

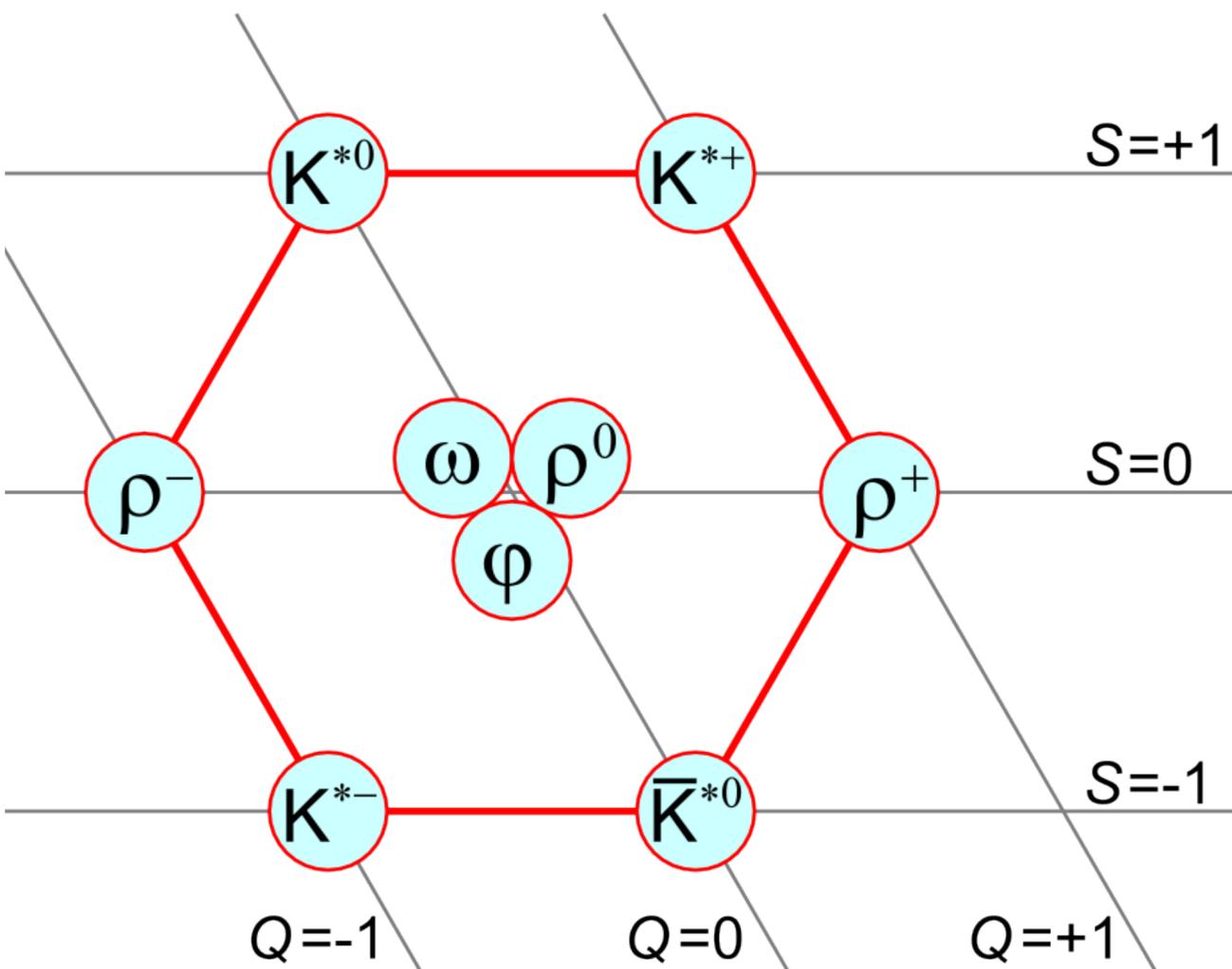
The Quest for Exotica

II Quantum numbers and decays

Arkaitz Rodas

Quark model

Wavefunction?

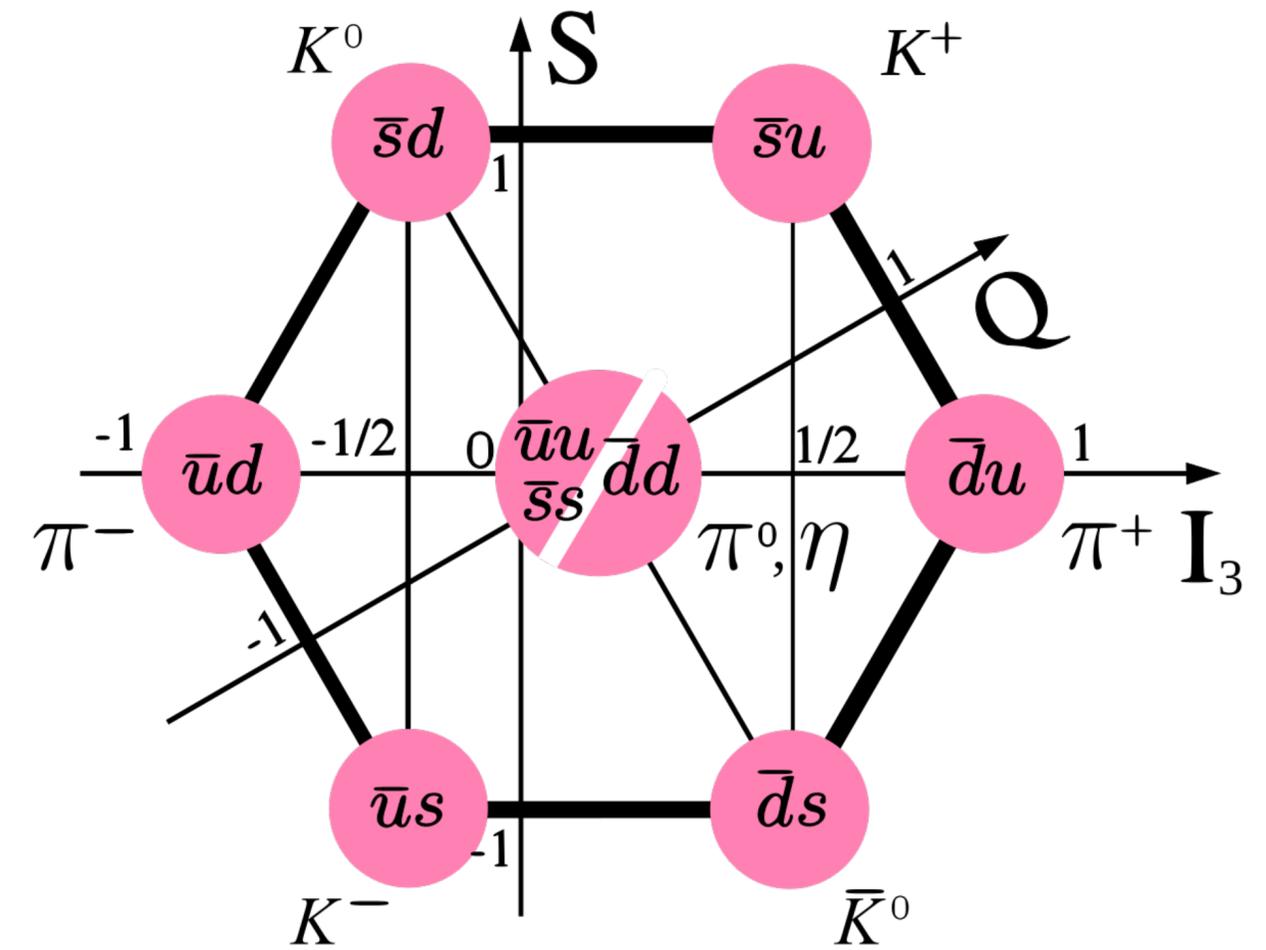


$$|\pi^+\rangle = |u\bar{d}\rangle$$

$$|\pi^-\rangle = |d\bar{u}\rangle$$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}} [|u\bar{u}\rangle - |d\bar{d}\rangle]$$

$$|q_1\bar{q}_2\rangle = \frac{1}{\sqrt{2}} [|q_1\bar{q}_2\rangle + |\bar{q}_2q_1\rangle]$$



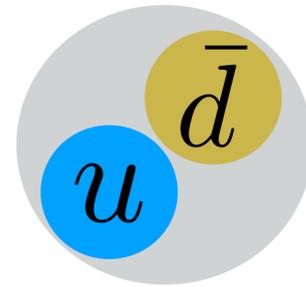
Quark model

Single q



$$|s; s_3\rangle |\vec{r}\rangle |q\rangle |c\rangle$$

Meson $\equiv q\bar{q}$



Basic QN

$$\frac{1}{\sqrt{2}} \left(1 - \hat{P}_{12}\right) |n\ell S J J_z\rangle |q_1 \bar{q}_2\rangle$$

$$\begin{aligned} |n\ell S J J_z\rangle &= \sum_{\ell S} \langle \ell S; \ell_z S_z | J J_z\rangle \sum_{s_1 s_2} |s_1 s_2; s_{1z} s_{2z}\rangle \langle s_1 s_2; s_{1z} s_{2z} | S S_z\rangle \int d^3 r |\vec{r}\rangle \langle \vec{r} | n\ell \ell_z\rangle \\ &= \sum_{s_{1z} s_{2z}} \sum_{\ell_z S_z} C_{J J_z; \ell_z S_z}^{\ell S} C_{S S_z; s_{1z} s_{2z}}^{s_1 s_2} |s_1 s_2; s_{1z} s_{2z}\rangle \int d^3 r R_{n\ell}(r) Y_{\ell}^{\ell_z}(\hat{n}) |\vec{r}\rangle \end{aligned}$$

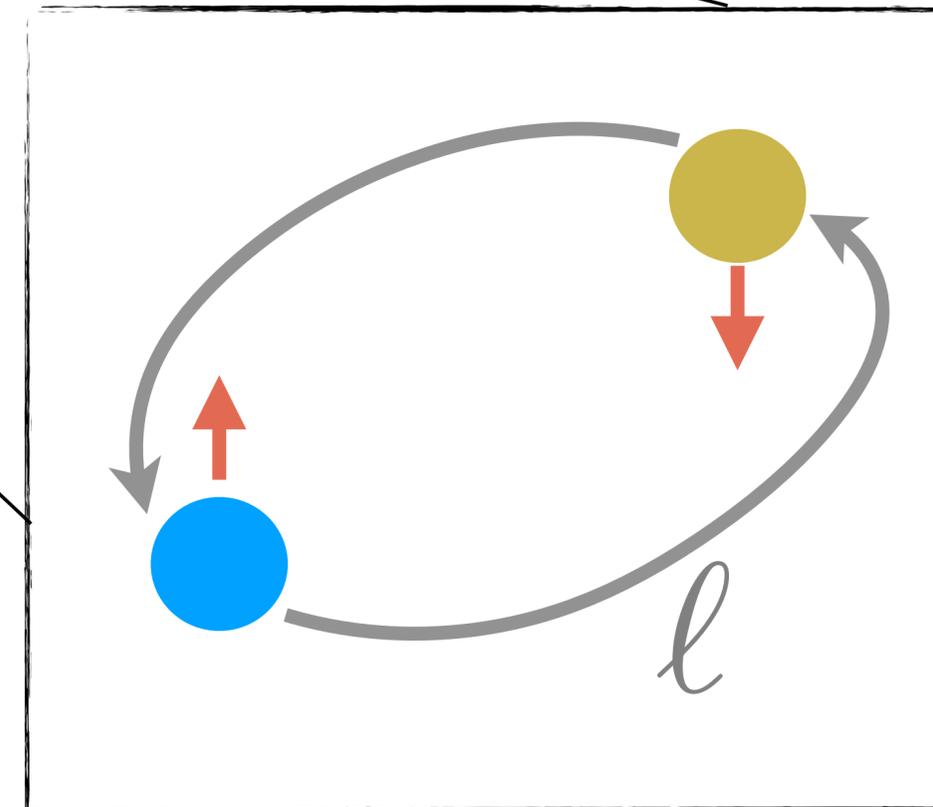
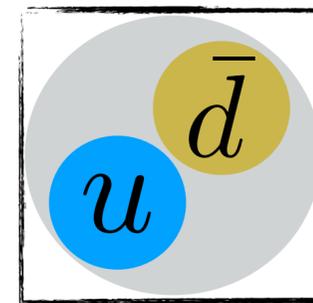
Quark model

Spectroscopic notation

$$|n^{2S+1} \ell_J (q_1 \bar{q}_2)\rangle = \frac{1}{\sqrt{2}} (1 - \hat{P}_{12}) |n \ell S J J_z\rangle |q_1 \bar{q}_2\rangle$$

$$= \frac{1}{\sqrt{6}} \sum_{s_{1z} s_{2z}} \sum_{\ell_z S_z} C_{JJ_z; \ell_z S_z}^{\ell S} C_{SS_z; s_{1z} s_{2z}}^{s_1 s_2} |s_1 s_2; s_{1z} s_{2z}\rangle$$

$$\times \int d^3 r R_{n\ell}(r) Y_{\ell}^{\ell_z}(\hat{n}) |\vec{r}\rangle \times [|q_1 \bar{q}_2\rangle + (-1)^{\ell+S} |\bar{q}_2 q_1\rangle]$$



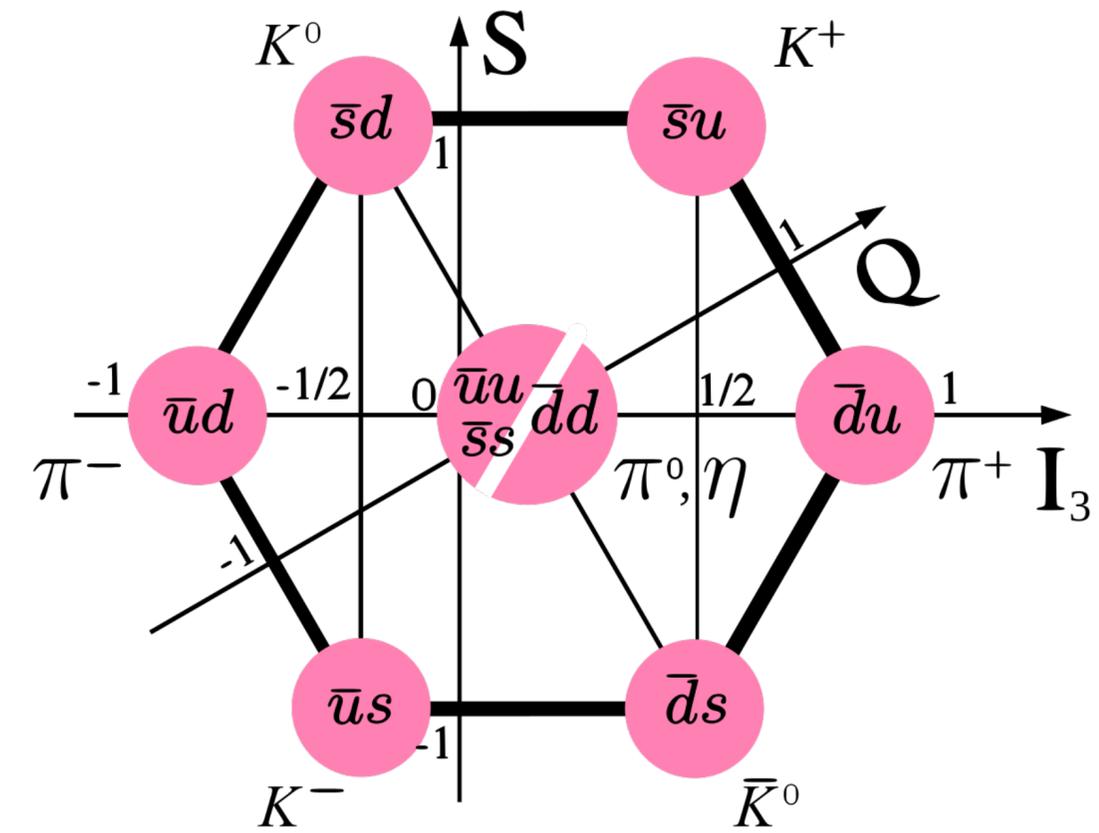
$$\hat{P} |n^{2S+1} \ell_J (q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J (q\bar{q})\rangle$$

$$\hat{C} |n^{2S+1} \ell_J (q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J (q\bar{q})\rangle$$

