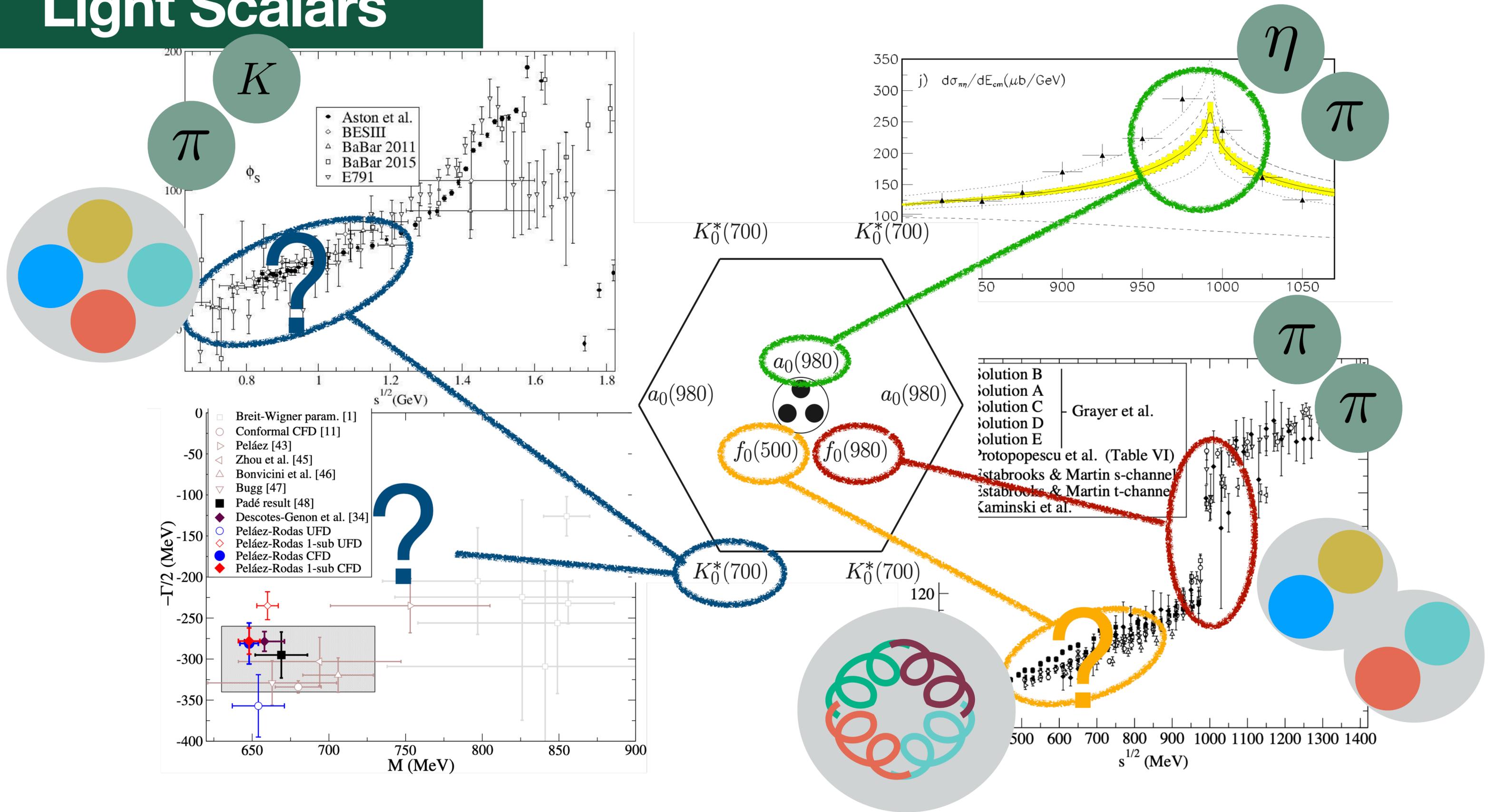


# The Quest for Exotica

## IV Dispersive analyses

Arkaitz Rodas

# Light Scalars

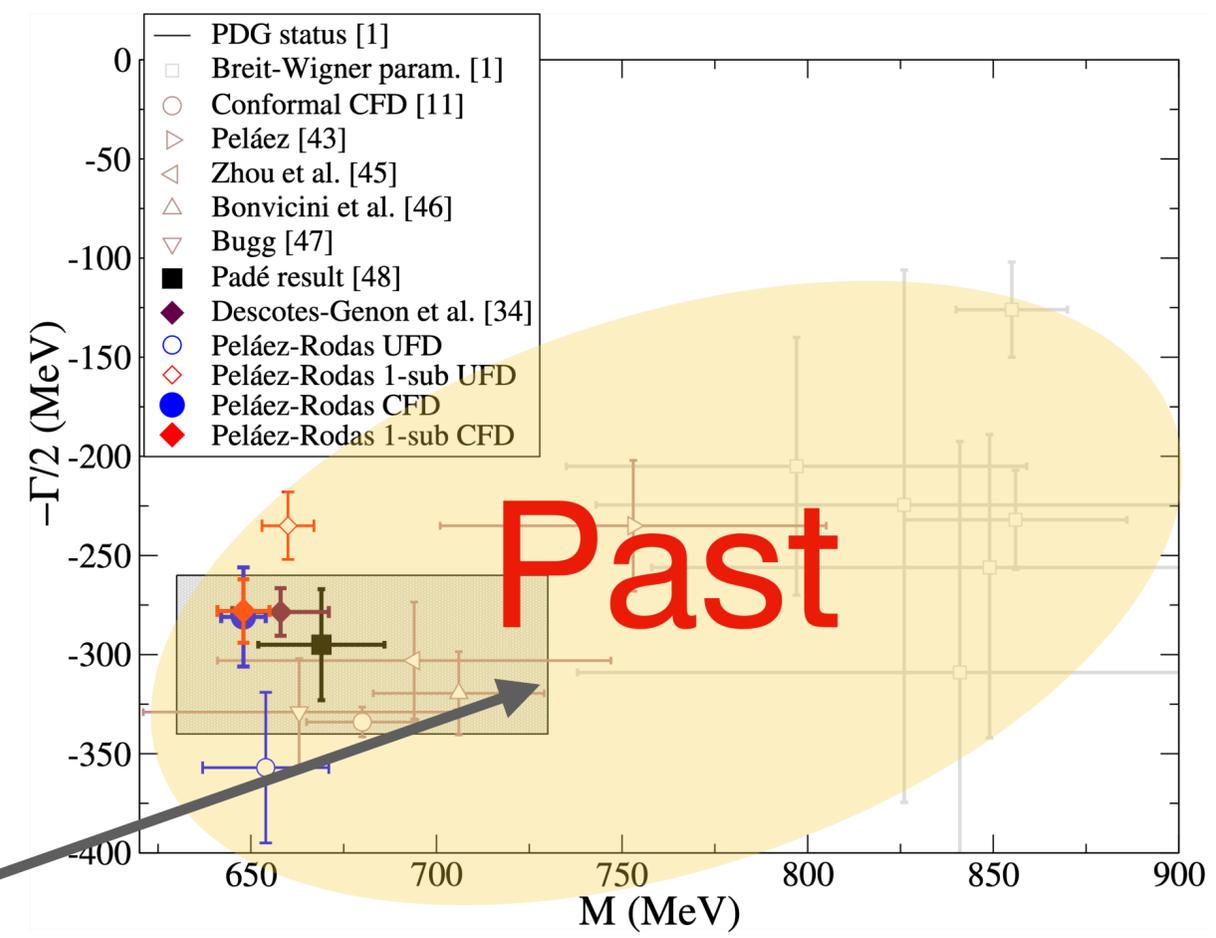


# Light Scalars

First step is determining them

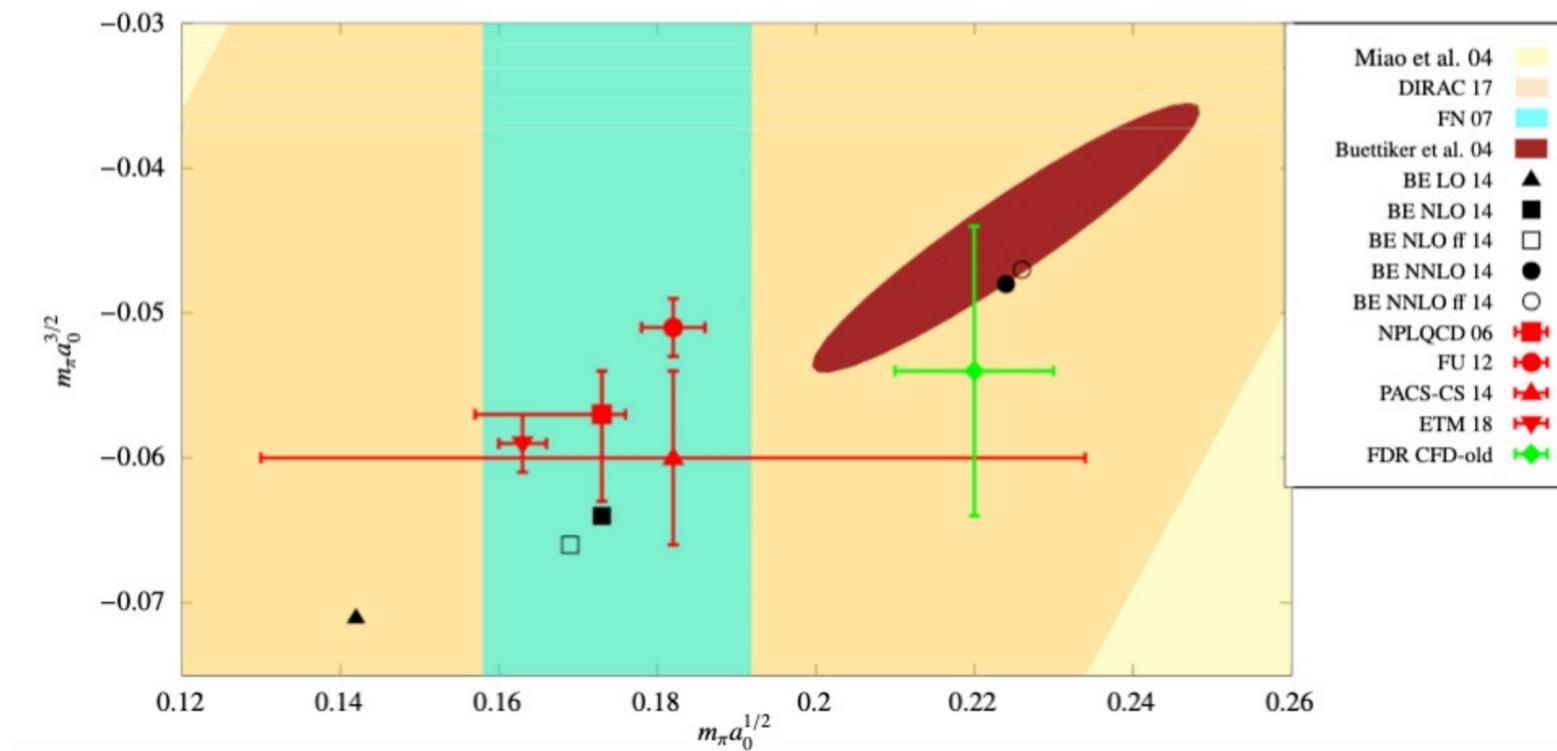
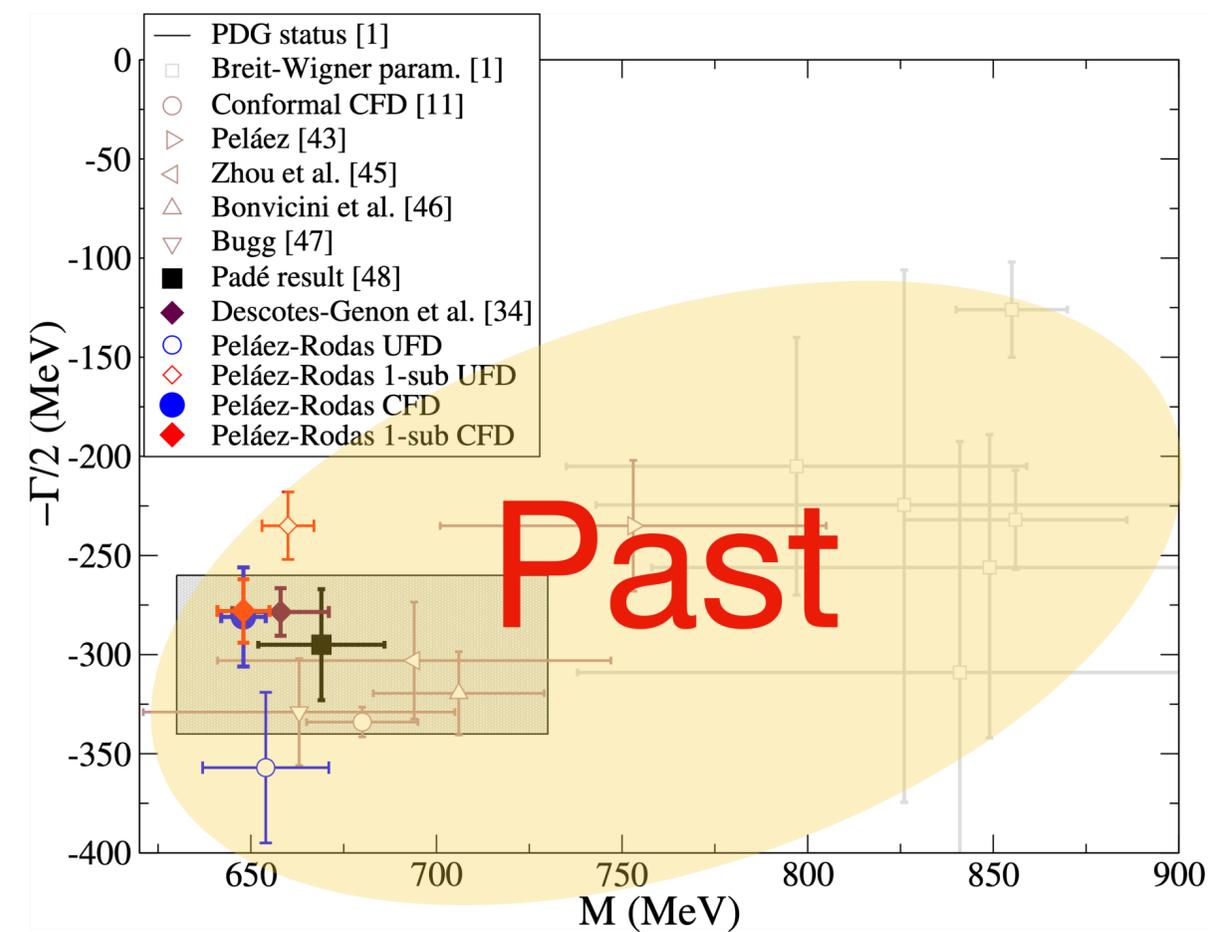
If fundamental parameters are wrong  $\rightarrow$  no bueno!

Well...



# Light Scalars

“We are beginning to think that  $\kappa$  should be classified along with flying saucers, the Loch Ness Monster, and the Abominable Snowman”  
 PDG 1967

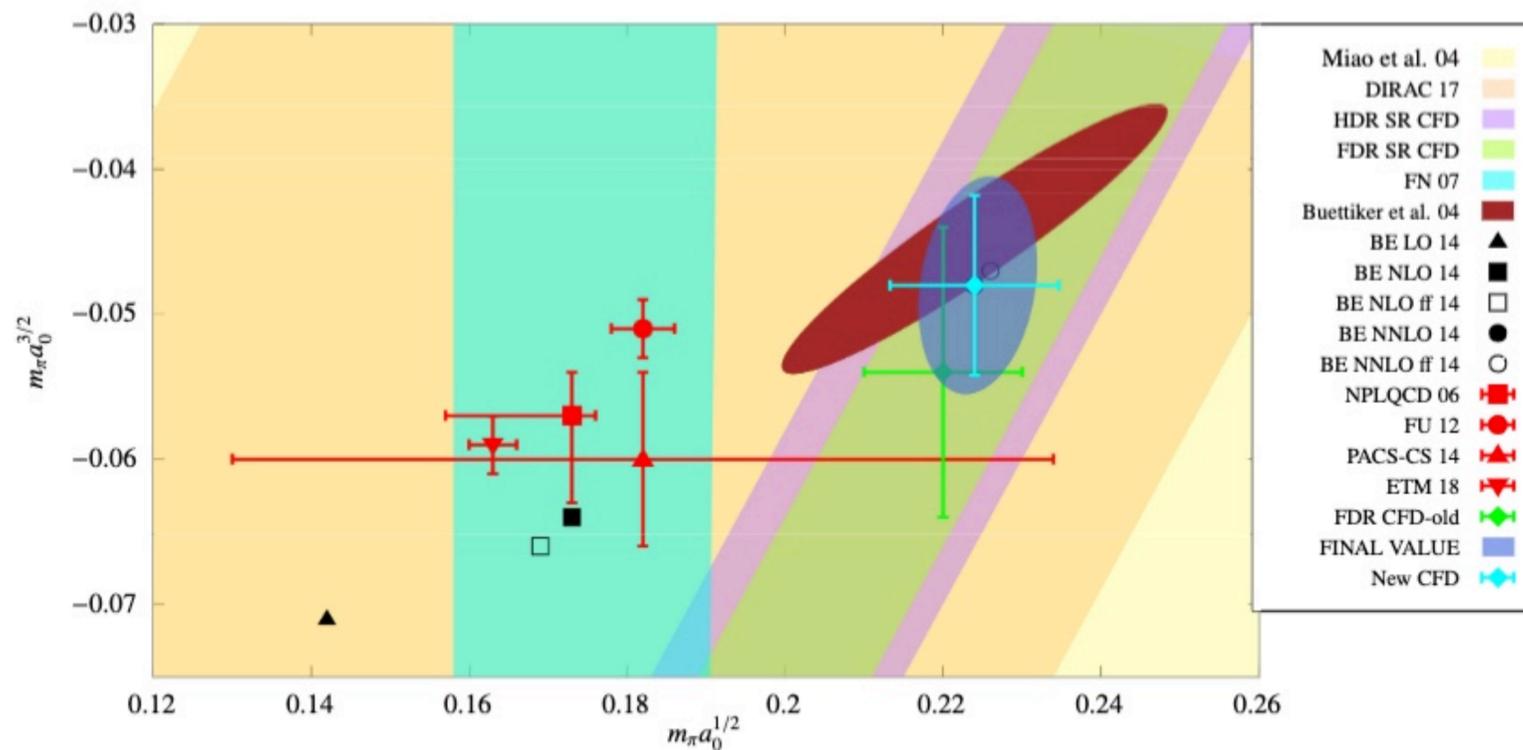
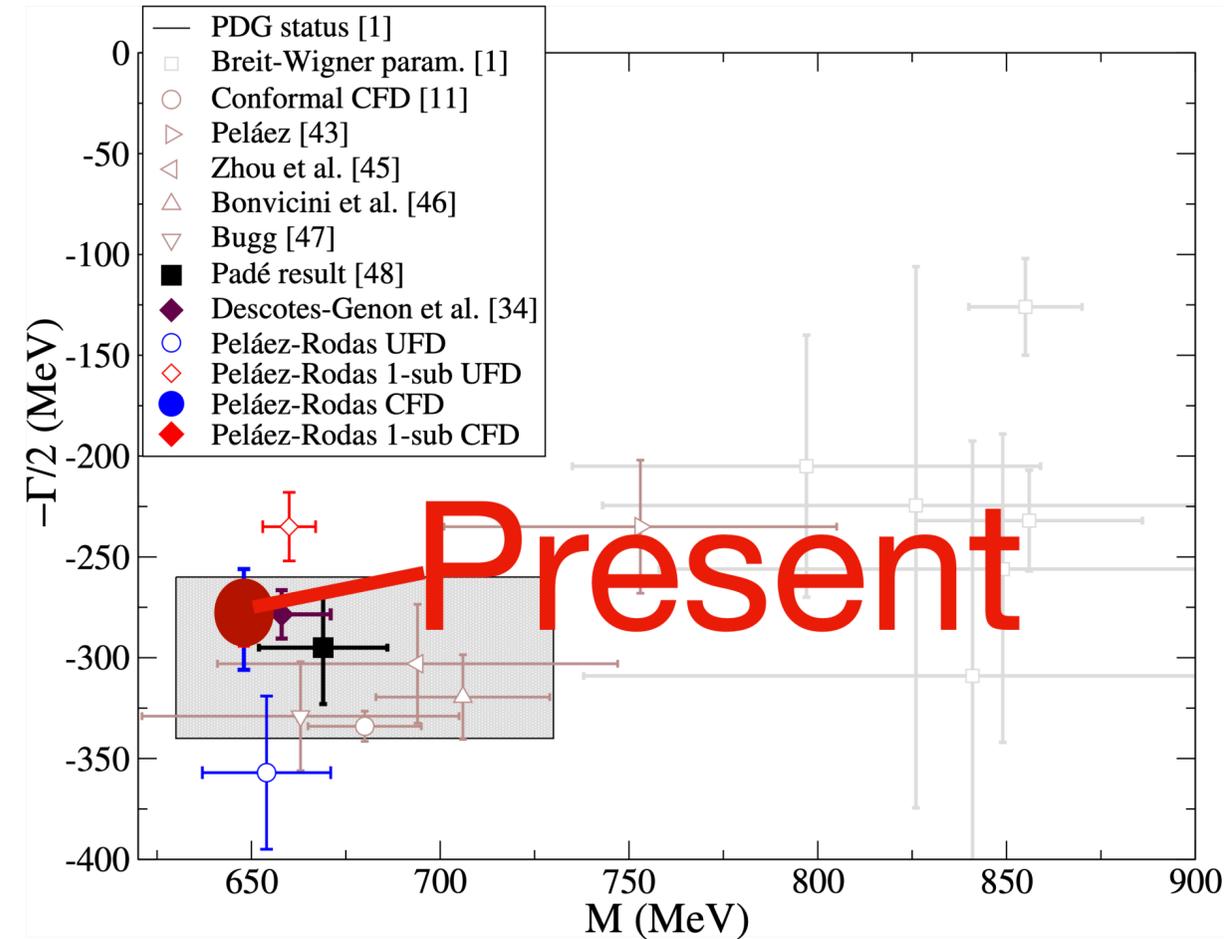


# Light Scalars

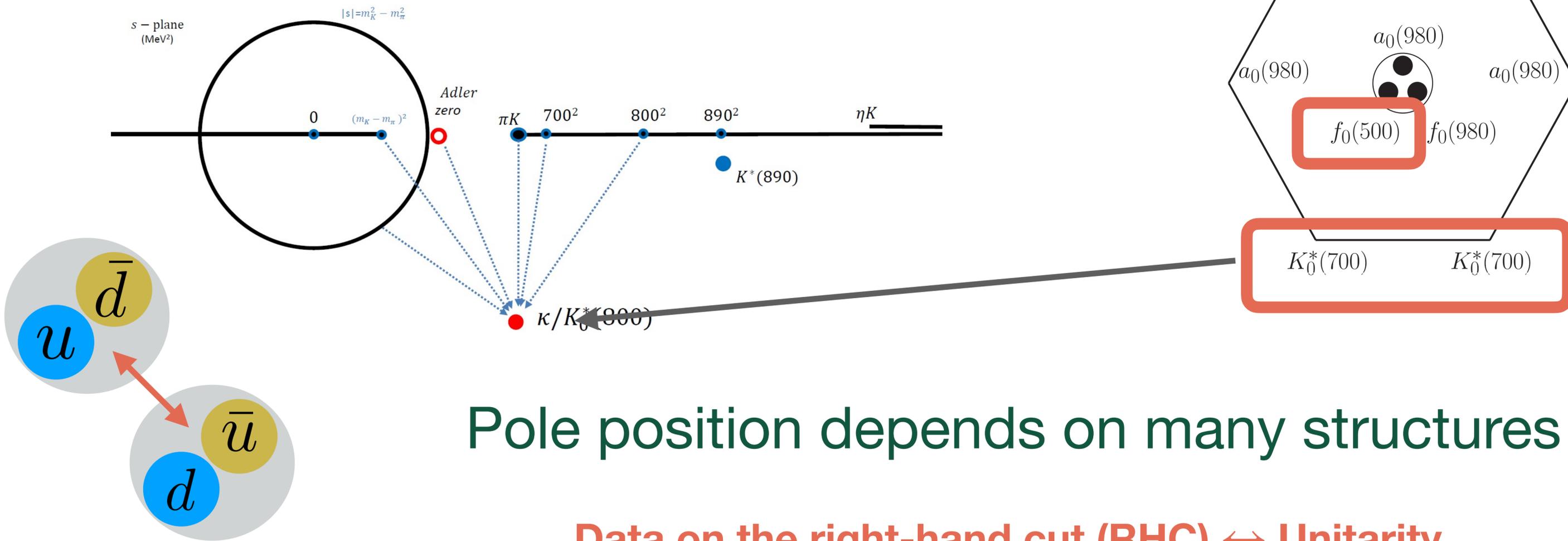
“ $\kappa$  accepted”

PDG 2021

$$t_\ell(s) = \frac{\sqrt{s}}{2q(s)} \sin \delta_\ell(s) e^{i\delta_\ell(s)} \propto a$$



