

# The Proton Radius

## – Extracted from J/Psi Production at 20+ GeV CEBAF

- “Mass radius of the proton” – Kharzeev’s talk at the INT workshop
- QCD factorization for exclusive J/Psi production with a diffractive proton
- Photo-production vs. lepton-production of J/Psi
- Exclusive single diffractive hard processes for extracting GPDs and 3D tomography
- Advantage of CEBAF at 20+ GeV

Jianwei Qiu

Jefferson Lab, Theory Center

# Kharzeev's talk at the INT workshop

## Mass radius of the proton

Dmitri Kharzeev

Based on: DK, Phys. Rev. D104 (2021) 5, 054015 [ arXiv:2102.00110]

+ ongoing work

### Outline

- Gravitational formfactors and the mass distribution
- Scale invariance and scale anomaly in QCD
- Measuring the mass radius of the proton in quarkonium photoproduction near the threshold
- The mass radius puzzle?

INT WORKSHOP INT-20R-77

## Origin of the Visible Universe: Unraveling the Proton Mass

June 13, 2022 - June 17, 2022

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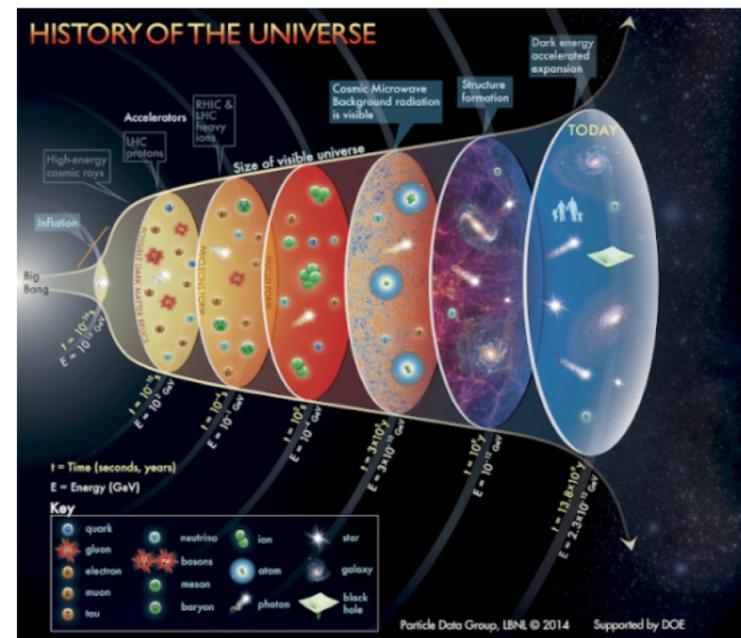
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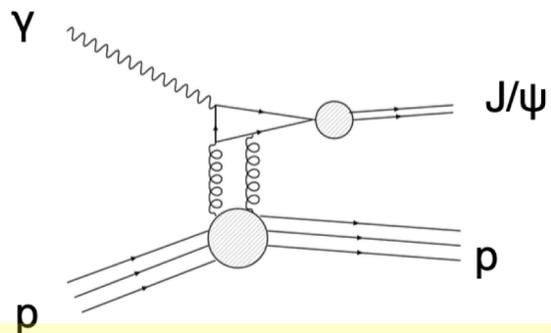


### OVERVIEW

**Note to applicants:** This is a "hybrid" workshop, meaning there will be a combination of virtual and in-person participants. In the COMMENTS section of the Application Form, please write [In-person], [Virtual], or [Either] to reflect your preferred mode of attendance. Please be aware that all In-person participants must show proof of vaccination against COVID-19 upon arrival to the INT.

# Kharzeev's talk at the INT workshop

## Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



Near threshold,  
dominance of

$$g^2 \mathbf{E}^a{}^2 = \frac{8\pi^2}{b} \theta_\mu^\mu + g^2 \theta_{00}^{(G)}$$

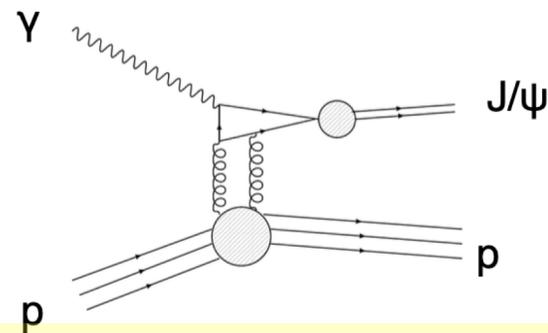
Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK '96; DK, Satz, Syamtomov, Zinovjev '99

Other approaches to threshold photoproduction:

Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19-'22;  
Ji, 2102.07830; Gao, Ji, Liu, 2103.11506; Sun, Tong, Yuan, 2103.12047...

## Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



Large minimum momentum transfer at threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23 \text{ GeV}^2 \simeq -(1.5 \text{ GeV})^2$$

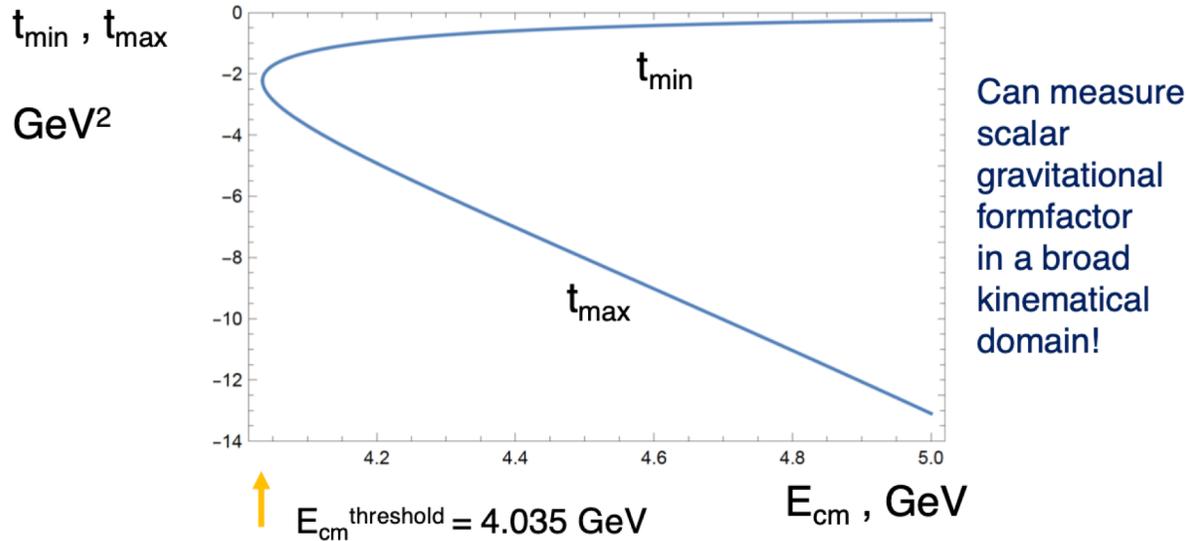
→ VDM questionable.

but, scanning the energy range near the threshold, we measure the scalar gravitational formfactor – can extract the proton mass distribution!

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# Kharzeev's talk at the INT workshop

## Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

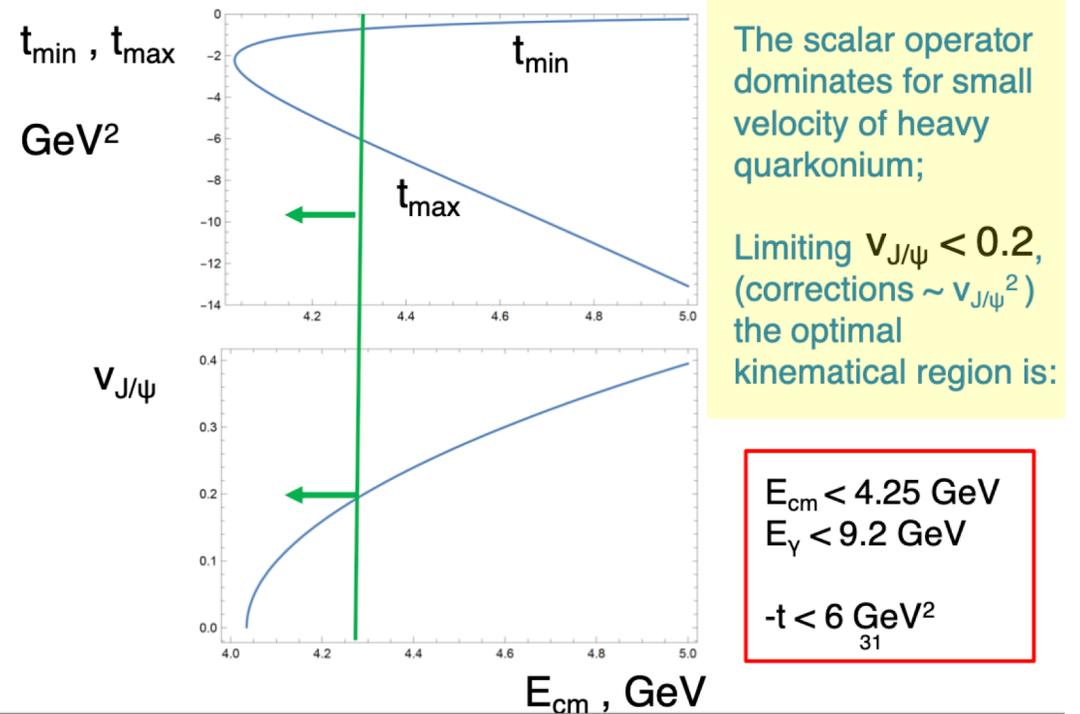


At threshold:

$$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23 \text{ GeV}^2 \simeq -(1.5 \text{ GeV})^2$$

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## Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton



# Kharzeev's talk at the INT workshop

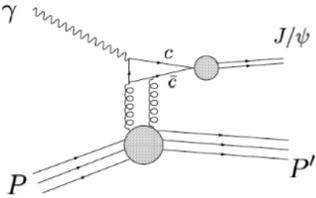
## Threshold photoproduction of quarkonium as a probe of mass distribution inside the proton

The amplitude:

$$\mathcal{M}_{\gamma P \rightarrow \psi P}(t) = -Qe c_2 2M \langle P' | g^2 \mathbf{E}^{a2} | P \rangle,$$



$$Qe = 2e/3$$



$$\mathcal{M}_{\gamma P \rightarrow \psi P}(t) = -Qe c_2 \frac{16\pi^2 M}{b} \langle P' | T | P \rangle$$