Quark model

States classified by J^{PC}

Not all QN allowed for $q\bar{q}$



Spin

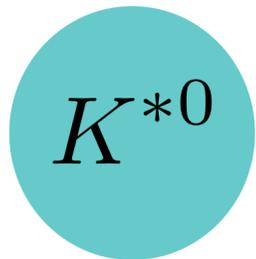
	J^{PC}	
	singlet	triplet
$l = 0$	0^{-+}	1^{--}
$l = 1$	1^{+-}	$(0, 1, 2)^{++}$
$l = 2$	2^{-+}	$(1, 2, 3)^{--}$

$$\hat{P} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

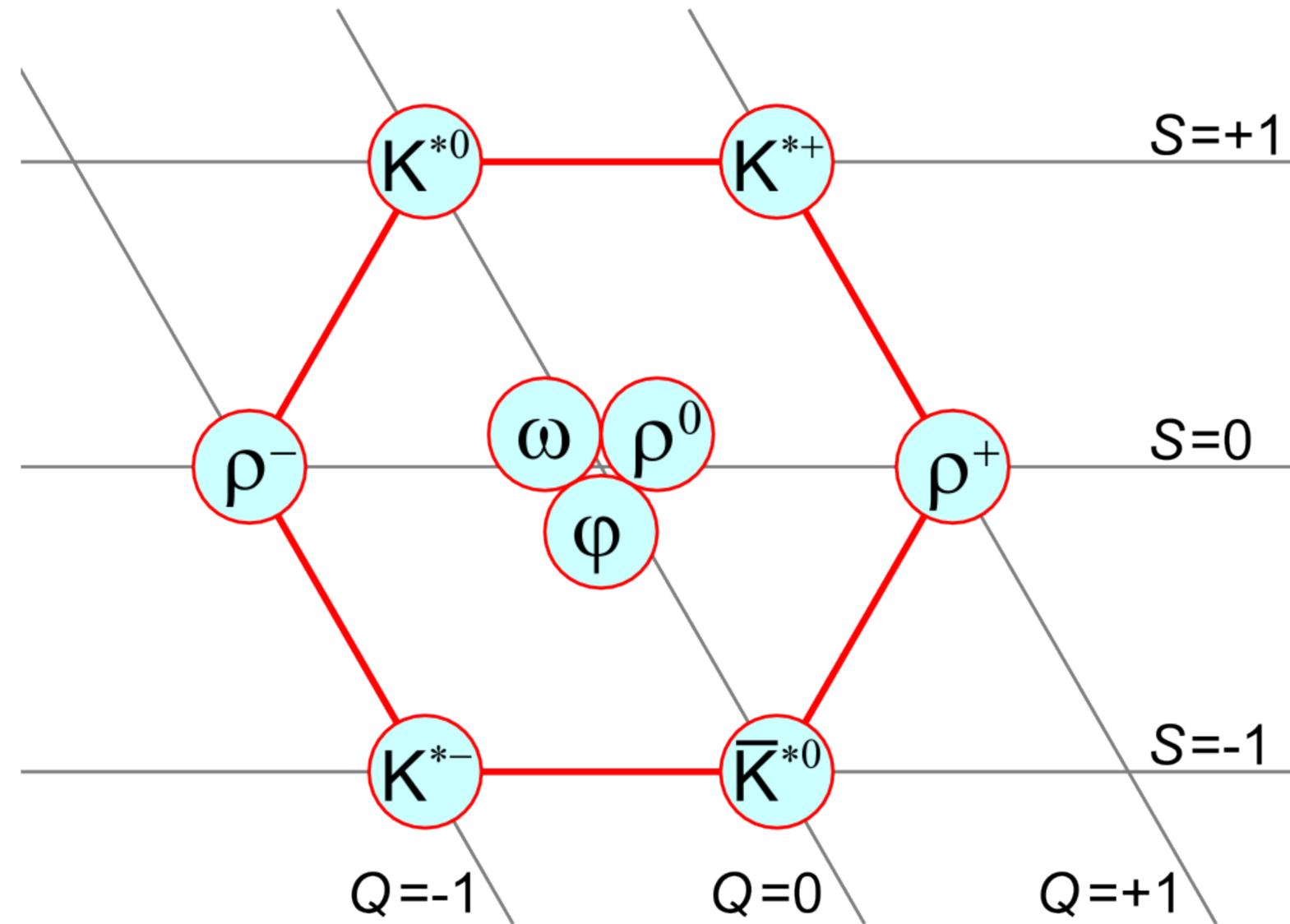
$$\hat{C} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

Quark model

Crucial for hadron physics



$$J^{PC} = 1^{--}$$

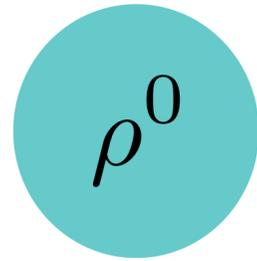


$$\hat{P} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

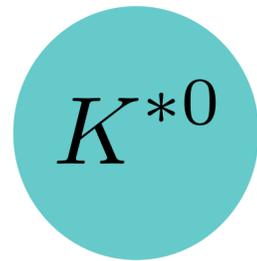
$$\hat{C} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

Quark model

Vector mesons



$$\frac{(u\bar{u} - d\bar{d})}{\sqrt{2}}$$

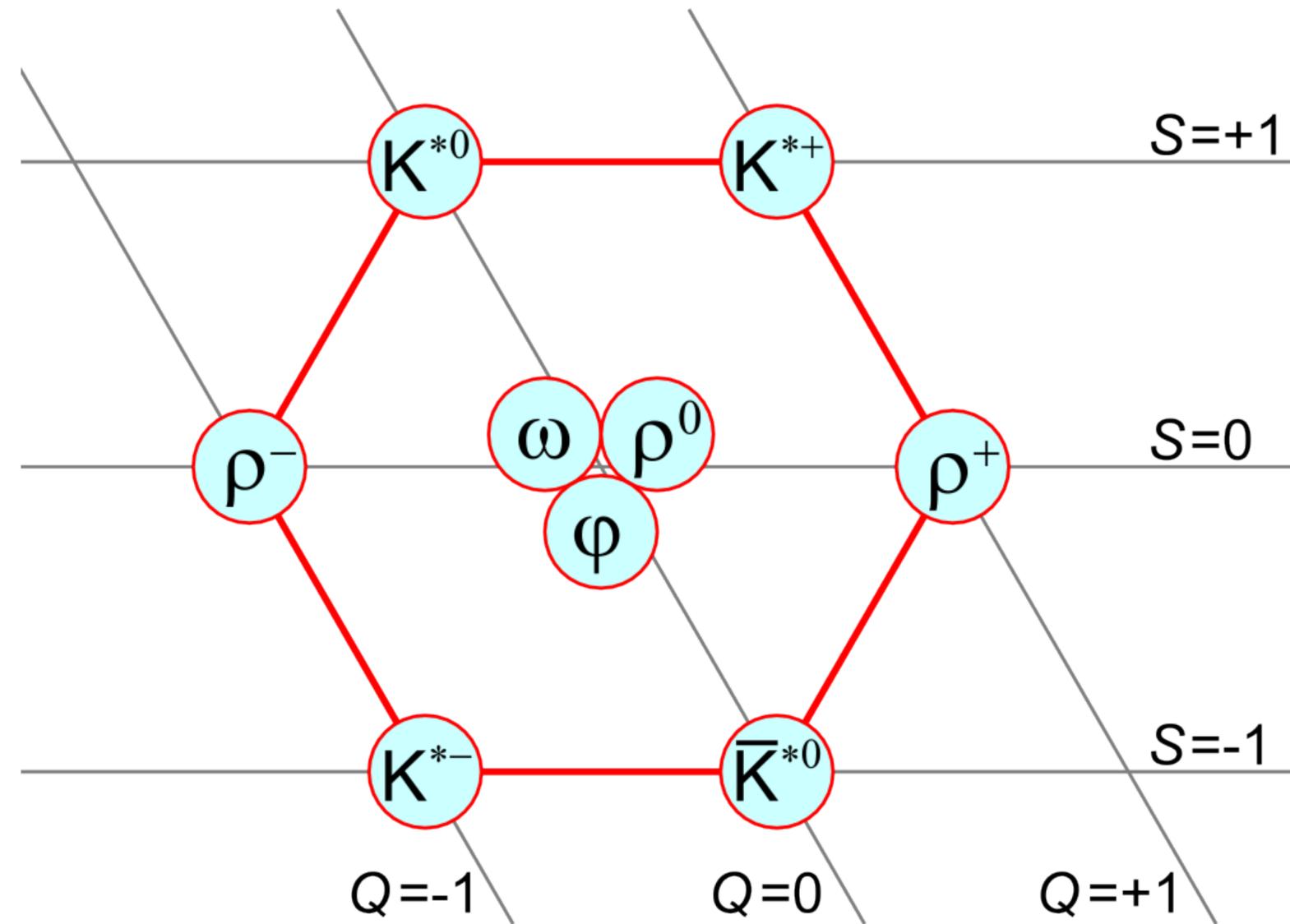


$$d\bar{s}$$



$$\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}}$$

$$J^{PC} = 1^{--}$$



$$\hat{P} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

$$\hat{C} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

Quark model

Coupling to photons

ρ^0

$$\Gamma(\rho) \propto \frac{1}{2} (Q_u - Q_d)^2 = \frac{1}{18} 9$$

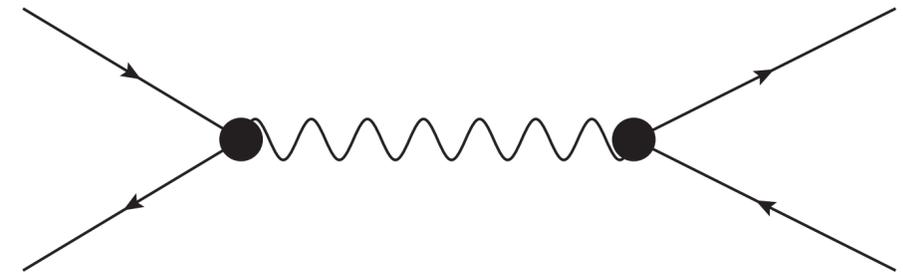
$J^{PC} = 1^{--}$

ϕ

$$\Gamma(\phi) \propto \frac{1}{2} Q_s^2 = \frac{1}{18} 2$$

ω^0

$$\Gamma(\omega) \propto \frac{1}{2} (Q_u + Q_d)^2 = \frac{1}{18}$$



Theoretical expectation

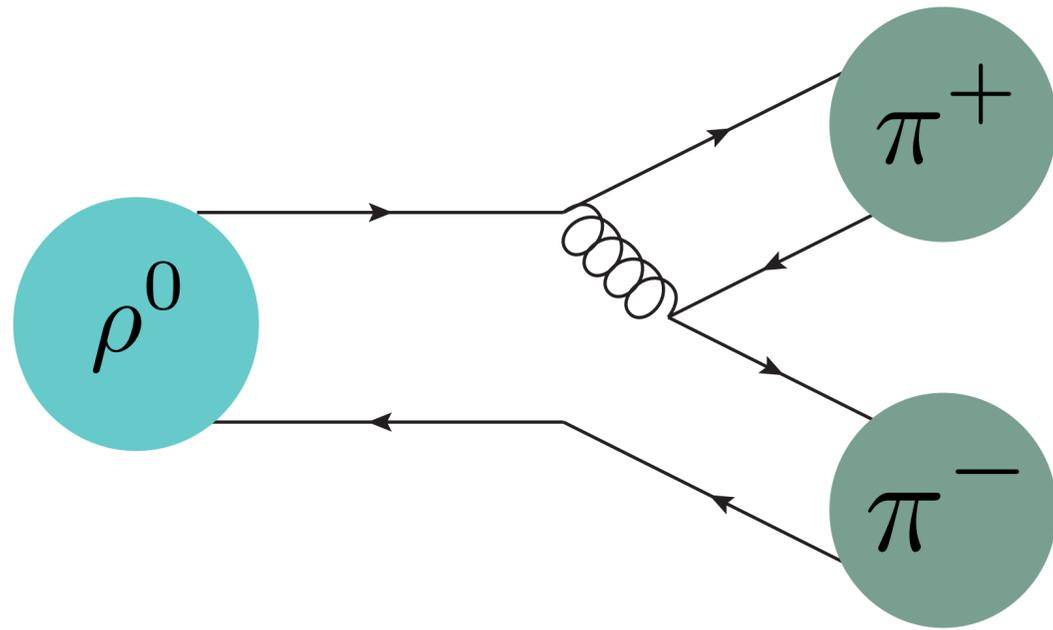
$$\Gamma_{\rho^0} : \Gamma_{\omega^0} : \Gamma_{\phi} = 9 : 1 : 2$$

Experiment

$$(8.8 \pm 2.6) : 1 : (1.7 \pm 0.4)$$

It works!

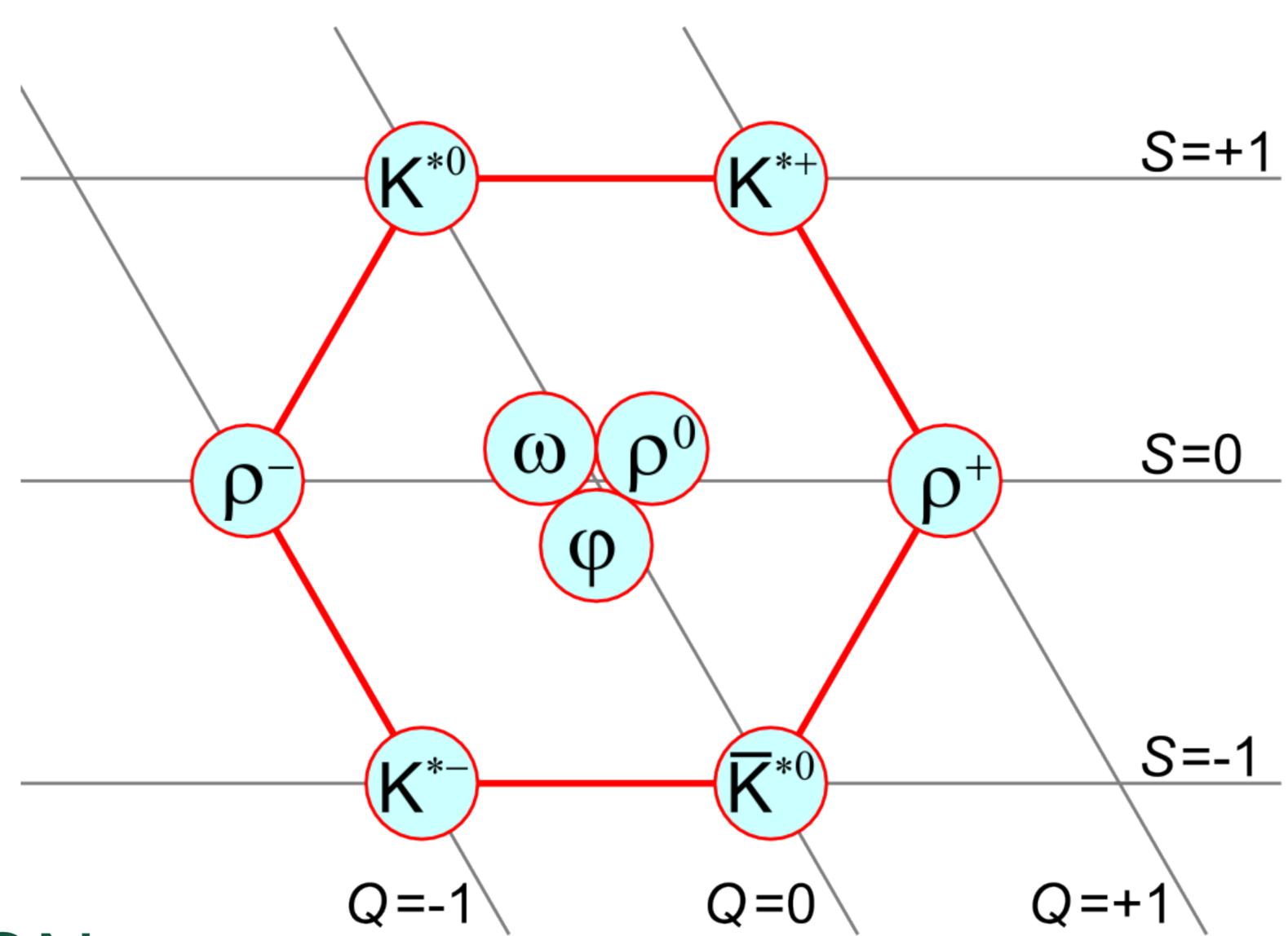
$I^G(J^{PC})$



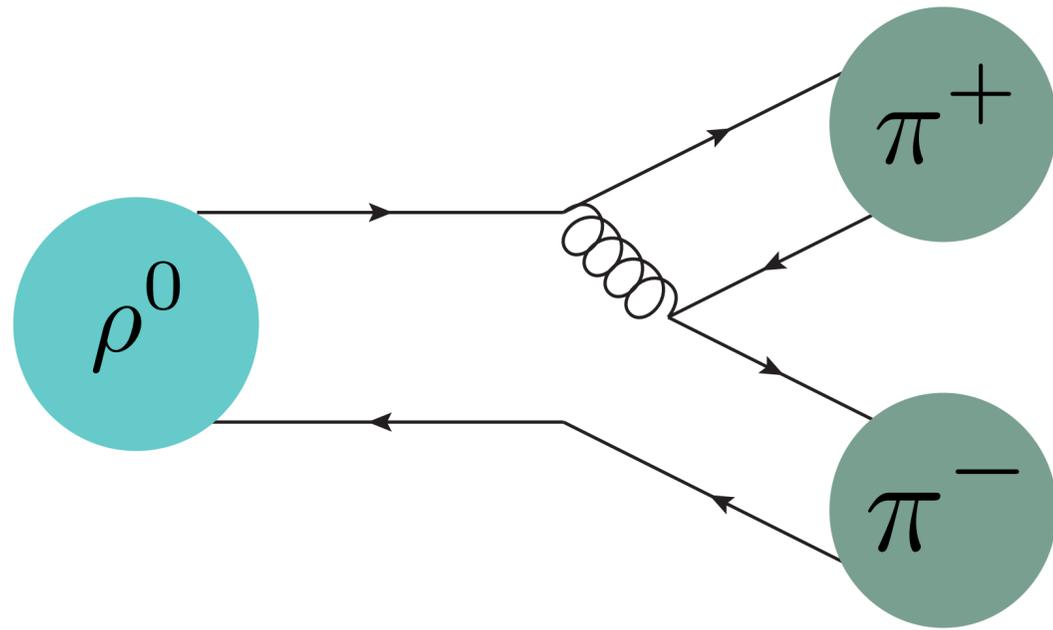
K^* ??

ω ??

