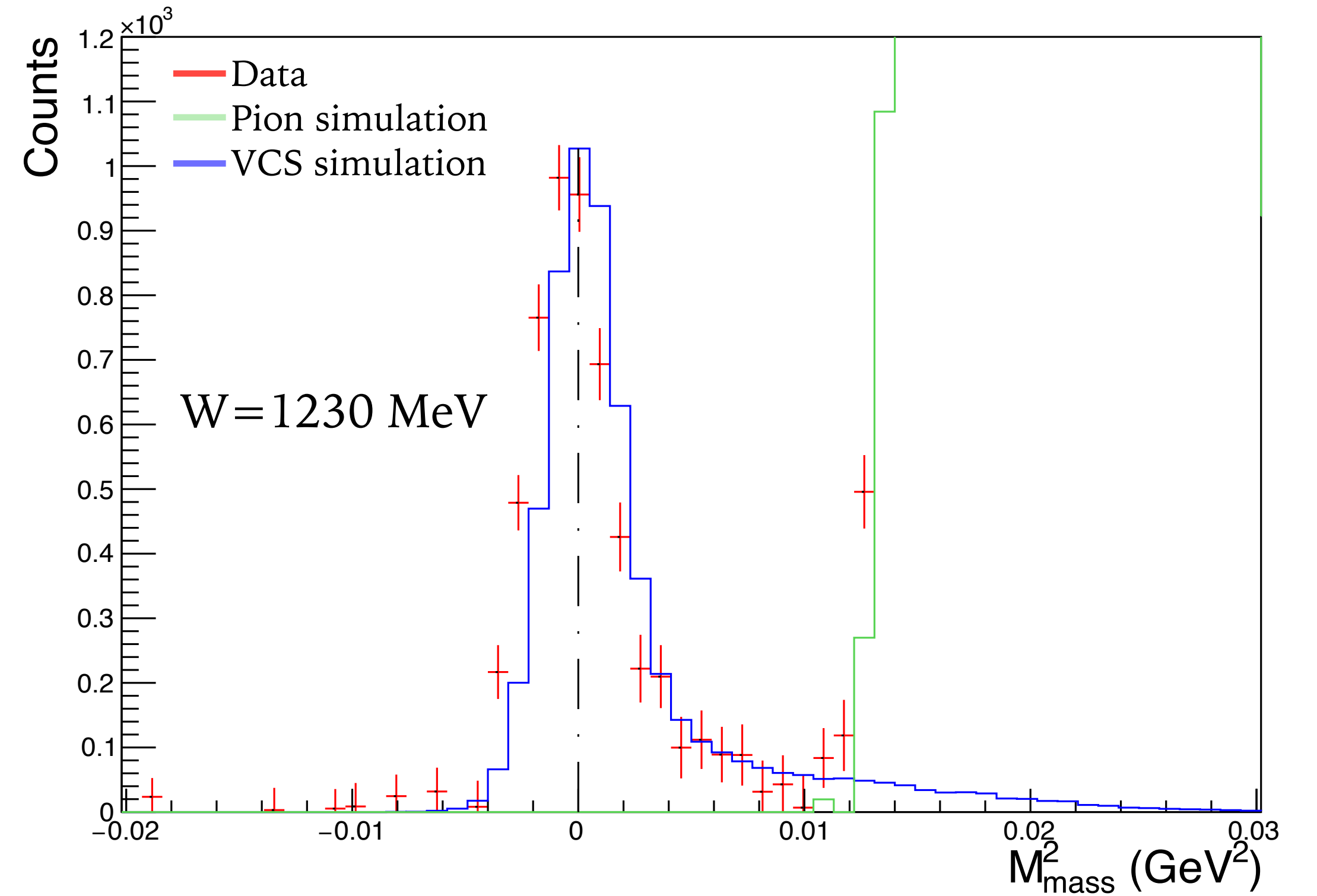
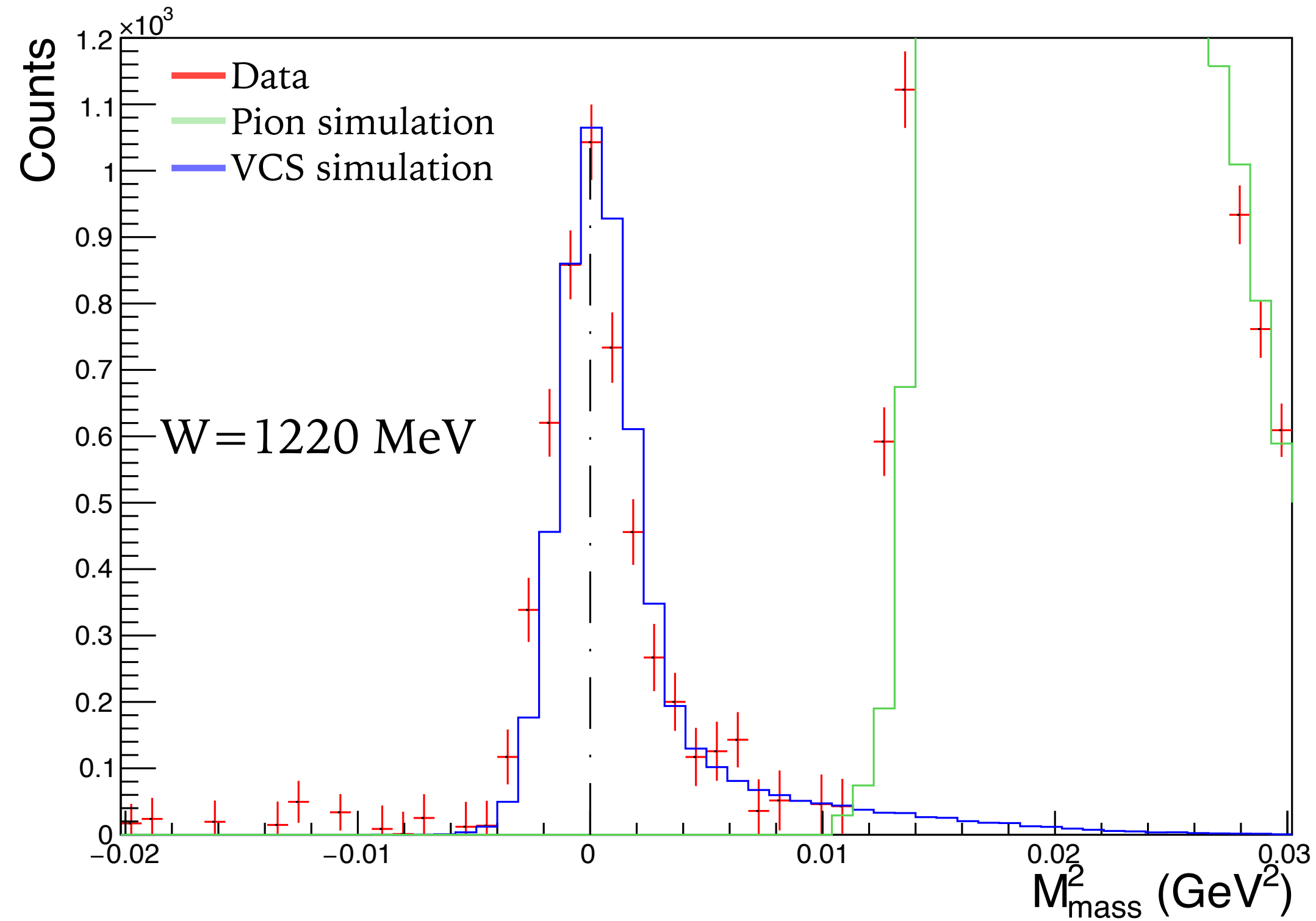
Pion Preliminary Analysis



