# $J/\psi$ near threshold in holographic QCD: A and D gravitational form factors

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# References

This talk is based on the references **2204.08857**, **2106.00722** (PRD), **2103.03186** (PRD), and **1910.04707** (PRD) in collaboration with **Ismail Zahed**

# Spin-2 (Gravitational) Form Factor of Proton

- the gravitational form factors of proton are defined as

$$\langle p_2|T^{\mu\nu}(0)|p_1\rangle = \overline{u}(p_2)\left(A(k)\gamma^{(\mu}p^{\nu)} + B(k)\frac{ip^{(\mu}\sigma^{\nu)\alpha}k_\alpha}{2m_N} + C(k)\frac{k^\mu k^\nu - \eta^{\mu\nu}k^2}{m_N}\right)u(p_1)$$

  with $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$, $k^2 = (p_2 - p_1)^2 = t$, $p = (p_1 + p_2)/2$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = i(\gamma^\mu\gamma^\nu - \eta^{\mu\nu})$, and the normalizations $\overline{u}u = 2m_N$ and $m_N \times \bar{u}(p_1)\gamma^\mu u(p_2) = \bar{u}(p_1)p^\mu u(p_2)$
- the energy-momentum tensor in the proton state is conserved and tracefull
- throughout, $D(k) = 4C(k)$ will be used interchangeably
- the spin-2 (gravitational) form factor of proton corresponds to

$$\left\langle p_2|\epsilon^{TT}_{\mu\nu}T^{\mu\nu}(0)|p_1\right\rangle = \overline{u}(p_2)\left(A(k)\epsilon^{TT}_{\mu\nu}\gamma^{(\mu}p^{\nu)}\right)u(p_1)$$

# Spin-0 (Scalar) Form Factor of Proton

- note that the spin-0 (scalar) form factor of proton corresponds to

$$\langle p_2|T^{\mu}_{\ \mu}(0)|p_1\rangle = \overline{u}(p_2)\left(A(k)\gamma^{\mu}p_{\mu} + B(k)\frac{ip^{\mu}\sigma_{\mu}^{\ \alpha}k_{\alpha}}{2m_N} - 3C(k)\frac{k^2}{m_N}\right)u(p_1)$$

- therefore, for $k=0$, we have $\langle p|\epsilon^{TT}_{\mu\nu}T^{\mu\nu}(0)|p\rangle = \langle p|T^{\mu}_{\ \mu}(0)|p\rangle$, for arbitrary values of $B(0)$ and $C(0)$
- in general $\langle p_2|\epsilon^{TT}_{\mu\nu}T^{\mu\nu}(0)|p_1\rangle \neq \langle p_2|T^{\mu}_{\ \mu}(0)|p_1\rangle$ unless $B(k)=C(k)=0$
- indeed in holographic QCD with degenerate mass spectrum for $2^{++}$ and $0^{++}$ glueballs, we have $B(k)=C(k)=0$
- but in lattice QCD with non-degenerate mass spectrum for $2^{++}$ and $0^{++}$ glueballs, we have $B(k)=0$, and $C(k)\neq 0$

# Holographic Gravitational Form Factors of Proton

- the holographic gravitational form factors of proton follow from the coupling of the irreducible representations of the metric fluctuations $h_{\mu\nu}$ to a bulk Dirac fermion
- and the bulk metric fluctuations can be decomposed in terms of the $2 \oplus 0$ invariant tensors [Kanitscheider:2008]

$$h_{\mu\nu}(k,z) \supset \left[\epsilon_{\mu\nu}^{TT} h(k,z)\right] + \left[\frac{1}{3}\eta_{\mu\nu} f(k,z)\right]$$

which is the spin-2 made of the transverse-traceless part $h$ plus the the spin-0 tracefull part $f$

# Spin-2 (Gravitational) Form Factor of Proton

- in holographic QCD, the gravitational form factors, can be computed using Witten diagrams in AdS

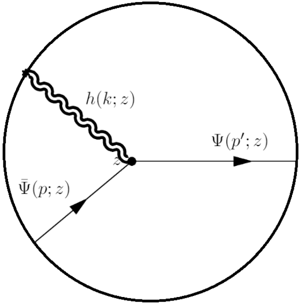



Figure: Witten diagram for the spin-2 gravitational form factor $\langle p_2|\epsilon^{TT}_{\mu\nu}T^{\mu\nu}(0)|p_1\rangle$ due to the exchange of spin-2 glueball resonances.