We need more QN



$I^G(J^{PC})$

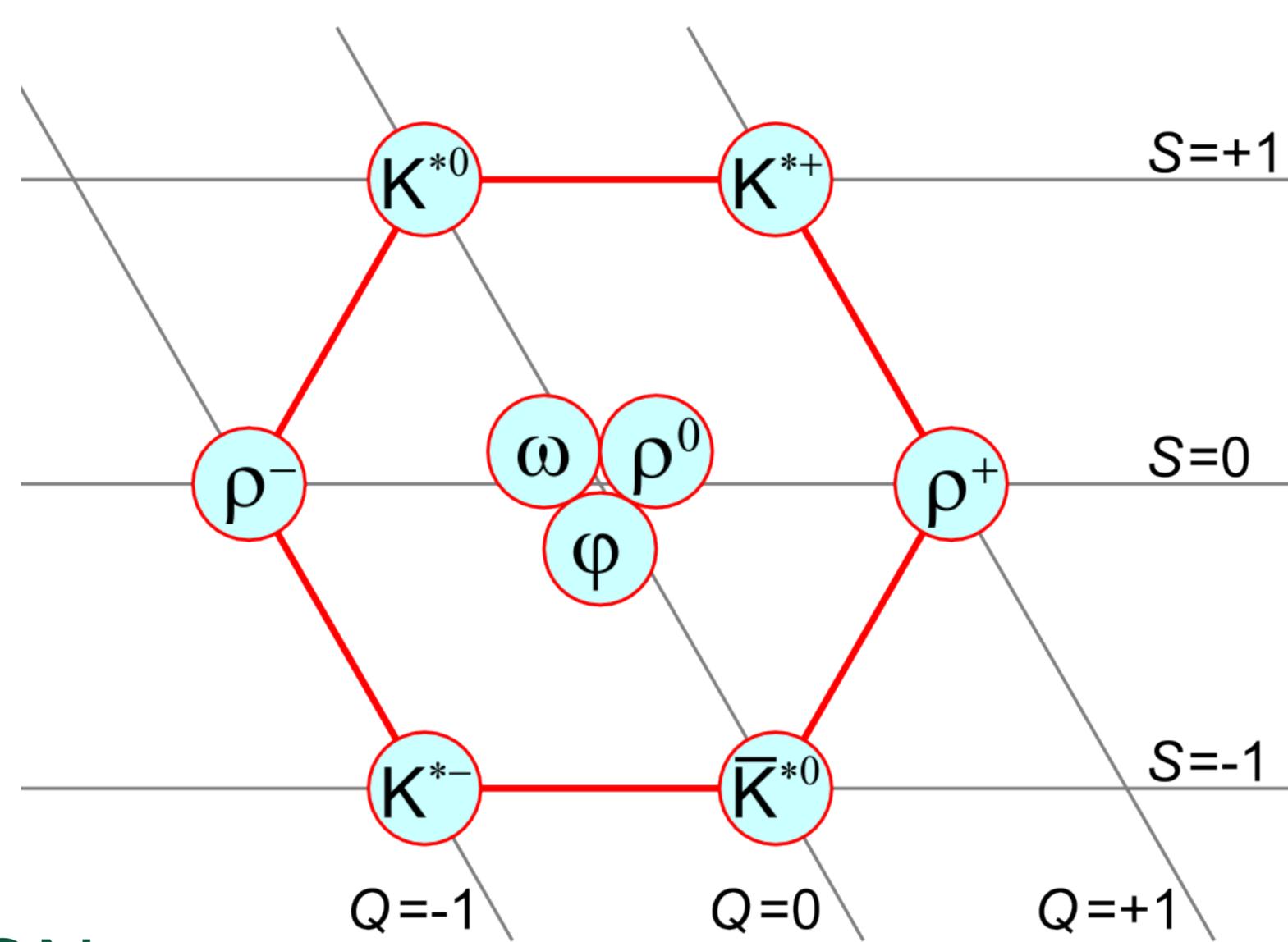


$J^{PC} = 1^{--}$

K^* ??

ω ??

We need more QN



Isospin??

$I^G(J^{PC})$

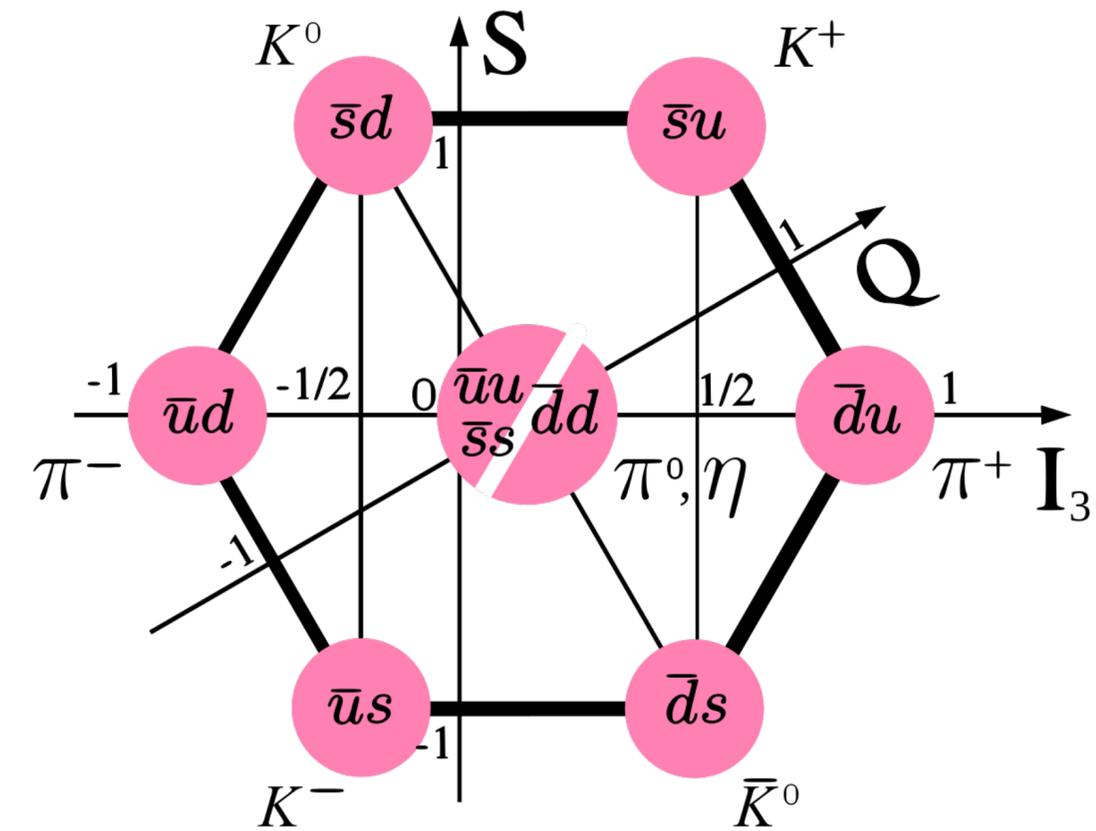
$$CI_y = +I_y C$$

Commutation

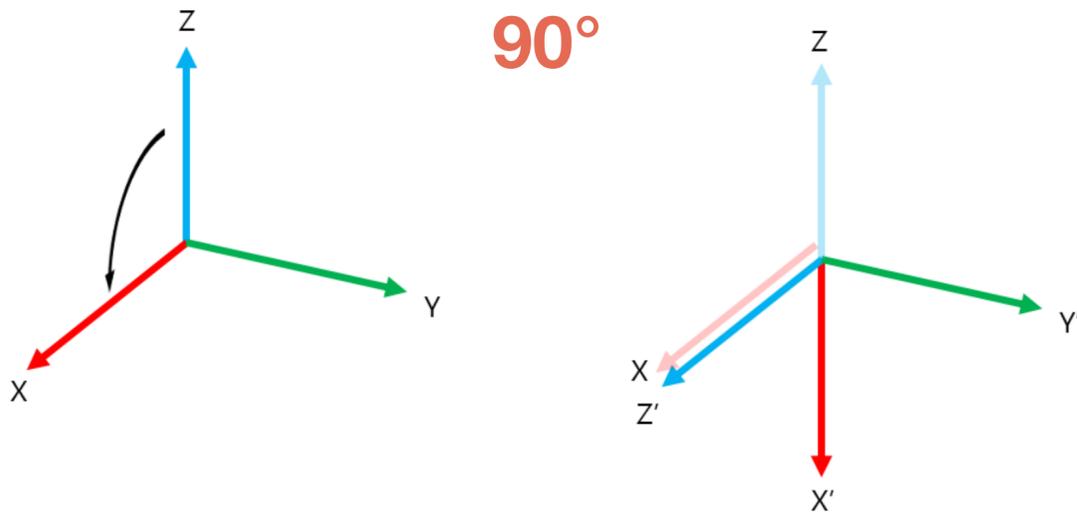
Rotation generator y axis

Rotation

$$\hat{R}_y(\pi) = \exp(-i\pi I_y)$$



90°

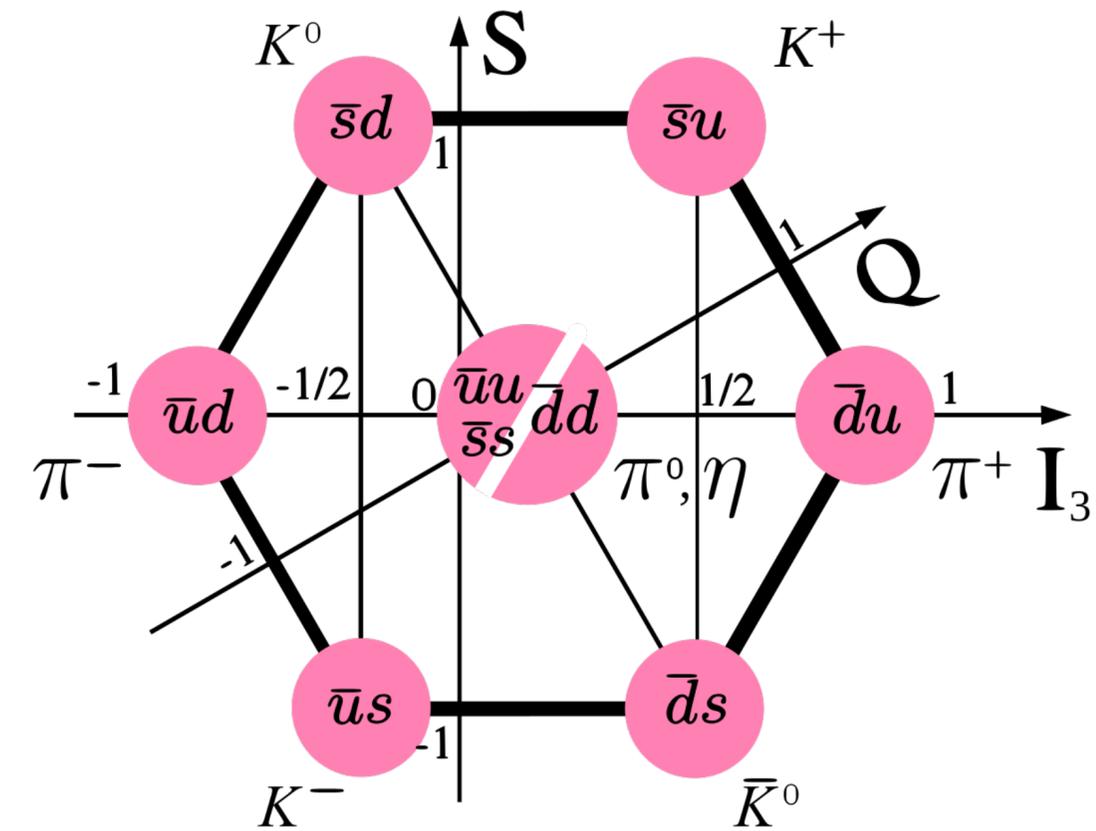


Commutation

$$\begin{aligned} \hat{R}_y(\pi) I_x &= -I_x \hat{R}_y(\pi) \\ \hat{R}_y(\pi) I_y &= +I_y \hat{R}_y(\pi) \\ \hat{R}_y(\pi) I_z &= -I_z \hat{R}_y(\pi) \end{aligned}$$

We classify particles with $I^G(J^{PC})$

$$\hat{G} \equiv \hat{C}\hat{R}_y(\pi) = \hat{R}_y(\pi)\hat{C} = \hat{C}e^{-i\pi\hat{I}_y}$$



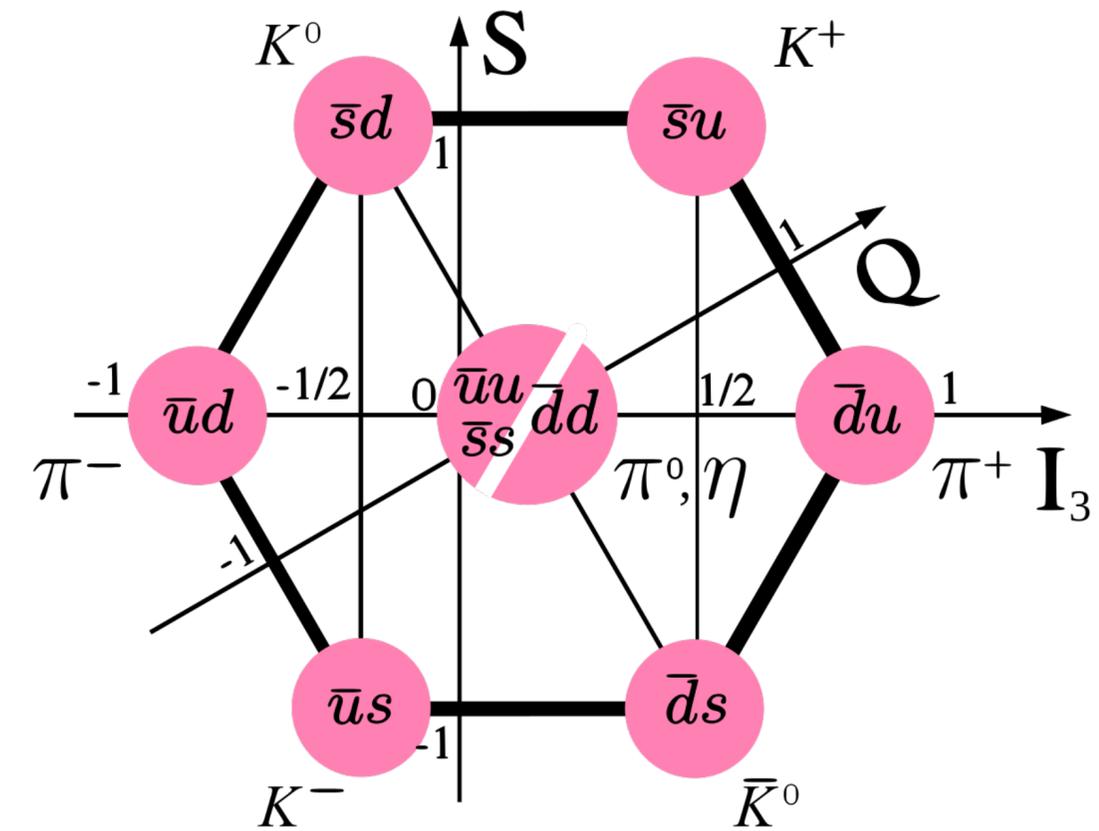
$$\hat{P} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

$$\hat{C} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

π^0

$$I^G(J^{PC}) = 1^-(0^-+)$$

We have omitted some results that have been superseded by later experiments. The omitted results may be found in our 1988 edition Physics Letters **B204** 1 (1988).



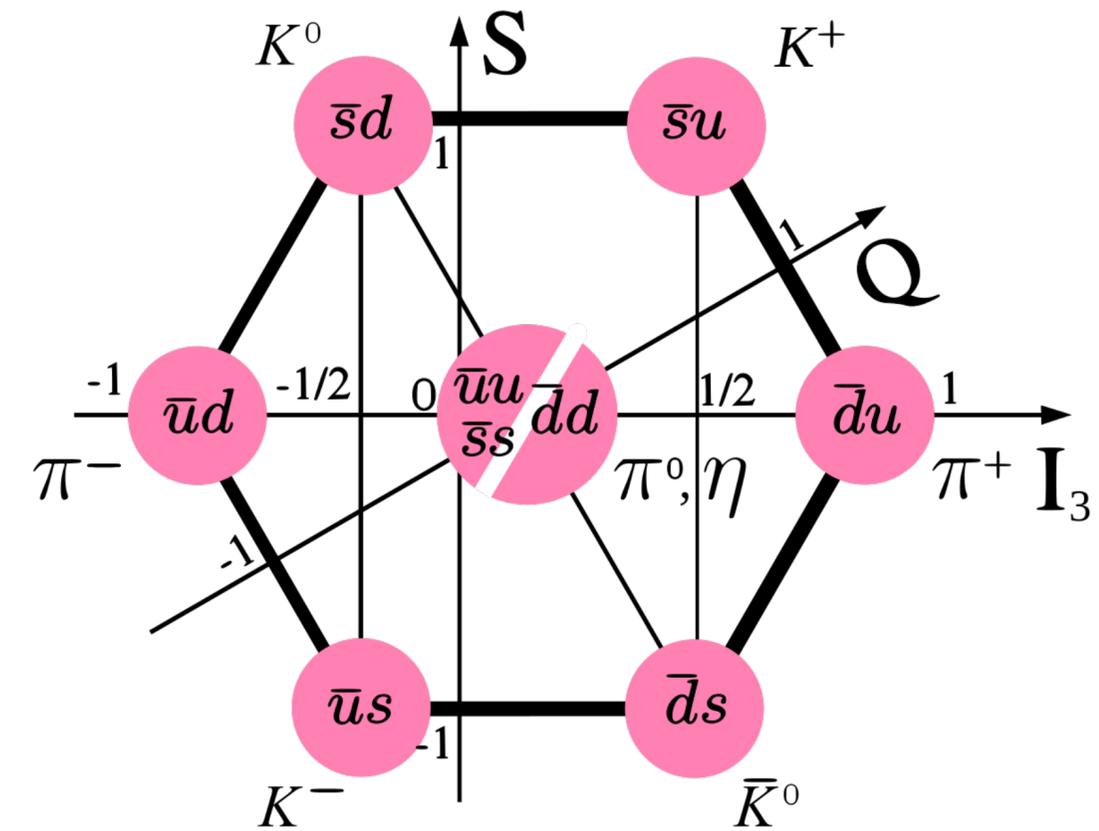
$$\hat{P} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+1} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

$$\hat{C} |n^{2S+1} \ell_J(q\bar{q})\rangle = (-1)^{\ell+S} |n^{2S+1} \ell_J(q\bar{q})\rangle$$

π^0

$$I^G(J^{PC}) = 1^-(0^-+)$$

We have omitted some results that have been superseded by later experiments. The omitted results may be found in our 1988 edition Physics Letters **B204** 1 (1988).