M1 - Magnetic dipole amplitude

Normalization wise good

VCS Analysis



Cuts:

$$\text{abs}(m^2) < 0.01 \text{ (GeV}^2\text{)}$$

$$\text{abs}(W - W_{\text{center}}) < 0.007 \text{ (GeV)}$$

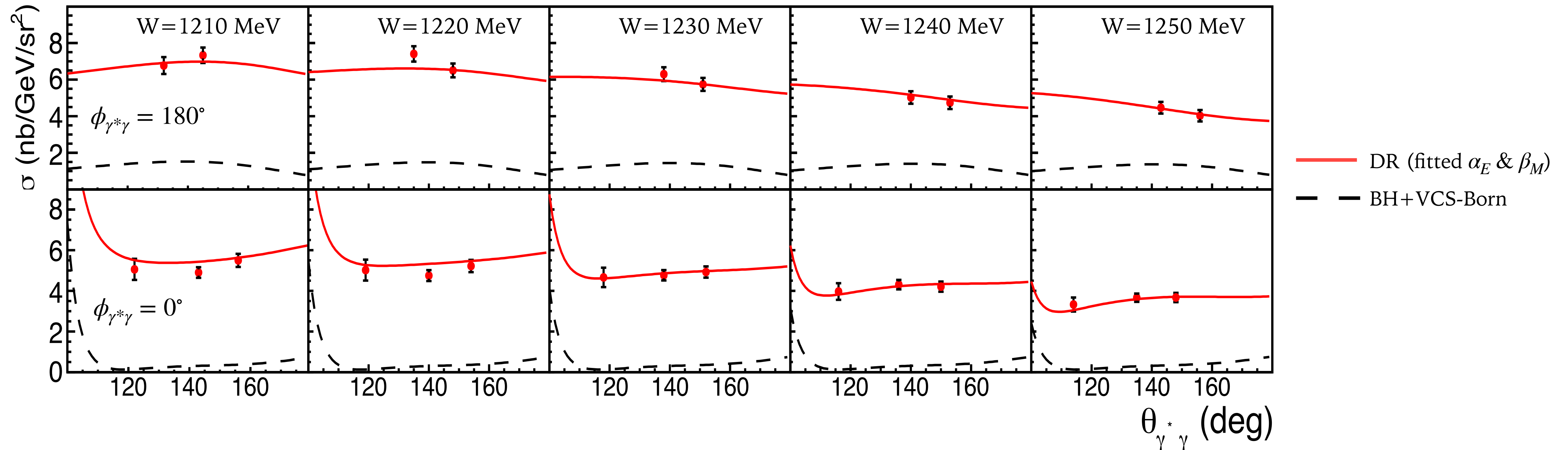
$$\text{abs}(Q^2 - 0.33) < 0.035 \text{ (GeV}^2\text{)}$$

$$\text{abs}(\theta_{pq} - \theta_{\text{center}}) < 4 \text{ (deg)}$$

$$\text{abs}(\phi_{pq} - \phi_{\text{center}}) < 25 \text{ (deg)}$$

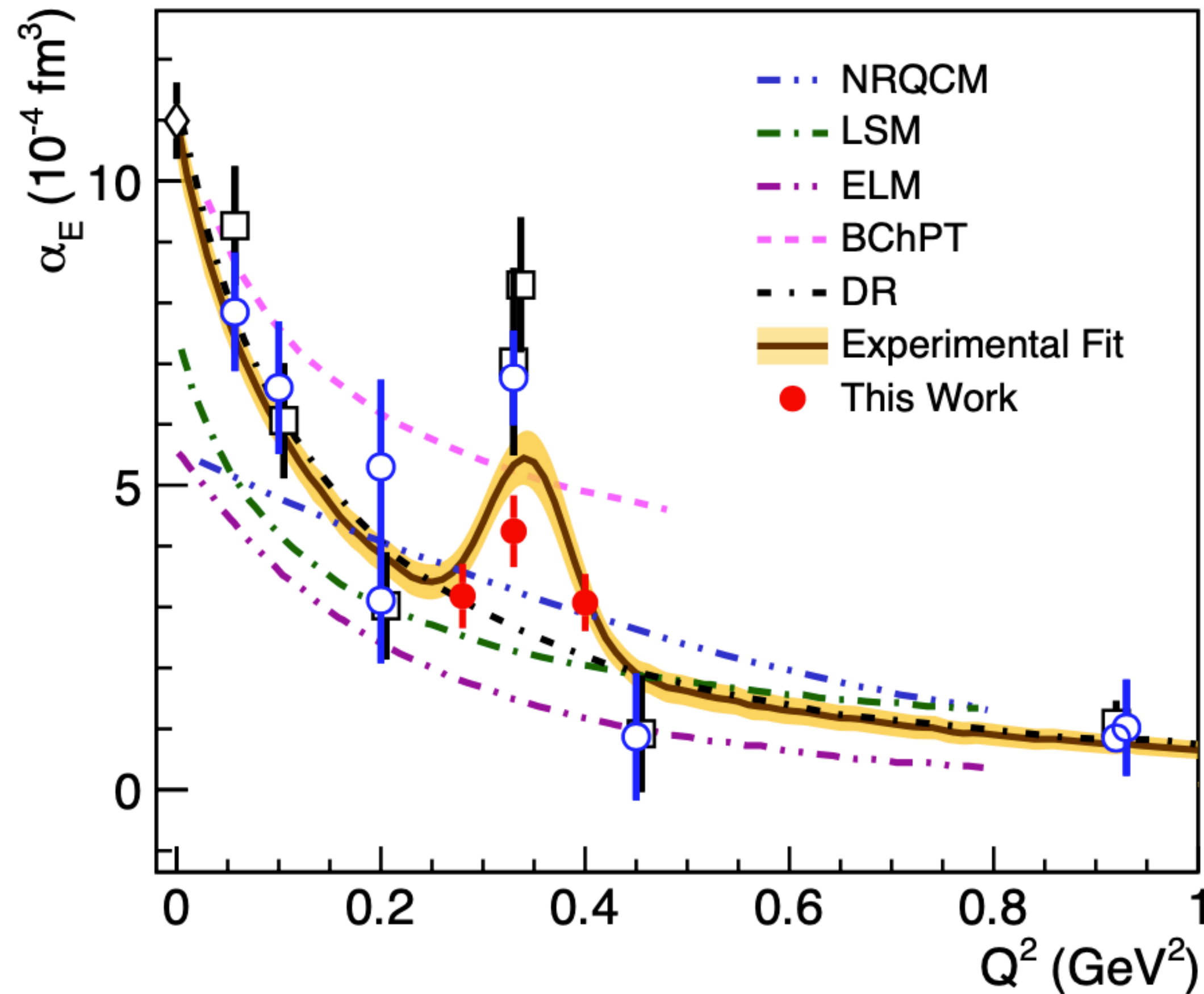
VCS Analysis

$Q^2=0.33$ (In-plane)

