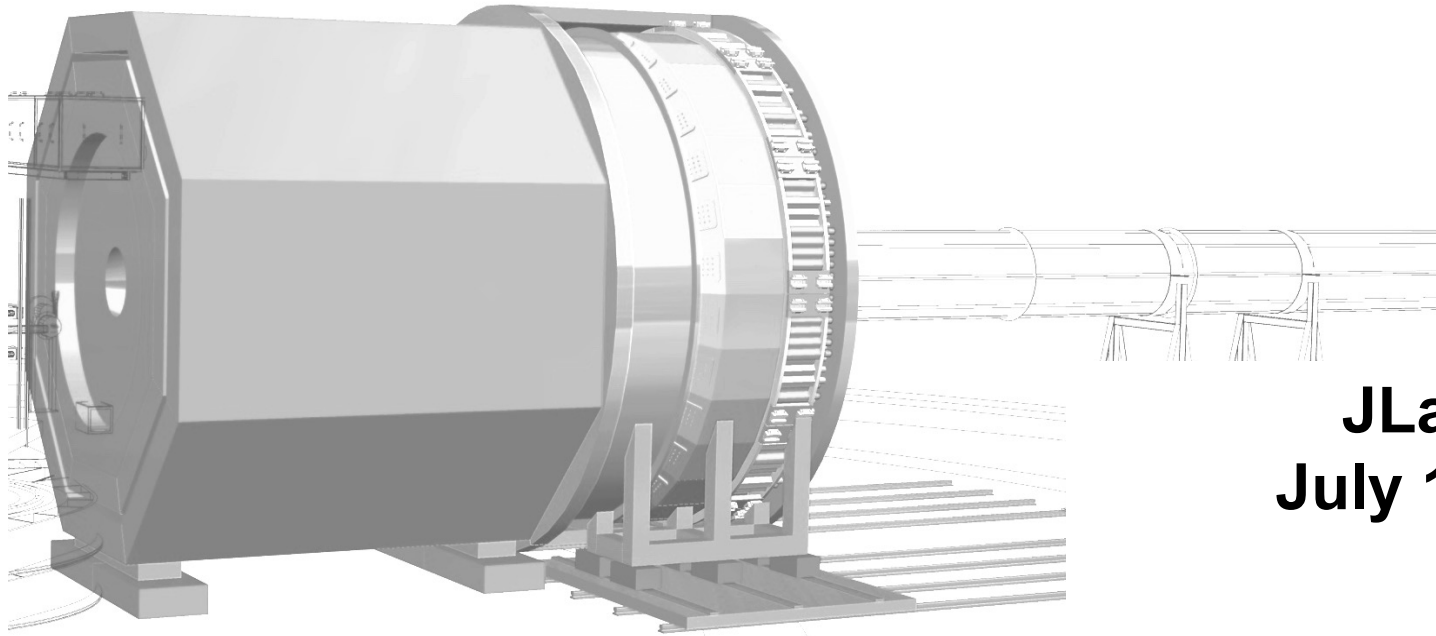


# *E12-10-006: Target Single Spin Asymmetry in Semi-Inclusive Deep-Inelastic $(e, e'\pi^\pm)$ Reaction on a Transversely Polarized $^3\text{He}$ Target at 8.8 and 11 GeV*



**JLab PAC50**  
**July 11- 15, 2022**

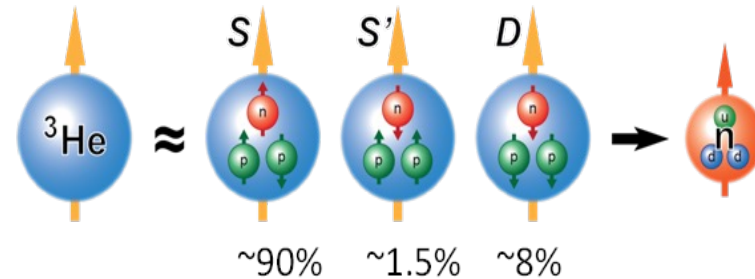
Haiyan Gao  
Duke University



On behalf of the E12-10-006  
and the SoLID Collaboration

# Outline

- Introduction
- SIDIS charged pion production from transversely polarized  $^3\text{He}$  target (“n”): E12-10-006:



- Run-group proposals
  - SIDIS di-hadron production: E12-10-006A
  - **SIDIS kaon production: E12-10-006D**
  - $A_y$  measurement: E12-10-006A
  - E12-10-006E on  $g_2^n$  and  $d_2^n$  measurements
  - **Deep exclusive meson production: E12-10-006B**
- Summary





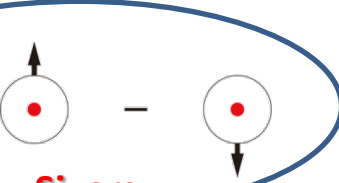


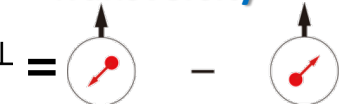
# Nucleon Structure from 1D to 3D – orbital motion & QCD dynamics

→ Nucleon Spin  
 → Quark Spin

		Quark polarization		
		Un-Polarized	Longitudinally Polarized	Transversely Polarized
Nucleon Polarization	U	$f_1 =$		$h_1^\perp =$ <b>Boer-Mulder</b>
	L		$g_1 =$ <b>Helicity</b>	$h_{1L}^\perp =$
	T	$f_{1T}^\perp =$ <b>Sivers</b>	$g_{1T}^\perp =$	$h_{1T} =$ <b>Transversity</b> $h_{1T}^\perp =$ <b>Pretzelosity</b>

# Nucleon Structure from 1D to 3D – orbital motion & QCD dynamics

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	T	$f_{1T}^\perp =$  <b>Sivers</b>	$g_{1T}^\perp =$ 	$h_{1T} =$  <b>Transversity</b> $h_{1T}^\perp =$  <b>Pretzelosity</b>

# TMDs – confined motion inside the nucleon

## Transversely Polarized Nucleon TMDs

- Nucleon Spin
- Quark Spin

## Transversity

$$h_{1T} = \begin{array}{c} \uparrow \\ \circ \uparrow \\ \text{---} \\ \circ \downarrow \\ \downarrow \\ \mathbf{S}_T \cdot \mathbf{S}_q \end{array}$$

## Relevant Vectors

- $\mathbf{S}_T$ : Nucleon Spin
- $\mathbf{s}_q$ : Quark Spin
- $\mathbf{k}_\perp$ : Quark Transverse Momentum
- $\mathbf{P}$ : Virtual photon 3-momentum (defines z-direction)

- $h_{1T}$  ( $h_1$ ) =  $g_1$  (no relativity)
- $h_{1T}$  → tensor charge (lattice QCD calculations)
- Connected to nucleon beta decay and EDM

## Sivers

$$f_{1T}^\perp = \begin{array}{c} \uparrow \\ \circ \bullet \\ \text{---} \\ \circ \bullet \\ \downarrow \\ \mathbf{S}_T \cdot \mathbf{k}_\perp \times \mathbf{P} \end{array}$$

- Nucleon spin - quark orbital angular momentum (OAM) correlation – zero if no OAM (model dependence)

## Pretzelosity

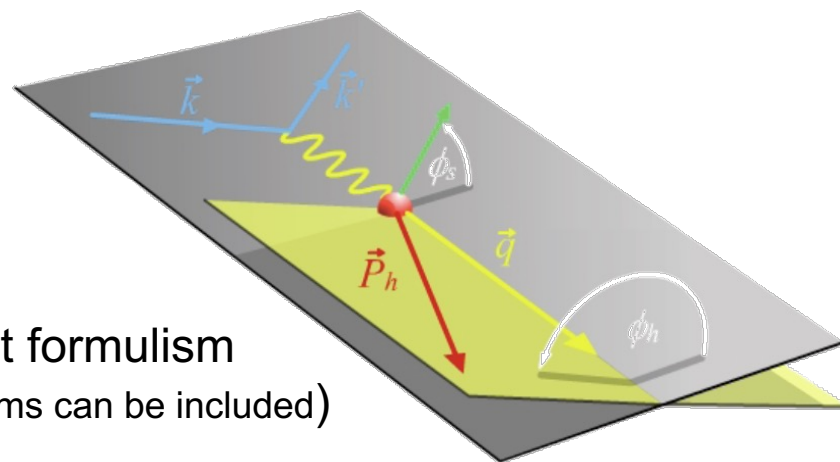
$$h_{1T}^\perp = \begin{array}{c} \uparrow \\ \circ \nearrow \\ \text{---} \\ \circ \searrow \\ \downarrow \\ \mathbf{S}_T \cdot [\mathbf{k}_\perp \mathbf{k}_\perp] \cdot \mathbf{s}_{qT} \end{array}$$

- Interference between components with OAM difference of 2 units (i.e., s-d, p-p) (model dependence)
- Signature for relativistic effect

# Separation of Collins, Sivers and Pretzelosity through angular dependence

SIDIS SSAs depend on 4-D variables ( $x, Q^2, z, P_T$ ) and small asymmetries demand **large acceptance + high luminosity** allowing for measuring symmetries in 4-D binning with precision!