Differential cross section:

$$\frac{d\sigma_{\gamma P \rightarrow \psi P}}{dt} = \frac{1}{64\pi s} \frac{1}{|\mathbf{p}_{\gamma cm}|^2} |\mathcal{M}_{\gamma P \rightarrow \psi P}(t)|^2$$

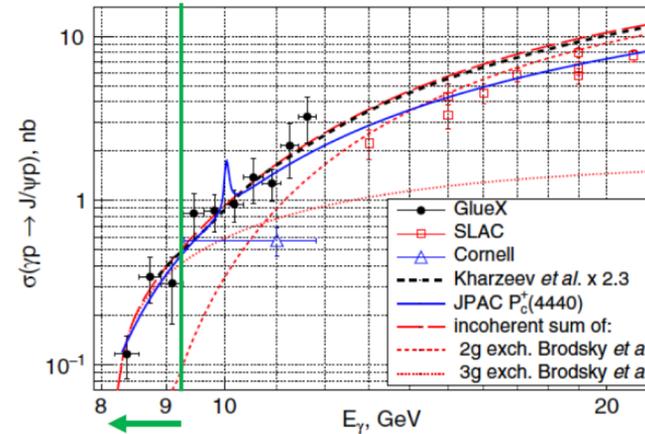
DK, PRD'21

$$\sigma_{\gamma P \rightarrow \psi P}(s) = \int_{t_{min}}^{t_{max}} dt \frac{d\sigma_{\gamma P \rightarrow \psi P}}{dt}, \quad 32$$

### First Measurement of Near-Threshold $J/\psi$ Exclusive Photoproduction off the Proton

A. Ali,<sup>10</sup> M. Amarian,<sup>22</sup> E. G. Anassontzis,<sup>2</sup> A. Austregesilo,<sup>3</sup> M. Baalouch,<sup>22</sup> F. Barbosa,<sup>14</sup> J. Barlow,<sup>7</sup> A. Barnes,<sup>3</sup> E. Barriga,<sup>7</sup> T. D. Beattie,<sup>23</sup> V. V. Berdnikov,<sup>17</sup> T. Black,<sup>20</sup> W. Boeglin,<sup>6</sup> M. Boer,<sup>4</sup> W. J. Briscoe,<sup>8</sup> T. Britton,<sup>14</sup> W. K. Brooks,<sup>24</sup> B. E. Cannon,<sup>7</sup> N. Cao,<sup>11</sup> E. Chudakov,<sup>14</sup> S. Cole,<sup>1</sup> O. Cortes,<sup>8</sup> V. Crede,<sup>7</sup> M. M. Dalton,<sup>14</sup> T. Daniels,<sup>20</sup> A. Deur,<sup>14</sup> S. Dobbs,<sup>7</sup> A. Dolgolenko,<sup>13</sup> R. Dotel,<sup>6</sup> M. Dugger,<sup>1</sup> R. Dzhygadlo,<sup>10</sup> H. Egiyan,<sup>14</sup> A. Ernst,<sup>7</sup> P. Eugenio,<sup>7</sup> C. Fanelli,<sup>16</sup> S. Fegan,<sup>8</sup> A. M. Foda,<sup>23</sup> J. Foote,<sup>12</sup> J. Frye,<sup>12</sup> S. Furlerot,<sup>14</sup> L. Gan,<sup>20</sup> A. Gasparian,<sup>19</sup> V. Gauzshtein,<sup>25,26</sup> N. Gevorgyan,<sup>27</sup> C. Gleason,<sup>12</sup> K. Goetzen,<sup>10</sup> A. Goncalves,<sup>7</sup> V. S. Goryachev,<sup>13</sup> L. Guo,<sup>6</sup> H. Hakobyan,<sup>24</sup> A. Hamdi,<sup>10</sup> S. Han,<sup>29</sup> J. Hardin,<sup>16</sup> G. M. Huber,<sup>23</sup> A. Hurley,<sup>28</sup> D. G. Ireland,<sup>9</sup> M. M. Ito,<sup>14</sup> N. S. Jarvis,<sup>3</sup> R. T. Jones,<sup>5</sup> V. Kakoyan,<sup>27</sup> G. Kalicy,<sup>8</sup> M. Kamel,<sup>9</sup> C. Kourkoumelis,<sup>2</sup> S. Kuleshov,<sup>24</sup> I. Kuznetsov,<sup>25,26</sup> I. Larin,<sup>15</sup> D. Lawrence,<sup>14</sup> D. I. Lersch,<sup>7</sup> H. Li,<sup>3</sup> W. Li,<sup>28</sup> B. Liu,<sup>11</sup> K. Livingston,<sup>9</sup> G. J. Lolos,<sup>23</sup> V. Lyubovitskij,<sup>25,26</sup> D. Mack,<sup>14</sup> H. Marukyan,<sup>27</sup> V. Matveev,<sup>13</sup> M. McCaughan,<sup>14</sup> M. McCracken,<sup>3</sup> W. McGinley,<sup>3</sup> J. McIntyre,<sup>3</sup> C. A. Meyer,<sup>3</sup> R. Miskimen,<sup>15</sup> R. E. Mitchell,<sup>12</sup> F. Mokaya,<sup>5</sup> F. Nerling,<sup>10</sup> L. Ng,<sup>7</sup> A. I. Ostrovidov,<sup>7</sup> Z. Papandreou,<sup>23</sup> M. Patsyuk,<sup>16</sup> P. Pauli,<sup>9</sup> R. Pedroni,<sup>19</sup> L. Pentchev,<sup>14,4</sup> K. J. Peters,<sup>10</sup> W. Phelps,<sup>8</sup> E. Pooser,<sup>14</sup> N. Qin,<sup>21</sup> J. Reinhold,<sup>6</sup> B. G. Ritchie,<sup>1</sup> L. Robison,<sup>21</sup> D. Romanov,<sup>17</sup> C. Romero,<sup>24</sup> C. Salgado,<sup>18</sup> A. M. Schertz,<sup>28</sup> R. A. Schumacher,<sup>3</sup> J. Schwiennie,<sup>10</sup> K. K. Seth,<sup>21</sup> X. Shen,<sup>11</sup> M. R. Shepherd,<sup>12</sup> E. S. Smith,<sup>14</sup> D. I. Sober,<sup>4</sup> A. Somov,<sup>14</sup> S. Somov,<sup>17</sup> O. Soto,<sup>24</sup> J. R. Stevens,<sup>28</sup> I. I. Strakovsky,<sup>8</sup> K. Suresh,<sup>23</sup> V. Tarasov,<sup>13</sup> S. Taylor,<sup>14</sup> A. Teymurazyan,<sup>23</sup> A. Thiel,<sup>9</sup> G. Vasileiadis,<sup>2</sup> D. Werthmüller,<sup>9</sup> T. Whitlatch,<sup>14</sup> N. Wickramaarachchi,<sup>22</sup> M. Williams,<sup>16</sup> T. Xiao,<sup>21</sup> Y. Yang,<sup>16</sup> J. Zarling,<sup>12</sup> Z. Zhang,<sup>29</sup> G. Zhao,<sup>11</sup> Q. Zhou,<sup>11</sup> X. Zhou,<sup>29</sup> and B. Zihlmann<sup>14</sup>

(GlueX Collaboration)



Need to focus on the threshold region!

$E_{cm} < 4.25$  GeV  
 $E_{\gamma} < 9.2$  GeV

# Kharzeev's talk at the INT workshop

## Threshold photoproduction of quarkonium: the effect of the scalar gravitational formfactor

The scalar gravitational formfactor can be constrained theoretically by using:

- i) dispersion relations;
- ii) low-energy theorems of broken scale invariance;
- iii) experimental data on  $\pi\pi$  phase shifts and scalar mesons

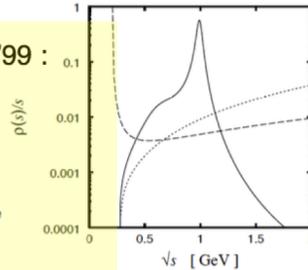
However, as a first step, can try a simple dipole formfactor of the type used for electromagnetic formfactor:

$$G(t) = \frac{M}{(1 - t/M_s^2)^2} \quad \text{radius} \quad \langle R_M^2 \rangle = \frac{6}{M} \left. \frac{dG}{dt} \right|_{t=0}$$

Dipole formfactor was also used for 2-gluon coupling in perturbative models

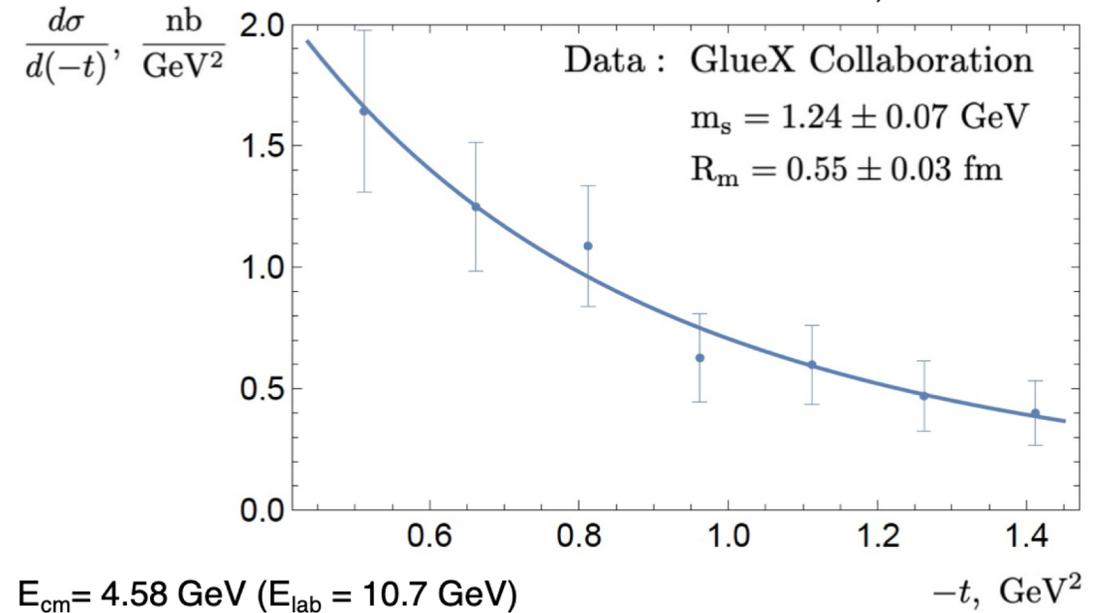
See e.g. Frankfurt, Strikman '02

See e.g. Fujii, DK'99 :



## Differential cross section

DK, arXiv:2102.00110



$$|c_2|^2 = 0.043 \pm 0.006 \text{ fm}^4$$

Lattice QCD, P. Shanahan, W. Detmold PRD'19:

$$m_s = 1.13 \pm 0.06 \text{ GeV} \quad (\text{Traceless gluon operator})$$

$$c_2 \sim \pi r_{c\bar{c}}^2 \quad r_{c\bar{c}} \simeq 0.1 \text{ fm}$$

# Kharzeev's talk at the INT workshop

## The proton mass radius

The r.m.s. "proton mass radius" from GlueX data:

DK, arXiv:2102.00110

$$R_m \equiv \sqrt{\langle R_m^2 \rangle} = 0.55 \pm 0.03 \text{ fm}$$

Compare to the proton charge radius:

$$\bar{R}_c \equiv \sqrt{R_c^2} = 0.8409 \pm 0.0004 \text{ fm}$$

See J. Bernauer, EPJ 234 (2020) for review

A more compact mass distribution? Need more data!