# Problem 1



Pole position depends on many structures

Data on the right-hand cut (RHC)  $\leftrightarrow$  Unitarity

Low energy expansion

Adler Zero

Left-hand cut

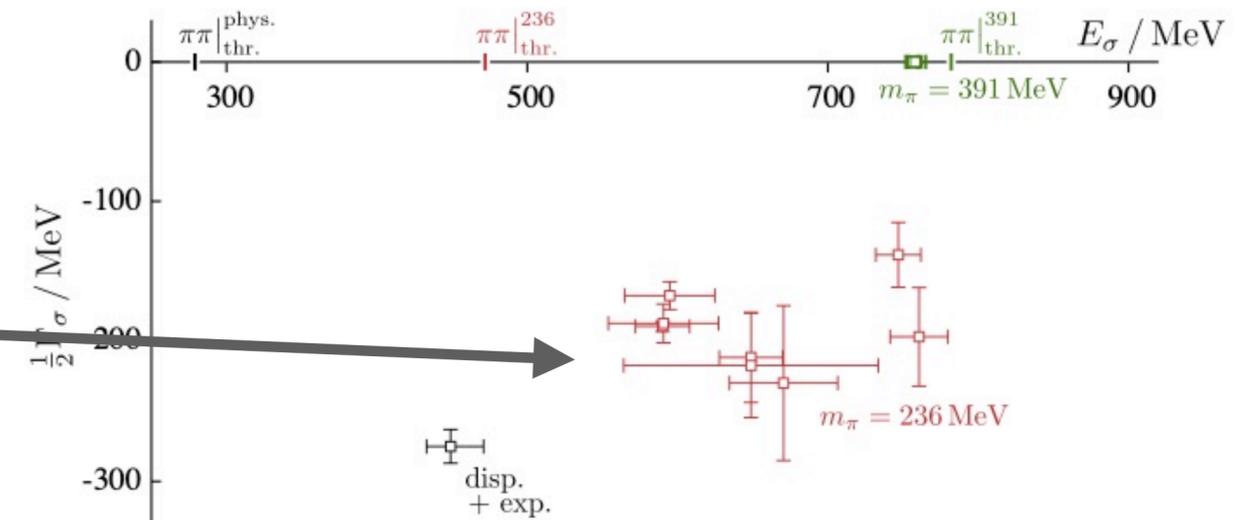
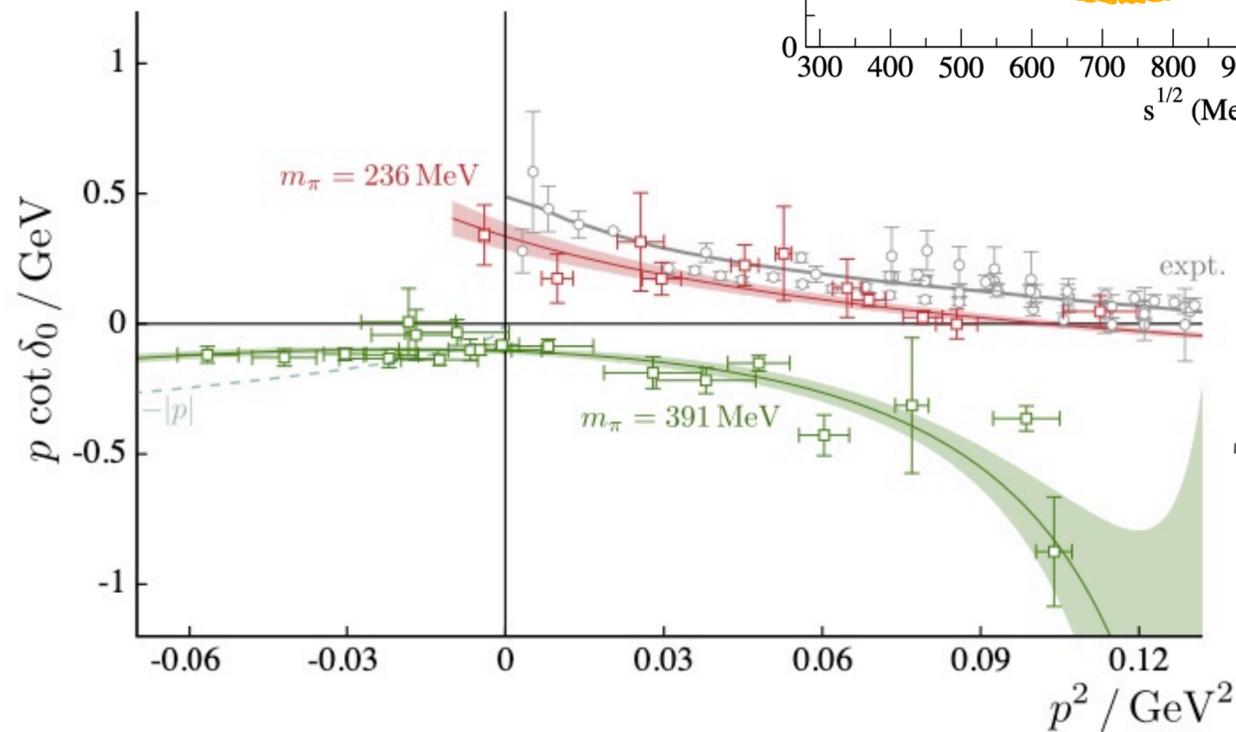
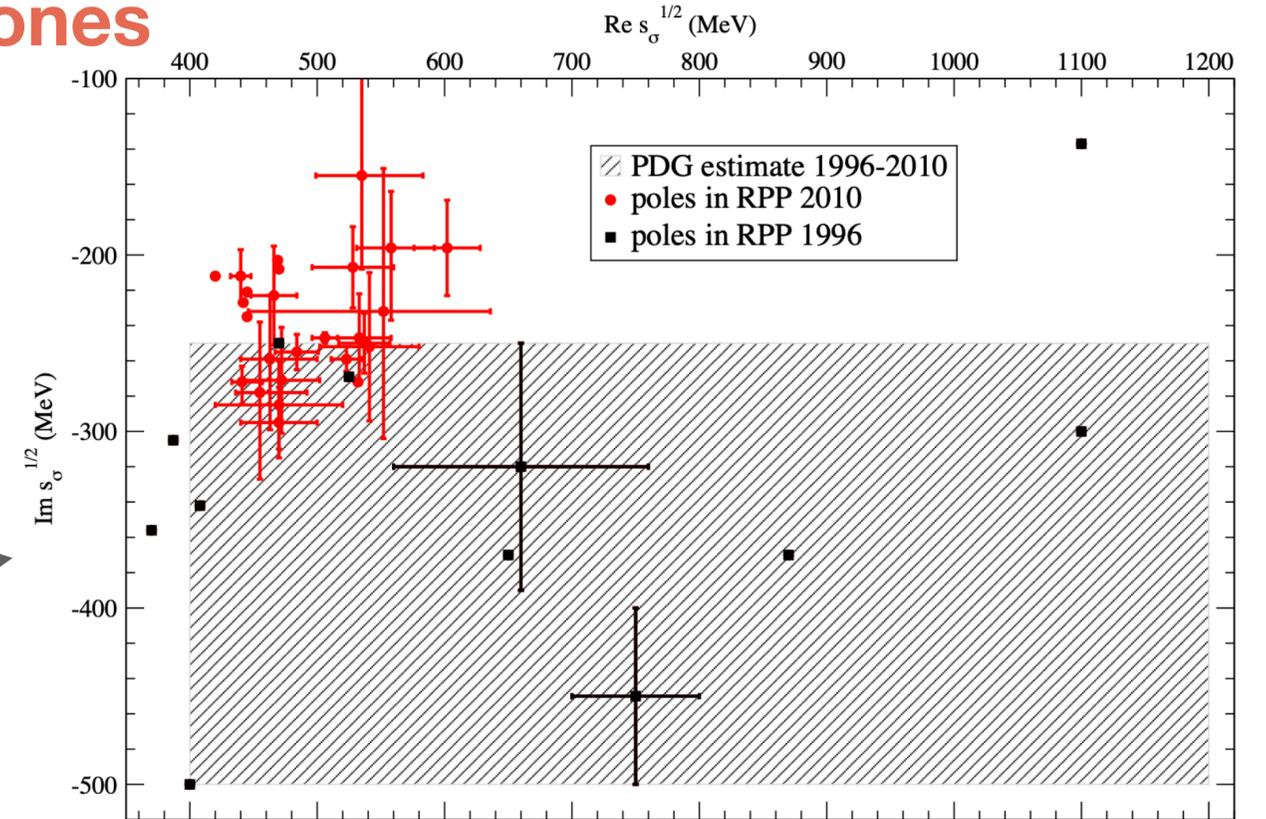
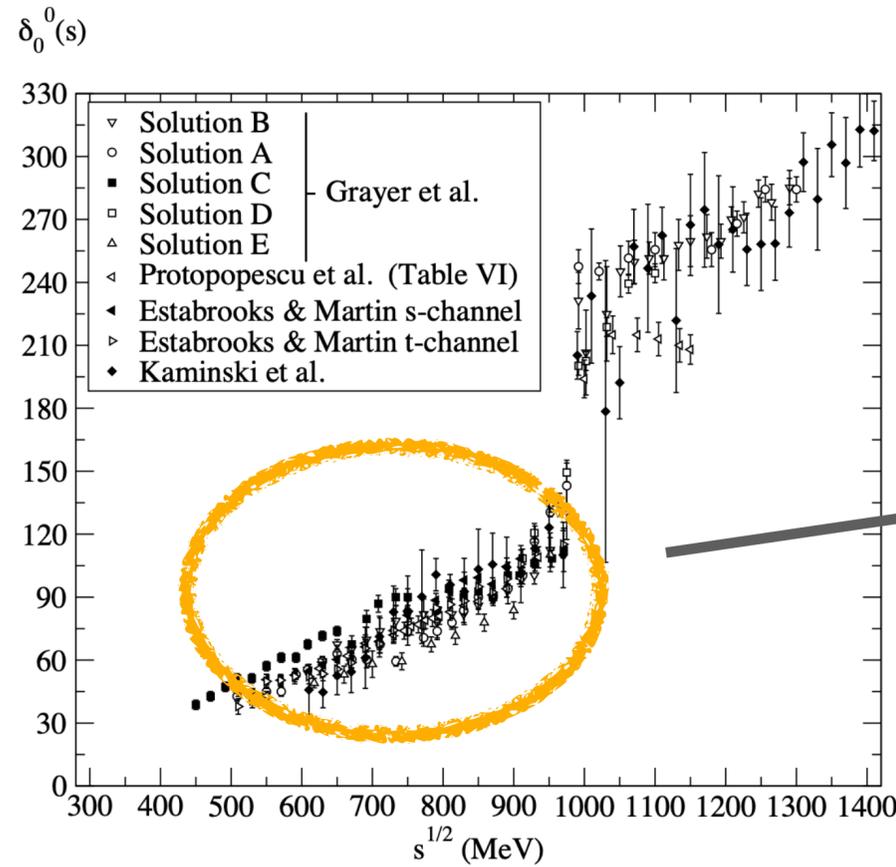
# Problem 1

# Models produce bad extractions

Even the best ones

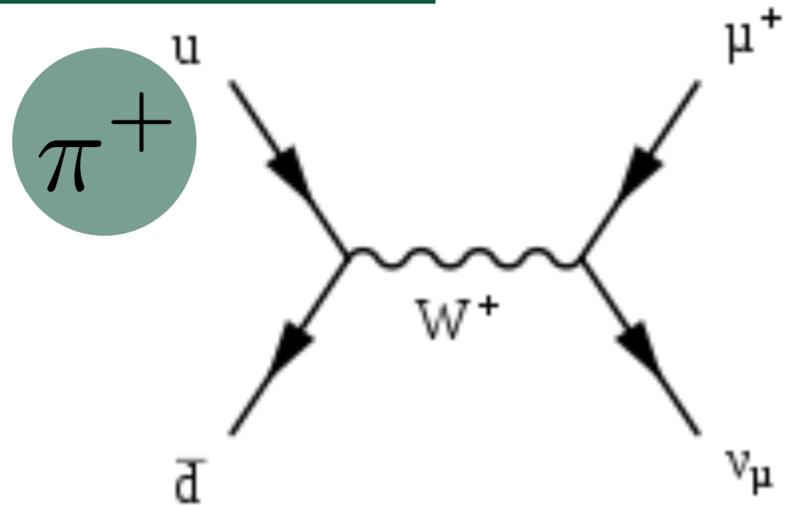
Experiment

had spec



# Problem 2

$$c\tau_{\pi^+} = 7.8 \text{ m}$$

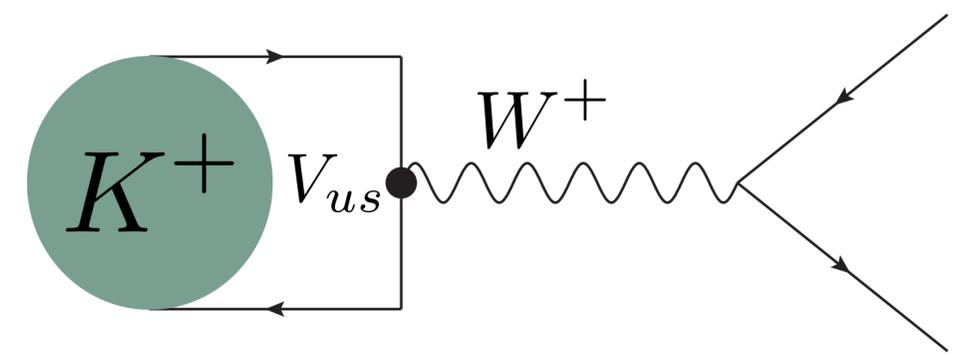


## Mesons decay

One beam (maybe)

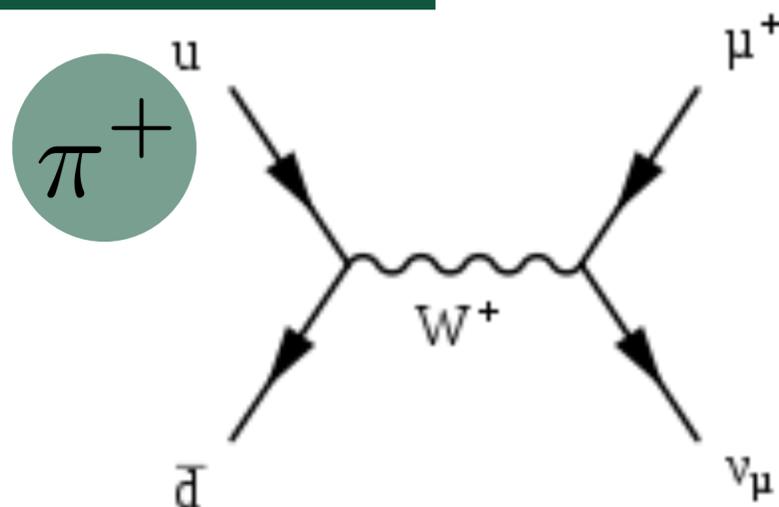
Not two beams

$$c\tau_K = 3.7 \text{ m}$$

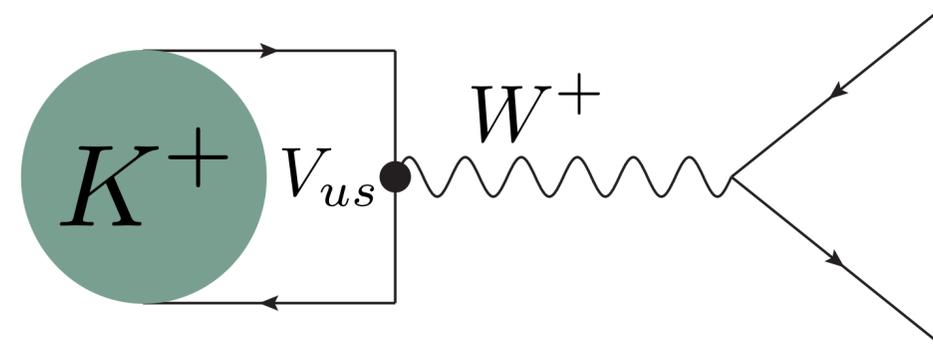


# Problem 2

$$c\tau_{\pi^+} = 7.8 \text{ m}$$



$$c\tau_K = 3.7 \text{ m}$$



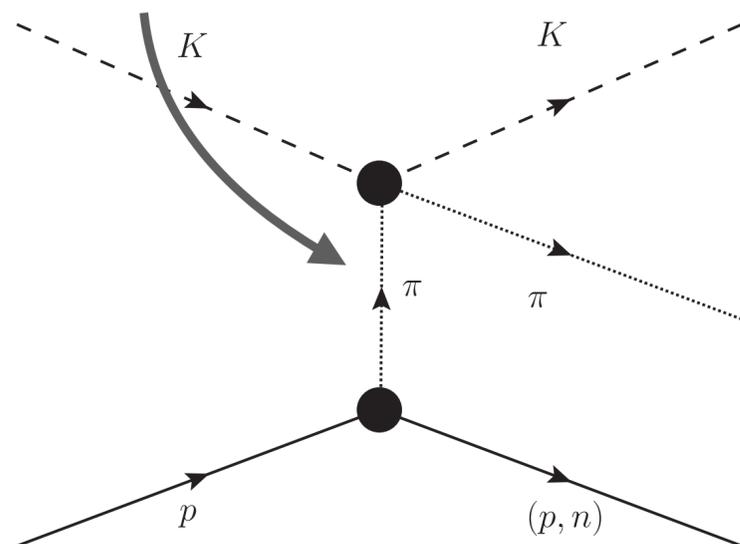
“Data” is not data

## Mesons decay

One beam (maybe)

Not two beams

### Virtual pion



For meson-meson data must be always modeled

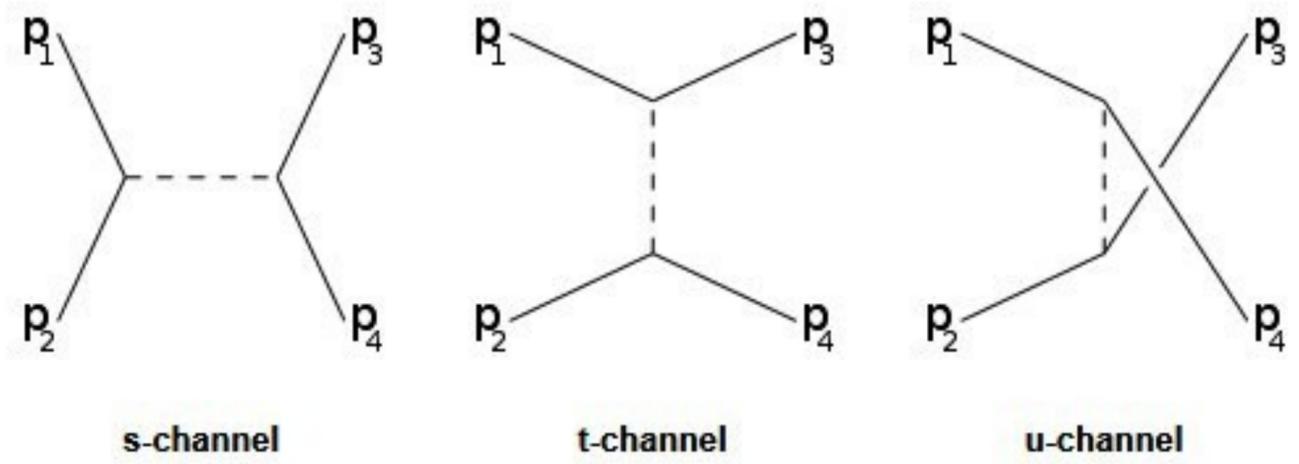
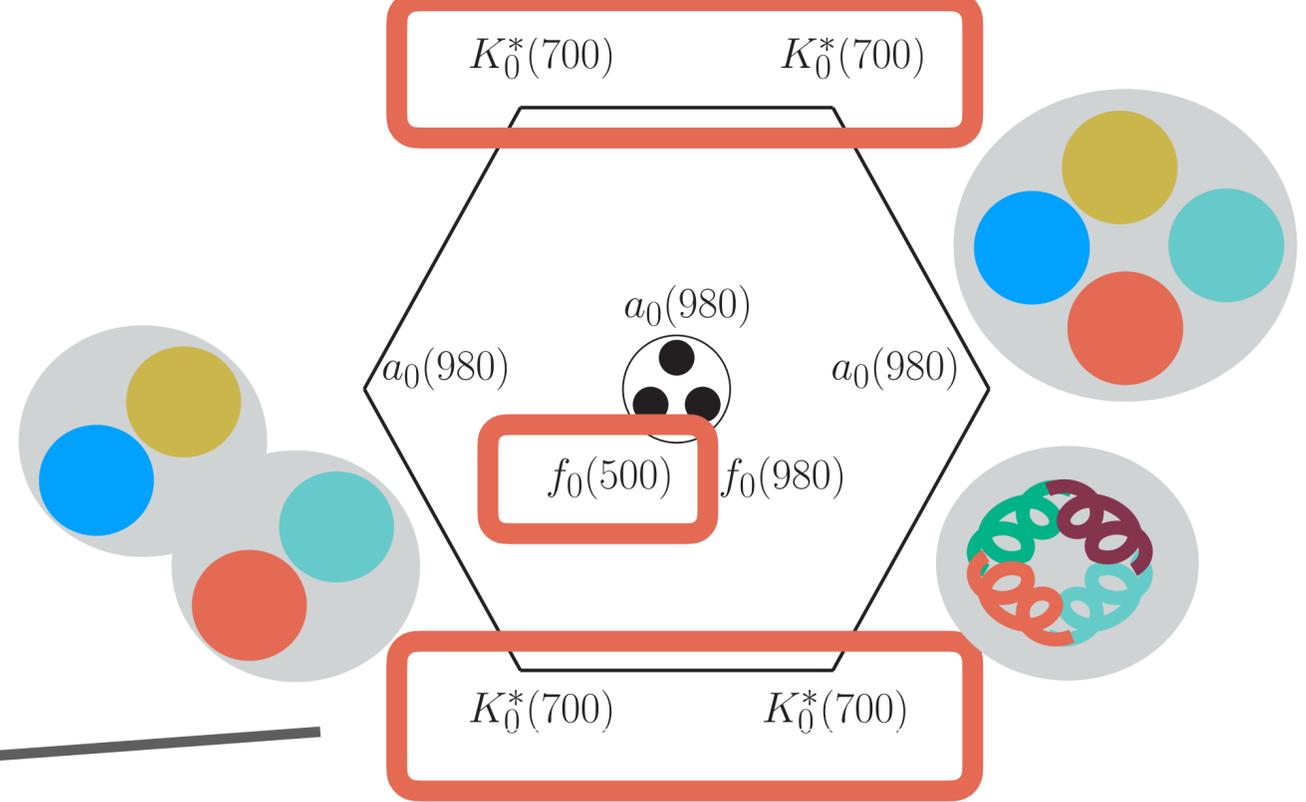
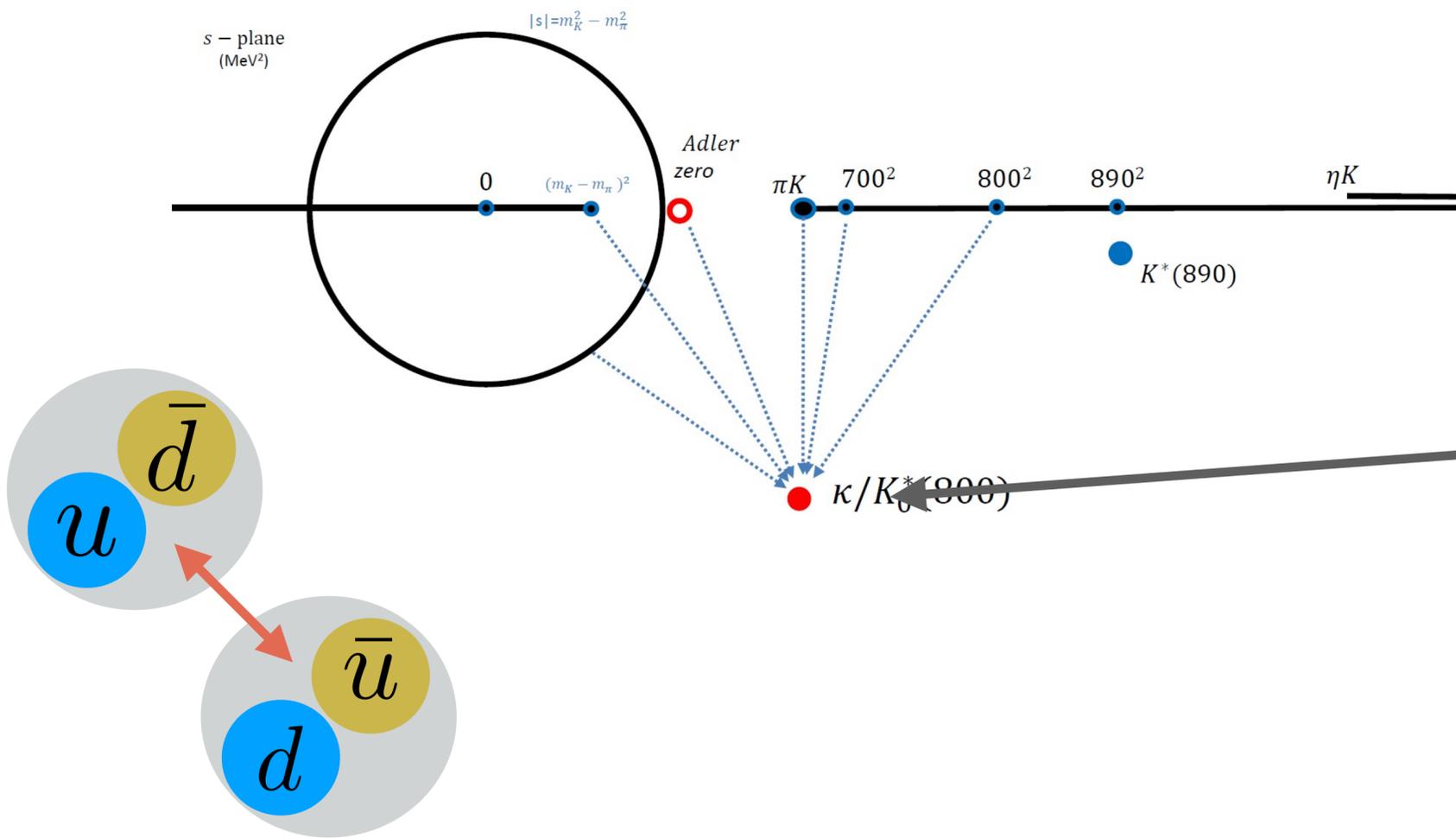
Some systematic uncertainties unknown

