The Quest for Exotica

III Amplitude Analyses

Arkaitz Rodas

WILLIAM & MARY
CHARTERED 1693

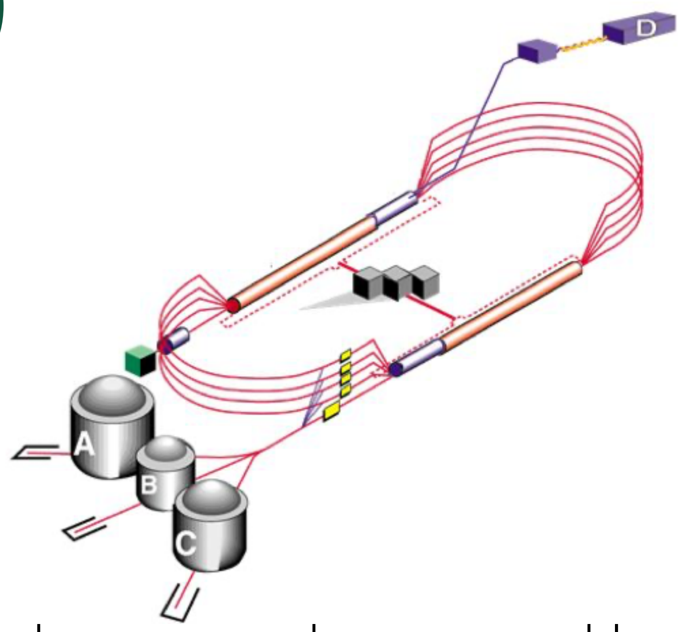
JPAC

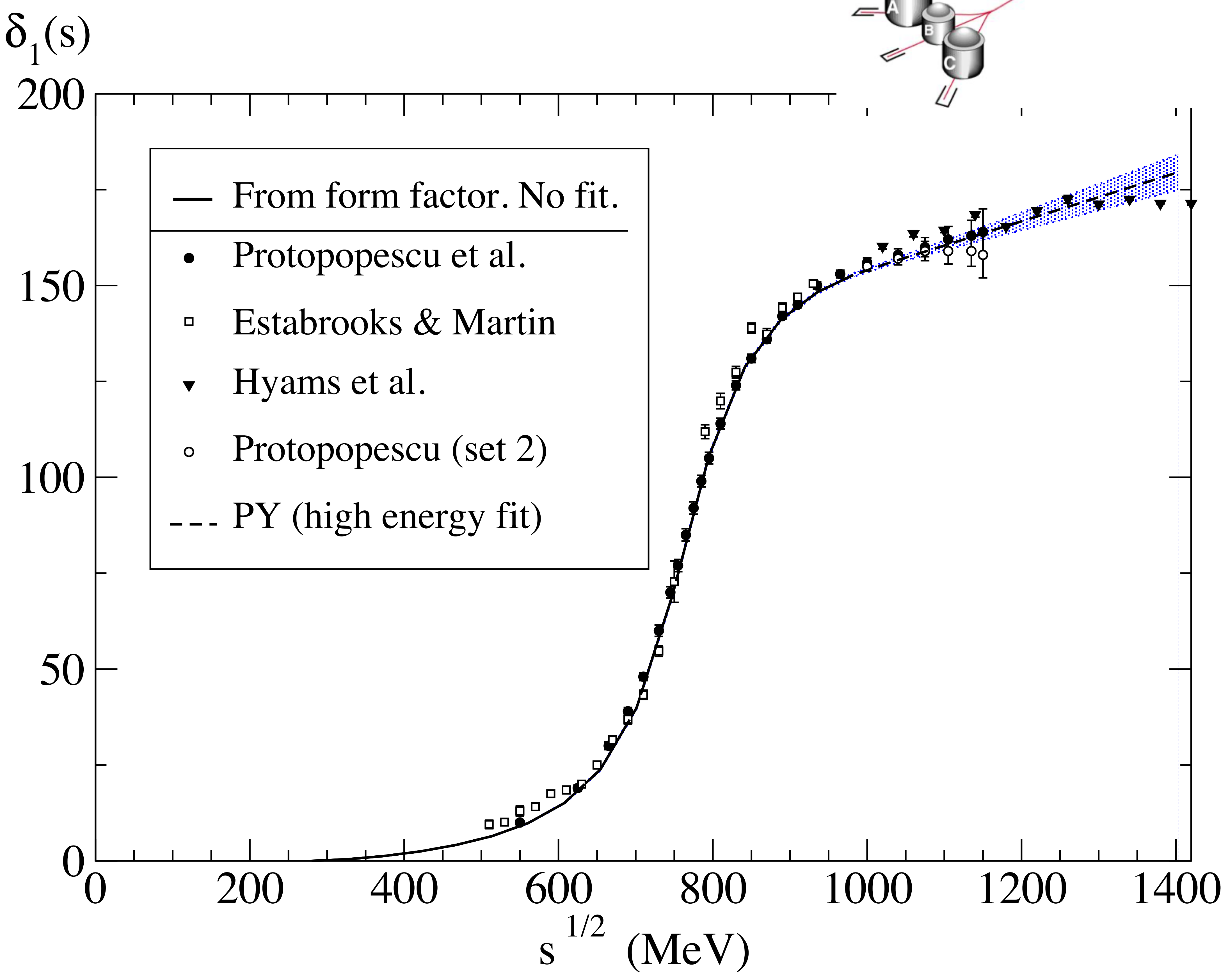
hadspec

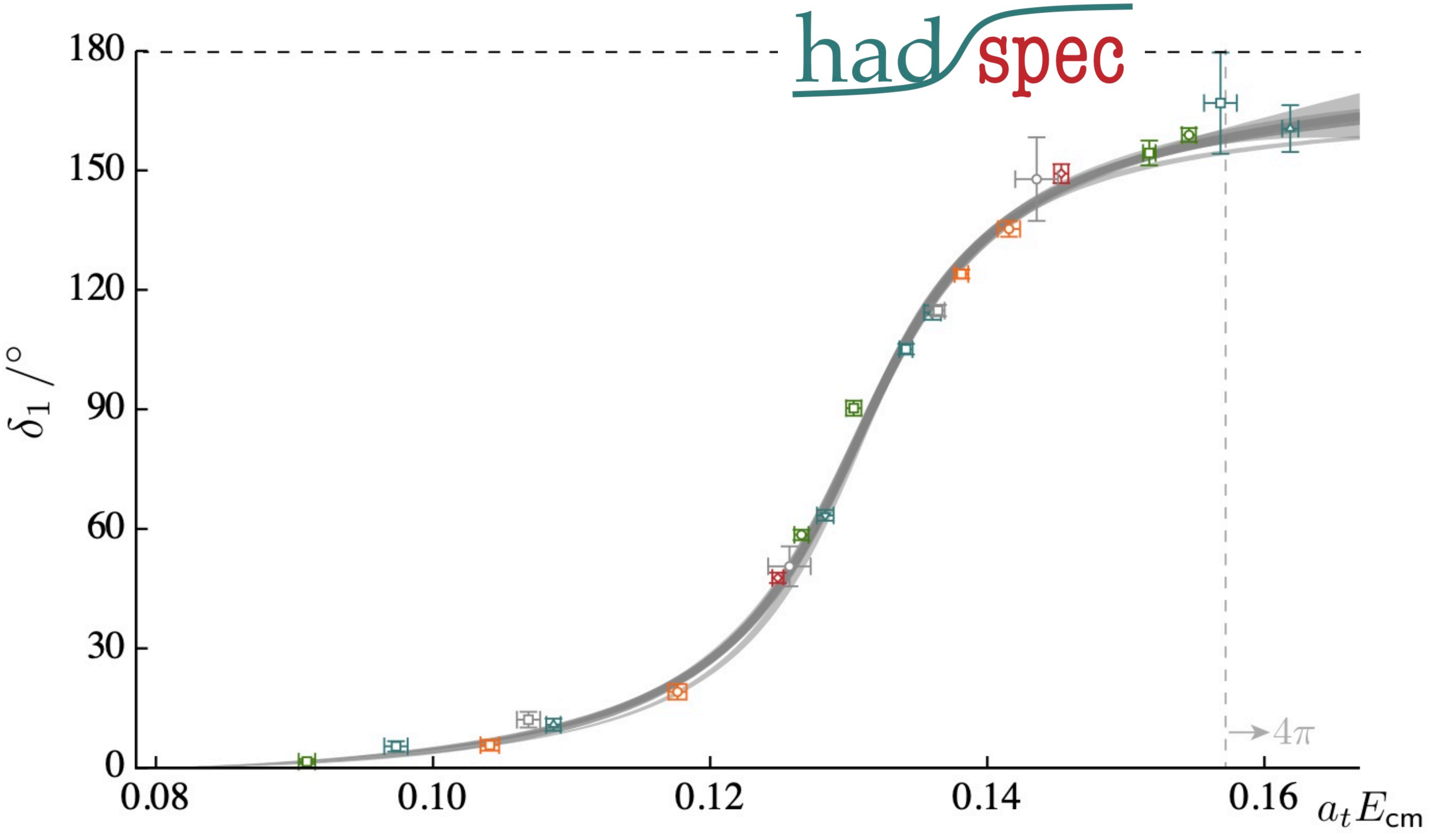
Jefferson Lab
Thomas Jefferson National Accelerator Facility

Decays

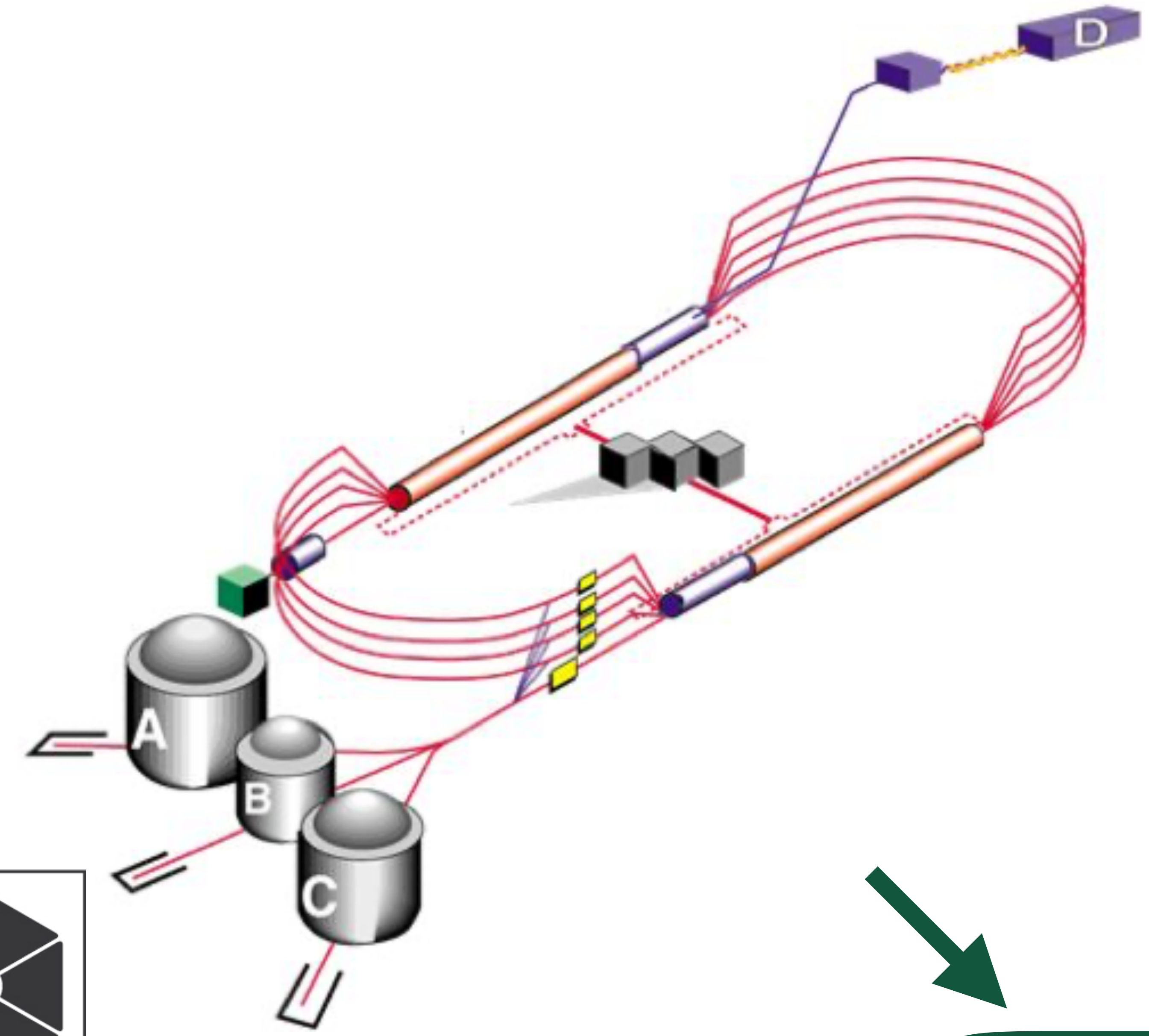
Experiment → $\delta(s)$







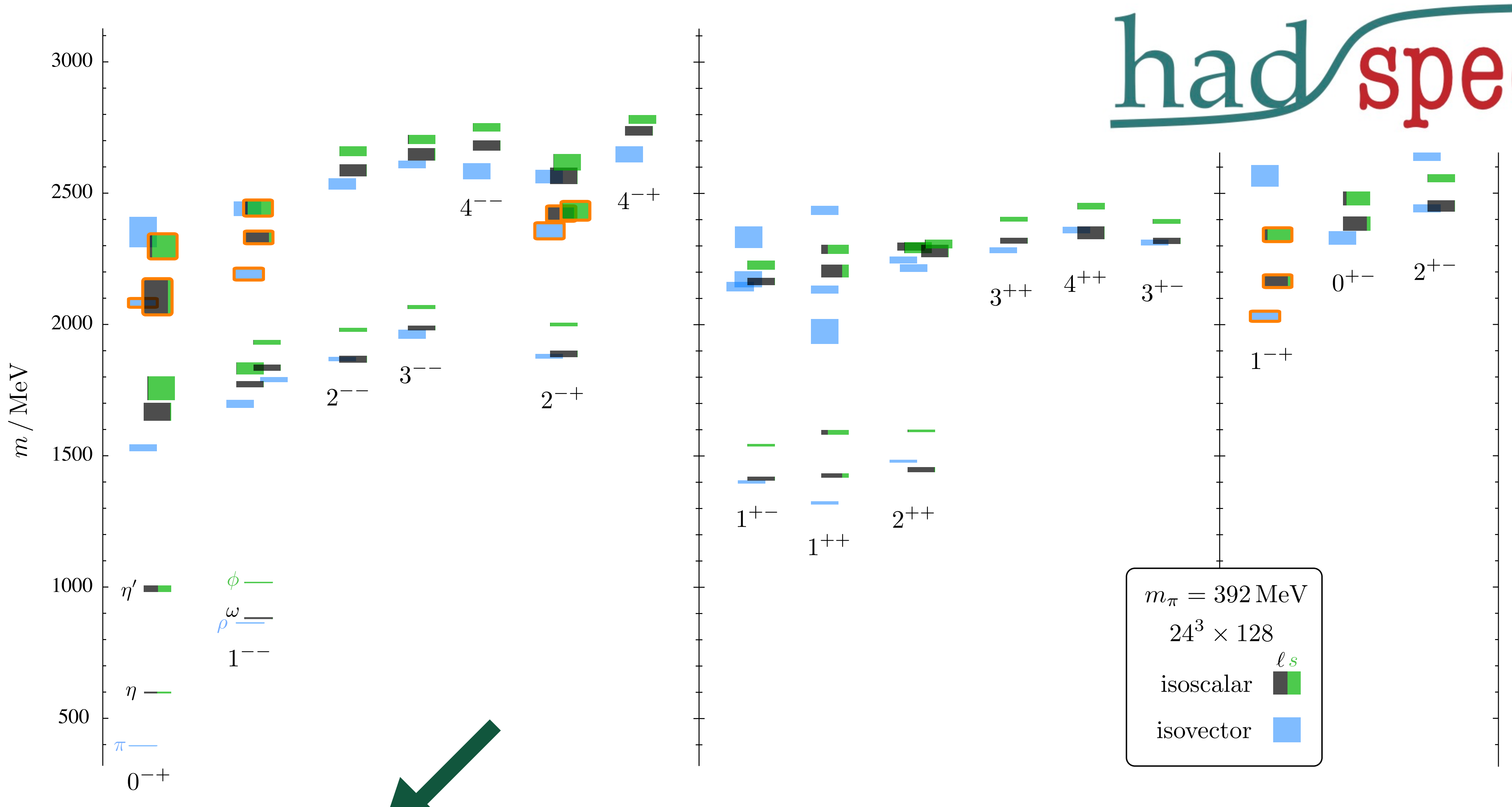
Lattice QCD → $\delta(s)$



GlueX

clas
CEBAF Large Acceptance Spectrometer

hadspec



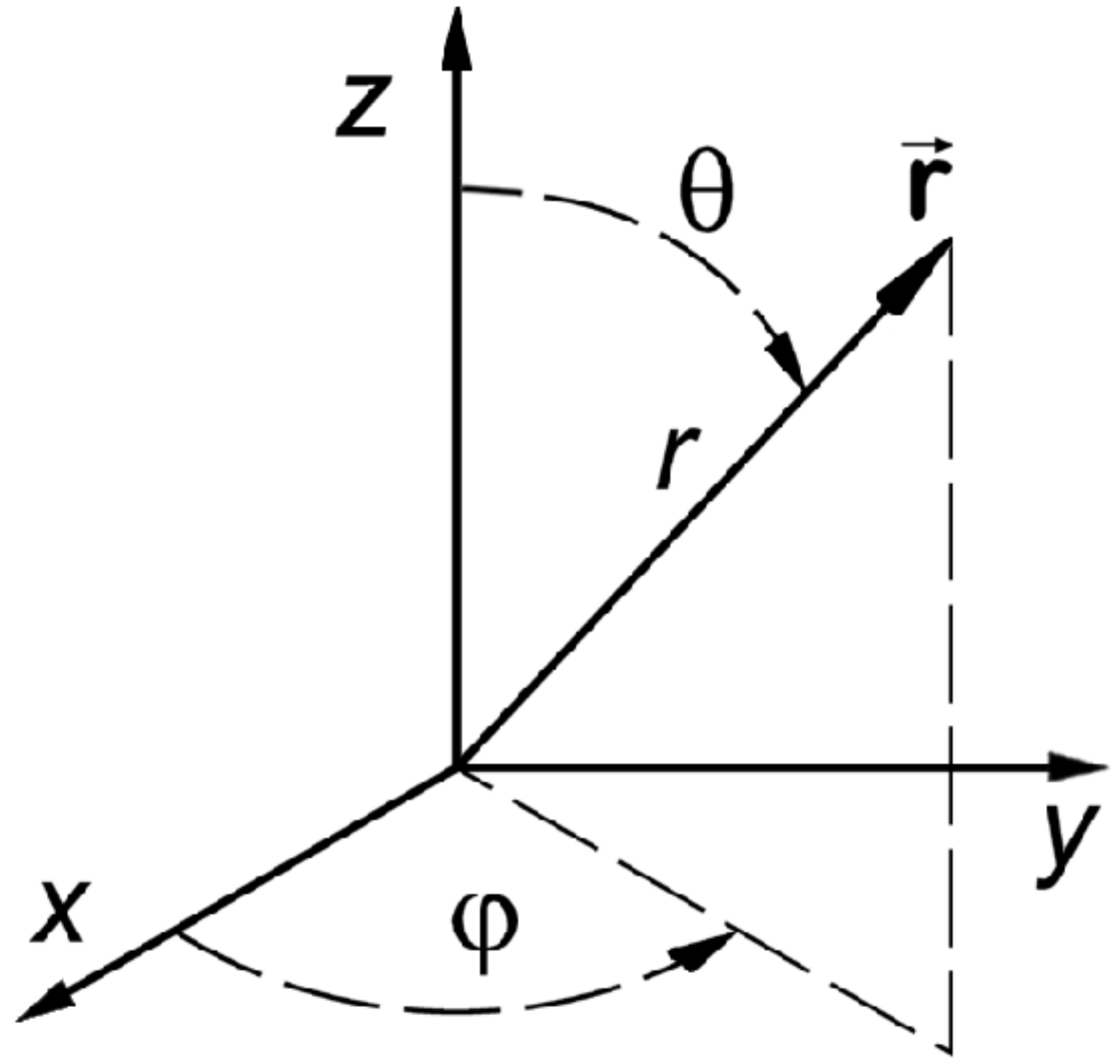
Amplitude analyses

- Unitarity
- Analiticity
- Crossing

QCD

Observables

Decays



$$|JM\rangle\,|p_{\mathrm{CM}}\rangle\,|\mu\rangle = \int d\Omega |\theta\phi\rangle\langle\theta\phi \mid JM\rangle\,|p_{\mathrm{CM}}\rangle\,|\mu\rangle$$

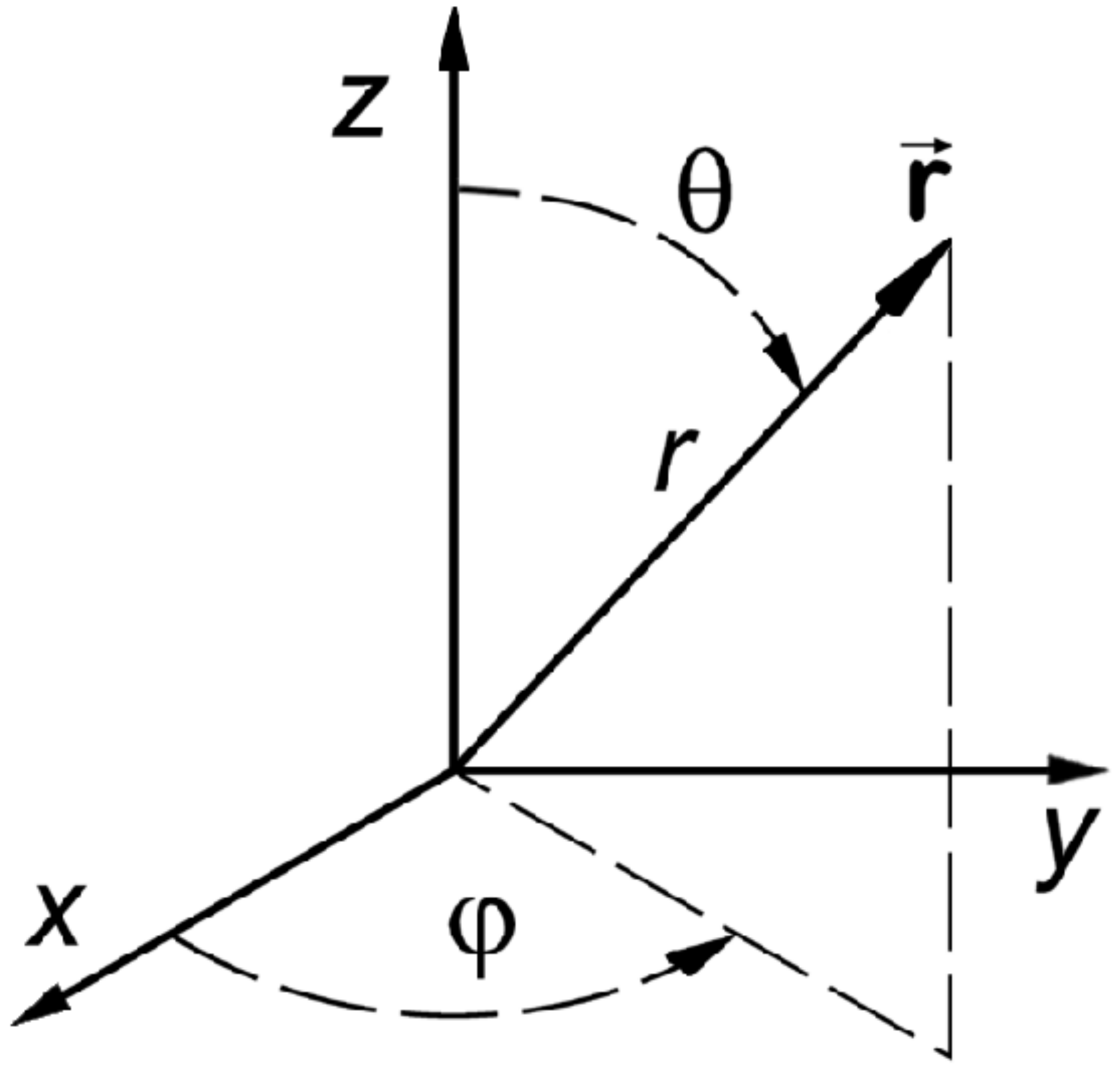
No spin

$$= \int d\Omega Y_J^M(\theta,\phi)|\theta\phi\rangle\,|p_{\mathrm{CM}}\rangle\,|\mu\rangle$$

$$t_J(s)$$

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$

Decays



$$|JM\rangle\,|p_{\rm CM}\rangle\,|\mu\rangle = \int d\Omega|\theta\phi\rangle\langle\theta\phi\mid JM\rangle\,|p_{\rm CM}\rangle\,|\mu\rangle$$

No spin

$$= \int d\Omega Y_J^M(\theta,\phi)|\theta\phi\rangle\,|p_{\rm CM}\rangle\,|\mu\rangle$$

$$t_J(s)$$

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$

Spherical harmonics

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x-iy)}{r}$$

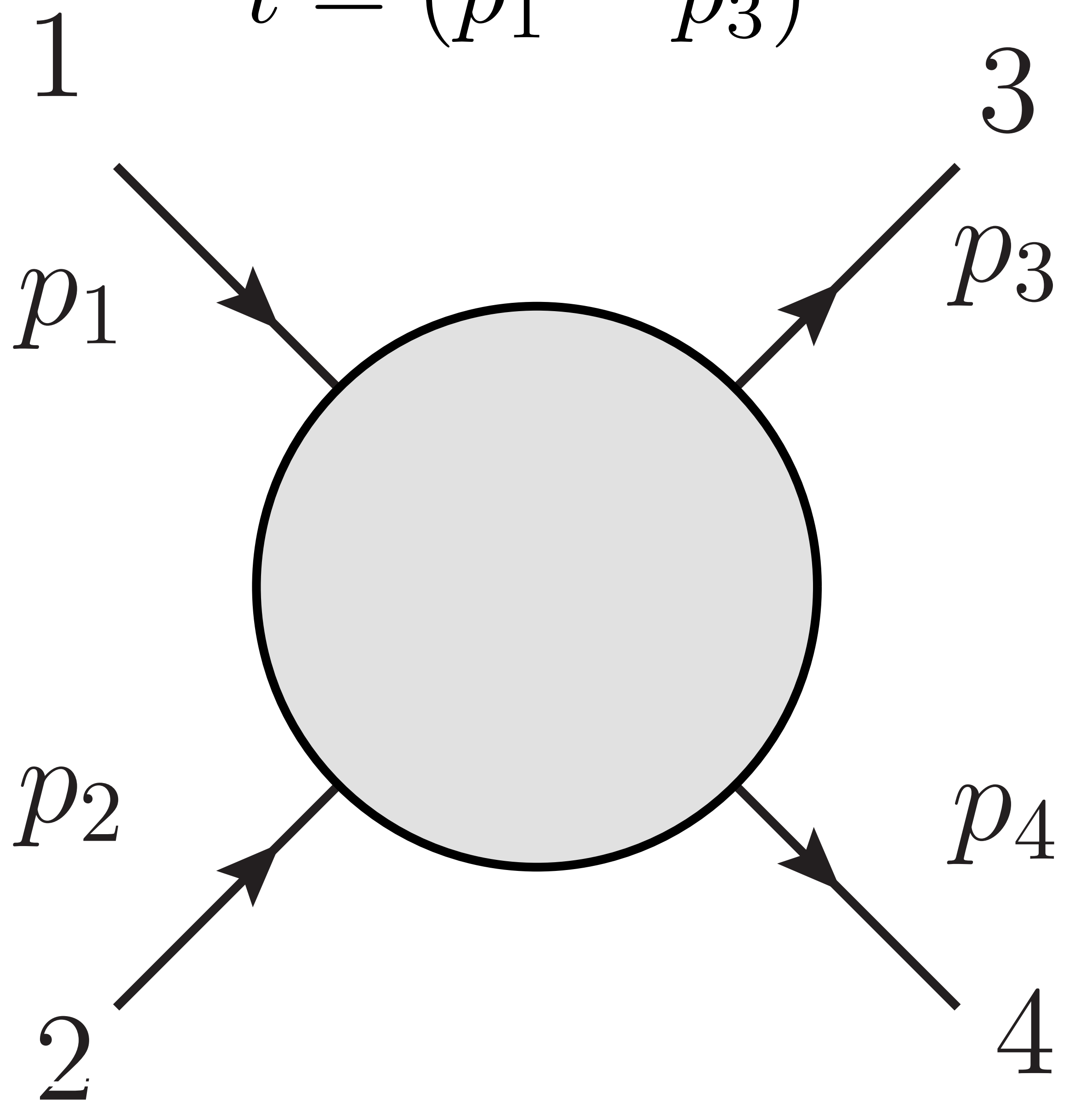
$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\frac{z}{r}$$

$$Y_1^{1}(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x+iy)}{r}$$

Scattering

$$t = (p_1 - p_3)^2$$

$$s = (p_1 + p_2)^2$$



Remember?

$$T(s,t) = 16K\pi \sum_{\ell} (2\ell + 1) P_\ell(\cos\theta) t_\ell(s)$$

Elastic unitarity $a \to a$

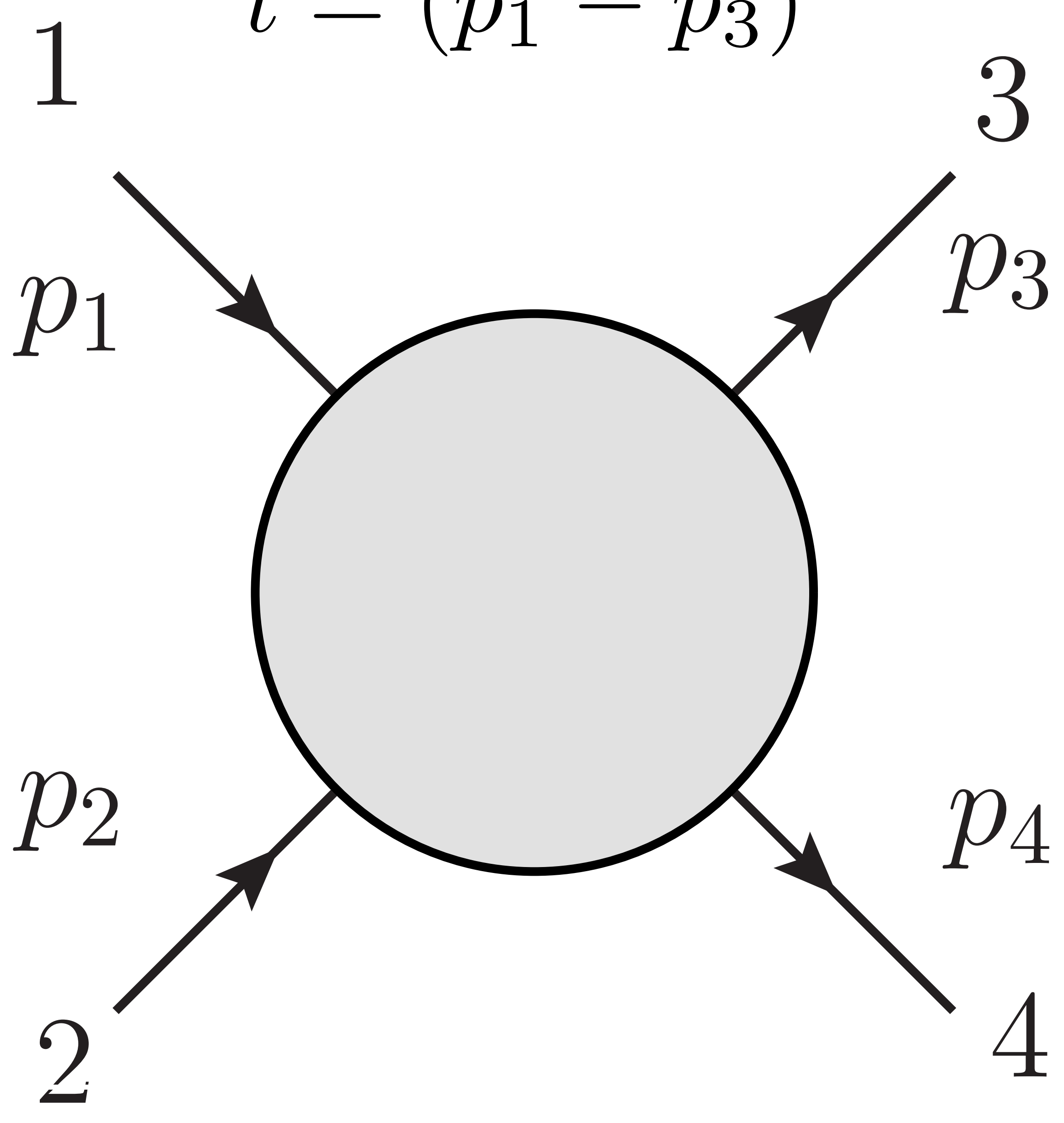
$$SS^\dagger = \mathbb{I} \longrightarrow |S_\ell(s)| = 1 \longrightarrow S_\ell(s) = e^{2i\delta_\ell(s)}$$

Scattering

Remember?

$$T(s,t) = 16K\pi \sum_{\ell} (2\ell+1) P_\ell(\cos\theta) t_\ell(s)$$

$$s = (p_1+p_2)^2 \qquad t = (p_1-p_3)^2$$



Elastic unitarity $a \to a$

$$SS^\dagger = \mathbb{I} \longrightarrow |S_\ell(s)| = 1 \longrightarrow S_\ell(s) = e^{2i\delta_\ell(s)}$$

$$S = 1 + 2iT$$

Exercise

$$\hat{t}_\ell(s) = \frac{e^{2i\delta_\ell(s)} - 1}{2i} = \sin\delta_\ell(s)\, e^{i\delta_\ell(s)}$$

$$t_\ell(s) = \frac{\sqrt{s}}{2q(s)} \hat{t}_\ell(s)$$

How do we parameterize?