## Theoretical uncertainties

- Higher dimensional operators (suppressed by  $1/m_c$ )
- Chiral limit (we omitted the scalar quark operator)
- Gluon operators with derivatives ( $\sim 5\%$  close to threshold)
- t-dependence of short-distance coefficient  $c_2$  ( $\sim t/4m_c^2$ )
- Dipole parameterization of formfactor

VALUE (fm)	DOCUMENT ID	TECN	COMMENT
<b>0.8409 ± 0.0004</b>	<b>OUR AVERAGE</b>		
0.833 ± 0.010	1 BEZGINOV 2019	LASR	2S-2P transition in H
0.831 ± 0.007 ± 0.012	2 XIONG 2019	SPEC	$e p \rightarrow ep$ form factor
0.84087 ± 0.00026 ± 0.00029	ANTOGNINI 2013	LASR	$\mu p$ -atom Lamb shift
... We do not use the following data for averages, fits, limits, etc. ...			
0.877 ± 0.013	3 FLEURBAEY 2018	LASR	1S-3S transition in H
0.8335 ± 0.0095	4 BEYER 2017	LASR	2S-4P transition in H
0.8751 ± 0.0061	MOHR 2016	RVUE	2014 CODATA value
0.895 ± 0.014 ± 0.014	5 LEE 2015	SPEC	Just 2010 Mainz data
0.916 ± 0.024	LEE 2015	SPEC	World data, no Mainz
0.8775 ± 0.0051	MOHR 2012	RVUE	2010 CODATA, $ep$ data
0.875 ± 0.008 ± 0.006	ZHAN 2011	SPEC	Recoil polarimetry
0.879 ± 0.005 ± 0.006	BERNAUER 2010	SPEC	$e p \rightarrow ep$ form factor

2020 Review of Particle Physics.  
P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Some day:  
p MASS RADIUS in PDG?

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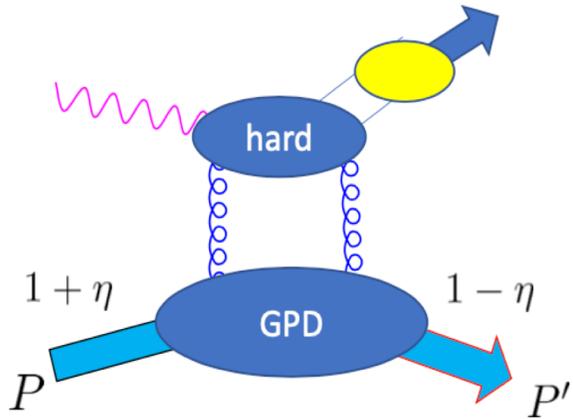
# Hatta's talk at this workshop

## Theory approach 4: QCD factorization

Light-cone dominance when  $Q^2 \rightarrow \infty$  or  $M_{QQ} \rightarrow \infty$

GPD factorization at high energy [Collins, Frankfurt, Strikman \(1996\)](#)  
[Ivanov, Schafer, Szymanowski, Krasnikov \(2004\)](#)

Let's assume factorization is valid and see what happens.



Amplitude proportional to **Compton form factor**

$$\int_{-1}^1 \frac{dx}{x} \left( \frac{1}{\eta - x - i\epsilon} - \frac{1}{\eta + x - i\epsilon} \right) H_g(x, \eta, t)$$

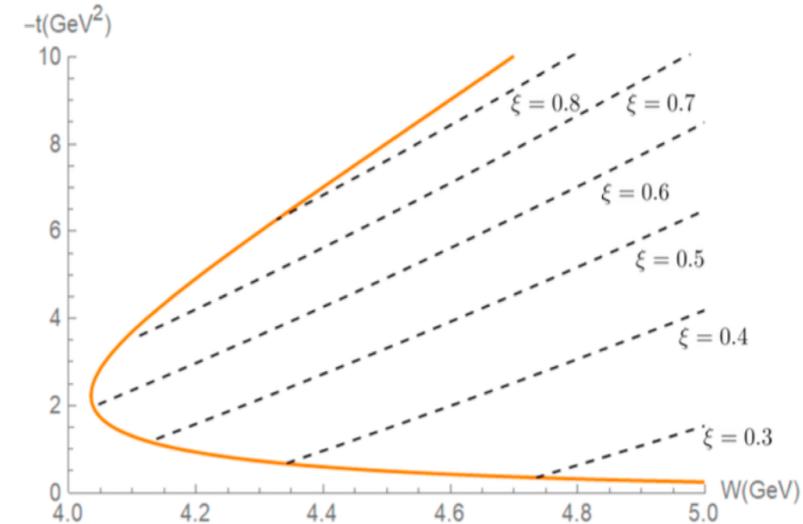
**Gluon GPD**

**Skewness**  $\eta = \frac{P^+ - P'^+}{P'^+ + P^+}$

**1 graviton  $\approx$  2 gluons ??**

## Skewness

$J/\psi$  Photoproduction



[Guo, Ji, Liu \(2021\)](#)

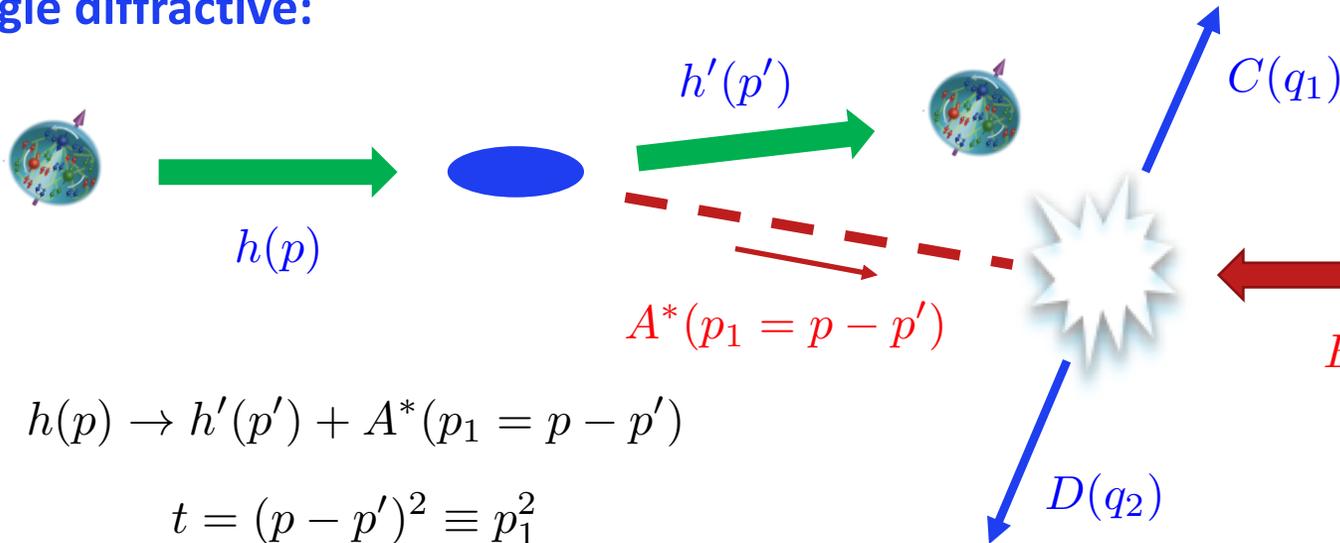
the ideal limits  $Q^2 \rightarrow \infty$  or  $m_V \rightarrow \infty$

# Extracting Two-gluon Correlation from Experiments

Qiu & Yu, 2022

## QCD factorization of diffractive $2 \rightarrow 3$ exclusive hard processes:

### Single diffractive:



### Hard probe: $2 \rightarrow 2$ high $q_T$ exclusive process

$$A^*(p_1) + B(p_2) \rightarrow C(q_1) + D(q_2)$$

$$|q_{1T}| = |q_{2T}| \gg \sqrt{-t}$$

### The single diffractive $2 \rightarrow 3$ exclusive hard processes:

$$h(p) + B(p_2) \rightarrow h'(p') + C(q_1) + D(q_2)$$

### Photo-production of J/Psi:

$$B(p_2) \rightarrow \gamma(p_2)$$

$$C(q_1) \rightarrow \ell(q_1), D(q_2) \rightarrow \bar{\ell}(q_2)$$

$$\text{with } (q_1 + q_2)^2 \sim M_{J/\psi}$$

### Lepton-production of J/Psi:

$$B(p_2) \rightarrow \ell(p_2)$$

$$C(q_1) \rightarrow \ell(q_1)$$

$$D(q_2) \rightarrow J/\psi(q_2)$$

$$\hookrightarrow \ell + \bar{\ell}$$

### Necessary condition for QCD factorization:

$$|q_{1T}| = |q_{2T}| \gg \sqrt{-t}$$

The state  $A^*(p_1)$  lives much longer

than  $2 \rightarrow 2$  hard exclusive collision!