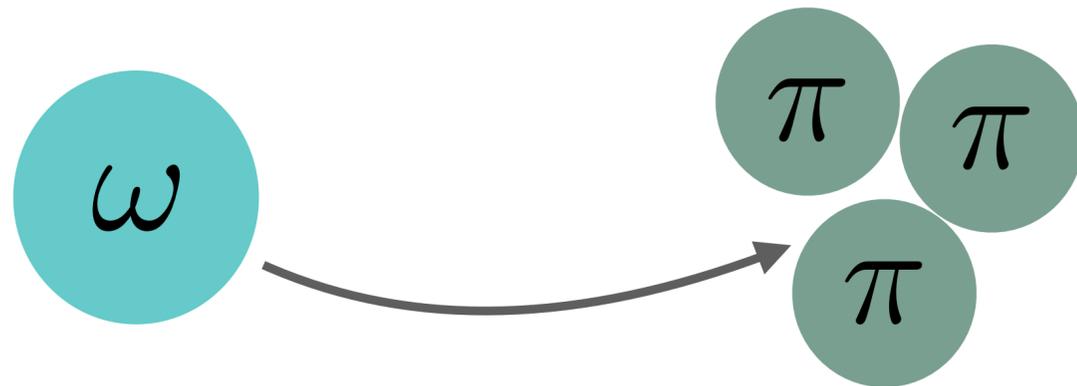
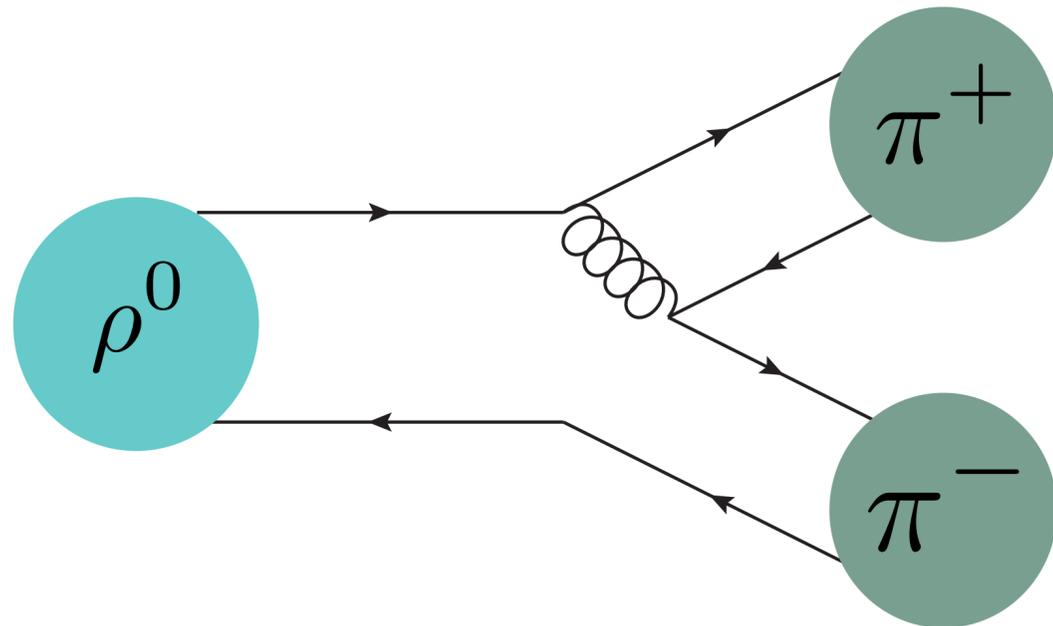
$$\hat{R}_y(\pi) = \begin{pmatrix} 0 & 0 & +1 \\ 0 & -1 & 0 \\ +1 & 0 & 0 \end{pmatrix} \text{ Pions}$$

$$G|\pi^+\rangle = C\hat{R}_y(\pi)|\pi^+\rangle = C|\pi^-\rangle = -|\pi^+\rangle$$

$$G = C(-1)^I$$

Questions??

Decays

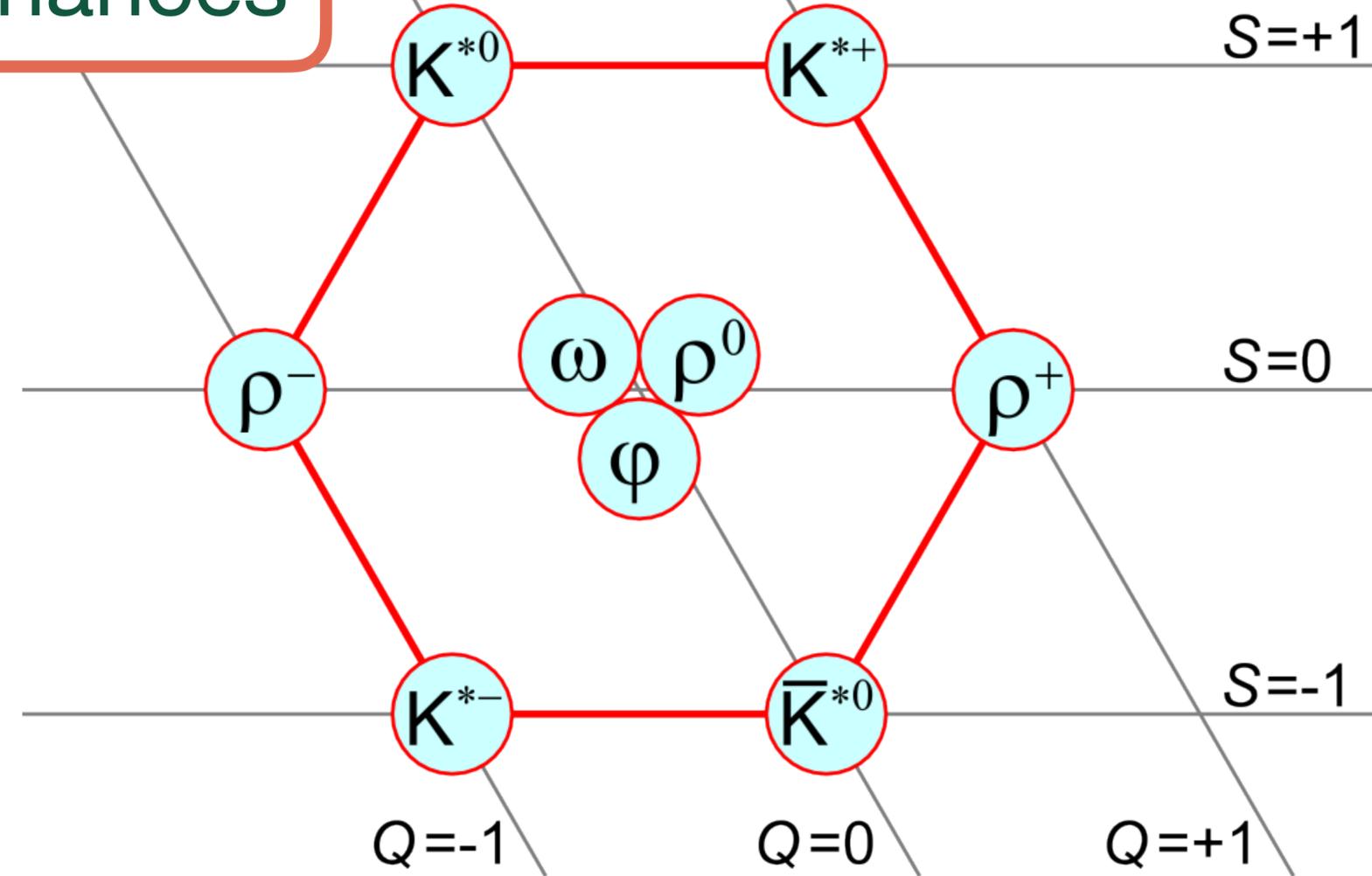


Resonances

$$I^G = 1^+$$

$$I = 1/2$$

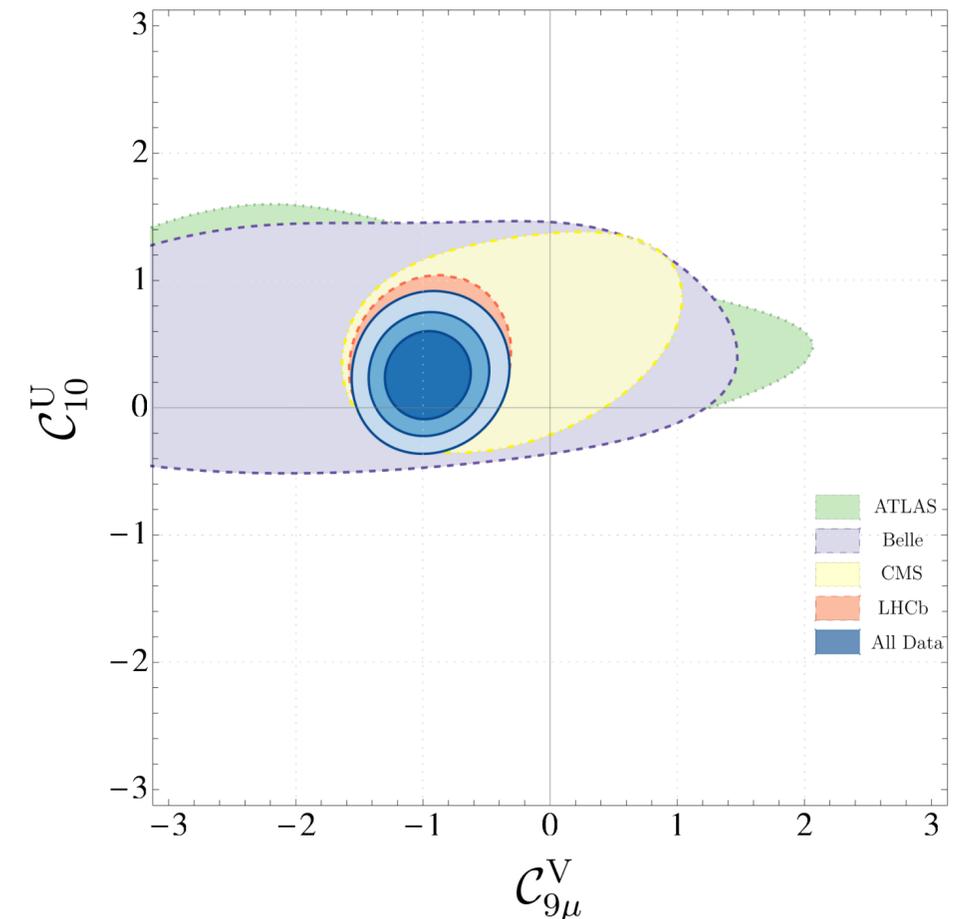
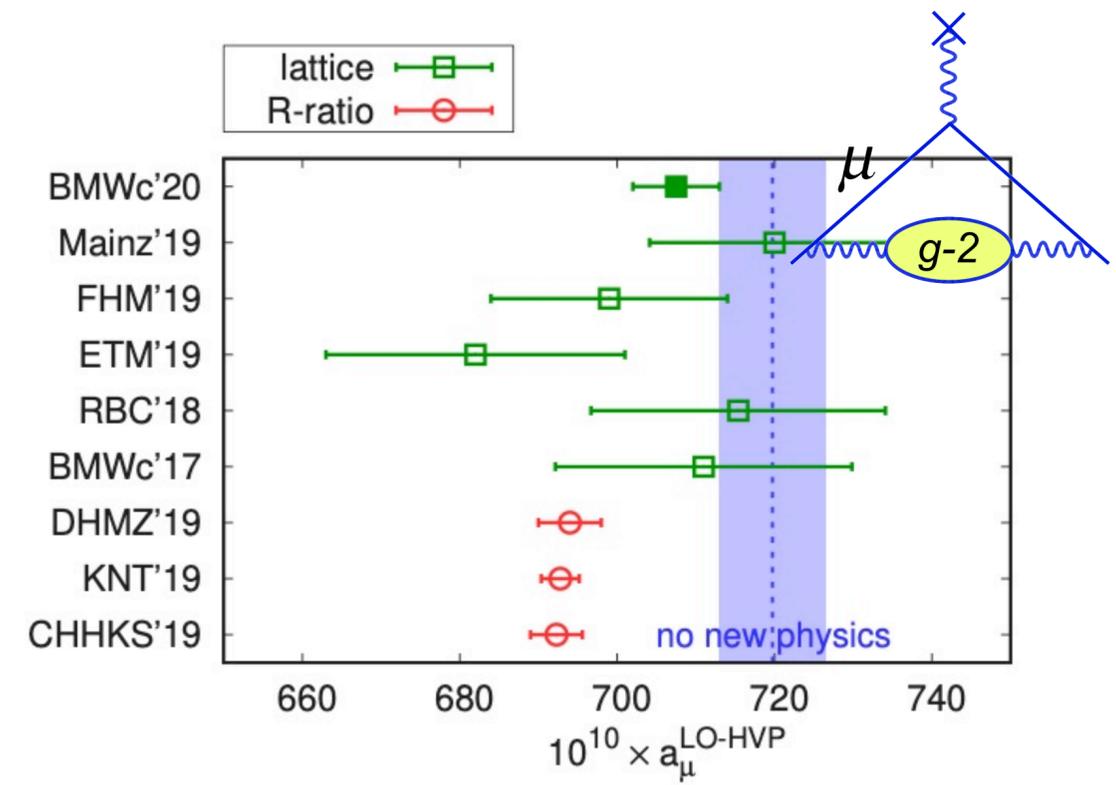
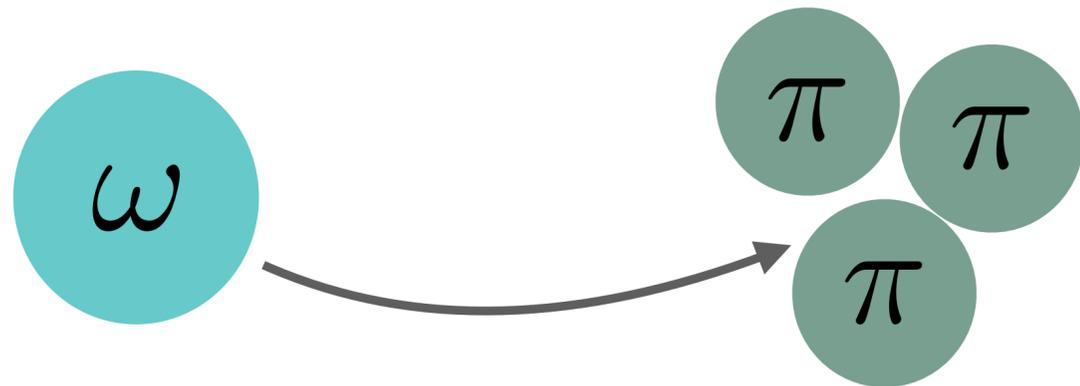
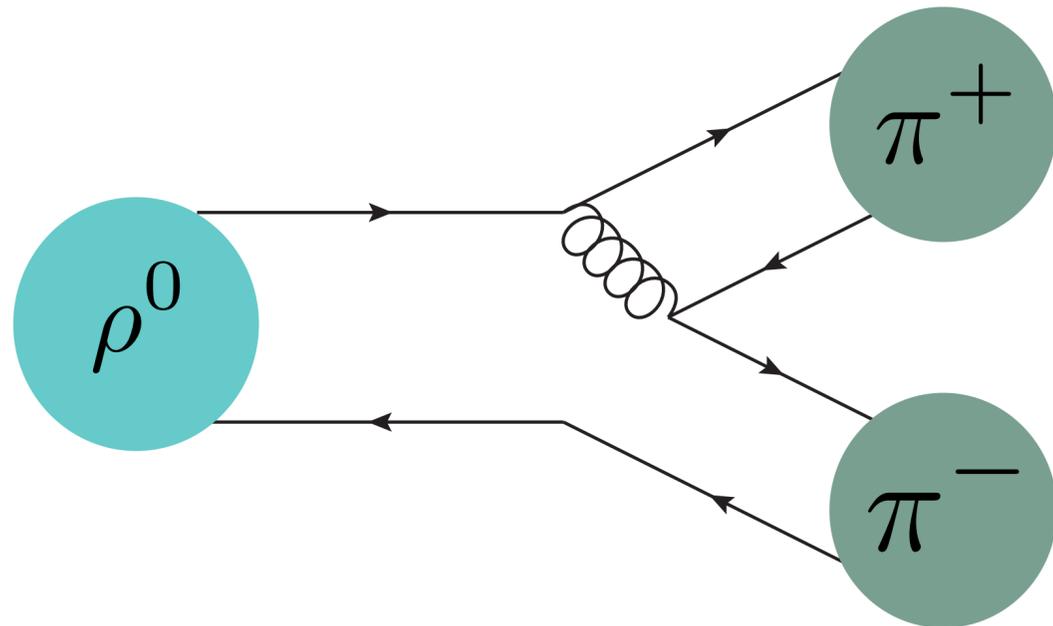
$$I^G = 1^-$$