( $2\pi$  azimuthal coverage)



$$A_{UT}(\phi_h, \phi_S) = \frac{1}{P_{t,pol}} \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}$$

Leading twist formulism  
(higher-twist terms can be included)

$$= \underbrace{A_{UT}^{Collins}}_{\text{purple}} \sin(\phi_h + \phi_S) + \underbrace{A_{UT}^{Pretzelosity}}_{\text{blue}} \sin(3\phi_h - \phi_S) + \underbrace{A_{UT}^{Sivers}}_{\text{green}} \sin(\phi_h - \phi_S)$$

$$\boxed{A_{UT}^{Collins}} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^\perp$$

Collins fragmentation function from  $e^+e^-$  collisions

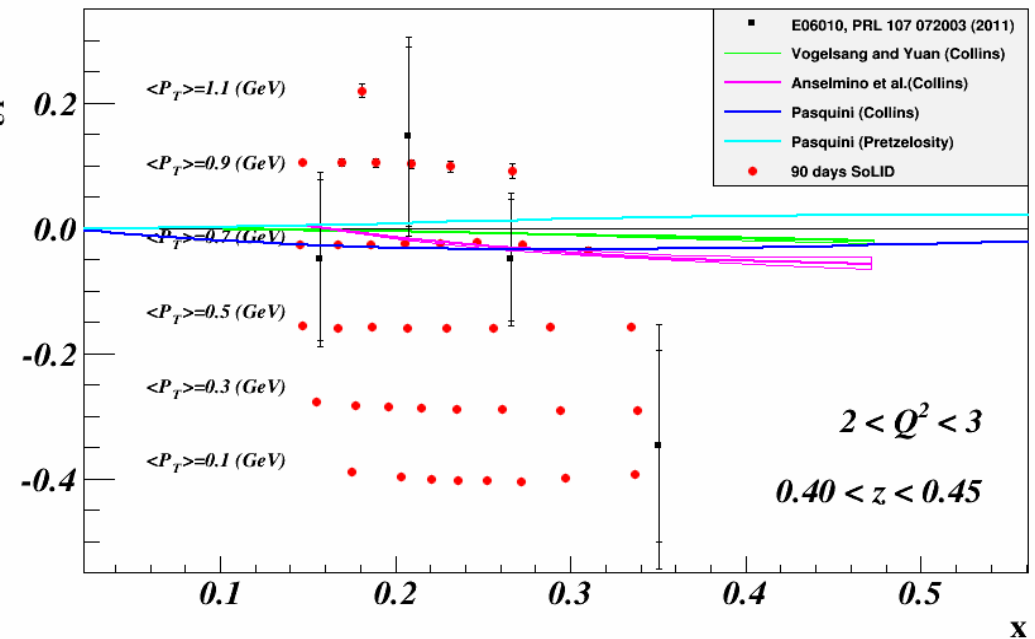
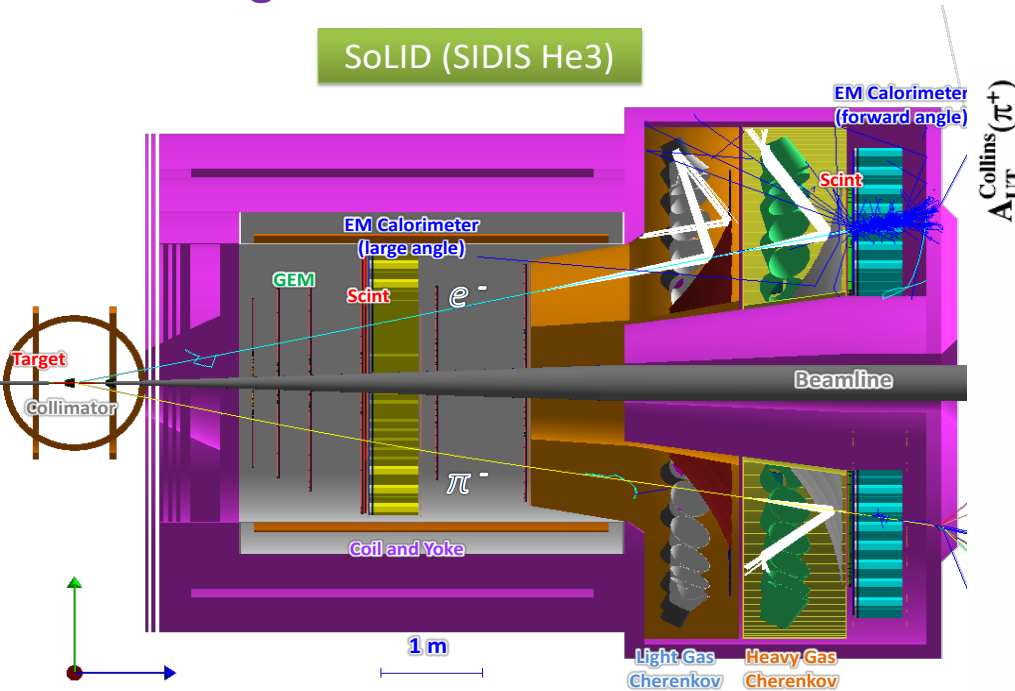
$$\boxed{A_{UT}^{Pretzelosity}} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^\perp \otimes H_1^\perp$$

$$\boxed{A_{UT}^{Sivers}} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^\perp \otimes D_1$$

Unpolarized fragmentation function

# QCD intensity frontier with SoLID: large-acceptance & high luminosity

**E12-10-006:** Single Spin Asymmetries on Transversely Polarized  $^3\text{He}$  @ 90 days  
**Rating A** Spokespersons: J.P. Chen, H. Gao (contact), J.C. Peng, X. Qian



SoLID-SIDIS program: Large acceptance, Full azimuthal coverage + High luminosity

- 4-D mapping of asymmetries with precision  
 $\Delta z = 0.05$ ,  $\Delta P_T = 0.2 \text{ GeV}$ ,  $\Delta Q^2 = 1 \text{ GeV}^2$ ,  $x$  bin sizes vary with median bin size 0.02 (typical stat. uncertainty for each bin:  $\delta A \leq 0.005$ )
- Constrain models and forms of TMDs, Tensor charge, ...
- Lattice QCD, QCD dynamics, models

*JLab 6-GeV experiment on SSA using  $A$  transversely polarized  $^3\text{He}$*   
*X. Qian et al., PRL107, 072003(2011)*

**SoLID: more than 1400 bins in  $x$ ,  $Q^2$ ,  $P_T$  and  $z$  for 11/8.8 GeV beam.**

# SoLID-SIDIS Measurements

- Deep inelastic kinematics at 8.8 GeV and 11 GeV incident electron beam energies
  - Coincidence detection of electrons and charged pions
  - Good electron PID and moderate charged pion PID
- Single and double spin asymmetries and flavor separation
  - $^3\text{He}$  target with both transverse and longitudinal in-beam polarizations of  $\sim 60\%$
  - Electron beam with polarization  $\sim 85\%$  allows both single and double spin asymmetries
- Small asymmetries, 4-dimensional binning and high precision require high luminosity (polarized)  $\sim 10^{36} \text{ cm}^{-2} \text{ s}^{-1}$  ( $^3\text{He}$ ), and large acceptance
- Extracting various azimuthal angular dependences and suppressing systematic uncertainties require full azimuthal coverage
- Four-dimensional binning in  $(x, z, Q^2 \text{ and } P_T)$ : requires reasonably good momentum and angular resolutions
  - GEM detectors provide excellent tracking capability
- The capability to handle high rates and backgrounds associated with high luminosity and large acceptance
  - DAQ rate: less than 100 KHz