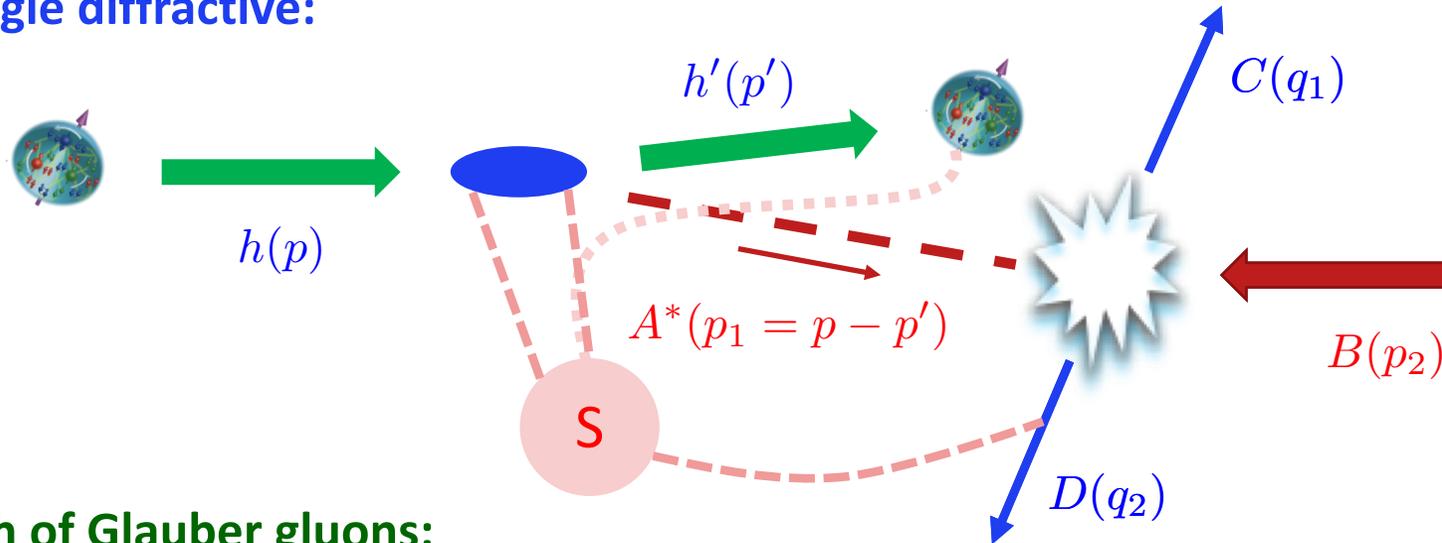
**Not necessarily sufficient!**

# Extracting Two-gluon Correlation from Experiments

Qiu & Yu, 2022

## QCD factorization of diffractive $2 \rightarrow 3$ exclusive hard processes:

### Single diffractive:



### Photo-production of J/Psi:

$$B(p_2) \rightarrow \gamma(p_2)$$

$$C(q_1) \rightarrow \ell(q_1)$$

with  $(q_1 + q_2)^2 \sim M_{J/\psi}$

### Lepton-production of J/Psi:

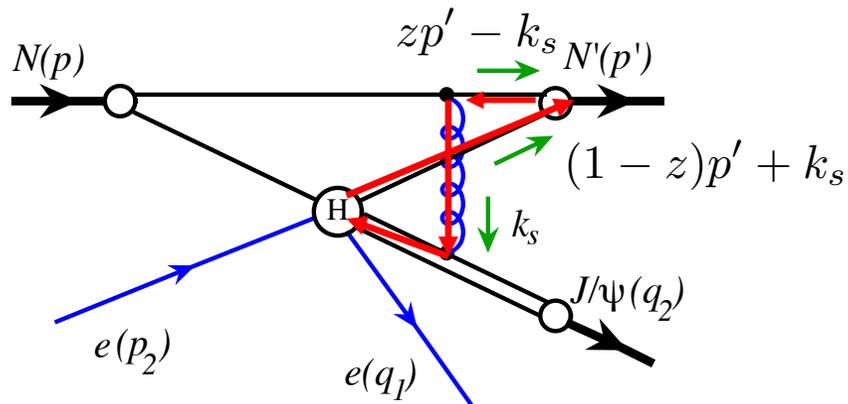
$$B(p_2) \rightarrow \ell(p_2)$$

$$C(q_1) \rightarrow \ell(q_1)$$

$$D(q_2) \rightarrow J/\psi(q_2)$$

$\hookrightarrow \ell + \bar{\ell}$

## Pinch of Glauber gluons:



### Gluons in the Glauber region:

$$k_s = (\lambda^2, \lambda^2, \lambda) Q$$

$$\frac{1}{((1-z)p' + k_s)^2 + i\epsilon} \rightarrow \frac{1}{k_s^- + i\epsilon}$$

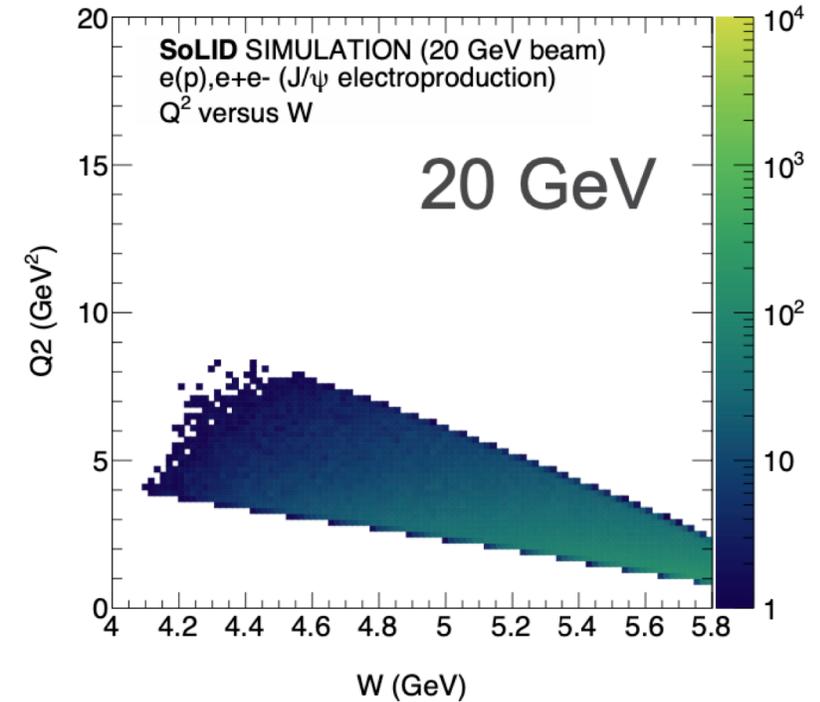
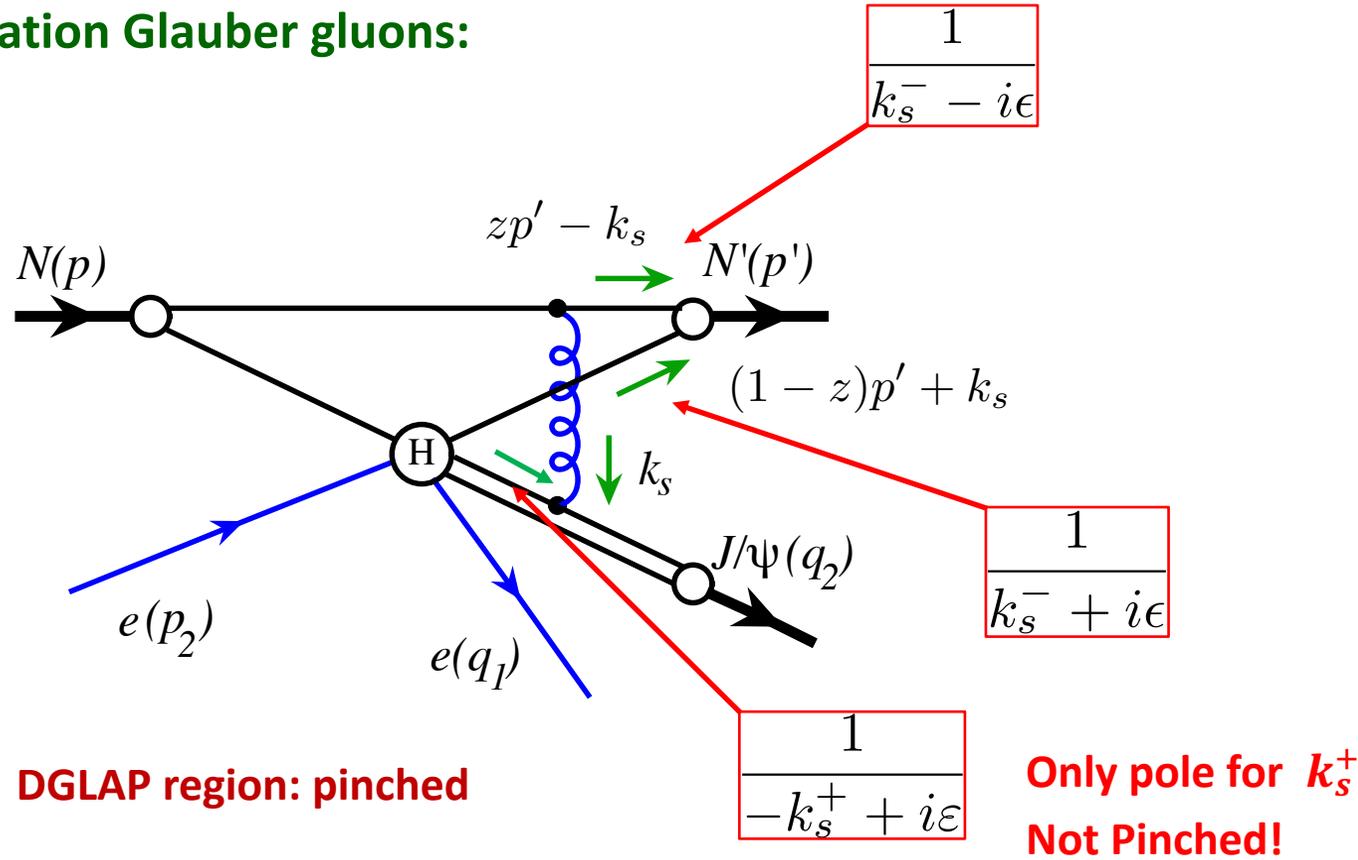
$$\frac{1}{(z p' - k_s)^2 + i\epsilon} \rightarrow \frac{1}{k_s^- - i\epsilon}$$