Lets assume that we are at threshold (q=0) or that the interaction is small

$$\mathrm{e}^{\mathrm{i}kL+2\mathrm{i}\delta(k)}$$

$$\delta_\ell \simeq aq$$

$$t_\ell(s) = \frac{\sqrt{s}}{2q(s)} \sin\delta_\ell(s) e^{i\delta_\ell(s)} \propto a$$

Total cross section??

How do we parameterize?

Lets assume that we are at threshold (q=0) or that the interaction is small

$$e^{ikL+2i\delta(k)}$$

$$\delta_\ell \simeq aq$$

$$t_\ell(s) = \frac{\sqrt{s}}{2q(s)} \sin\delta_\ell(s) e^{i\delta_\ell(s)} \propto a$$

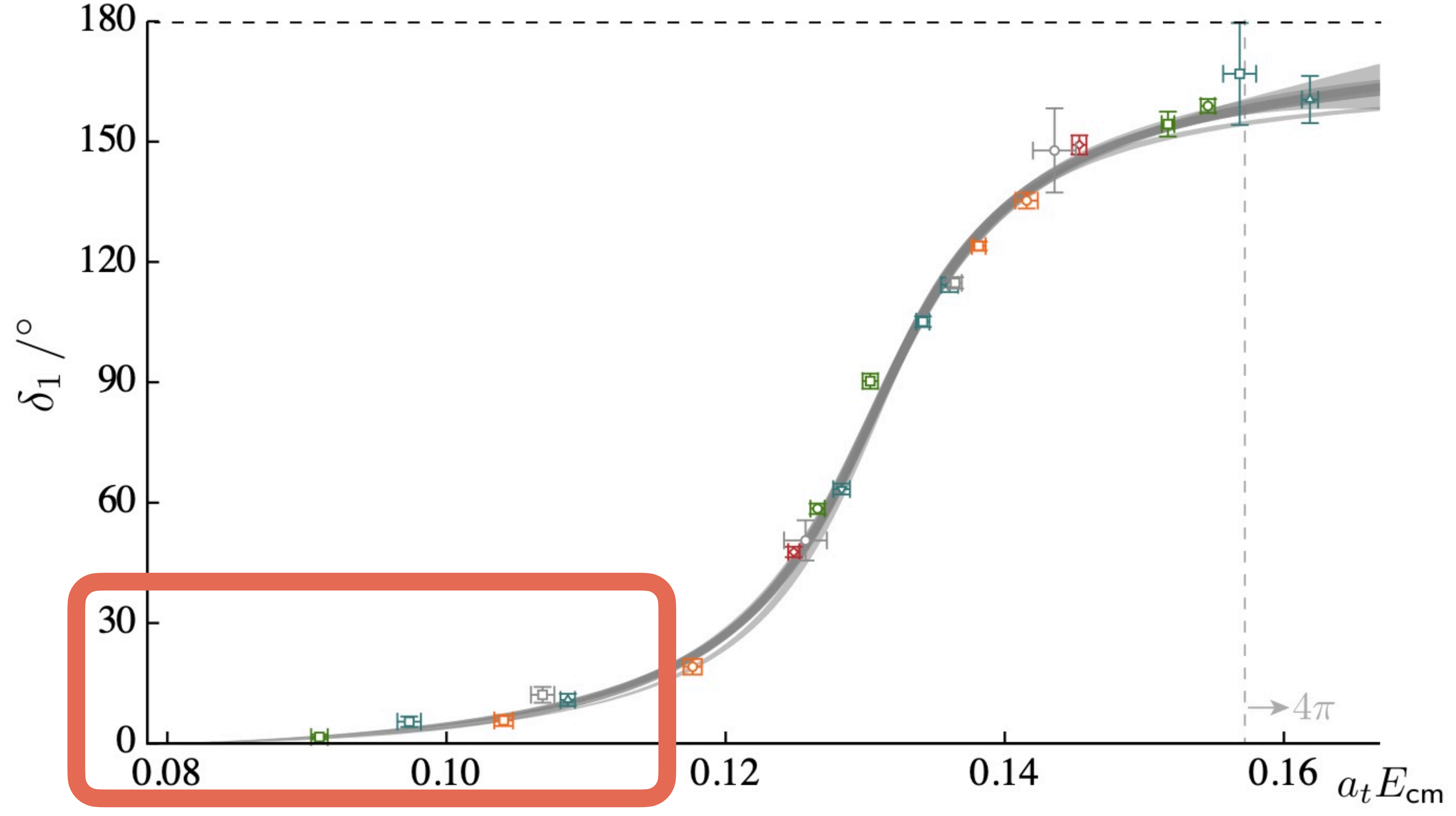
Total cross section??

$$Im\ t_\ell(s) \propto \sigma \simeq 4\pi a^2$$

Scattering

Low energy expansion

$$t_\ell(s) = \sum_{n=0}^{N} (a_n q^{2n})$$



Does a polynomial contain a pole?

$$t_\ell(s) \simeq \frac{-M\Gamma}{M^2 - s - iM\Gamma}$$

Is a polynomial unitary??

$$Im\, t_\ell(s) = \rho(s)|t_\ell(s)|^2$$

Scattering

General elastic param.

$$\rho(s) = \frac{\sqrt{s}}{2q(s)}$$

Phase space

By virtue of unitarity

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$

Freedom to choose

$$q(s)\cot\delta_\ell(s) = 1/a$$

Scattering

General elastic param.

$$\rho(s) = \frac{\sqrt{s}}{2q(s)}$$

Phase space

By virtue of unitarity

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$

Freedom to choose

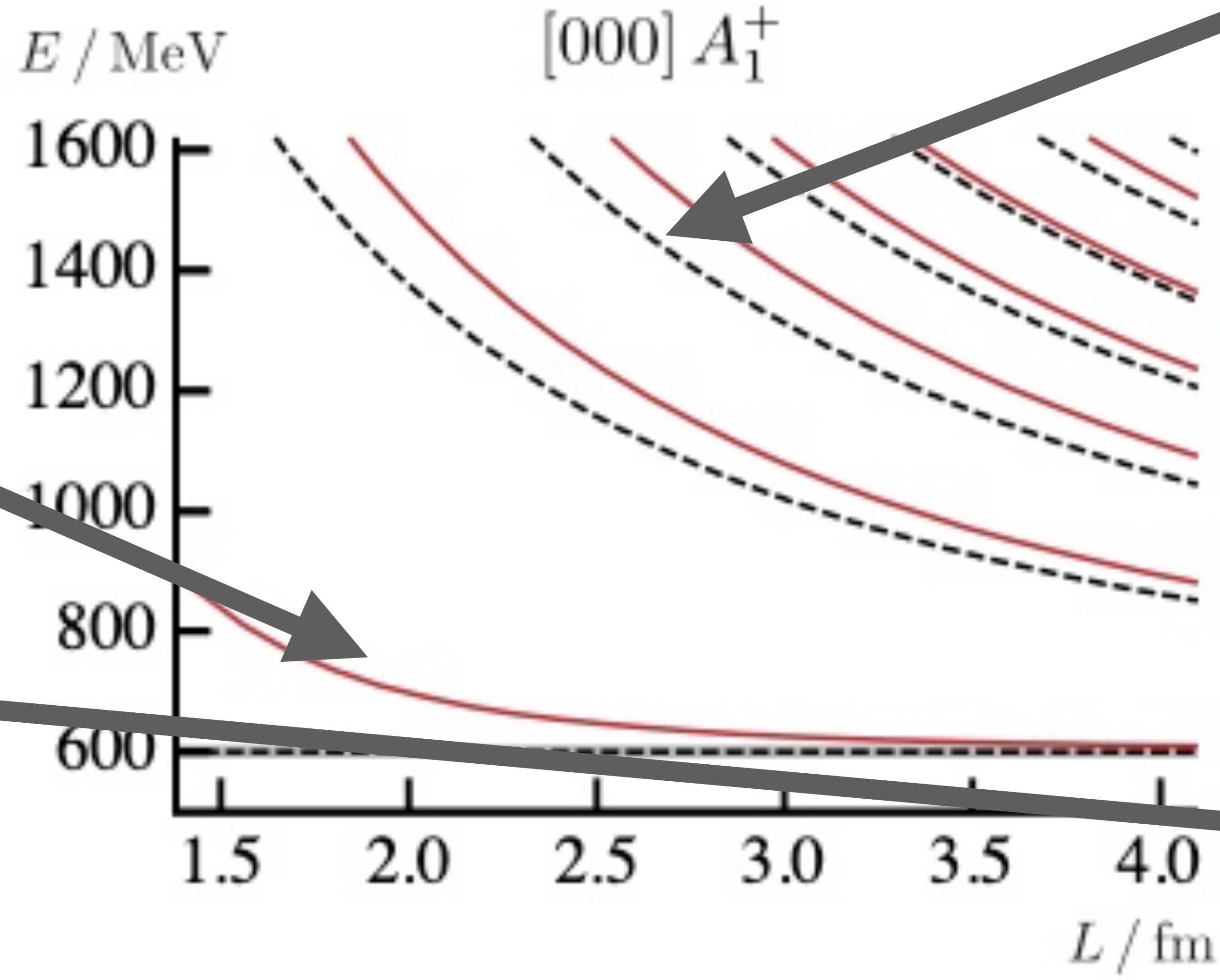
$$q(s)\cot\delta_\ell(s) = 1/a$$

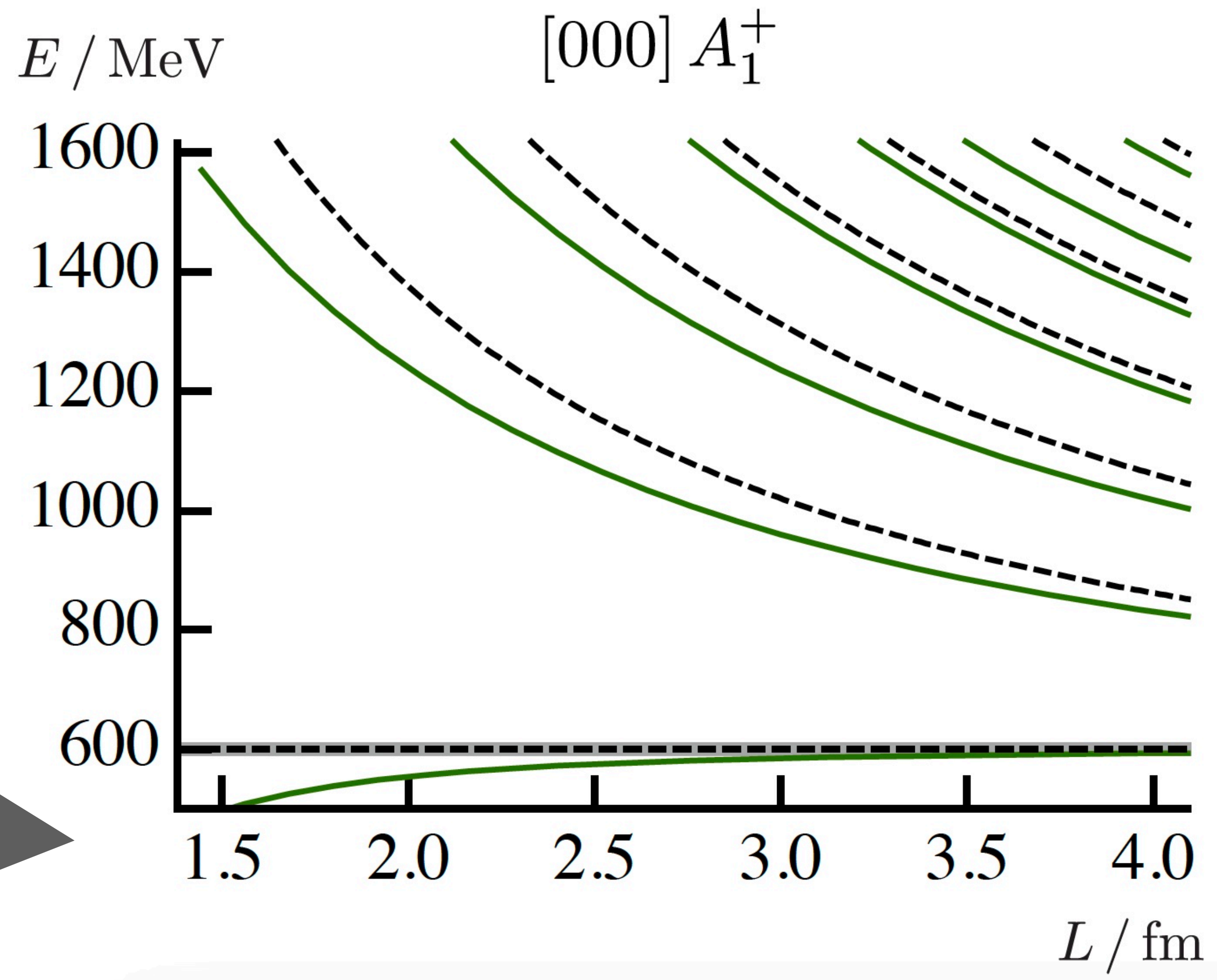
Non-interacting

How does it look in a box?

$a < 0 \rightarrow$ repulsion

$a > 0 \rightarrow$ atraction





Scattering

Low energy expansion

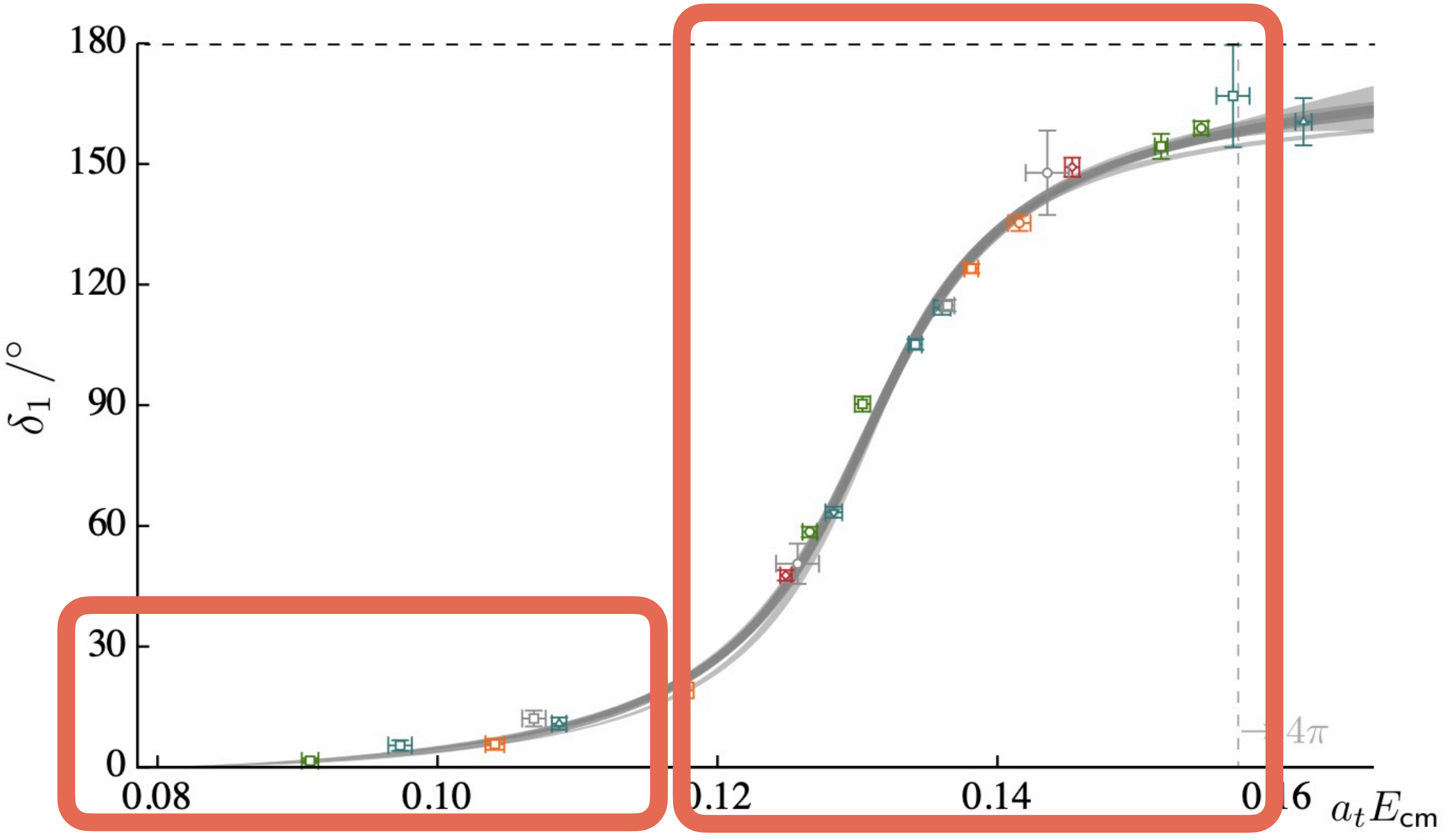
$$q(s)\cot\delta_\ell(s) = \sum_{n=0}(a_n q^{2n})$$

This one can contain a pole

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$

Pole at

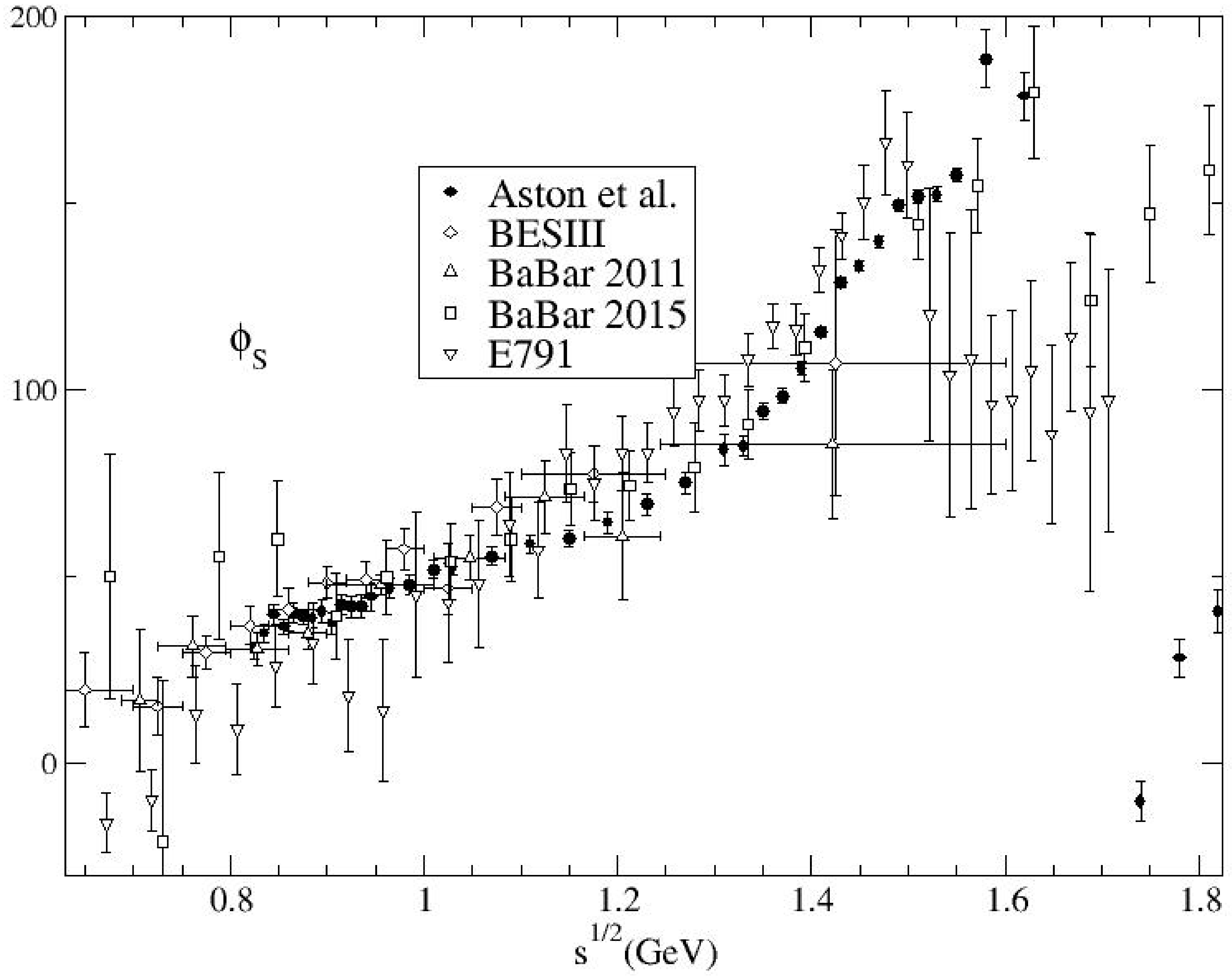
$$cot\delta_\ell(s) = i$$

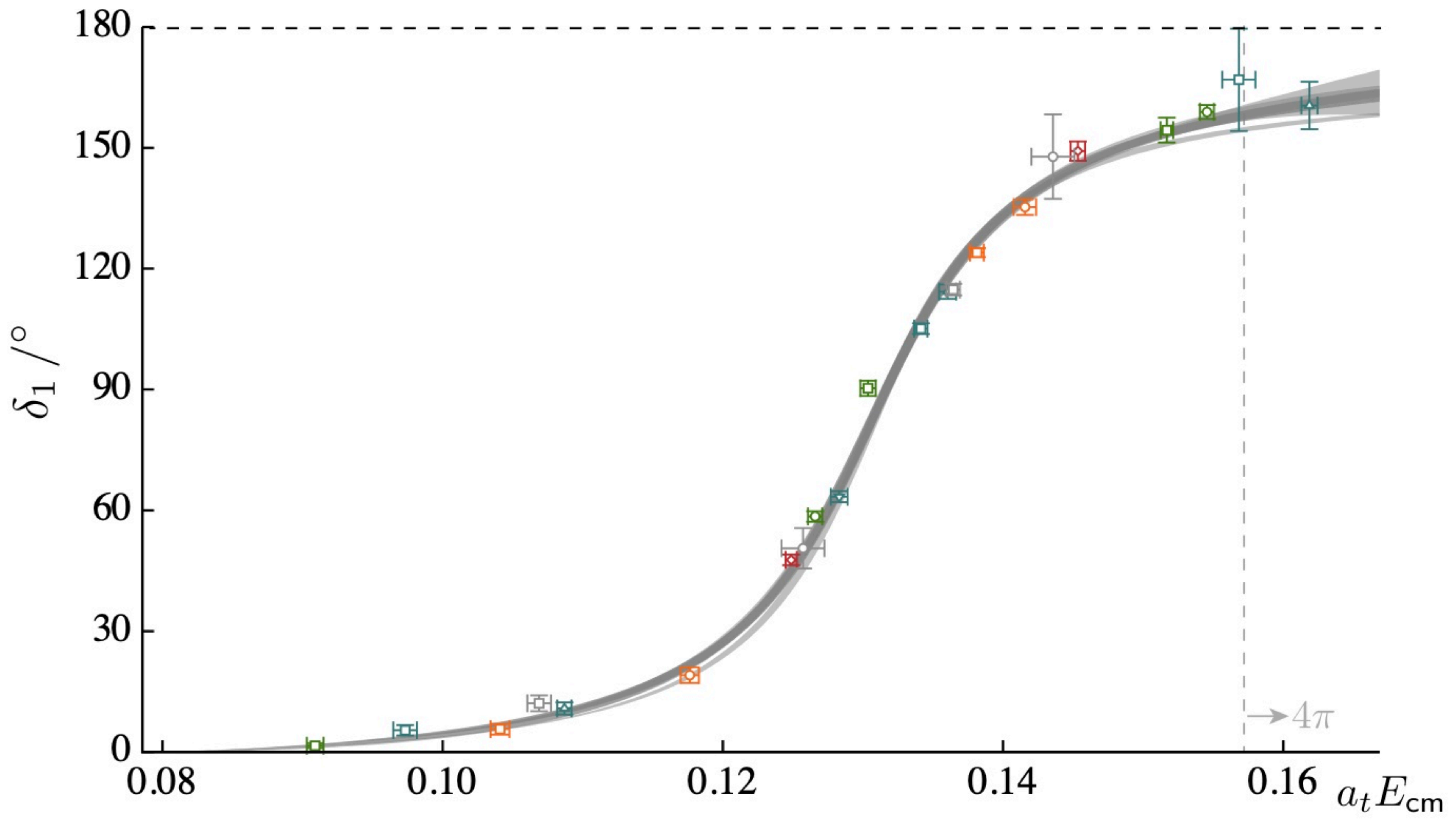


Scattering

Low energy expansion

$$q(s) \cot \delta_\ell(s) = \sum_{n=0} (a_n q^{2n})$$





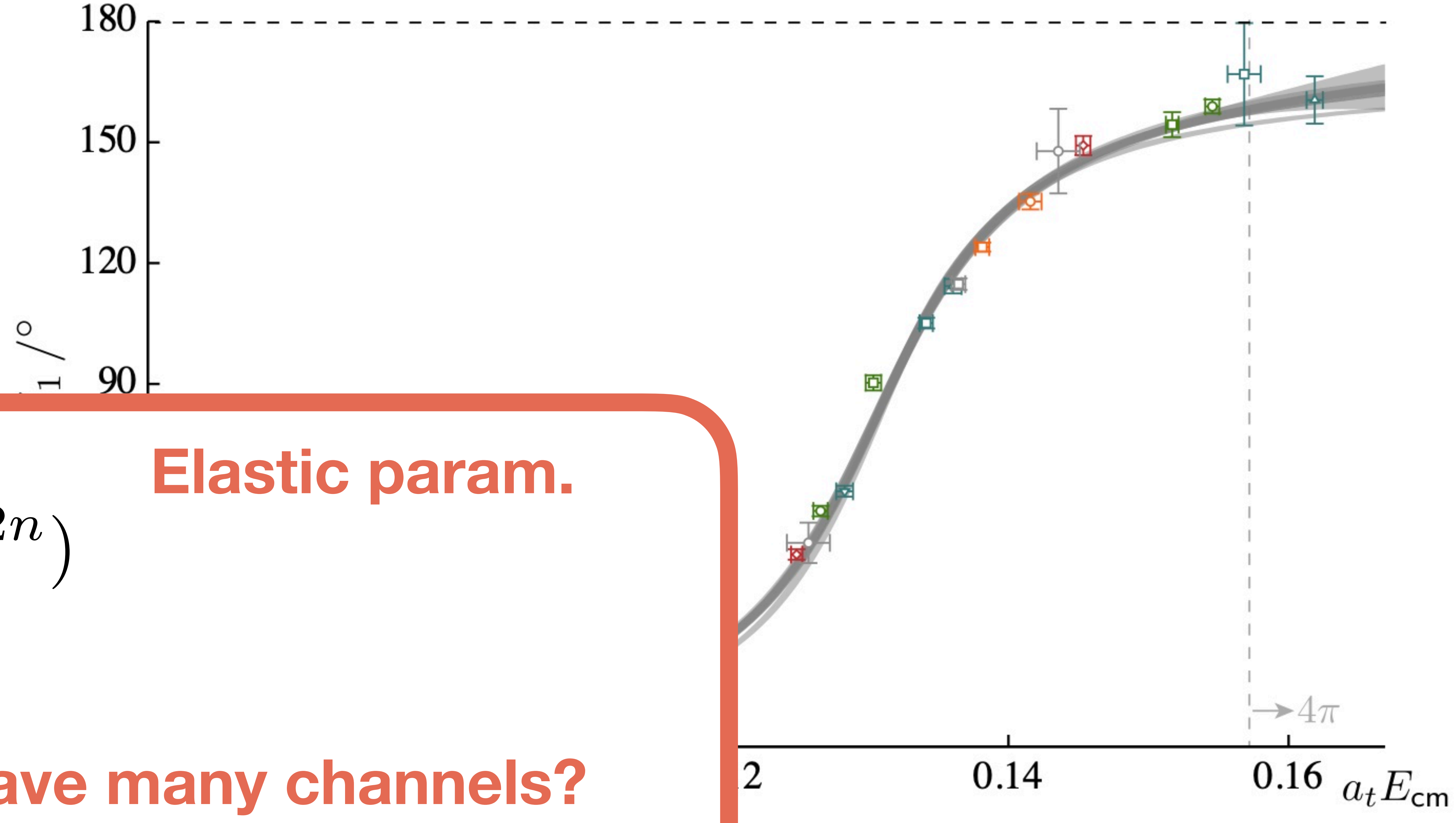
Simple, yet powerful

$$t_\ell(s) = \frac{1}{\rho(s) \cot \delta_\ell(s) - i\rho(s)}$$

Scattering

Low energy expansion

$q(s)\cot$



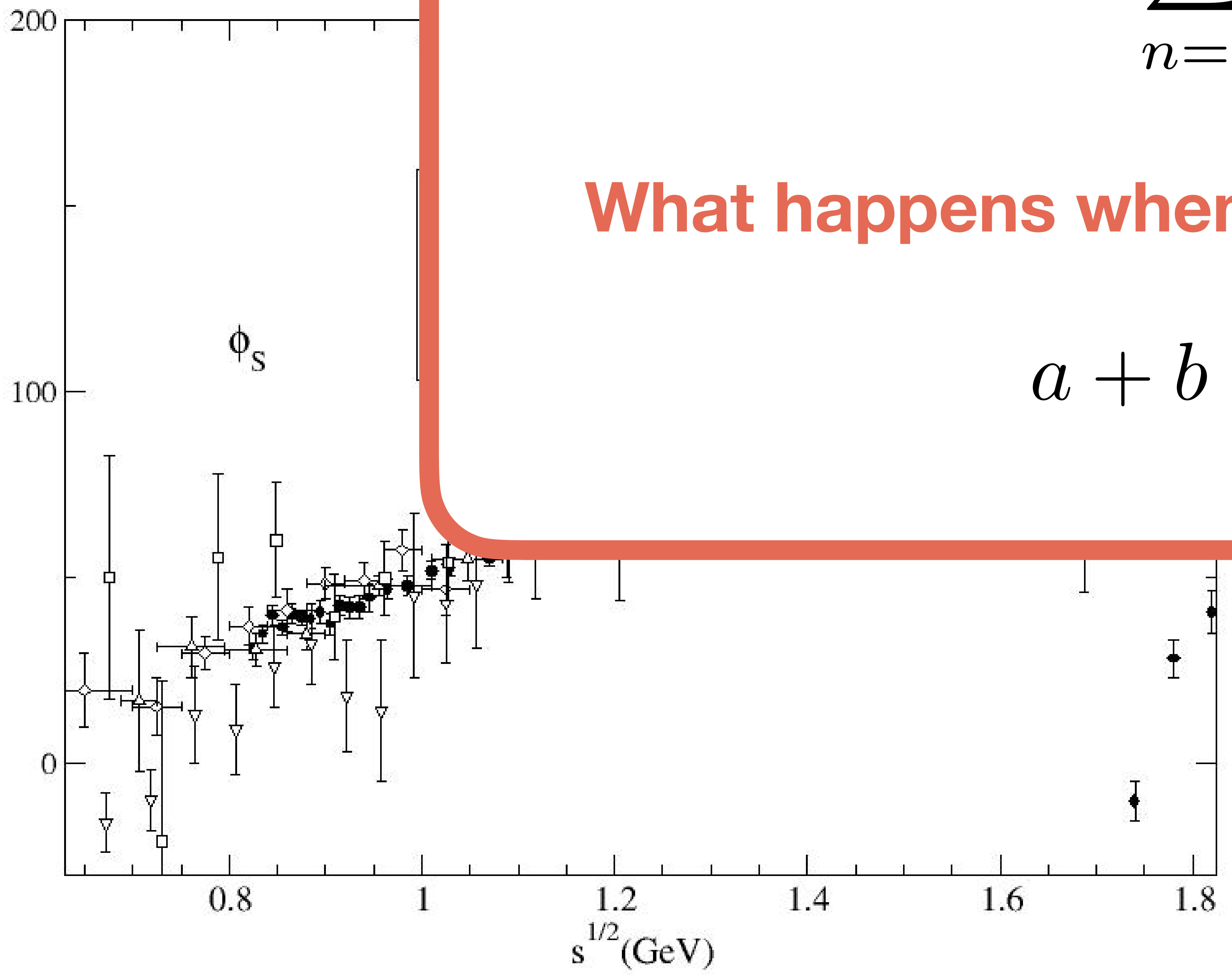
Elastic param.