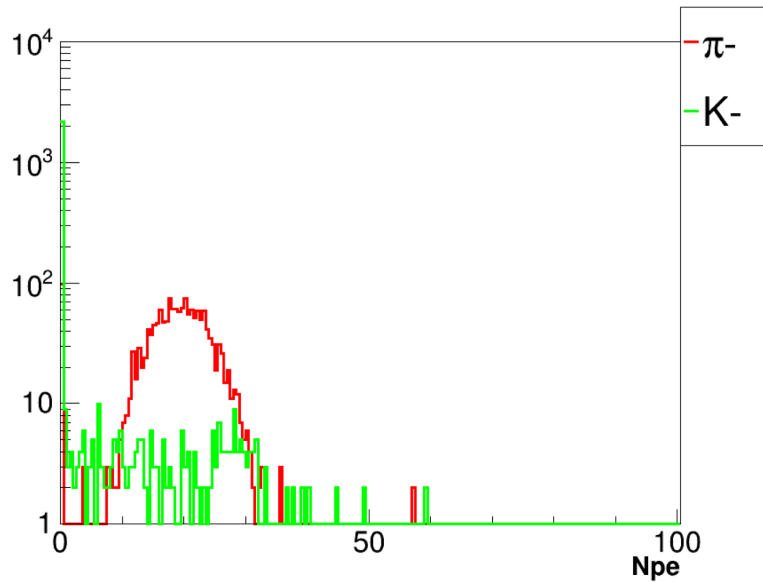


# Key Parameters and Requirement for SIDIS

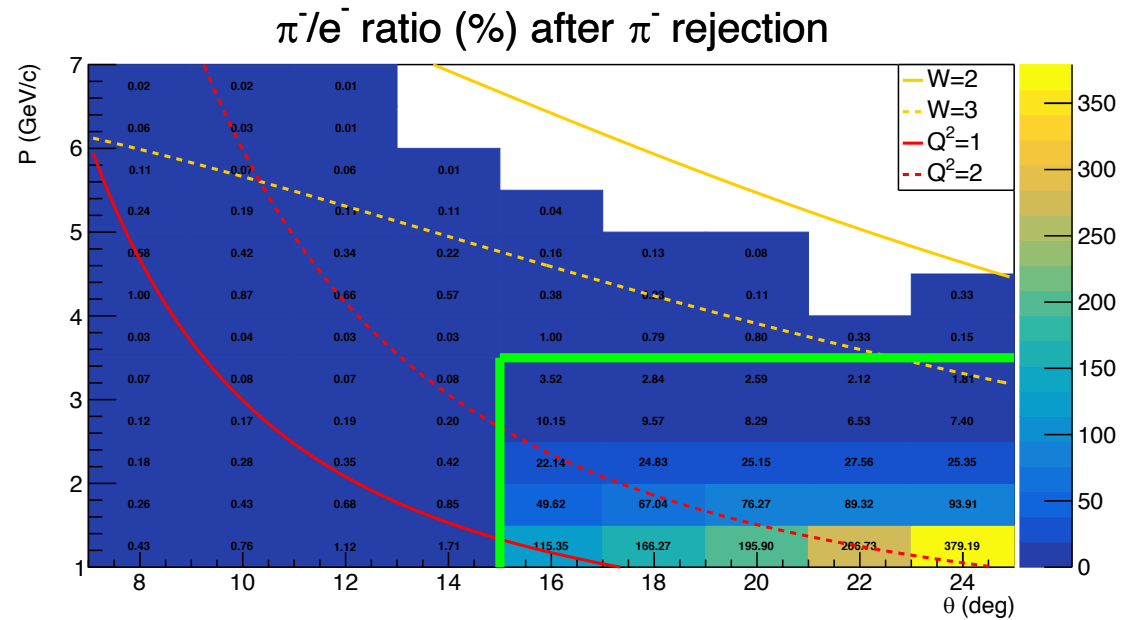
Key parameters	SIDIS $^3\text{He}$
Reaction channel	$(e, e' \pi^\pm)$
Unpol. Luminosity ( $/\text{cm}^2/\text{s}$ )	$1e37$
Momentum coverage (GeV/c)	1-7
Momentum resolution	$\sim 2\%$
Polar angular coverage (deg)	8 - 24
Polar/Azimuthal angle resolution	2/6 mr
PID ( $e^-$ )	Detection eff. $> 90\%$ Pion contam. $< 1\%$
PID ( $\pi^\pm$ )	Detection eff. $> 90\%$ Kaon contam. $< 1\%$
Trigger type	Coincidence $e^- \pi^\pm$
Expected DAQ rate	$< 100$ kHz
Backgrounds	$(e, \pi^- \pi^\pm)$ $(e, e' K^\pm)$
Major Requirements	Radiation hardness Kaon contamination DAQ

# SoLID-SIDIS and Subsystems

Heavy gas Cherenkov: pion efficiency  $\sim 90\%$   
with kaon rejection factor of 10

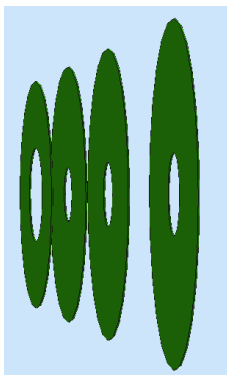


Combined light gas Cherenkov and  
Calorimeter detector performance

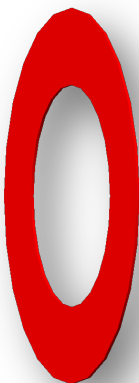


SIDIS&J/ $\psi$  :

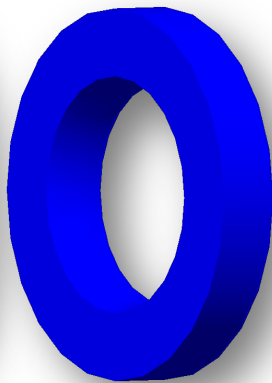
4xGEMs



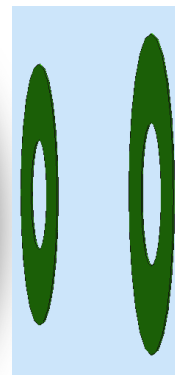
LASPD



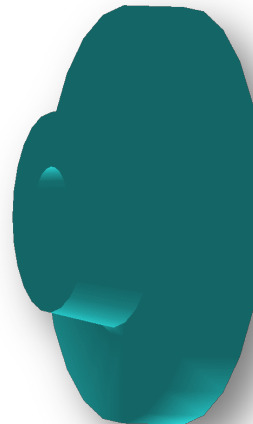
LAEC



2xGEMs



LGC



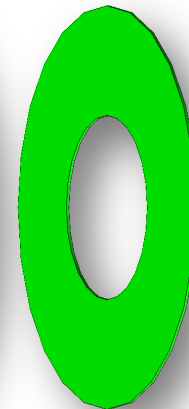
HGC



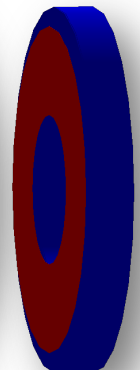
FASPD



MRPC



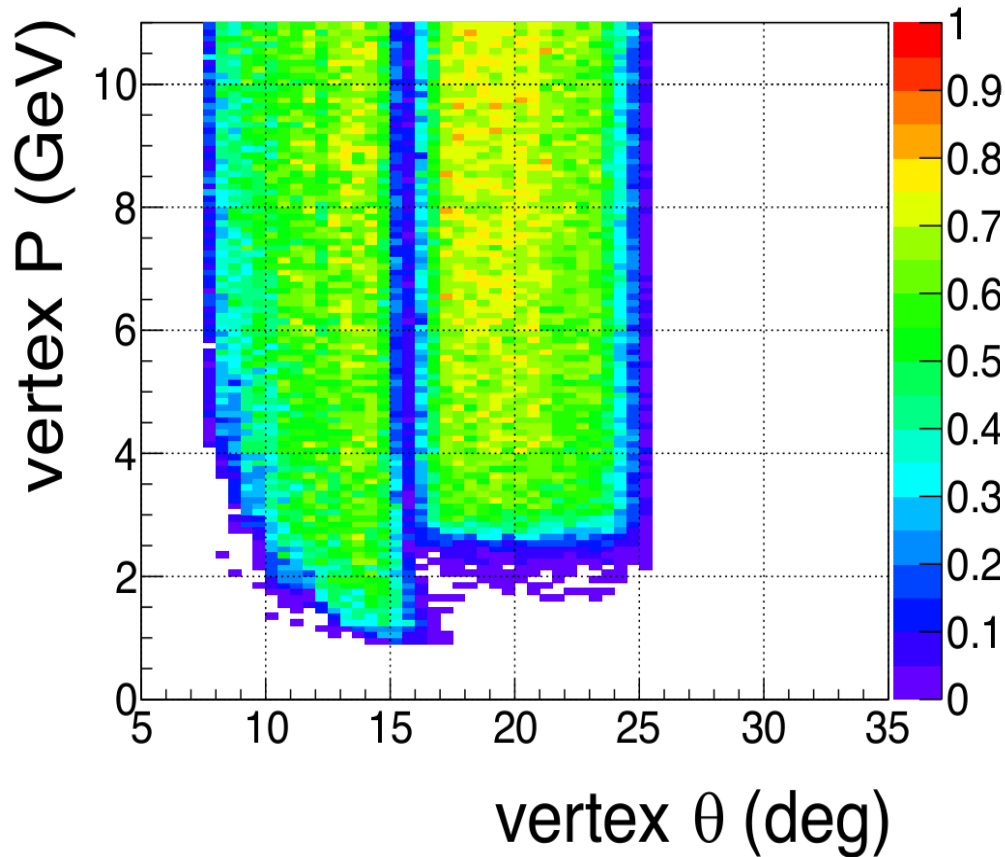
FAEC



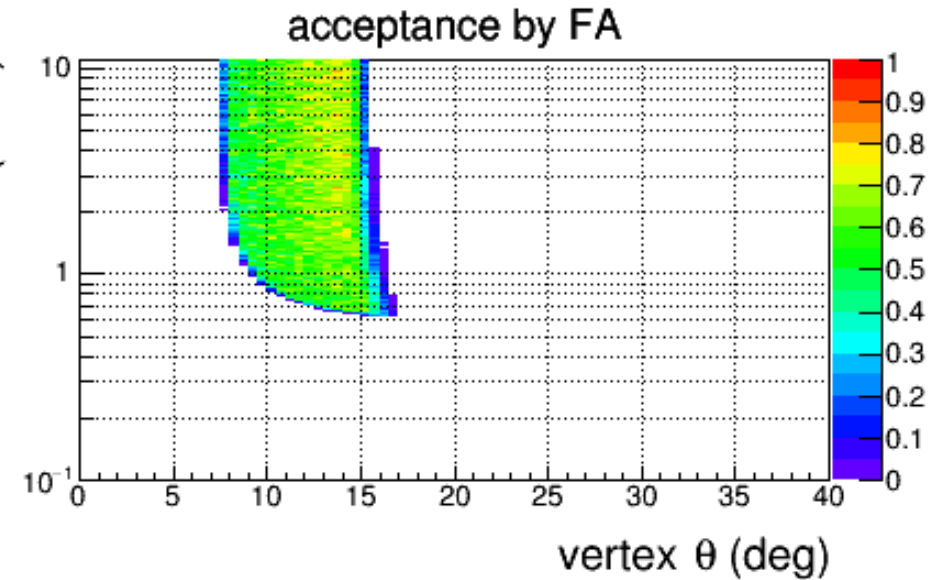
MRPC for kaon and improved pion identification

# SoLID-SIDIS acceptance & efficiency

SIDIS electron acceptance & efficiency



$\pi$  acceptance



charged pion acceptance:  $8^\circ < \theta < 15^\circ$ ,  
full  $2\pi$  azimuthal angle

Combined effect of acceptance and efficiency  
(except tracking)

# SoLID-SIDIS: Systematic Uncertainties

## Addressing various systematic uncertainties

- *Raw asymmetries*: control the syst. uncertainties corresponding to detector efficiencies (time dependent part) by monitoring the single  $e^-$ ,  $\pi^+$ ,  $\pi^-$  rates
- *Target polarization*: knowledge of the target pol. at 3% level  $\rightarrow$  a 3% rel. syst. uncertainty of the SSAs
- *Random coincidence*: obtained from the signal to noise ratio and background within 6 ns timing window
- *Diffraction meson*: the pion contribution from the diffractive production decay estimated based on HERMES tuned Pythia at SoLID SIDIS kinematics
- *Radiative corrections*: the effect simulated with HAPRAD (QED one-loop level) & more (see next talk)
- *Detector resolution*: estimated based on the track fitting studies
- *Nuclear effects*: estimated based on theoretical calculations of the neutron SSA extraction at SoLID SIDIS kinematics

# Same Beam Time Request

- We request the same approved beam time as before:  
**90 (days)**
- 69 days on the transversely polarized  $^3\text{He}$  target
- 10 days for a dedicated study of the x-z factorization with Hydrogen and Deuterium using a reference target cell
- 3 days with a longitudinal target polarization to study the systematics of potential  $A_{UL}$  contamination
- 8 days of overhead time for regular target polarization measurements, spin flip, target changes, beam energy change, etc.