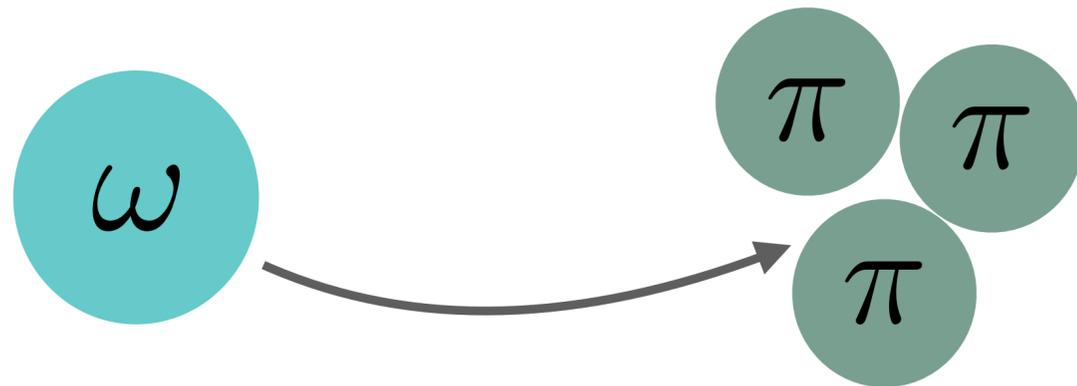
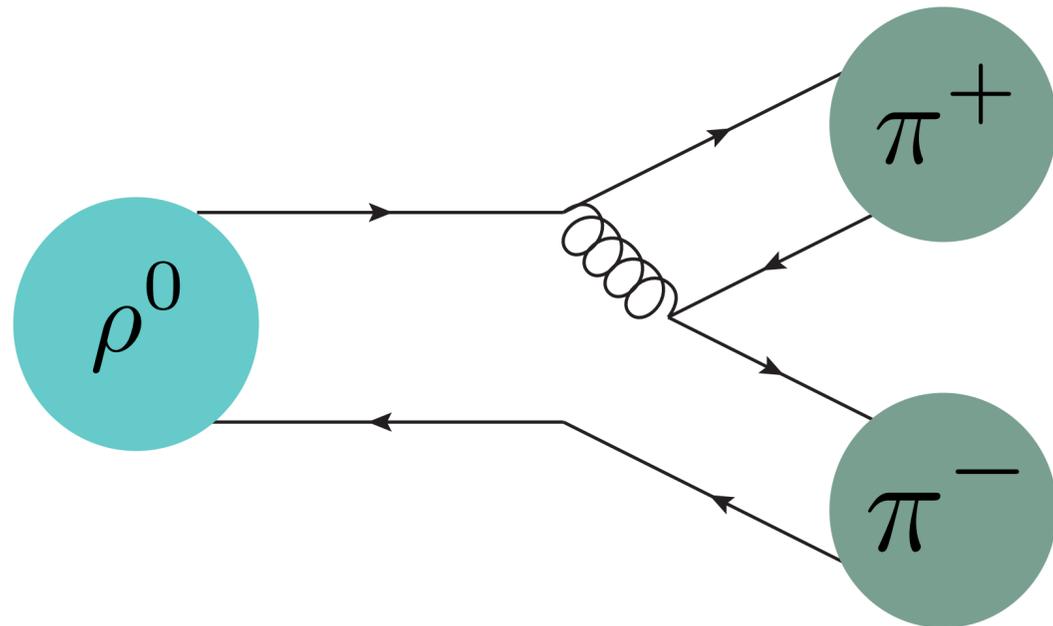
$$G = C(-1)^I$$

Decays

Very relevant



Decays



Resonances

Too short lived

$$c\tau_\rho = 1.3 \text{ fm}$$

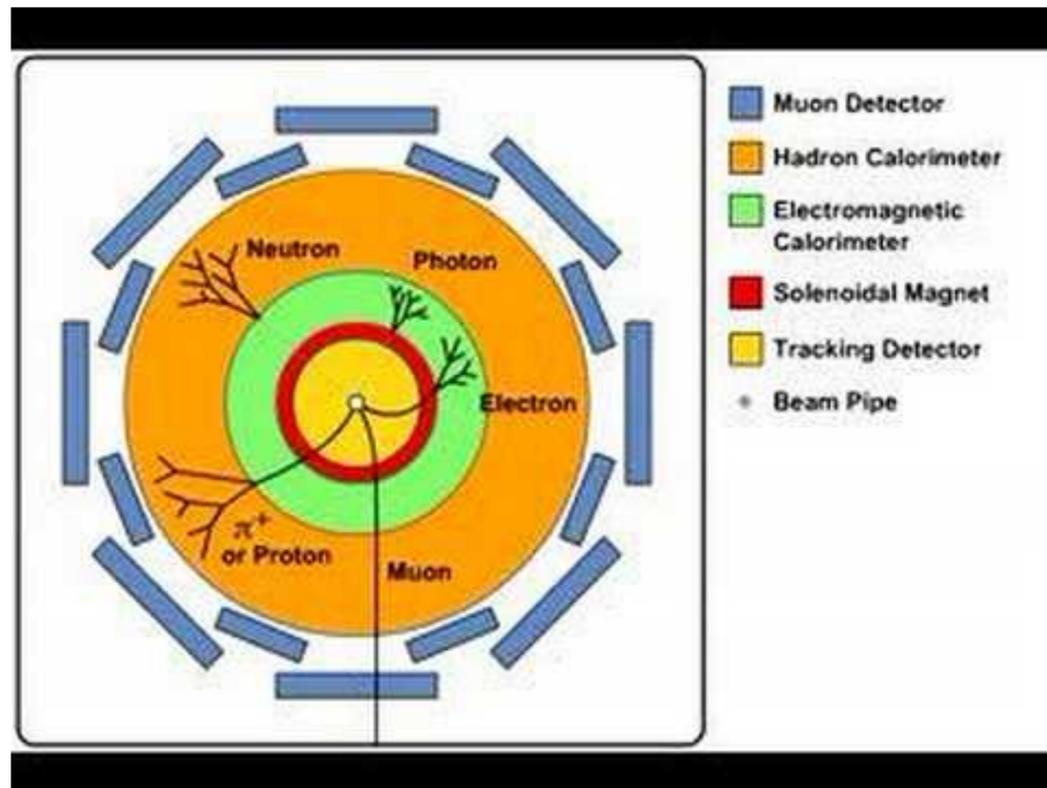
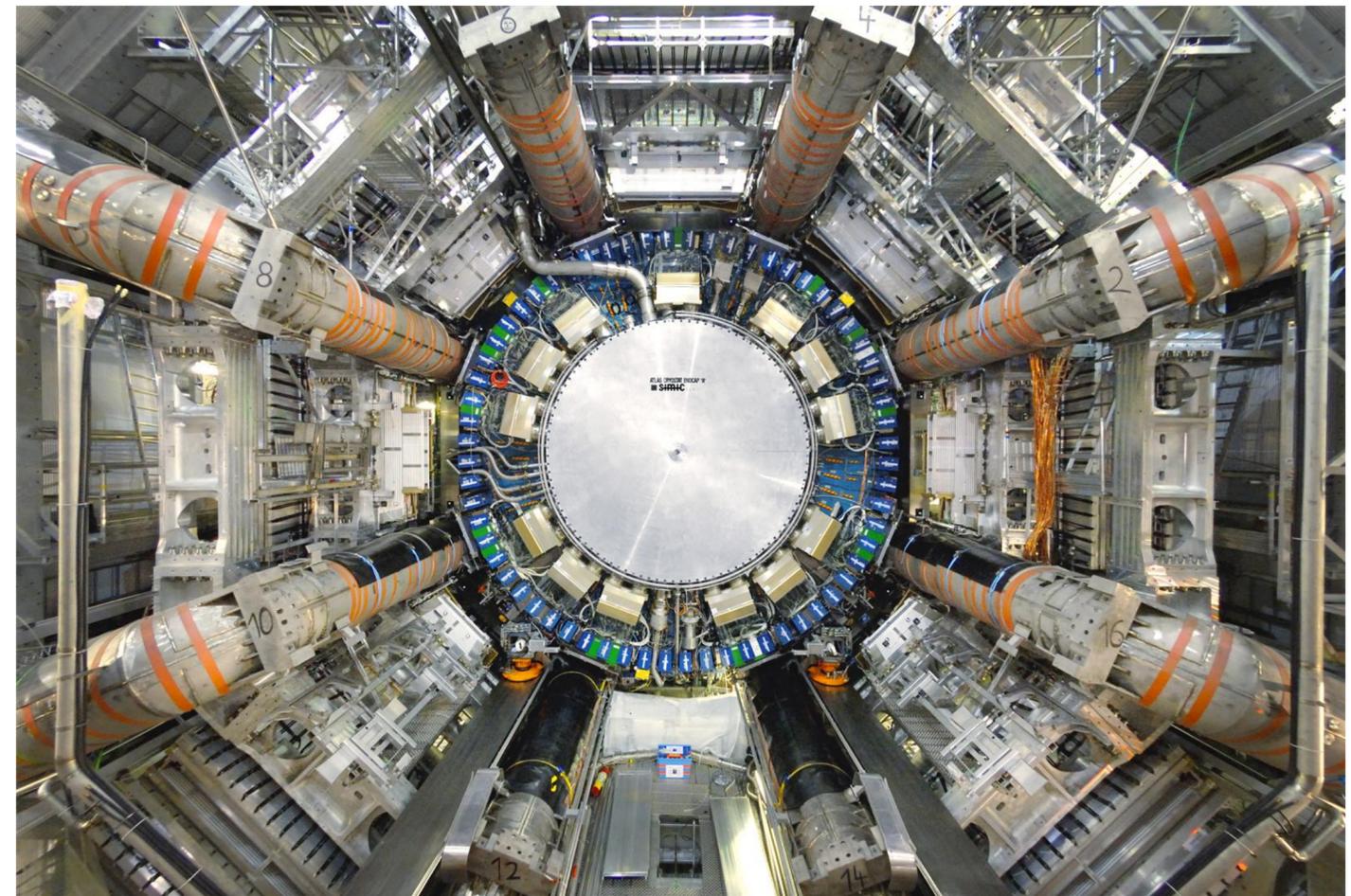
$$c\tau_{K^*} = 3.8 \text{ fm}$$