$$q(s)\cot\delta_\ell(s) = \sum_{n=0}(a_n q^{2n})$$

What happens when we have many channels?

$$a + b \to \sum_i c_i$$

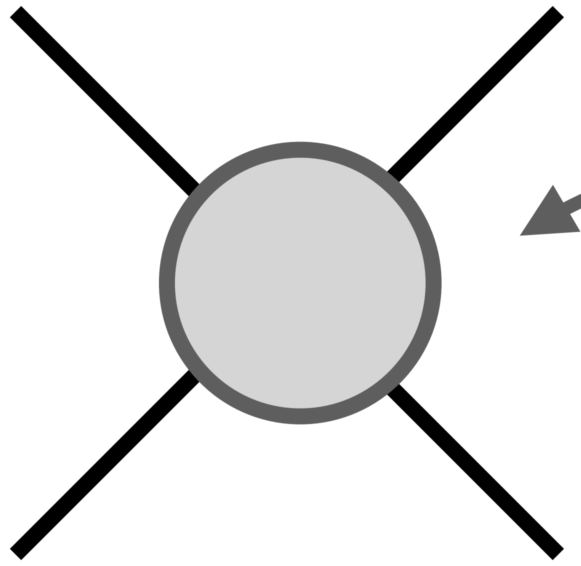
$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$



K-matrix

Effective for coupled-channel problems

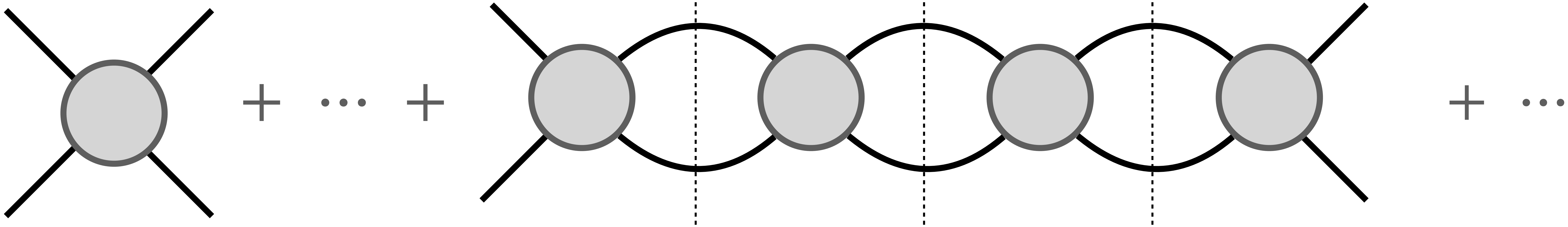
First-order approx.



$$t_\ell(s) \simeq K_\ell(s)$$

Higher orders

$$t_\ell(s) \simeq K_\ell(s) + K_\ell(s) i\rho(s) K_\ell(s) + K_\ell(s) i\rho(s) K_\ell(s) i\rho(s) K_\ell(s) + \ldots$$



Geometric series

$$t_\ell(s) = K_\ell(s)\,\left[1 - i\rho(s) K_\ell(s)\right]^{-1} \qquad t_\ell(s) = \left[K_\ell^{-1}(s) - i\rho(s)\right]^{-1}$$

K-matrix

Basic requirements

Time reversal

$$\langle a|S|b\rangle \equiv \langle b|S|a\rangle \quad \longrightarrow \quad K_\ell(s) = K_\ell^T(s)$$

Unitarity

$$\longrightarrow \quad K_\ell(s) = K_\ell^\dagger(s)$$

$$S_\ell(s) S_\ell^\dagger(s) = 1$$

$$S_\ell(s) = \frac{1 + i\rho(s)K_\ell(s)}{1 - i\rho(s)K_\ell(s)}$$

K-matrix

Combining the two together

$$\left.\begin{aligned} K_\ell(s) &= K_\ell^T(s) \\ K_\ell(s) &= K_\ell^\dagger(s) \end{aligned}\right\}$$

In the elastic region we can recover

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)} \longrightarrow \boxed{K_\ell^{-1}(s) = \rho(s)\cot\delta_\ell(s)}$$

K-matrix

Combining the two together

$$\left.\begin{array}{l} K_\ell(s) = K_\ell^T(s) \\ \\ K_\ell(s) = K_\ell^\dagger(s) \end{array}\right\} \quad K_\ell(s) = K_\ell^*(s)$$

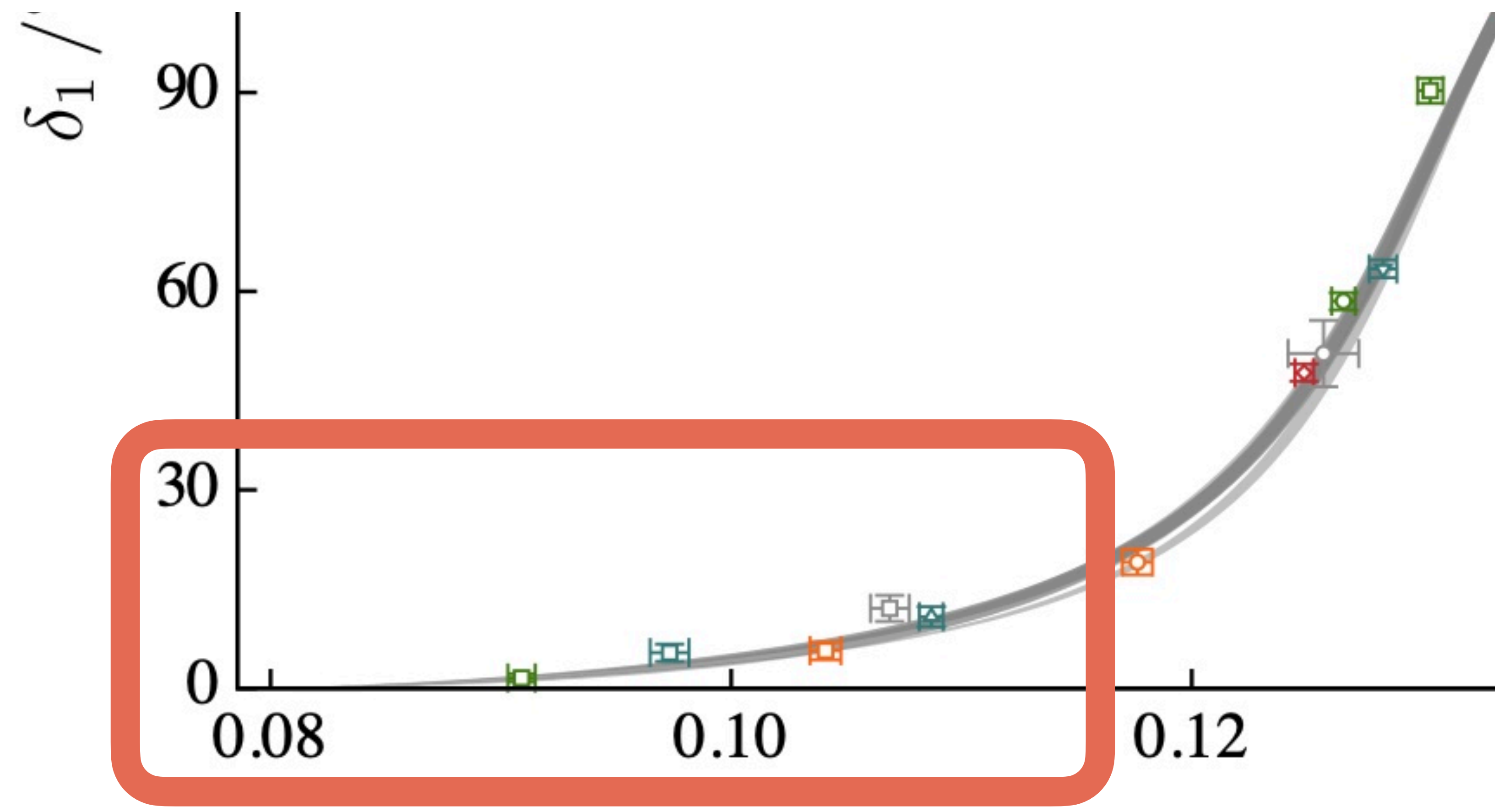
The K-matrix is real !!

In the elastic region we can recover

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)} \longrightarrow \boxed{K_\ell^{-1}(s) = \rho(s)\cot\delta_\ell(s)}$$

K-matrix

$$K_\ell(s) = \rho(s) \cot \delta_\ell(s)$$

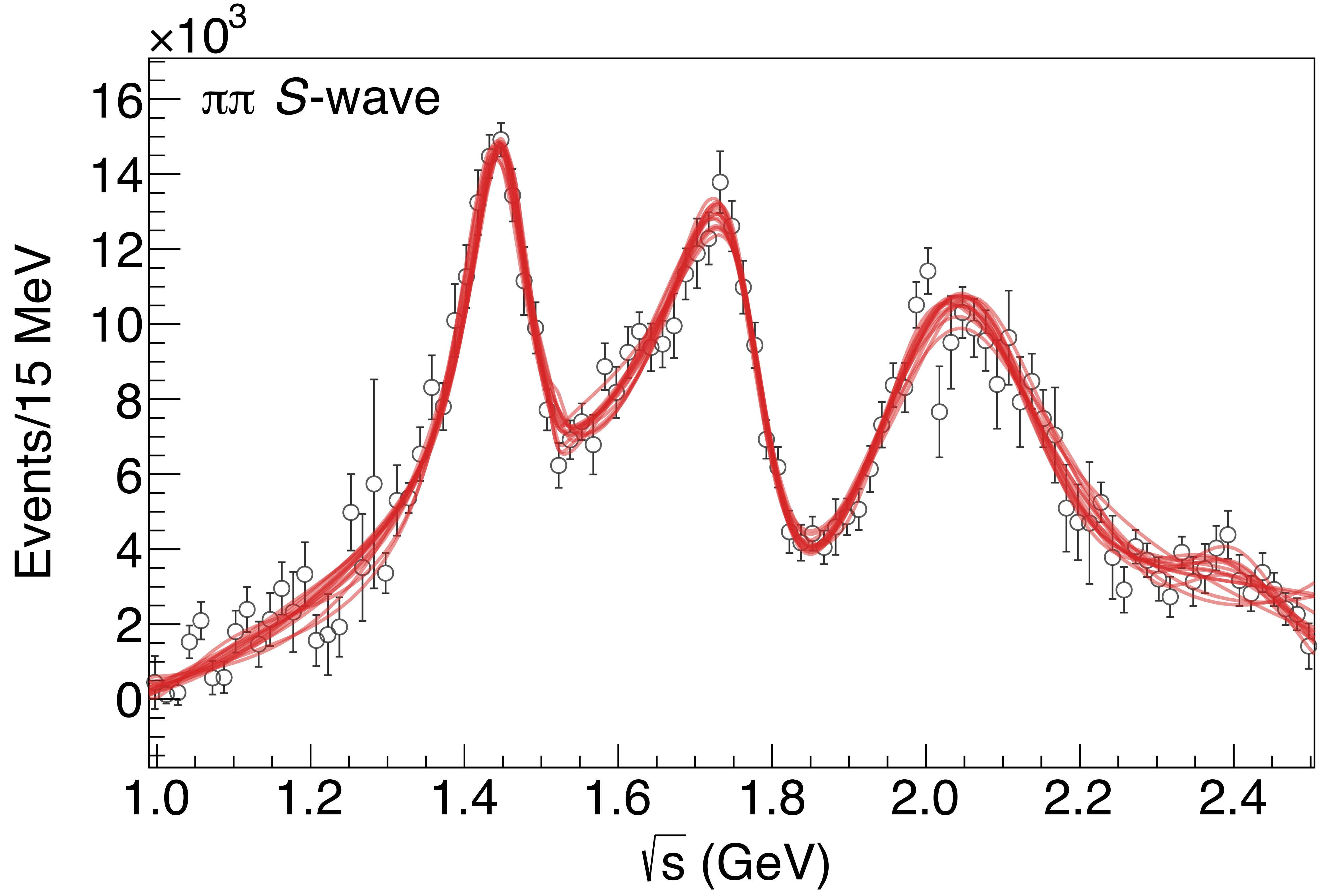


Toolkit to amplitude fitting

Good for easy data $$K_\ell(s) = \sum_{n=0} (a_n q^{2n})$$

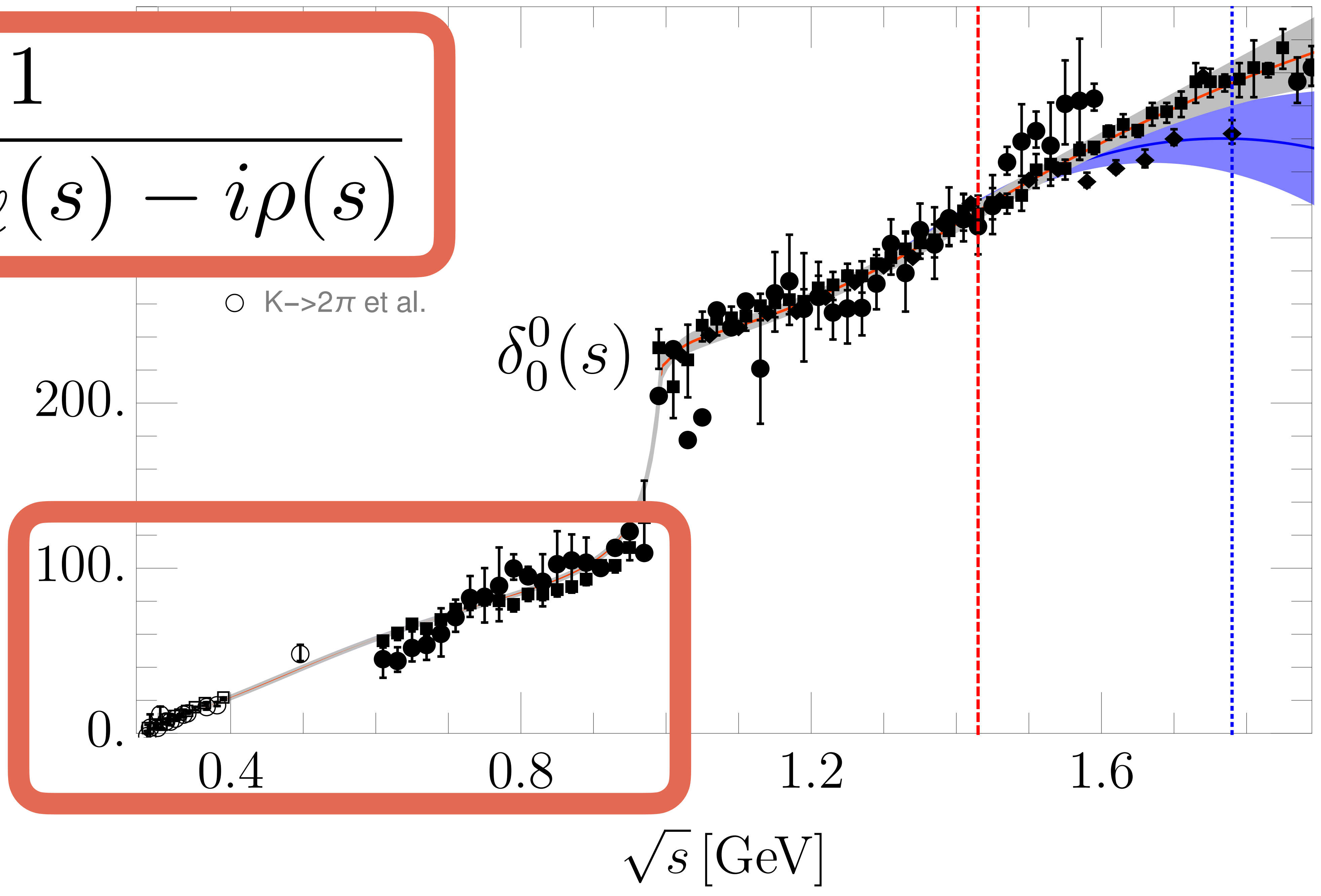
Good for overlapping resonances

$$K_\ell^{i\,j}(s) = \sum_{n=0} \frac{g_n^i g_n^j}{m_n^2 - s} + P^{i\,j}(s)$$



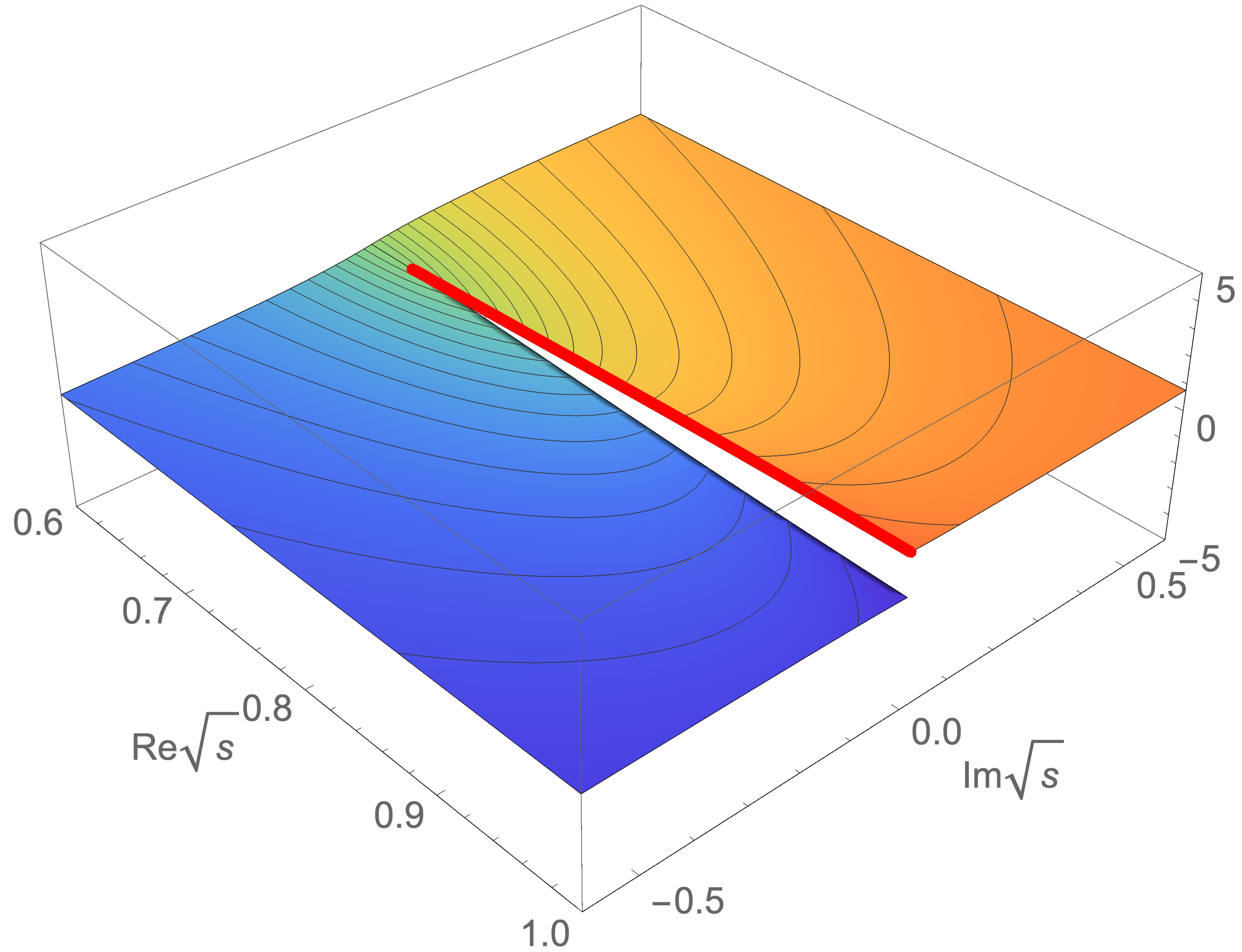
Scattering

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$



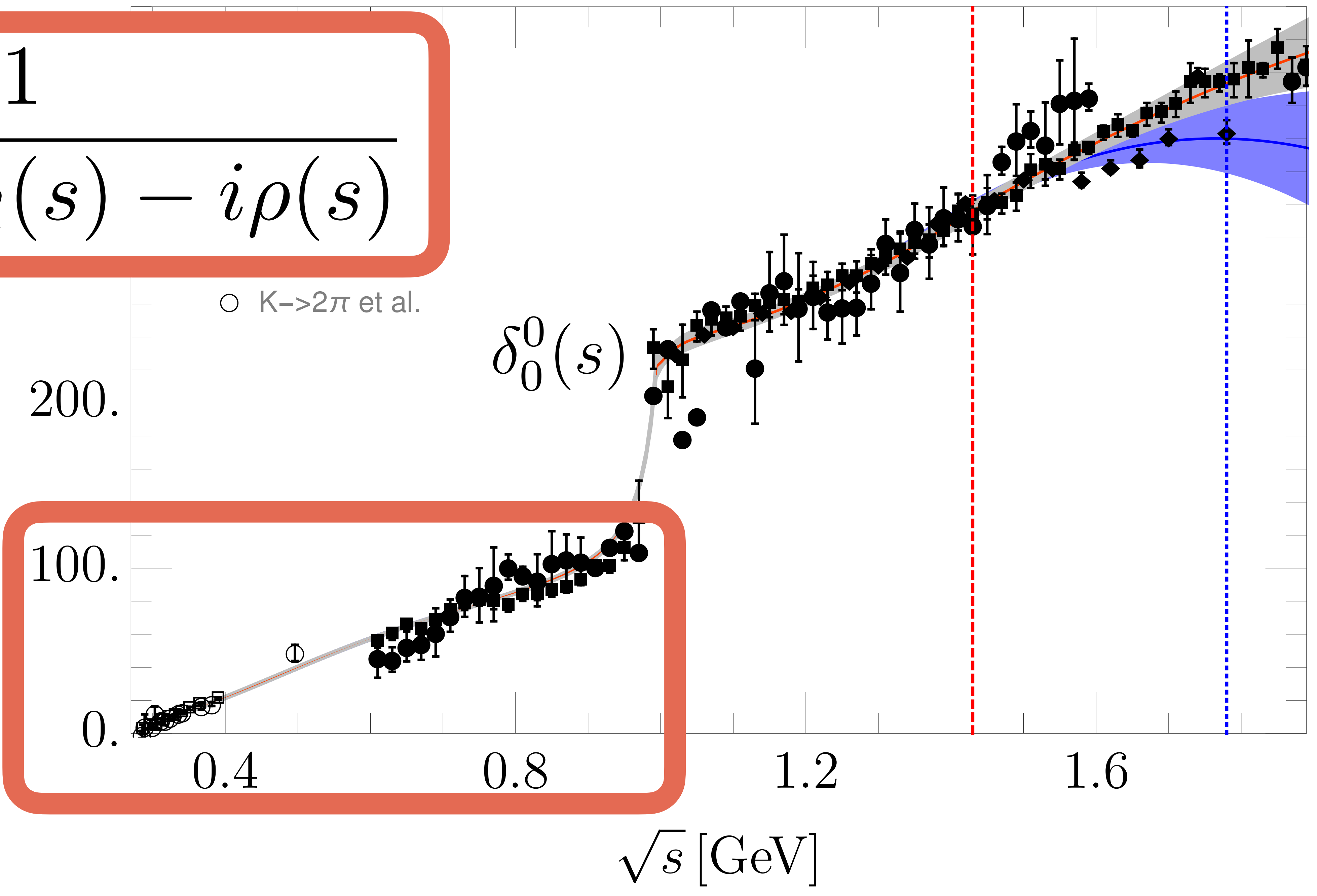
My fits

I sheet clear of poles



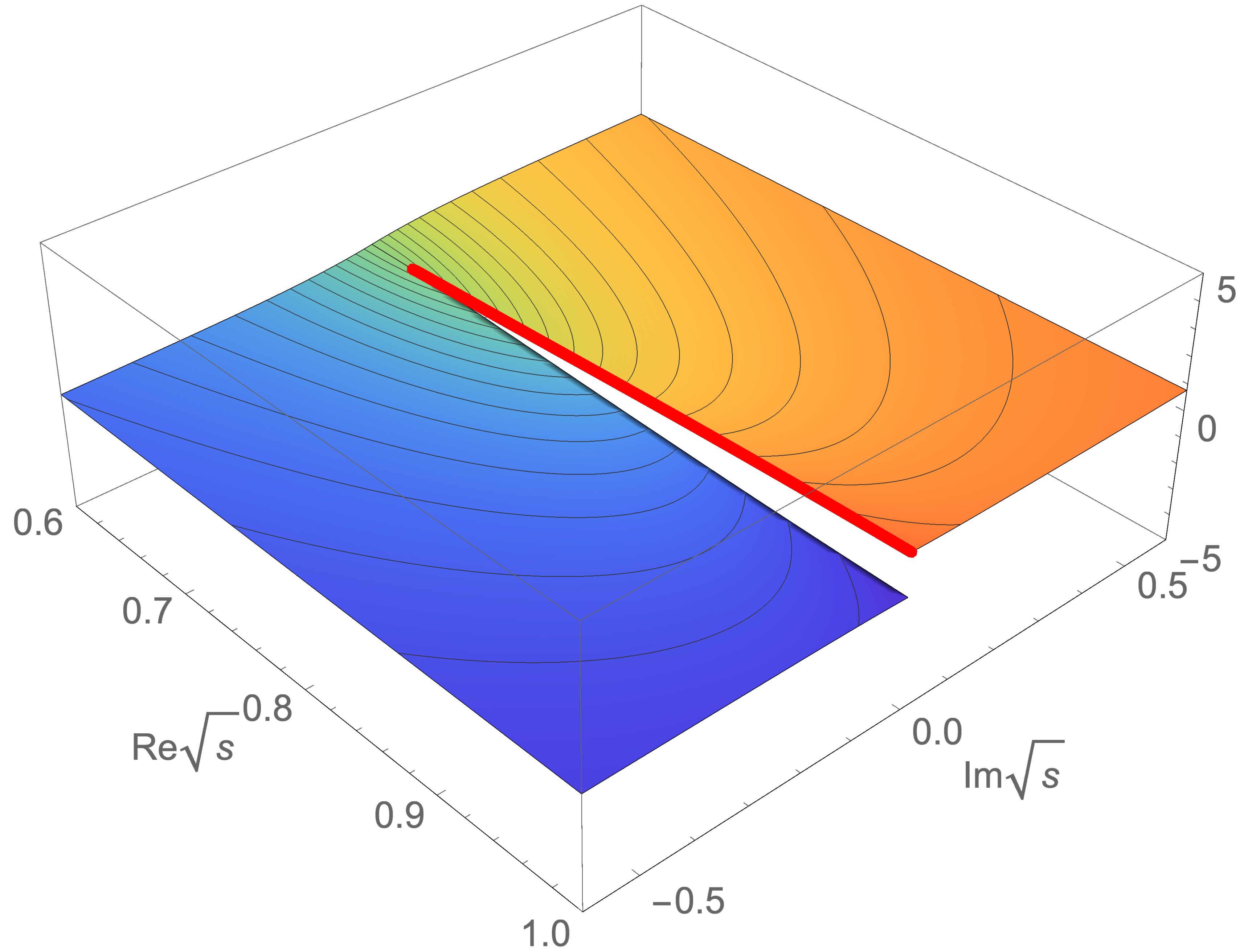
Scattering

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$



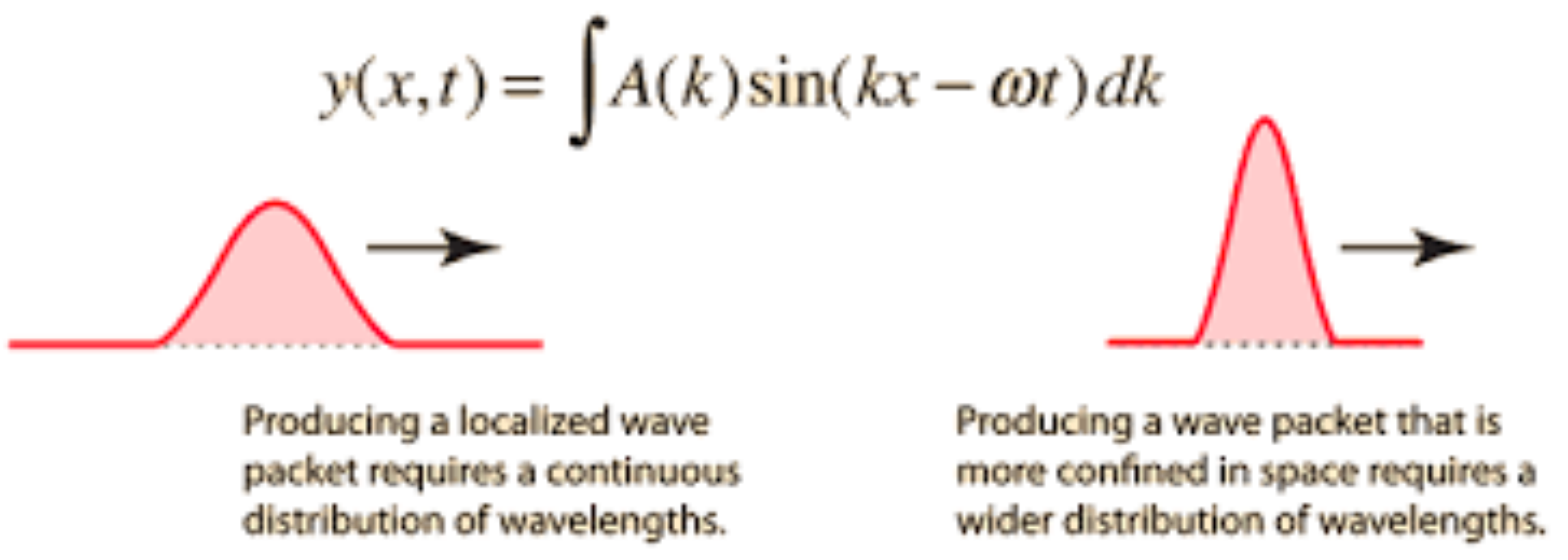
My fits

I sheet clear of poles



Analiticity

Scattering



Incoming wavepacket

$$\Phi_{in}(r,t) \equiv -\int_0^\infty dE\, A(E) e^{-ikr-iEt}$$

Scattered wavepacket

Remember?