# SoLID SIDIS Projection

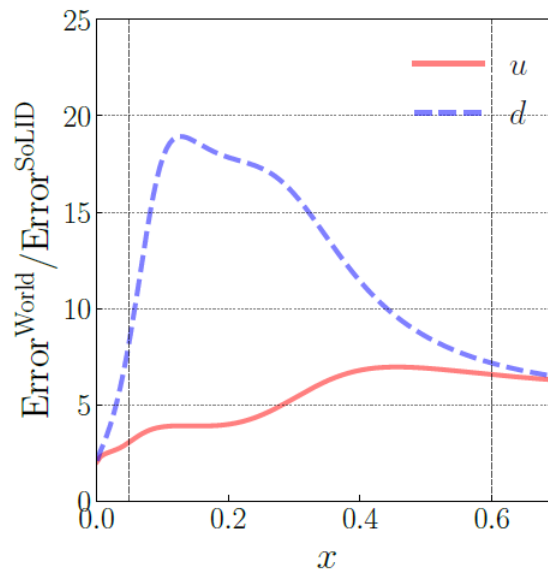
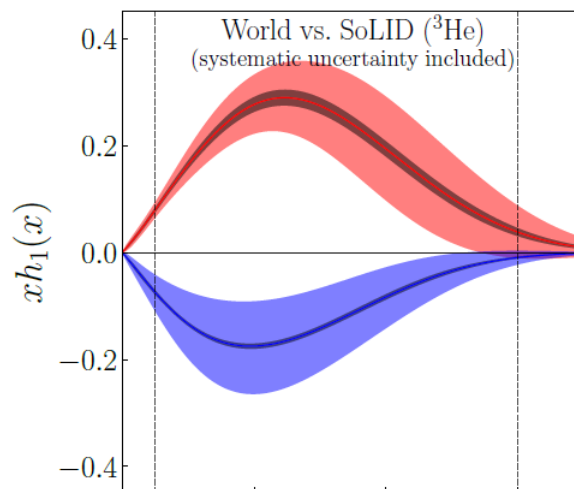
## Compare SoLID $^3\text{He}$ with World Data

- Fit Collins and Sivers asymmetries in SIDIS and  $e^+e^-$  annihilation
- World data from HERMES, COMPASS
- $e^+e^-$  data from BELLE, BABAR, and BESIII
- Monte Carlo method is applied
- Including both systematic and statistical uncertainties

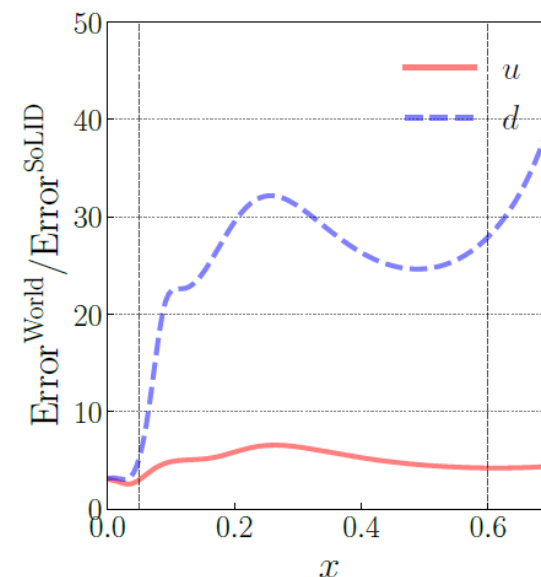
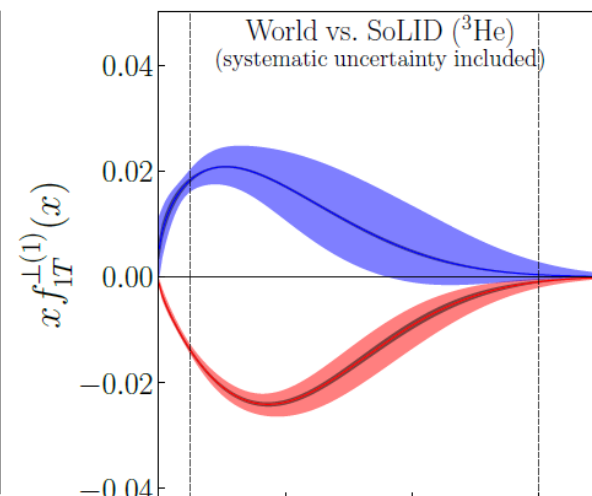
World data according to SoLID preCDR (2019) <https://solid.jlab.org/experiments.htm>

D'Alesio et al., Phys. Lett. B 803 (2020) 135347  
Anselmino et al., JHEP 04 (2017) 046

### Transversity

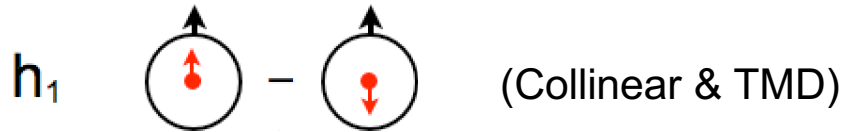


### Sivers



# Transversity and Tensor Charge

## Transversity distribution

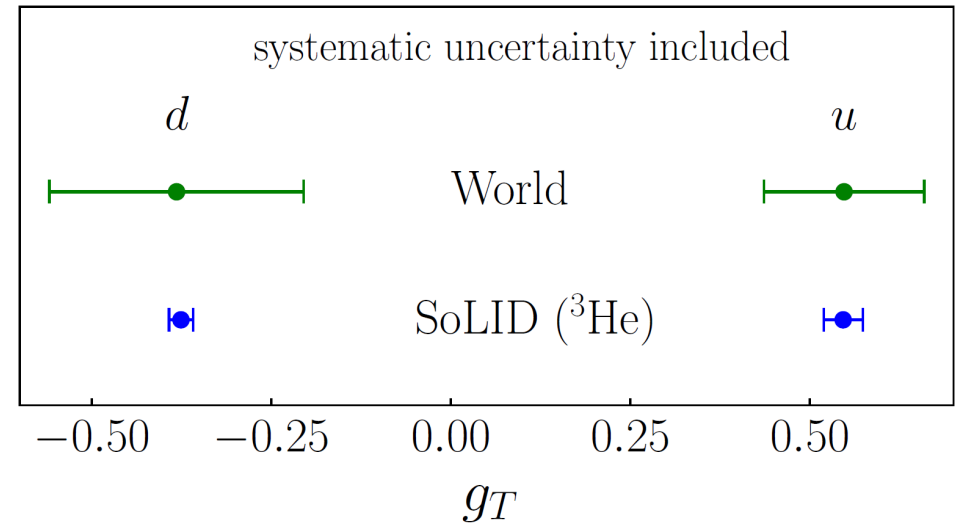


- Chiral-odd, unique for the quarks
- No mixing with gluons, simpler evolution effect
- Tensor charge:

$$\langle P, S | \bar{\psi}_q i\sigma^{\mu\nu} \psi_q | P, S \rangle = g_T^q \bar{u}(P, S) i\sigma^{\mu\nu} u(P, S)$$

$$g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)] dx$$

- A fundamental QCD quantity dominated by valence quarks
- Precisely calculated on the lattice
- Difference from nucleon axial charge is due to relativity



$g_T$ Flavor separation	World data	SoLID ( $^3\text{He}$ )
$u / d$ value	0.548 / -0.382	0.547 / -0.376
$u / d$ uncertainty	0.112 / 0.177	0.027 / 0.017

### SoLID projection: statistical and systematic uncertainties included

- Tensor charge also connected to neutron and proton EDMs, unique opportunity for SM tests and new physics

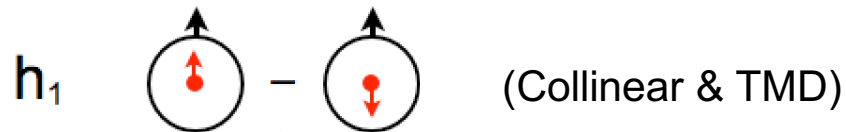
$$d_n = g_T^d d_u + g_T^u d_d + g_T^s d_s$$