$$c\tau_\omega = 22.5 \text{ fm}$$

Decays

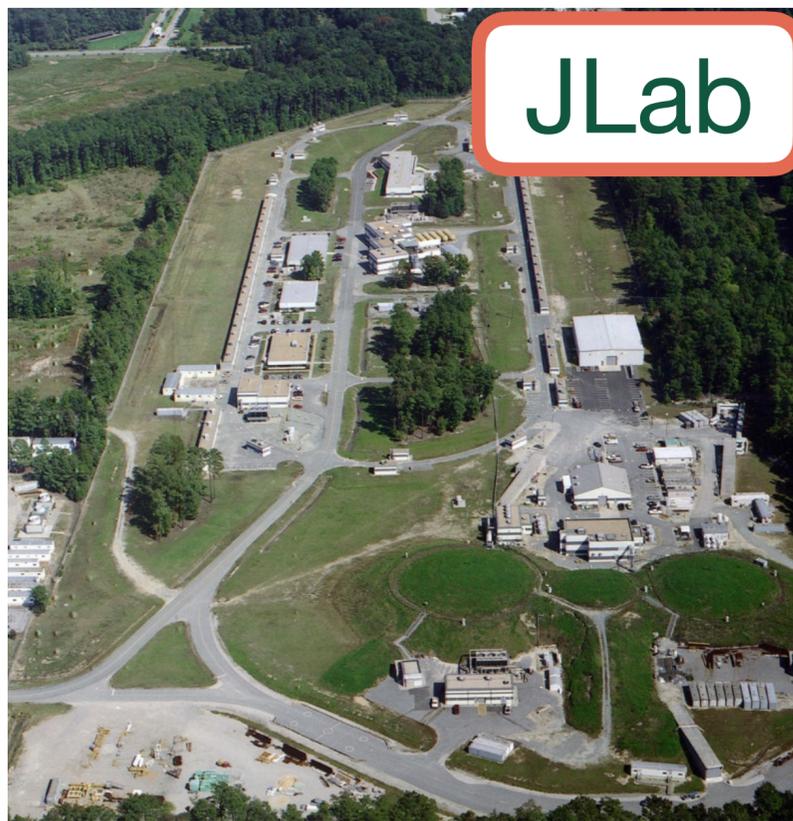
Detecting particles

Resonances leave tracks

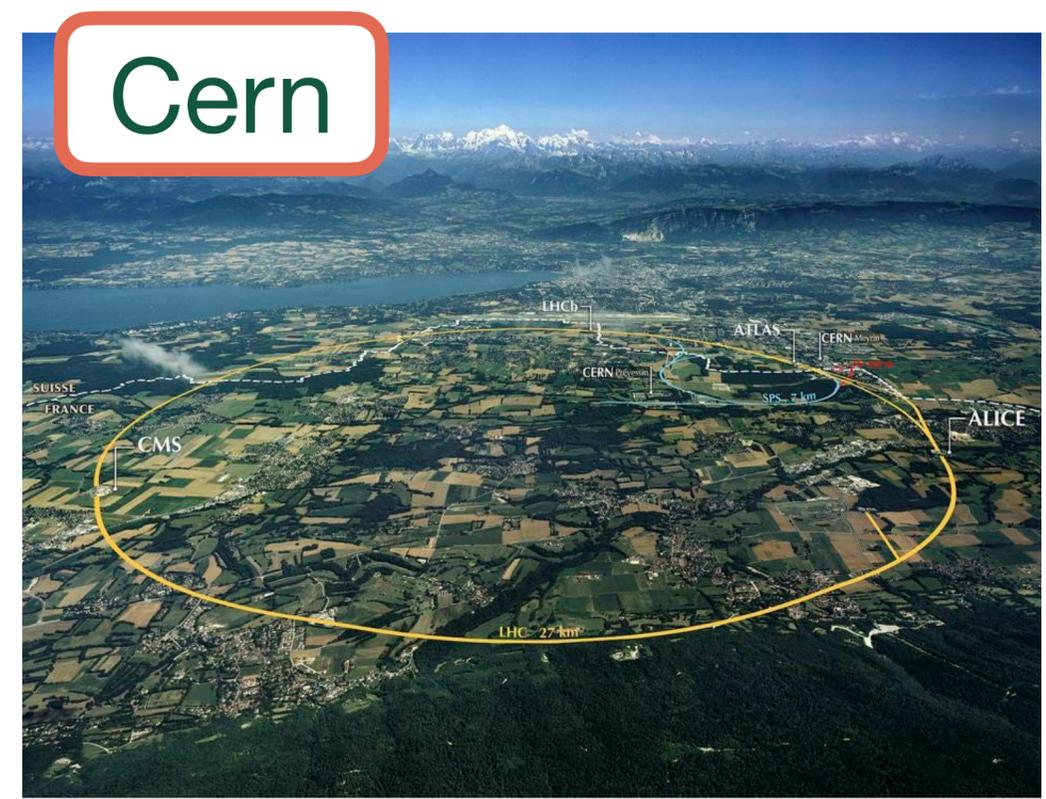


We use these huge apparatus to detect their decays

A computer counts the # hits

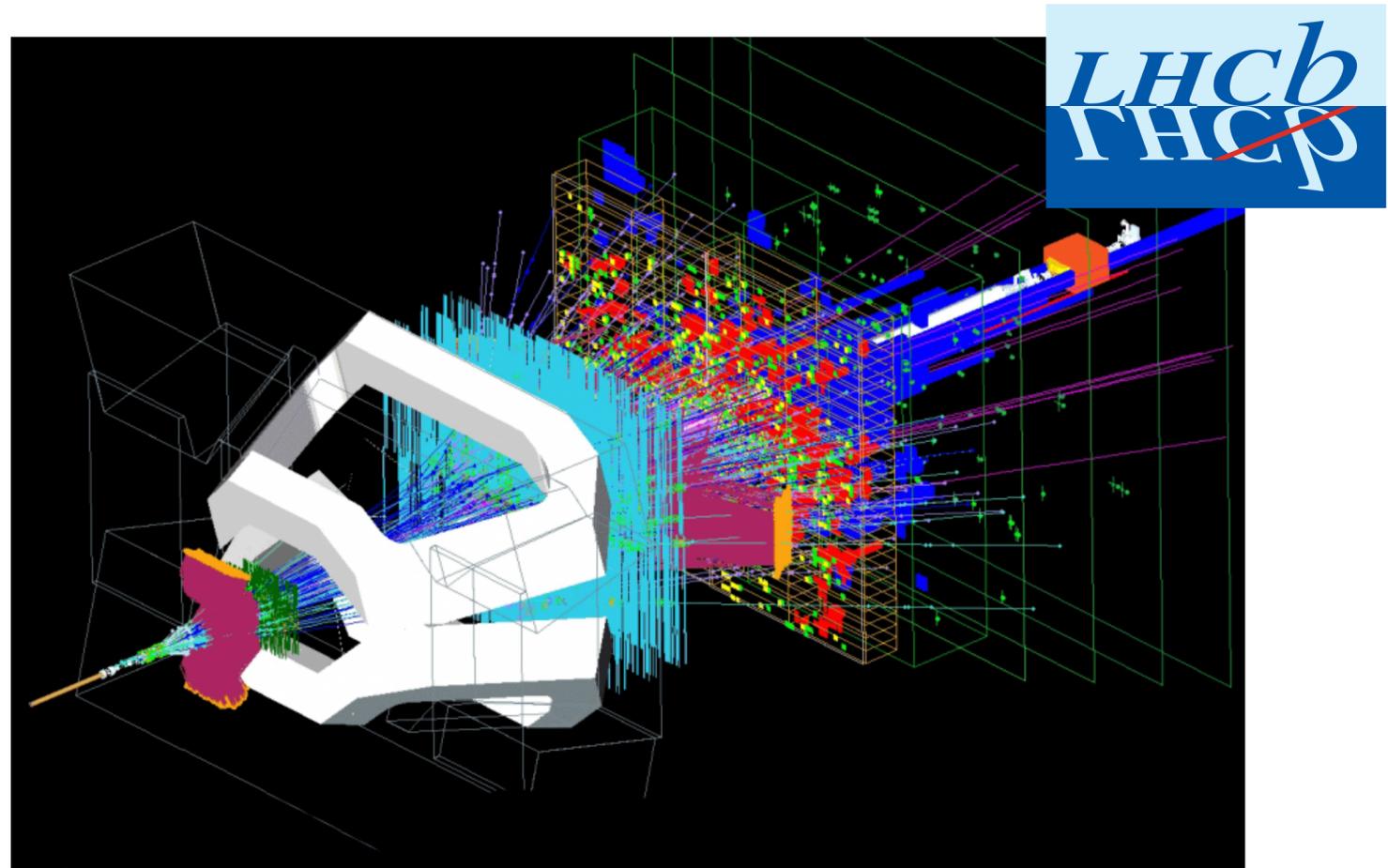


JLab



Cern

1. Accelerate particles
2. Smash them together
3. Collect their products

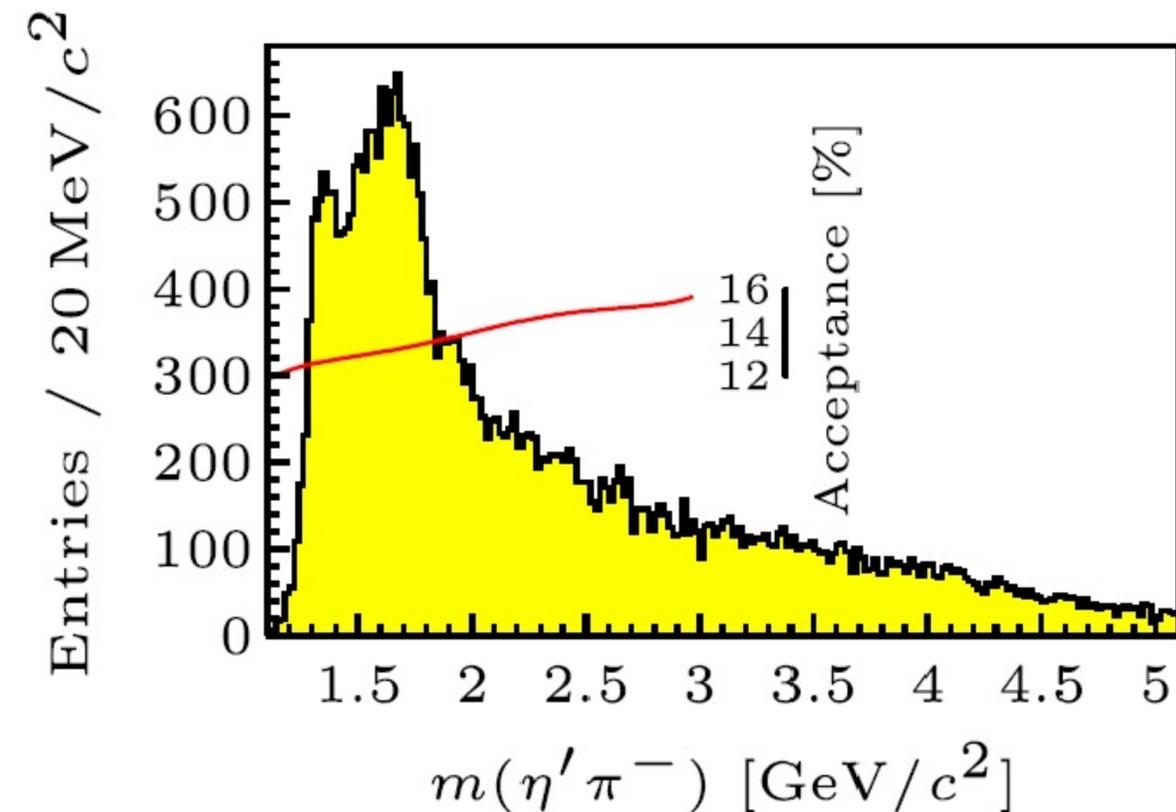
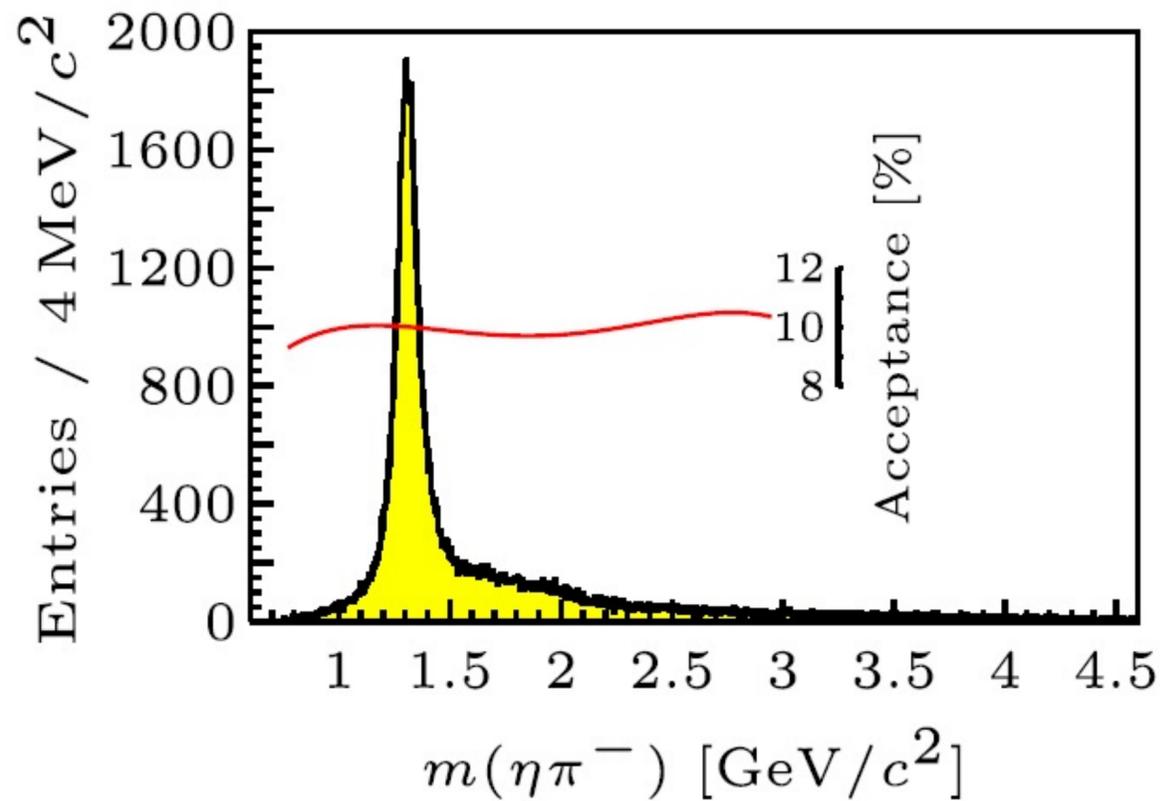


Decays

hits \rightarrow Probability of the decay

$$P = |\Psi|^2$$

We can link these hits to the amplitude

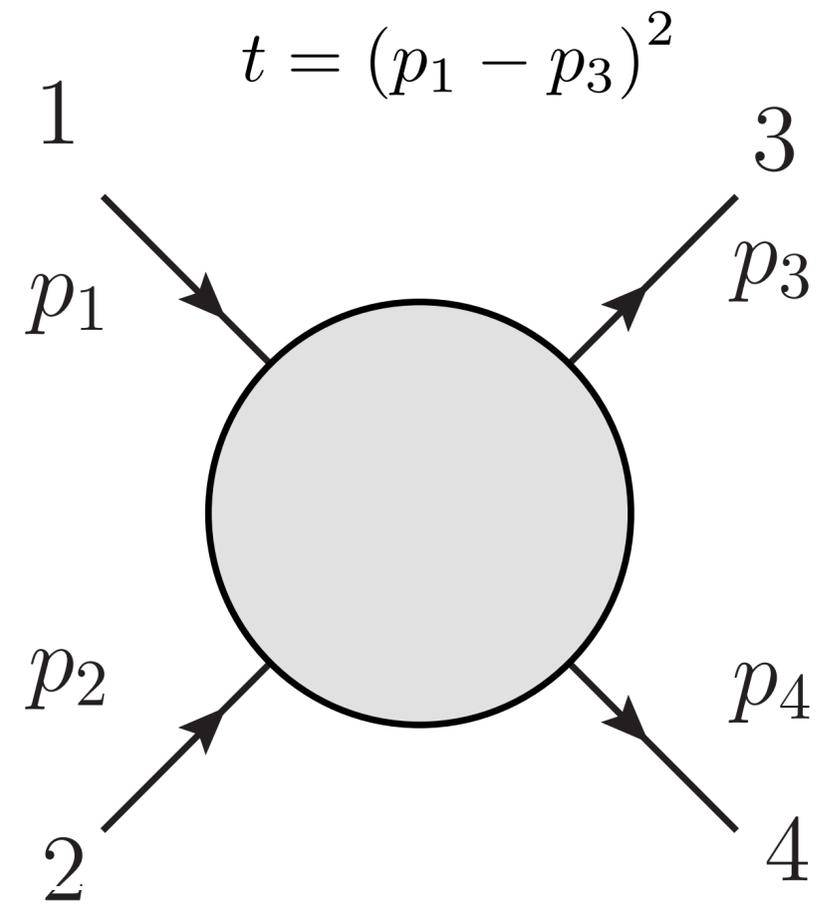


Decays

$\alpha \rightarrow \beta$ transition

$$\mathcal{S}_{\alpha\beta}(s) = \langle \beta, \text{out} | S | \alpha, \text{in} \rangle$$

$$s = (p_1 + p_2)^2$$



“100% of the times something will happen”

A. Piloni

Unitarity

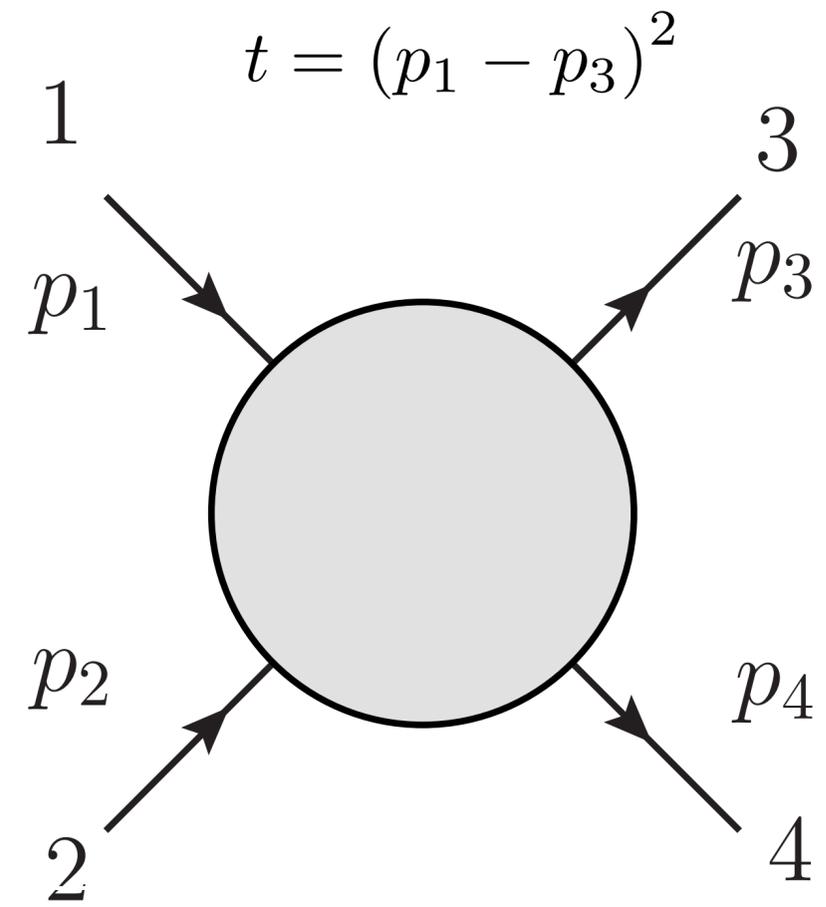
$$S S^\dagger = \mathbb{I}$$

Decays

$\alpha \rightarrow \beta$ transition

$$\mathcal{S}_{\alpha\beta}(s) = \langle \beta, \text{out} | S | \alpha, \text{in} \rangle$$

$$s = (p_1 + p_2)^2$$



“100% of the times something will happen”

Us

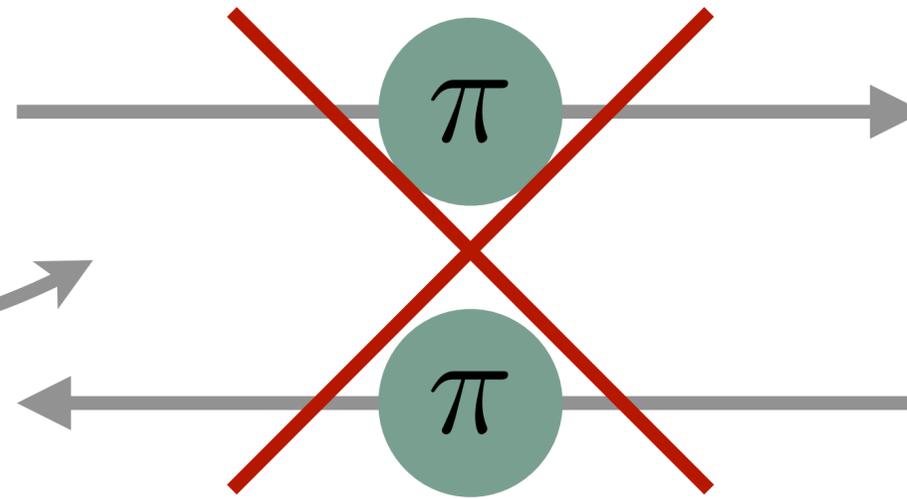
Unitarity

$$SS^\dagger = \mathbb{I}$$

Decays

$$S = \mathbb{I} + iT$$

Trivial



$$SS^\dagger = \mathbb{I}$$

Unitarity from $S \rightarrow T$

$$SS^\dagger = \mathbb{I}$$

$$T - T^\dagger = iTT^\dagger$$

Optical theorem

We will prove

$$\text{Im } T_{ii}(s, t = 0) \propto \sum_n T_{in} T_{ni}^\dagger \propto \sigma_{\text{tot}}^i$$

Remember, $J(\ell)$, is conserved

$$T(s, t) = 16K\pi \sum_{\ell} (2\ell + 1) P_{\ell}(\cos \theta) t_{\ell}(s)$$

Partial waves

How does an eigenstate evolve

$$\Psi(t) = \Psi(0)e^{-iMt}$$

$$|\Psi(t)|^2 = |\Psi(0)|^2$$

Lives forever → Bound State

How does a quasi-eigenstate evolve

$$\Psi(t) = \Psi(0)e^{-iMt - t\Gamma/2}$$

$$|\Psi(t)|^2 = |\Psi(0)|^2 e^{-\Gamma t}$$

Lives for $1/\Gamma$ → Resonance

“Fourier transform”



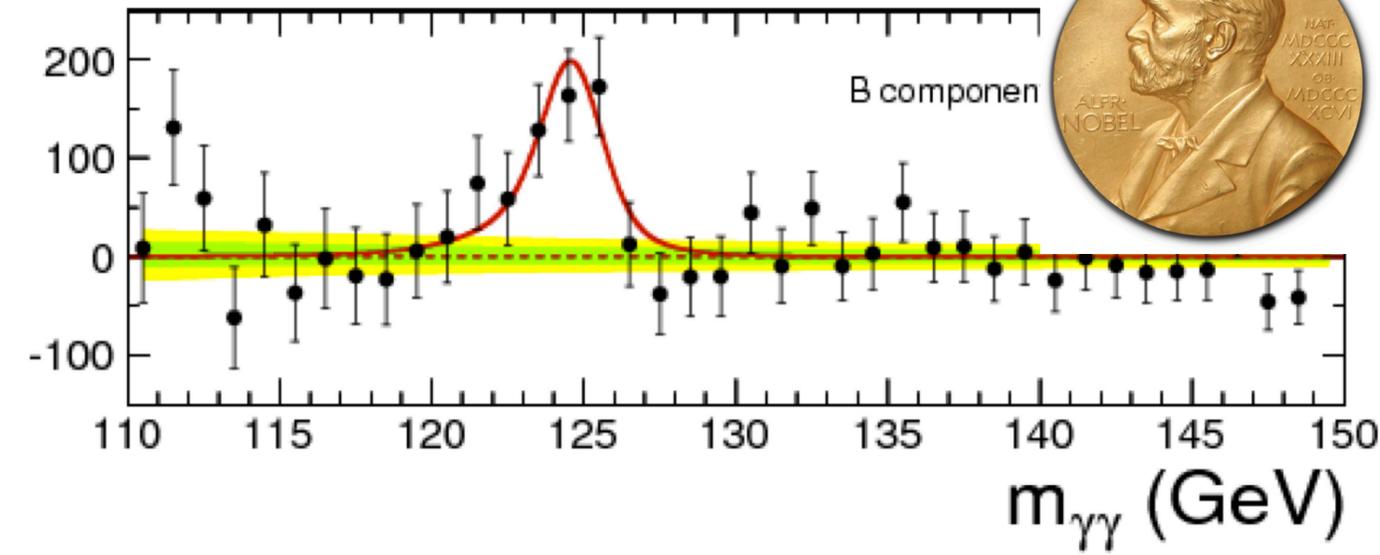
$$t_{\ell}(E) \simeq \frac{\Gamma/2}{M - E + i\Gamma/2}$$

Decays

Light spectrum



What's this ?