# Spin-2 (Gravitational) Form Factor of Proton

- the holographic spin-2 (gravitational) form factor of proton is given by [Abidin and Carlson:2009]

$$
\begin{aligned}
A(K,\kappa_T) &\equiv \frac{1}{2}\int dz\,\sqrt{g}\,e^{-\phi}z\,\left(\psi_R^2(z)+\psi_L^2(z)\right)\times\sum_{n=0}^{\infty}\frac{\sqrt{2}\kappa F_n\psi_n(z)}{K^2+m_n^2}\,,\\
&= \frac{1}{2}\int dz\,\sqrt{g}\,e^{-\phi}z\,\left(\psi_R^2(z)+\psi_L^2(z)\right)\times h(K,z)\,,\\
&= A(0)\times 6\times\frac{\Gamma\left(2+\frac{a_K}{2}\right)}{\Gamma\left(4+\frac{a_K}{2}\right)}\times {}_2F_1\left(3,\frac{a_K}{2};\frac{a_K}{2}+4;-1\right)\,,
\end{aligned}
$$

with $a_K = K^2/8\kappa_T^2$, $\kappa_T = 0.388$ GeV, and $A(0) = 0.430$

- for numerical convenience we can approximate the spin-2 (gravitational) form factor of proton by tripole form as

$$
A(K,m_{TT}) = \frac{A(0)}{\left(1+\frac{K^2}{m_{TT}^2}\right)^3}\,,
$$

with $m_{TT} = 1.612$ GeV, and $A(0) = 0.430$

# Spin-0 (Scalar) Form Factor of Proton

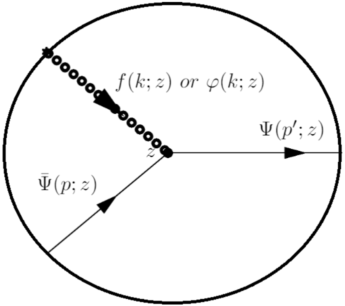



Figure: Witten diagram for the spin-0 gravitational form factor $\langle p_2 | T^{\mu}_{\ \mu}(0) | p_1 \rangle$ due to the exchange of spin-0 glueball resonances.

# Spin-0 (Scalar) Form Factor of Proton

- we have found the spin-0 (scalar) form factor of proton to be

$$D(K,\kappa_T) \equiv \frac{1}{2}\int dz\,\sqrt{g}\,e^{-\phi}z\,\left(\psi_R^2(z)+\psi_L^2(z)\right)\times\sum_{n=0}^{\infty}\frac{4}{3}\frac{p_1^2}{k^2}\times\frac{\sqrt{2}\kappa F_n\psi_n(z)}{-k^2+m_n^2} = -\frac{4}{3}\frac{m_N^2}{K^2}\times A(K,\kappa_T)\,,$$

- assuming non-degenerate spectrum for $0^{++}$ and $2^{++}$ glueballs we have

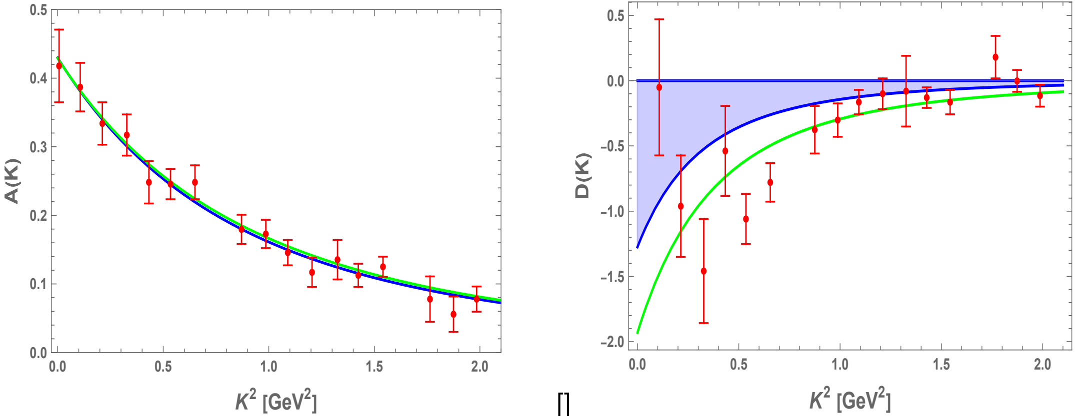
$$D(K,\kappa_T)\rightarrow D(K,\kappa_T,\kappa_S) = -\frac{4}{3}\frac{m_N^2}{K^2}\times\left[A(K,\kappa_T)-A_S(K,\kappa_S)\right]\,,$$