# Solution?

## Amplitude analyses

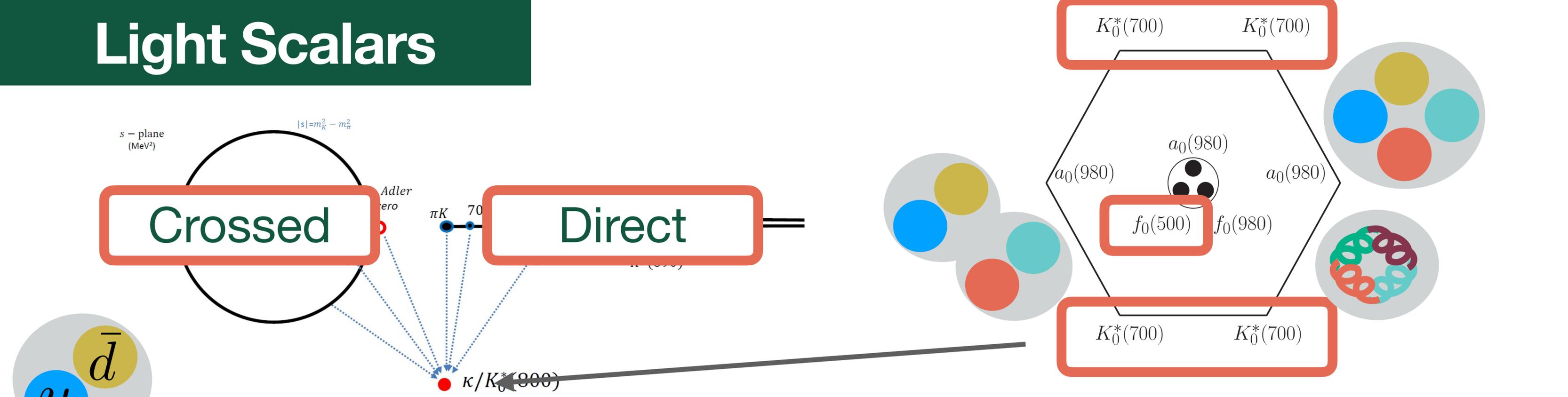
- Unitarity
- Analiticity
- Crossing

# Light Scalars

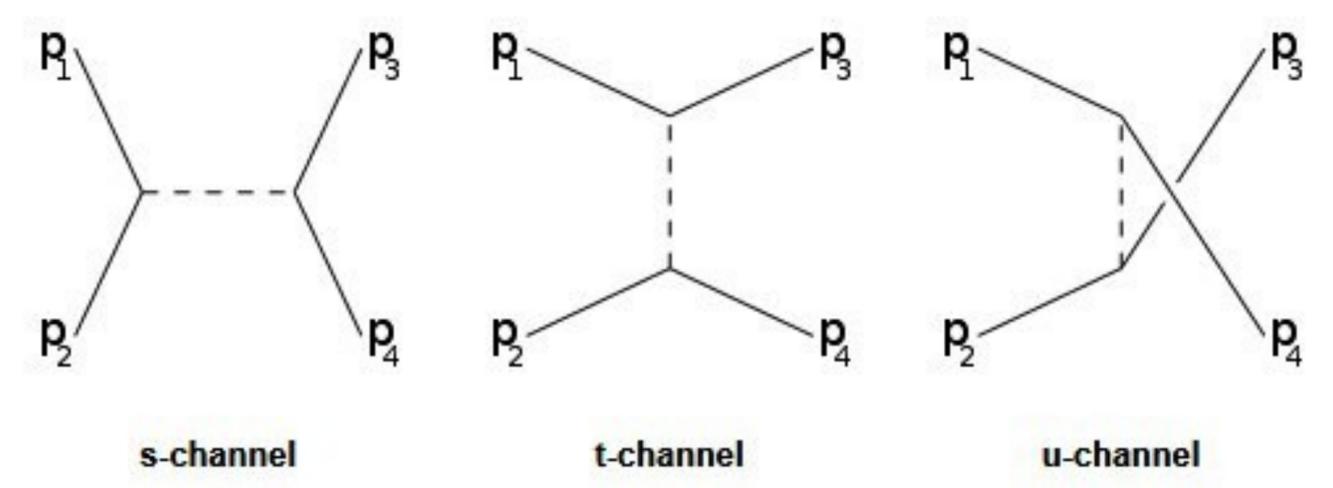


$$T(s, t, u) = \begin{cases} T_{12 \rightarrow 34}(s, t, u), & s \geq (m_1 + m_2)^2, & t \leq 0, & u \leq 0, \\ T_{1\bar{3} \rightarrow \bar{2}4}(t, s, u), & t \geq (m_1 + m_3)^2, & s \leq 0, & u \leq 0, \\ T_{1\bar{4} \rightarrow 3\bar{2}}(u, t, s), & u \geq (m_1 + m_4)^2, & s \leq 0, & t \leq 0. \end{cases}$$

# Light Scalars



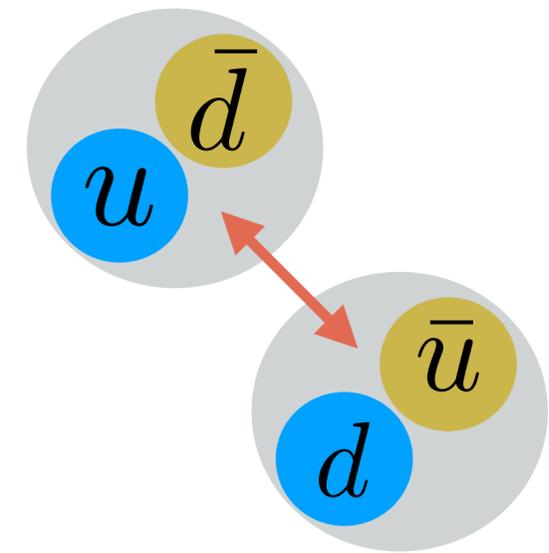
We need to relate particle  $\leftrightarrow$  anti-particle



$$T(s, t, u) = \begin{cases} T_{12 \rightarrow 34}(s, t, u), & s \geq (m_1 + m_2)^2, & t \leq 0, & u \leq 0, \\ T_{1\bar{3} \rightarrow \bar{2}4}(t, s, u), & t \geq (m_1 + m_3)^2, & s \leq 0, & u \leq 0, \\ T_{1\bar{4} \rightarrow 3\bar{2}}(u, t, s), & u \geq (m_1 + m_4)^2, & s \leq 0, & t \leq 0. \end{cases}$$

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

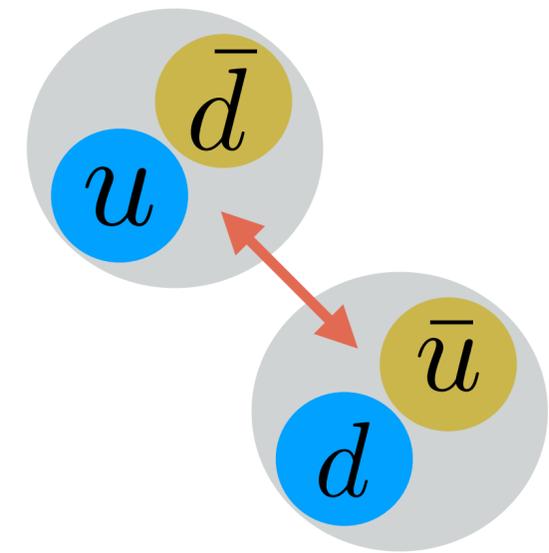
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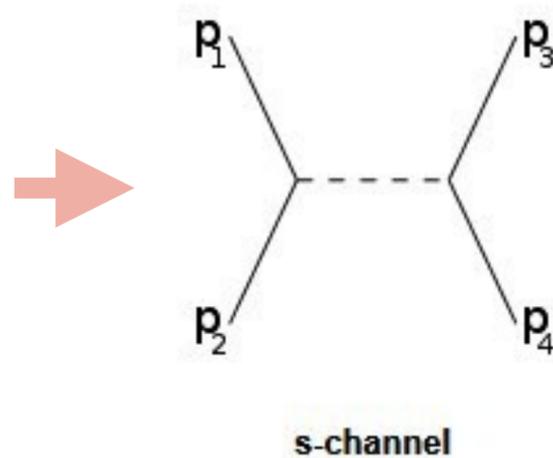
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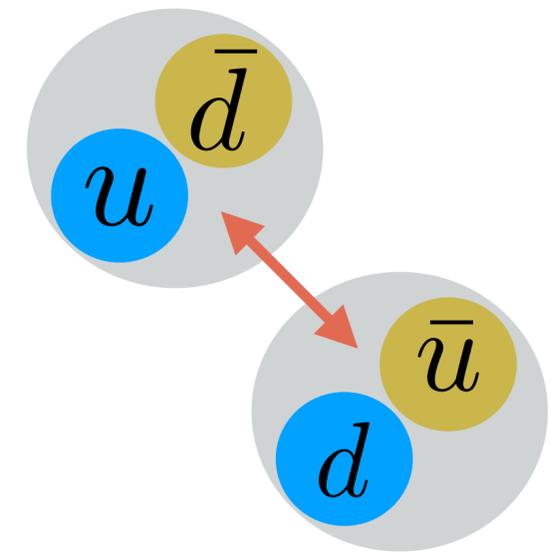
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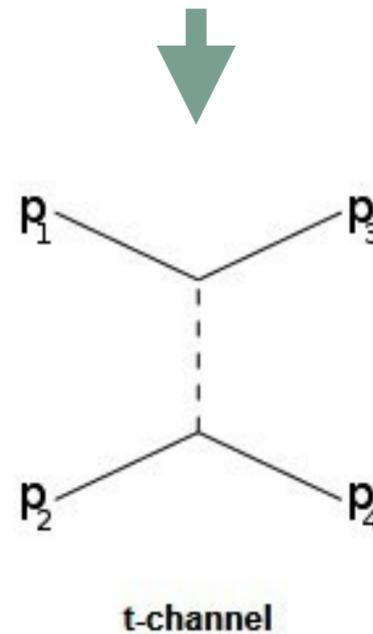
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$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

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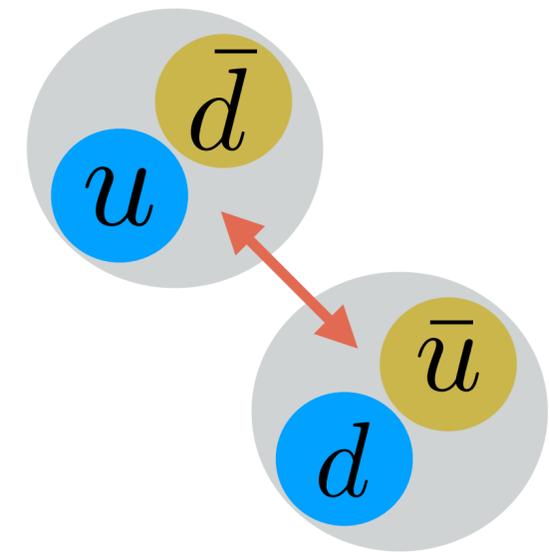
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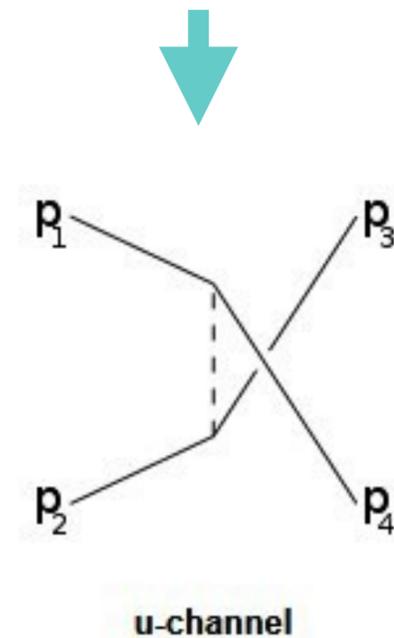
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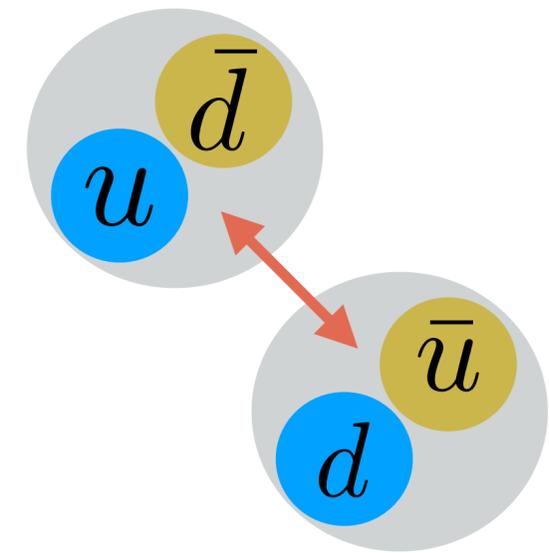
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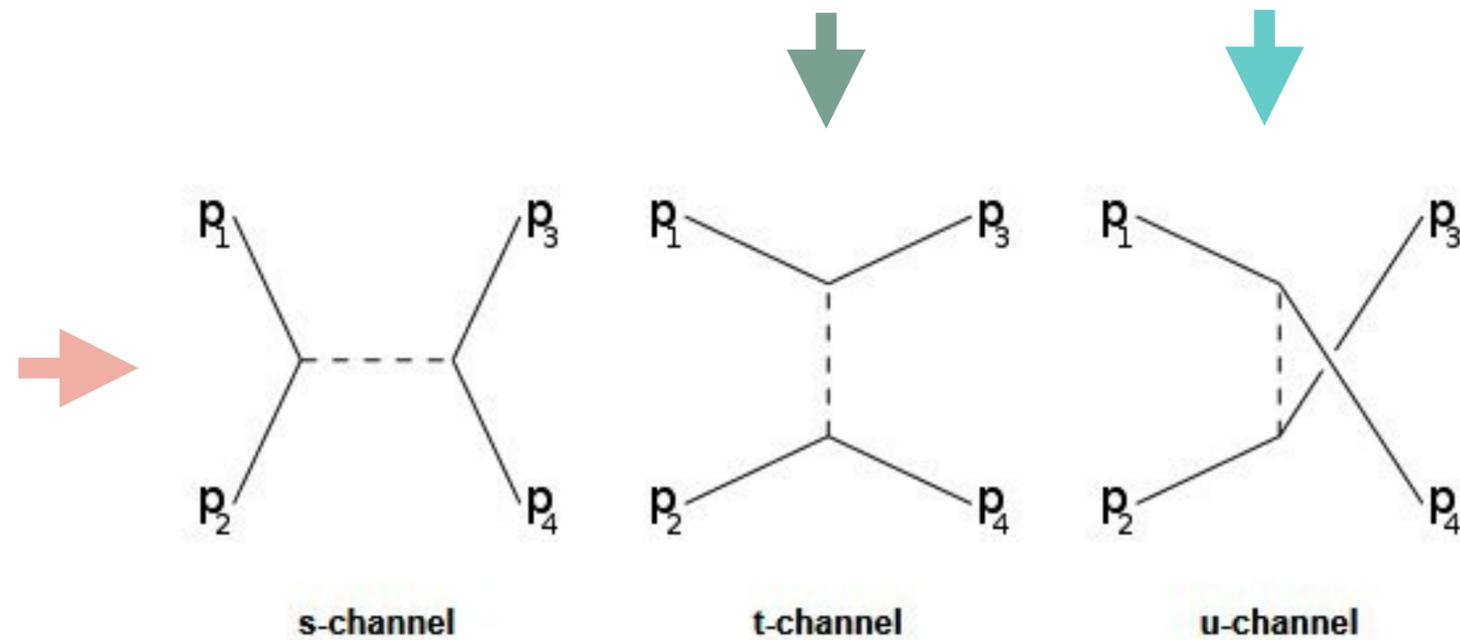
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Crossing

$$T^{I_s}(s, t, u) = \sum_{I_t} C_{su} T^{I_u}(u, t, s)$$

# Dispersion relations

$$m_3 = m_1$$

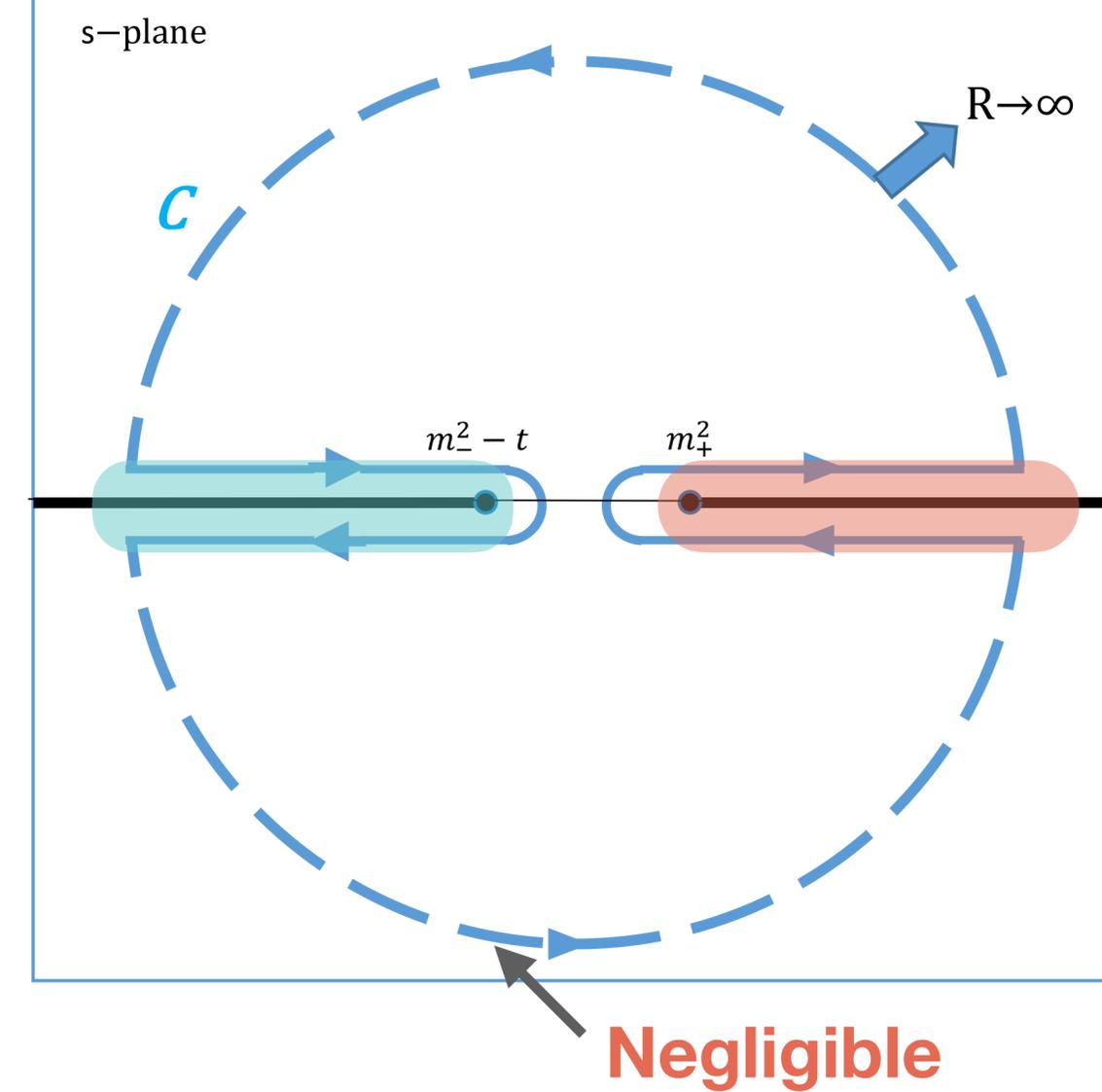
$$m_4 = m_2$$

Cauchy

$$t(z) = \oint_C \frac{t(z')}{z' - z} dz'$$

$$m_+ = (m_1 + m_2)^2$$

$$m_- = (m_1 - m_2)^2$$



# Dispersion relations

$$m_+ = (m_1 + m_2)^2$$

$$m_- = (m_1 - m_2)^2$$

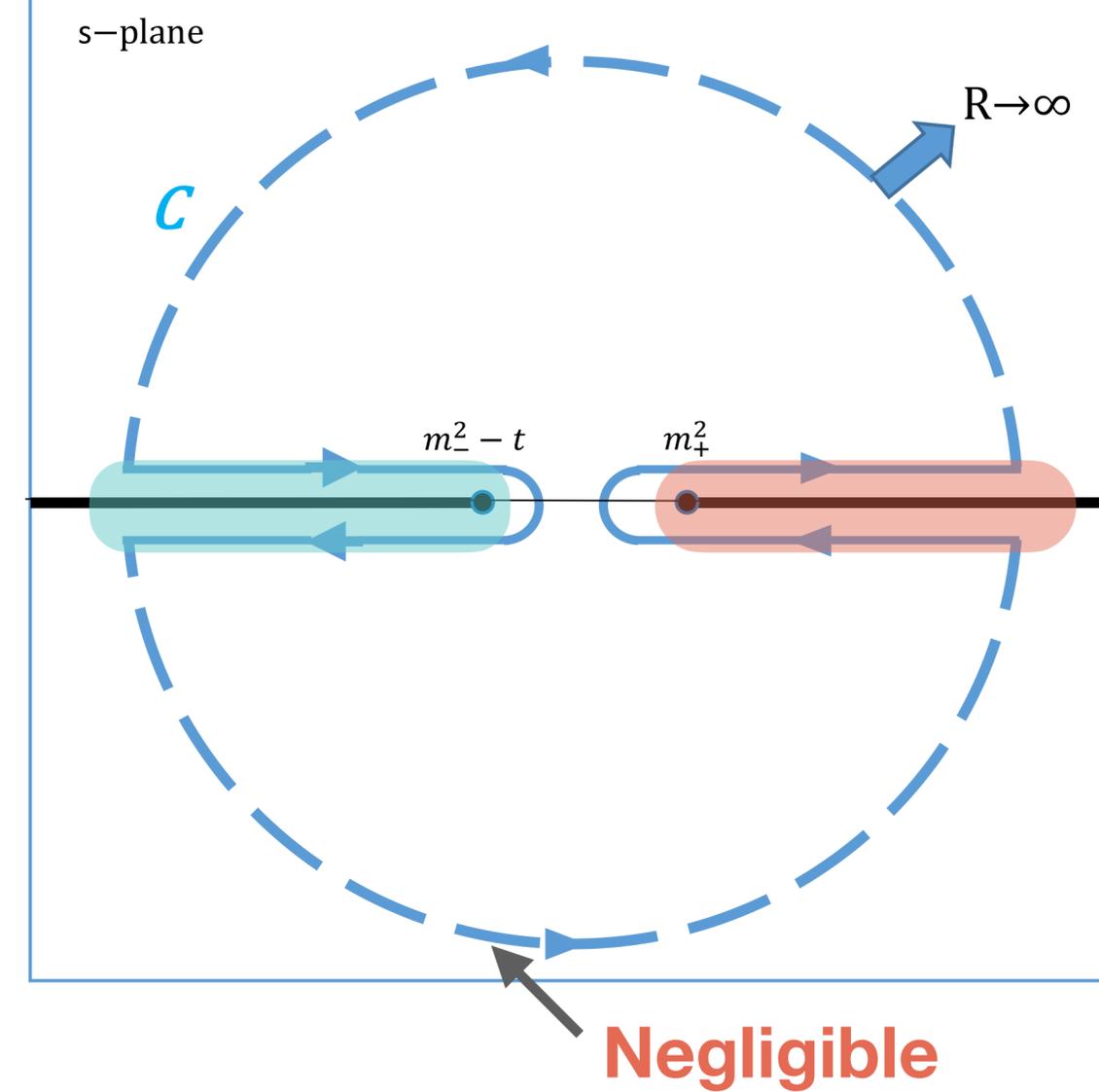
$$m_3 = m_1$$

$$m_4 = m_2$$

Cauchy

$$t(z) = \oint_C \frac{t(z')}{z' - z} dz'$$

$$T(s, t) = \frac{1}{\pi} \int_{sth}^{\infty} ds' \frac{\text{Im}T(s', t)}{s' - s} + \text{LHC}$$



# Dispersion relations

$$m_+ = (m_1 + m_2)^2$$

$$m_- = (m_1 - m_2)^2$$

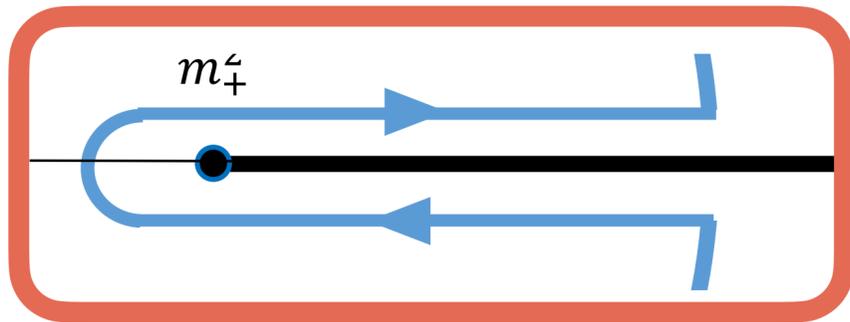
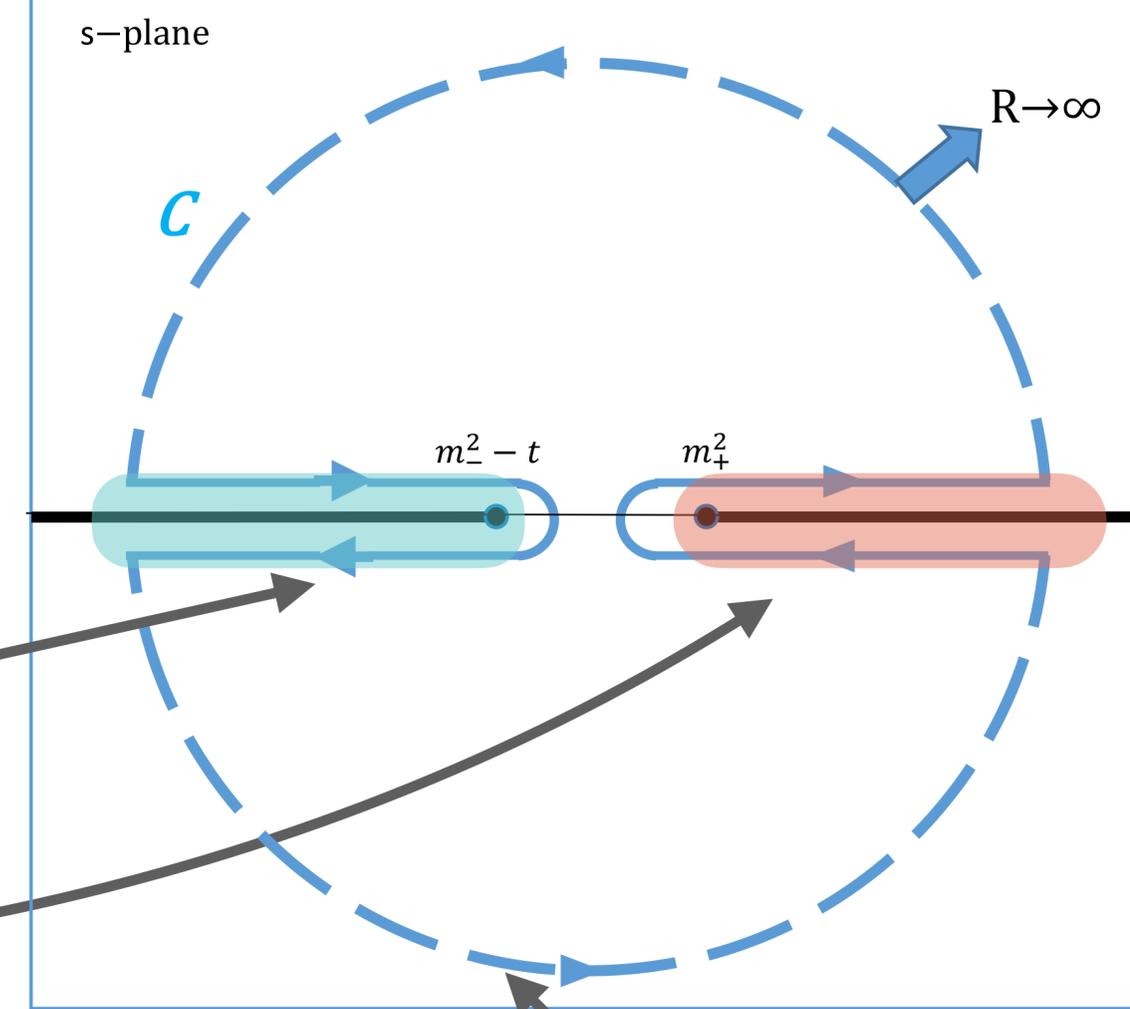
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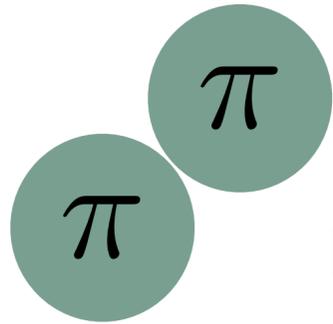


$$T(s' + i\epsilon, t) - T(s' - i\epsilon, t) = 2i Im T(s' + i\epsilon, t) \equiv 2i Im T(s', t)$$