$$\Phi_{sc}(r,t) = \int_0^\infty dE\, A(E)[S(E)-1]e^{ikr-iEt} = 2\pi \int_0^\infty dE\, A(E) e^{-ikr-iEt} G(r,E)$$

Redefinition

$$\underbrace{\Phi_{sc}(r,t)}_{Effect} = \int_{-\infty}^{\infty} dt' g(r,t-t')\, \underbrace{\Phi_{in}(r,t')}_{Cause}$$

Scattering

THE THEORY MUST BE CAUSAL !!

$$\underbrace{\Phi_{sc}(r,t)}_{Effect} = \int_{-\infty}^{\infty} dt' g(r, t-t')\, \underbrace{\Phi_{in}(r,t')}_{Cause}$$

Scattering

THE THEORY MUST BE CAUSAL !!

$$\underbrace{\Phi_{sc}(r,t)}_{Effect} = \int_{-\infty}^{\infty} dt' g(r, t-t') \underbrace{\Phi_{in}(r,t')}_{Cause}$$

So that

$$g(\tau) = 0$$
$$\tau = t - t' < 0$$

$$G(r,E) = \frac{1}{2\pi}\int_0^{\infty} d\tau g(r,\tau) e^{iE\tau}$$

Scattering

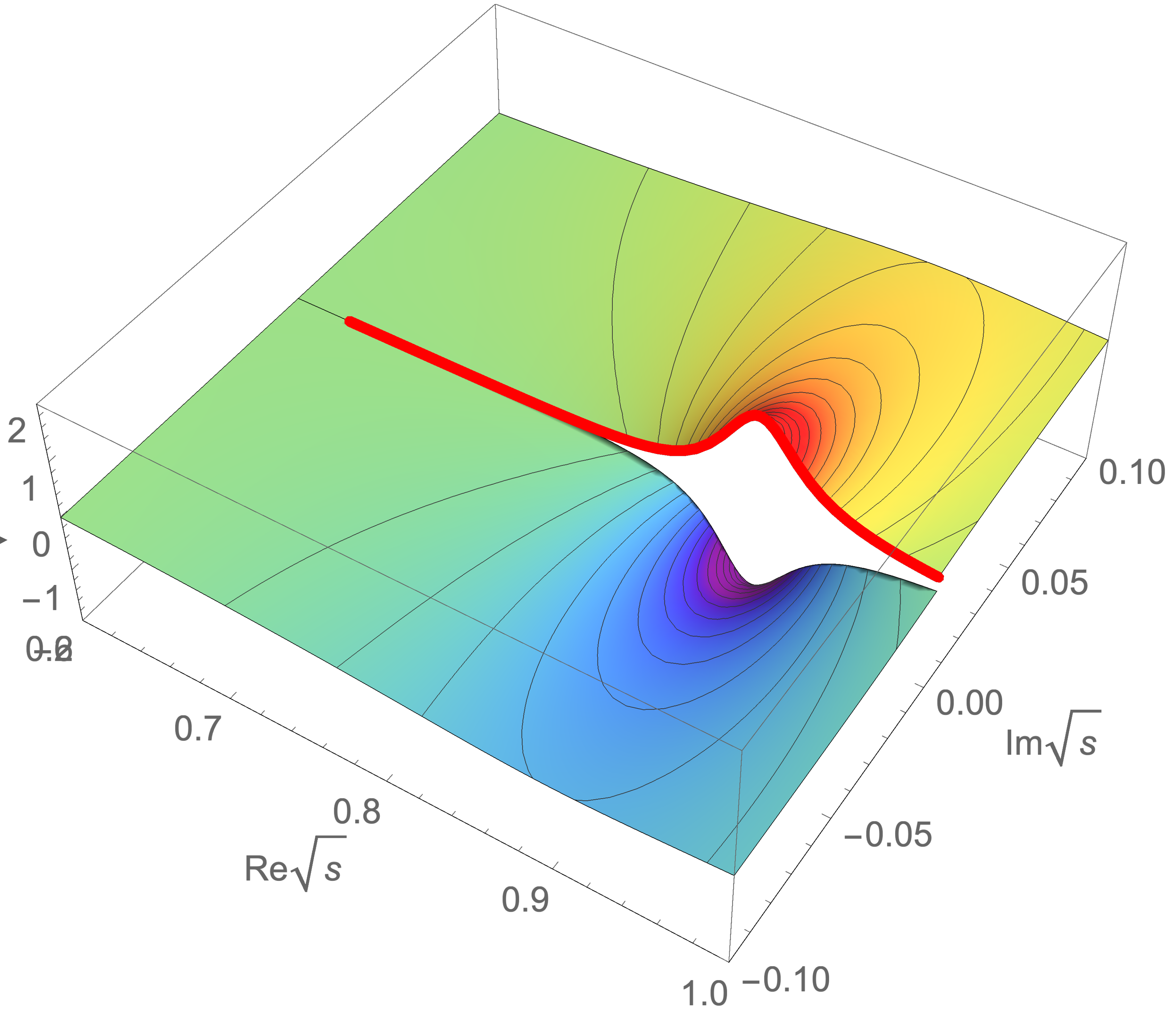
$$\Phi_{sc}(r,t) = 2\pi \int_0^\infty dE\, A(E) e^{-ikr-iEt} G(r,E)$$

For E in $\mathbb{C}^+$ → E^2 in $\mathbb{C}$ → G is well behaved

$$G(r,E) = \frac{1}{2\pi}\int_0^\infty d\tau g(r,\tau) e^{iE\tau}$$

$$e^{(i\phi)^2} = e^{2(i\phi)}$$

No poles in I sheet



But for bound states $\Psi(t) = \Psi(0)e^{-iMt}$

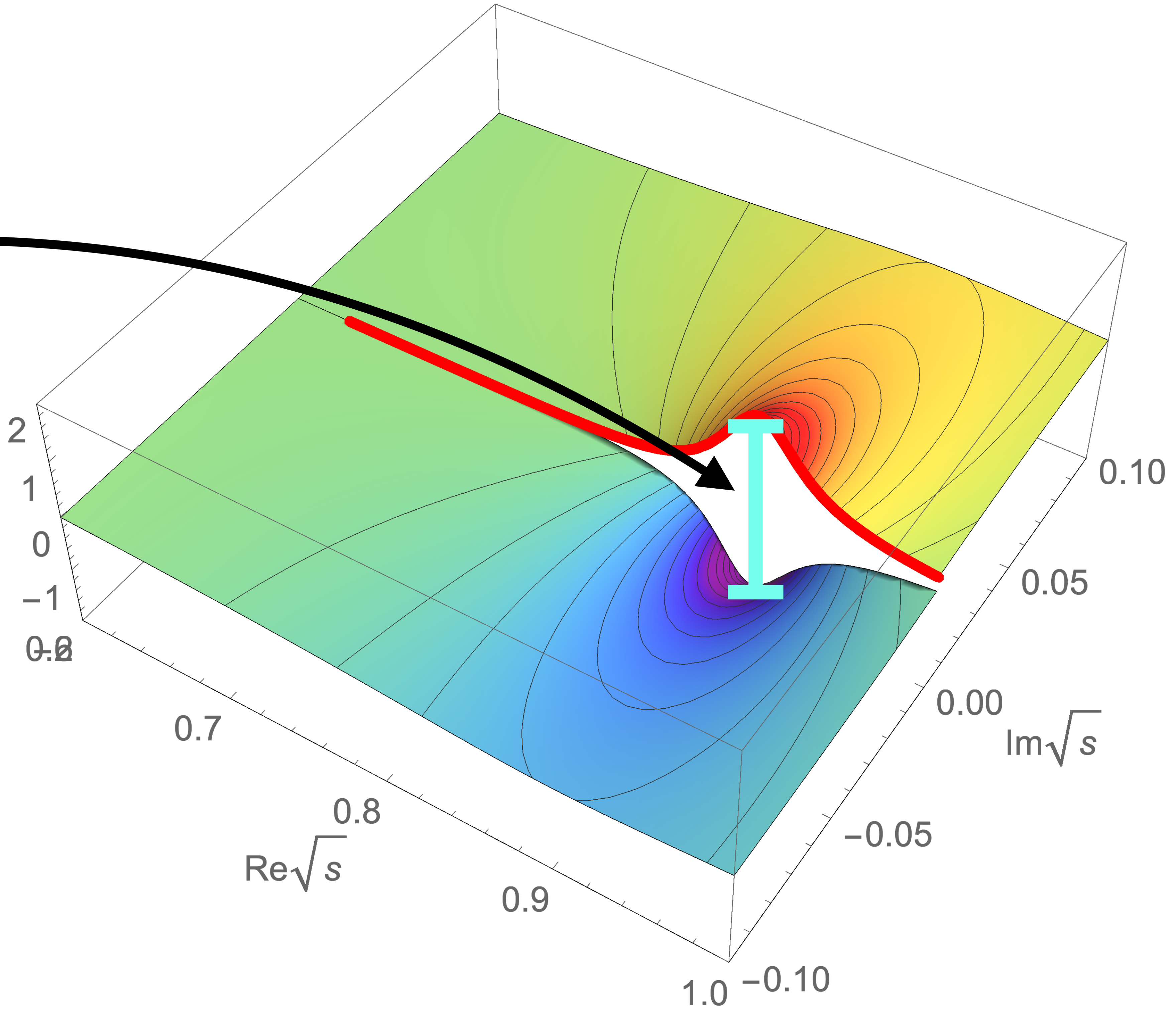
Schwarz reflection principle

In mathematics, the **Schwarz reflection principle** is a way to extend the domain of definition of a complex analytic function, i.e., it is a form of analytic continuation. It states that if an analytic function is defined on the upper half-plane, and has well-defined (non-singular) real values on the real axis, then it can be extended to the conjugate function on the lower half-plane. In notation, if $F(z)$ is a function that satisfies the above requirements, then its extension to the rest of the complex plane is given by the formula,

$$S_\ell(s^*) = S_\ell^*(s) \qquad t_\ell(s^*) = t_\ell^*(s)$$

Above - below the real axis

$$\begin{cases} t_\ell(s) - t_\ell(s^*) = t_\ell(s) - t_\ell^*(s) \\ t_\ell(s) - t_\ell^*(s) \equiv 2i\,\mathrm{Im}\,t_\ell(s) \end{cases}$$



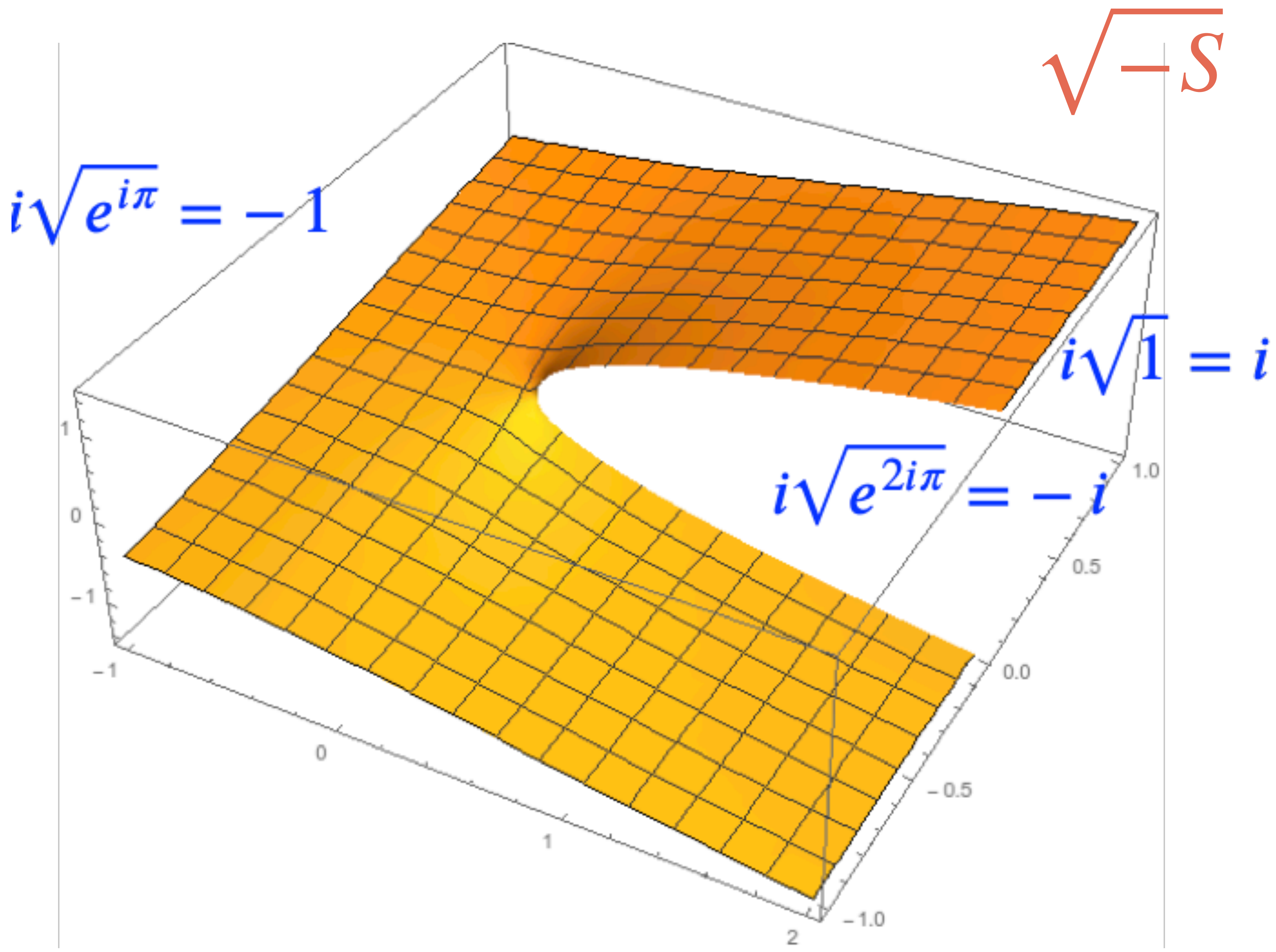
Scattering

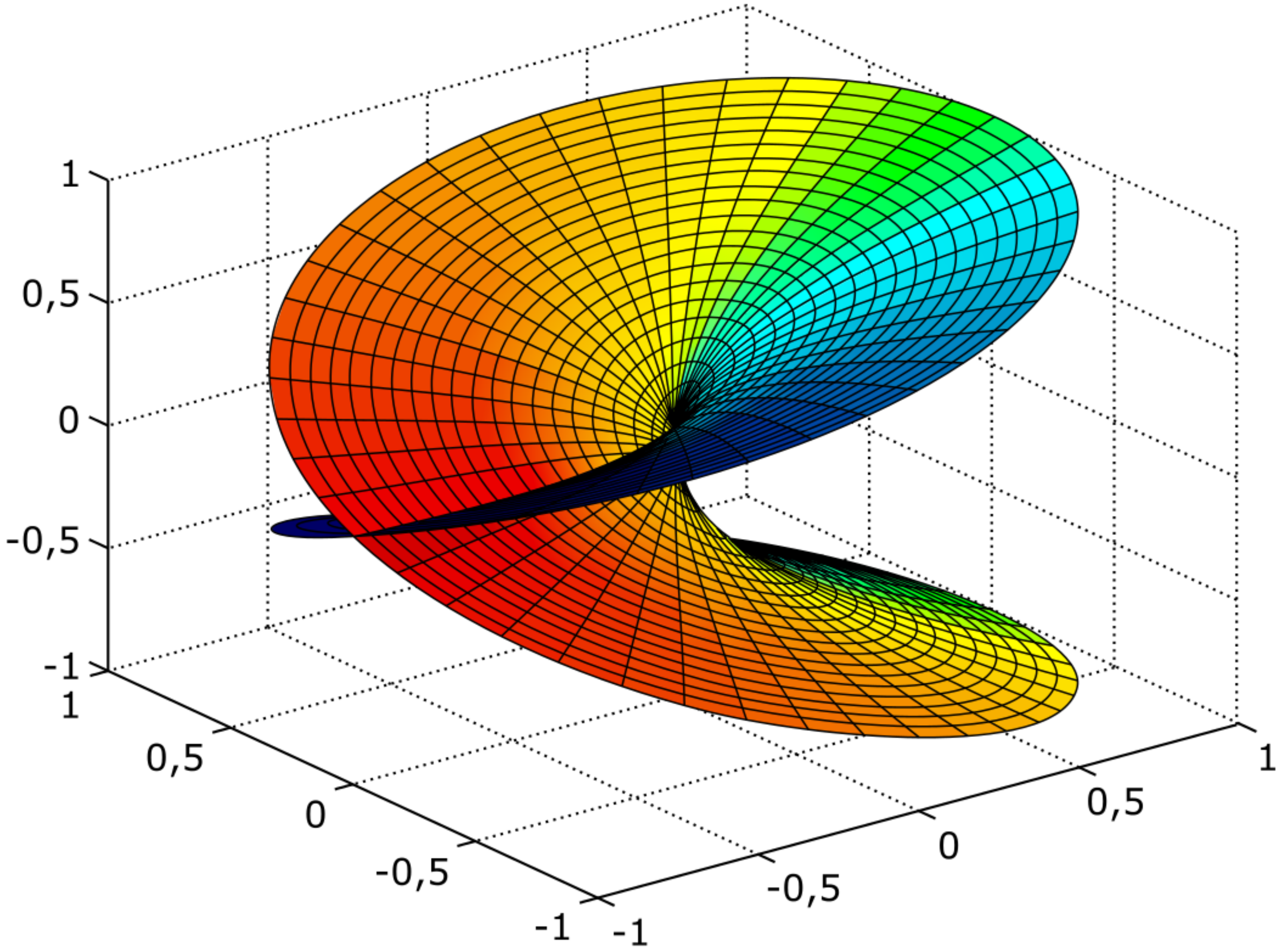
Unitarity $T - T^\dagger = iTT^\dagger$

Complex function with $Im\, f(Re(s)) \neq 0$ → Branch cut

$$\mathrm{Im}\, T_{i\,i}(s, t=0) \propto \sum_n T_{i\,n} T^\dagger_{n\,i} \propto \sigma^i_{\mathrm{tot}}$$