Z. Ye *et al*, Phys. Lett. B 767, 91 (2017)

H. Gao, T. Liu and Z. Zhao, PRD 97, 074018 (2018)

# Transversity and Tensor Charge

## Transversity distribution

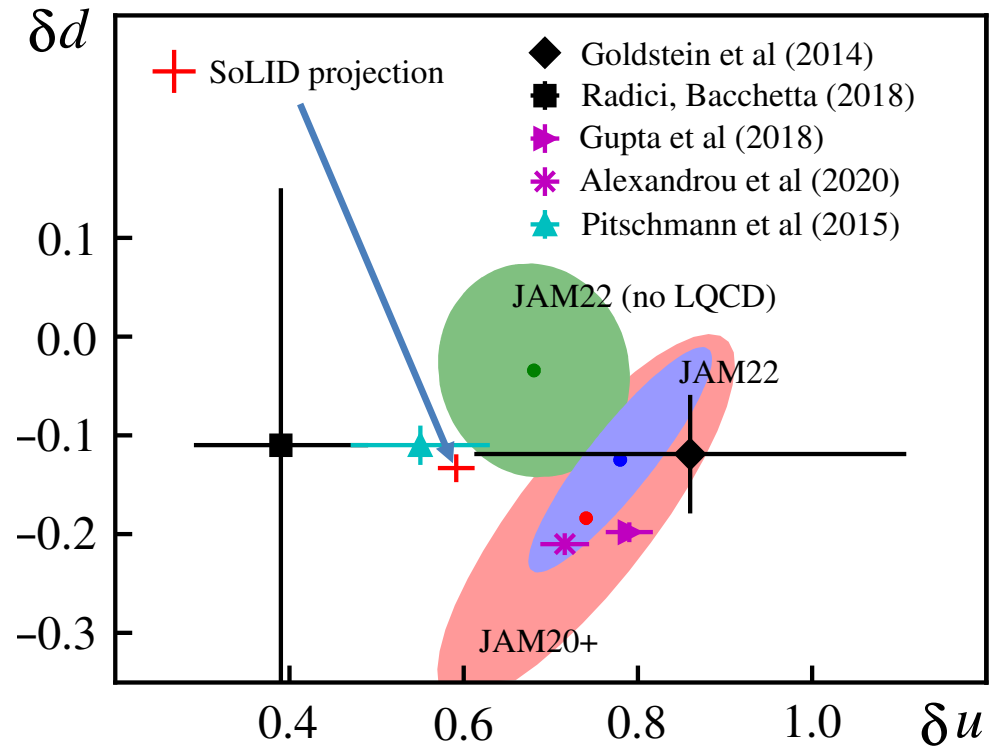


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- A fundamental QCD quantity dominated by valence quarks
- Precisely calculated on the lattice
- Difference from nucleon axial charge is due to relativity
- SoLID measurements allows for high-precision test of LQCD predictions
- Global analysis including LQCD (PRL 120 (2018) 15, 152502)



Combining E12-10-006 & E12-11-108

**SoLID projection: statistical and systematic uncertainties included (shifted for visibility)**

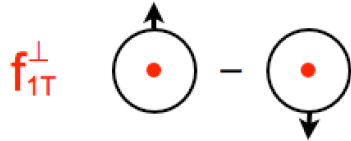
J. Cammarota et al, PRD 102, 054002 (2020) (JAM20+)

L. Gamberg et al., arXiv:2205.00999 (JAM22)



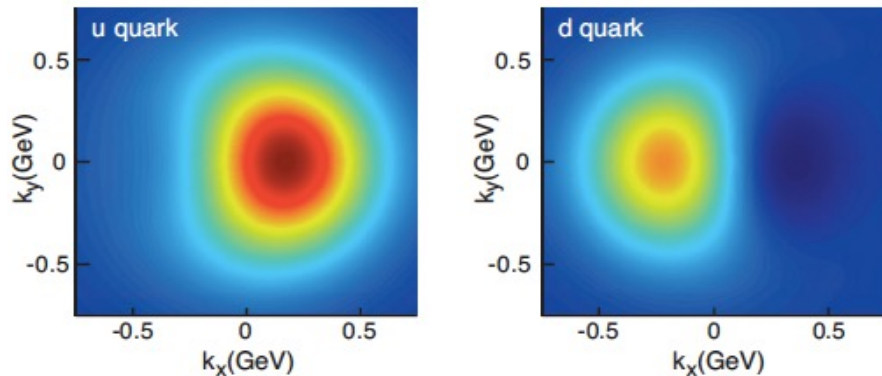
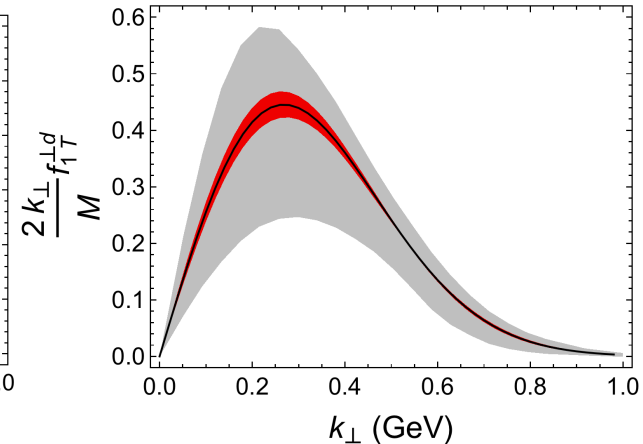
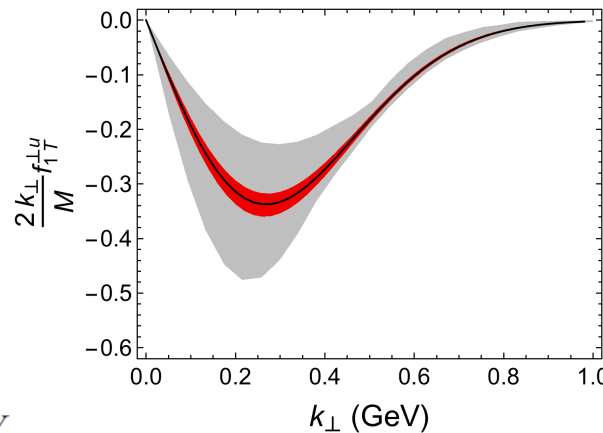
# TMDs – confined motion inside the nucleon

## Sivers distribution



naively time-reversal odd

$$f_{1T}^{\perp q}(x, k_\perp) \Big|_{\text{SIDIS}} = -f_{1T}^{\perp q}(x, k_\perp) \Big|_{\text{DY}}$$



$$f_{q/p\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, k_\perp) - f_{1T}^{\perp q}(x, k_\perp) \frac{\hat{\mathbf{P}} \times \mathbf{k}_\perp \cdot \mathbf{S}}{M}$$

$$\langle \mathbf{k}_\perp \rangle = -M \int dx f_{1T}^{\perp(1)}(x) (\mathbf{S} \times \hat{\mathbf{P}})$$

Parametrization by M. Anselmino et al., EPJ A 39, 89 (2009)  
 SoLID projection with transversely polarized n/p

Nucleon spin - quark orbital angular momentum (OAM) correlation  
 – zero if no OAM (collinear, massless quarks)

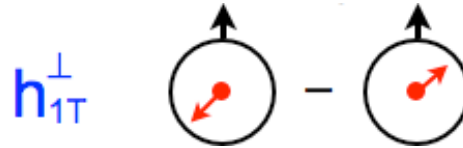
	$\langle k_\perp \rangle^u$	$\langle k_\perp \rangle^d$
<span style="display: inline-block; width: 15px; height: 15px; background-color: grey; margin-right: 5px;"></span>	$96_{-28}^{+60}$ MeV	$-113_{-51}^{+45}$ MeV
<span style="display: inline-block; width: 15px; height: 15px; background-color: red; margin-right: 5px;"></span>	$96_{-2.4}^{+2.8}$ MeV	$-113_{-1.7}^{+1.3}$ MeV

Combining E12-10-006 & E12-11-108

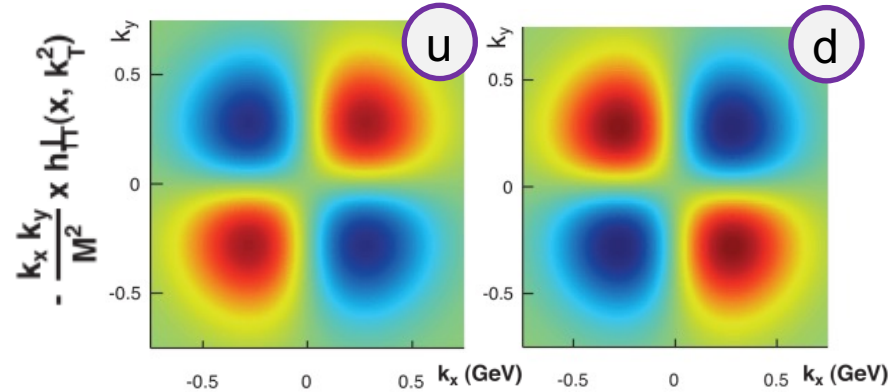
Exact finding is model dependent but SoLID impact is model-independent!