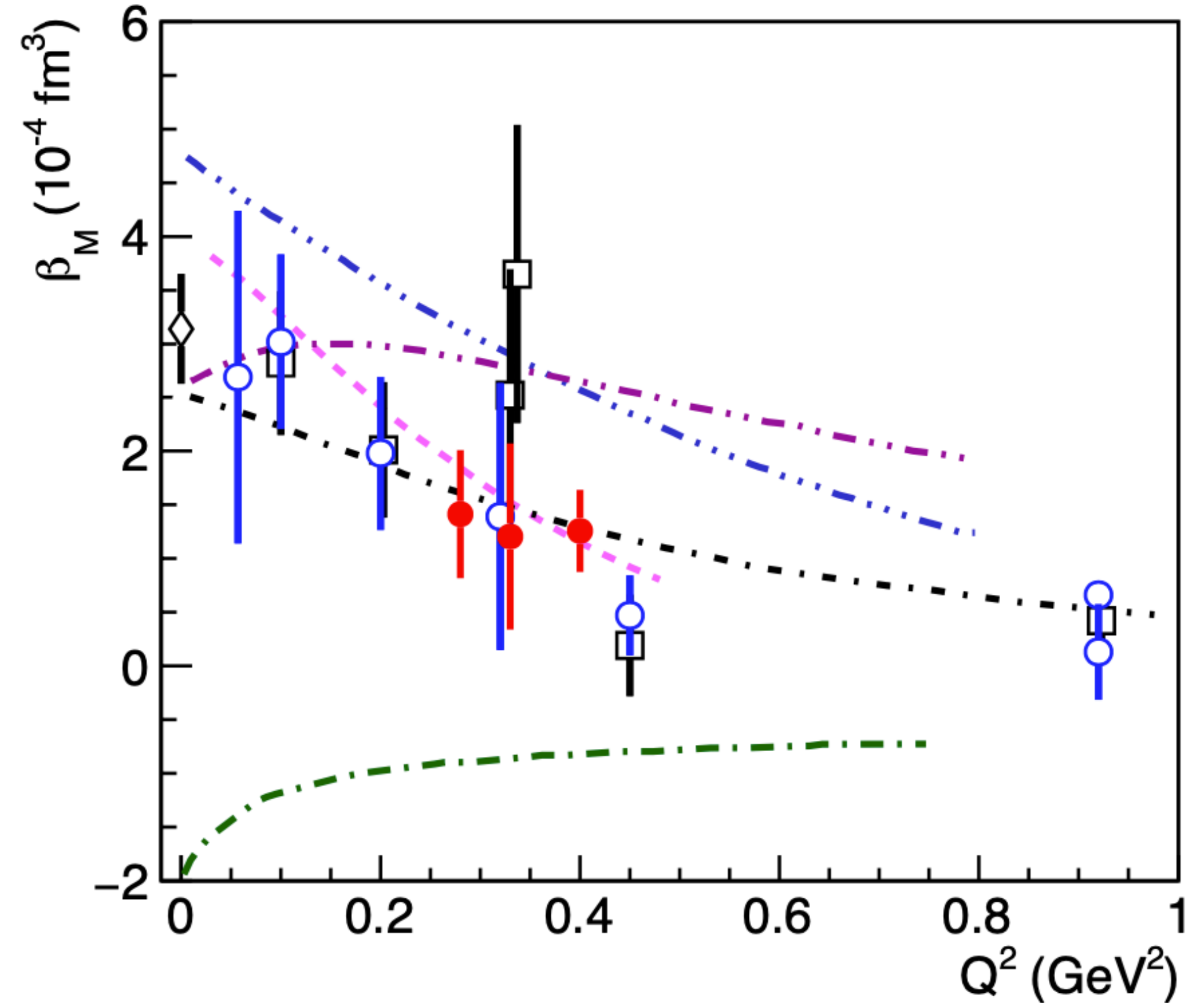


- Example of VCS in-plane cross-sections at $Q^2 = 0.33$ for different W bins
- Fitted final α_E & β_M are based on full data set for both in-plane and out-of-plane

VCS Analysis

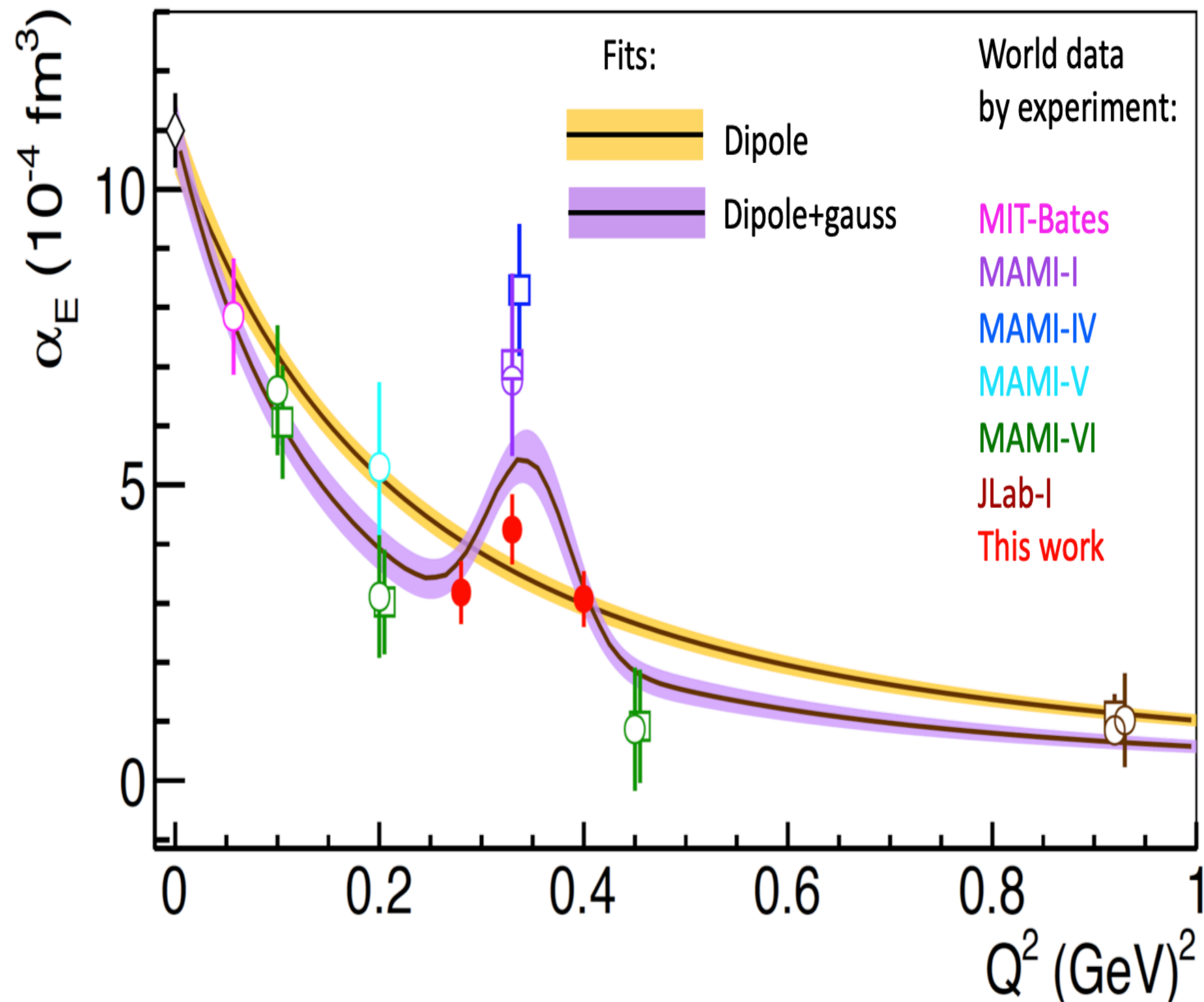


- **Local α_E enhancement in the measured region**
- **A smaller α_E magnitude with improved precision**



- β_M has a smooth Q^2 -dependence
- Near cancellation because of dia- and para-magnetic effects