# Dispersion relations

$$m_+ = (m_1 + m_2)^2$$

$$m_- = (m_1 - m_2)^2$$



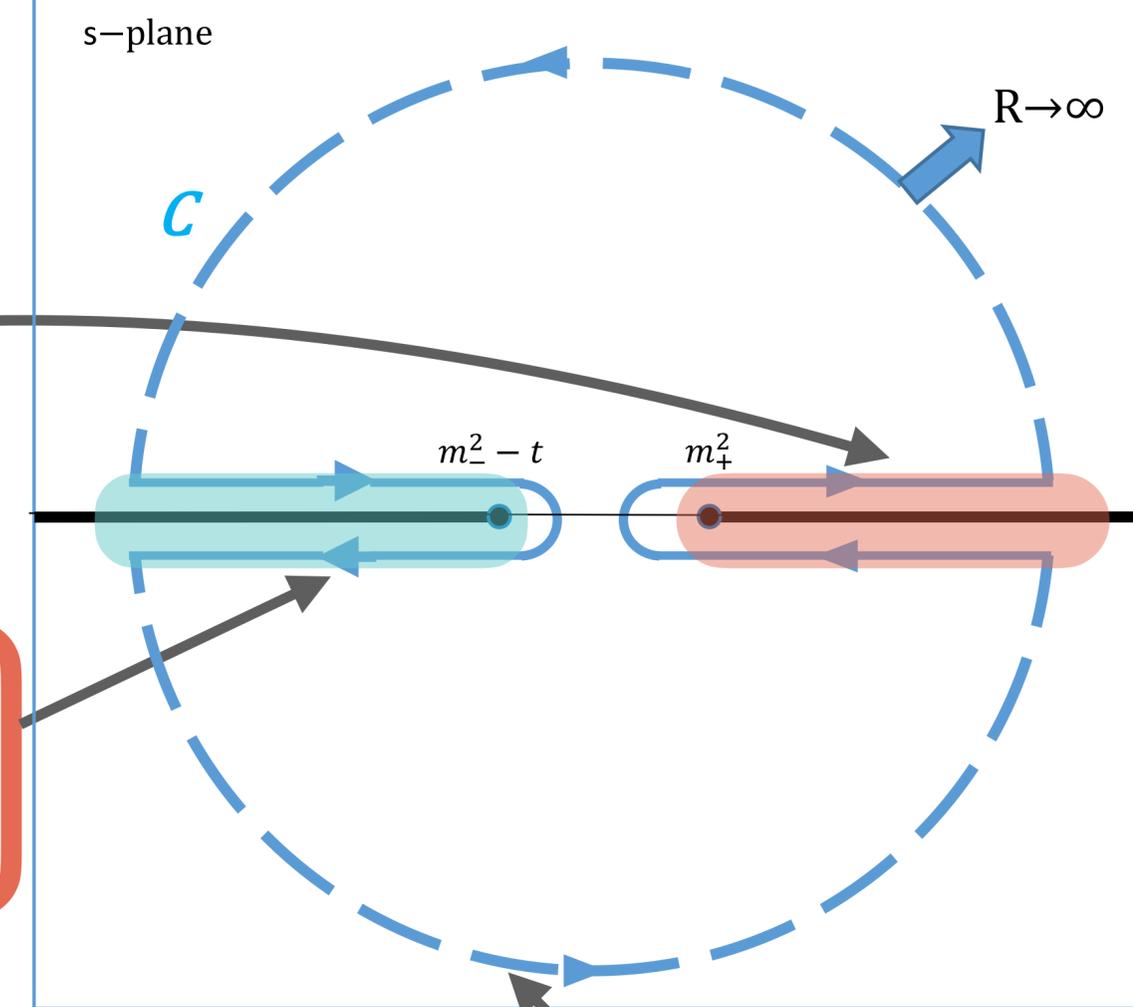
For  $\pi\pi$

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im } T(s', t)}{(s' - s)} + \frac{1}{\pi} \int_{-t}^{-\infty} ds' \frac{\text{Im } T(s', t)}{(s' - s)}$$

Change of variables

$$s' \rightarrow u' = 4m_\pi^2 - t - s'$$

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \left( \frac{\text{Im } T(s', t)}{(s' - s)} ds' + \frac{\text{Im } T(4m_\pi^2 - s' - t, t, u')}{(u' - u)} du' \right)$$



Negligible

# Dispersion relations

$$m_+ = (m_1 + m_2)^2$$

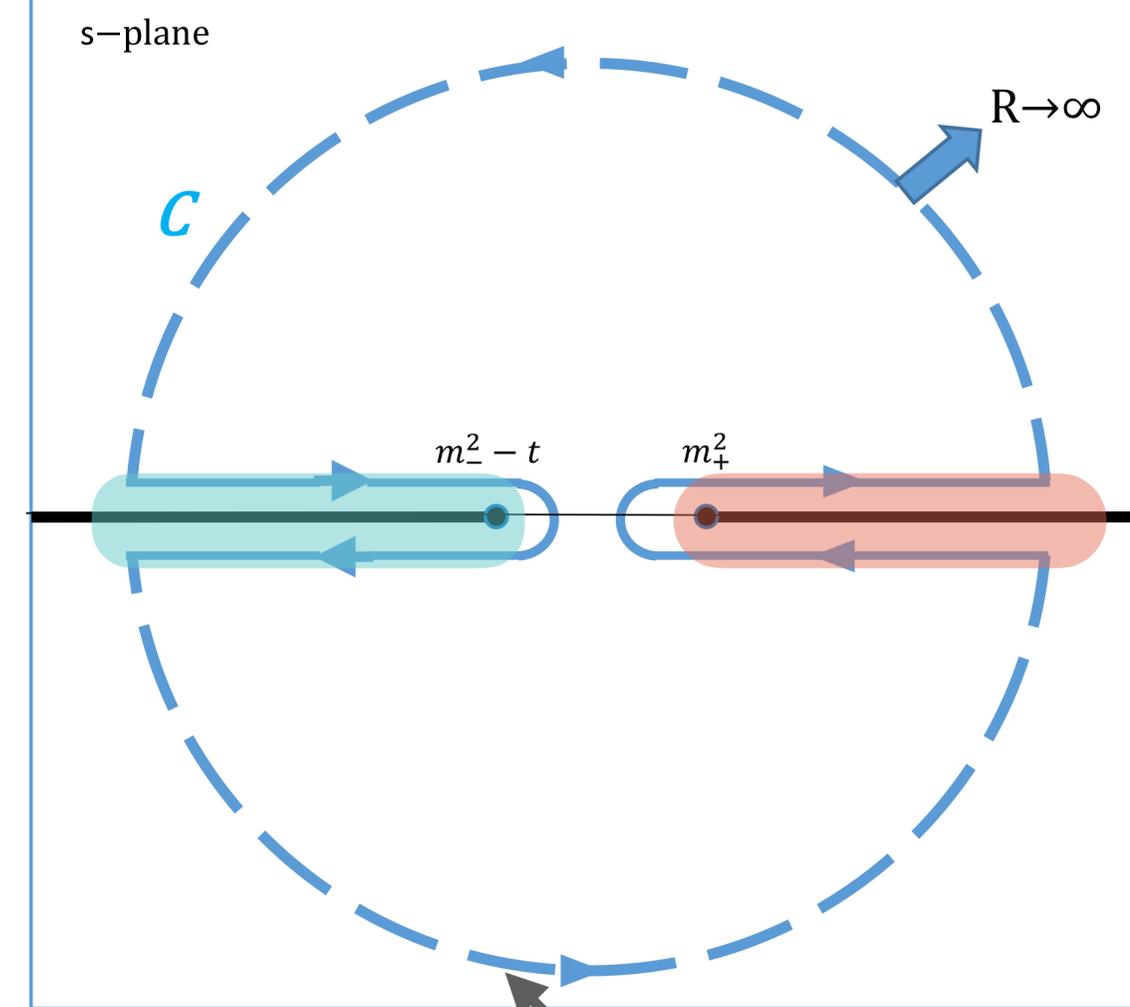
$$m_- = (m_1 - m_2)^2$$

## Last step: Crossing

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \left( \frac{\text{Im} T(s', t)}{(s' - s)} ds' + \frac{\text{Im} T(4m_\pi^2 - s' - t, t, u')}{(u' - u)} du' \right)$$

$$T^{I_s}(s, t, u) = \sum_{I_t} C_{su} T^{I_u}(u, t, s)$$

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \left( \frac{\text{Im} T(s', t)}{(s' - s)} + \frac{\sum C_{su}^{I'} \text{Im} T^{I'}(s', t)}{(s' - u)} \right)$$



**Negligible**

# Dispersion relations

$$m_+ = (m_1 + m_2)^2$$

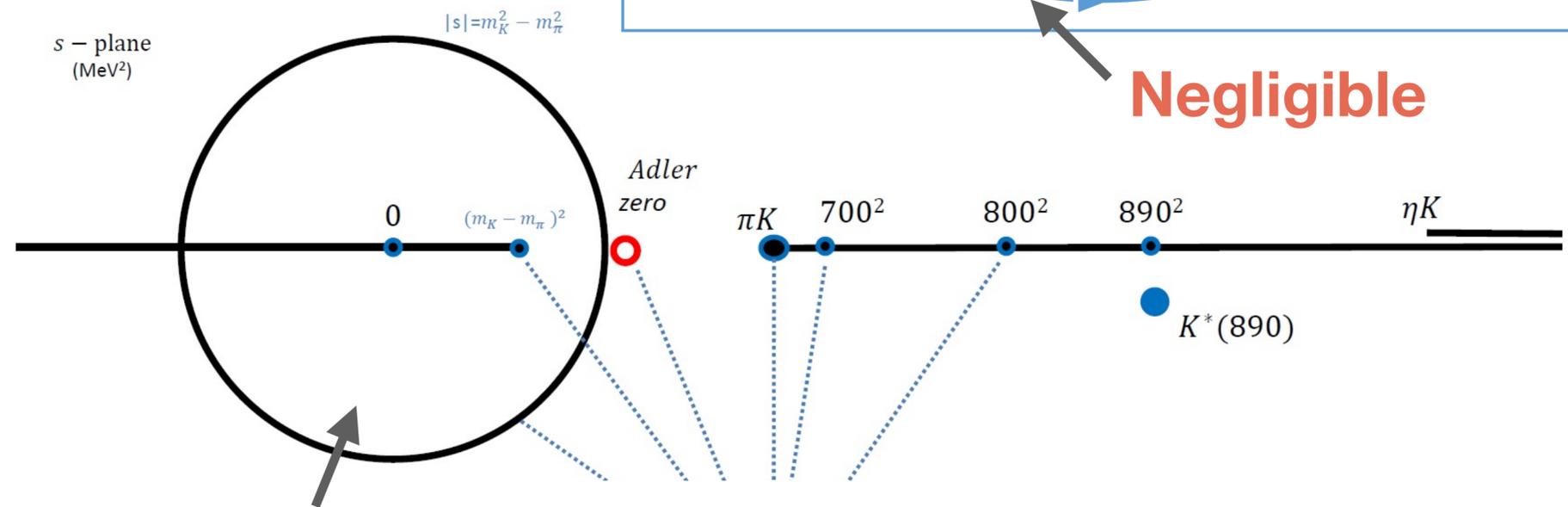
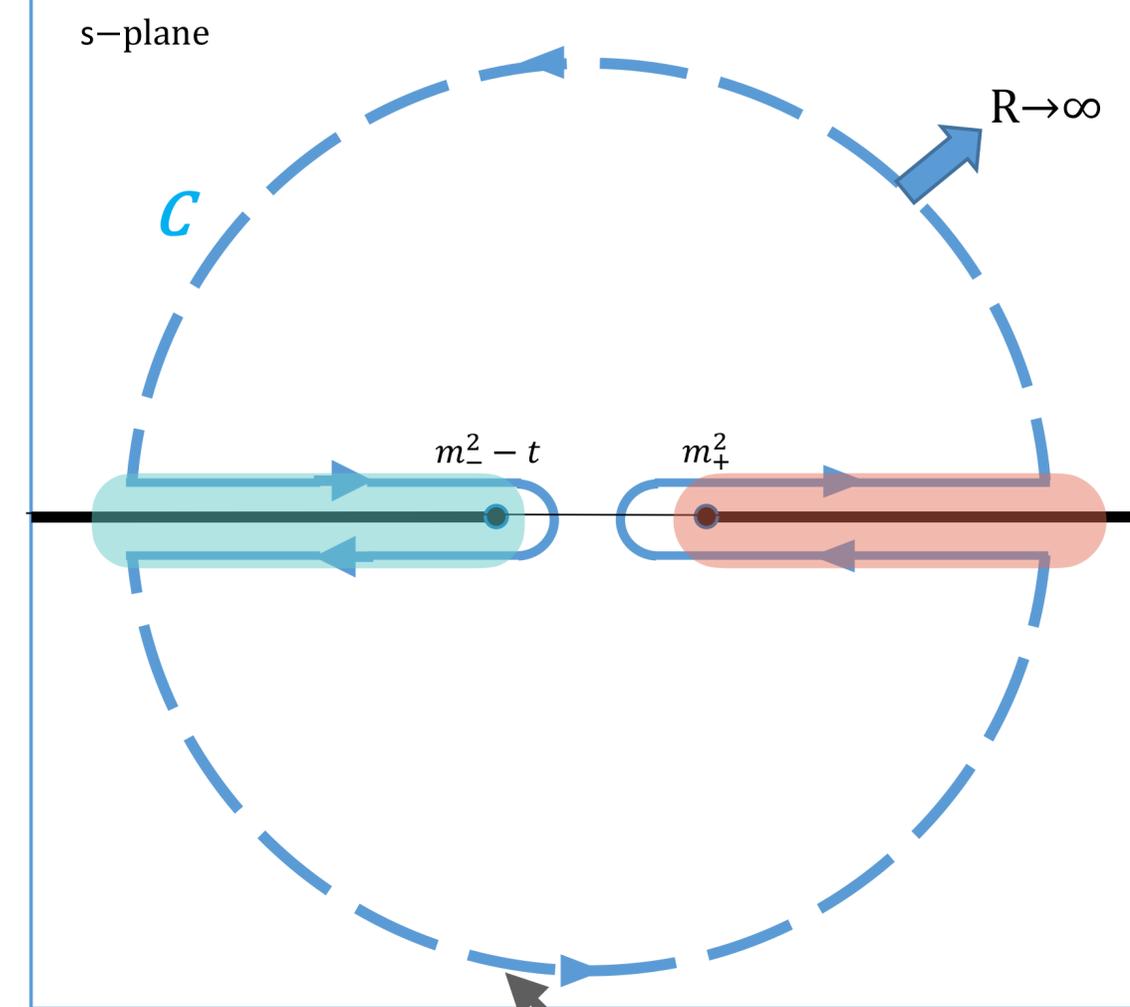
$$m_- = (m_1 - m_2)^2$$

Final DR for an amplitude

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \left( \frac{\text{Im}T(s', t)}{(s' - s)} + \frac{\sum C_{s'u}^{I'} \text{Im}T^{I'}(s', t)}{(s' - u)} \right)$$

Partial waves are more cumbersome

$$T(s, t) = 16K\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}(s)$$



Negligible

*Fit* → *In*

*DR* → *Out*

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \left( \frac{\text{Im}T(s', t)}{(s' - s)} + \frac{\sum C_{s u}^{I'} \text{Im}T^{I'}(s', t)}{(s' - u)} \right)$$

$$T(s, t) = 16K\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}(s)$$

## Model-independent

Use all data available

Prune/fit data

Obtain your DRs

Make *Fit* → *In* *DR* → *Out* compatible

### Amplitude analyses

- Unitarity
- Analyticity
- Crossing

# $\pi K$ scattering

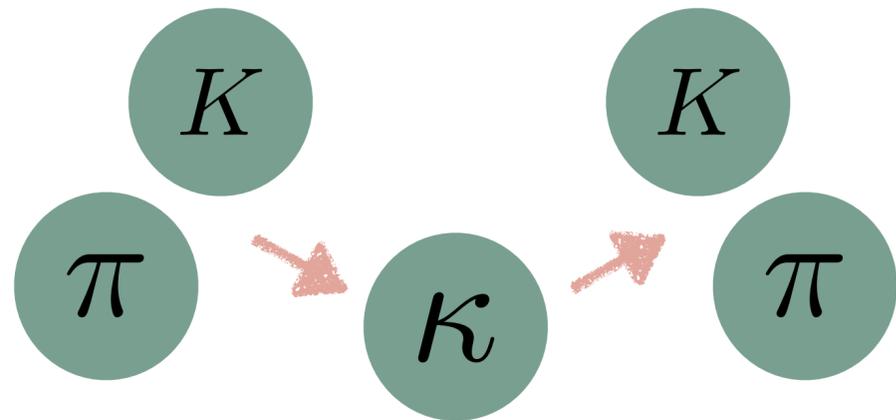
$t = 0$  for simplicity

$T \rightarrow F$  (literature)

Fit  $\rightarrow$  In

DR  $\rightarrow$  Out

$$F^+(s) = F^+(s_{th}) + \frac{(s - s_{th})}{\pi} \int_{s_{th}}^{\infty} ds' \left[ \frac{\text{Im}F^+(s')}{(s' - s)(s' - s_{th})} + \frac{\text{Im}F^+(s')}{(s' + s - 2\Sigma_{\pi K})(s' + s_{th} - 2\Sigma_{\pi K})} \right]$$



Data

$$f_{\ell}^I(s) = \frac{1}{\rho(s) \cot \delta_{\ell}^I(s) - i\rho(s)}$$

$$T(s, t) = 16K\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}(s)$$

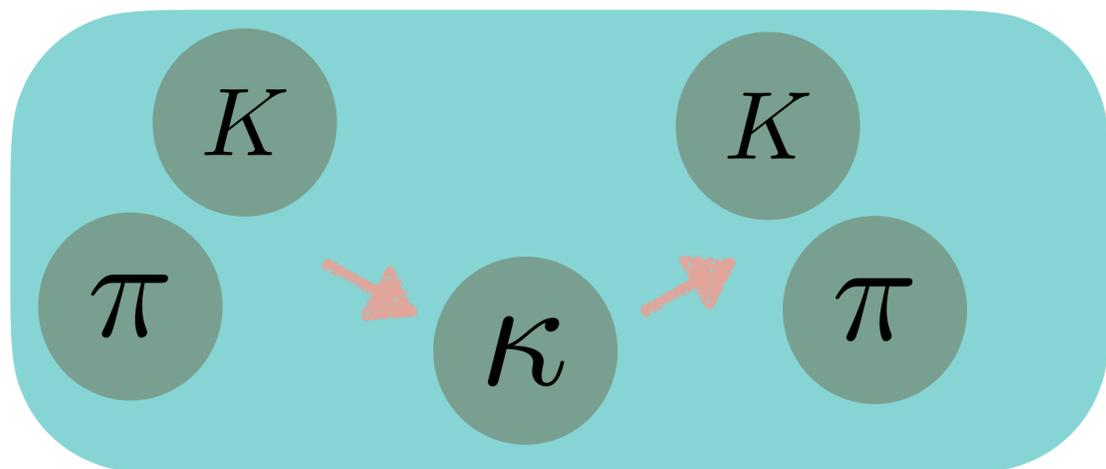
$$F^I(s, t) = 16\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) f_{\ell}^I(s)$$

Remember?

$$\cot \delta_{\ell}^I(s) = \frac{\sqrt{s}}{2q^{2\ell+1}} \sum B_n \omega(s)^n$$

# $\pi K$ scattering

$$I = 1/2, 3/2$$



**Direct**

$$F^+(s, t) = \frac{1}{3} F^{1/2}(s, t) + \frac{2}{3} F^{3/2}(s, t) = \frac{G^{I_t=0}(t, s)}{\sqrt{6}}$$

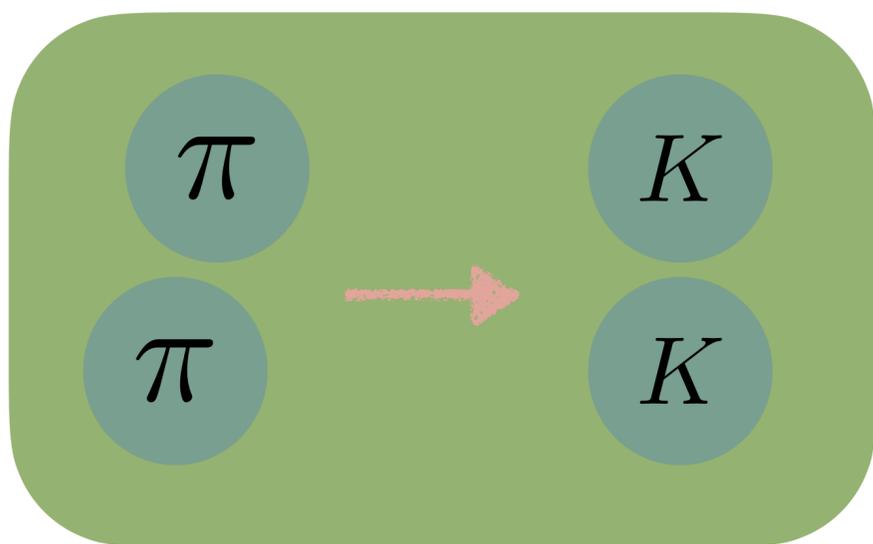
**Crossed**

$$F^-(s, t) = \frac{1}{3} F^{1/2}(s, t) - \frac{1}{3} F^{3/2}(s, t) = \frac{G^{I_t=1}(t, s)}{2}$$

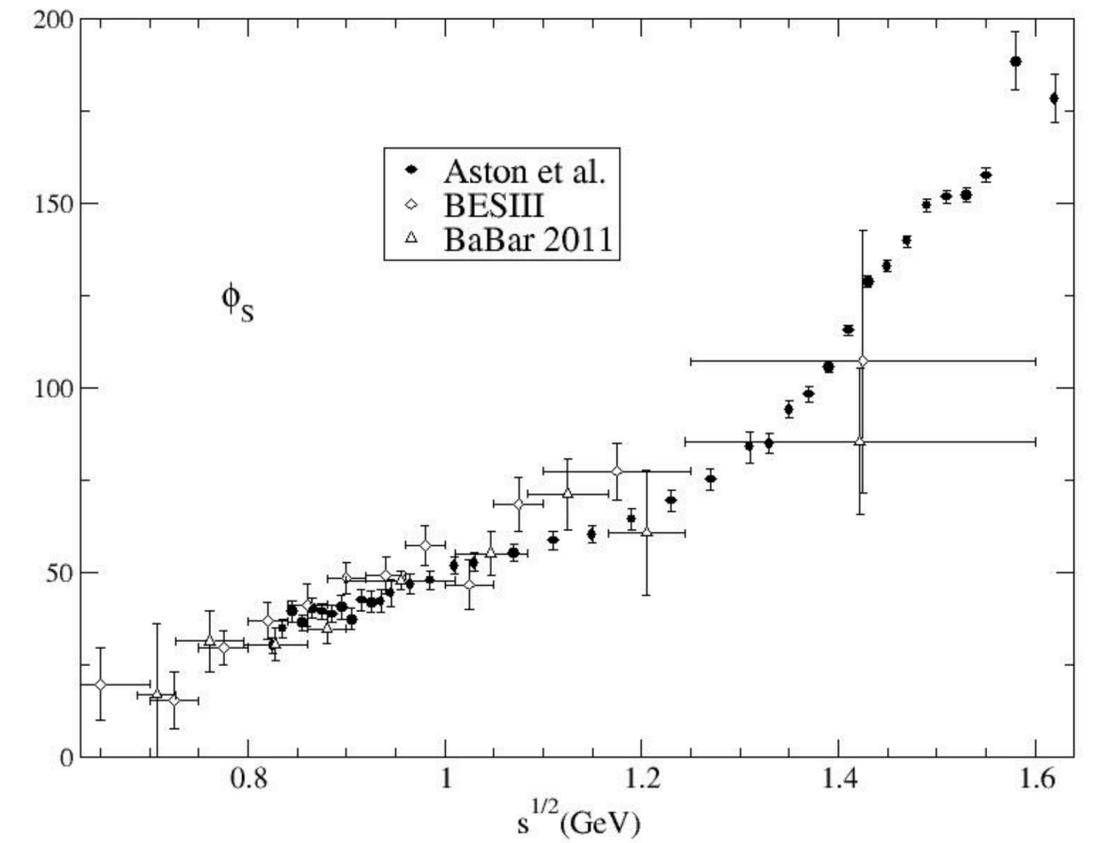
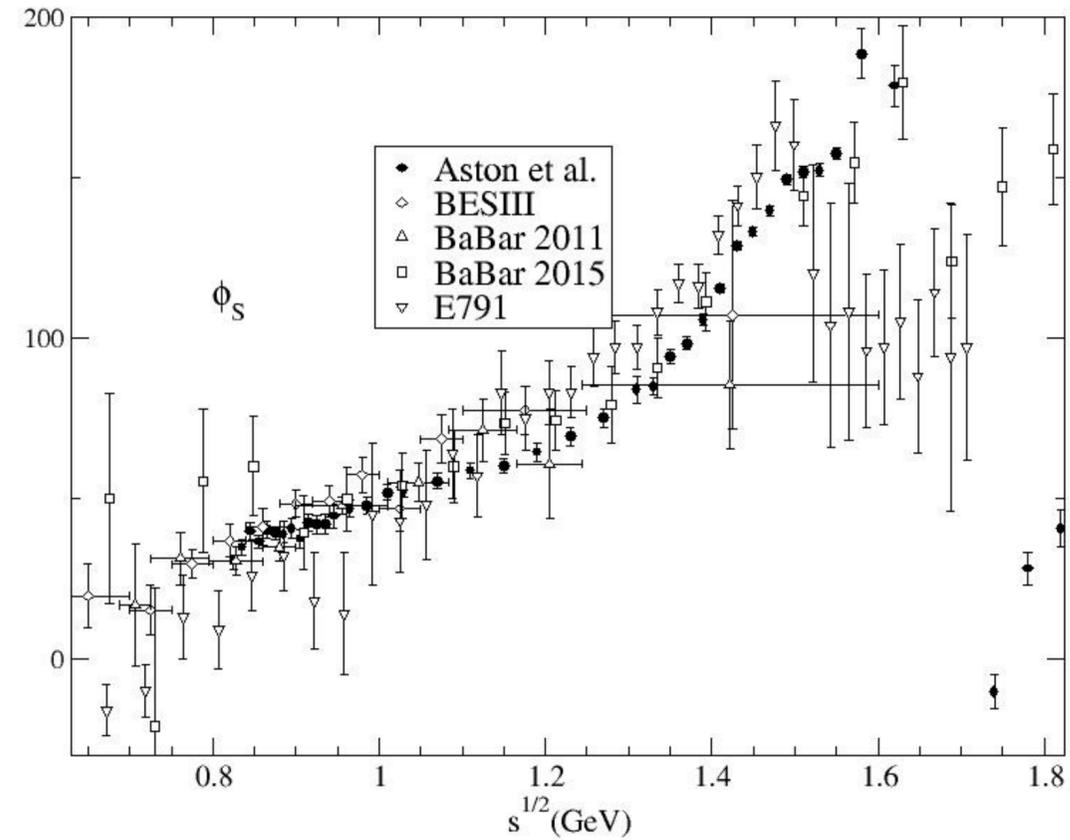
## Partial wave expansions

$$F^I(s, t) = 16\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) f_{\ell}^I(s)$$

$$G^I(t, s) = 16\pi \sqrt{2} \sum_{\ell} (2\ell + 1) (q_{\pi} q_K)^{\ell} g_{\ell}^I(t) P(z_t(s))$$