# TMDs – confined motion inside the nucleon

## Pretzelocity distribution



- Chiral-odd, no gluon analogy
- Quadrupole modulation of parton density in the distribution of transversely polarized quarks in a transversely polarized nucleon
- Measuring the difference between helicity and transversity (relativistic effects)



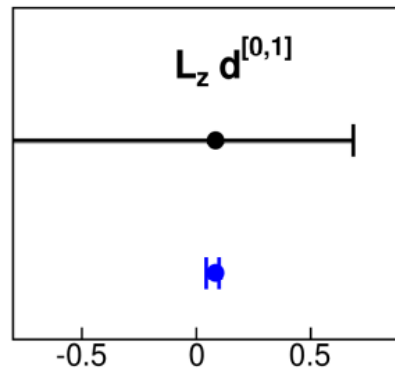
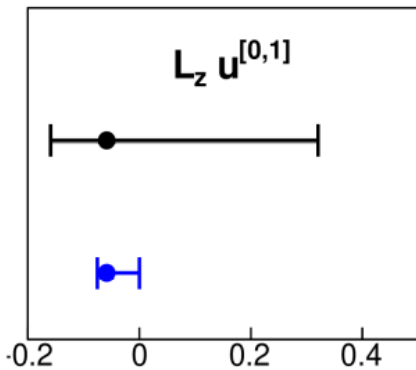
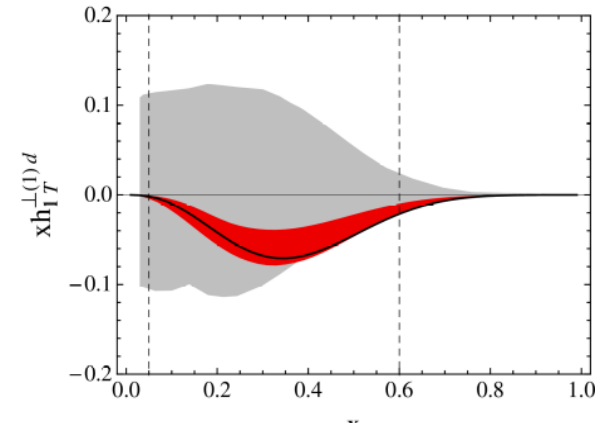
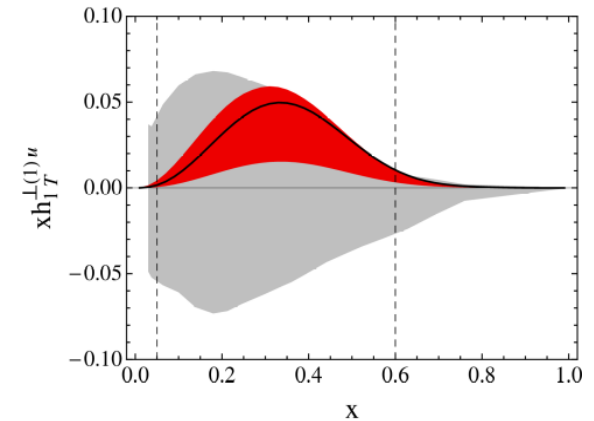
Images from PRD 91 034010 (2015)

Parametrization by C. Lefky et al., PRD 91, 034010 (2015)

SoLID projection with transversely polarized n and p data

### Relation to OAM (canonical)

$$L_z^q = - \int dx d^2\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{2M^2} h_{1T}^{\perp q}(x, k_\perp) = - \int dx h_{1T}^{\perp(1)q}(x)$$



Lefky and Prokudin  
PRD 91, 034010 (2015)

SoLID projection

# ***SoLID run group experiments with E12-10-006***

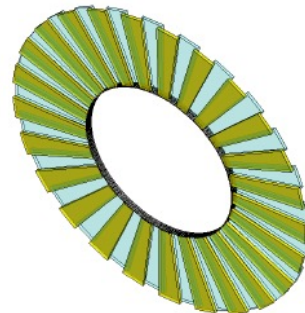
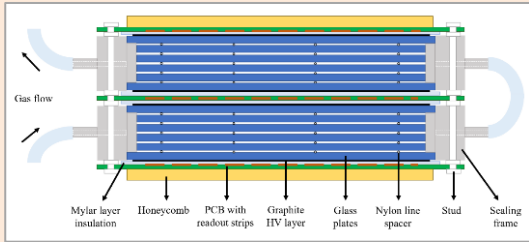
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- **SIDIS Dihadron with Transversely Polarized  $^3\text{He}$**   
J.-P. Chen, A. Courtoy, H. Gao, A. W. Thomas, Z. Xiao, J. Zhang, Approved as run group (E12-10-006A)
- **SIDIS in Kaon Production with Transversely Polarized  $^3\text{He}$**   
T. Liu, S. Park, Z. Ye, Y. Wang, Z.W. Zhao, Approved as run group (E12-10-006D)
- **Ay with Transversely Polarized  $^3\text{He}$**   
T. Averett, A. Camsonne, N. Liyanage, Approved as run group (E12-10-006A)
- **$g_2^n$  and  $d_2^n$  with Transversely and Longitudinally Polarized  $^3\text{He}$**   
C. Peng, Y. Tian, Approved as run group (E12-10-006E)
- **Deep exclusive  $\pi^-$  Production with Transversely Polarized  $^3\text{He}$**   
Z. Ahmed, G. Huber, Z. Ye, Approved as run group (E12-10-006B)

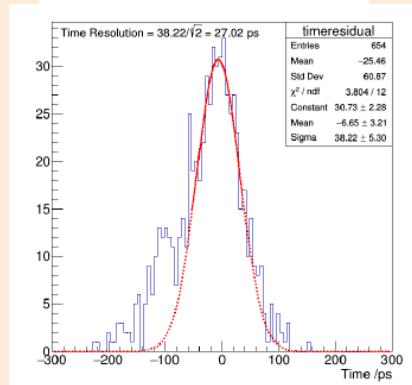
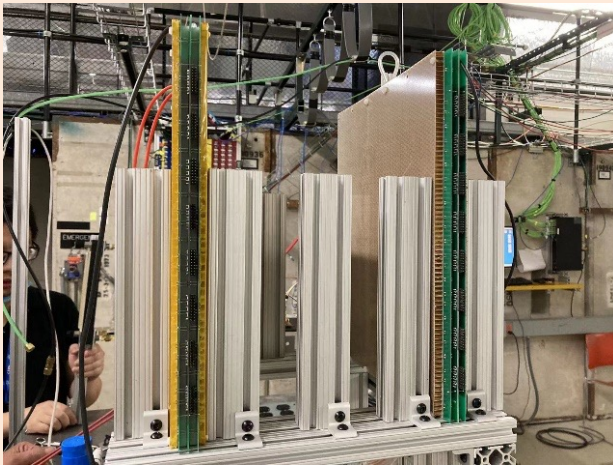
## ■ $K^\pm$ production in SIDIS using both the transversely polarized $^3\text{He}$ and $\text{NH}_3$ Targets

- ✓ Extract  $K^\pm$  Collins, Sivers and other TMD asymmetries
- ✓ Flavor decomposition of u, d and sea quarks' TMDs
- ✓ Kaon detection: 30ps MRPC

### Kaon Identification: 30ps TOF + Veto of HGC

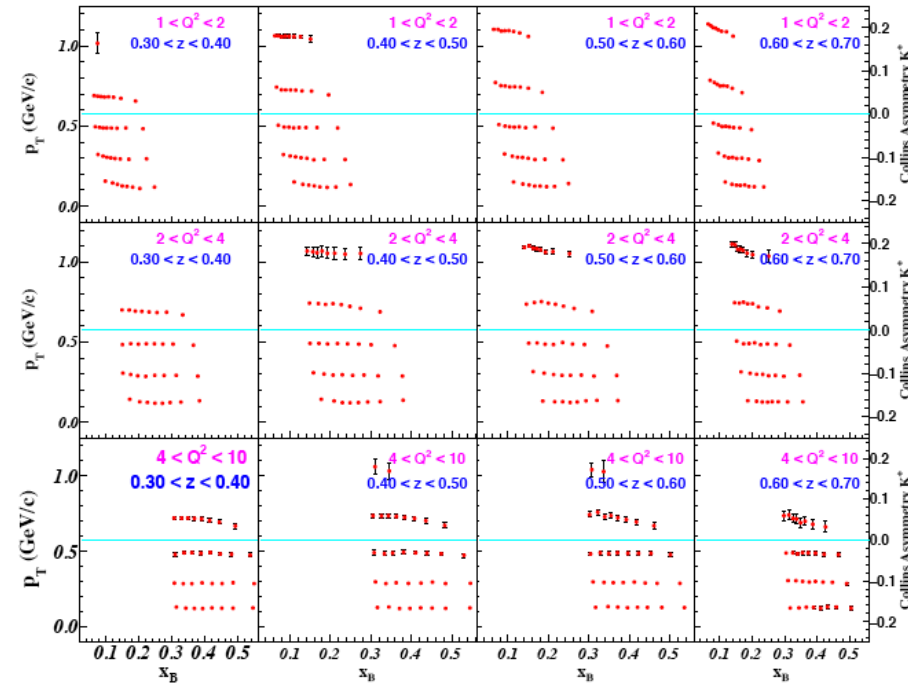


(a) The layout of the MRPC



MRPC beam test at Fermilab (Tsinghua+UIC)