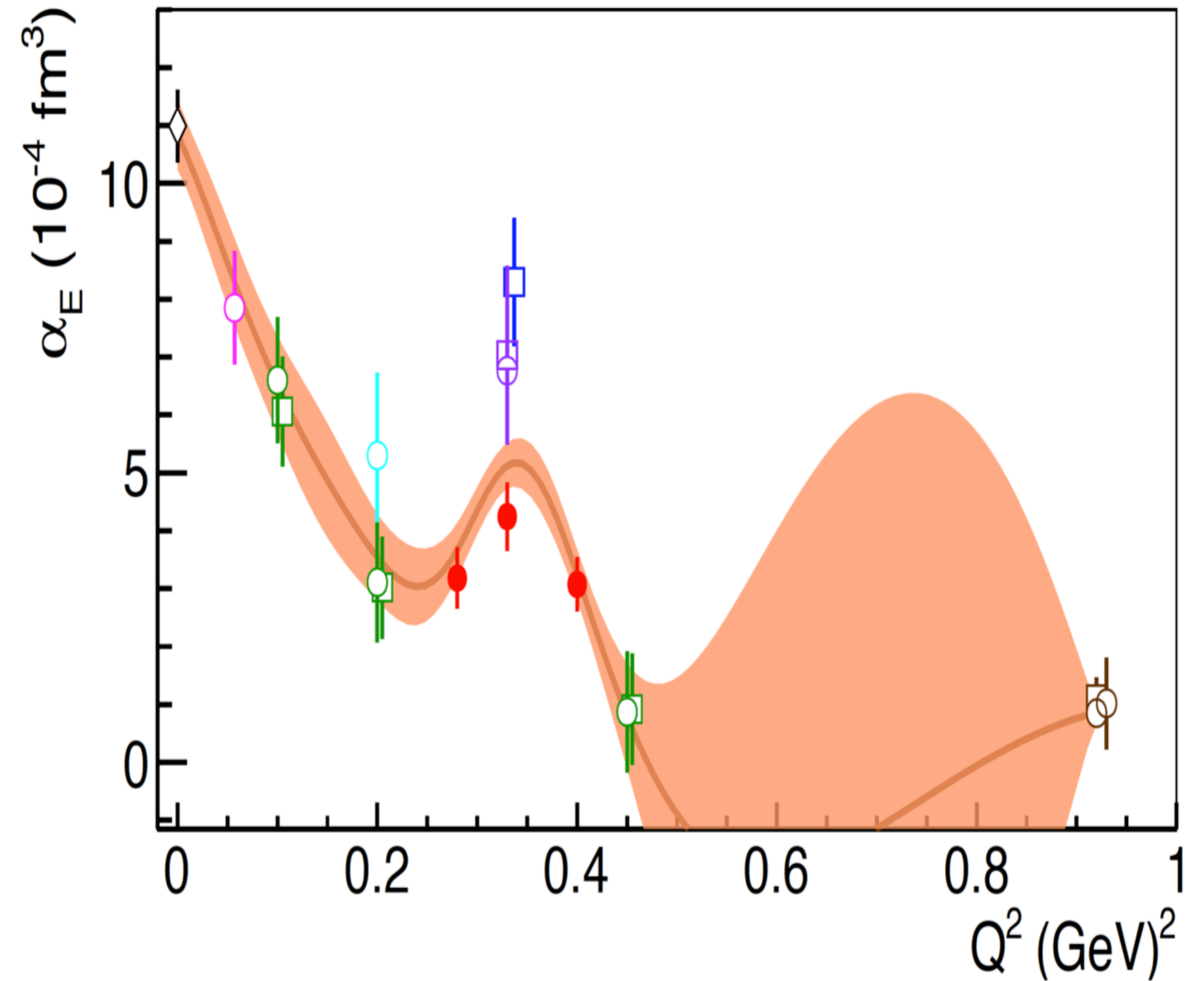
VCS Analysis



• Fits to the data using predefined functional form

- Dipole fit – $\chi^2_v = 3.7$
- Dipole+gauss fit – $\chi^2_v = 1.9$

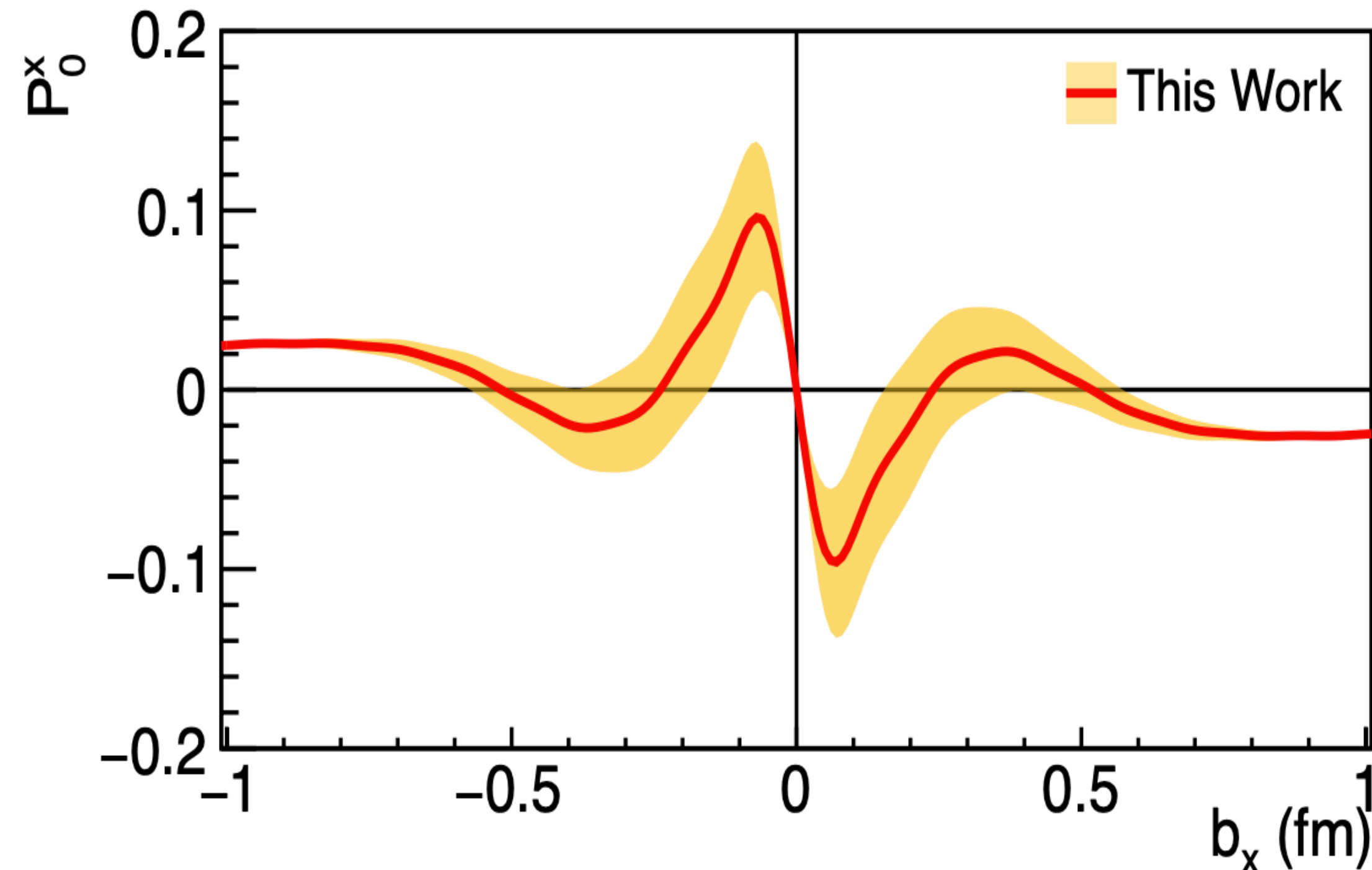
Figure credit: Hamza Atac



- Fits are based on a data-driven technique that assumes no direct underlying functional form
- The error bands correspond to the total uncertainty at 1σ level

Figure credit: Michael Paolone

Induced Polarization in the Proton



Induced polarization P_0^x in the proton versus the transverse position b_x when subject to an EM field with *photon polarization along the x-axis* for $b_y = 0$

$$\vec{P}_0(\vec{b}) = \hat{b} \int_0^\infty \frac{dQ}{(2\pi)} Q J_1(b Q) A(Q^2)$$