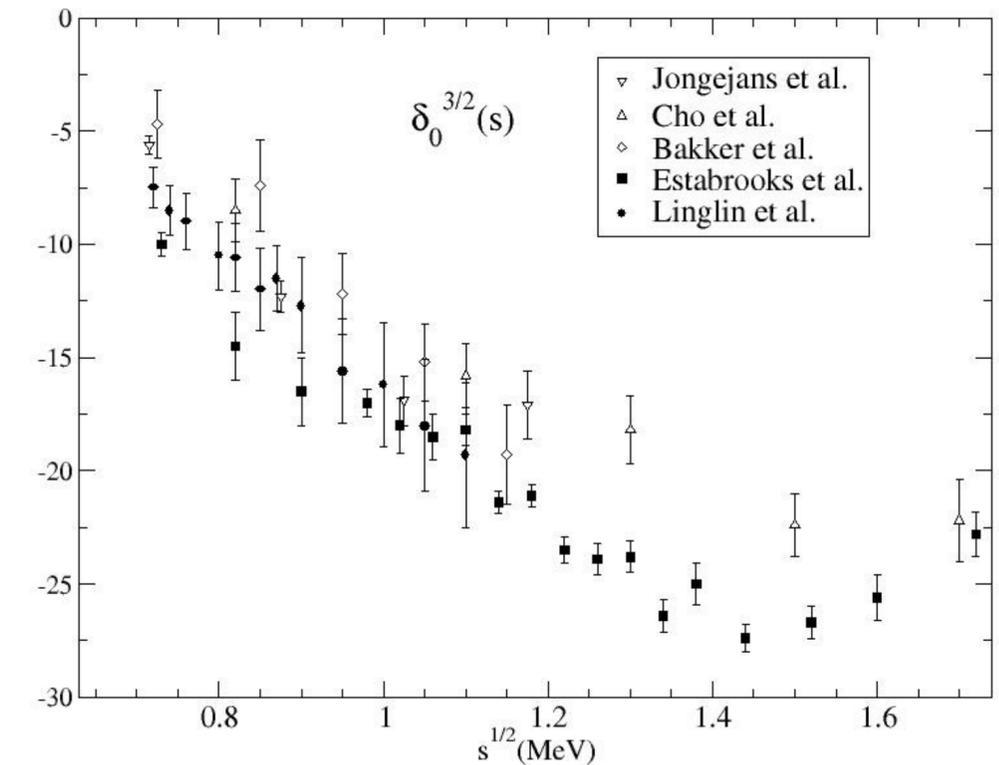
# $\pi K$ scattering



**Use all data available**

**Most of it produced in the 80's**

**Mixture of production/semileptonic/decays**

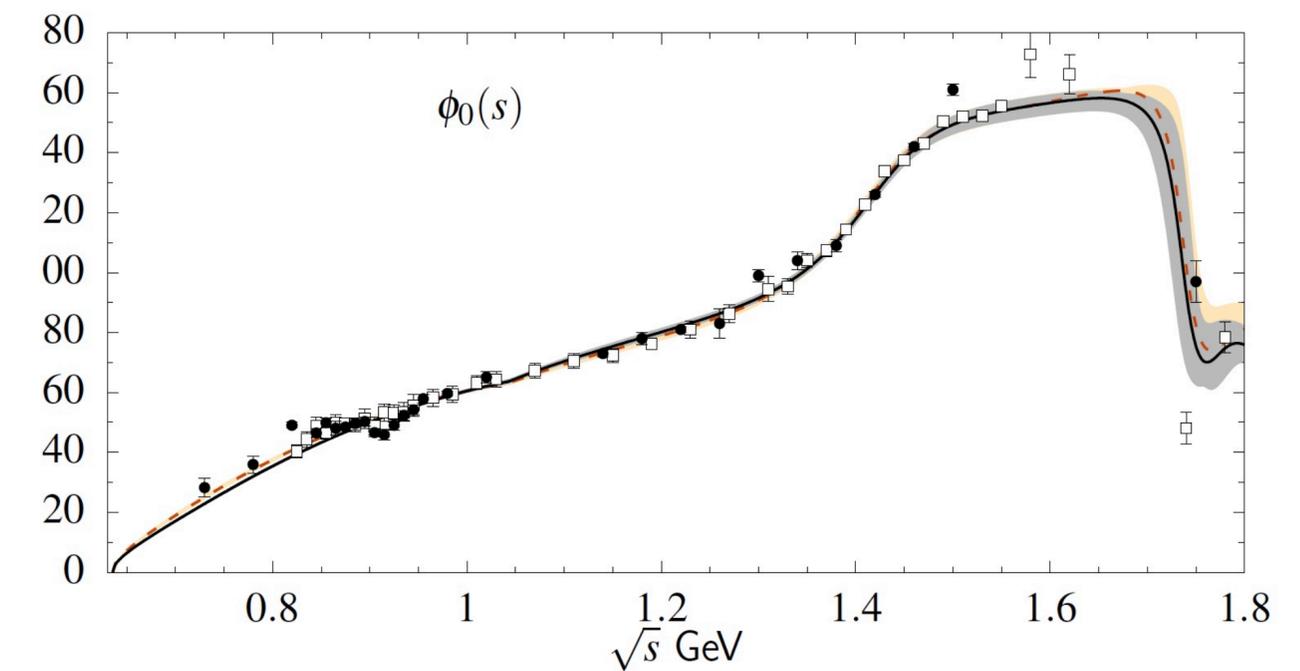
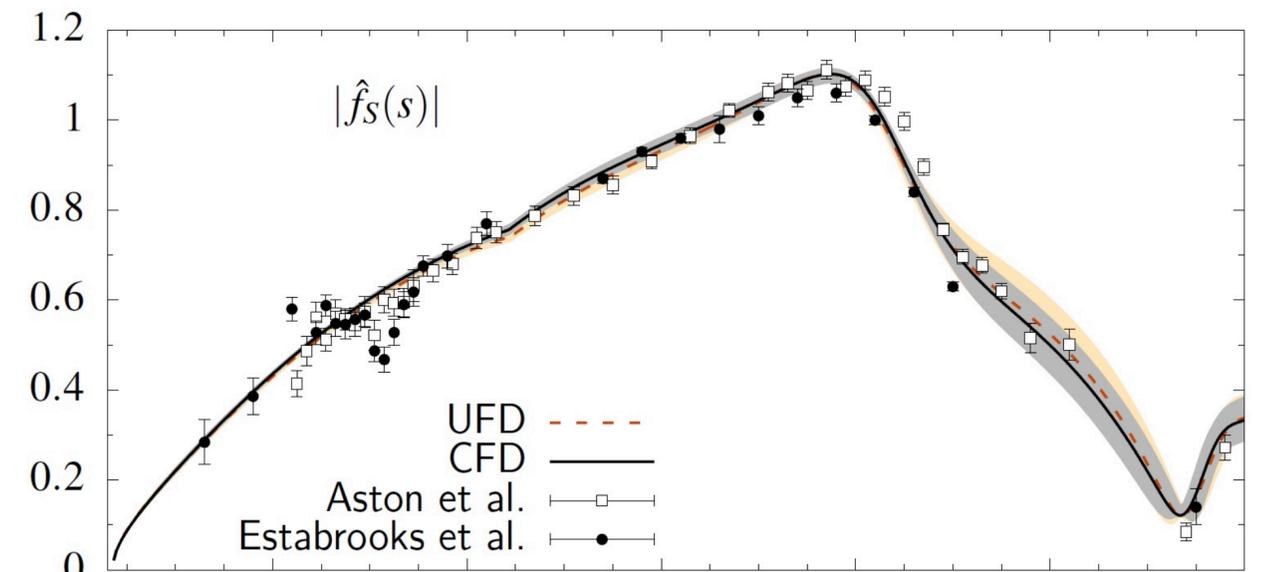
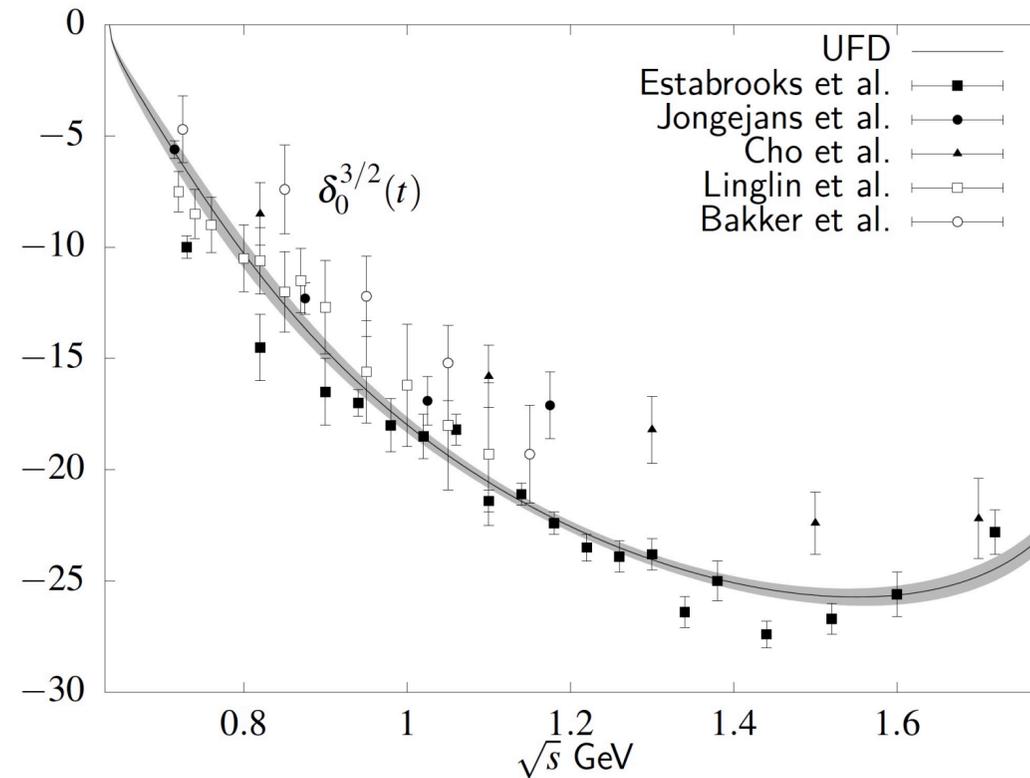


# $\pi K$ scattering



Prune/fit data

Best experiment  $\rightarrow$  SLAC



Best param.

$$\cot \delta_\ell^I(s) = \frac{\sqrt{s}}{2q^{2\ell+1}} \sum B_n \omega(s)^n$$

# $\pi K$ scattering



Obtain your DRs

## Amplitude DR

$$F^+(s) = F^+(s_{th}) + \frac{(s - s_{th})}{\pi} \int_{s_{th}}^{\infty} ds' \left[ \frac{\text{Im}F^+(s')}{(s' - s)(s' - s_{th})} + \frac{\text{Im}F^+(s')}{(s' + s - 2\Sigma_{\pi K})(s' + s_{th} - 2\Sigma_{\pi K})} \right]$$

## Partial wave DRs

Scalar

$$f_0^+(s) = a_0^+ + \frac{1}{\pi} \sum_l \left( \int_{s_{th}}^{\infty} ds' K_{0l}^+(s, s') \text{Im}f_l^+(s') + \int_{4m_\pi^2}^{\infty} dt' G_{0,2l}^+(s, t') \text{Im}g_{2l}^0(t') \right)$$

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{1}{\pi} \sum_l \left( \int_{m_+^2}^{\infty} ds' G_{0,l}^+(t, s') \text{Im}f_l^+(s') + t \int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G_{0,2l}^0(t, t') \text{Im}g_{2l}^0(t') \right)$$

# $\pi K$ scattering

The dispersive result is unique (provided some decent data constrains)

arXiv:hep-ph/0005042

○ Make *Fit*  $\rightarrow$  *In* *DR*  $\rightarrow$  *Out* compatible

**Scalar**

$$f_0^+(s) = a_0^+ + \frac{1}{\pi} \sum_l \left( \int_{s_{th}}^{\infty} ds' K_{0l}^+(s, s') \text{Im} f_l^+(s') + \int_{4m_\pi^2}^{\infty} dt' G_{02l}^+(s, t') \text{Im} g_{2l}^0(t') \right)$$

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{1}{\pi} \sum_l \left( \int_{m_+^2}^{\infty} ds' G_{0,l}^+(t, s') \text{Im} f_l^+(s') + t \int_{4m_\pi^2}^{\infty} \frac{dt'}{t'} G_{0,2l}^0(t, t') \text{Im} g_{2l}^0(t') \right)$$

# $\pi K$ scattering



Make

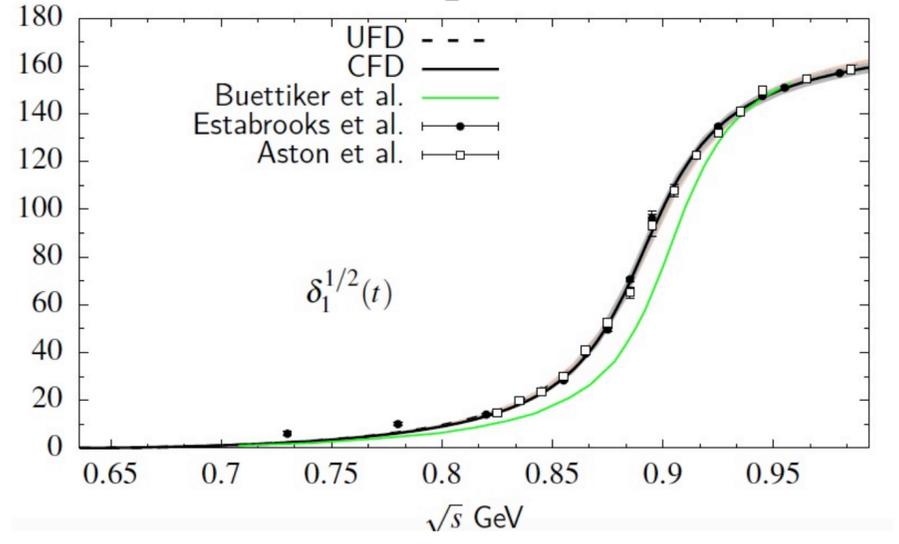
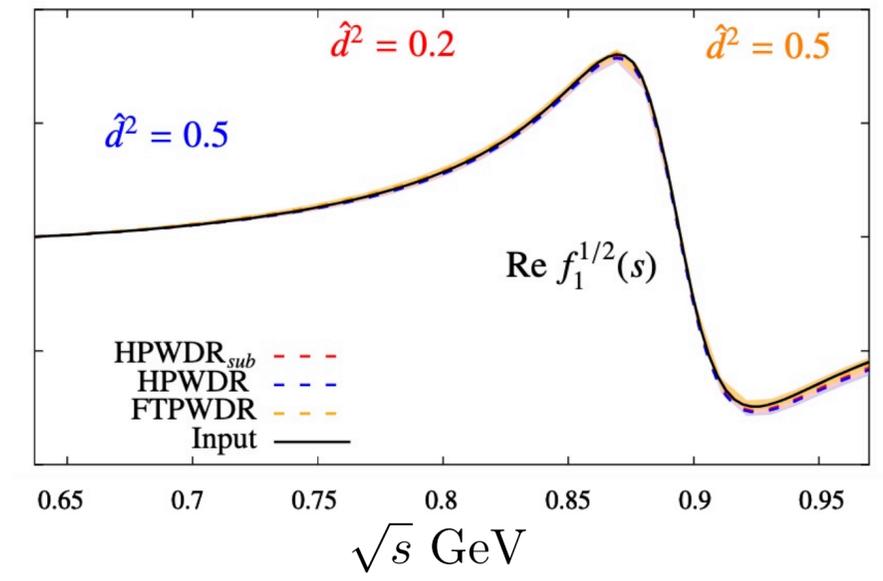
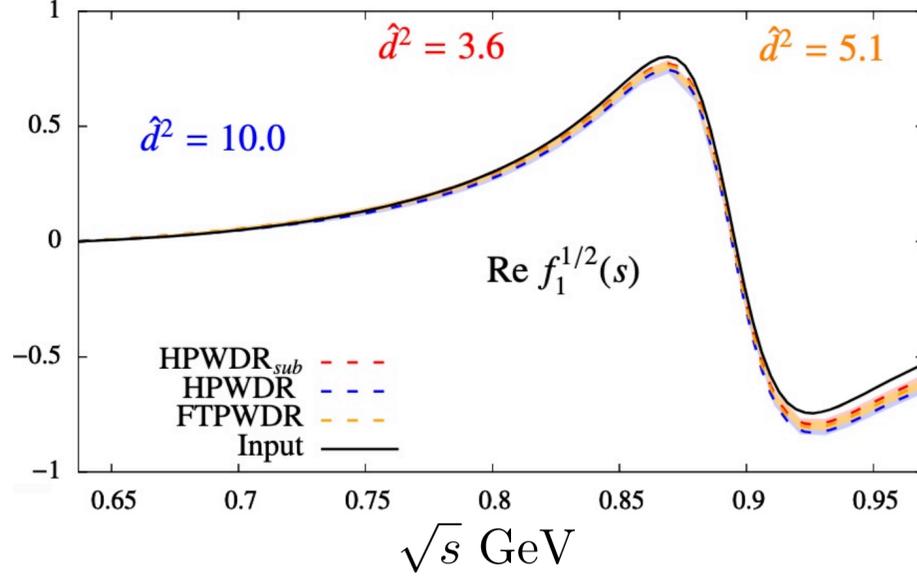
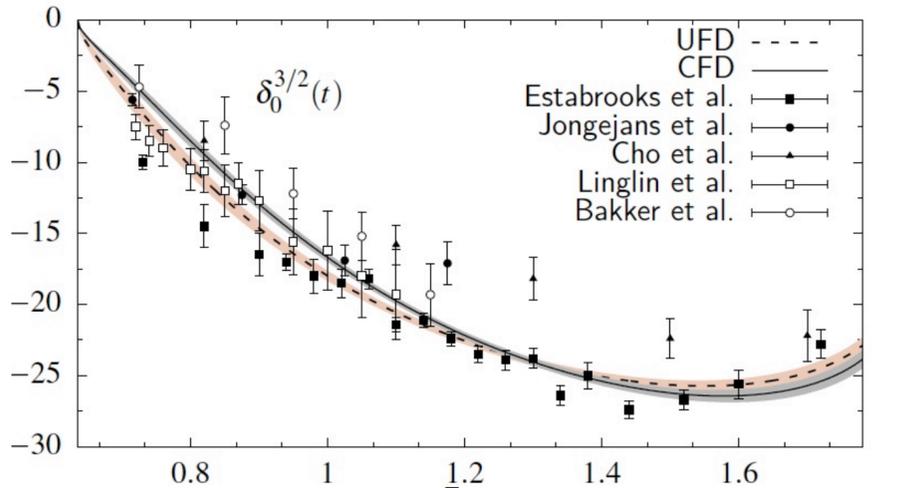
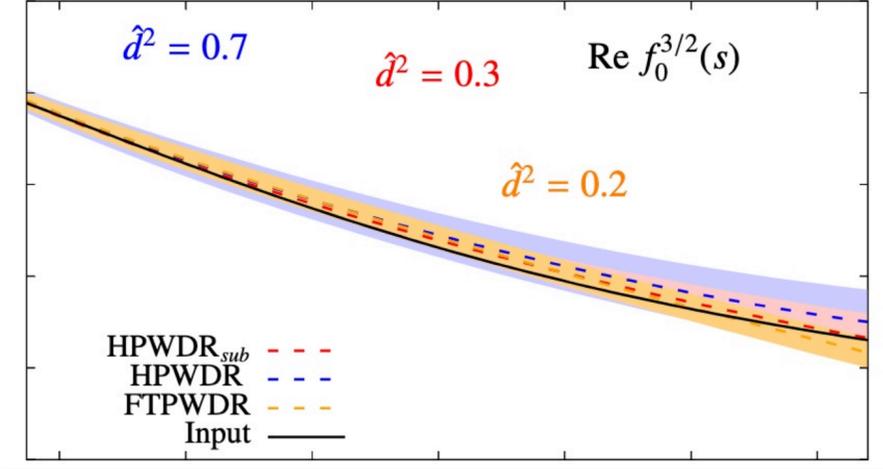
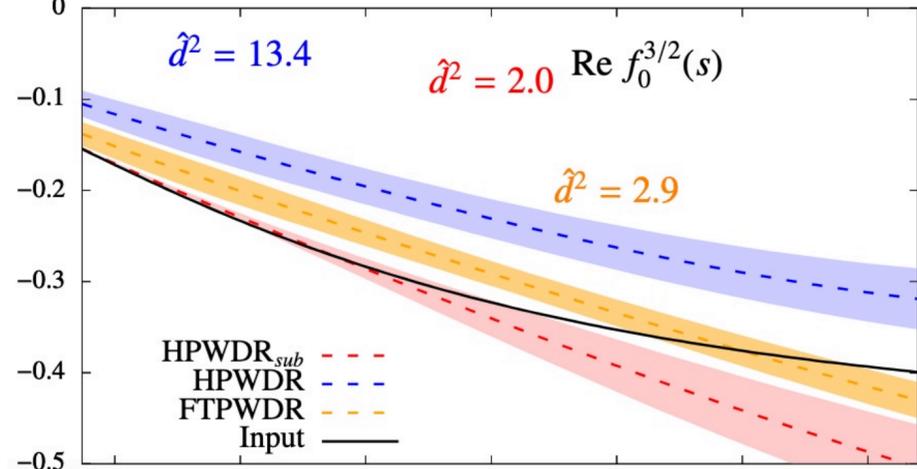
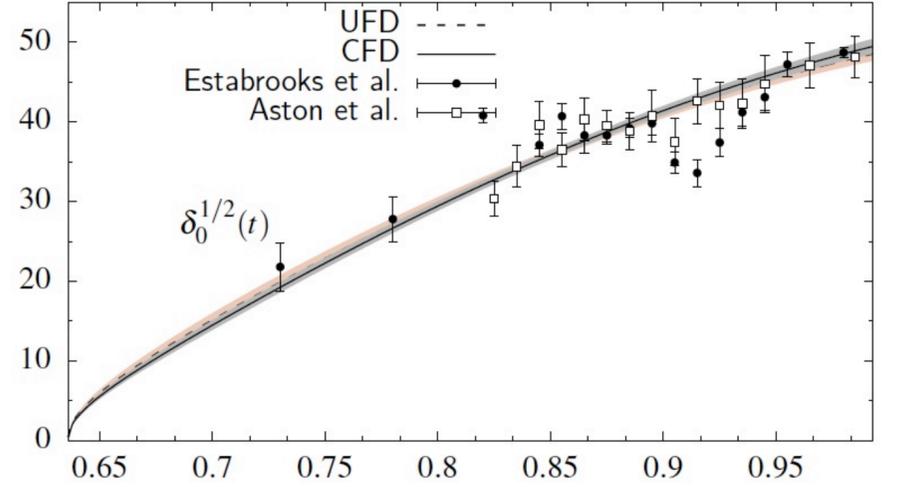
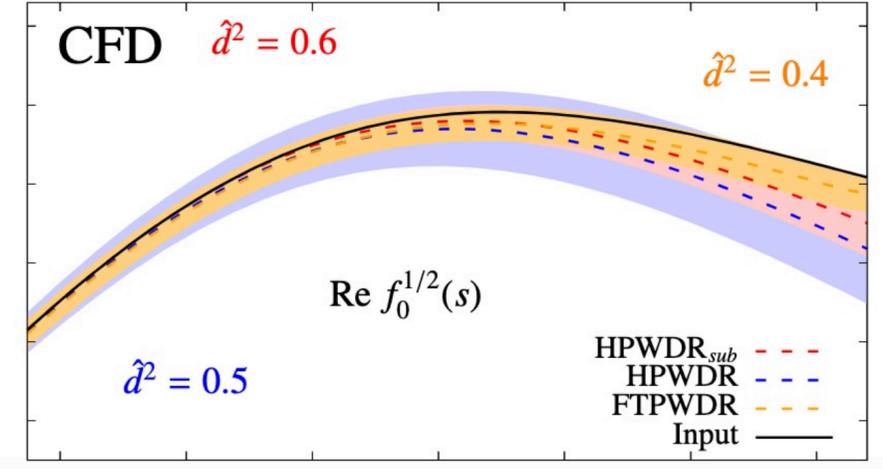
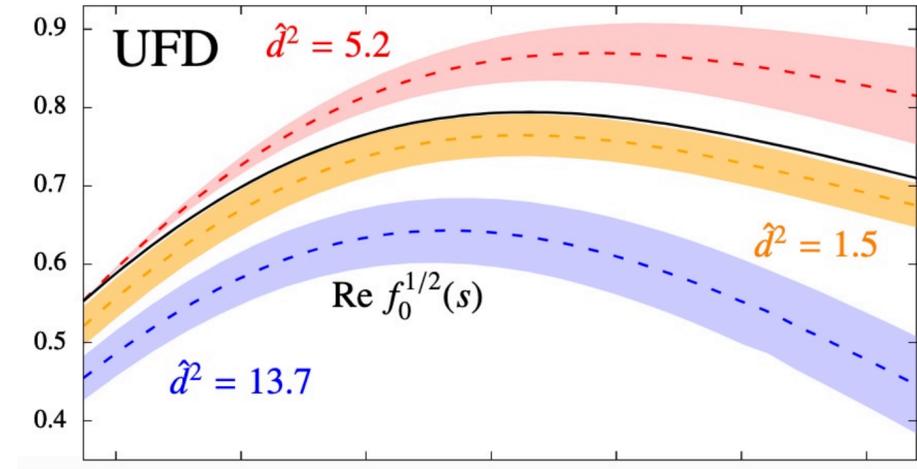
*Fit*  $\rightarrow$  *In*