  with $\kappa_T = 0.388$ GeV, $\kappa_S = 0.217$ GeV, $A(0) = 0.430$, and $D(0) = -1.275$
- for numerical convenience we can also approximate the spin-0 (scalar) form factor of proton by tripole form as

$$D(K,m_{SS}) = \frac{D(0)}{\left(1+\frac{K^2}{m_{SS}^2}\right)^3}\,,$$

  with $m_{SS} = 0.963$ GeV, and $D(0) = -1.275$

# Gravitational Form Factors of Proton

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The $A$ and D form factor from the recent lattice QCD result [Pefkou:2021] (red lattice data points), and our holographic fit with $\kappa_T = 0.388$ GeV, $\kappa_S = 0.217$ GeV, $A(0) = 0.430$, and $D(0) = -1.275$ (solid-blue curve). The solid-green curve is the tripole lattice fit.

# Scalar and Mass Radii of Proton

- the gluonic scalar mass radius $r_{GS}$ of proton (derived from the trace of its energy-momentum tensor) can be defined as [Ji:2021]

$$\langle r_{GS}^2 \rangle = -\frac{6}{A_S(0)} \left( \frac{dA_S(K)}{dK^2} \right)_0 \hbar^2 c^2$$

where

$$A_S(K) \equiv A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{3K^2}{4m_N^2} D(K) \,.$$

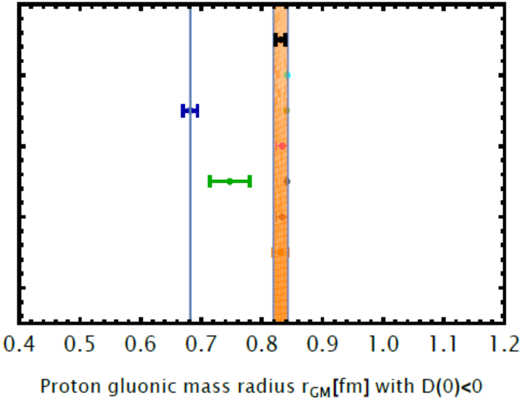
- the gluonic mass radius $r_{GM}$ of proton (derived from 00 component of its energy-momentum tensor) can also be defined as [Ji:2021]

$$\langle r_{GM}^2 \rangle = -\frac{6}{A_M(0)} \left( \frac{dA_M(K)}{dK^2} \right)_0 \hbar^2 c^2$$

where

$$A_M(K) \equiv A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{K^2}{4m_N^2} D(K) \,.$$

# Scalar and Mass Radii of Proton

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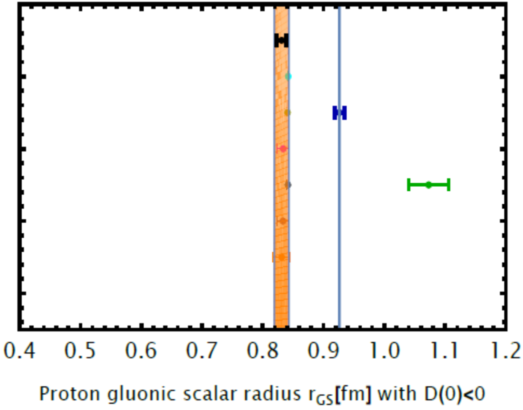


[] The blue data points at $(r_{GM}, r_{GS}) = (0.682 \pm 0.012 \text{ fm}, 0.926 \pm 0.008 \text{ fm})$ are determined by using our holographic A and D form factors, and the green data point at $(r_{GM}, r_{GS}) = (0.747 \pm 0.033 \text{ fm}, 0.926 \pm 0.008 \text{ fm})$ is determined using the lattice tripole fit to the lattice A and D form factor data [Pefkou:2021].

# Electroproduction of heavy mesons near threshold

- the spin-2 (gravitational) form factor can be measured from photo or electroproduction of heavy vector mesons ($J/\psi$ or $\Upsilon$) near threshold

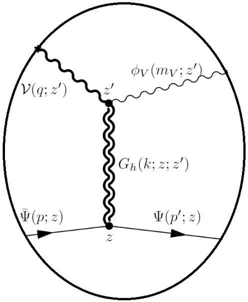


Figure: Witten diagram for the diffractive photoproduction of vector mesons with a bulk wave function $\phi_V$. The thick lines or thick wiggles represent the propagators of summed over vector meson or glueball resonances. The thin lines or thin wiggles correspond to a single vector meson and proton.