**Pinched!**

# Extracting Two-gluon Correlation from Experiments

Qiu & Yu, 2022

## Factorization Glauber gluons:



## Deformation out of the Glauber region:

$$k_s^+ \rightarrow k_s^+ - i\mathcal{O}(Q) \quad \longrightarrow \quad k_s \sim (1, \lambda^2, \lambda)Q$$

Collinear region

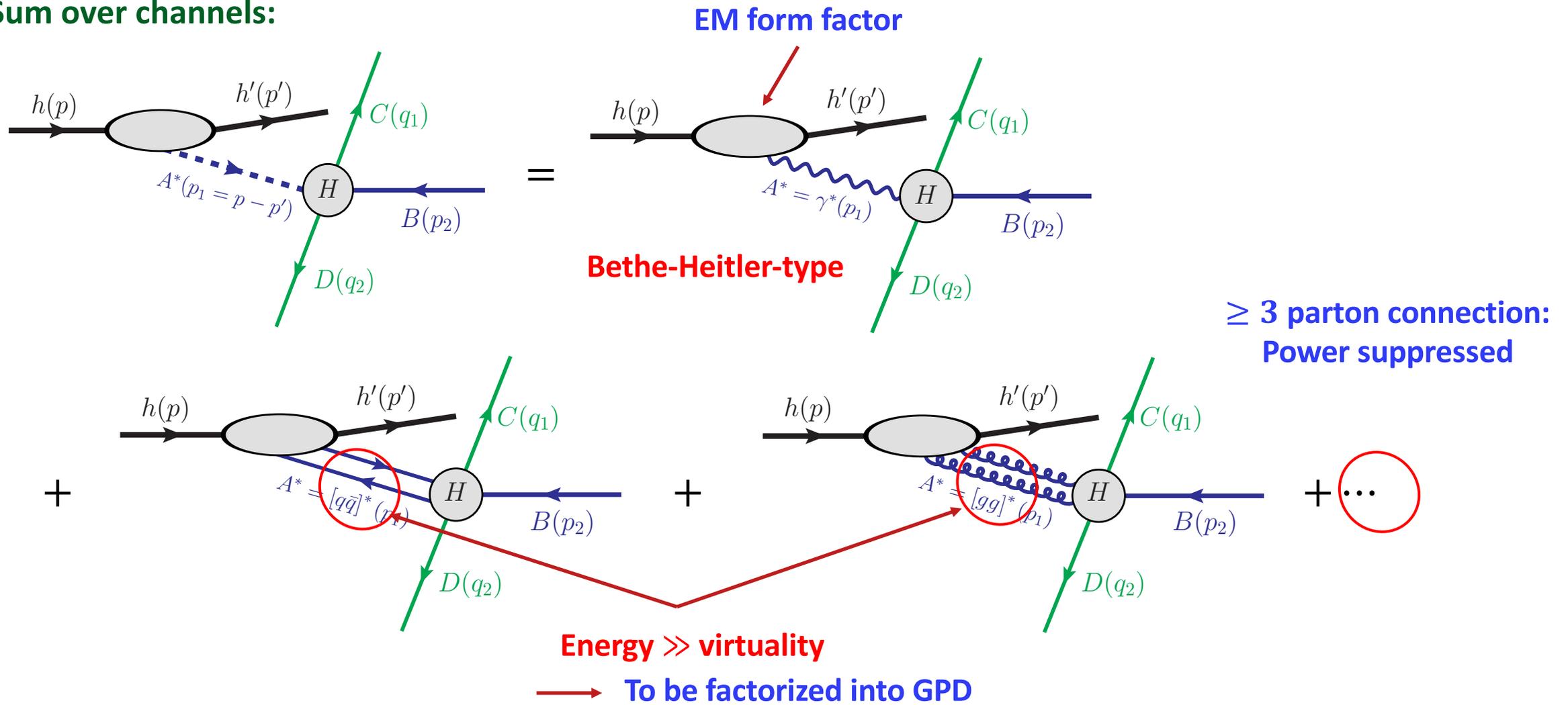
## Factorize all soft gluon attached to J/ψ jet

$$\longrightarrow \quad q_{2T} \gtrsim m_{J/\psi}$$

Higher beam energy!

# Factorization in the Two-stage Paradigm

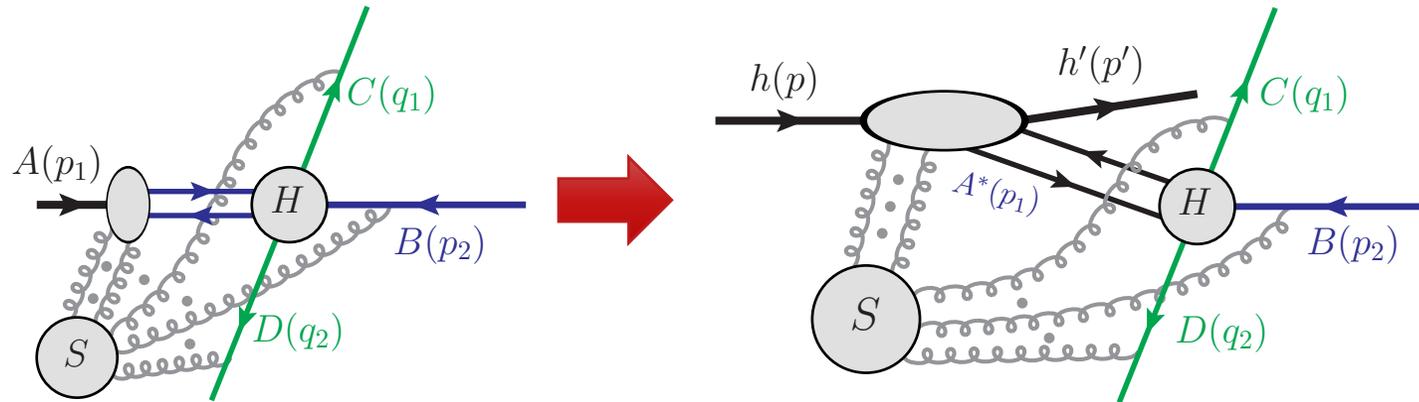
□ Sum over channels:



# Factorization in the Two-stage Paradigm

Qiu & Yu, 2205.07846 (JHEP in press)

## Factorization for 2-parton channel factorization:

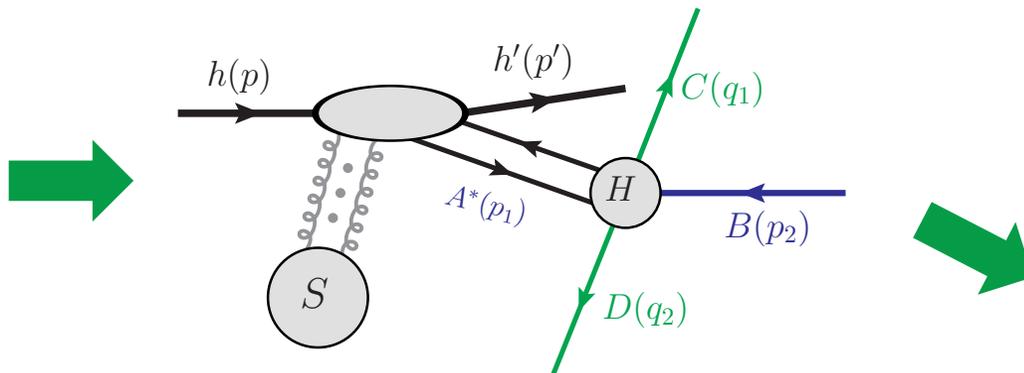


**Only complication:**  
 $k_s^-$  is **pinched** in Glauber region for DGLAP region.

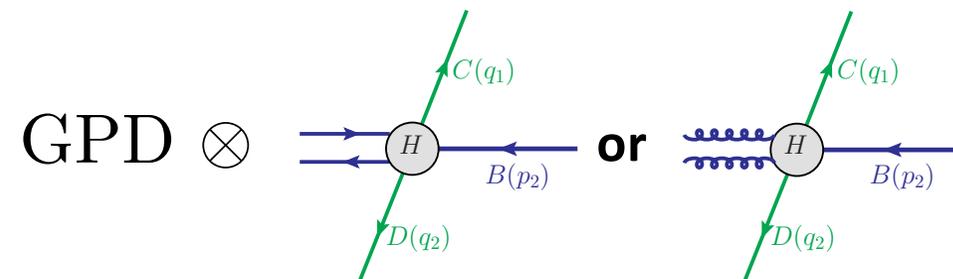


$k_s^+ \mapsto k_s^+ \pm i\mathcal{O}(Q)$   
**Glauber**  $\rightarrow$   **$h$ -collinear region**

## Soft gluons cancel for the meson-initialized process if $C$ and $D$ are mesons:



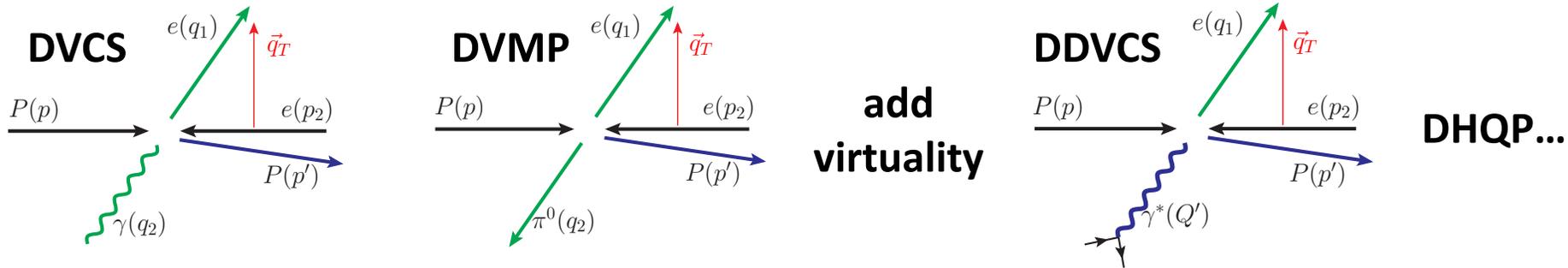
Soft gluons are no longer pinched and can be deformed into  $h$ -collinear region