*DR*  $\rightarrow$  *Out*

compatible

Vary the fits a bit

$f_l^I(s)$



*Fit* → *In*

*DR* → *Out*

$$T(s, t) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \left( \frac{\text{Im}T(s', t)}{(s' - s)} + \frac{\sum C_{s u}^{I'} \text{Im}T^{I'}(s', t)}{(s' - u)} \right)$$

$$T(s, t) = 16K\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}(s)$$

## Model-independent

✓ **Use all data available**

✓ **Prune/fit data**

✓ **Obtain your DRs**

✓ **Make** *Fit* → *In* *DR* → *Out* **compatible**

**Amplitude analyses**

- Unitarity
- Analiticity
- Crossing

# $\pi K$ scattering

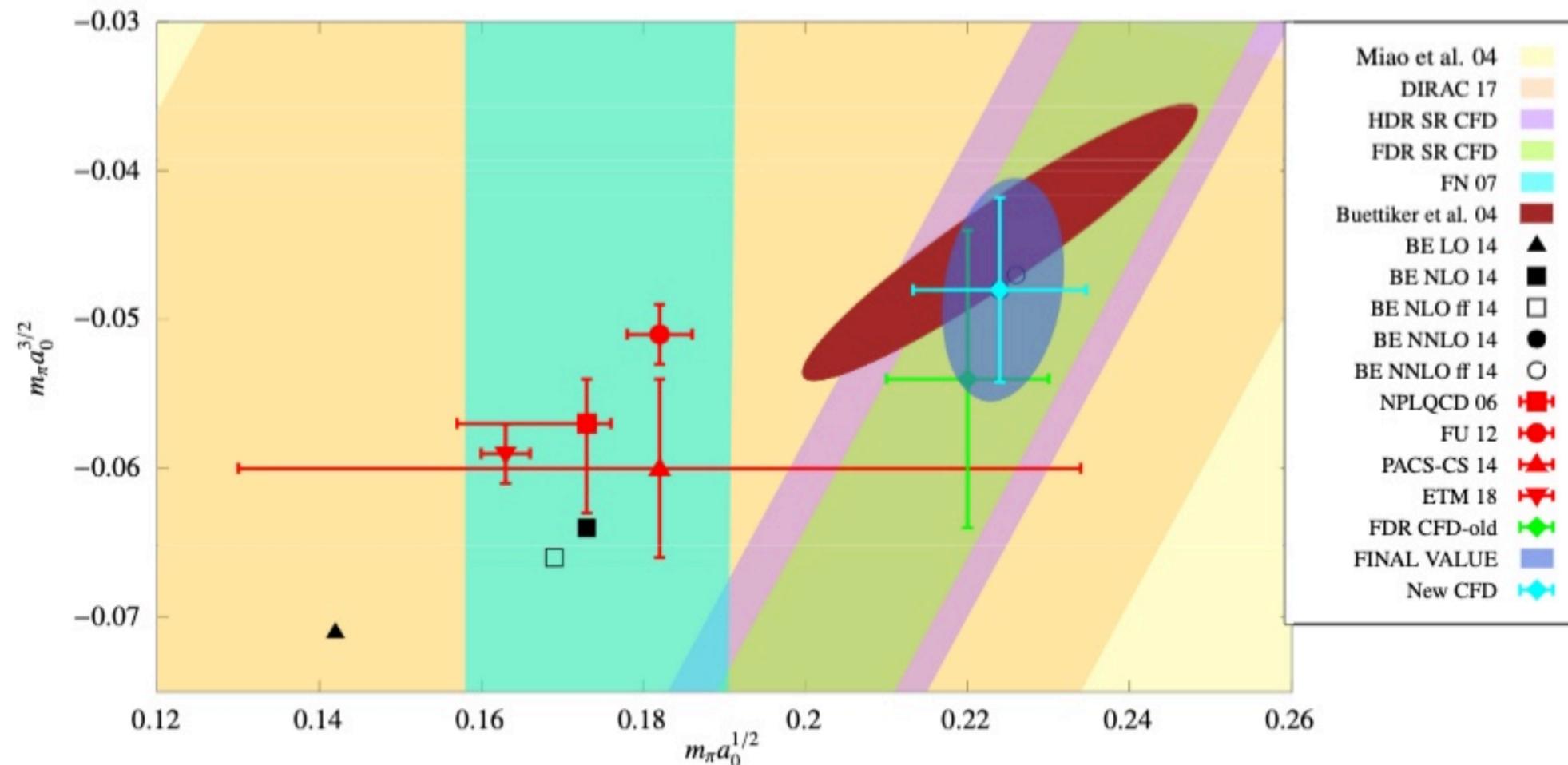
## Model-independent results

### Dispersive scattering lengths

$$a_0^- = \frac{m_\pi m_K}{2\pi^2 m_+} \int_{m_+^2}^{\infty} \frac{\text{Im } F^-(s')}{(s' - m_-^2)(s' - m_+^2)} ds'$$

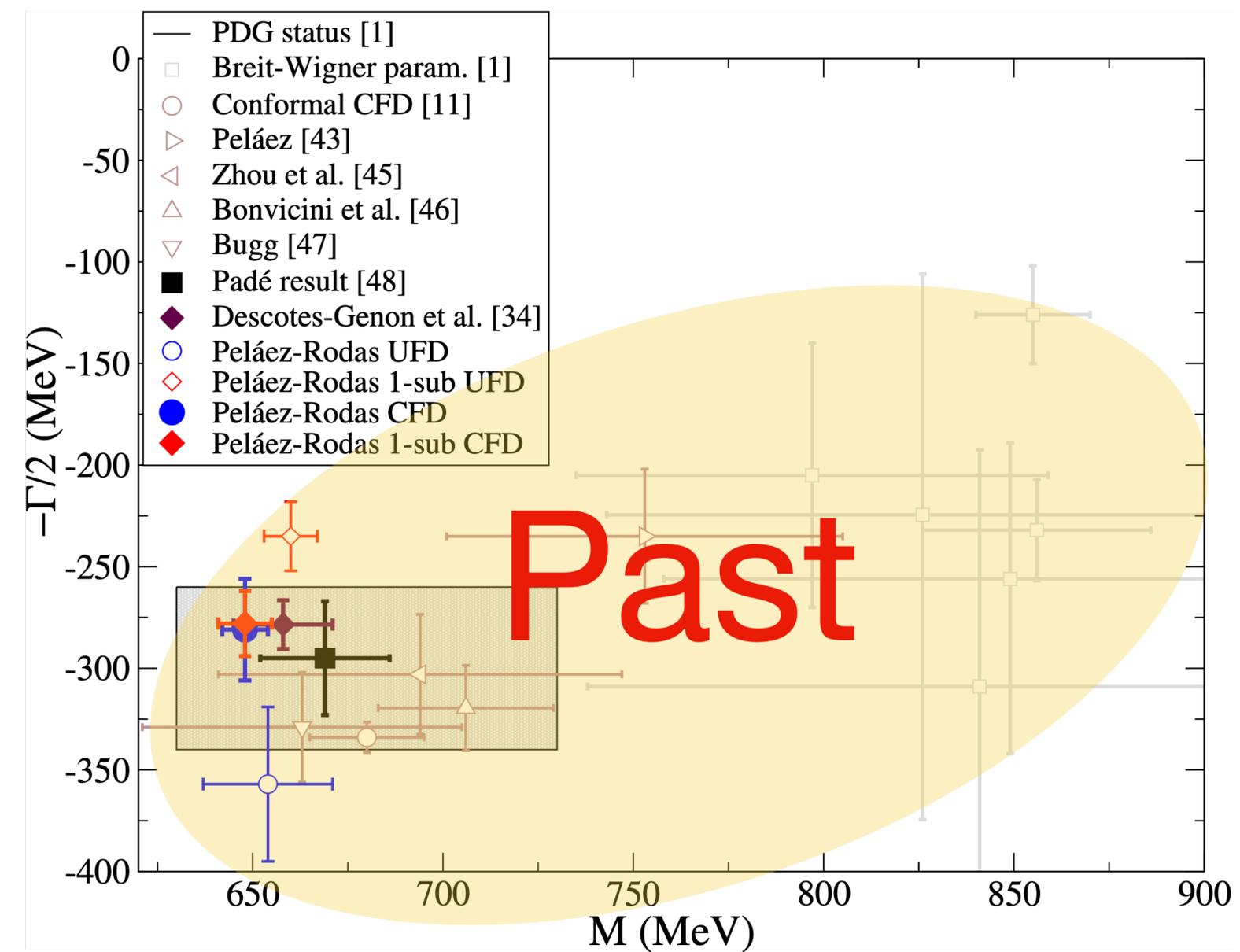
	FINAL	CFD	Paris group
$a_0^{1/2}$	$0.224 \pm 0.008$	$0.224 \pm 0.011$	$0.224 \pm 0.022$
$a_0^{3/2}$	$-0.0480 \pm 0.0056$	$-0.048 \pm 0.006$	$-0.0448 \pm 0.0077$

**Tension between lattice QCD  
and dispersive results!!**



# $\pi K$ scattering

## Model-independent $\kappa$ extraction



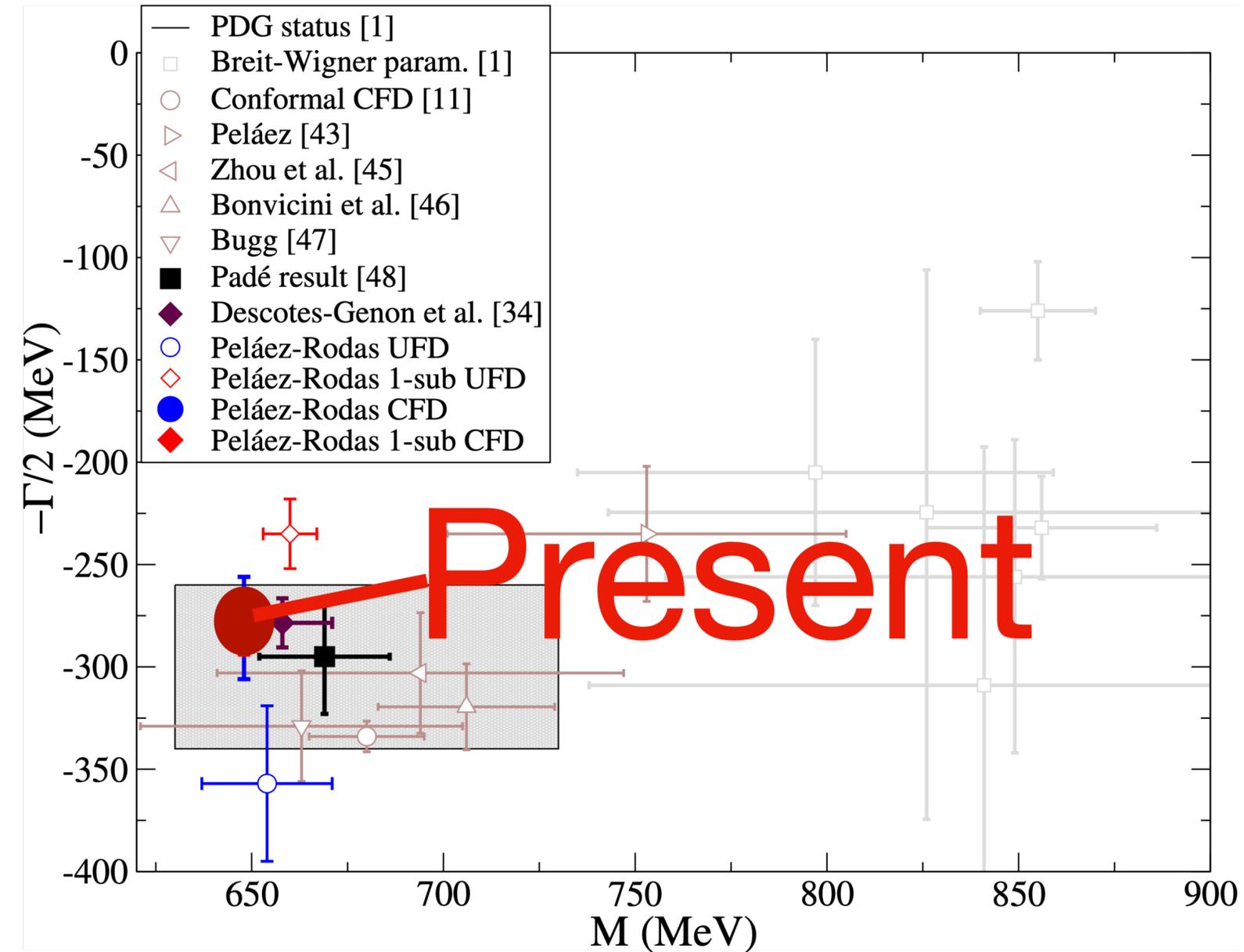
# $\pi K$ scattering

## Model-independent $\kappa$ extraction

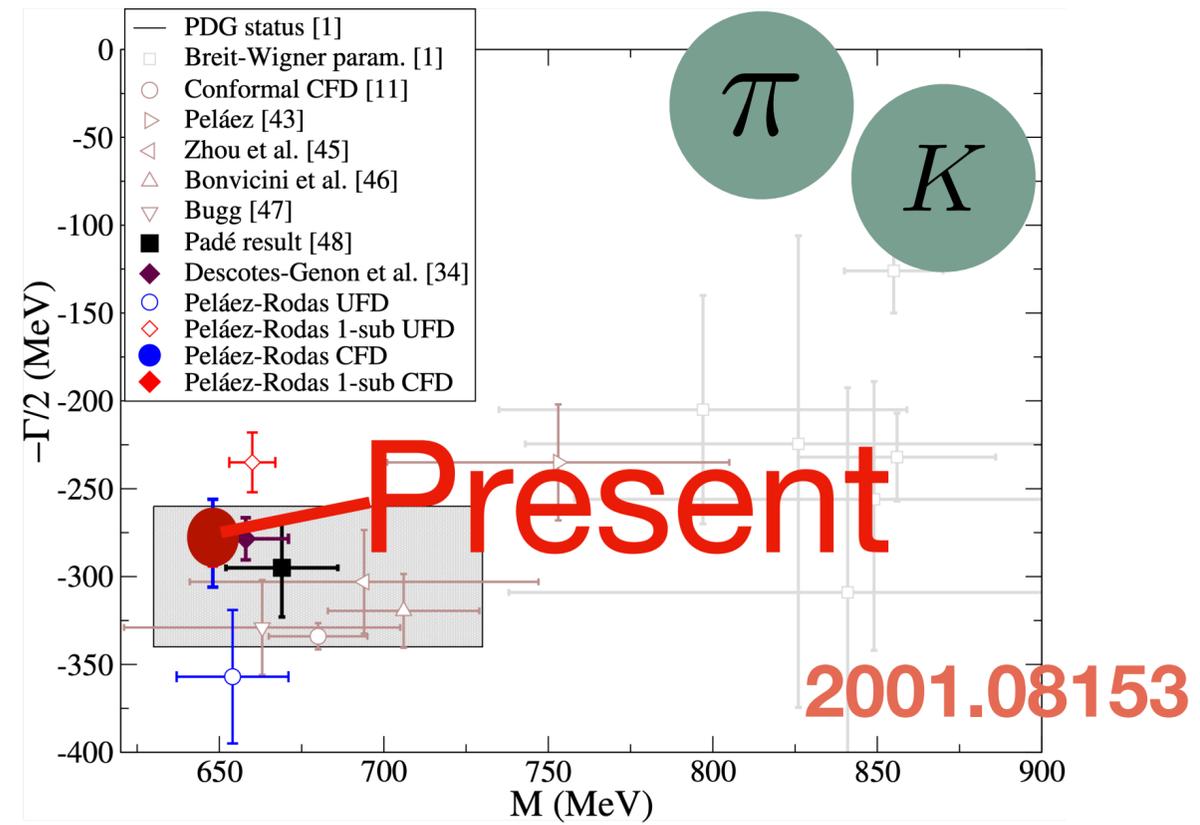
All errors (that we knew about) included

## Extremely precise result

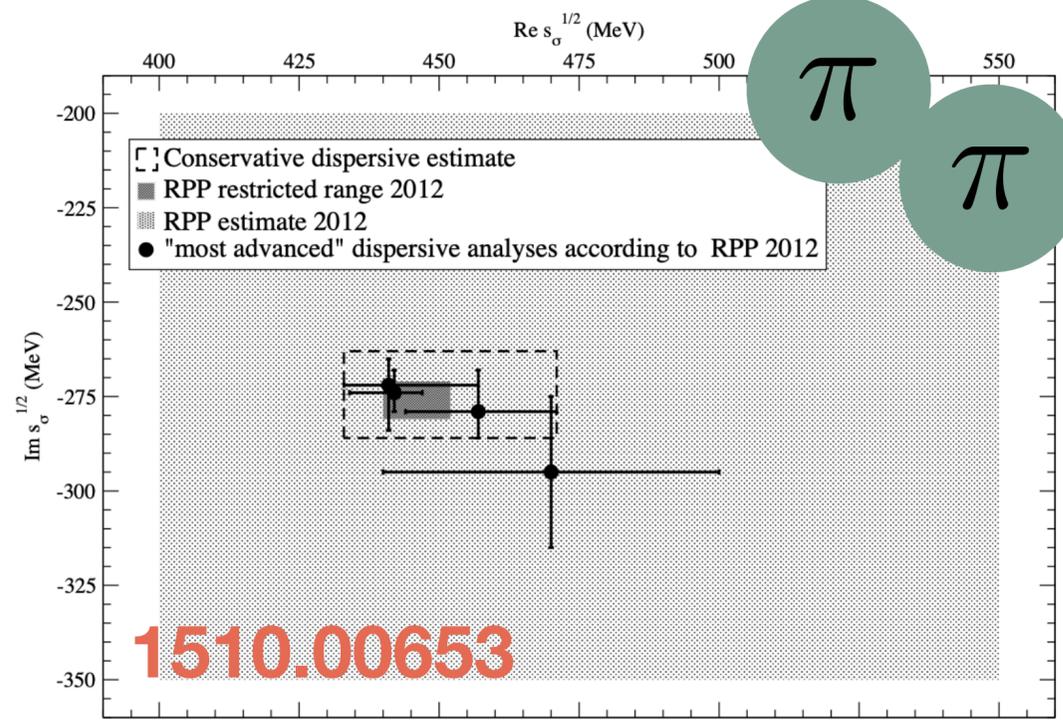
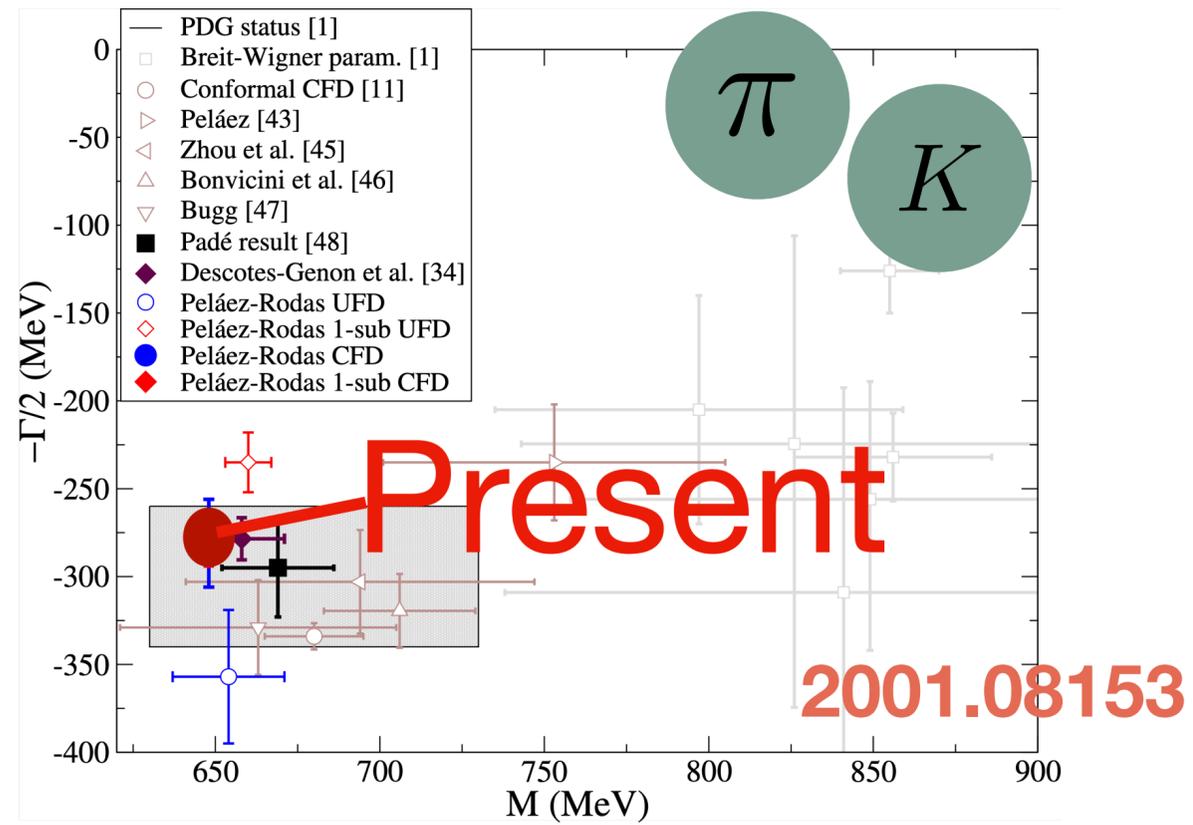
Most other deters.  $\rightarrow$  several times more “noisy”