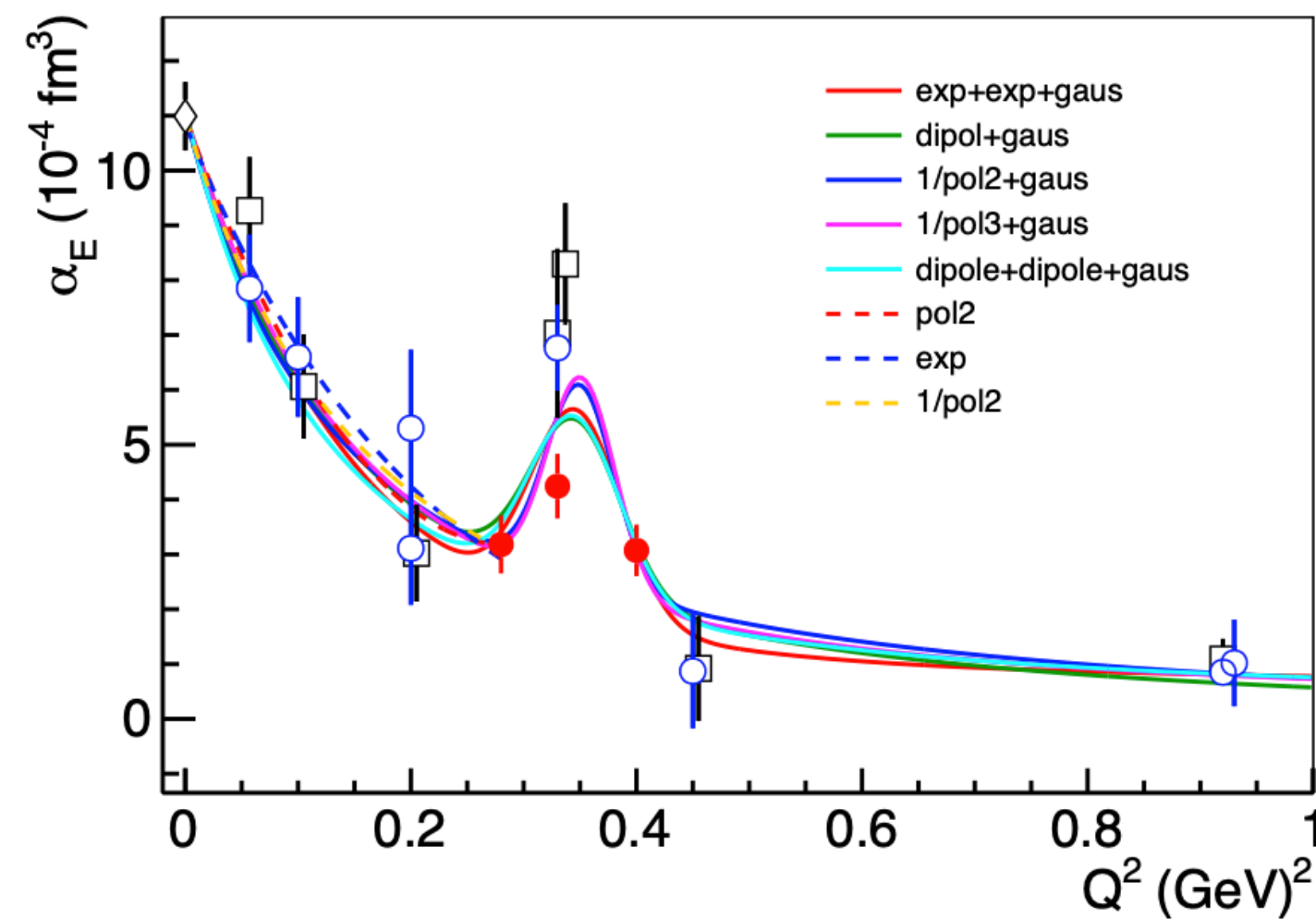
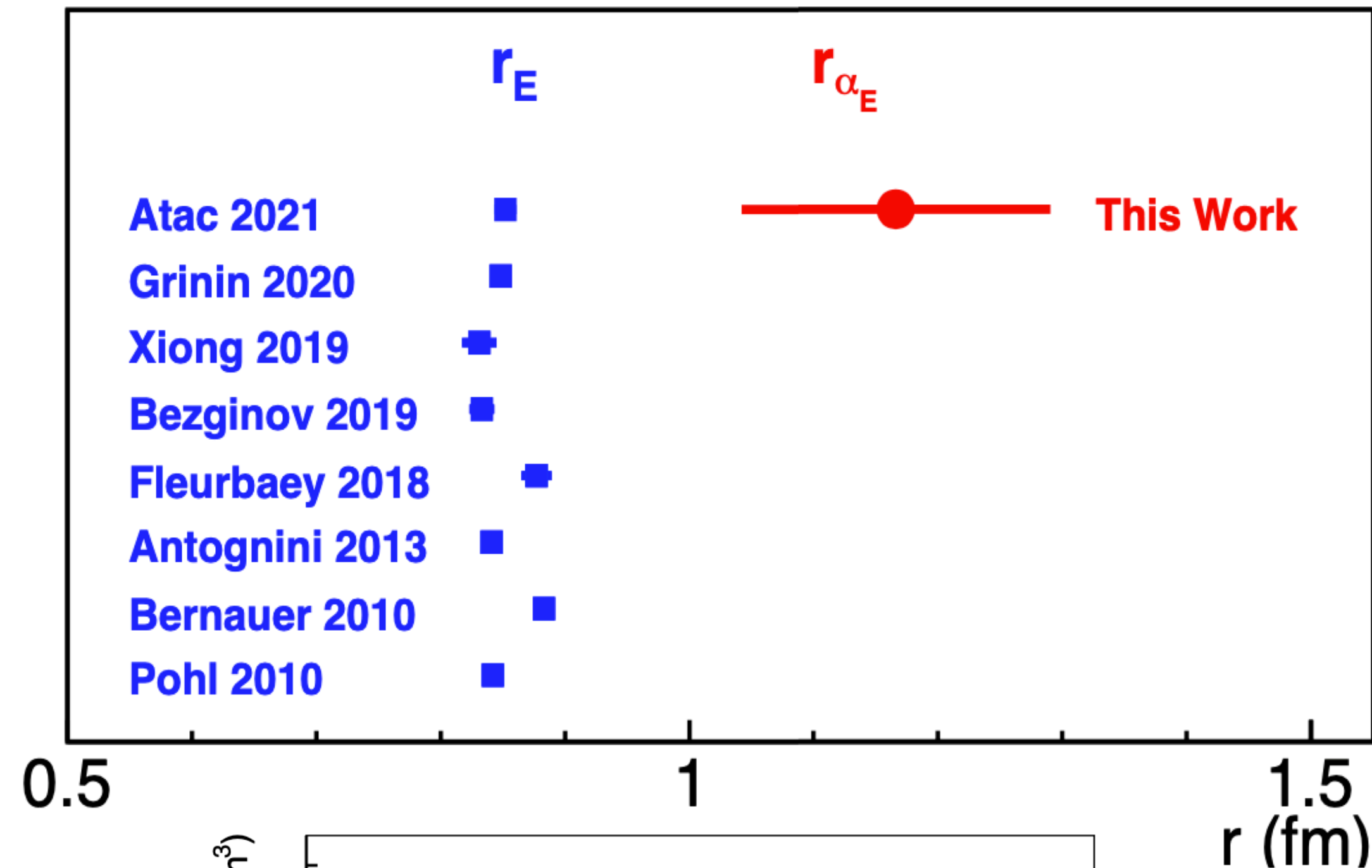
– \vec{b} is the transverse position, $\hat{b} = \vec{b}/|b|$

– J_1 is the 1st order Bessel function

– $A(Q^2)$ is a function of the GPs

- **Spatial distribution of P_0^x has distinct structure:**
 - **First change of sign – ~ 0.25 fm**
 - **A secondary maximum – ~0.35 fm**

Proton Mean Square Electric Polarizability radius



The proton mean square electric polarizability radius:

$$\langle r_{\alpha_E}^2 \rangle = \frac{-6}{\alpha_E(0)} \cdot \frac{d}{dQ^2} \alpha_E(Q^2) \Big|_{Q^2=0}$$

• The slope of α_E at $Q^2 = 0$ determined from fits of world data

• Multiple functional forms were used to provide a reliable fit

• $\alpha_E(0)$ from the most recent measurement – *Phys.Rev.Lett.* 128 (2022) 13, 132503

• $\langle r_{\alpha_E}^2 \rangle = 1.36 \pm 0.29 \text{ fm}^2$

• Larger than the proton mean square charge radius

Figure credit: Hamza Atac

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Summary

- **GPs are fundamental structure constants**
- **Data at $Q^2 = 0.33 \text{ GeV}^2$ implies a non-trivial structure**
- **New JLab E12-15-001 experiment measured α_E and β_M with improved precision at $Q^2 = 0.28 \text{ \& } 0.33 \text{ \& } 0.40 \text{ GeV}^2$**
- **A local enhancement of α_E is observed which is currently not accounted for in the theory**
- **More data points at $Q^2 > 0.4 \text{ GeV}^2$ will be proposed and measured**