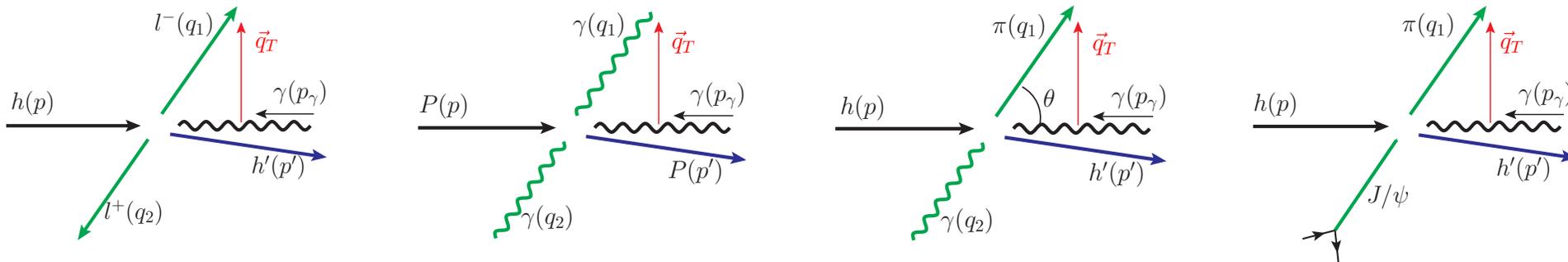
# Classification of these New Processes for GPDs

Qiu & Yu, 2205.07846 (JHEP in press), 2207.xxxxx

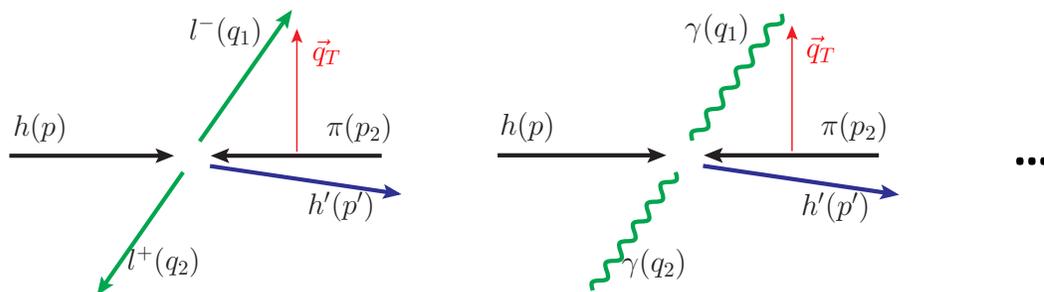
## □ Electro-production (JLab, EIC, ...)



## □ Photo-production (JLab Hall-D, ...)



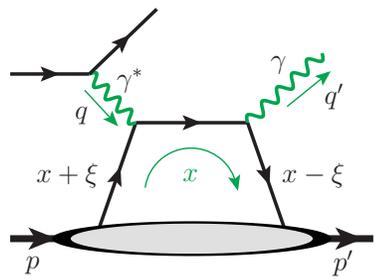
## □ Meson-production (AMBER, J-PARC, ...)



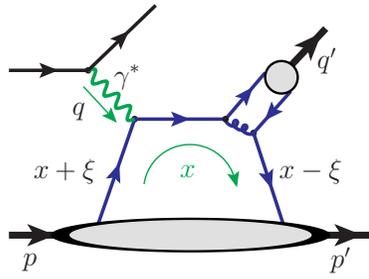
One can easily list more processes by using mesons, adding virtuality...

# GPDs – QCD Tomography

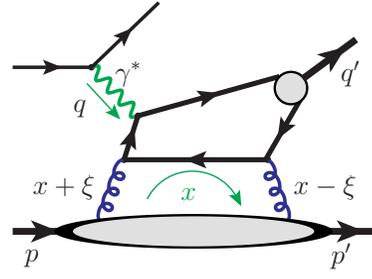
□ Imagining spatial distribution of quarks and gluons:



DVCS:  $Q^2 \gg |t|$



DVMP



DVQP



$$\frac{d\sigma}{dt}$$

$$t = (p - p')^2$$



**Factorization**

$$Q^2 \gg |t|$$

**GPDs:**  $f_{i/h}(x, \xi, t; \mu)$



**F.T.**  $t_T$  to  $b_T$

**at**  $\xi \propto (p - p')^+ \rightarrow 0$

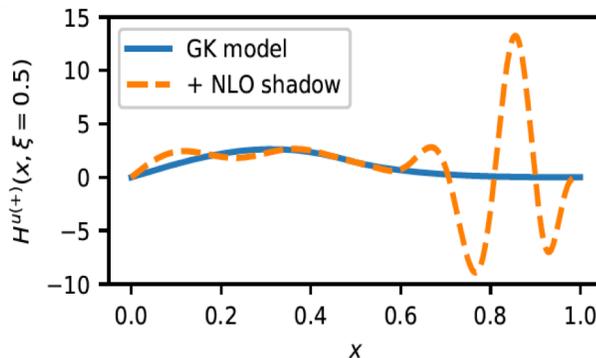
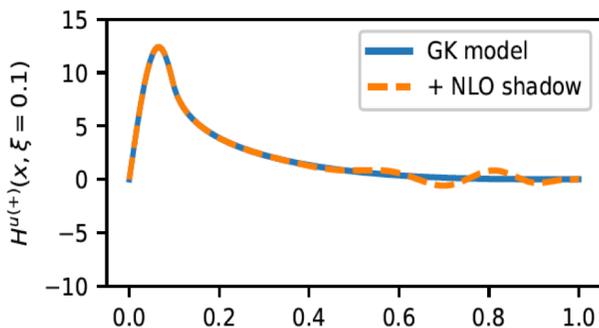
□ Proton radii of quark and gluon spatial distribution,  $r_q(x)$  &  $r_g(x)$

Should  $r_q(x) > r_g(x)$ , or vice versa? Could  $r_g(x)$  saturate as  $x \rightarrow 0$  ?

...

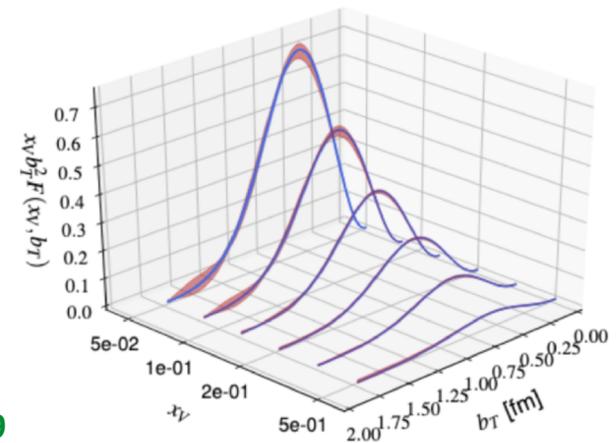
□ But, all these observables are not very sensitive to the  $x$ -dependence!

Sensitive to the total momentum of the pair, not the relative momentum



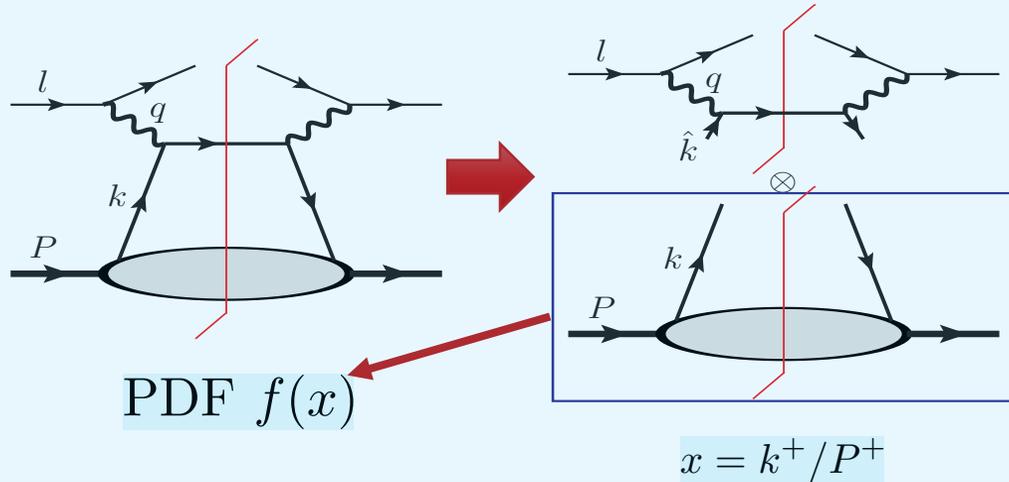
**Blue and dashed  
Fit the same CFFs !**