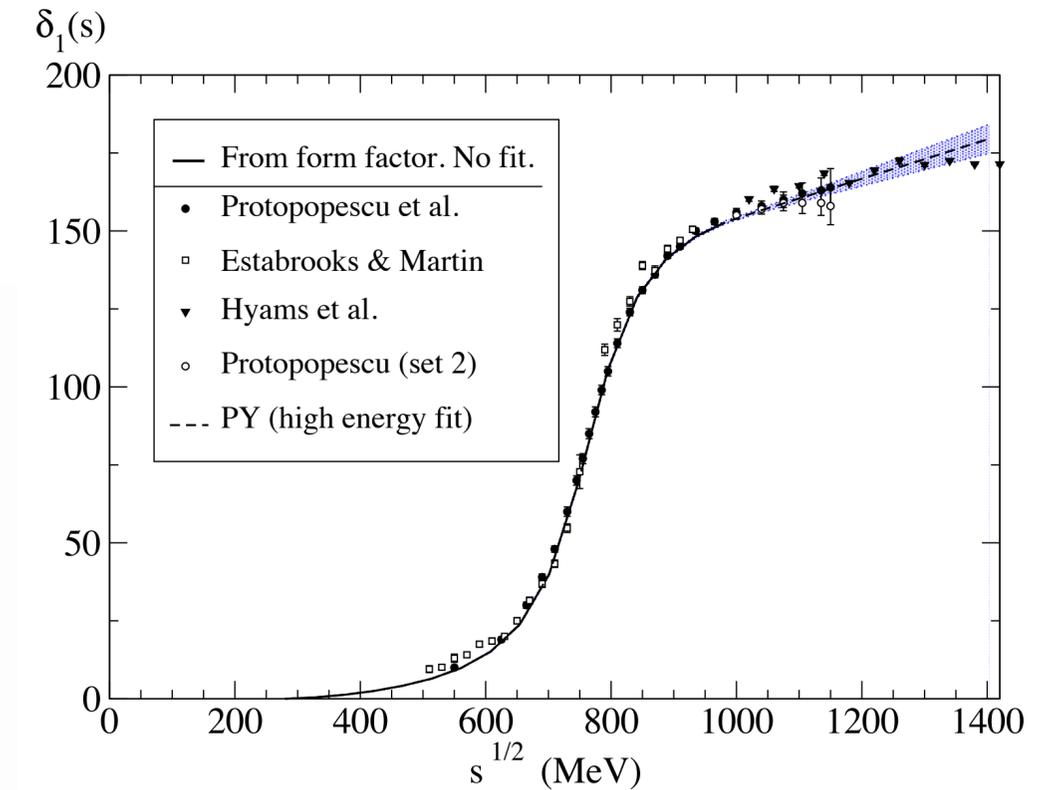
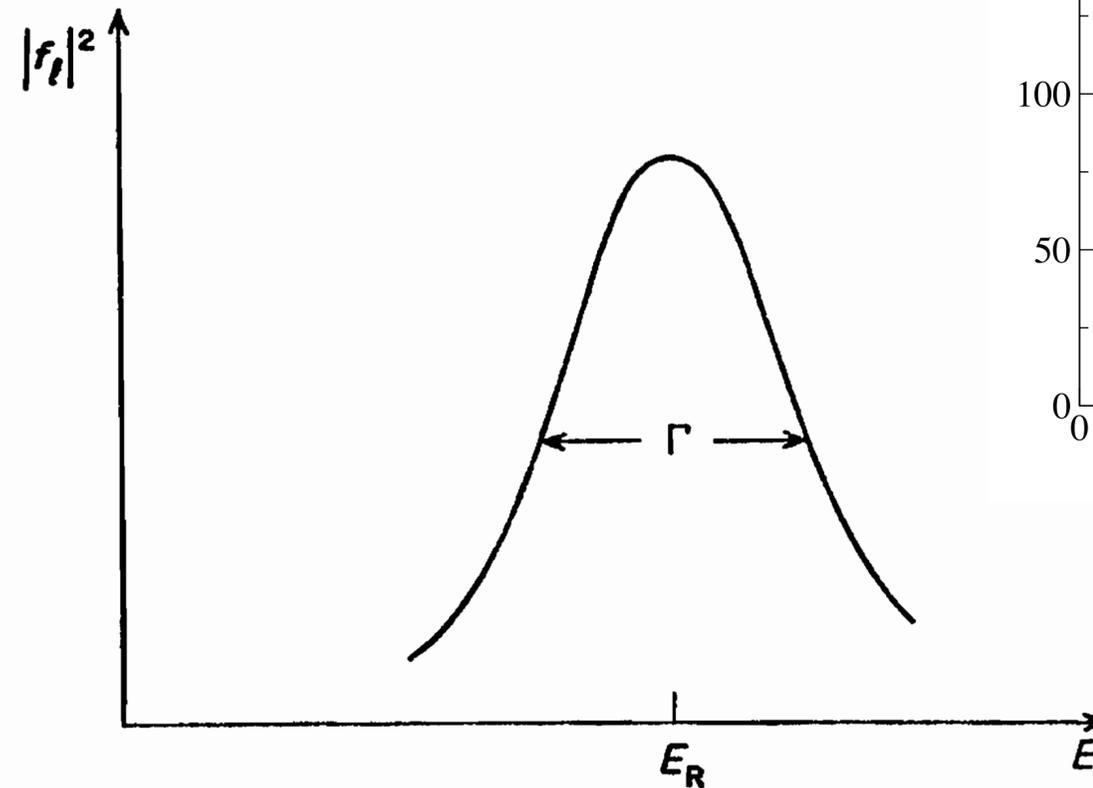


Relativistic Breit-Wigner (BW) formula

$$t_\ell(s) \simeq \frac{-M\Gamma}{M^2 - s - iM\Gamma}$$

Elastic $a \rightarrow a$ region

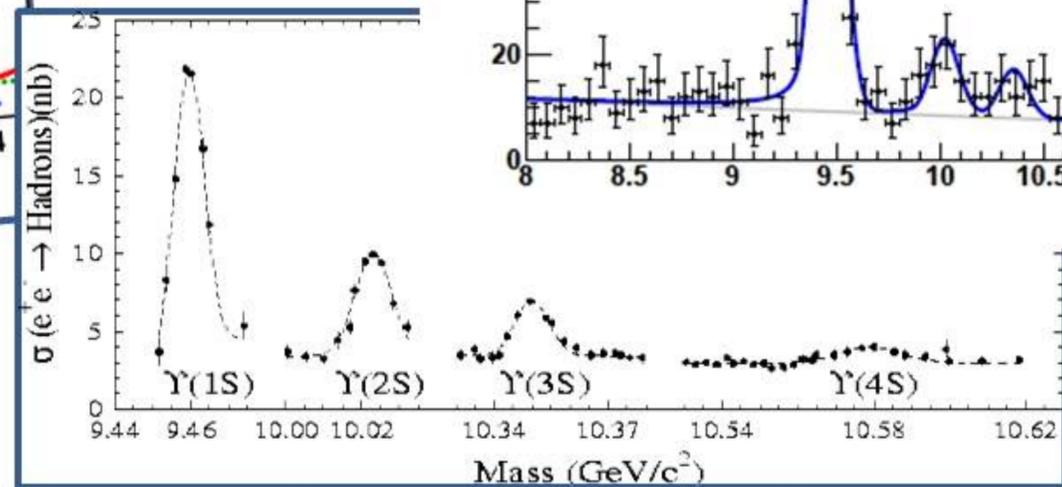
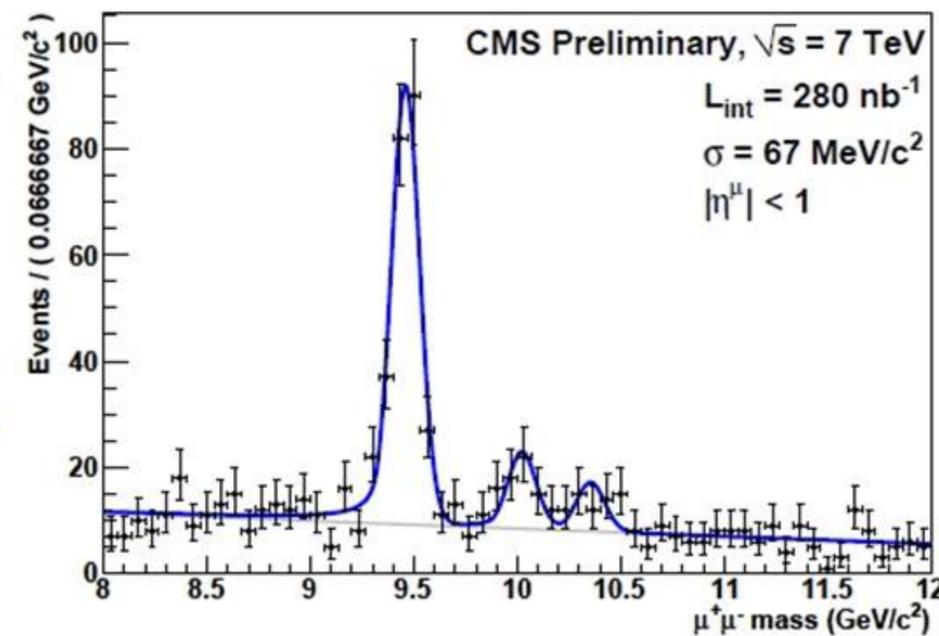
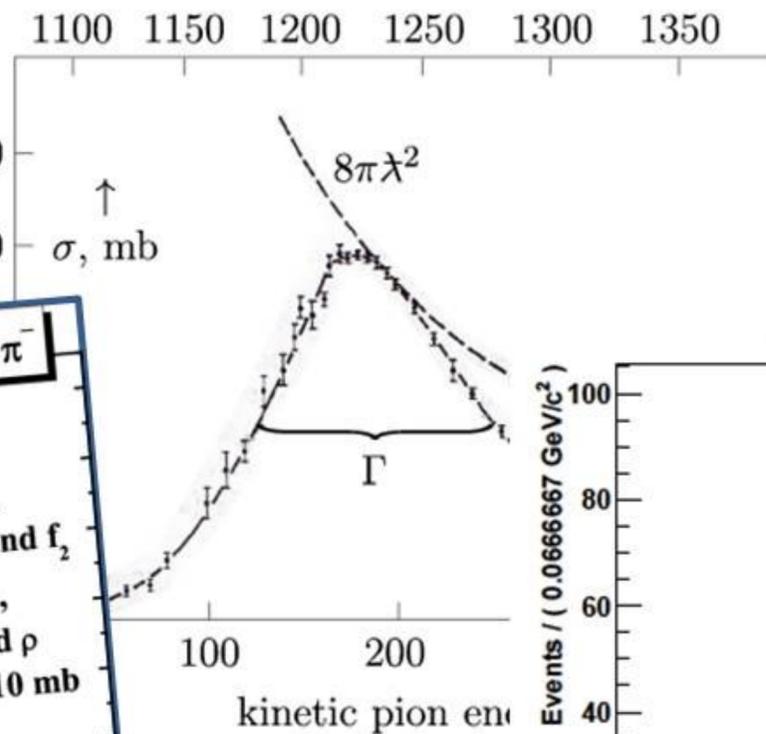
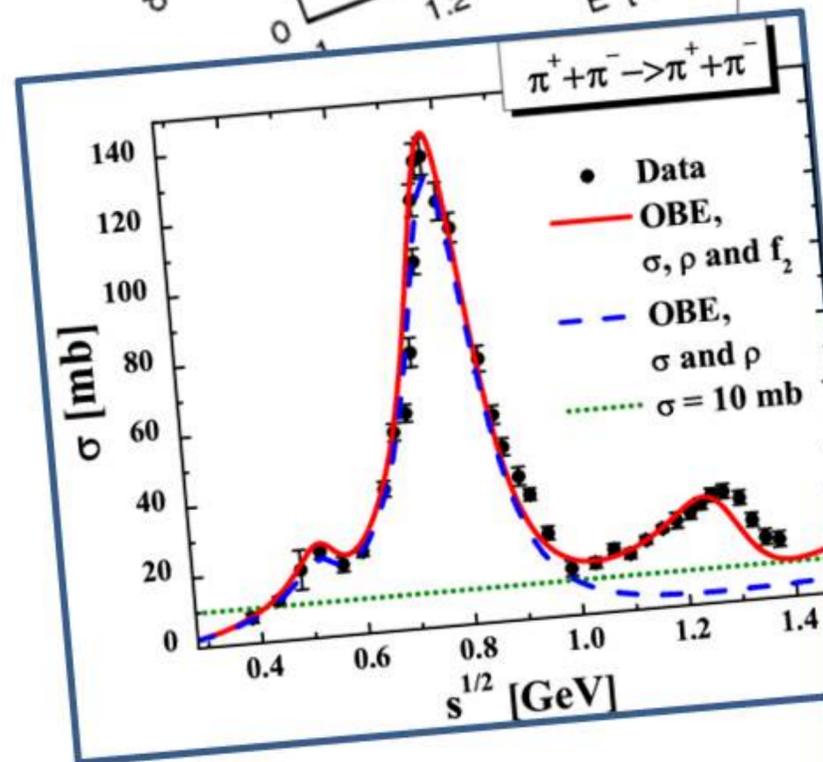
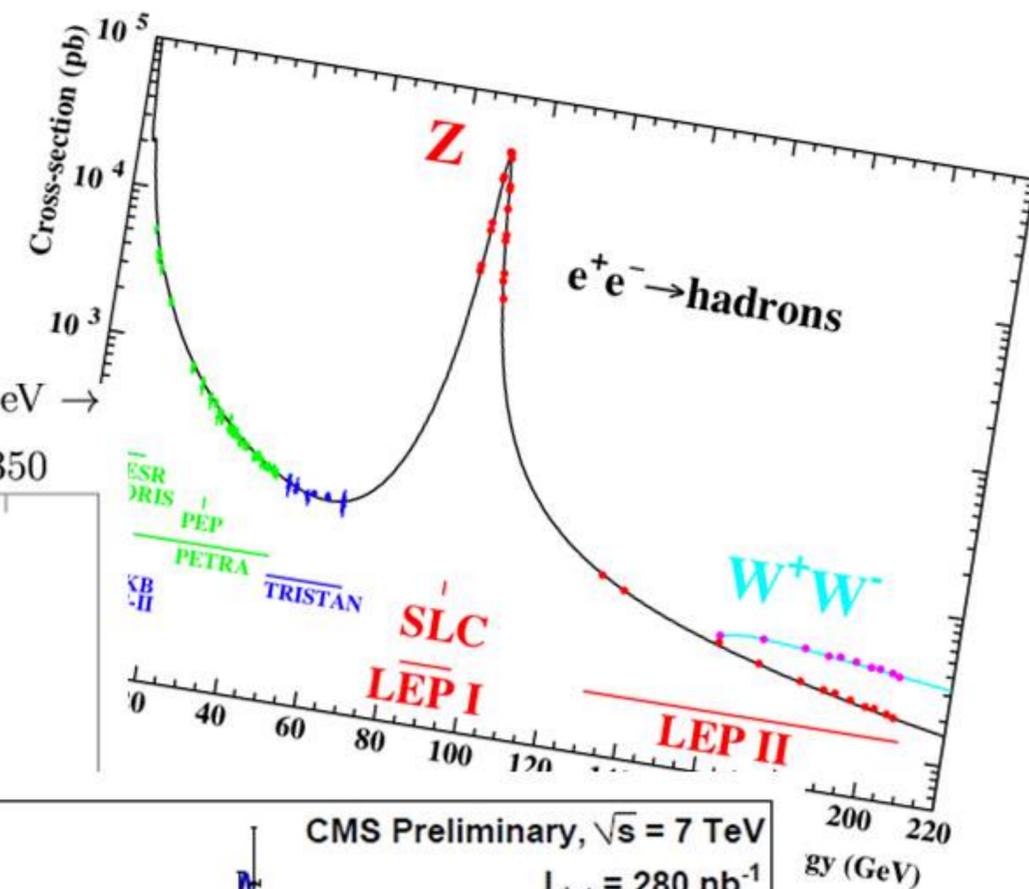
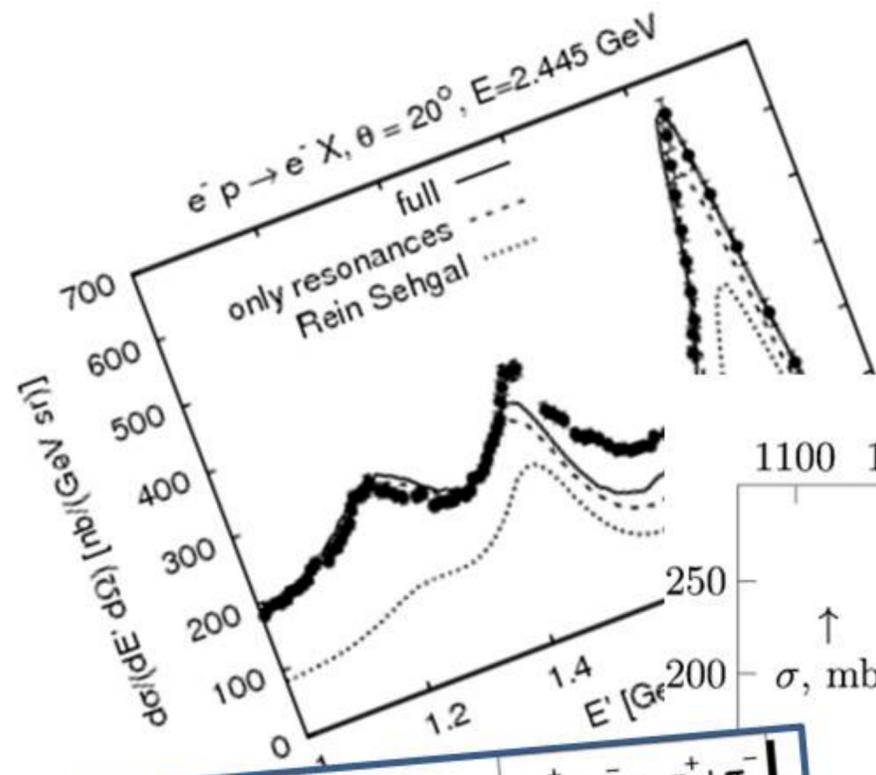
$$t_{\ell,el}(s) = |t_{\ell,el}(s)| e^{i\delta_\ell(s)}$$