The VCS paper is submitted to Nature and currently under review

People

Zulkaida Akbar, [Hamza Atac](#), Vladimir Berdnikov, Deepak Bhetuwal, Debaditya Biswas, [Marie Boer](#),
[Alexandre Camsonne](#), Jian-Ping Chen, Eric Christy, Arthur Conover, Markus Diefenthaler, Burcu Duran,
Dipangkar Dutta, Rolf Ent, [Dave Gaskell](#), Carlos Ayerbe Gayoso, Ole Hansen, Florian Hauenstein, Nathan
Heinrich, William Henry, Tanja Horn, Joshua Hoskins, Garth Huber, Shuo Jia, [Mark Jones](#), Sylvester
Joosten, Abishek Karki, Stephen Kay, Vijay Kumar, [Ruonan Li](#), Xiaqing Li, Wenliang Li, Anusha
Habarakada Liyanage, [Dave Mack](#), [Simona Malace](#), Pete Markowitz, Mike McCaughan, Hamlet Mkrtchyan,
Casey Morean, Mireille Muhoza, Amrendra Narayan, [Michael Paolone](#), Melanie Rehfuss, [Brad Sawatzky](#),
Andrew Smith, Greg Smith, [Nikolaos Sparveris](#), Richard Trotta, Carlos Yero, Xiaochao Zheng, Jingyi Zhou

Spokespersons

Run Coordinators

Post-docs

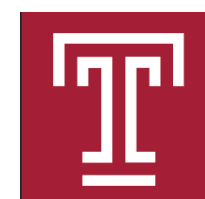
Graduate student

Thank You & Question Time

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Ruonan Li

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 Jefferson Lab



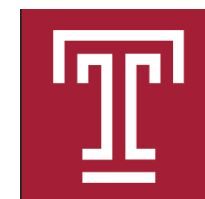
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Backup Slides

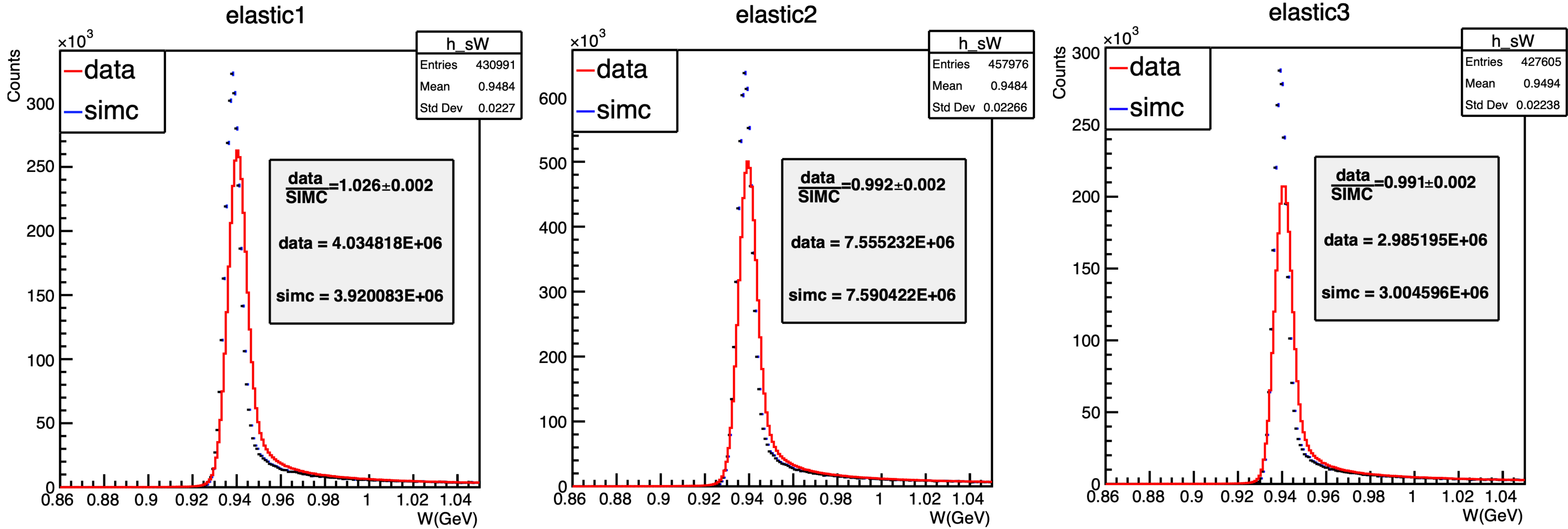
Temple University
Ruonan Li

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Elastic



Kinematic	θ_e°	$P_e(GeV/c)$	θ_p°	$P_p(GeV/c)$
Elastic I	10.76	4.193	61.16	0.893
Elastic II	10.41	4.214	61.95	0.863
Elastic III	9.64	4.259	63.76	0.795