**Phys.Rev. D103 (2021) 114019**



# Inclusive Process vs. Exclusive Process

## Deeply Inelastic Scattering (DIS):



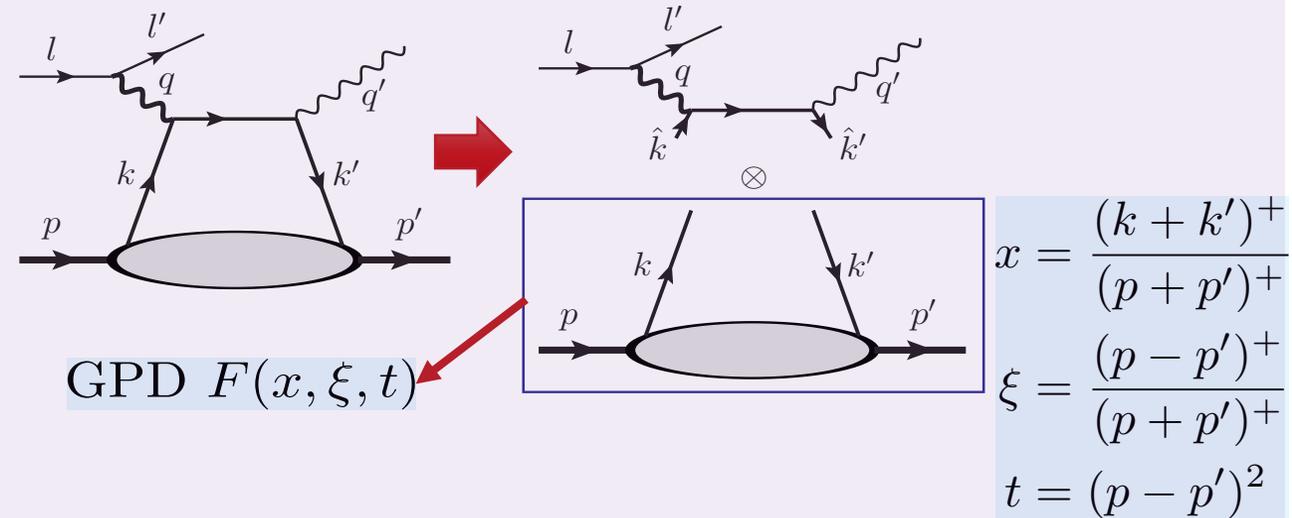
**Cross section:** Cut diagrams

$$\sigma_{\text{DIS}} \simeq \int_{x_B}^1 dx f(x) \hat{\sigma}(x/x_B)$$

- PDF  $\sim$  probability
- At LO:  $x = x_B$
- Beyond LO:  $x \in [x_B, 1]$

**x-dependence:** Part of measurement

## Deeply Virtual Compton Scattering (DVCS):



**Amplitude:** Uncut diagrams

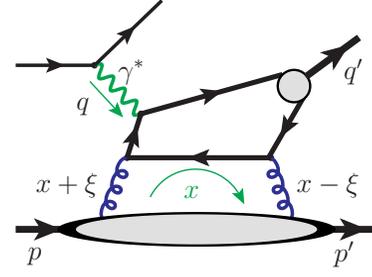
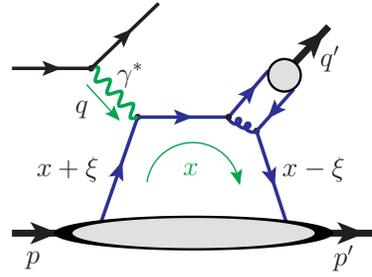
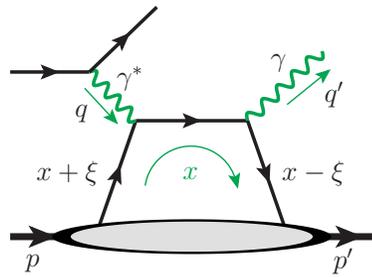
$$\mathcal{M}_{\text{DVCS}}(\xi, t) \simeq \int_{-1}^1 dx F(x, \xi, t) \hat{\mathcal{M}}(x, \xi)$$

- GPD  $\sim$  amplitude
- $k^+ = (x + \xi) P^+$  is loop momentum
- At any order:  $x \in [-1, 1]$

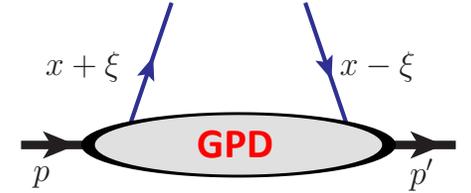
**x-dependence:** Hard to measure

# GPDs – QCD Tomography

## Imagining spatial distribution of quarks and gluons:



Factorization



$$i\mathcal{M} \sim \int_{-1}^1 dx F(x, \xi, t) \cdot C(x, \xi; Q/\mu)$$

## Sensitivity to $x$ comes from $C(x, \xi; Q/\mu)$ :

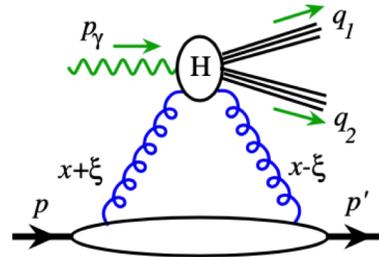
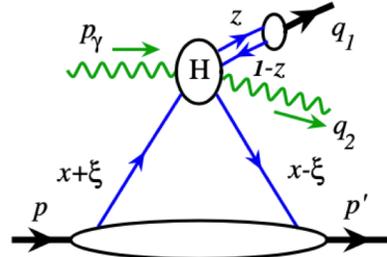
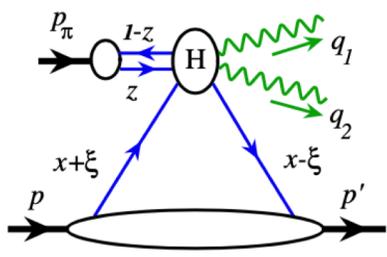
**DVCS:**  $C(x, \xi; Q/\mu) \propto \frac{1}{x - \xi + i\epsilon} \longrightarrow i\mathcal{M} \propto \int_{-1}^1 dx \frac{F(x, \xi, t)}{x - \xi + i\epsilon} \equiv "F_0(\xi, t)"$

$x \sim$  loop momentum

$t, \xi \sim$  directly measured

Not ideal if:  $C(x, \xi; Q/\mu) \Rightarrow C_Q(Q/\mu) \cdot C_x(x) \cdot C_\xi(\xi)$

## Exclusive massive pair production with high- $P_T$ (two-scale observables):



**Hard scale:**  $q_T \gg \Lambda_{\text{QCD}}$   
in  $p_\pi - (p - p')$  frame

**Soft scale:**  $t = (p - p')^2$

**Factorization:**  $q_T \gg \sqrt{|t|}$

Introduced by G. Duplancic et al.  
JHEP 11 (2018) 179

Introduced by Y. Hatta et al.  
Phys.Rev.Lett. 116 (2016) 202301

# Exclusive Photo-Production of a $\pi\gamma$ Pair

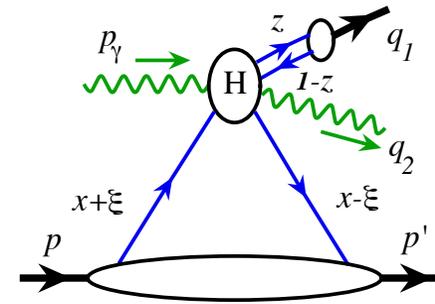
□ **Process:**  $\gamma(p_\gamma) + h(p) \rightarrow \pi^\pm(q_1) + \gamma(q_2) + h'(p')$

Introduced by G. Duplancic et al. [JHEP 11 (2018) 179],  
No contribution from gluon GPDs

□ **Factorization:**

Proved to be valid when  $q_T \gg \sqrt{|t|} \gtrsim \Lambda_{\text{QCD}}$

□ **Cancellation of unwanted propagators &  $\cos\theta$  dependence:**



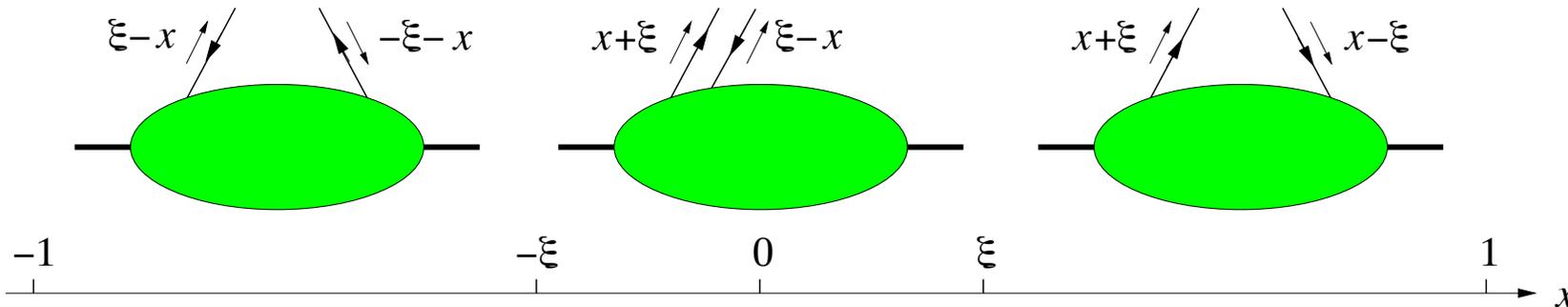
Hall D at JLab

$$\frac{d\sigma}{dt d\xi dq_T^2} \quad \text{or} \quad \frac{d\sigma}{dt d\xi d\cos\theta}$$

$$\text{Re } O_{++} = (e_1 - e_2)^2 \left[ \frac{1 - \cos\theta}{1 + \cos\theta} \cdot P \frac{x + z - 2xz}{2xz(1-x)(1-z)} \right] + (e_1^2 - e_2^2) \left[ \frac{2}{1 - \cos\theta} \cdot P \frac{x - z}{xz(1-x)(1-z)} \right]$$

$$- e_1 e_2 P \frac{1 - \cos\theta}{xz(1-x)(1-z)} \cdot \frac{(xz + (1-x)(1-z))(x(1-x) + z(1-z))}{(2(1-x)(1-z) - (1 + \cos\theta)xz)(2xz - (1 + \cos\theta)(1-x)(1-z))}$$

□ **Sensitive to ERBL region (complementary)**



Also sensitive to DA  
in the bulk region.

# Exclusive $\pi^0\gamma$ Pair Production

## Phenomenology:

$$\frac{d\sigma}{d|t| d\xi d\cos\theta_\pi d\phi_\pi} = \frac{|\mathcal{A}|^2}{32 s (2\pi)^4 (1 + \xi)^2}$$

$$\frac{1}{2} |\overline{\mathcal{A}}|^2 = \left(\frac{2\pi\alpha_s}{s} f_\pi\right)^2 \left(\frac{C_F}{N_c}\right)^2 \left(\frac{1 + \xi}{\xi}\right)^2 (1 - \xi^2)$$

$$\times \left[ |O_{+++}^{[\tilde{H}]}|^2 + |O_{+-}^{[\tilde{H}]}|^2 + |\tilde{O}_{+++}^{[H]}|^2 + |\tilde{O}_{+-}^{[H]}|^2 \right]$$

### Factorized helicity amplitude:

$$O_{\lambda\lambda'}^{[\tilde{H}]} = \sum_q \int_{x_L}^{x_R} dx \int_0^1 dz \tilde{H}^q(x, \xi, t) \phi_\pi^q(z) O_{\lambda\lambda'}^q(x, z)$$

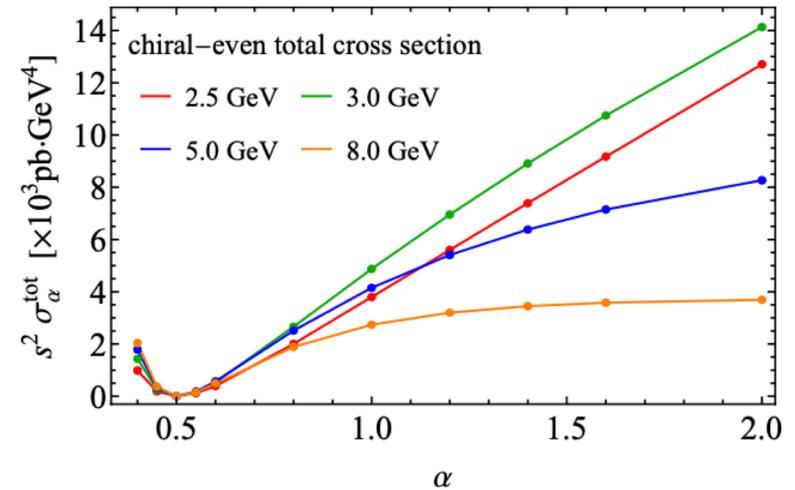
### Pion distribution amplitude:

$$\phi_{\pi^0}^d(z) = \phi_{\pi^0}^u(z) = \frac{1}{\sqrt{2}} \frac{z^\alpha (1-z)^\alpha}{B(1+\alpha, 1+\alpha)}, \quad (\alpha > 0)$$