Multi-valued function

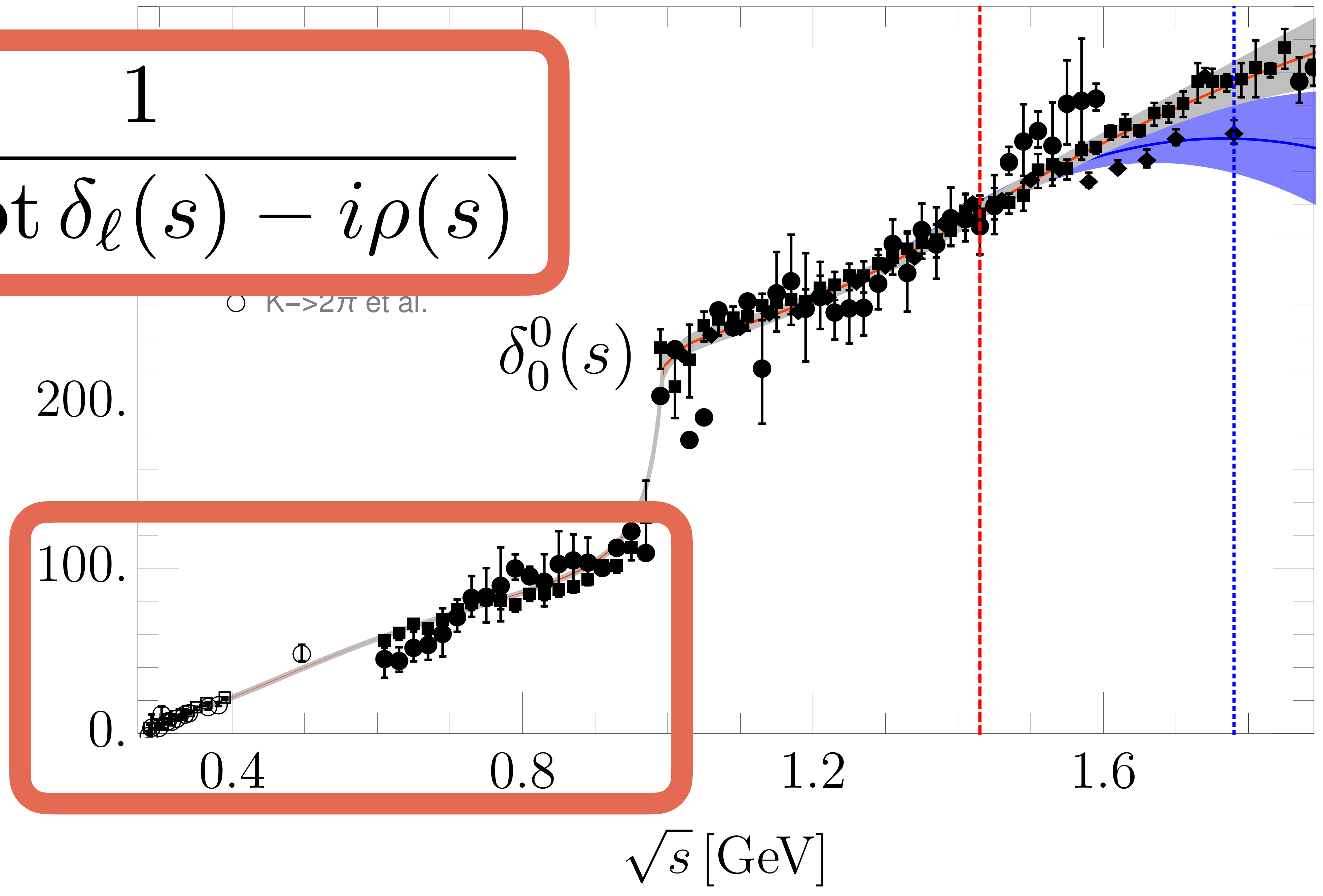




Scattering

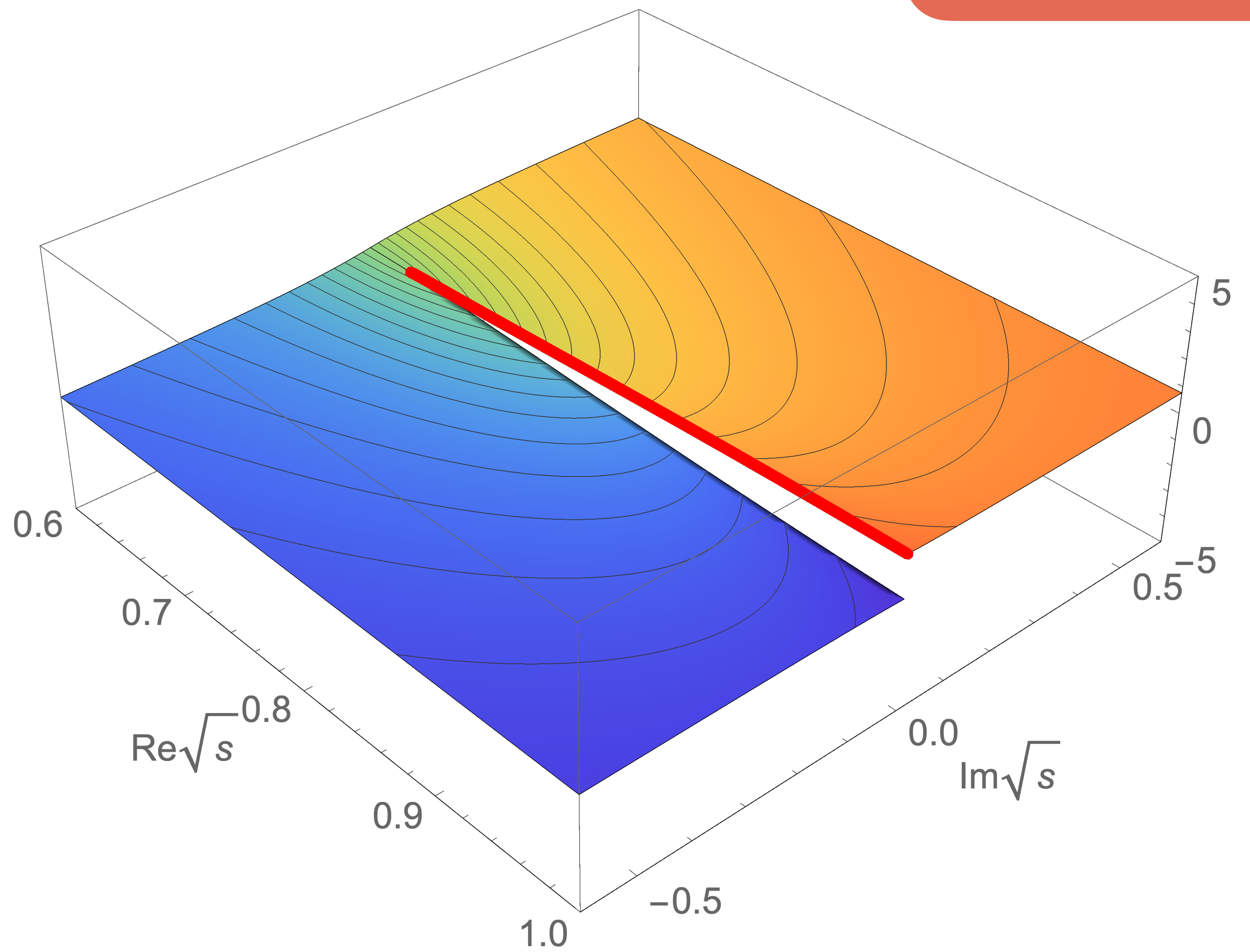
My fits

$$t_\ell(s) = \frac{1}{\rho(s)\cot\delta_\ell(s) - i\rho(s)}$$



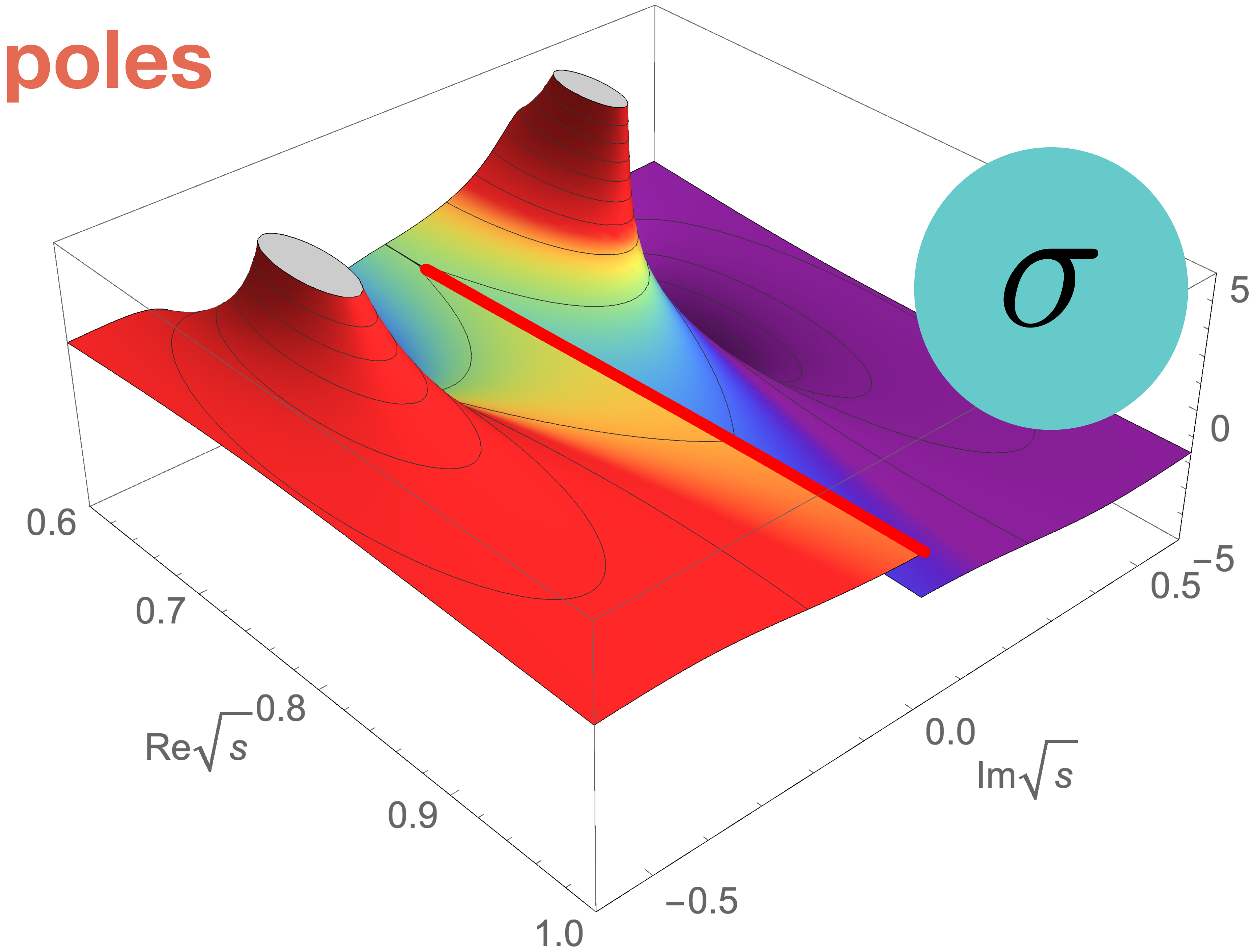
I sheet clear of poles

Analiticity



II sheet poles

Unitarity



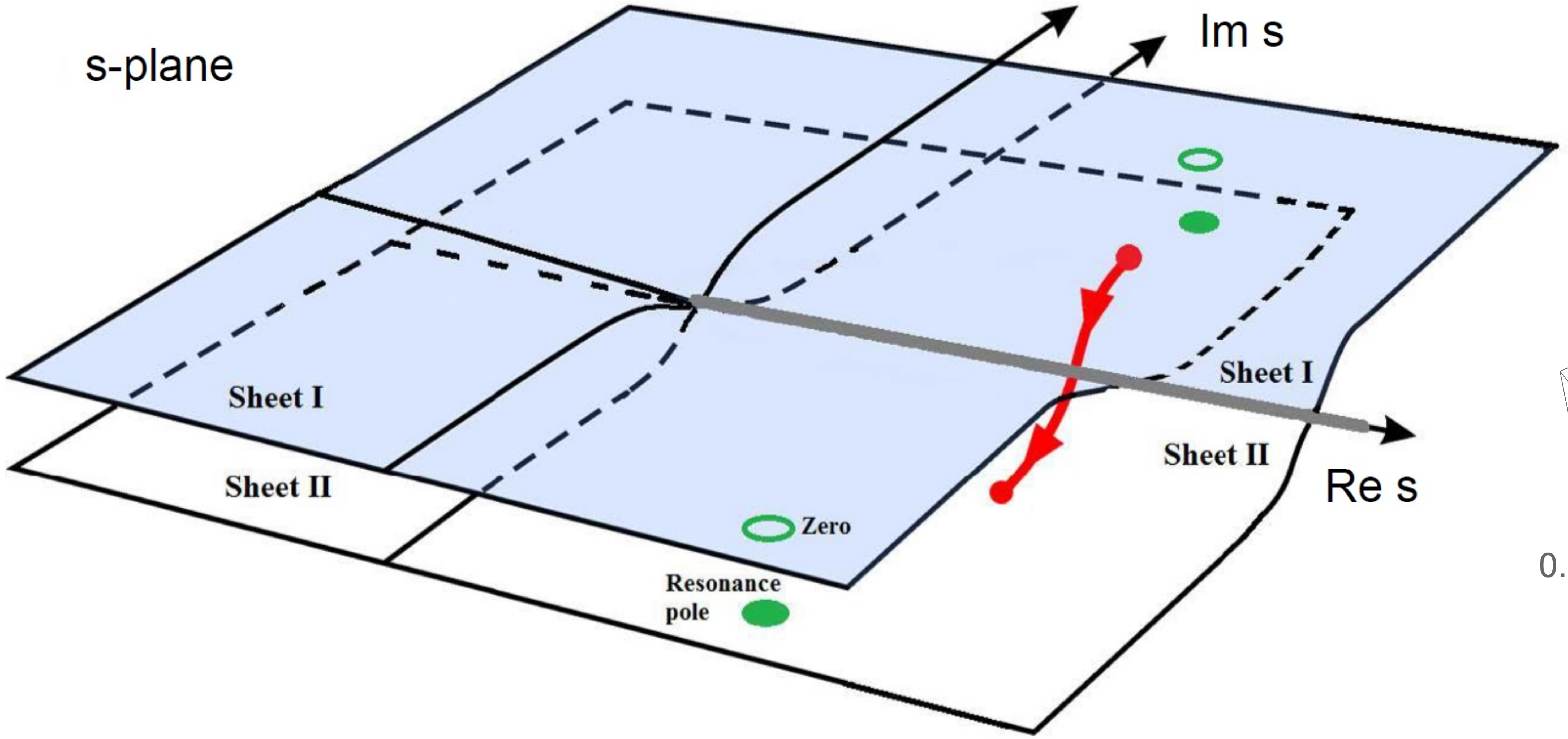
Scattering

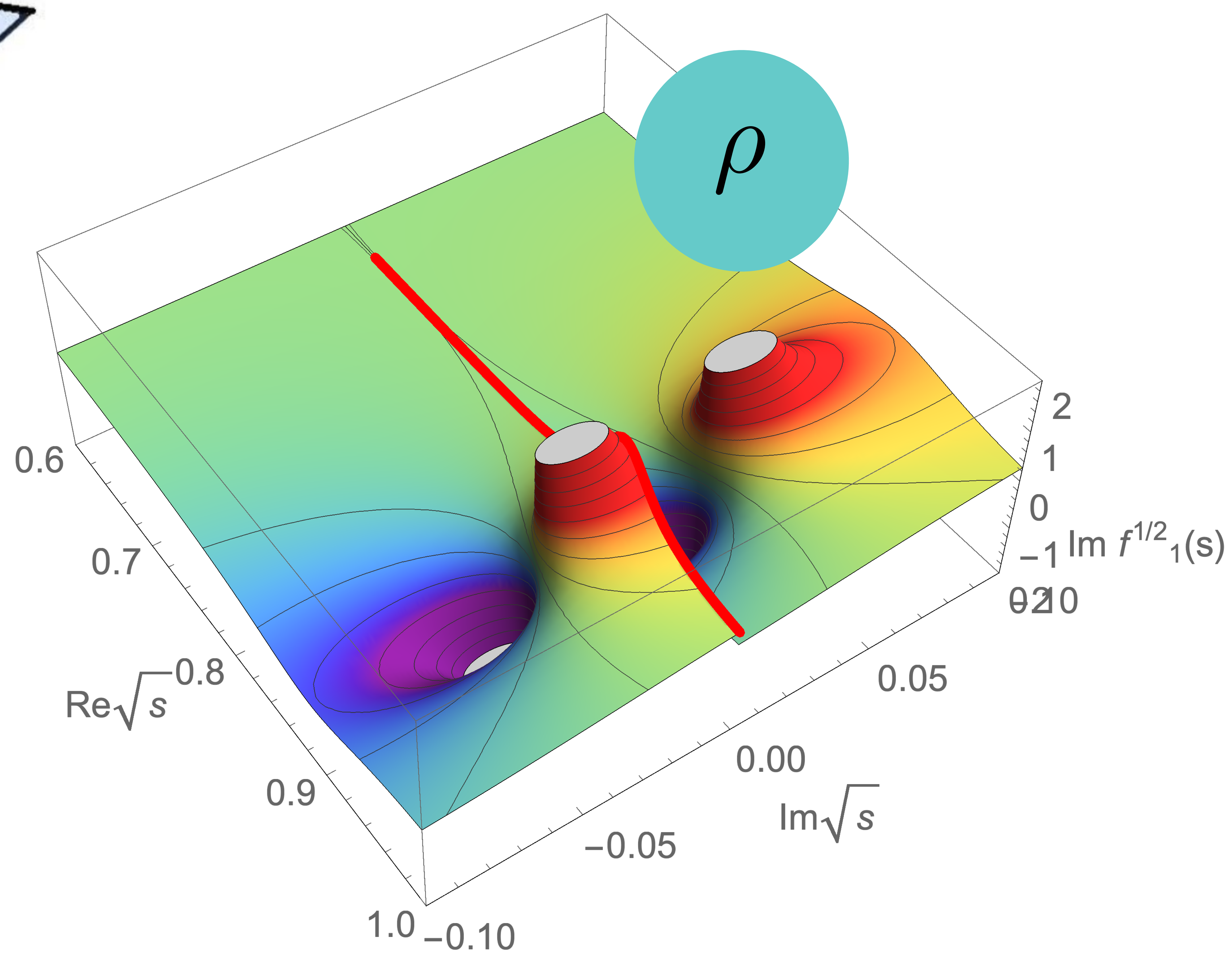
Pole in II sheet ↔ 0 in I sheet

Unitarity

Exercise

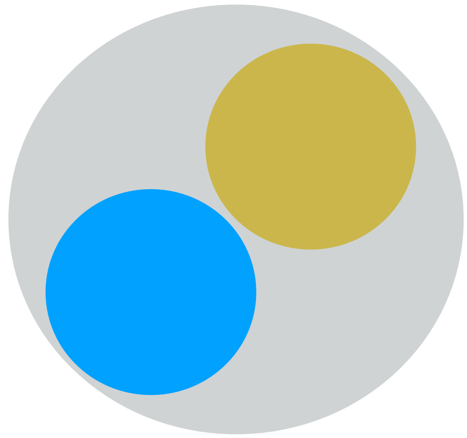
$$S_\ell^{II}(s) = \frac{1}{S_\ell^{I}(s)}$$



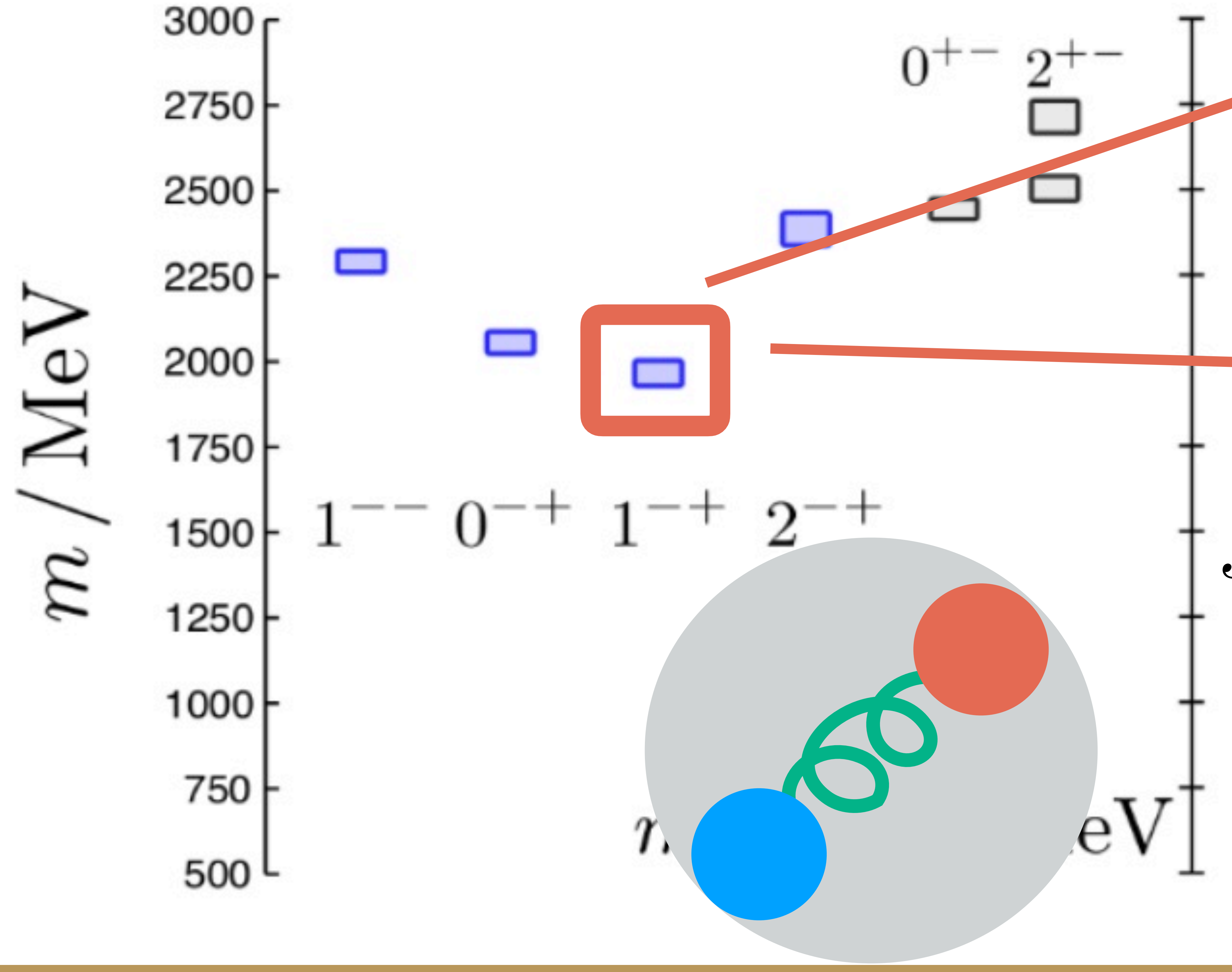


Questions??

Hybrid



	J^{PC} singlet	J^{PC} triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$



$$J^{PC}(\pi_1) = 1^{-+}$$

• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^{\pm}$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^{\pm}$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^{\pm}$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^{\pm}$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^{\pm}$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^{\pm}$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^{\pm}$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^{\pm}$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^{\pm}$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^{\pm}$	$0(0^+)$
• $D_{s1}(2460)^{\pm}$	$0(1^+)$
• $D_{s1}(2536)^{\pm}$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^{\pm}$	$0(1^-)$
$D_{s1}^*(2860)^{\pm}$	$0(1^-)$
$D_{s3}^*(2860)^{\pm}$	$0(3^-)$
$D_{sJ}(3040)^{\pm}$	$0(?^?)$

BOTTOM ($B = \pm1$)

	$I(J^P)$
• $B^{\pm}$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^{\pm}/B^0$ ADMIXTURE	
• $B^{\pm}/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

	$I(J^P)$
• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^{\pm}$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

	$I(J^P)$
• B_c^+	$0(0^-)$
$B_c(2S)^{\pm}$	$0(0^-)$

$c\bar{c}$ (+ possibly non-$q\bar{q}$ states)

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$

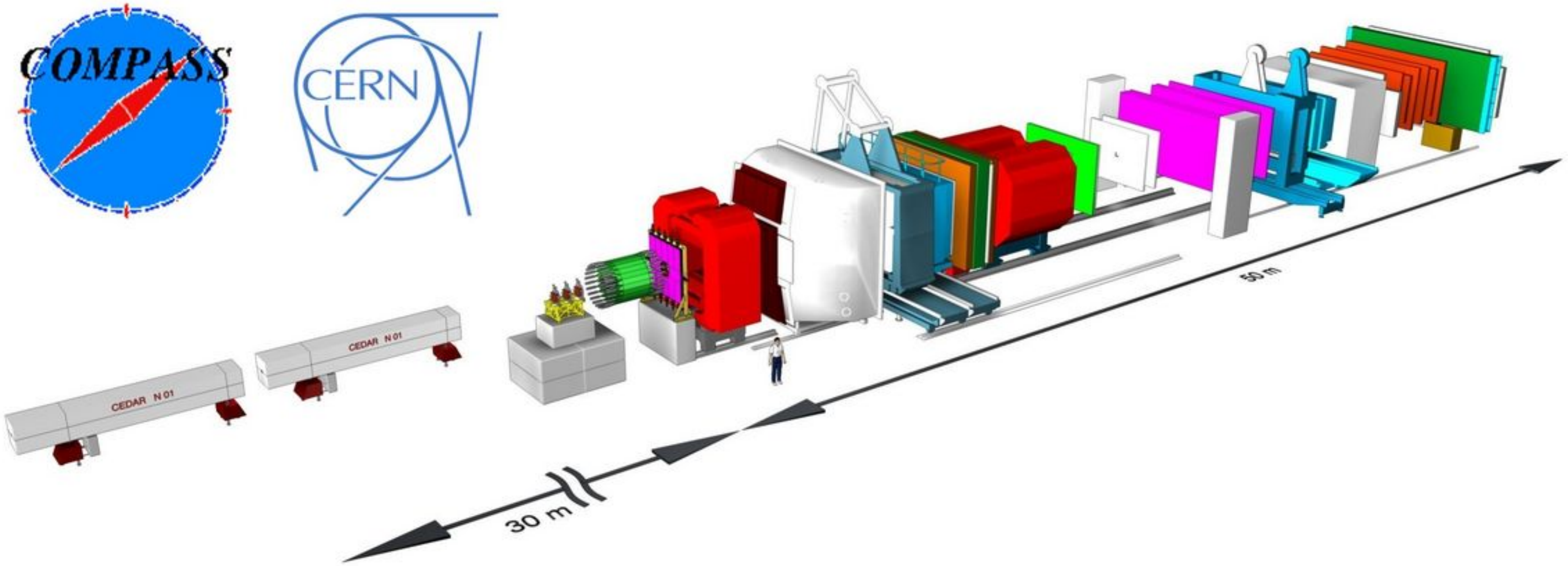
$c\bar{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^{\pm}$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^{\pm}$	$1^-(?^{?+})$
$X(4055)^{\pm}$	$1^+(?^{?-})$
$X(4100)^{\pm}$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^{\pm}$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\bar{b}$ (+ possibly non-$q\bar{q}$ states)

	$I^G(J^{PC})$
• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

J^{PC} dictates final states



$$J^{PC}(\pi_1) = 1^{-+}$$

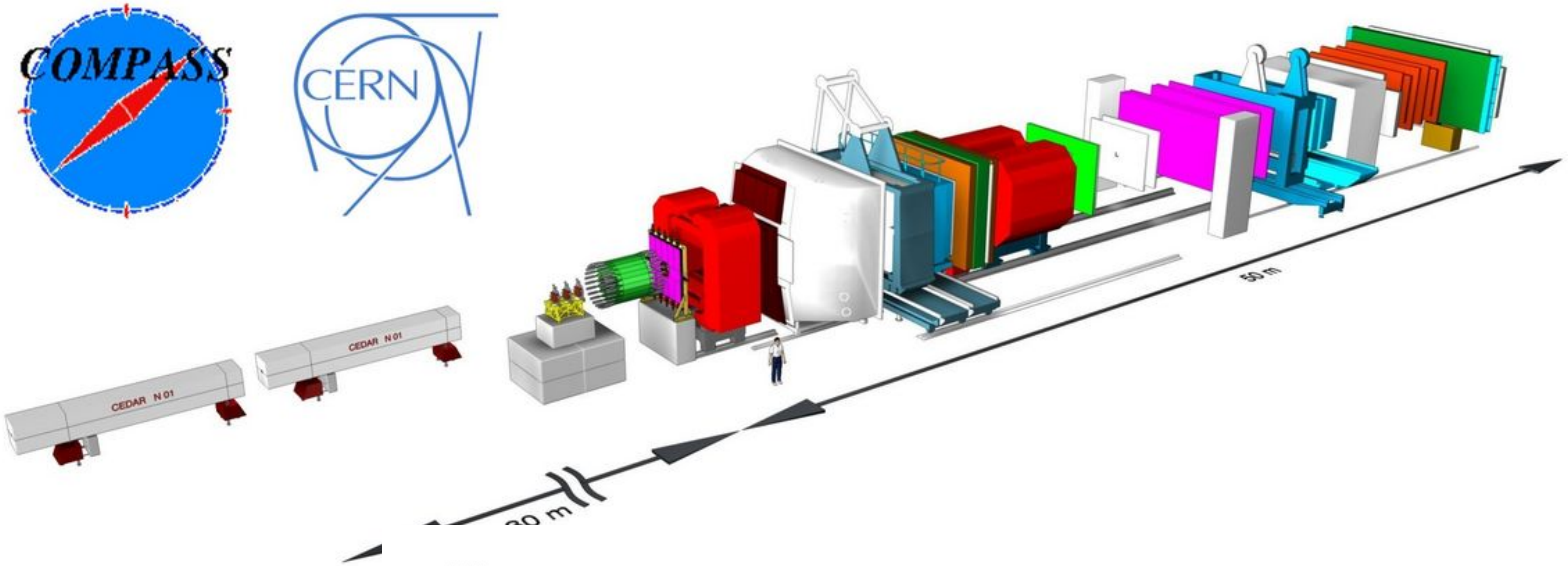
Looking at lightest final states

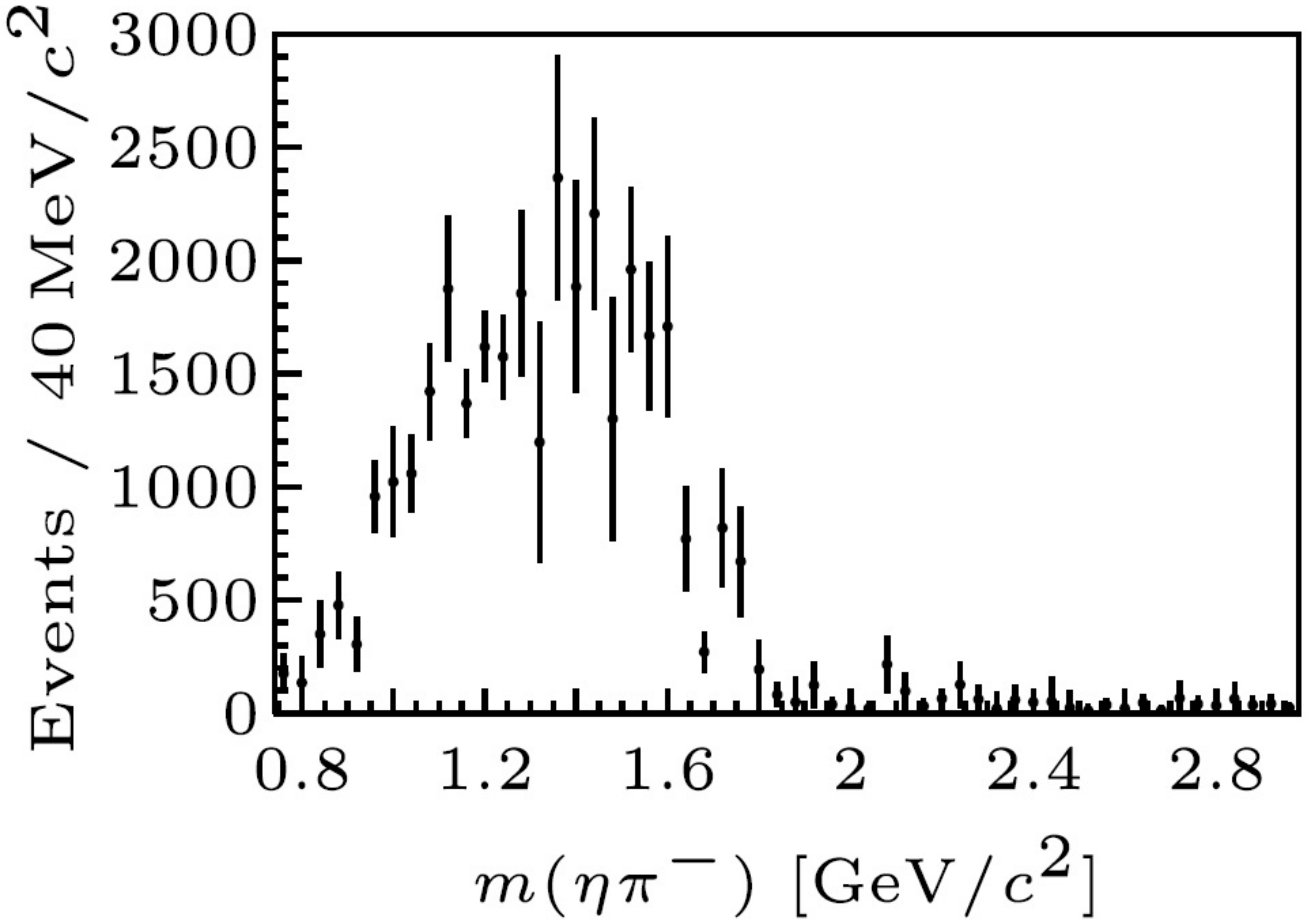
$$J^{PC}(\pi) = 0^{-+} \qquad \frac{1}{\sqrt{2}}\left(|\pi^+\pi^-\rangle + (-1)^J|\pi^-\pi^+\rangle\right) \qquad J^{PC}(\pi\pi) = 0^{++},\, 1^{--},\, 2^{++}$$

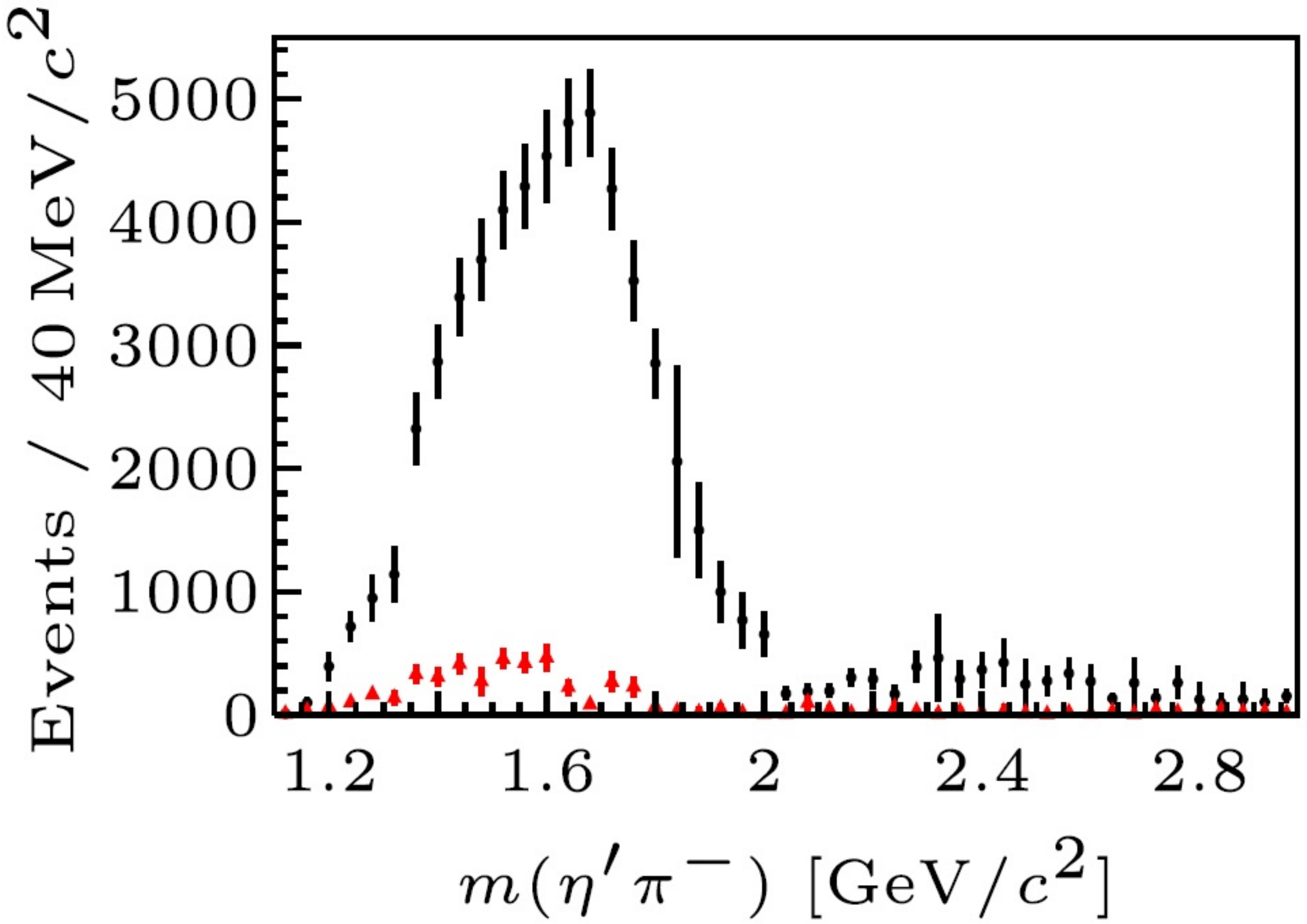
$$J^{PC}(\eta^{(\prime)}) = 0^{-+} \qquad |\pi\eta^{(\prime)}\rangle \qquad J^{PC}(\pi\eta^{(\prime)}) = 0^{++},\, 1^{-+},\, 2^{++}$$

J^{PC} dictates final states

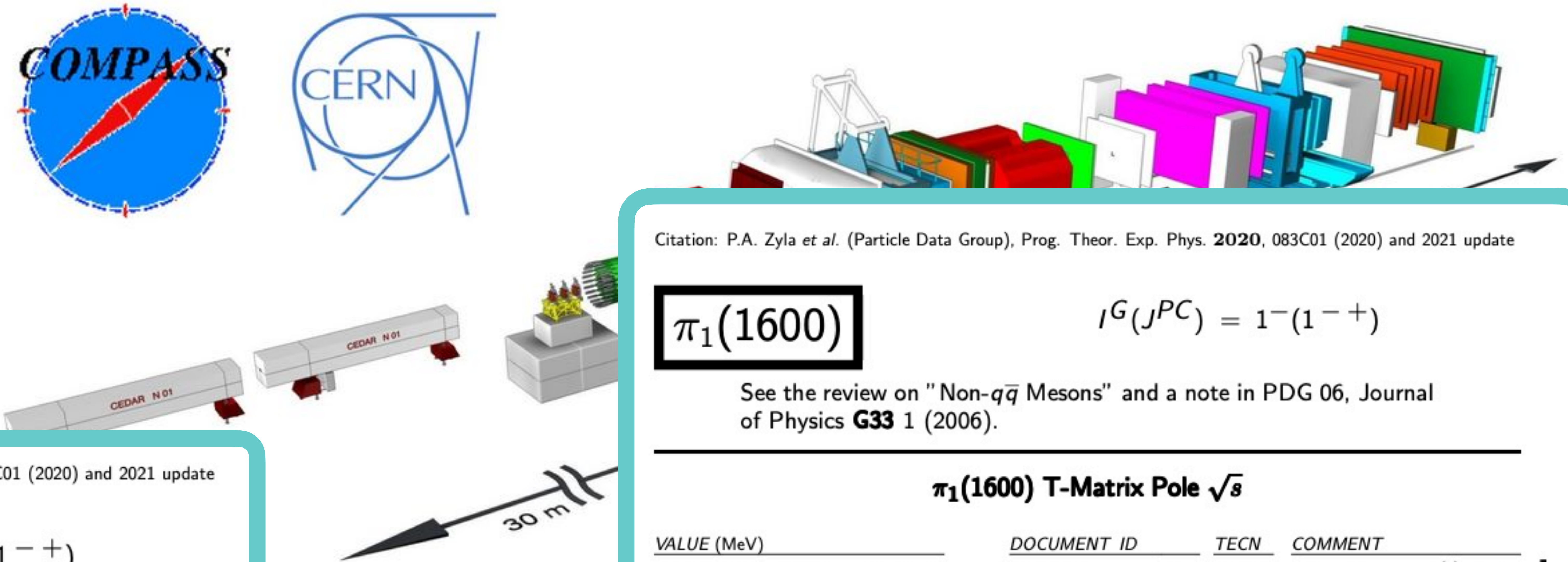
$$J^{PC}(\pi_1) = 1^{-+}$$







At odds with theory!!



Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update

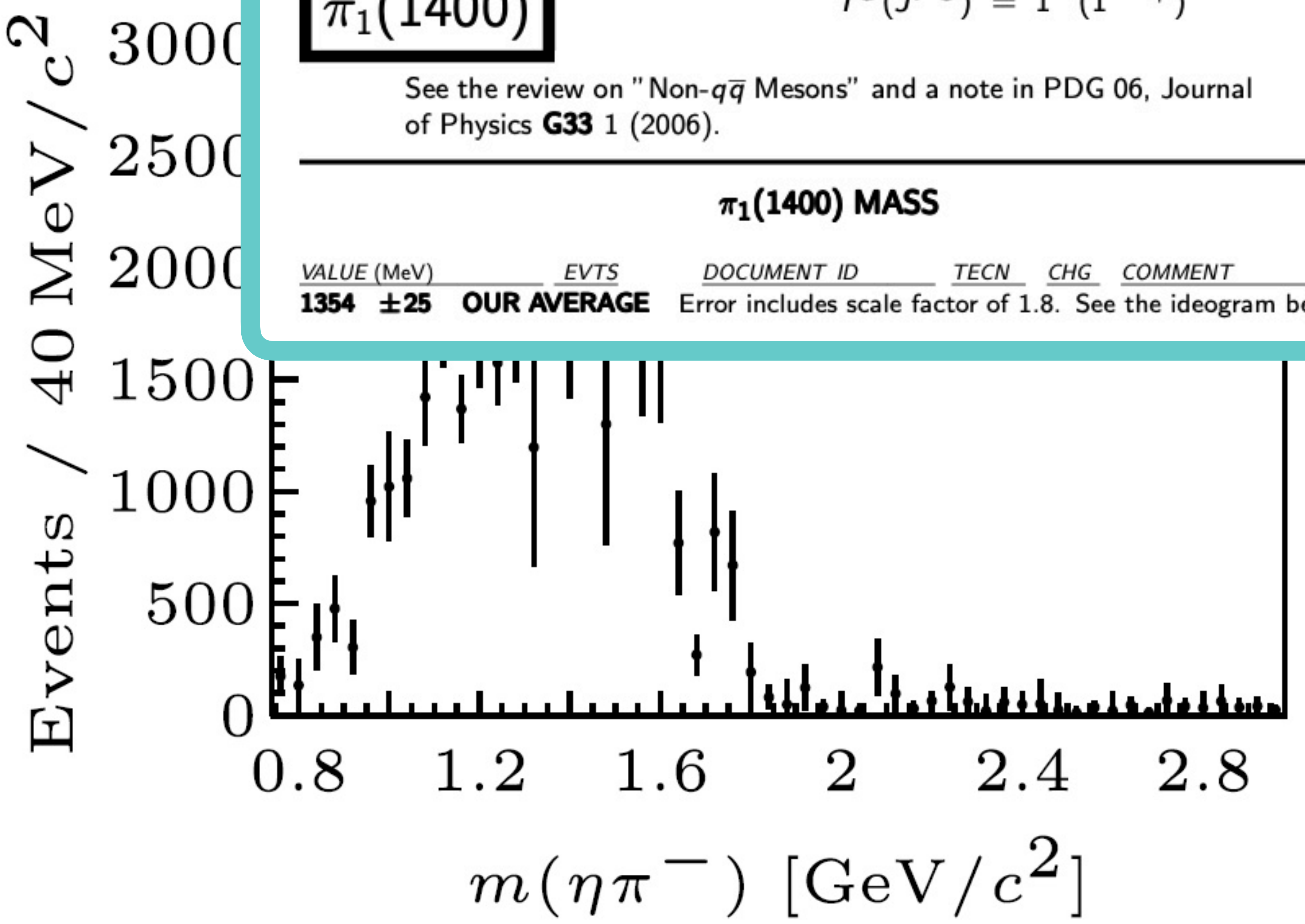
$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Non-$q\bar{q}$ Mesons" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1400)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
1354 ±25	**OUR AVERAGE**	Error includes scale factor of 1.8. See the ideogram below.			