# Electroproduction of heavy mesons near threshold

- the differential cross section for electroproduction of heavy vector mesons ($J/\psi$ or $\Upsilon$), near threshold, is given by

$$\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_T,\epsilon'_T)}{dt} \propto \mathcal{I}^2(Q,M_{J/\Psi}) \times \left(\frac{s}{\tilde{\kappa}_N^2}\right)^2 \times \left[A(t)+\eta^2 D(t)\right]^2$$

$$\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_L,\epsilon'_L)}{dt} \propto \frac{1}{9} \times \frac{Q^2}{M_{J/\Psi}^2} \times \mathcal{I}^2(Q,M_{J/\Psi}) \times \left(\frac{s}{\tilde{\kappa}_N^2}\right)^2 \times \left[A(t)+\eta^2 D(t)\right]^2$$

# Electroproduction of heavy mesons near threshold

- where we defined the transition form factor that controls the $Q$ dependence as

$$\mathcal{I}(Q, M_{J/\Psi}) = \frac{\mathcal{I}(0, M_{J/\Psi})}{\frac{1}{6}\times\left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}}+3\right)\left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}}+2\right)\left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}}+1\right)},$$

with $\mathcal{I}(0, M_{J/\Psi}) = \frac{g_5 f_{J/\Psi}}{4M_{J/\Psi}}$, and $\tilde{\kappa}_{J/\Psi} = 2^{-3/4}\sqrt{g_5 f_{J/\Psi} M_{J/\Psi}}$

# Electroproduction of heavy mesons near threshold

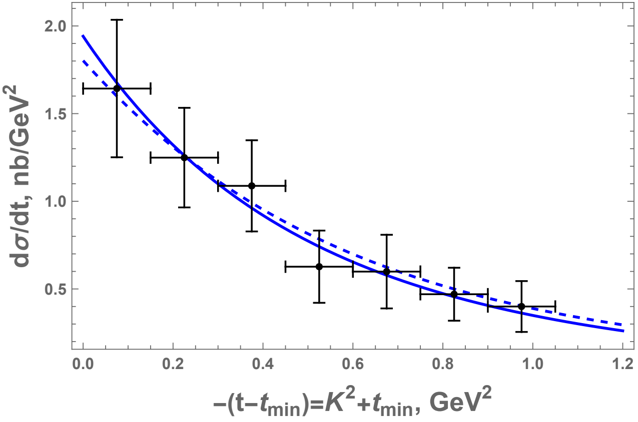


Figure: Holographic differential cross section for $J/\Psi$ photoproduction at $E_\gamma = 10.72$ GeV. The solid-blue curve is with $\mathcal{N} \times e = 2.311\,\mathrm{nb\,GeV^{-2}}$, $\kappa_T = \kappa_S = 0.388$ GeV, and $D(0) = 0$. The dashed-blue curve is with $\mathcal{N} \times e = 2.032\,\mathrm{nb\,GeV^{-2}}$, $\kappa_T = 0.388$ GeV, $\kappa_S = 0.217$ GeV, and $D(0) = -1.275$. The data is from GlueX [GlueX:2019].

# Electroproduction of heavy mesons near threshold

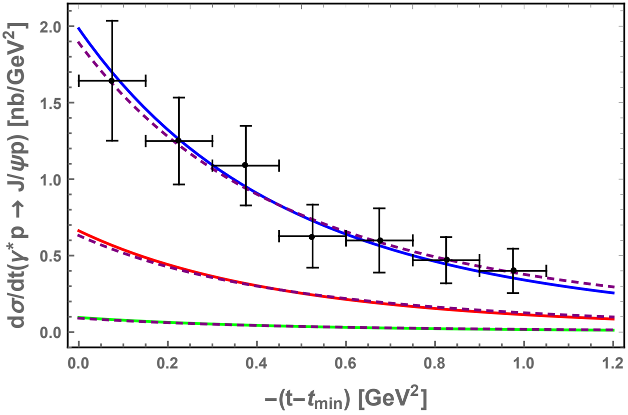


Figure: The variation of the total differential cross section with $t$ and $Q^2$ for $s = 21\ GeV^2$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV, and $A^2(0) \times \tilde{n}' = 30.7944\ [nb/GeV^2]$.The blue curve is for $Q = 0$. The red curve is for $Q = 1.2\ GeV$. The green curve is for $Q = 2.2\ GeV$. The data is from GlueX collaboration at JLab in 2019.