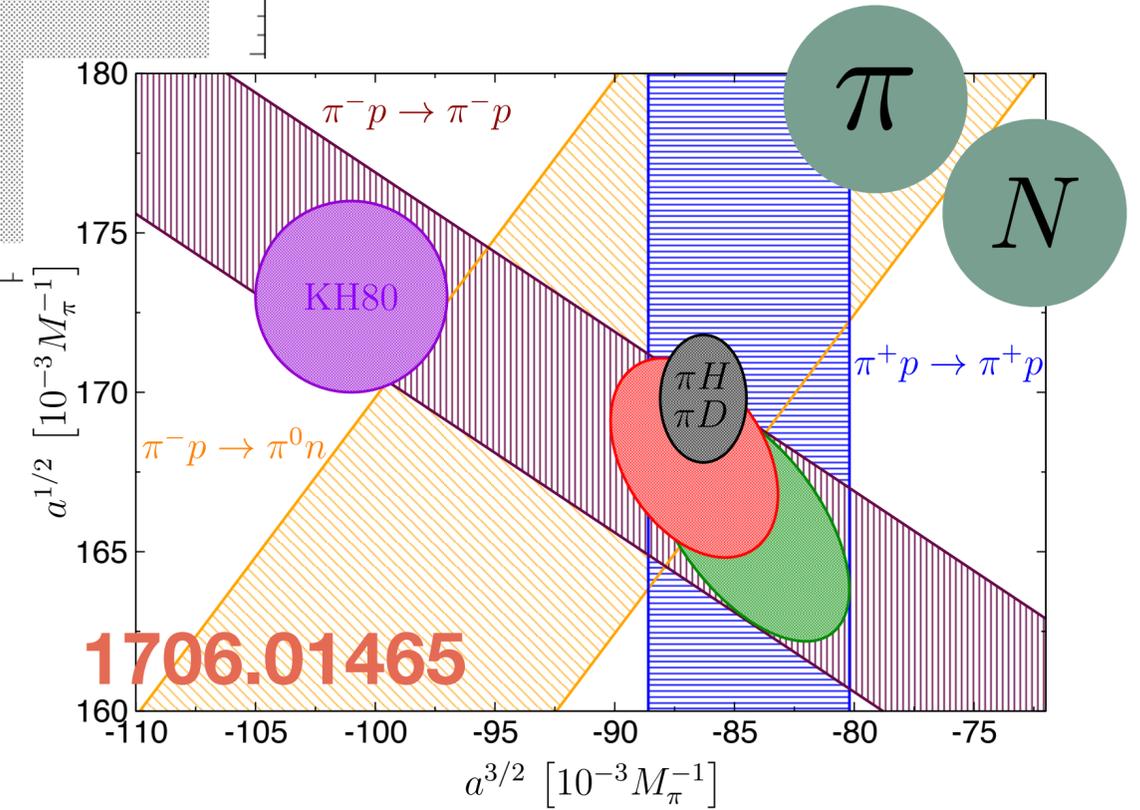
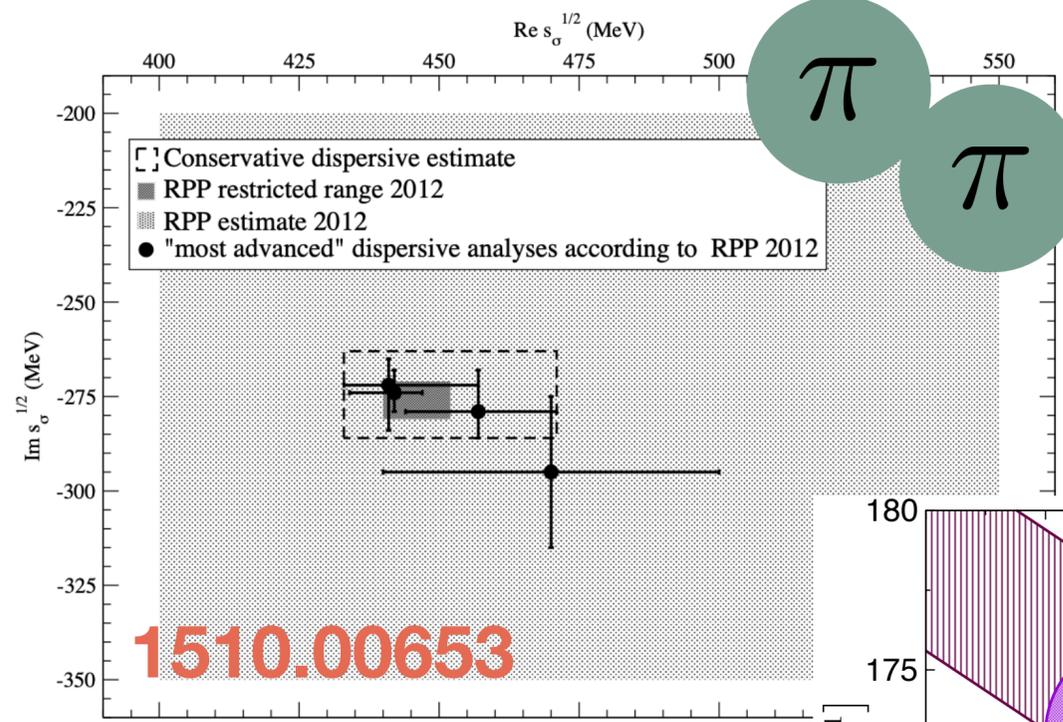
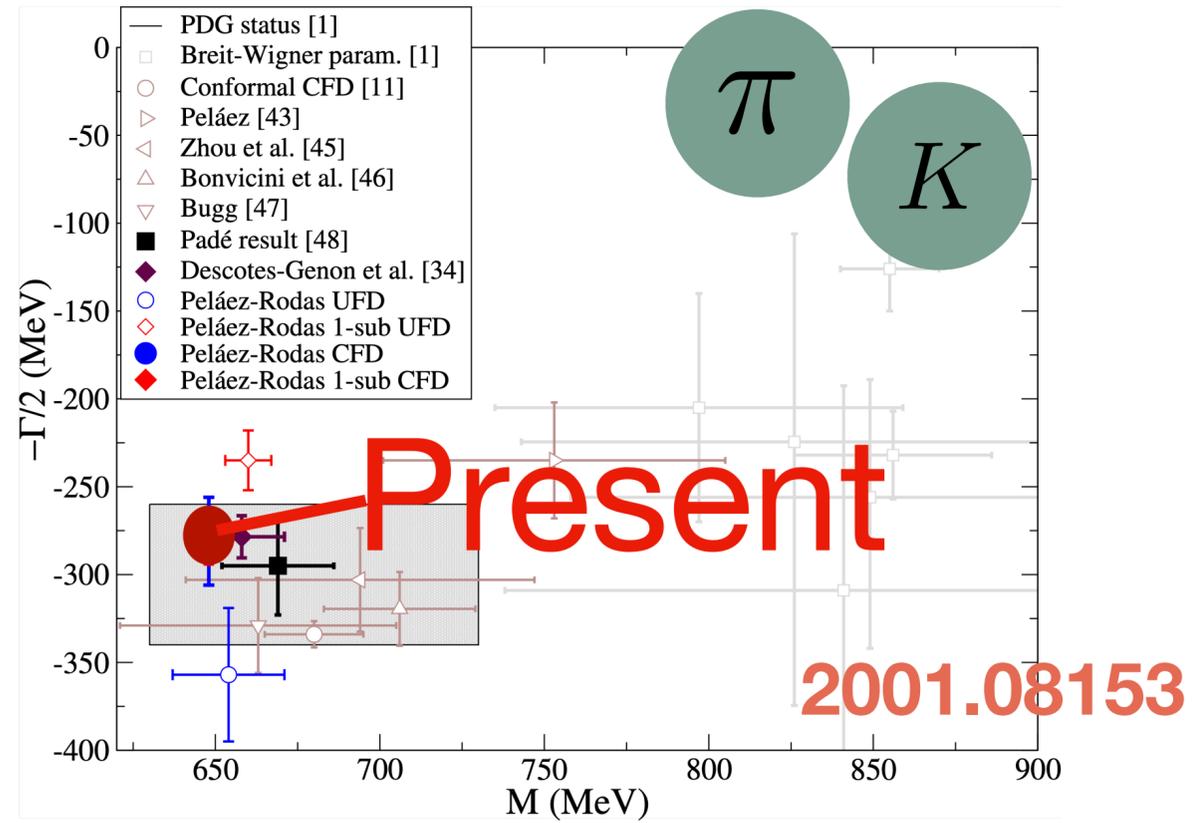
# Dispersive analyses



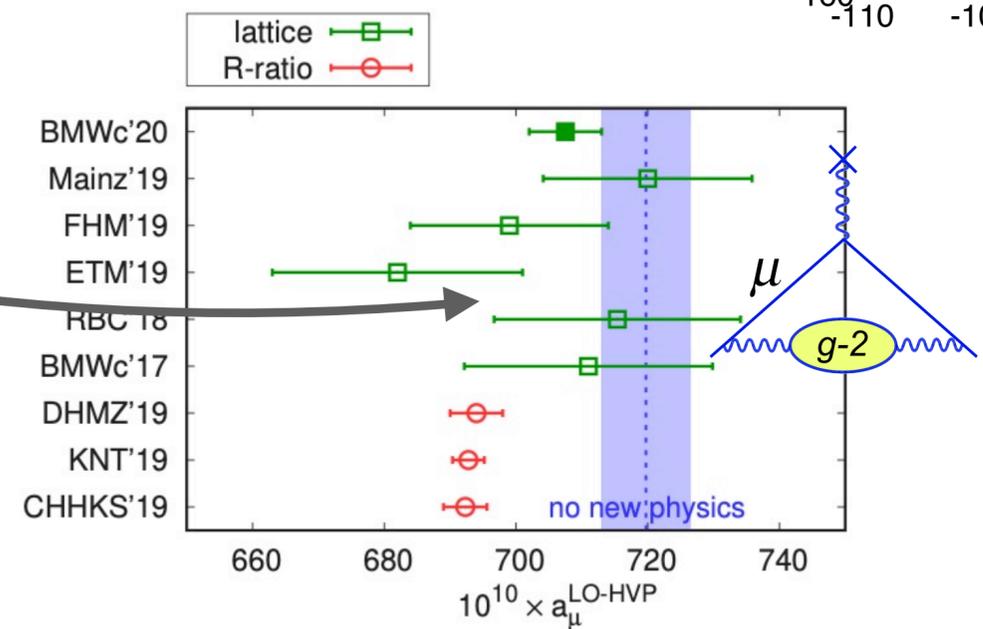
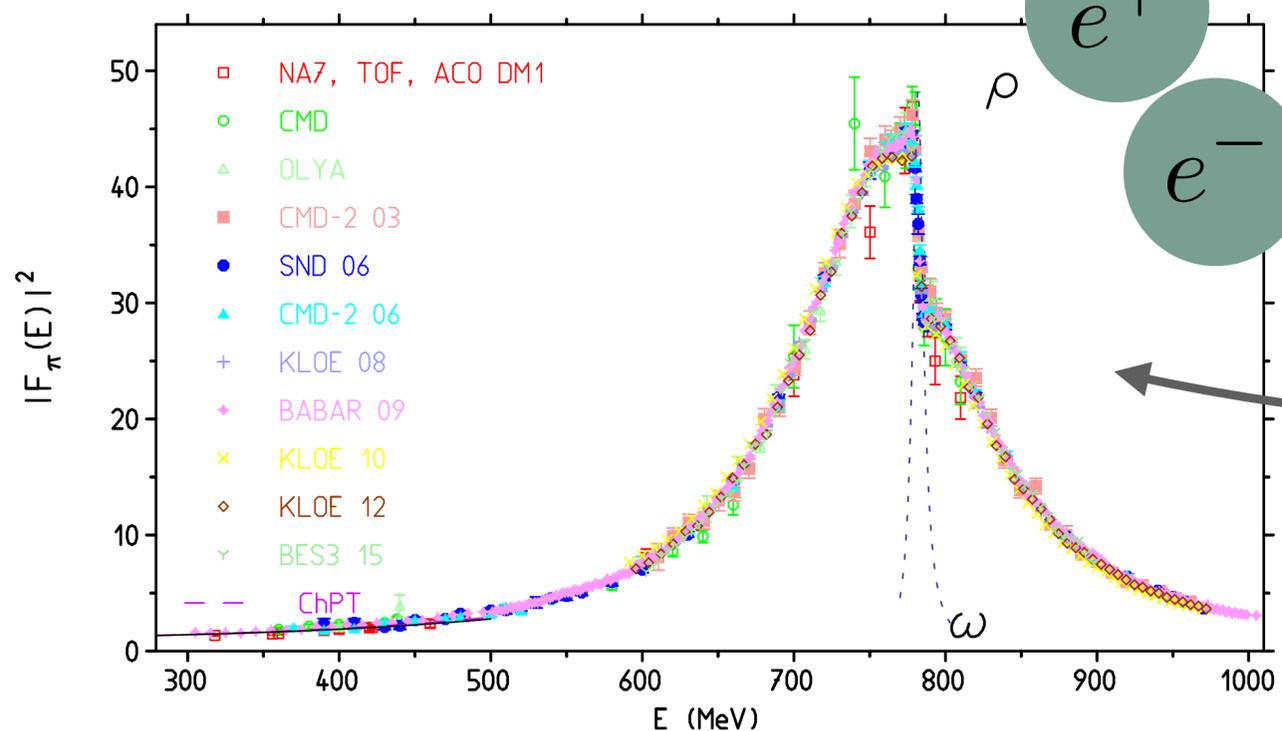
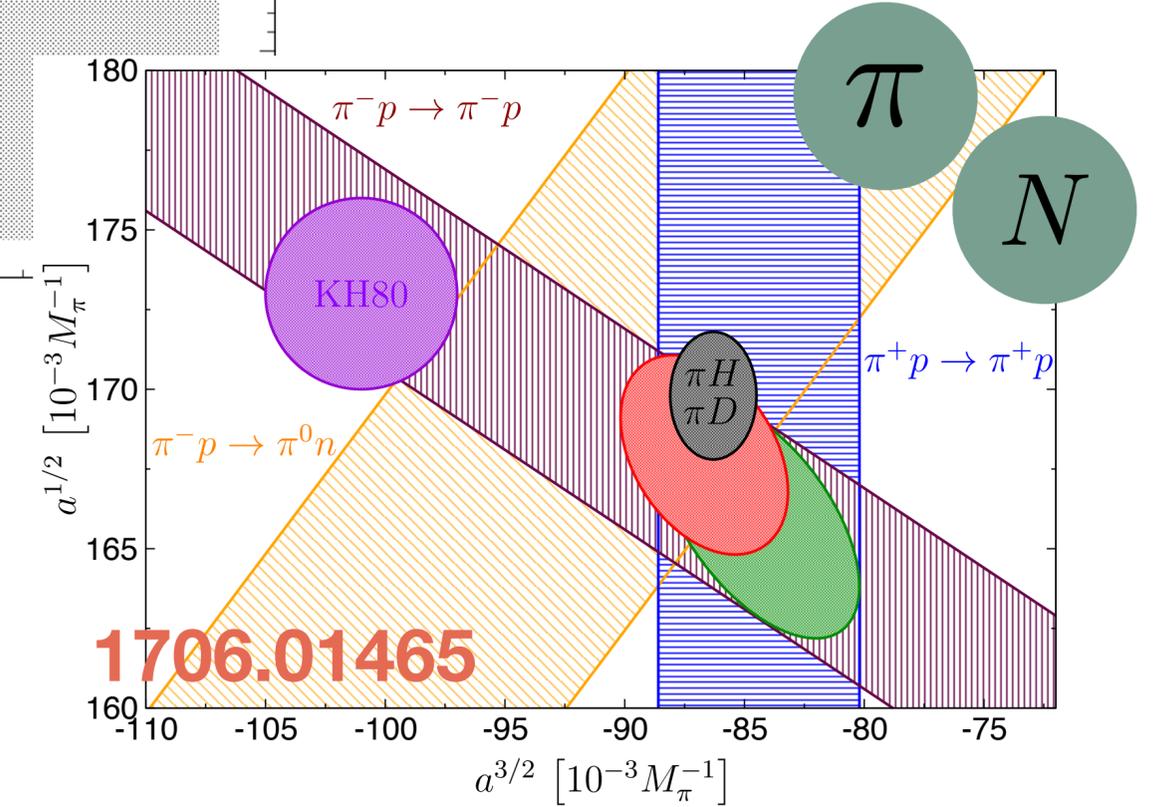
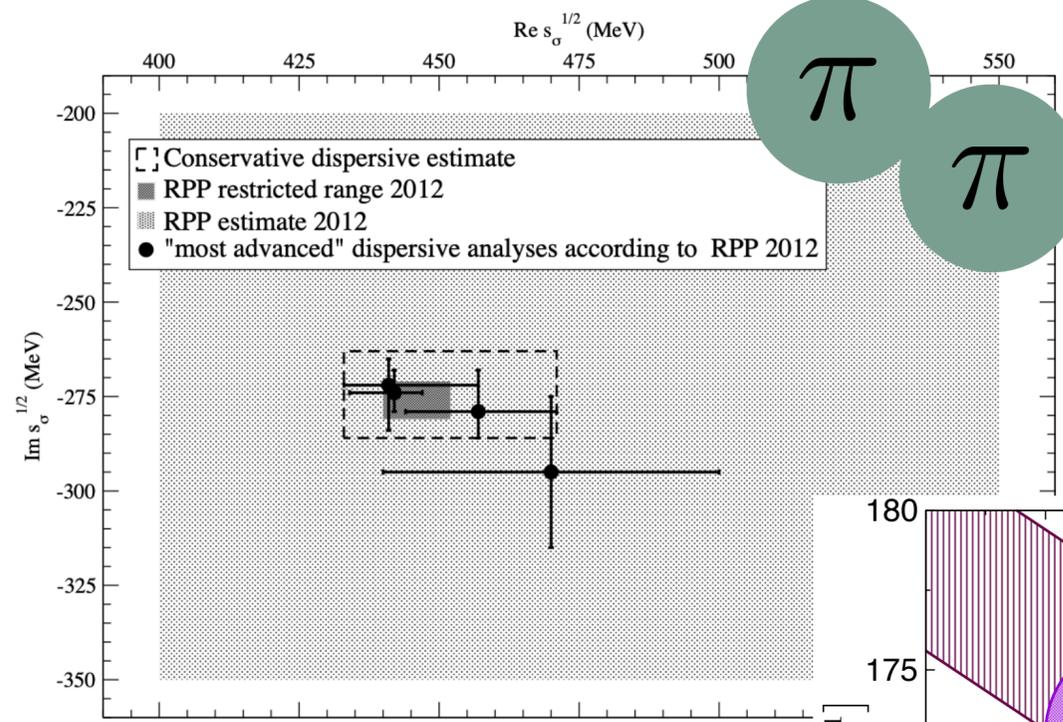
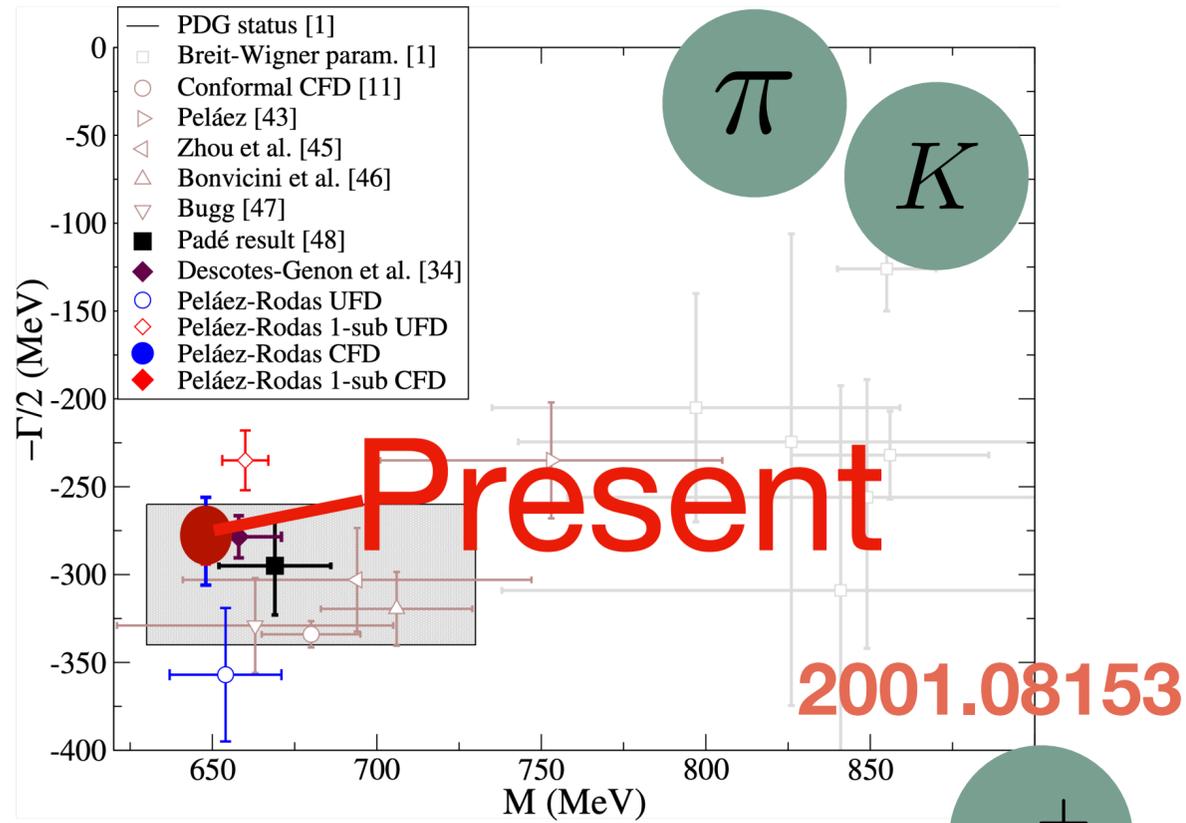
# Dispersive analyses



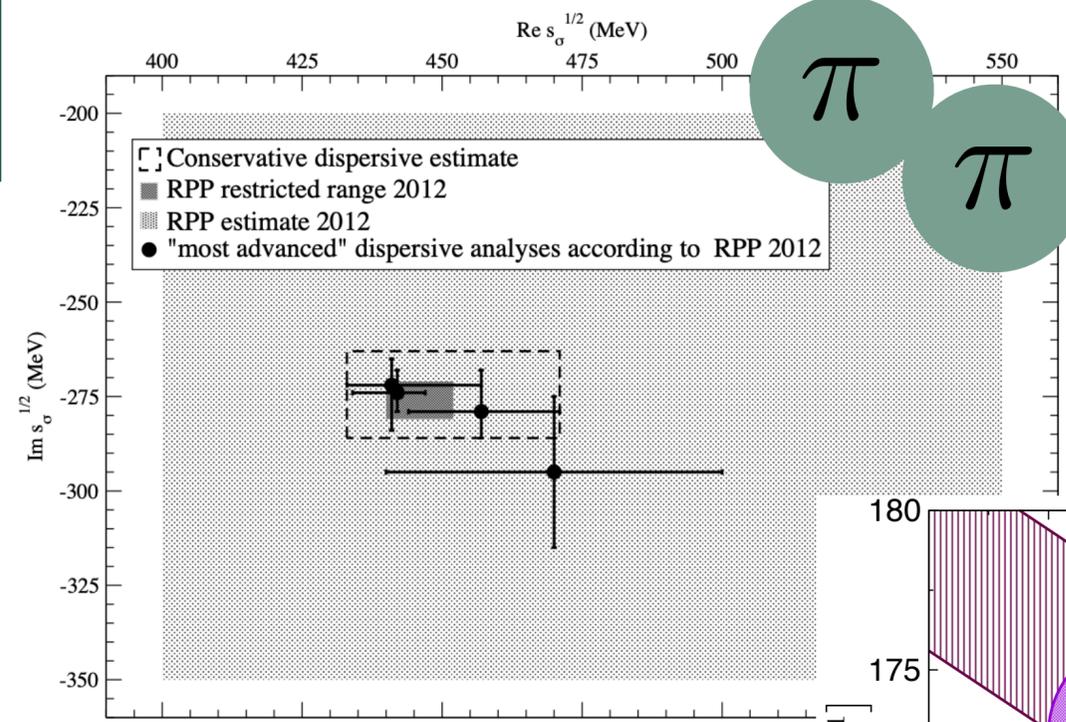
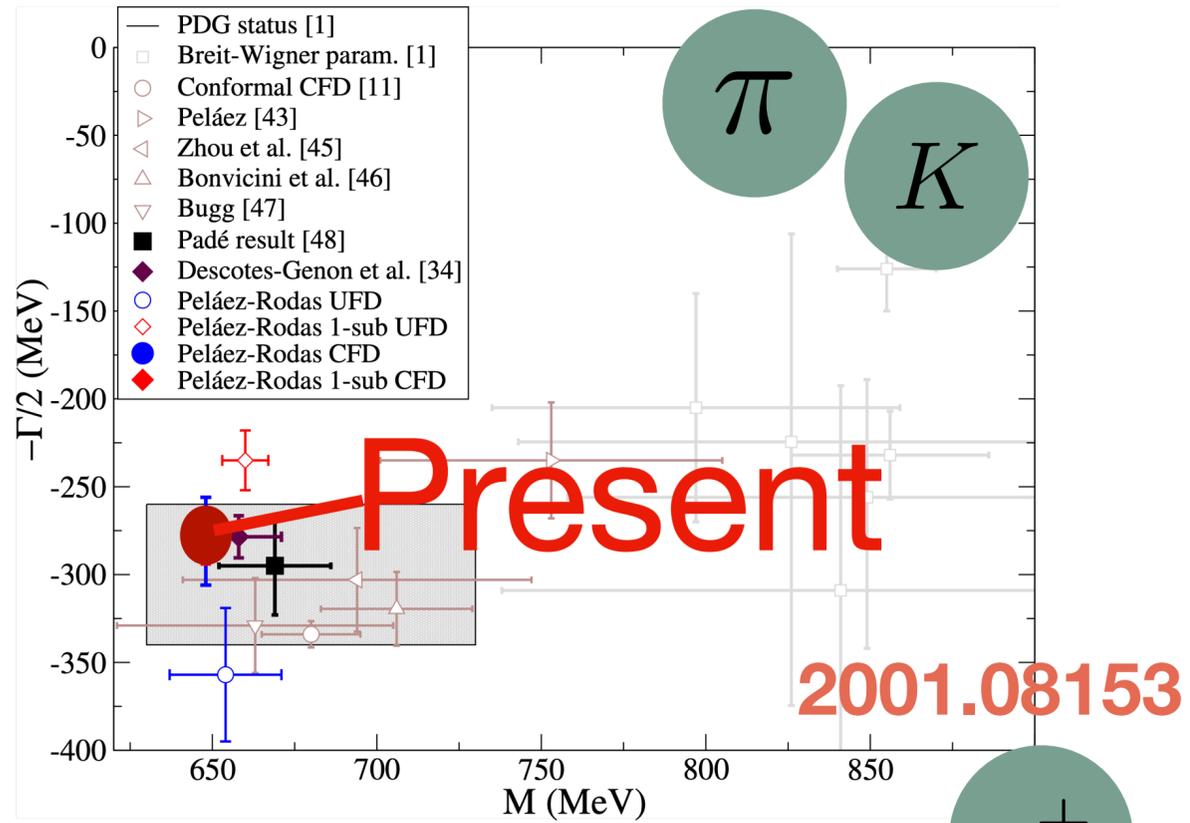
# Dispersive analyses