## Projection on Collins $K^+$



(a)  $\bar{n}(e, e' K^+) X$

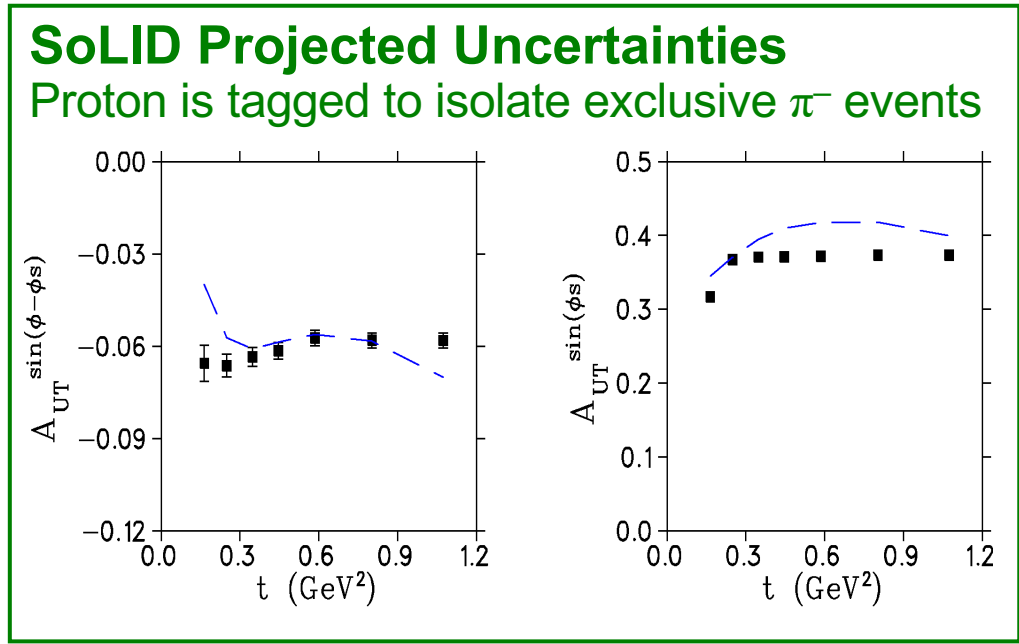
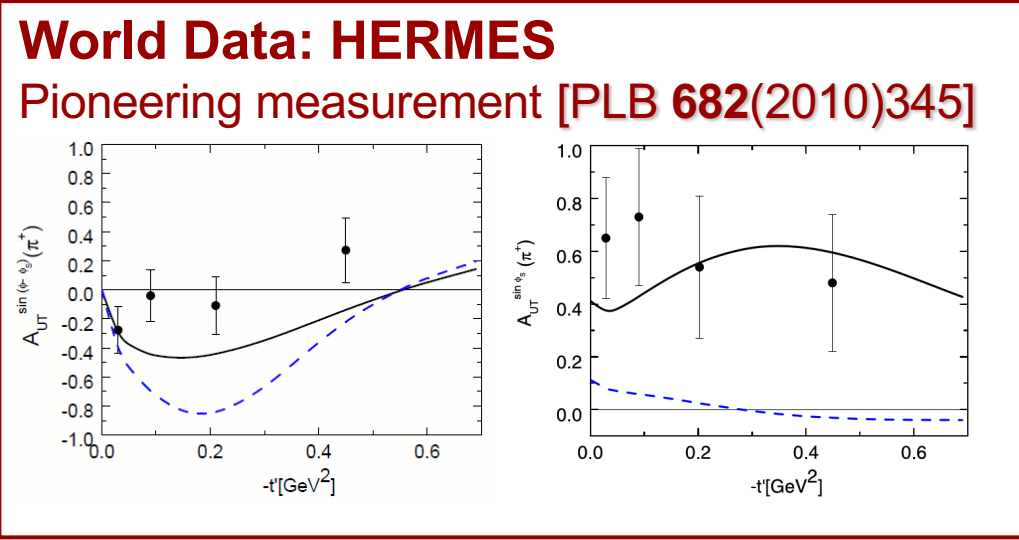
# E12-10-00B: Deep Exclusive $\pi^-$ from Transversely Polarized $n$

- Azimuthal modulations of Transverse Single Spin Asymmetry allow access to different GPDs:

- $\sin(\beta=\phi-\phi_s)$  moment sensitive to helicity-flip GPD  $\tilde{E}$
- $\sin(\phi_s)$  moment sensitive to transversity GPDs

$\vec{n}(e, e' \pi^-) p$  with transversely polarized  $^3\text{He}$

$$\langle A_{UT} \rangle = \frac{1}{P \cdot \eta_n \cdot d} \left( \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \right)$$



**SoLID's large acceptance and high luminosity well-suited to this measurement**  
**■ World unique, cannot be done anywhere else!**

# Jeopardy Proposals: Four questions PAC considers

- Is there any new information that would affect the scientific importance or impact of the Experiment since it was originally proposed? Global analyses of data from various experiments, theoretical progress (esp. lattice) and EIC make this experiment more compelling and further highlight the impact of SoLID
- If the Experiment has already received a portion of its allocated beam .....N/A
- What is the status of the collaboration in terms of institutes, committed staff, and prospective students? 270+ collaborators, 70+ institutions from 13 countries with many students' contributions over the years
- Should the remaining beam time allocation and experiment grade be reconsidered? Yes, the same beam requested and approved as before. The Physics is more compelling and therefore remains at the highest rating.

# Summary

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- SoLID: A **large acceptance** device which can handle **very high luminosity** allows for full exploitation of JLab12 potential  
→ pushing the limit of QCD intensity frontier
- SoLID SIDIS experiment E12-10-006 with transversely polarized  $^3\text{He}$  (&NH<sub>3</sub>) target allows for 4-D mapping of SSAs with high precision in valence quark region complementary and synergistic to EIC
- A diverse set of approved run-group experiments further enhances the impact of this experiment
- We have a mature pre-conceptual design with expected performance meeting the physics requirements and
- Completed the DOE science review (March 8-10, 2021) successfully

**E12-10-006 is more compelling today than it was originally approved 10+ years ago!**

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# BACKUP slides



# E12-10-006A: SIDIS Dihadron with Transversely Polarized $^3\text{He}$

$$A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}(x, y, z, M_h, Q) = \frac{1}{|S_T|} \frac{\frac{8}{\pi} \int d\phi_R d\cos\theta \sin(\phi_R + \phi_S) (d\sigma^\uparrow - d\sigma^\downarrow)}{\int d\phi_R d\cos\theta (d\sigma^\uparrow + d\sigma^\downarrow)}$$

$$= \frac{\frac{4}{\pi} \varepsilon \int d\cos\theta F_{UT}^{\sin(\phi_R+\phi_S)}}{\int d\cos\theta (F_{UU,T} + \varepsilon F_{UU,L})}$$

where

$$F_{UU,T} = x f_1(x) D_1(z, \cos\theta, M_h) \quad ,$$

$$F_{UU}^{\cos\phi_R} = -x \frac{|R| \sin\theta}{Q} \frac{1}{z} f_1(x) \tilde{D}^\triangleleft(z, \cos\theta, M_h) \quad ,$$

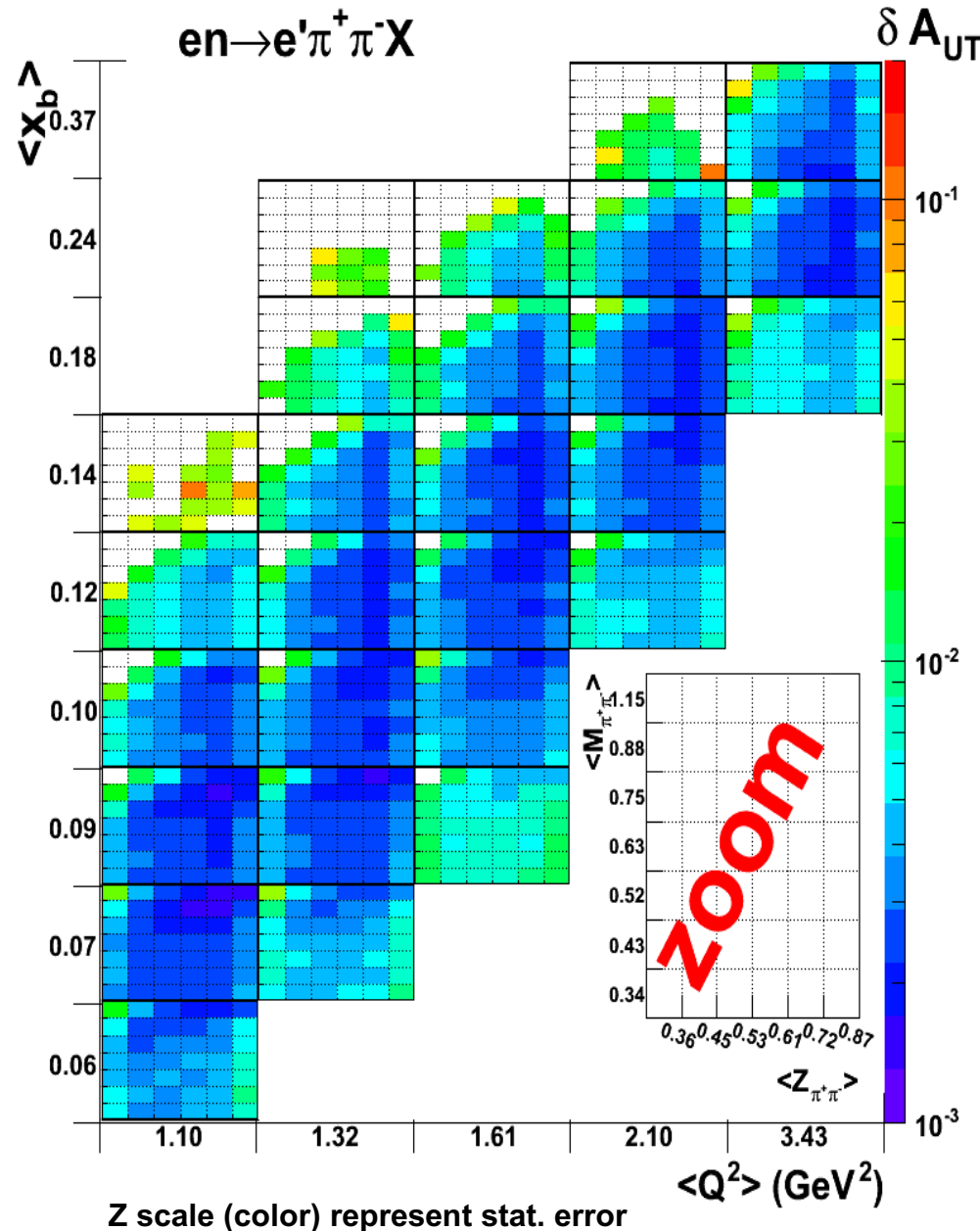
$$F_{UT}^{\sin(\phi_R+\phi_S)} = x \frac{|R| \sin\theta}{M_h} h_1(x) H_1^\triangleleft(z, \cos\theta, M_h^2) \quad ,$$

$$|R| = \frac{1}{2} \sqrt{M_h^2 - 2(M_1^2 + M_2^2) + (M_1^2 - M_2^2)^2}$$

This is what we proposed to measure. The transversity ( $h_1$ ) is in a linear framework with the DiFFs, which makes it relatively easy to extract comparing to single SIDIS analysis...