# Electroproduction of heavy mesons near threshold

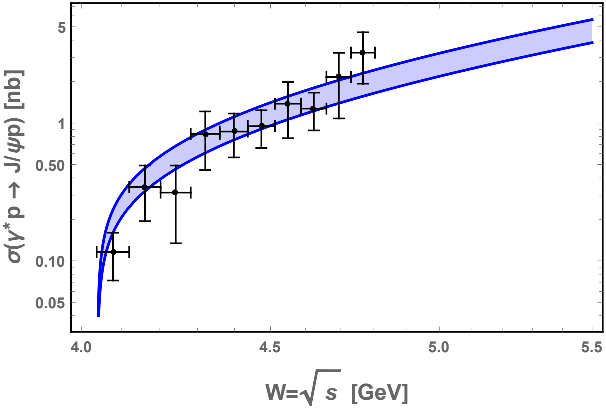


Figure: The total cross section near threshold, for $e = 0.3$, $m_N = 0.94$ GeV, $\tilde{\kappa}_N = 0.350$ GeV, $f_{J/\Psi} = 0.405$ GeV, $M_{J/\Psi} = 3.1$ GeV, $A^2(0) \times \tilde{n} = 240 \pm 47$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV, and $Q^2 = 0$. The data is from GlueX collaboration at JLab in 2019.

# Electroproduction of heavy mesons near threshold

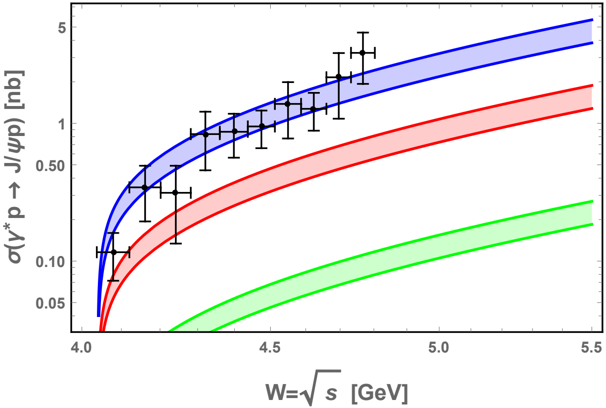


Figure: The variation of the total cross section (near threshold) with $Q^2$ and $\sqrt{s}$. The blue band is for $Q^2 = 0$ (the data is from GlueX in 2019), the red band is for $Q^2 = 1.2^2$ GeV$^2$, the green band is for $Q^2 = 2.2^2$ GeV$^2$.

# Electroproduction of heavy mesons near threshold

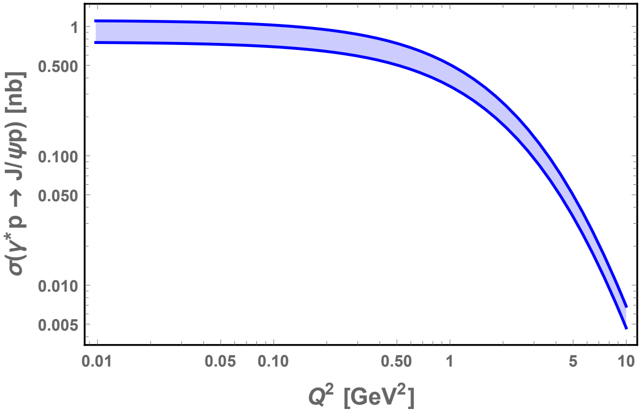


Figure: The variation of the total cross section with $Q^2$ (near threshold), for $\tilde{\kappa}_N = 0.350$ GeV, $s = W^2 = 4.4^2$ GeV$^2$, $e = 0.3$, $f_{J/\Psi} = 0.405$ GeV, $M_{J/\Psi} = 3.1$ GeV, $A^2(0) \times \tilde{n} = 240 \pm 47$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV.

# Electroproduction of heavy mesons near threshold

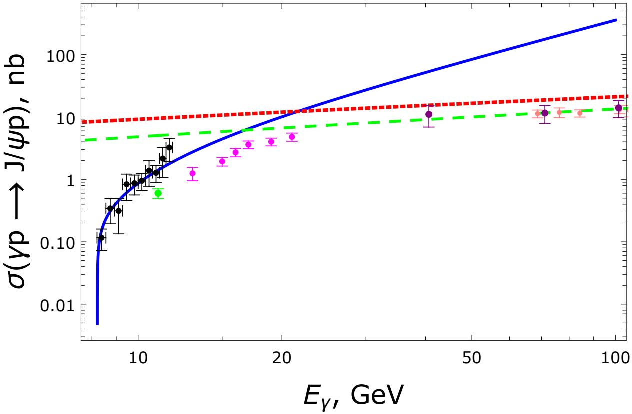


Figure: The total cross section for $J/\Psi$ photoproduction.

# Thank You!