Citation: P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020) and 2021 update

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Non-$q\bar{q}$ Mesons" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

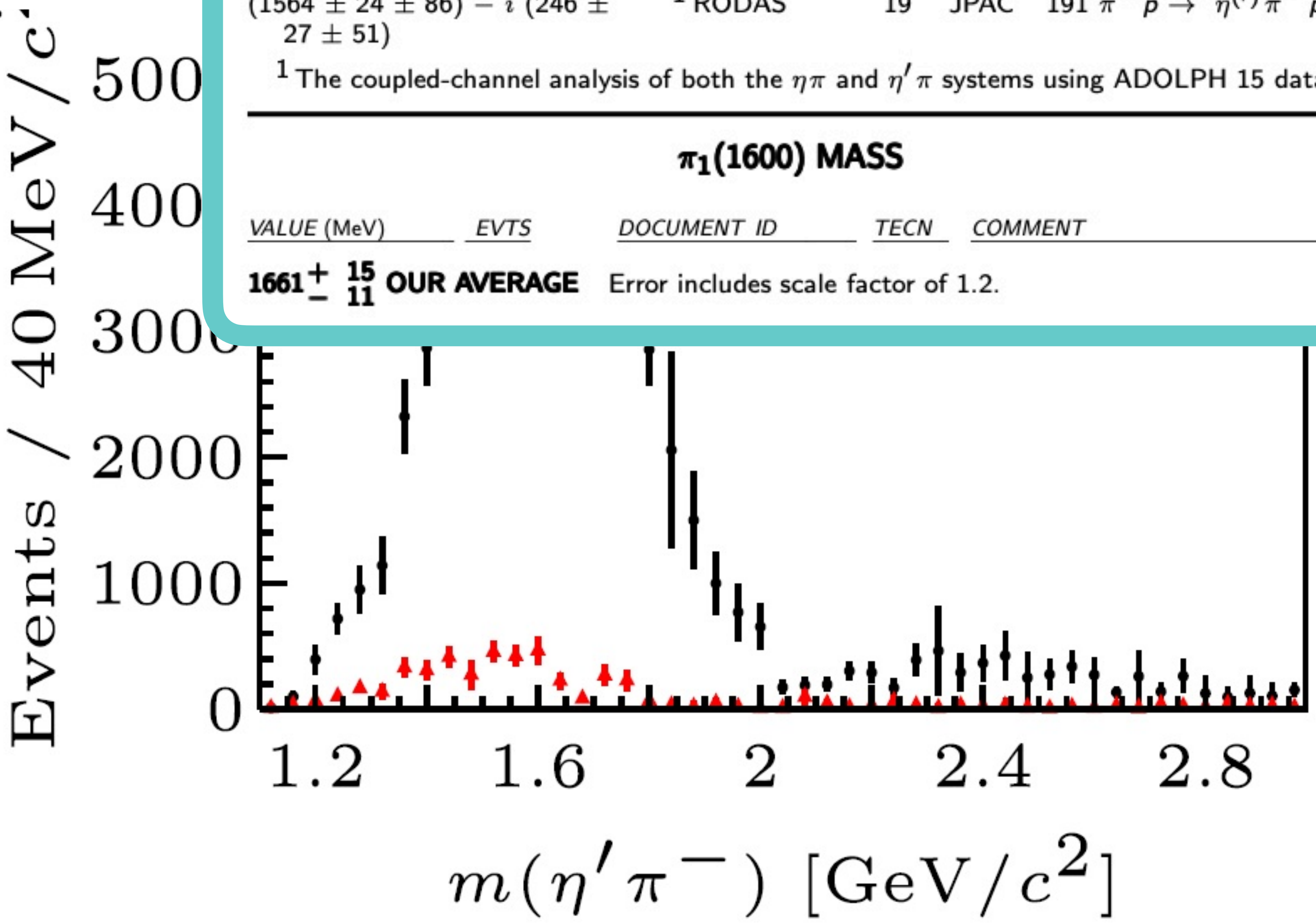
$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$(1564 \pm 24 \pm 86) - i\,(246 \pm 27 \pm 51)$	[1] RODAS	19	JPAC	191 $\pi^- p \to \eta^{(\prime)} \pi^- p$

[1] The coupled-channel analysis of both the $\eta\pi$ and $\eta'\pi$ systems using ADOLPH 15 data.

$\pi_1(1600)$ MASS

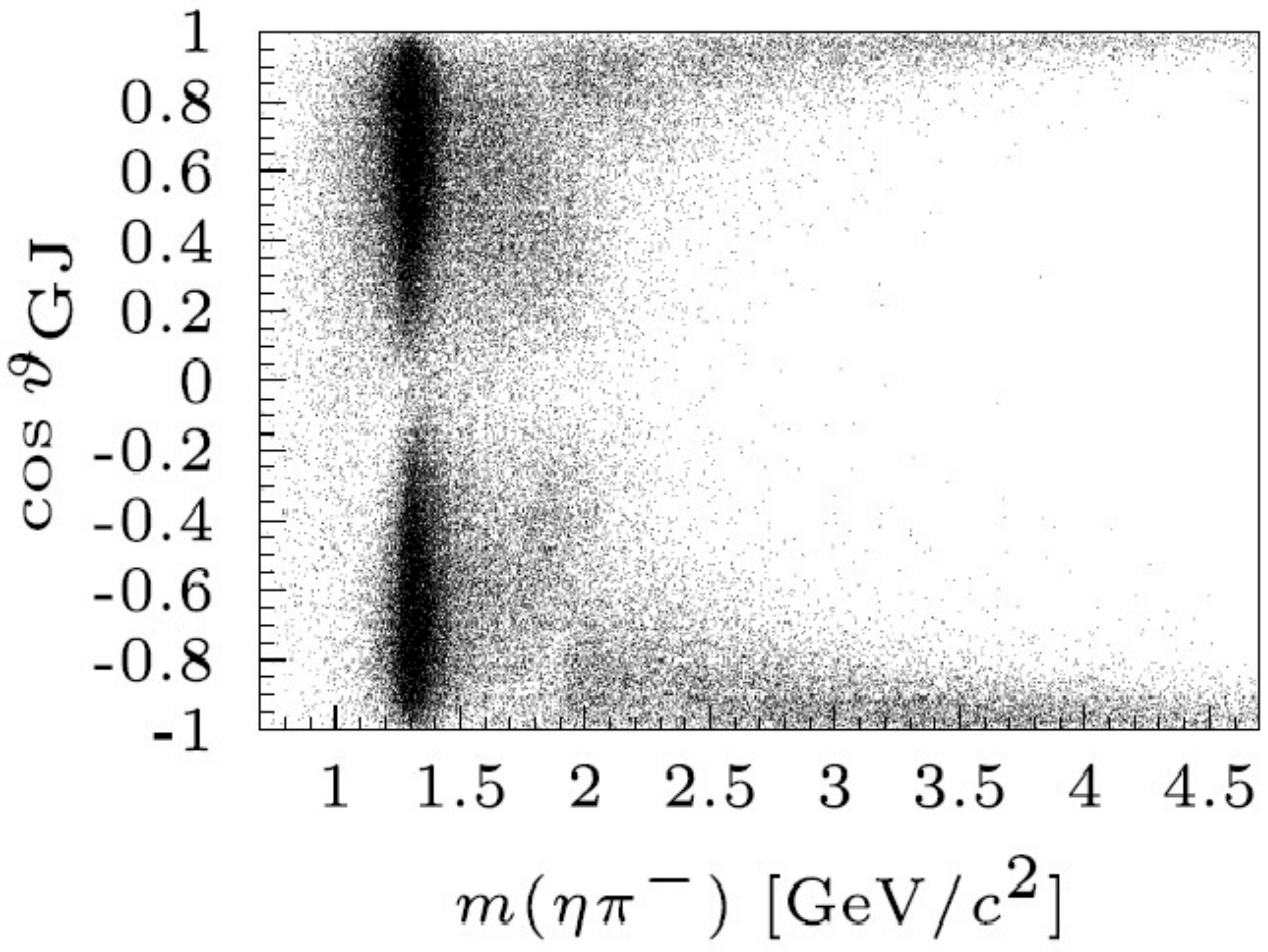
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
1661^{+15}_{-11}	**OUR AVERAGE**	Error includes scale factor of 1.2.		

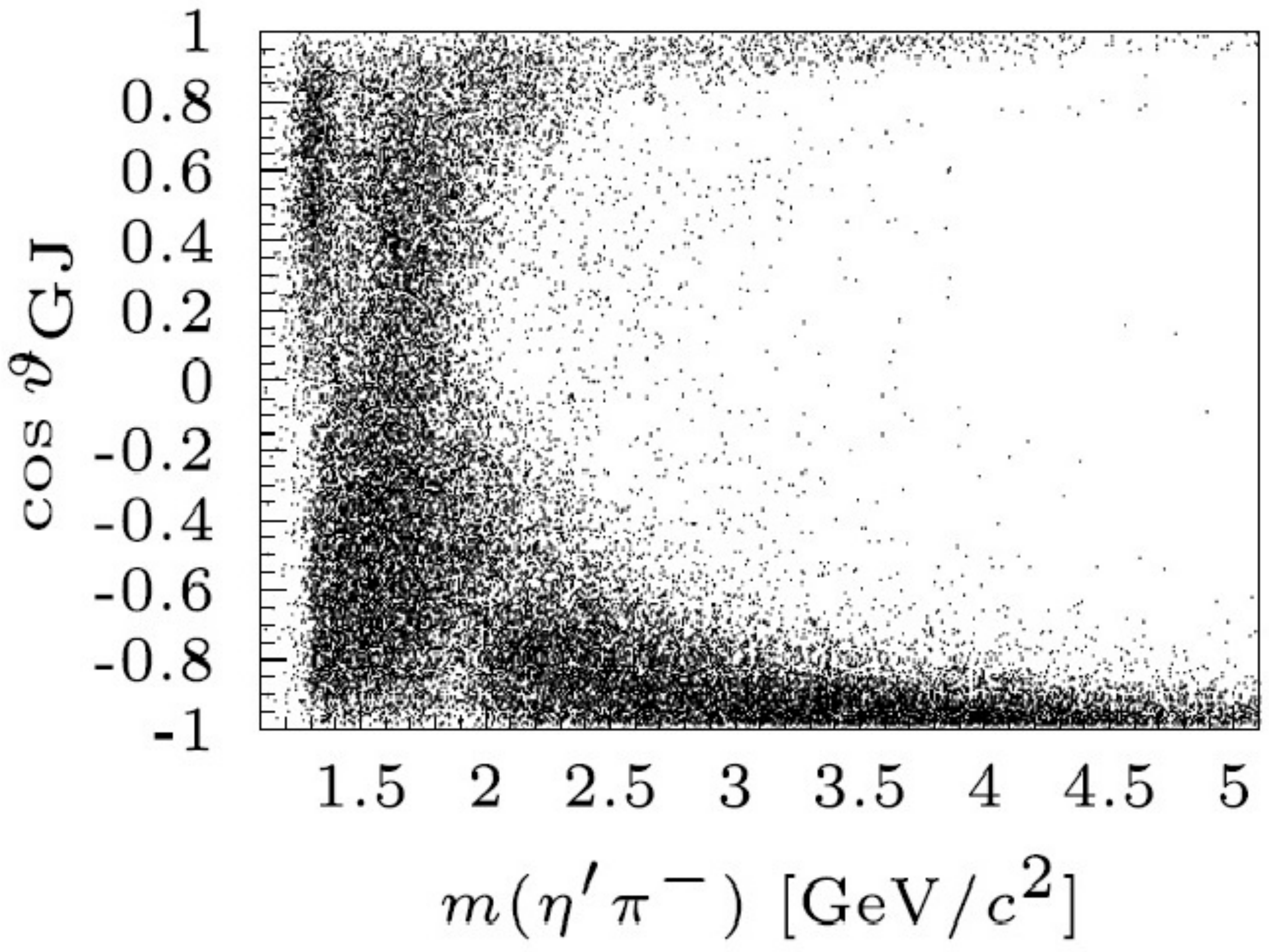


Hybrid

Remember

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$





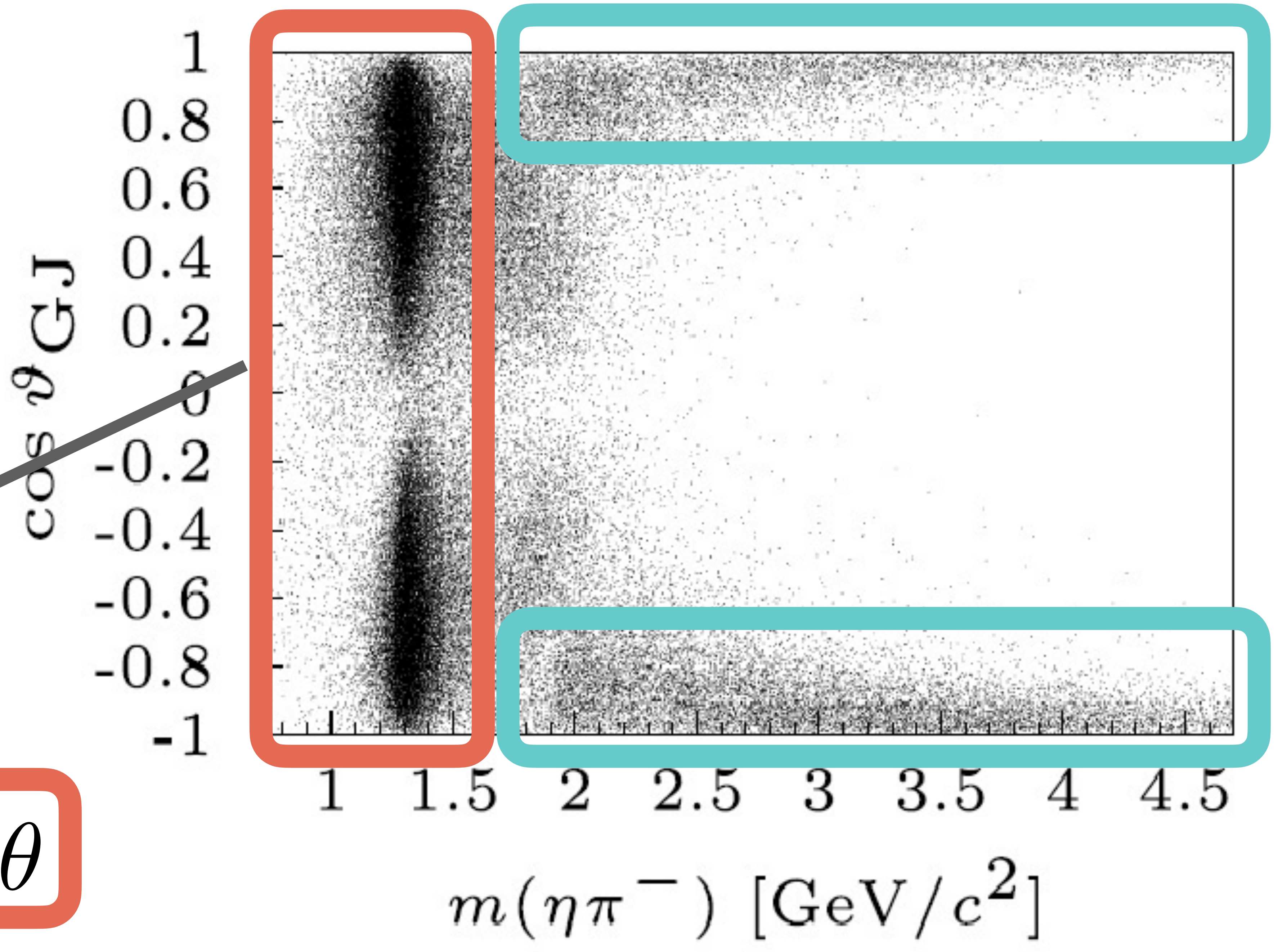
Hybrid

Remember?

$$I(\theta,\phi) \propto \left|Y_J^M(\theta,\phi)\right|^2$$

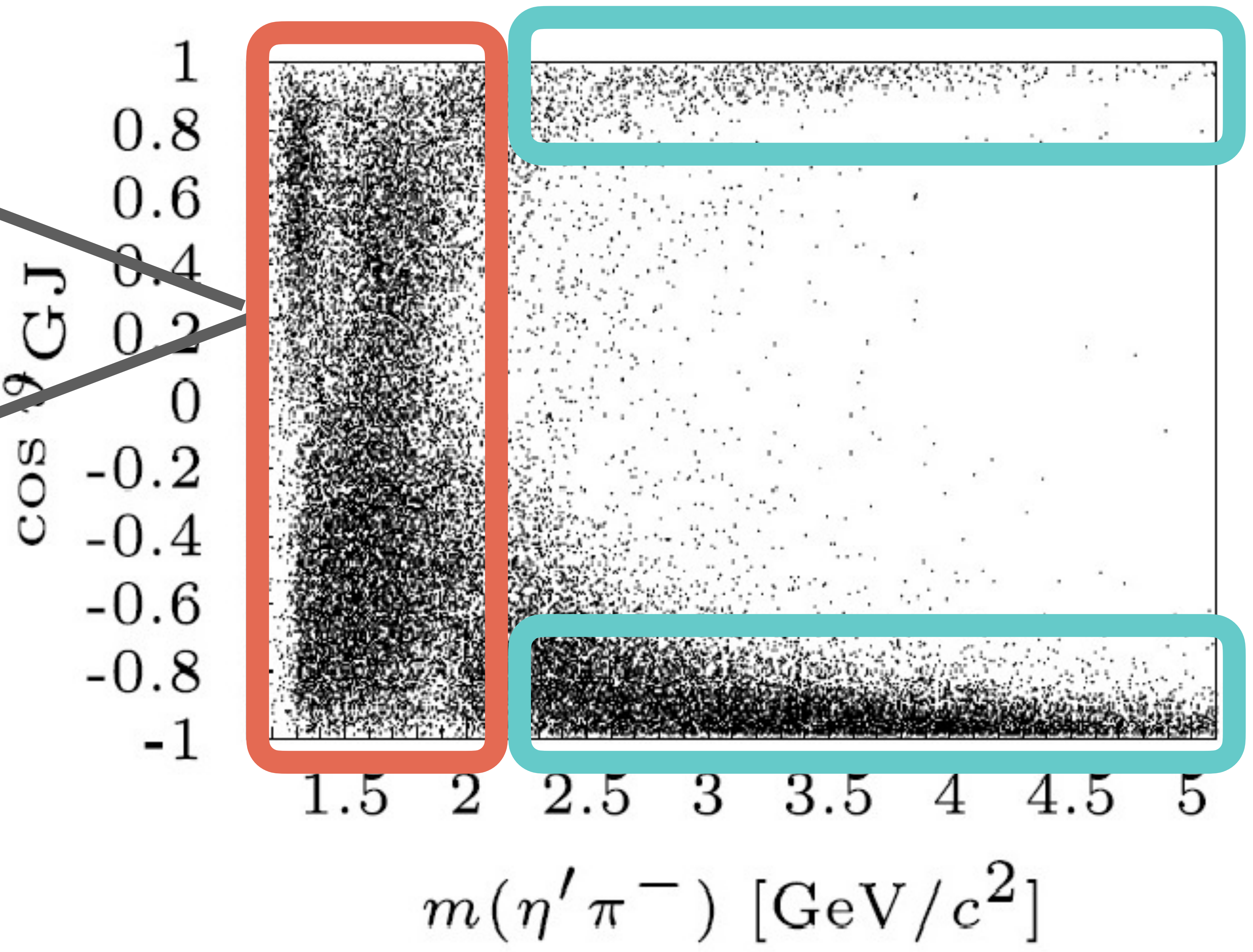
Tensor waves

$$J = 2\ M = \pm 1 \rightarrow I \propto \cos^2\theta \sin^2\theta$$

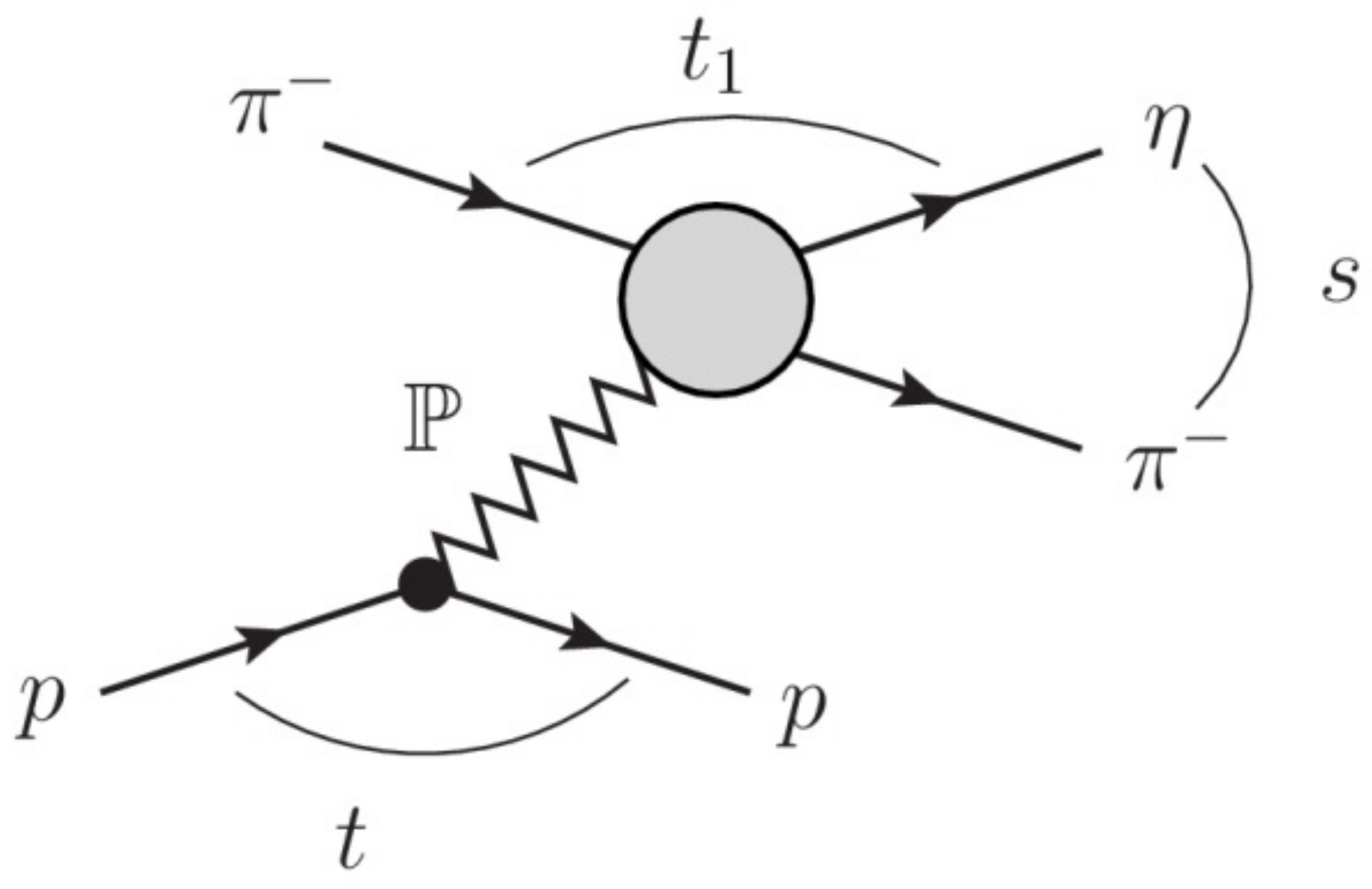


Vector waves

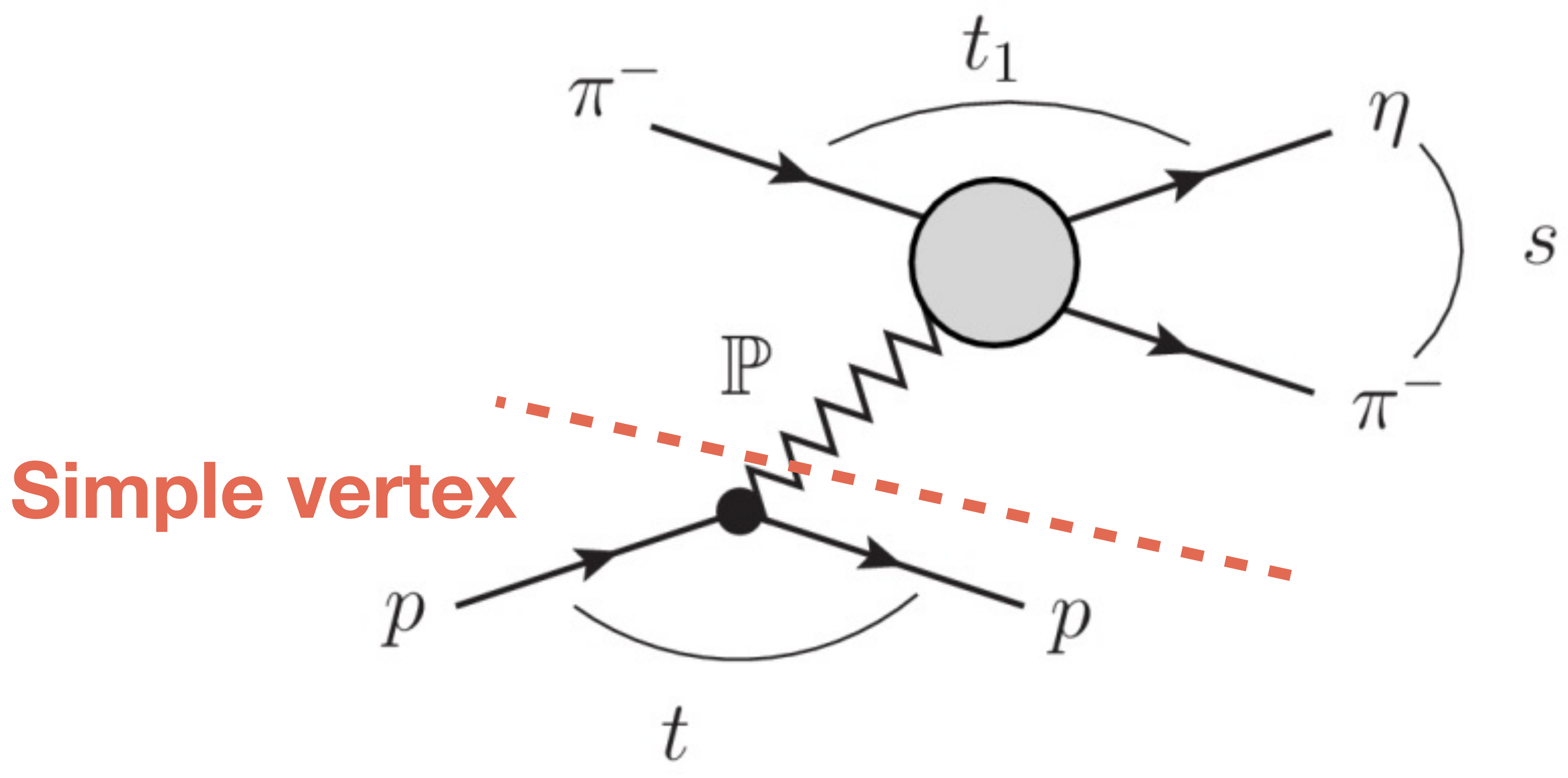
$$J = 1\ M = \pm 1 \rightarrow I \propto \sin^2\theta$$



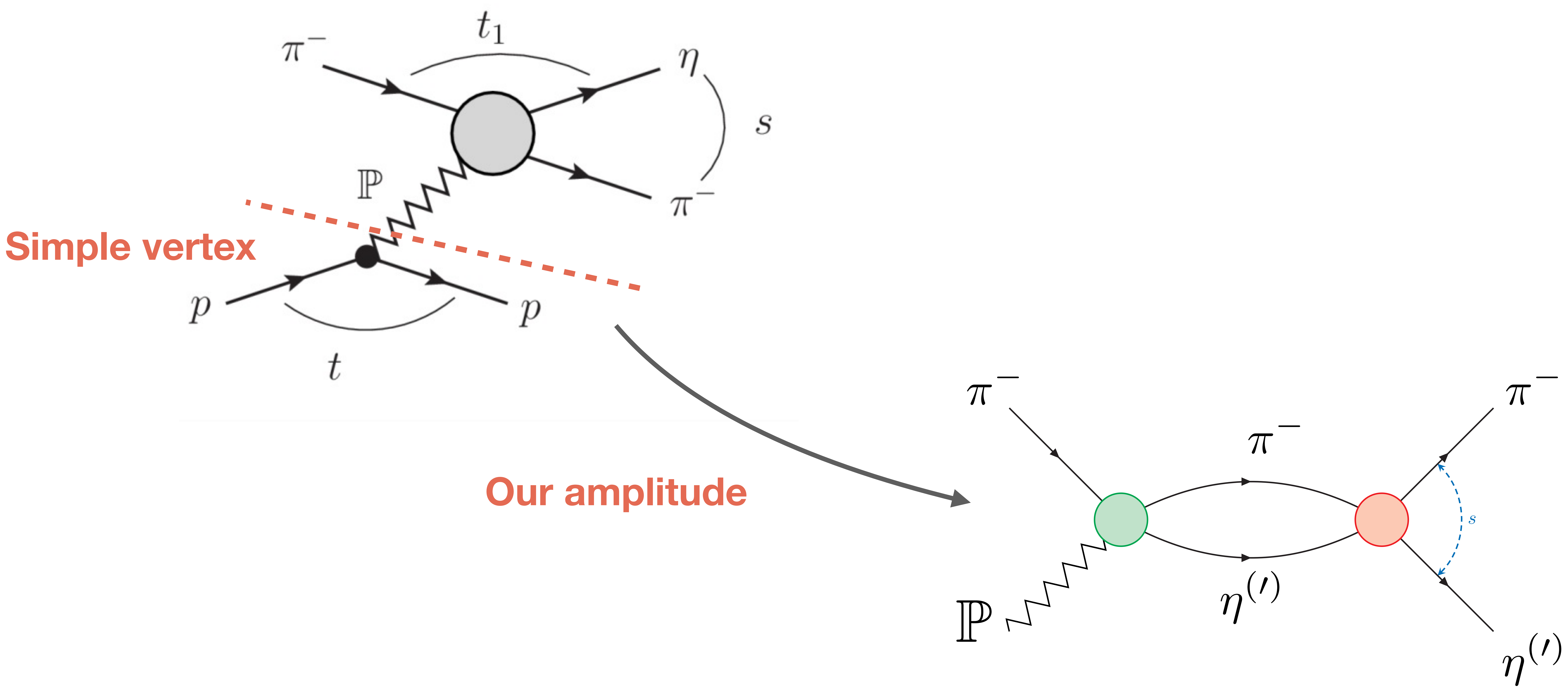
Hybrid Measurement



Measurement



Measurement

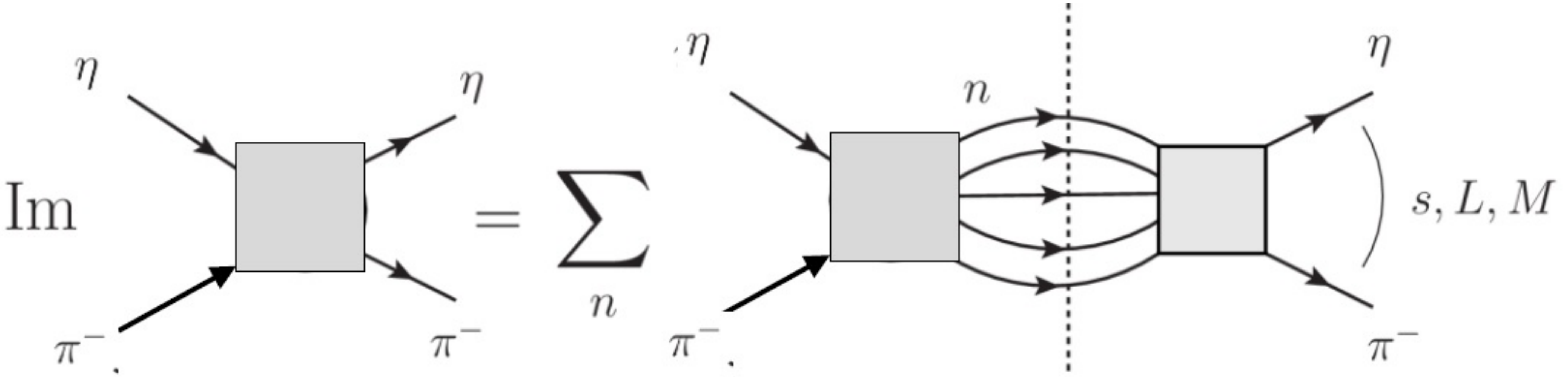


Hybrid

$$t_\ell(s) = [K_\ell^{-1}(s) - i\rho(s)]^{-1}$$

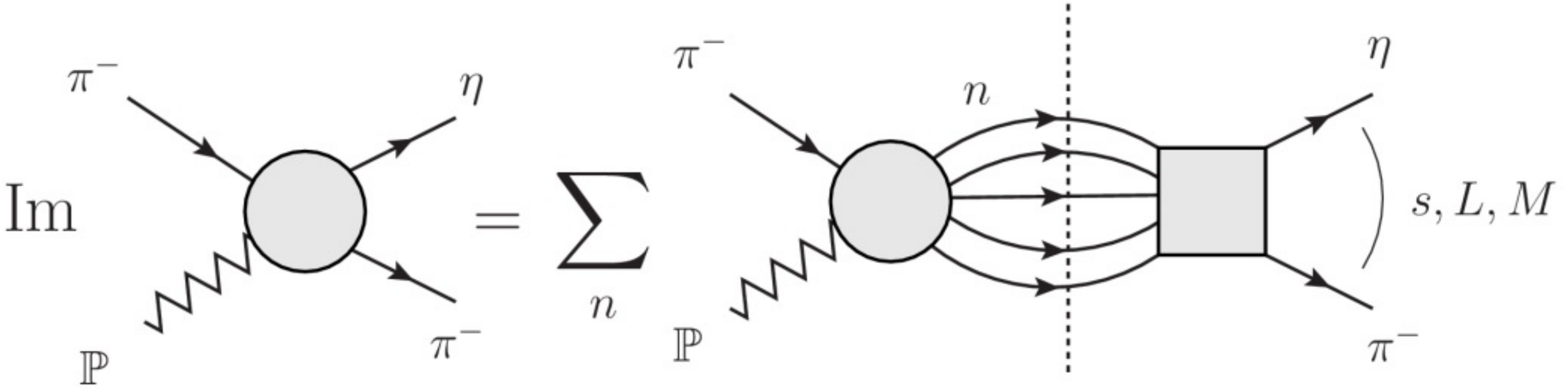
Remember?