Cuts:
 $0.85 < W < 1.05$

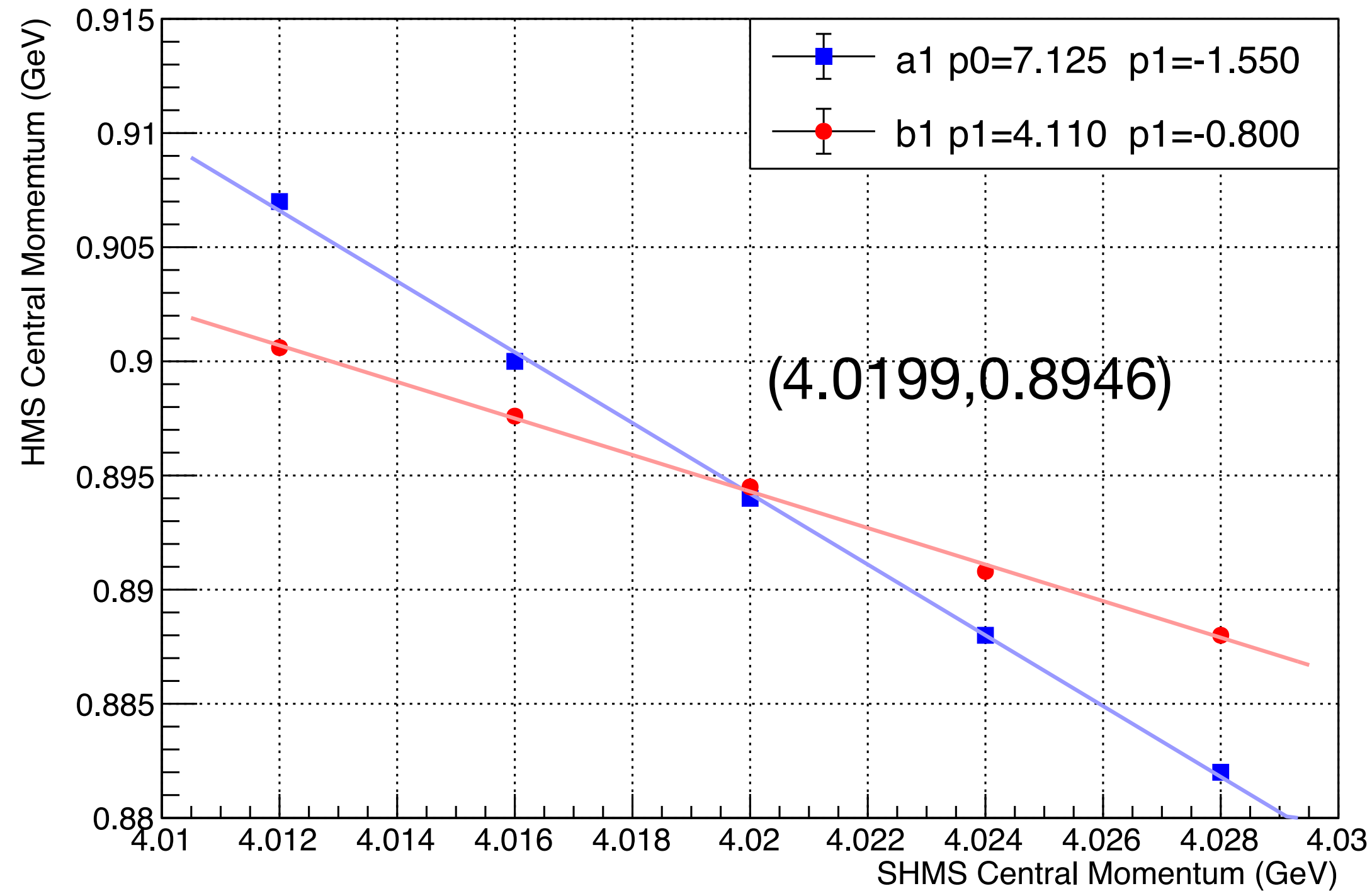
Energy Calibration

Spectrometer: Same momentum, Different HMS theta

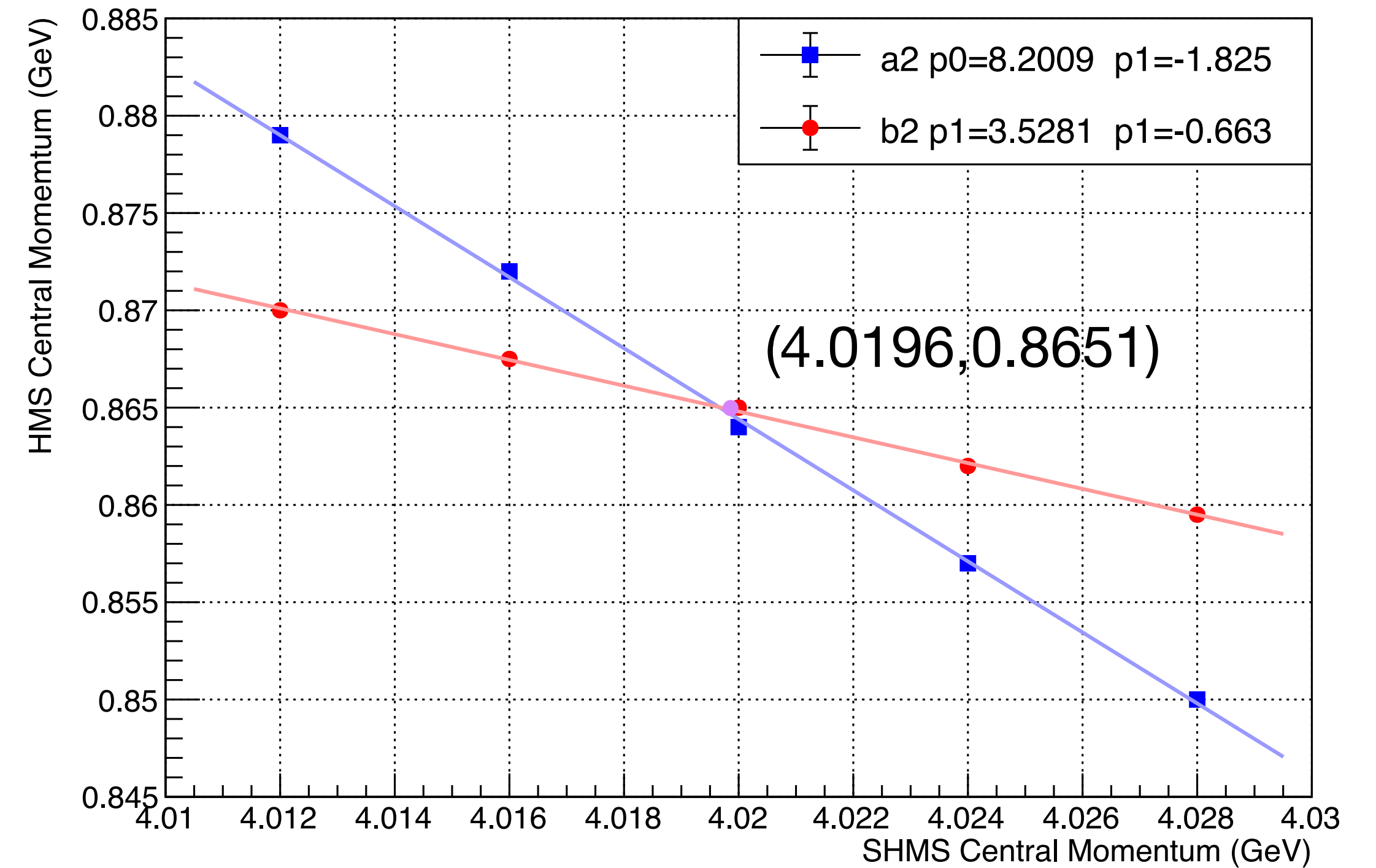
	SHMS_p	SHMS_th	HMS_p	HMS_th
Kin1a	4.034	7.69	0.893	37.33
Kin1b	4.034	7.69	0.893	51.40

	SHMS_p	SHMS_th	HMS_p	HMS_th
Kin2a	4.034	7.69	0.863	33.52
Kin2b	4.034	7.69	0.863	55.22