Only for statistic error illustration in the right:

- 48 days of 11 GeV data on polarized  $^3\text{He}$  target
- Lumi= $10^{36}$  (n)/s/cm $^2$
- Wide  $x_b$  and  $Q^2$  coverages
- Measure transversity via  $\pi^+\pi^-$  dihadron channel

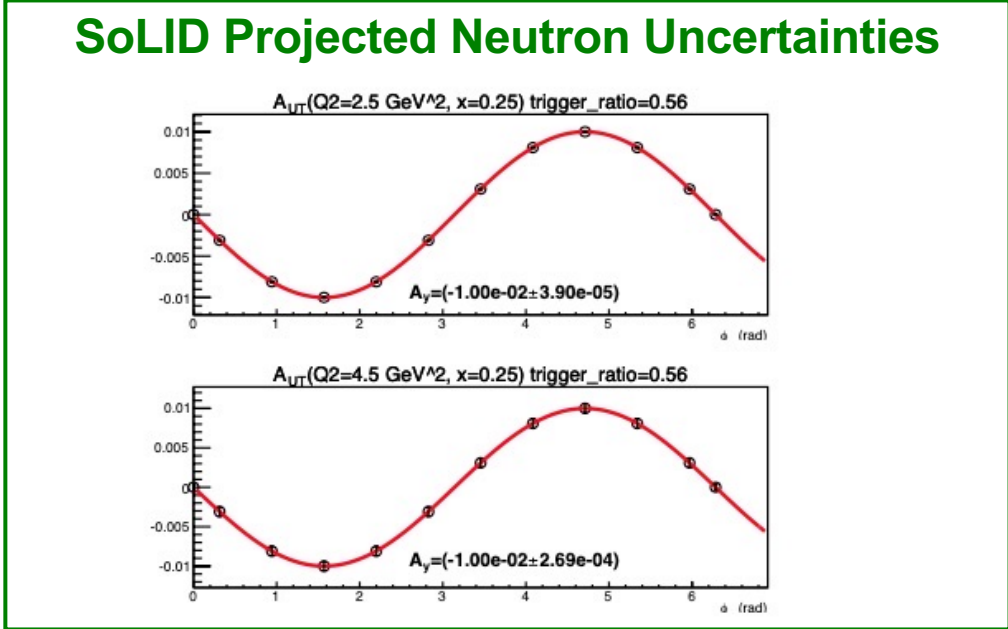
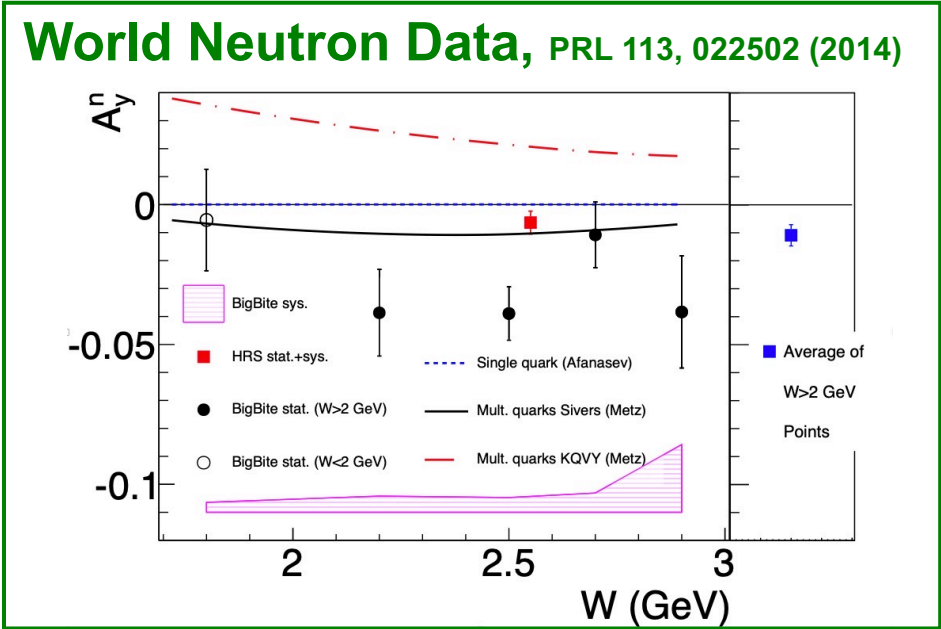


Combine with proton data can do flavor separation

# E12-11-108A/E12-10-006A: Transversely Polarized Target Single Spin Asymmetry, $A_y$ : Accessing TPEX through inclusive scattering from protons and neutrons

$$\vec{N}(e, e')X \quad \langle A_{UT} \rangle = \frac{1}{P \cdot \eta_n \cdot d} \left( \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \right) \quad A_{UT} = A_y \cos \varphi$$

- No contribution from single photon exchange
- Leading contribution from TPEX – direct access
- Measure using polarized  $\text{NH}_3$  and  $^3\text{He}$  targets during SIDIS
  - Theoretical calculations predict both positive and negative neutron asymmetry
  - Current data consistent with input from Siver's, inconsistent with Drell-Yan



SoLID's large acceptance and high luminosity well-suited to this measurement

■ World unique, cannot be done anywhere else!

# E12-10-006E: A Precision Measurement of Inclusive $g_{2n}$ and $d_{2n}$

Inclusive scatterings of longitudinally polarized electrons @11 GeV and 8.8 GeV off transversely and longitudinally polarized  $^3\text{He}$  targets.

□  $g_2$ : carries information of quark-gluon interaction ( $x > 0.1$  and  $1.5 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$ )

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y}$$

Wandzura-Wilczek relation

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(y, Q^2) \frac{dy}{y}$$

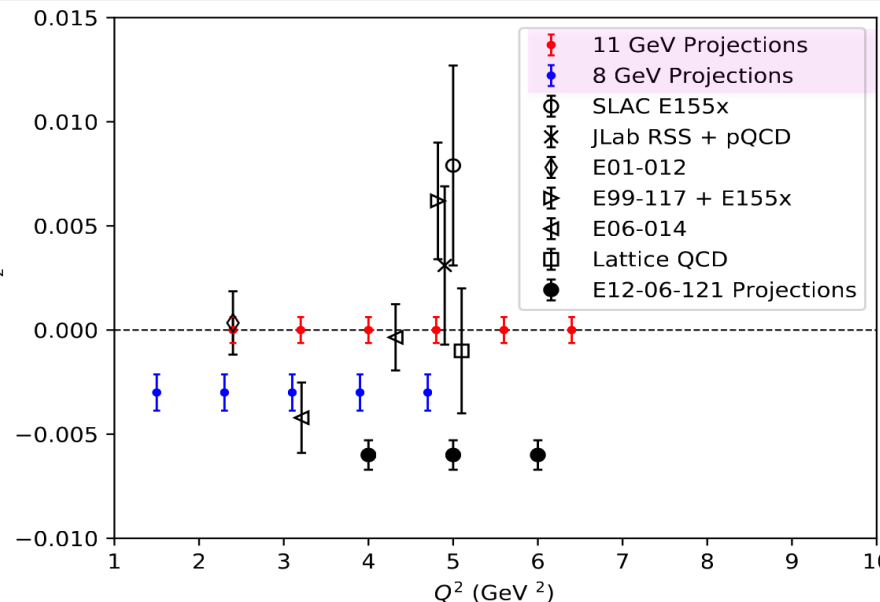
quark transverse momentum contribution

twist-3 part which arises from quark-gluon interactions

□  $d_2$ : the  $x^2$  moment of  $\bar{g}_2(x, Q^2)$

$$d_2(Q^2) = 3 \int_0^1 x^2 [g_2(x, Q^2) - g_2^{WW}(x, Q^2)] dx = \int_0^1 x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] dx$$

- Calculable on the Lattice.
- A clean way to access twist-3 contribution  $d_2$
- Dominated by high  $x$  data because of weighting



- ✧  $d_2$  projection to the region of  $Q^2 < 6.5 \text{ GeV}^2$
- ✧  $x_{\min} > 0.4$  to obtain  $d_2$
- ✧ Assigned 15% error for the unmeasured region
- ✧ Statistic and systematic errors combined
- ✧ Systematic errors dominate