# Dispersive analyses

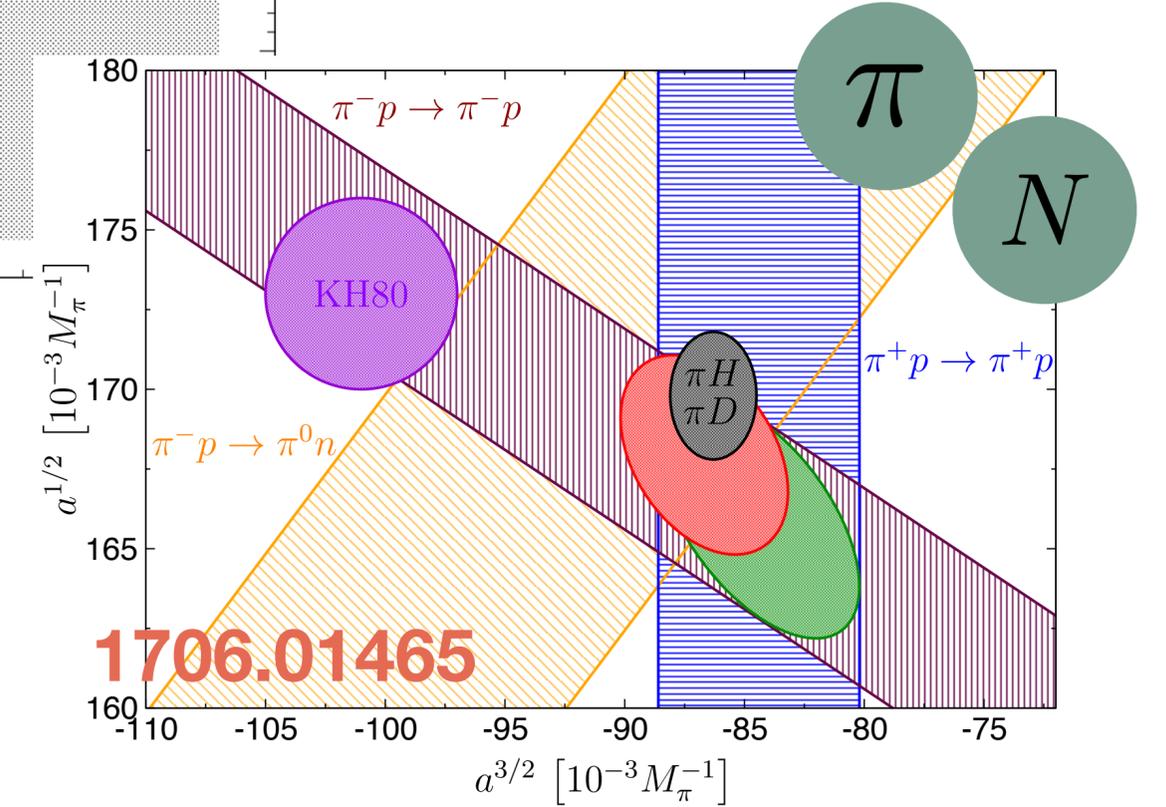


# Dispersive analyses



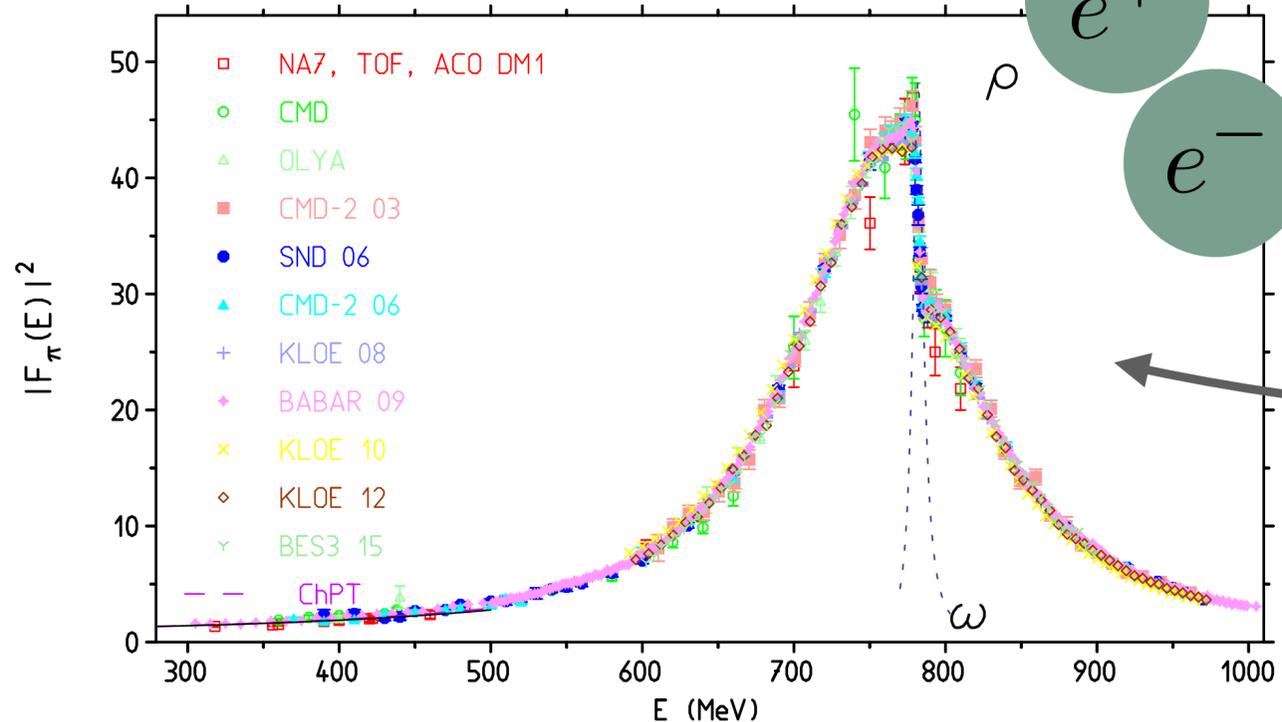
$\pi$

$\pi$



$\pi$

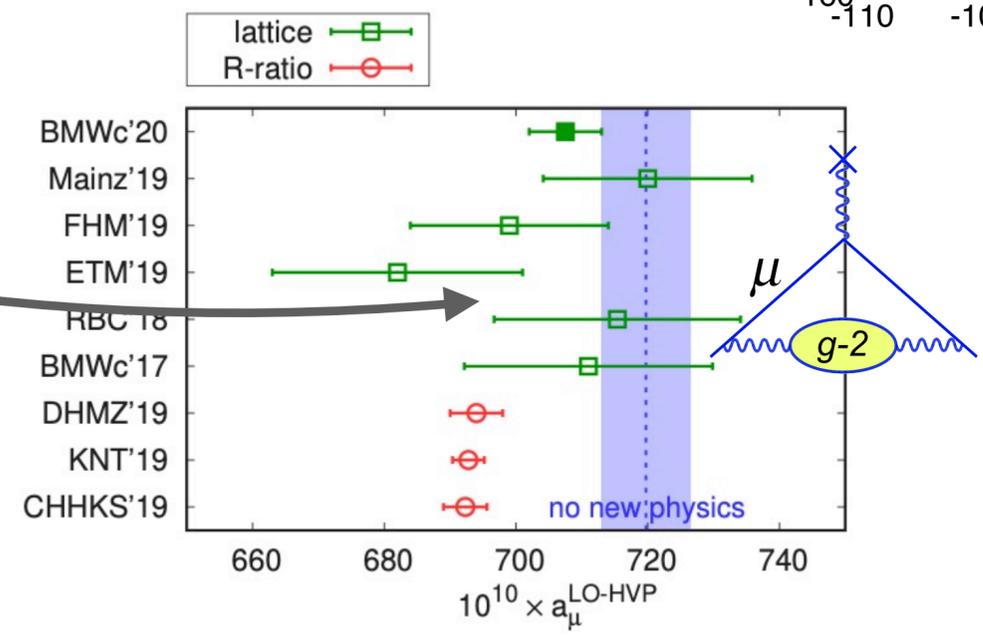
$N$



2006.04822

$e^+$

$e^-$



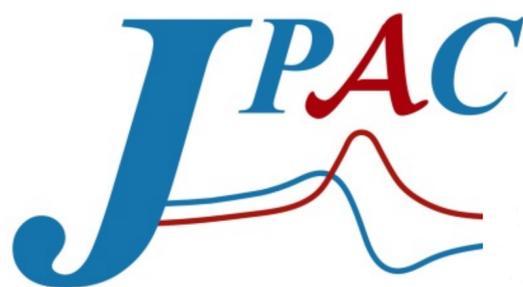
And others...

Like it?

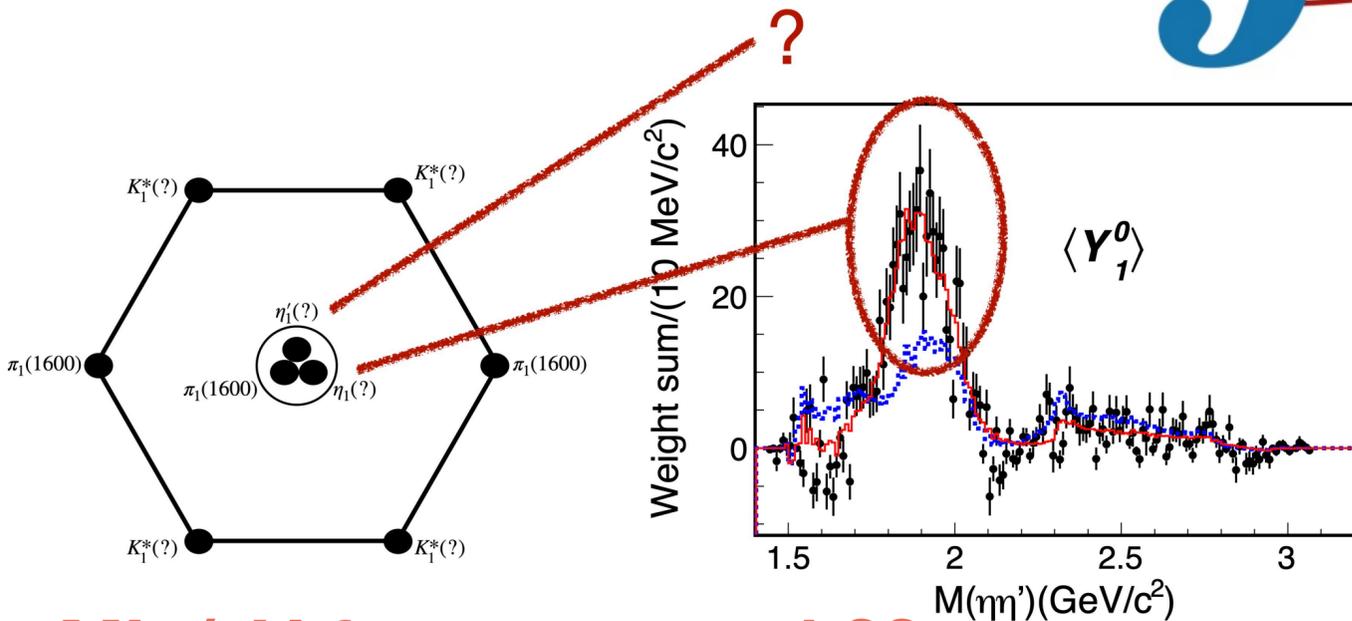
Like it?

Join us!!

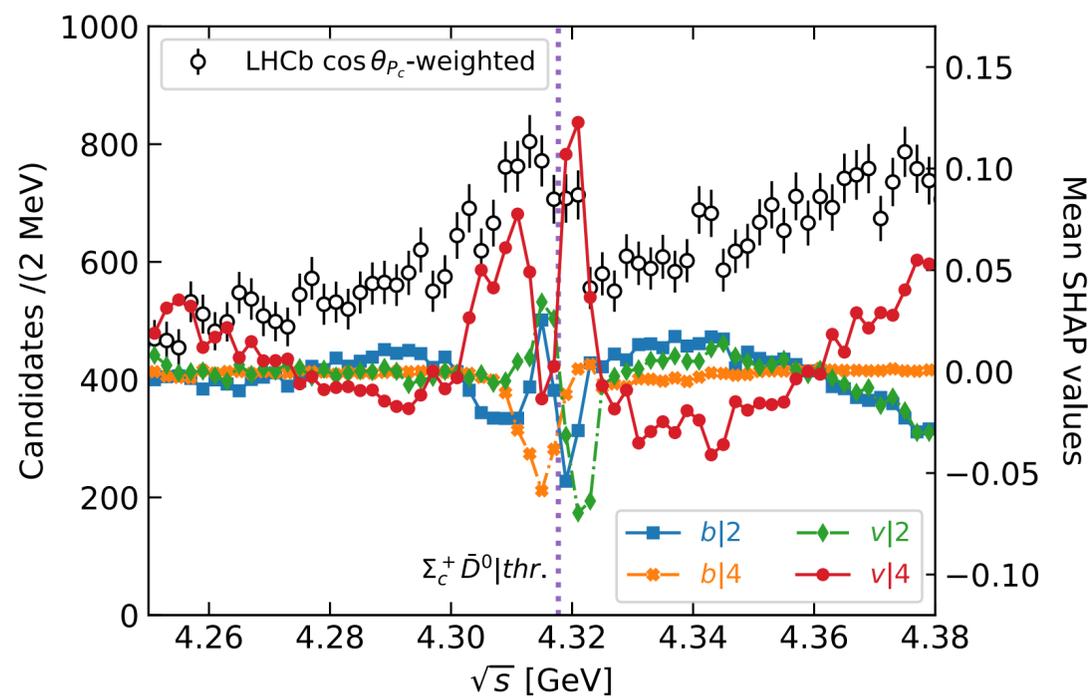
# Future with us



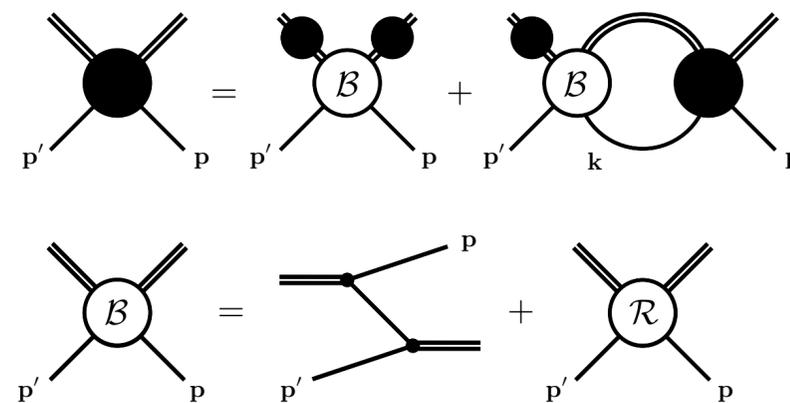
## Looking for other hybrids



## ML / AI for pentaquark??

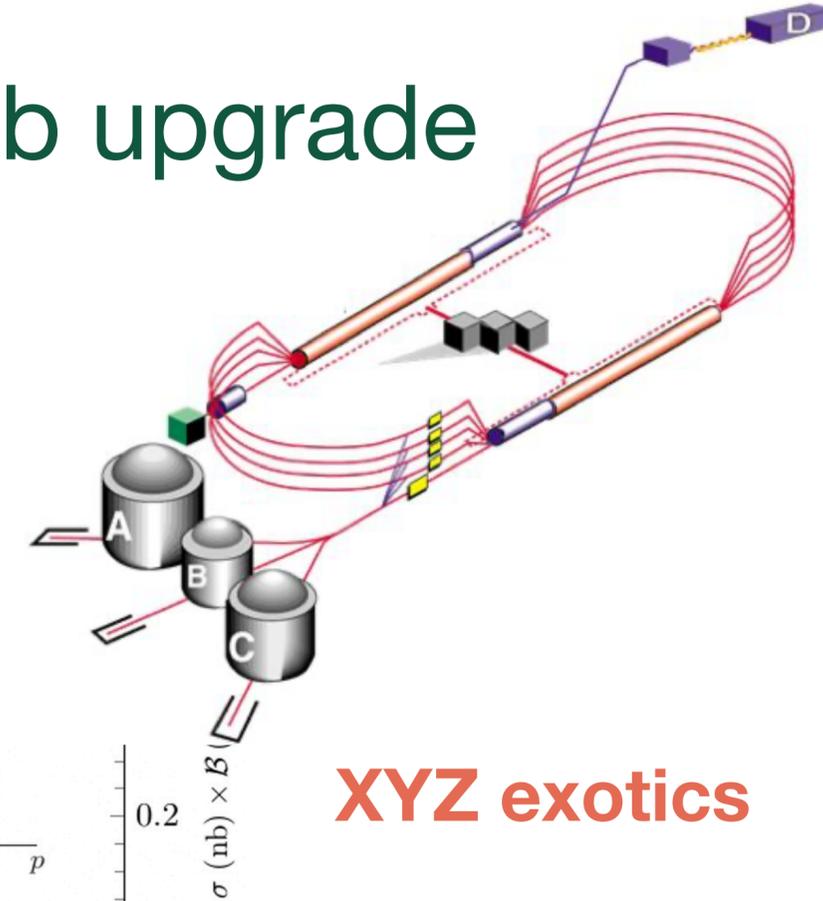
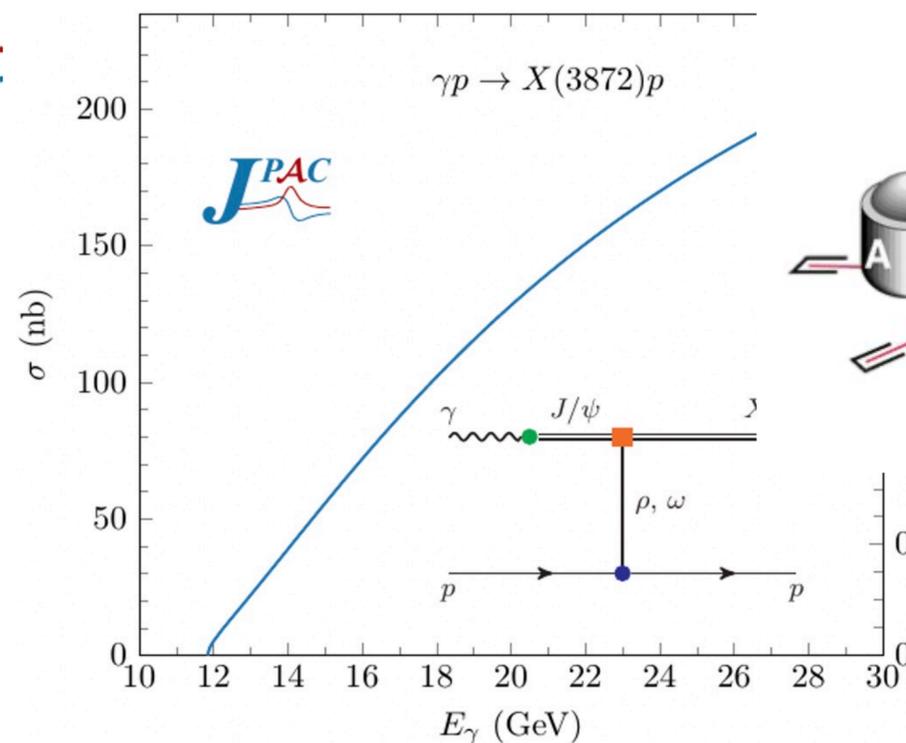


## Developing 3b formalism



And more...

## JLab upgrade



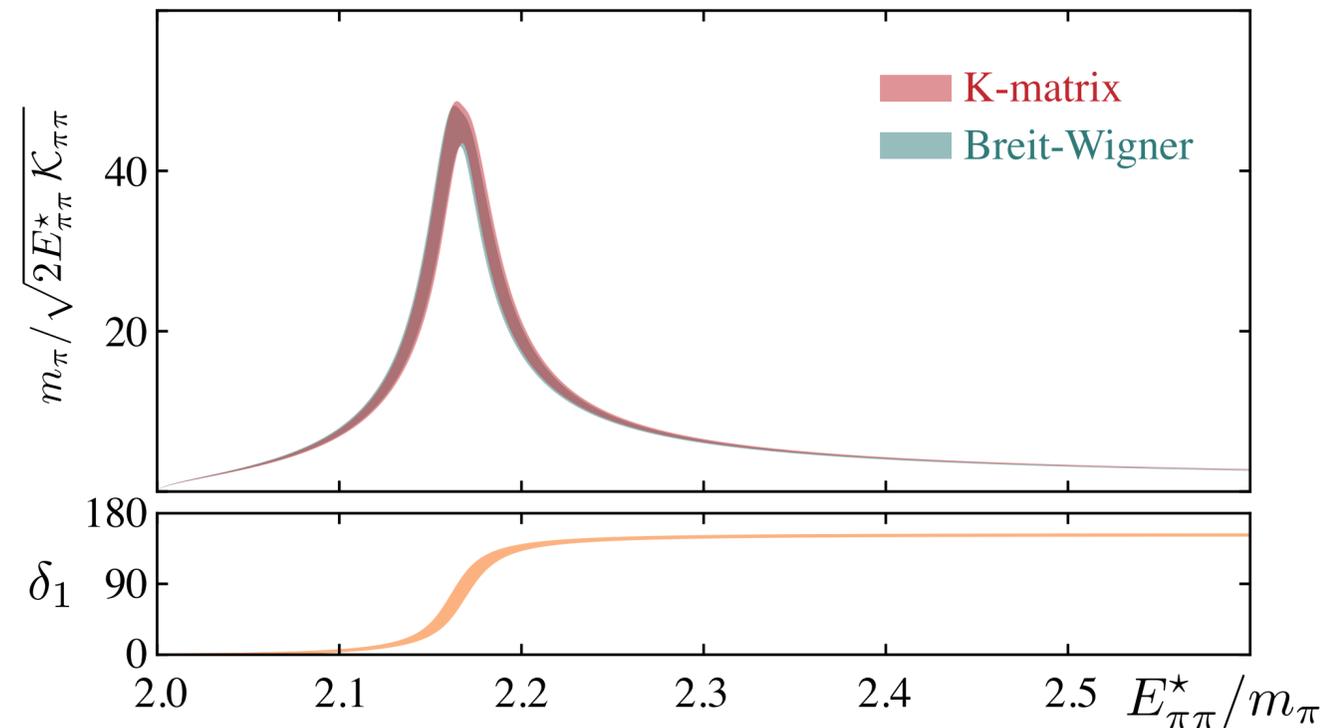
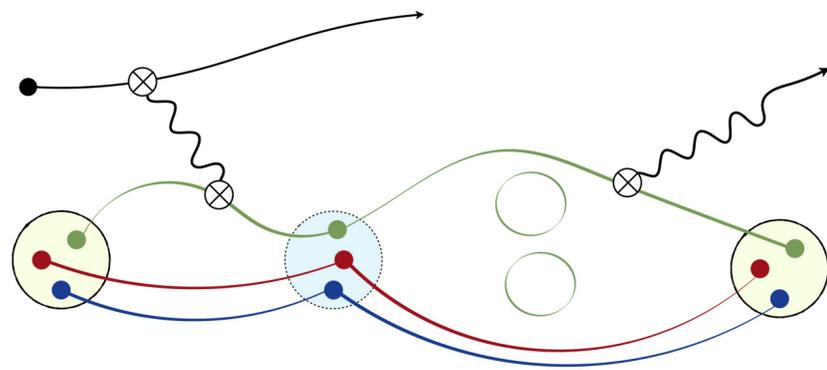
XYZ exotics

# Future with us

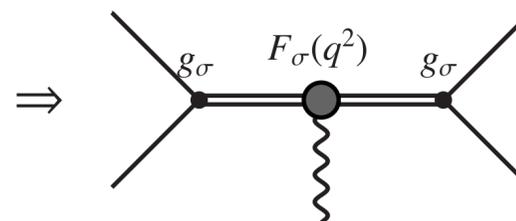
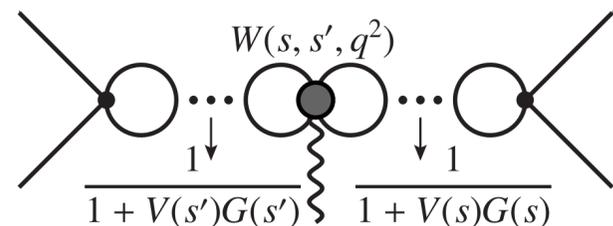
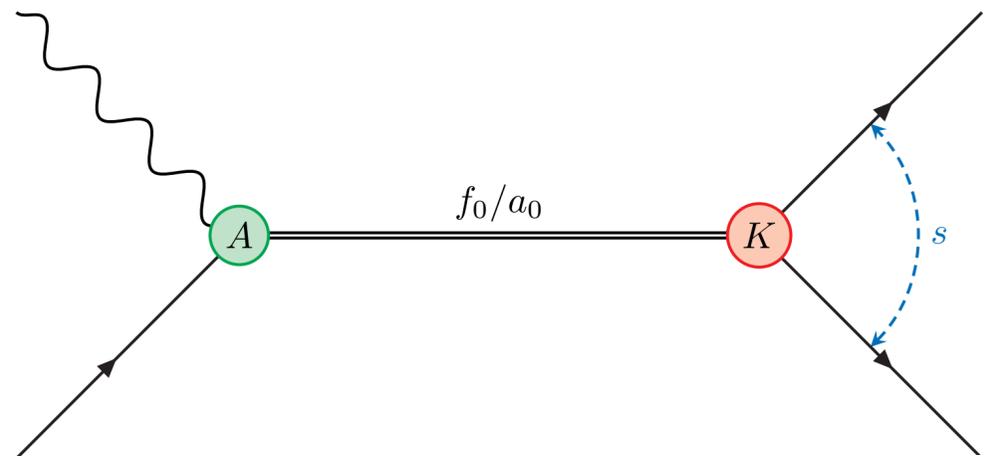
# had spec

## What's inside?

$\gamma$  to shed some light

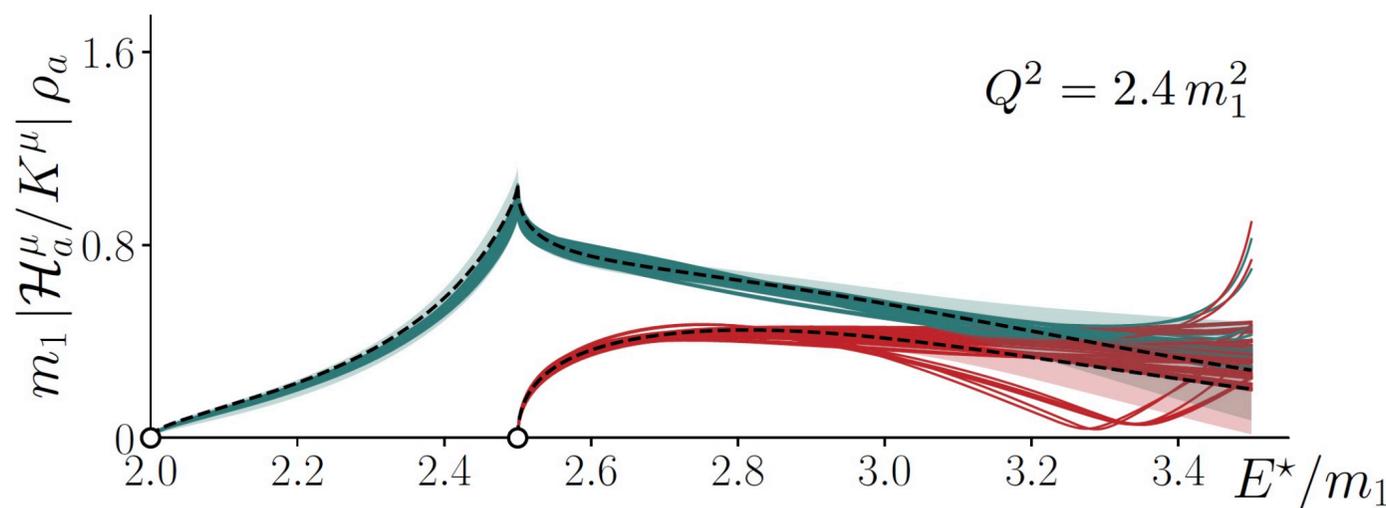


## Flavor structure



$\gamma \rightarrow$  charge radius

$h \rightarrow$  mass radius??



$$Q^2 = 2.4 m_1^2$$

And more...

# Questions??