Kin 1a & Kin 1b

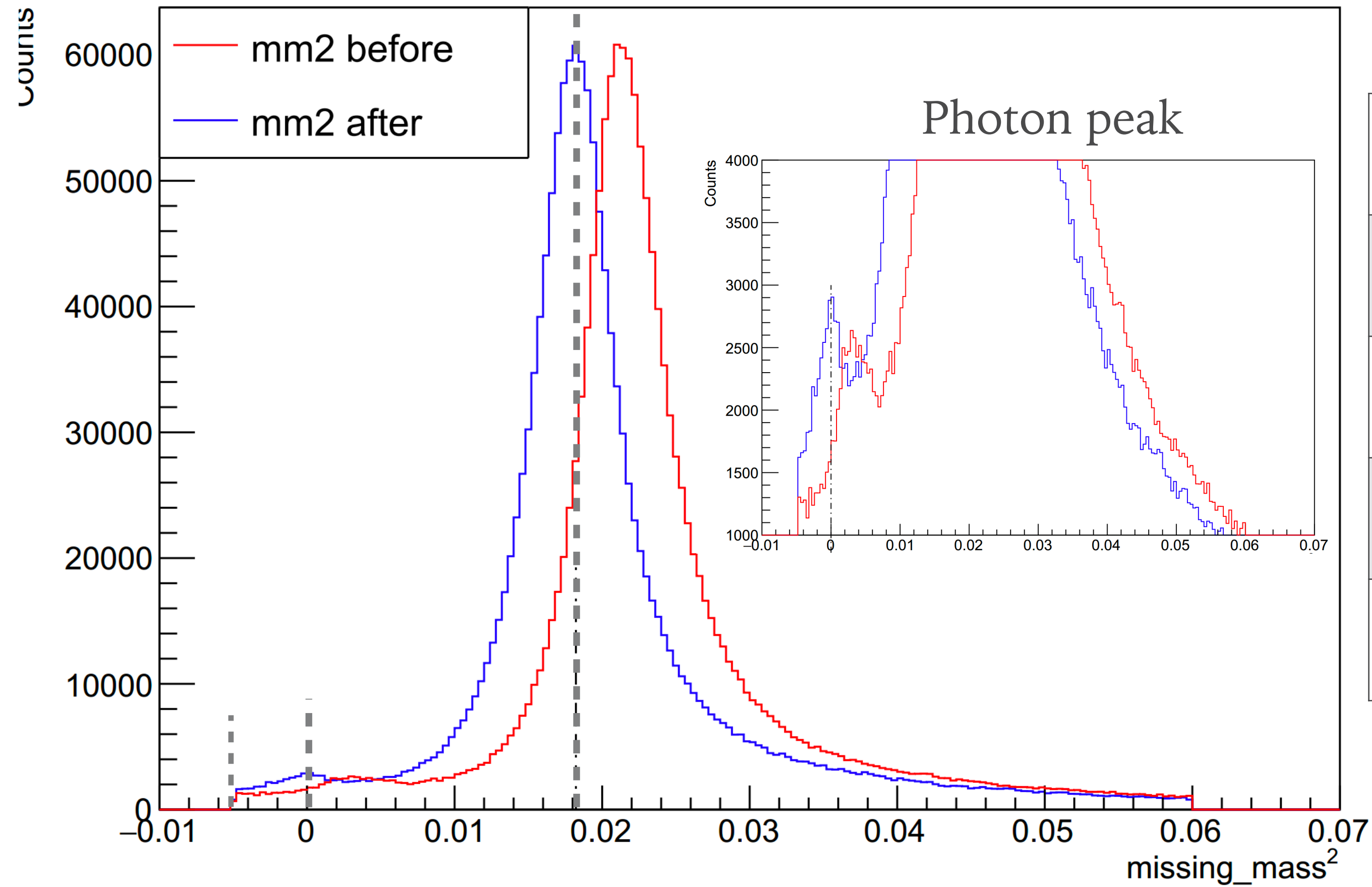


Kin 2a & Kin 2b



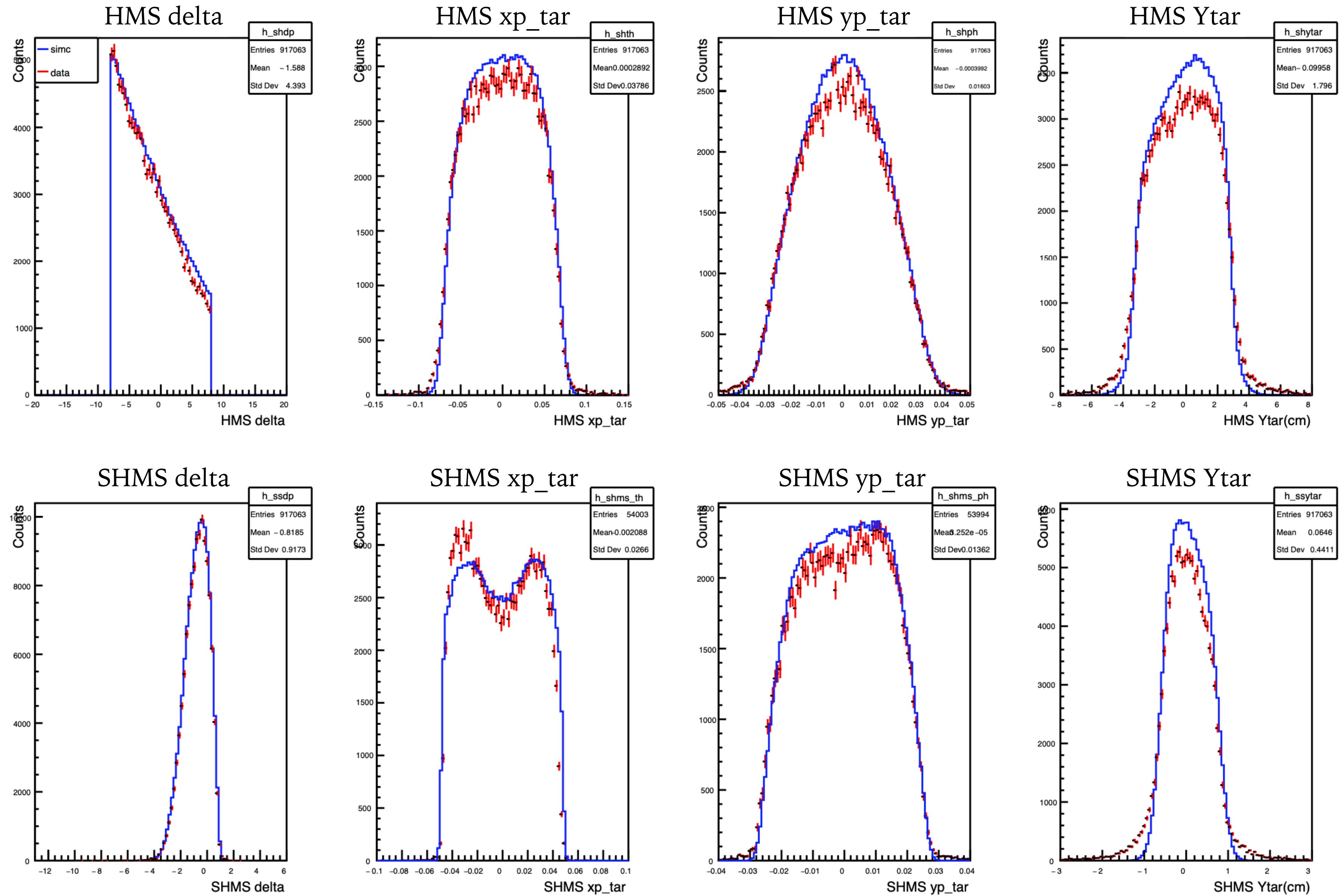
Energy Calibration

KIN 2b MM2



	SHMS_p Cal	SHMS_p Exp	Offset	HMS_p Cal	HMS_p Exp	Offset
Kin1a	4.0199	4.034	0.003	0.8946	0.893	0.002
Kin1b	4.0199	4.034	0.003	0.8946	0.893	0.002
Kin2a	4.0196	4.034	0.004	0.8651	0.863	0.002
Kin2b	4.0196	4.034	0.004	0.8651	0.863	0.002

Pion Preliminary Analysis



HCANA UPDATE

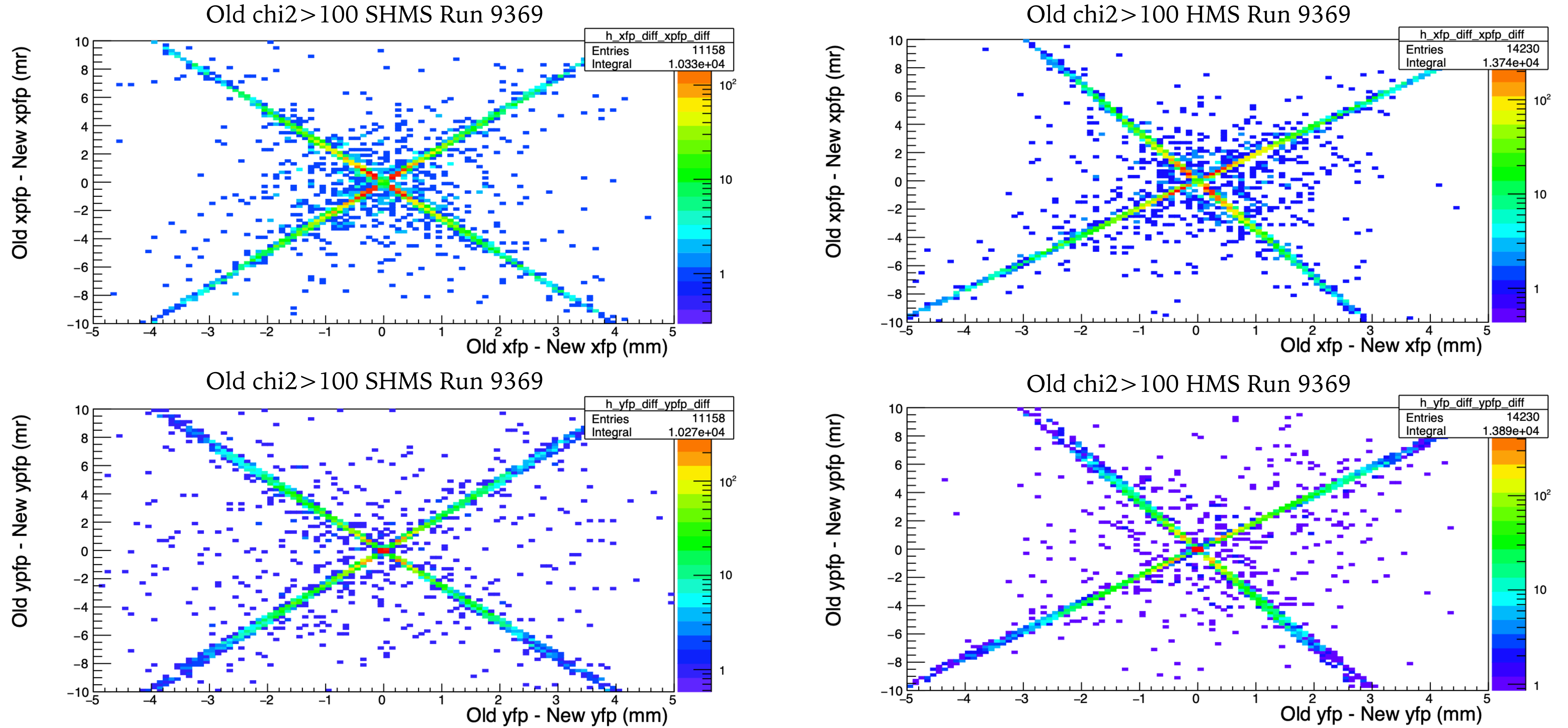


Figure Credit: Mark Jones