$$Im\, t_\ell(s) = \rho(s)|t_\ell(s)|^2$$



$t_\ell(s)$ **contains the pole**

$$Im\, a_\ell(s) = \rho(s) t_\ell^*(s) a_\ell(s)$$



$a_\ell(s)$ **is simple**

$$a_{\ell\, i}(s) \propto \sum_k n_k^\ell(s) \left[D^\ell(s)^{-1} \right]_{ki}$$

Hybrid

$$t_\ell(s) = \left[K_\ell^{-1}(s) - i\rho(s)\right]^{-1}$$

$$D_{ki}^J(s) = \left[K^J(s)^{-1}\right]_{ki} - \frac{s}{\pi}\int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s'-s-i\epsilon)}$$

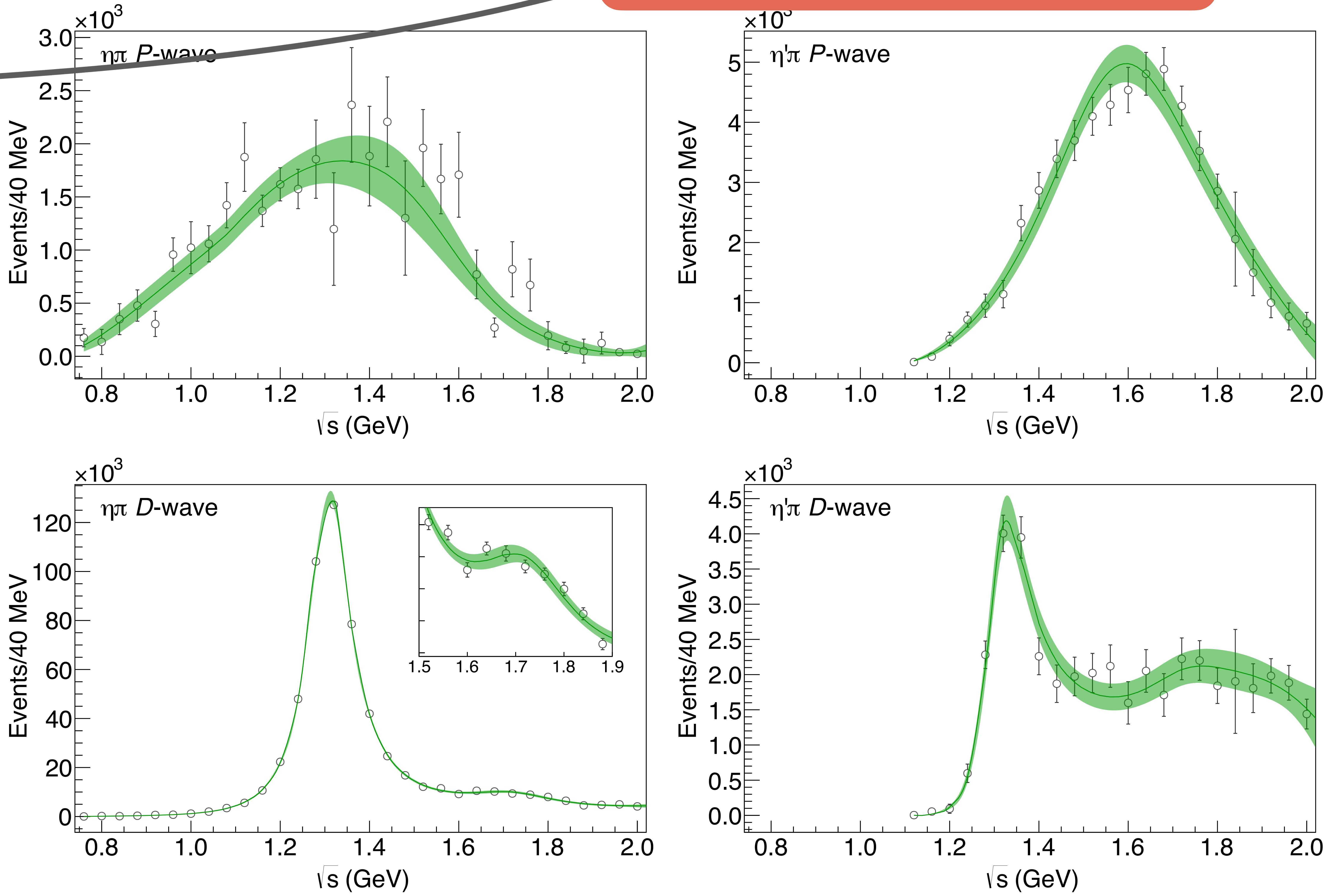
Later

Very good fit to data

Describing two diff. peaks

$\eta\pi \to$ **D wave**

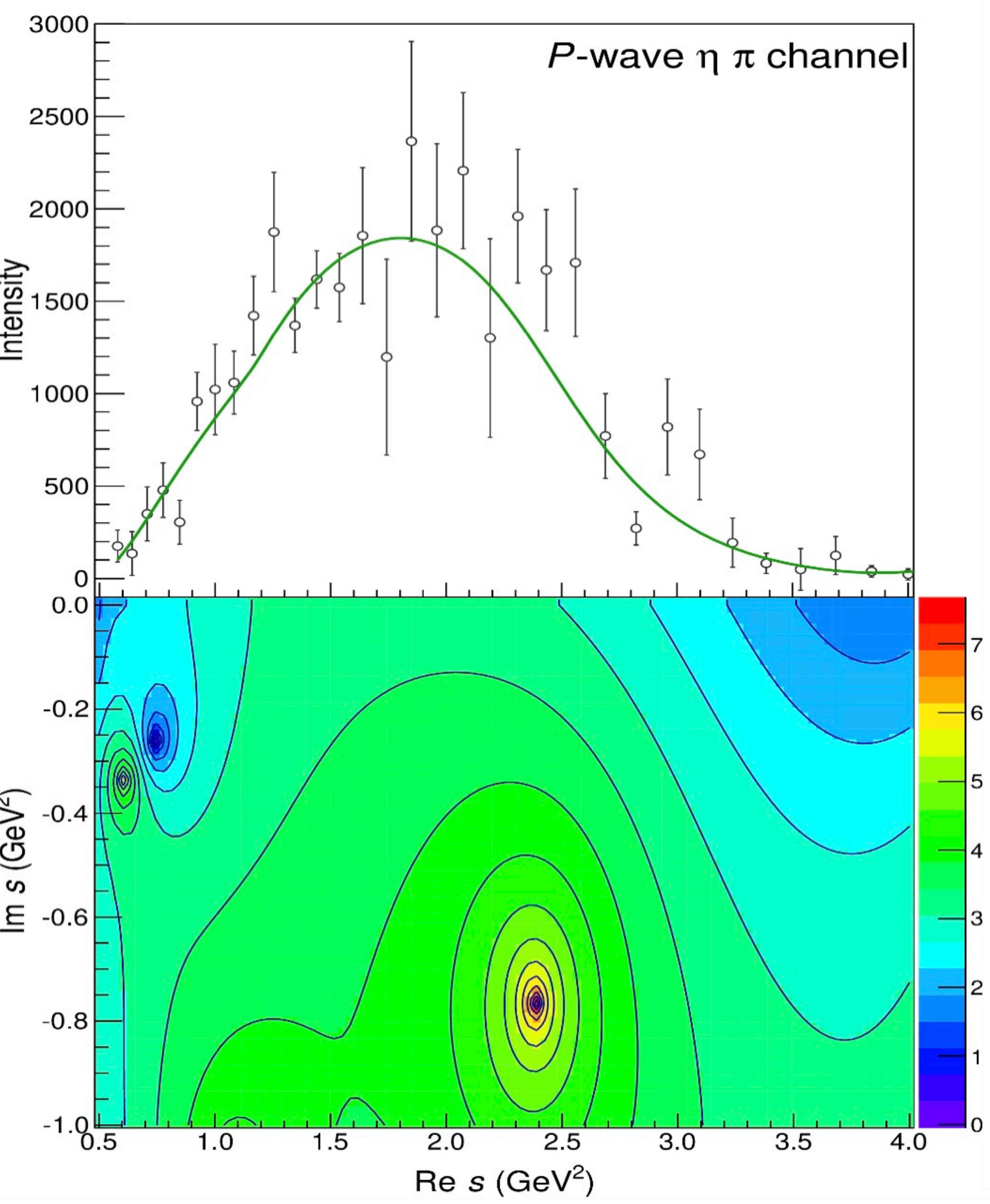
$\eta'\pi \to$ **Both P, D**

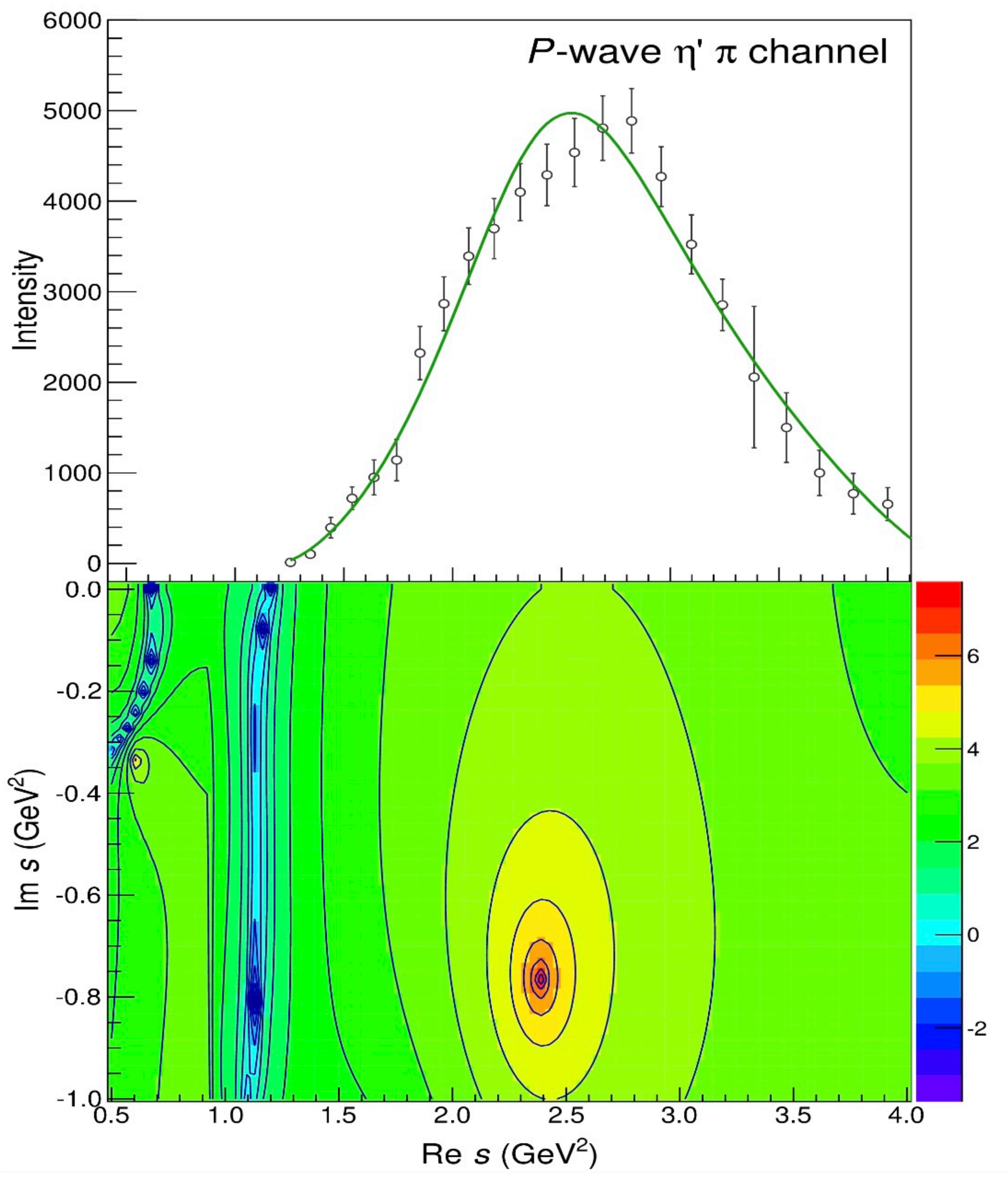


Hybrid

Only one pole

Reconciles for the first time theory ⟷ experiment





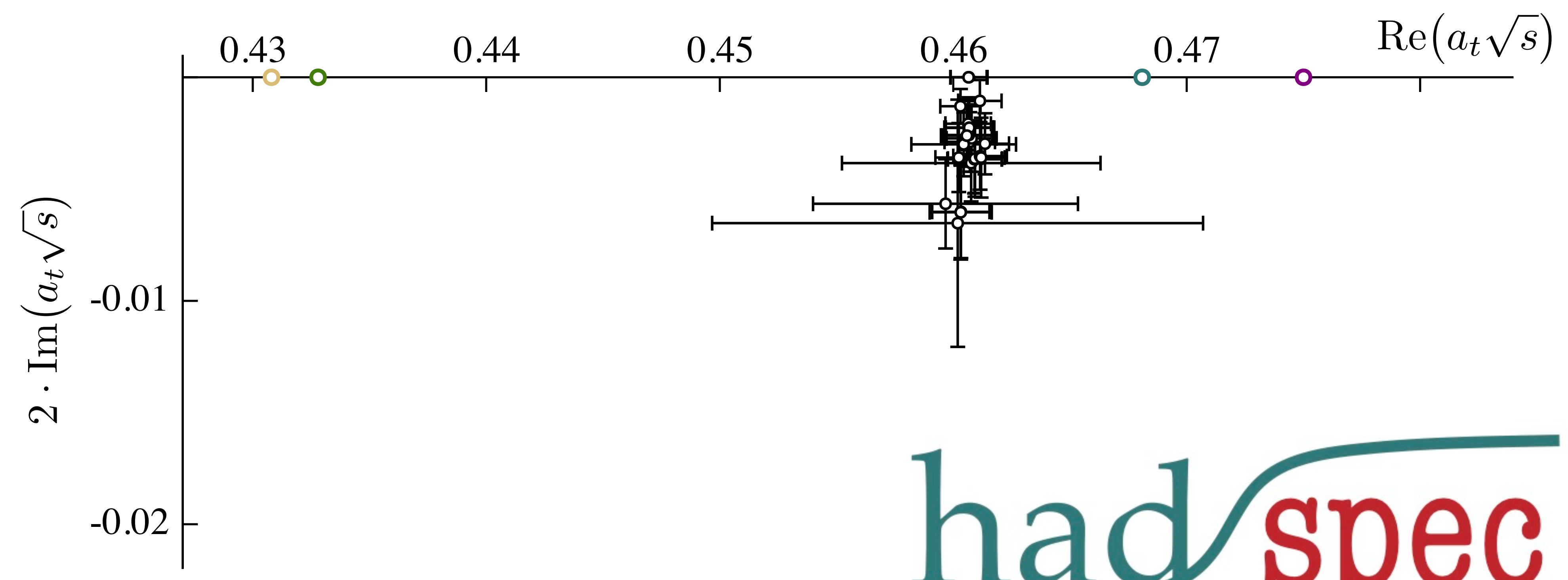
Exp. + pheno. confirmed 2

Investigation of the lightest hybrid meson candidate with a coupled-channel analysis of $\bar{p}p$-, $\pi^- p$- and $\pi\pi$-Data

B. Kopf (Ruhr U., Bochum), M. Albrecht (Ruhr U., Bochum), H. Koch (Ruhr U., Bochum), M. Küßner (Ruhr U., Bochum (main)), J. Pychy (Ruhr U., Bochum) et al. (Aug 26, 2020)

Published in: *Eur.Phys.J.C* 81 (2021) 12, 1056 • e-Print: 2008.11566 [hep-ph]