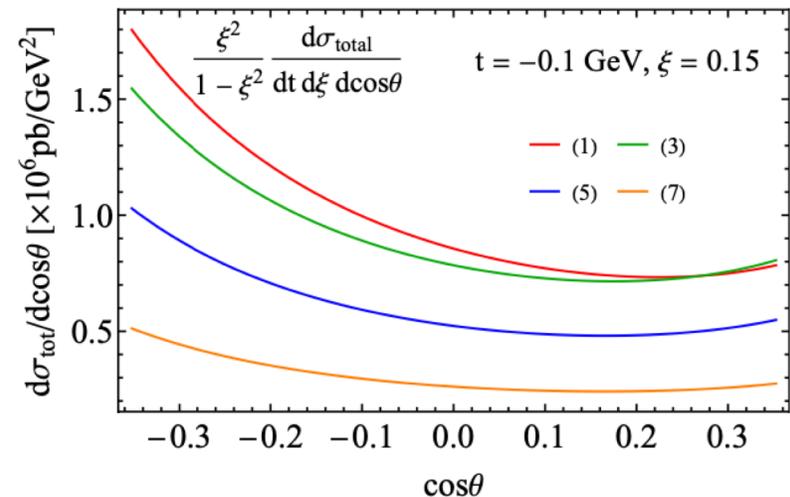
### Model GPDs = simplified GK model:

- Taking  $n_i = 0$
- Parametrizing the forward limit as  $x^a(1-x)^b$
- Neglecting the D-term

## Sensitivity on DAs (total – $q_T > 1$ GeV):



## Sensitivity on GPDs ( $\alpha = 0.63$ ):



# Exclusive $\pi^0\gamma$ Pair Production

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$$\times \left[ |O_{+++}^{[\tilde{H}]}|^2 + |O_{+-}^{[\tilde{H}]}|^2 + |\tilde{O}_{+++}^{[H]}|^2 + |\tilde{O}_{+-}^{[H]}|^2 \right]$$

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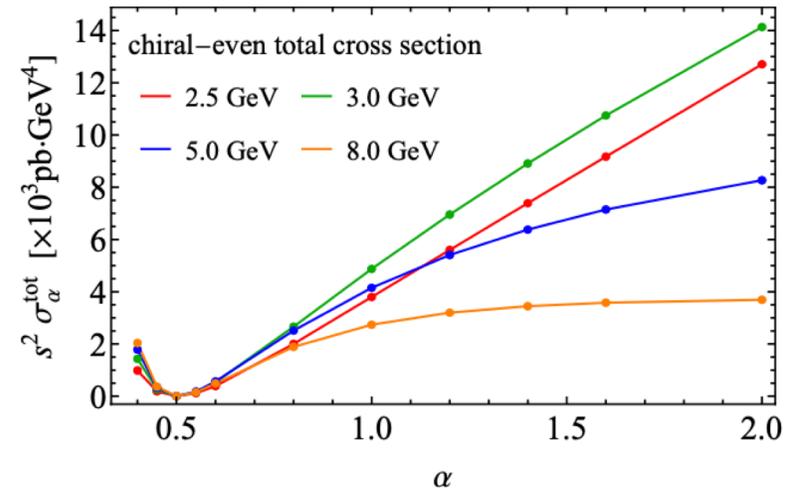
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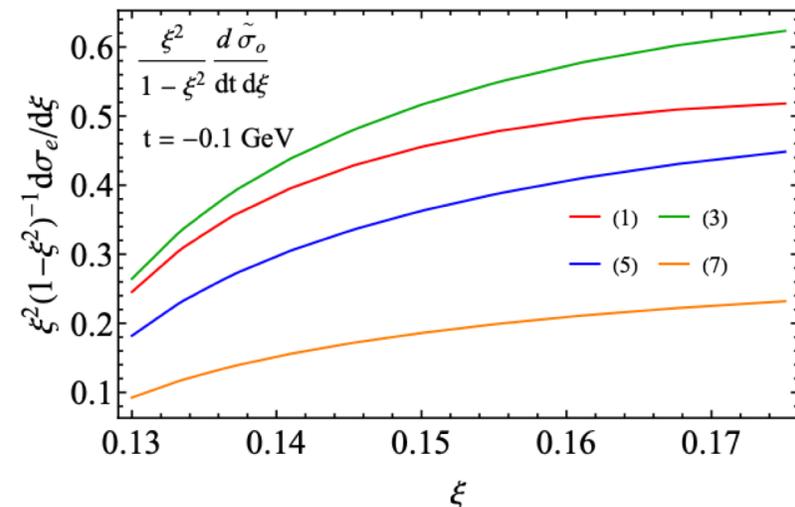
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## Sensitivity on GPDs ( $\alpha = 0.63$ ):



# Summary and Outlook

- Summarized the part of Kharzeev's talk at the INT on probing mass radius from quarkonium photo-production near threshold
  
- Introduced the diffractive  $2 \rightarrow 3$  exclusive hard processes for extracting GPDs, ...
  - Provide both necessary and sufficient conditions for leading power factorization
  - Including existing/known processes for extracting GPDs, ..
  - Identify new factorizable exclusive processes for GPDs more sensitive to x-dependence

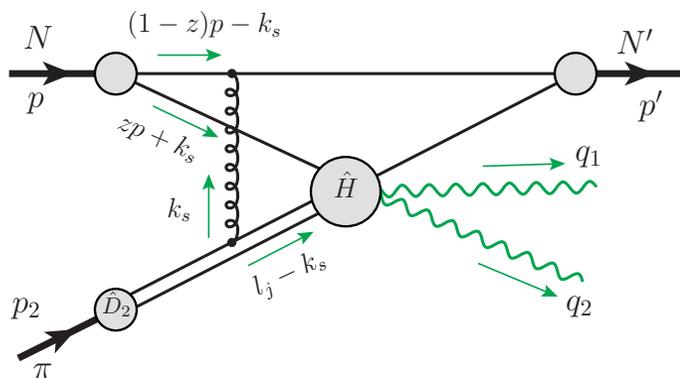
*Introduce a path forward to identify more factorizable exclusive processes for extracting GPDs, and multi-parton correlations, ...*
  
- QCD factorization requires a sufficient  $P_T$  of  $J/\psi$  in the lab frame (or  $Q^2$ ) for lepto-production
  - 20+ GeV is sufficient for exploring gluon GPDs from lepto-production of  $J/\psi$  at high  $Q^2$

**Thanks!**

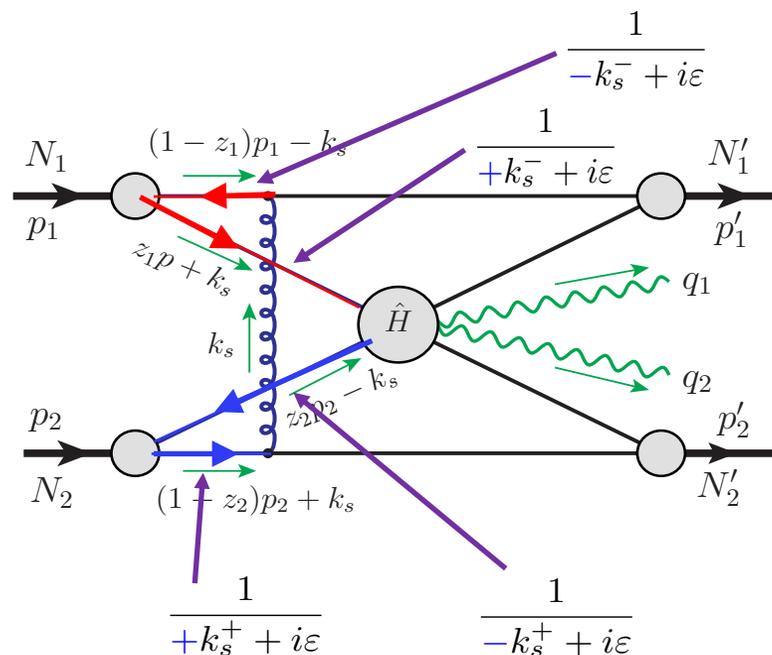
# Why single diffractive?

## □ Double diffractive process

### Glauber pinch for diffractive scattering

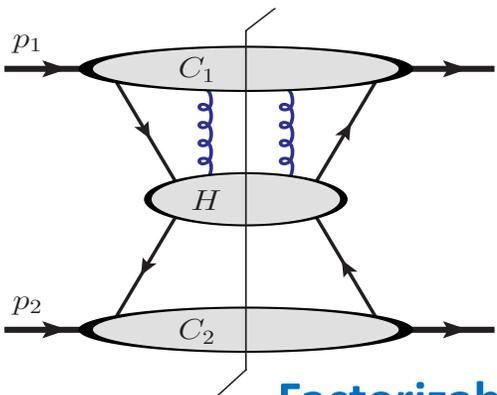


Factorizable if all pion momentum flows into hard part

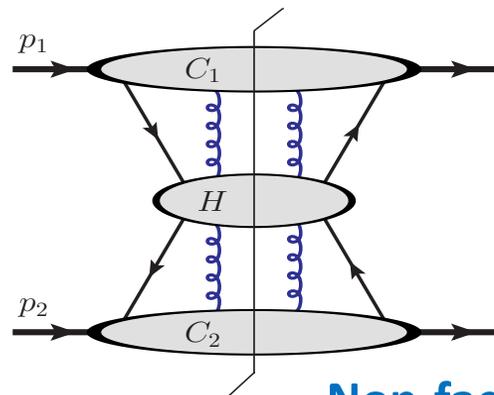


Both  $k_s^+$  and  $k_s^-$  are pinched in Glauber region!

## □ Compare: Drell-Yan process at high twist:



Factorizable



Non-factorizable

Only the 1<sup>st</sup> sub-leading twist is factorizable!

Qiu & Sterman, NPB, 1991