Lattice QCD confirmed

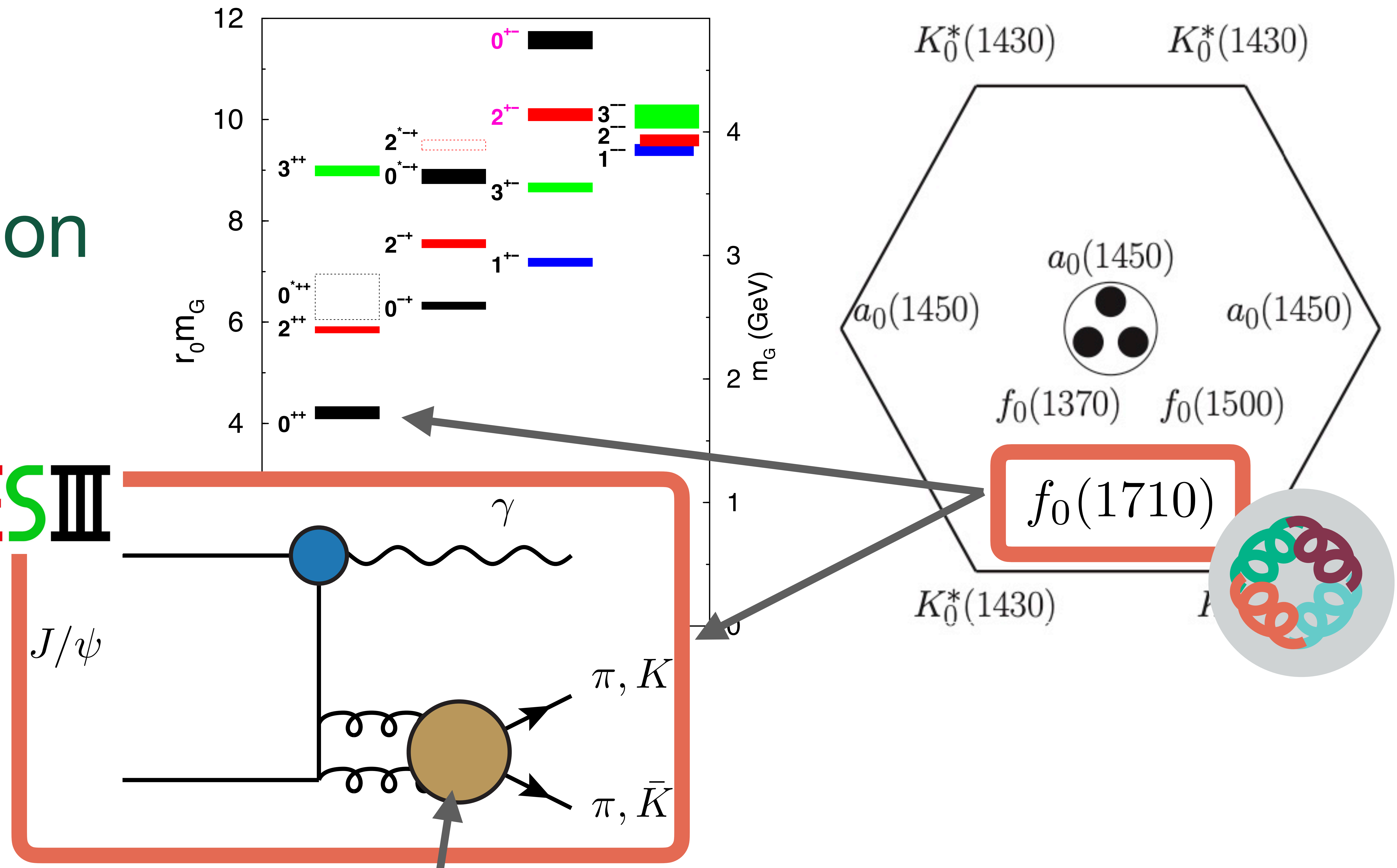


Glueball

Lattice QCD expectation

Glueball around 2 GeV

Golden channel in BESIII

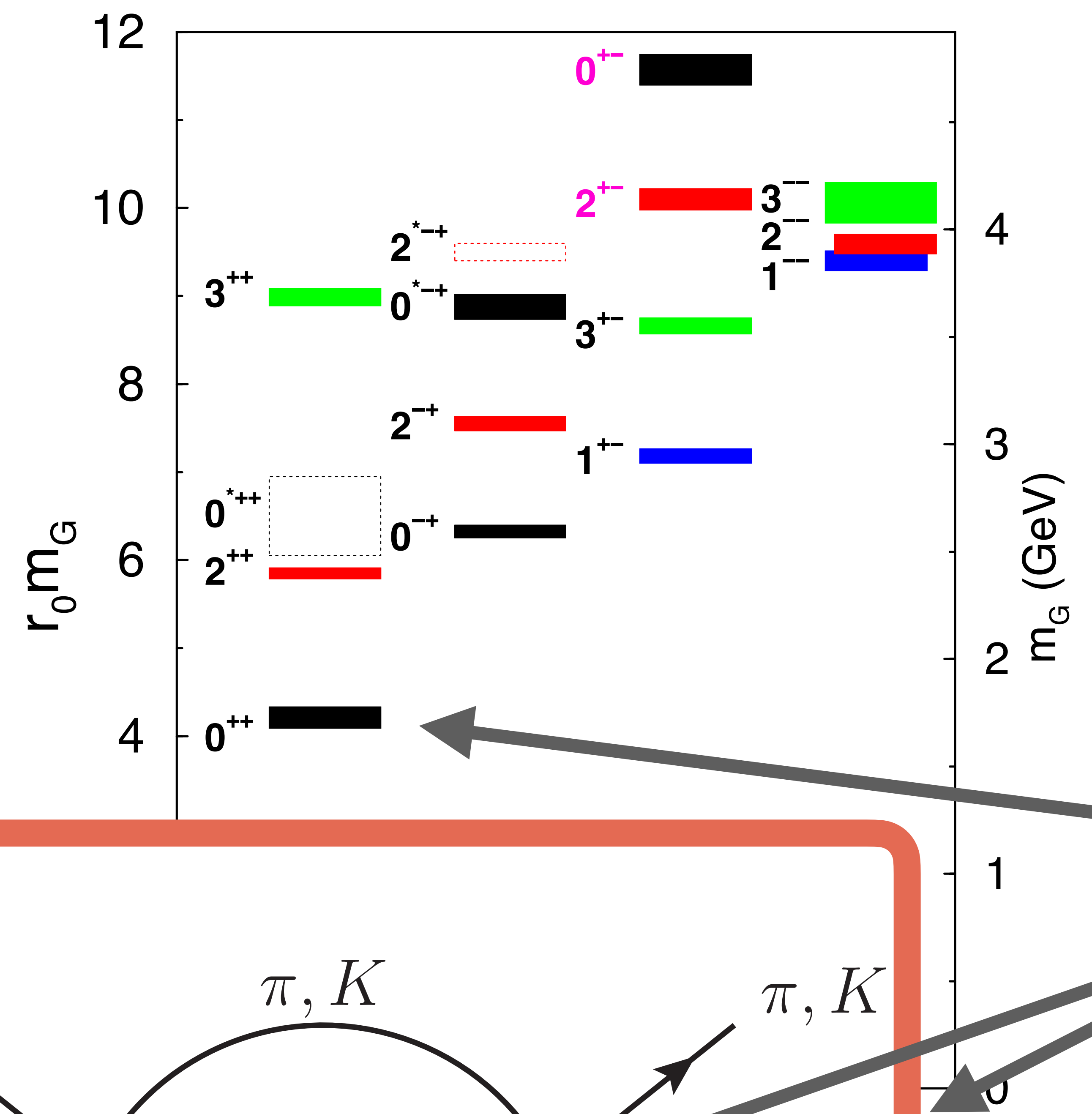


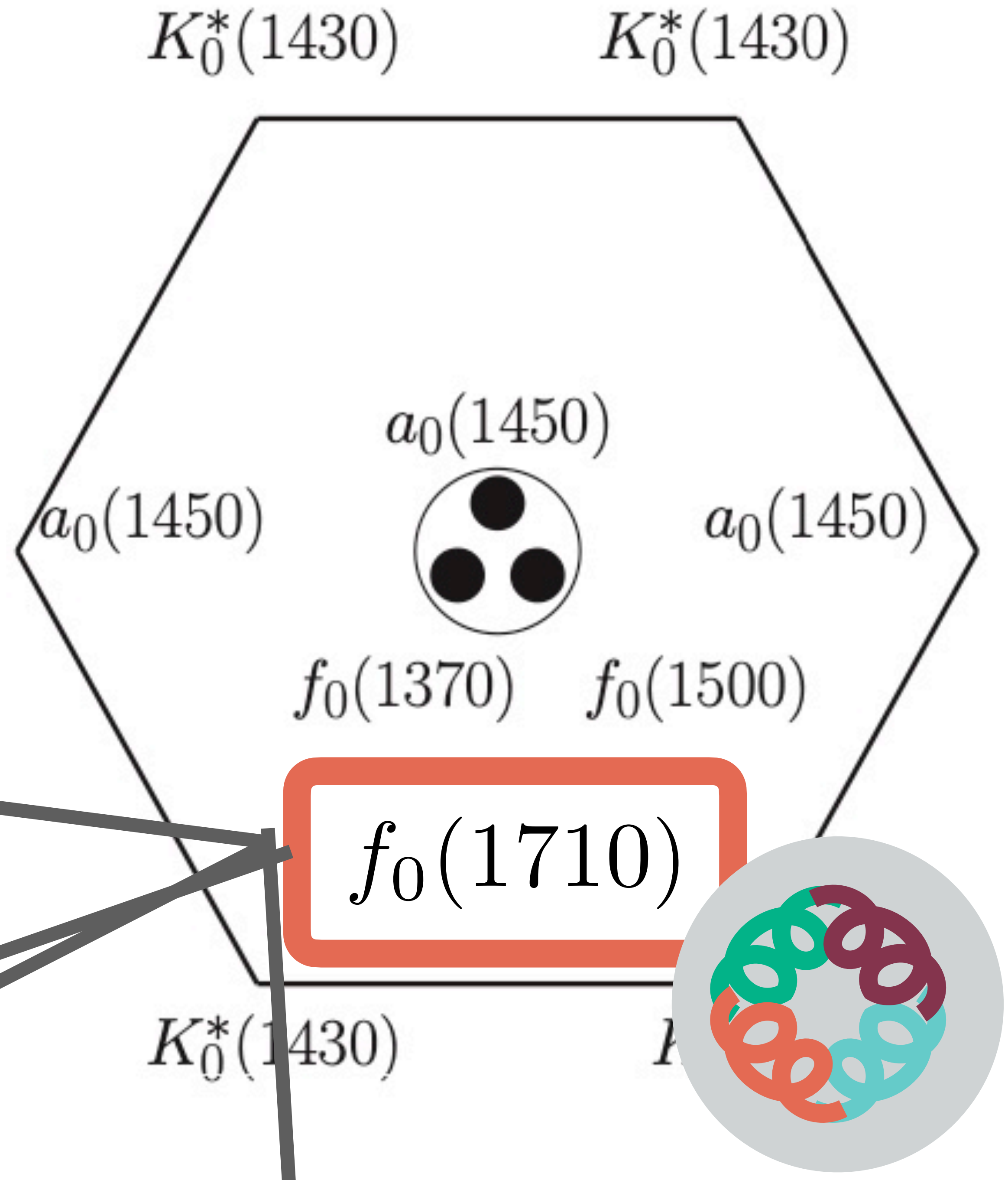
Stronger coupling here → glueball

Glueball

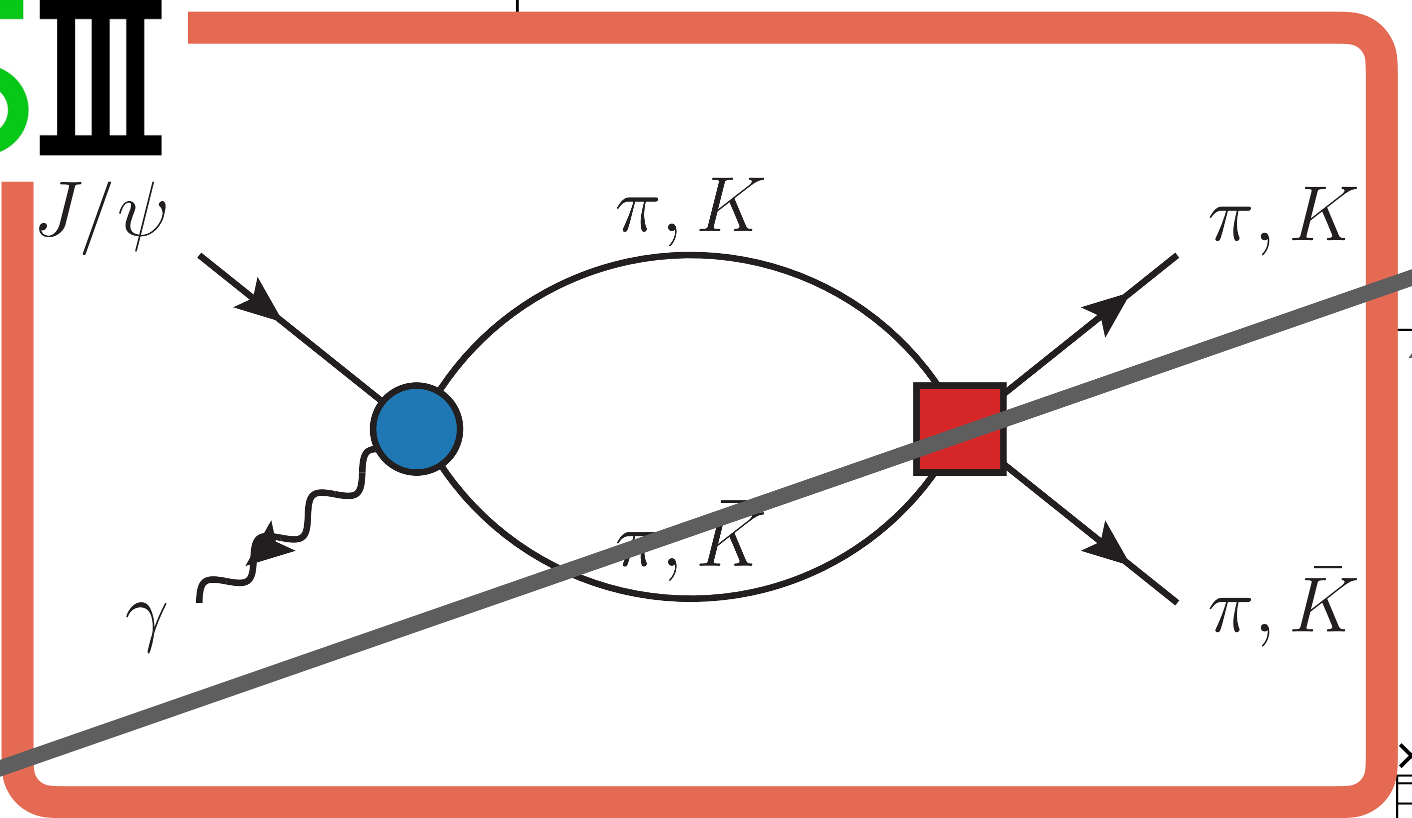
Lattice QCD expectation

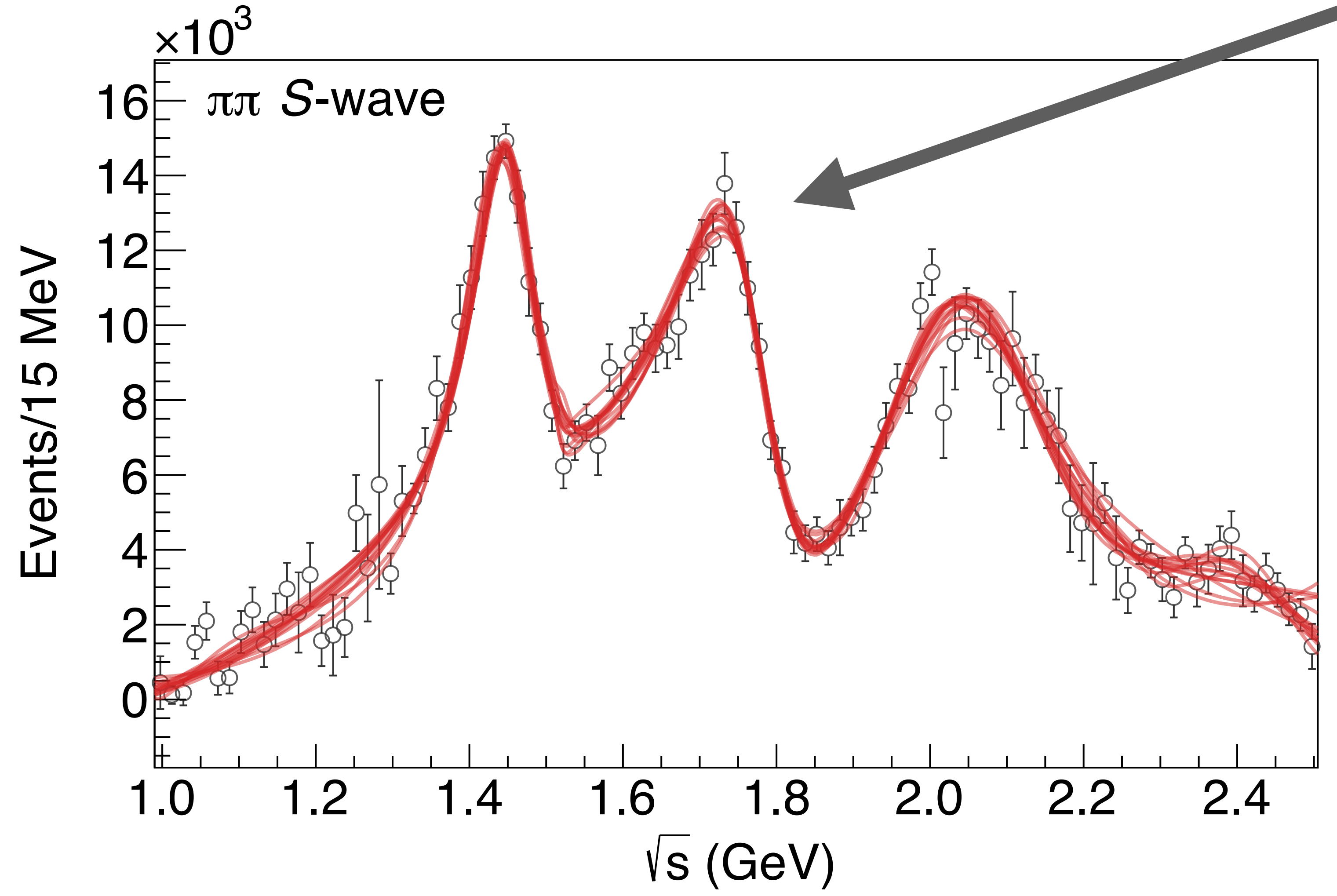
Glueball around 2 GeV

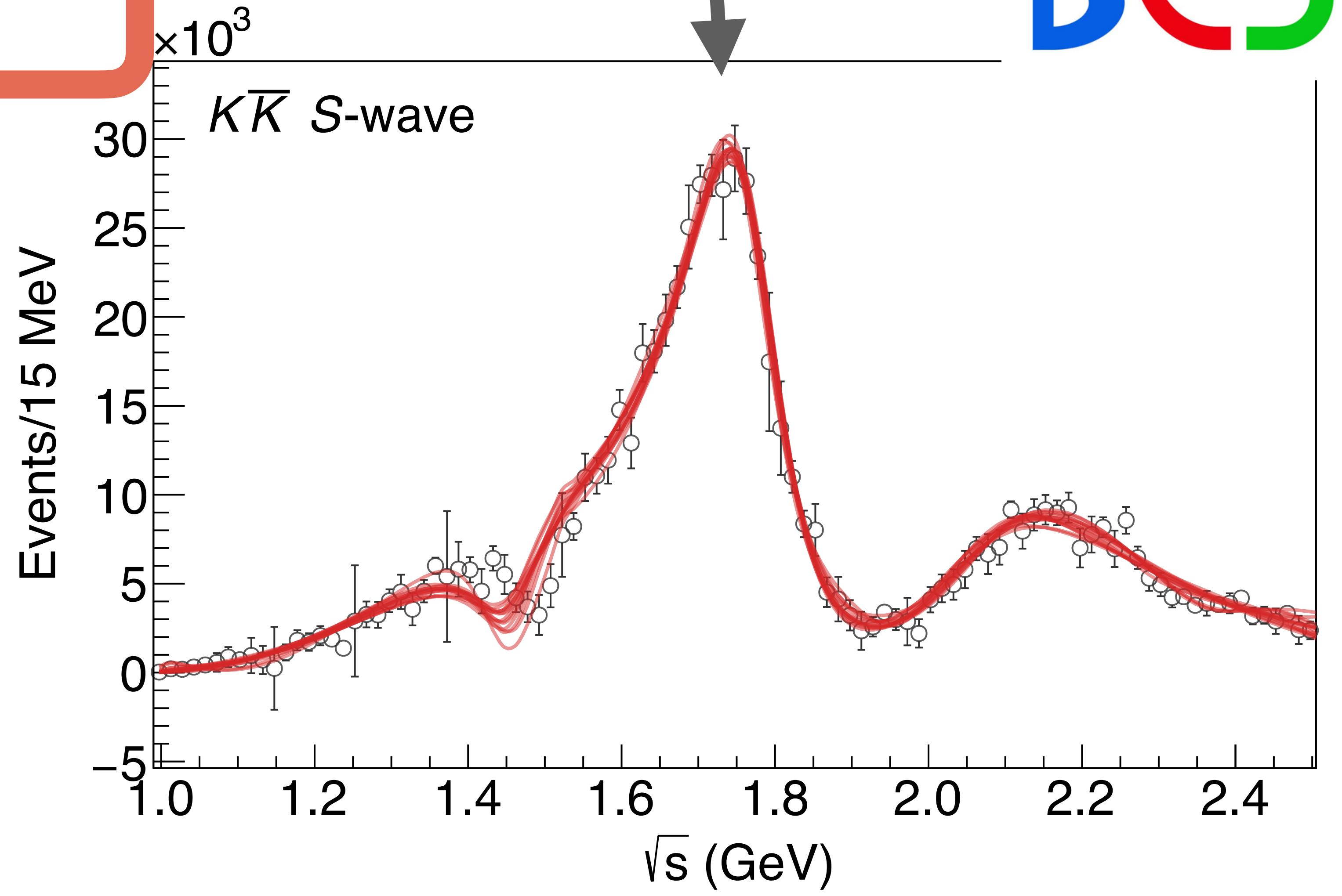




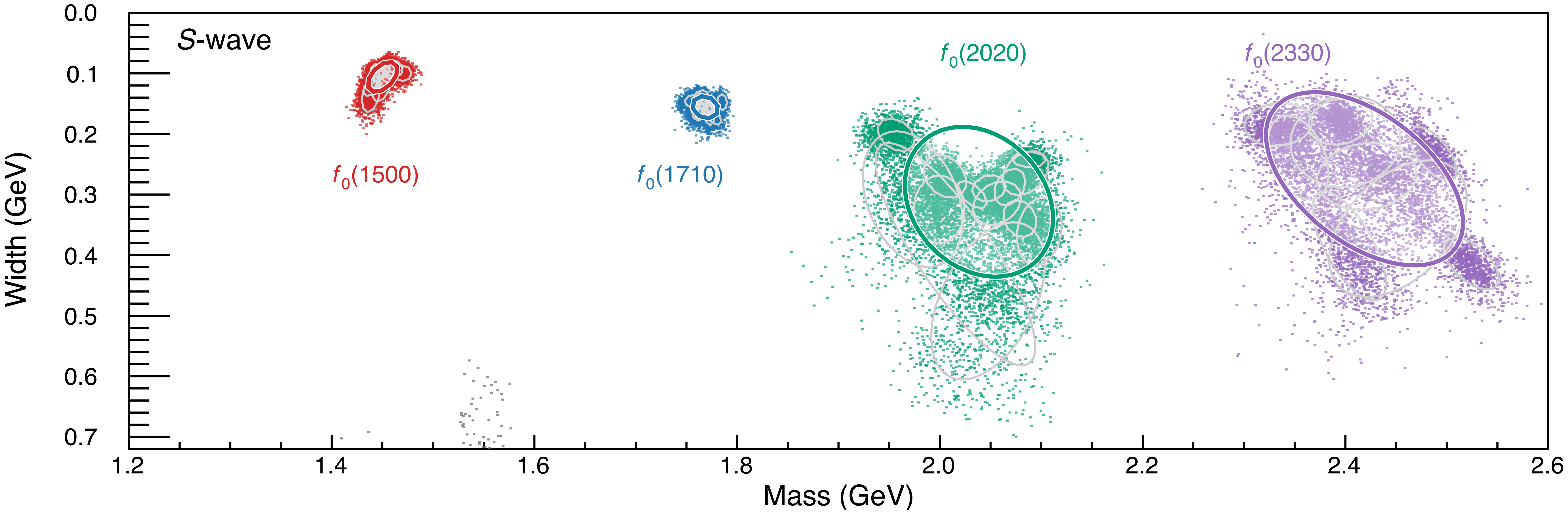
Same as before

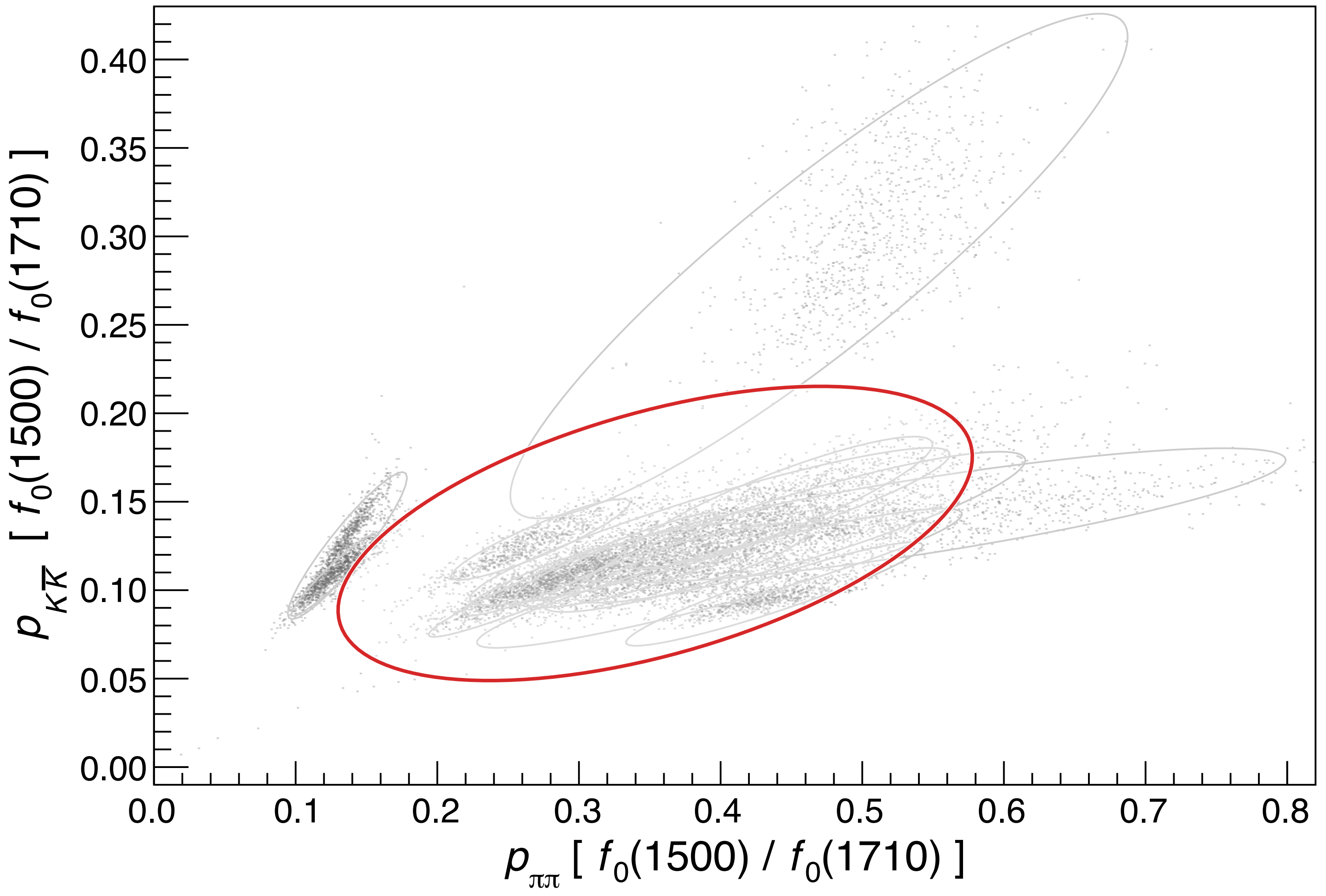


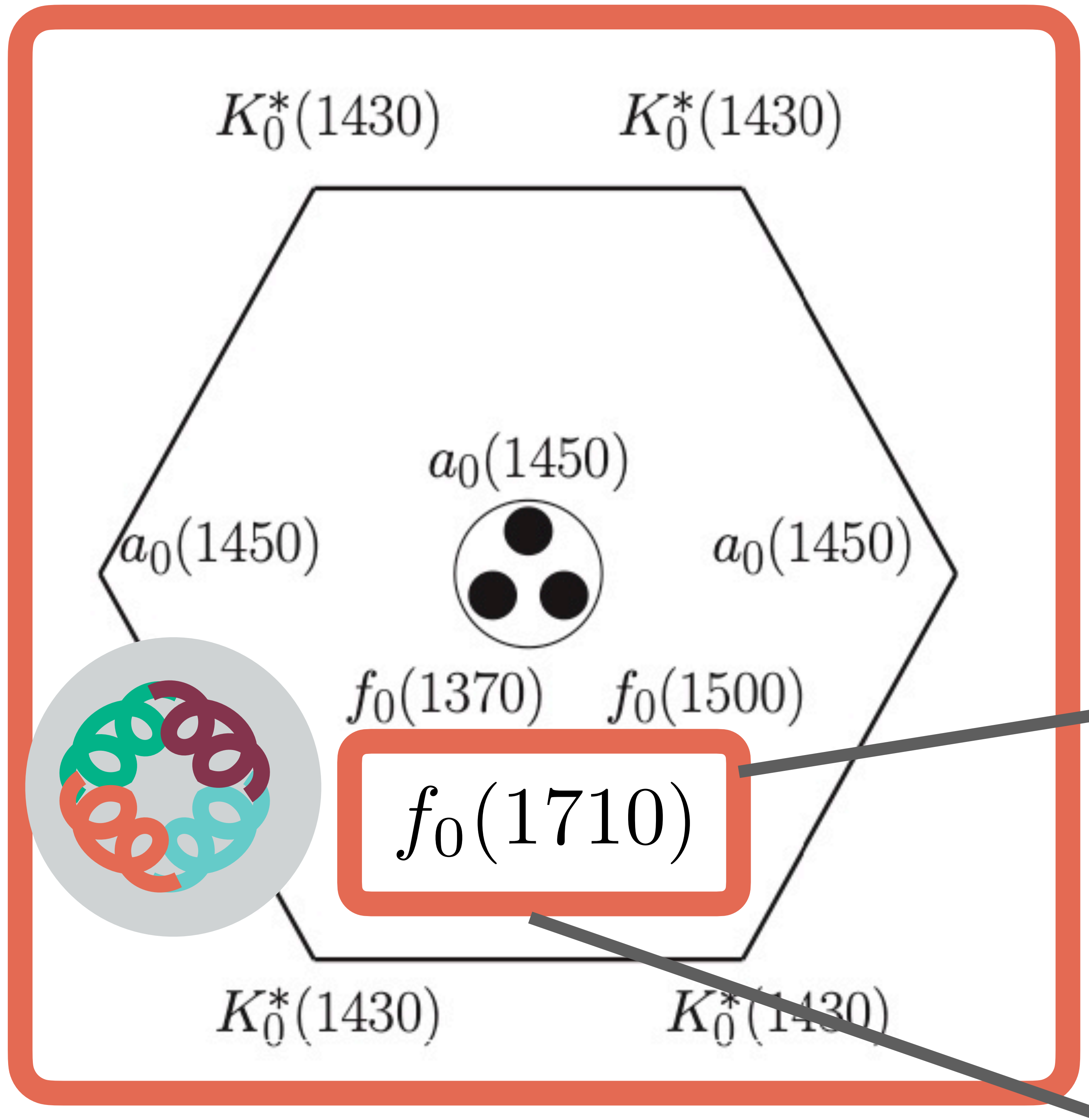




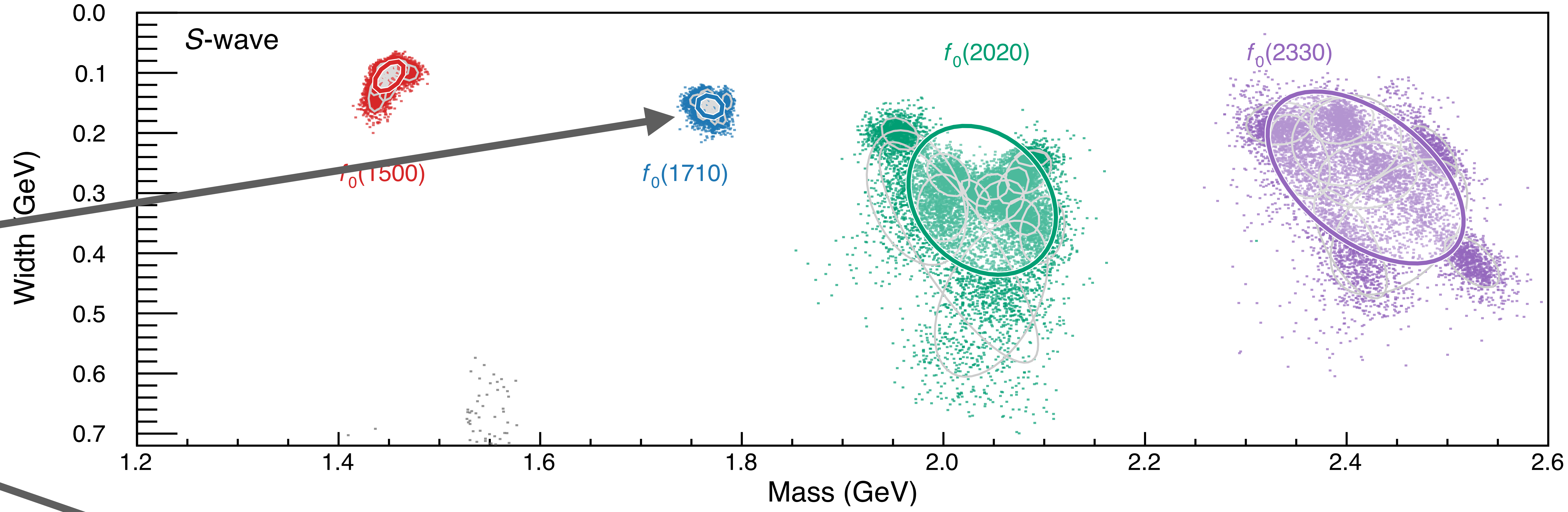
We determined 7 particles with precision

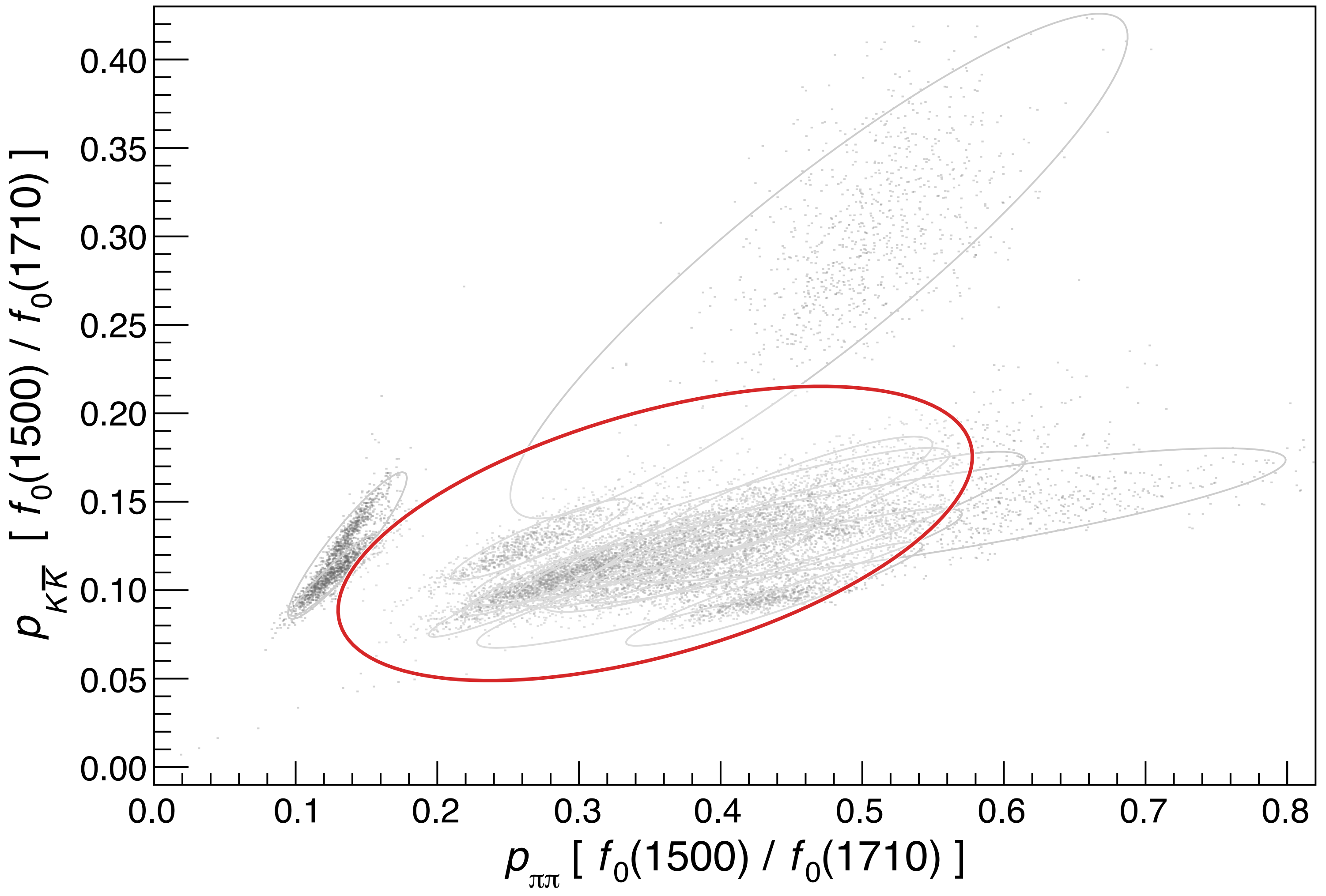






We determined 7 particles with precision





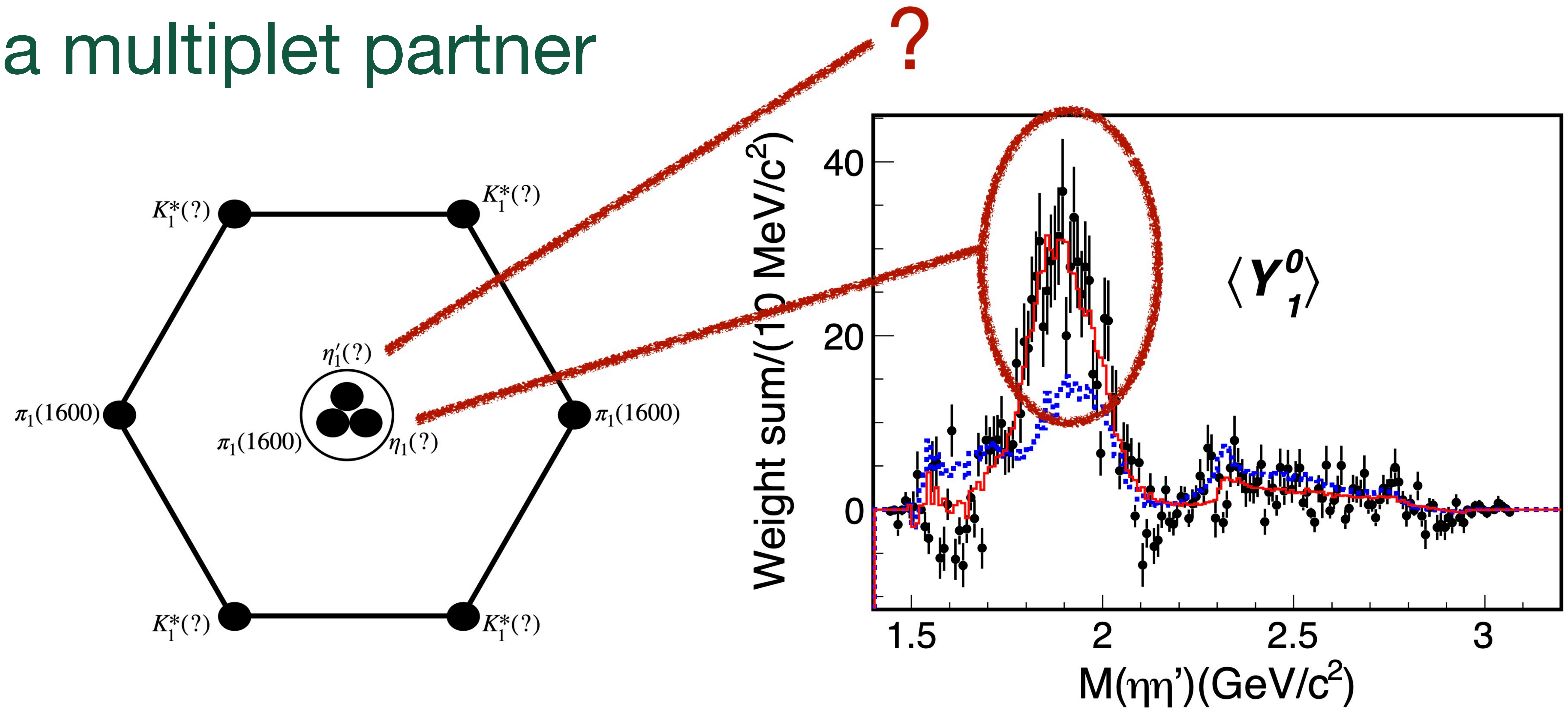
Highest "glue" coupling $\rightarrow f_0(1710)$

Points to a predominant glueball nature

Questions??

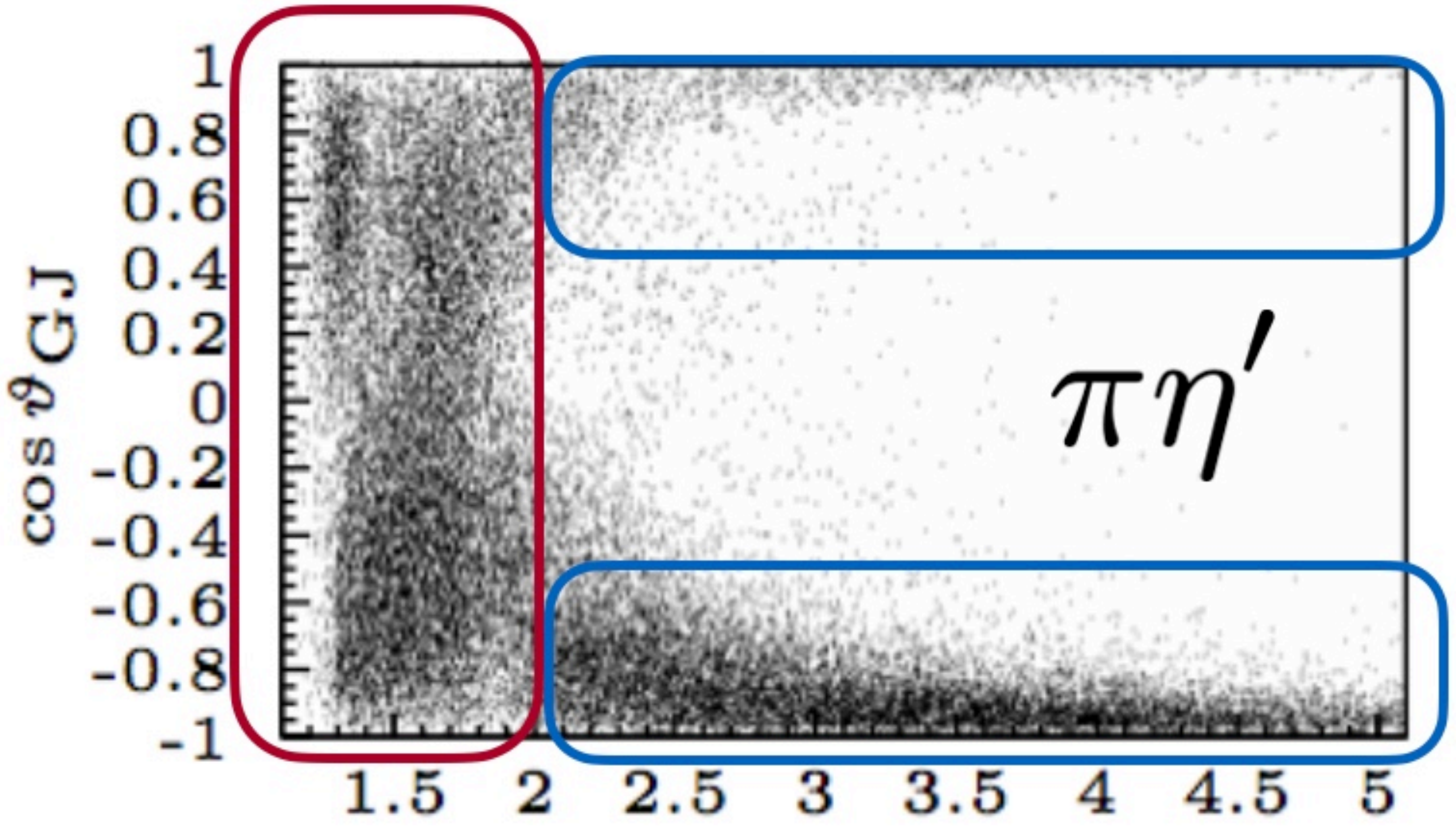
Hybrid

We have a multiplet partner



The second one should also be here

Kinematics are more challenging



Mixed in data