Decays

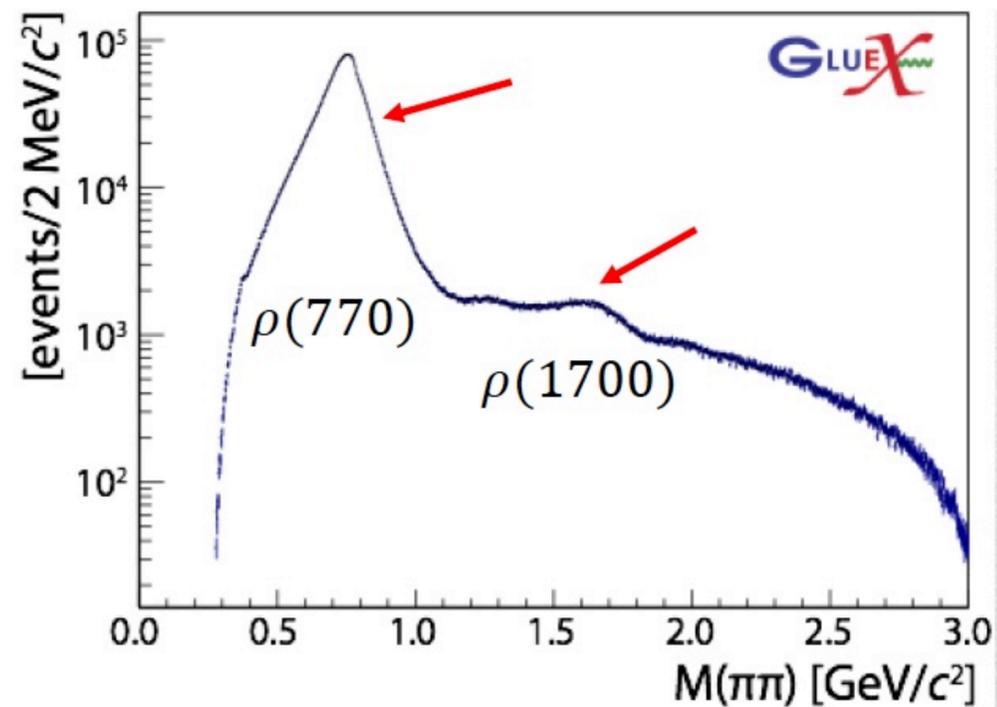
The BW has worked
wonders in
many cases

Life is easy...
sometimes

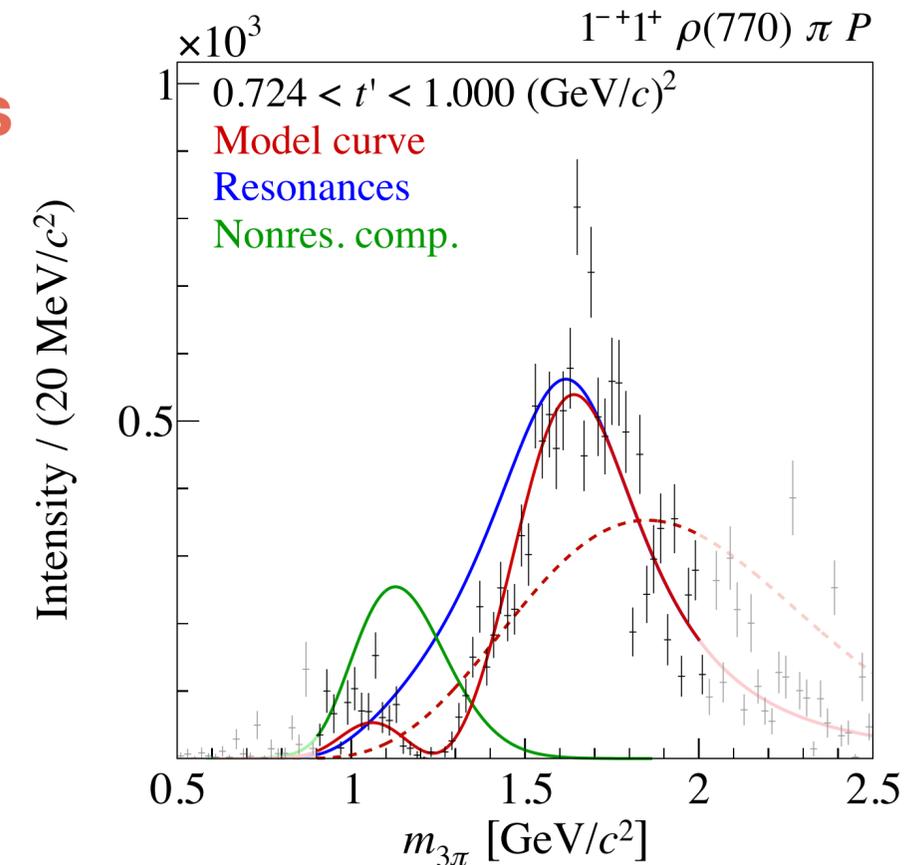


This is not so easy

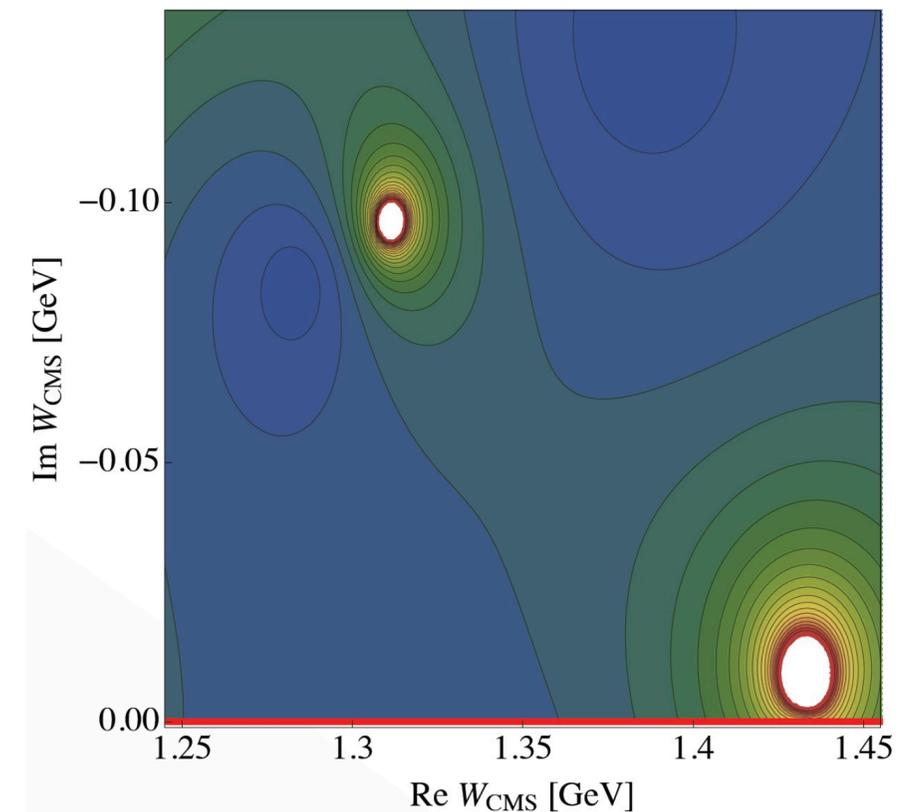
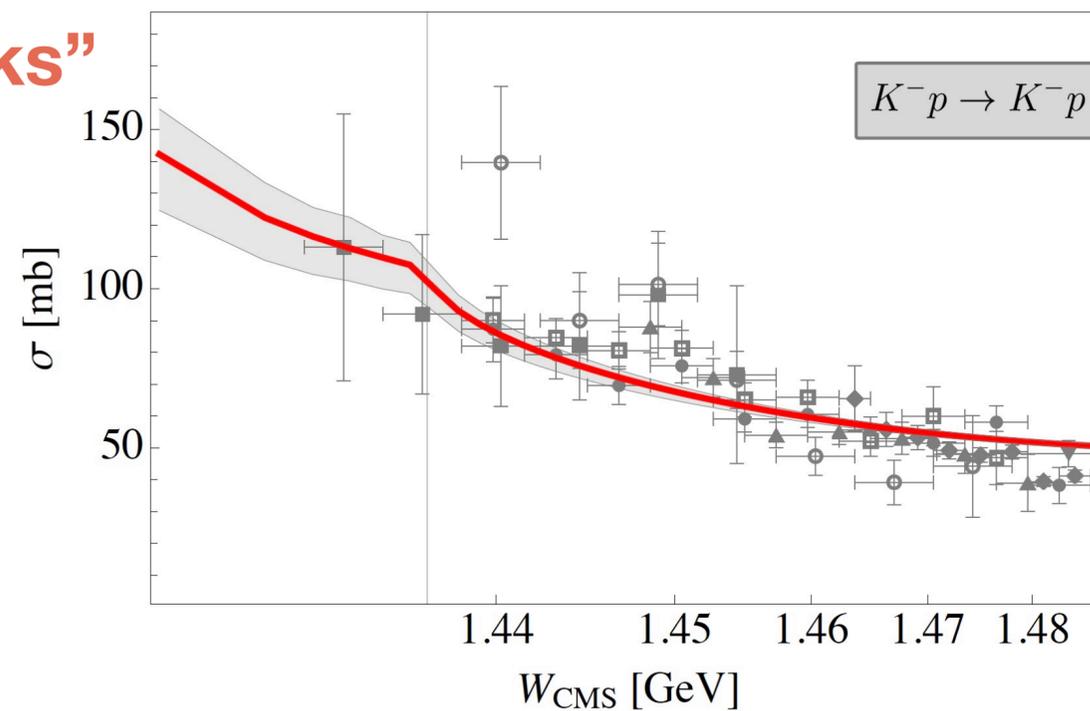
Inelastic
Large background



Three particles

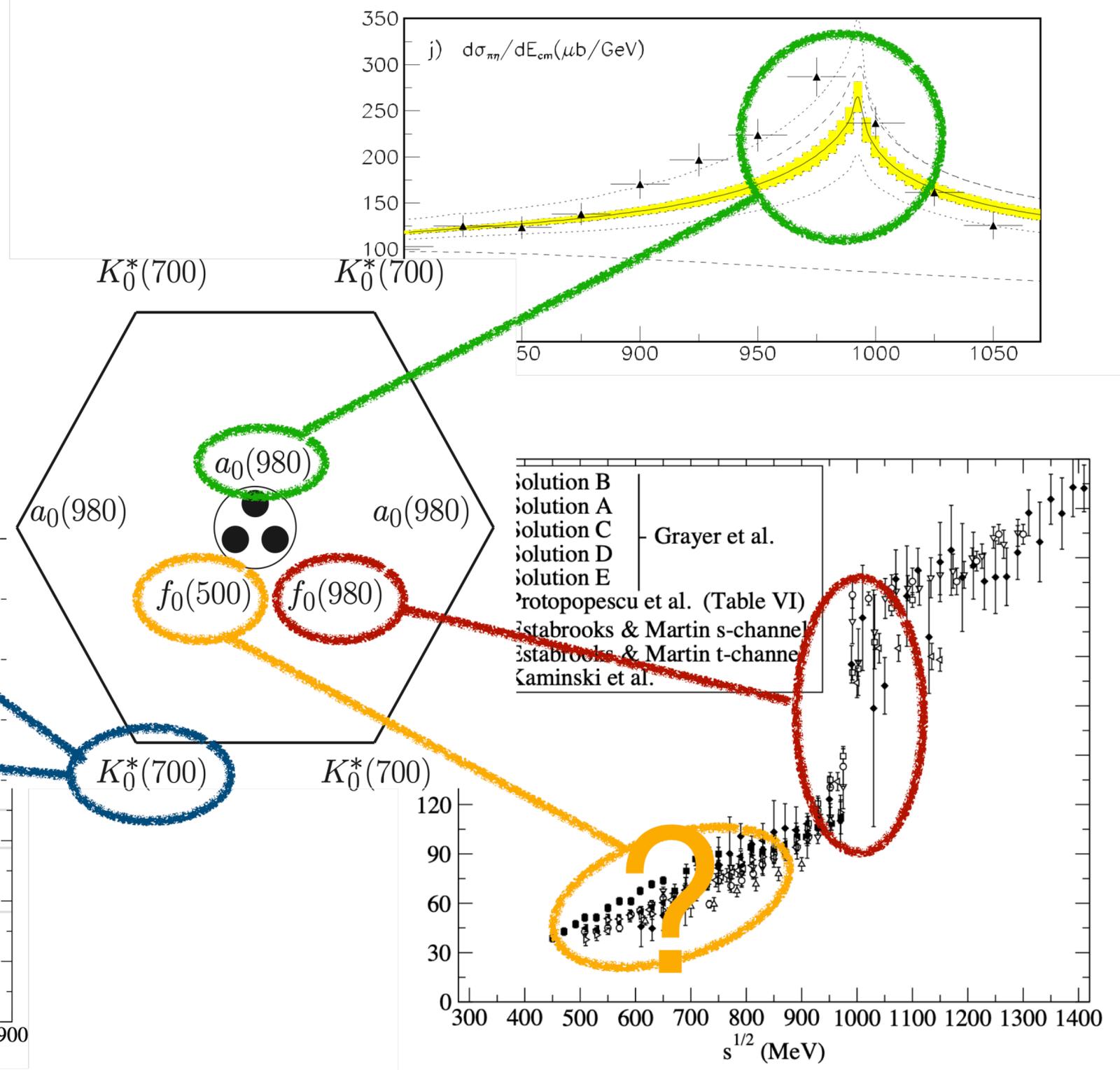
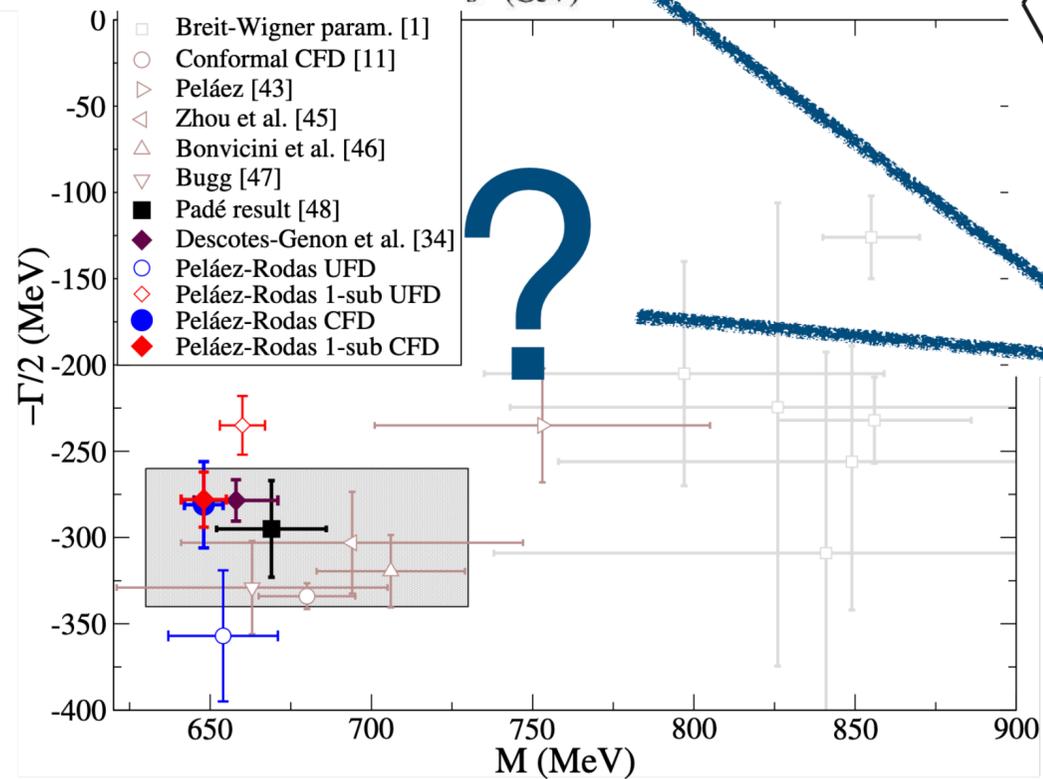
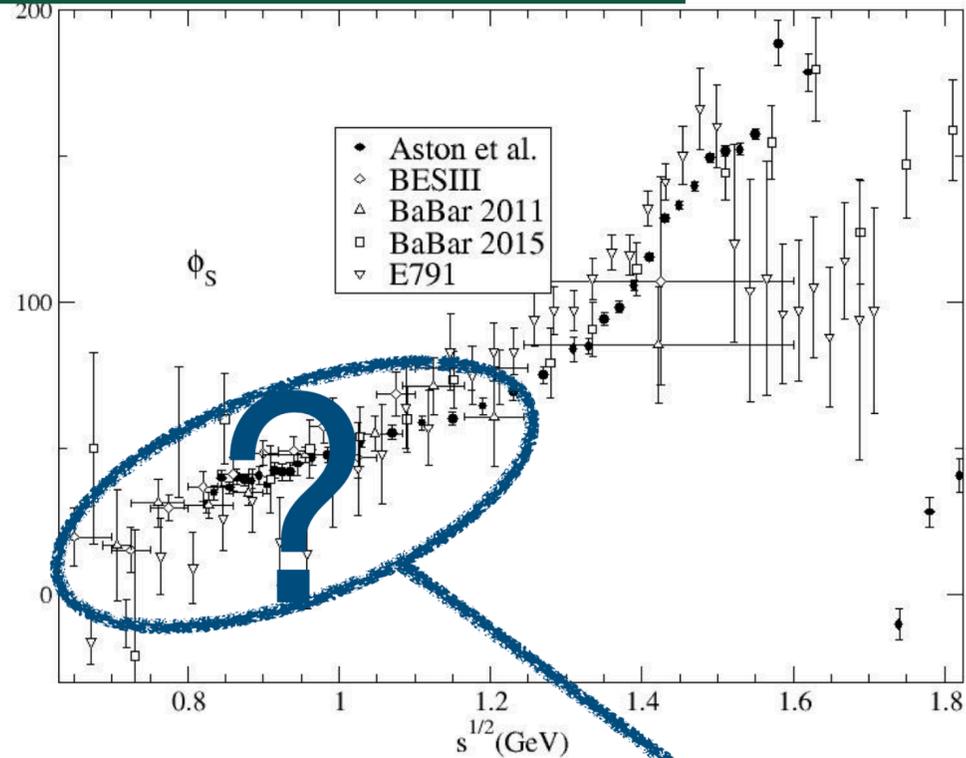


Not “simple peaks”



Decays

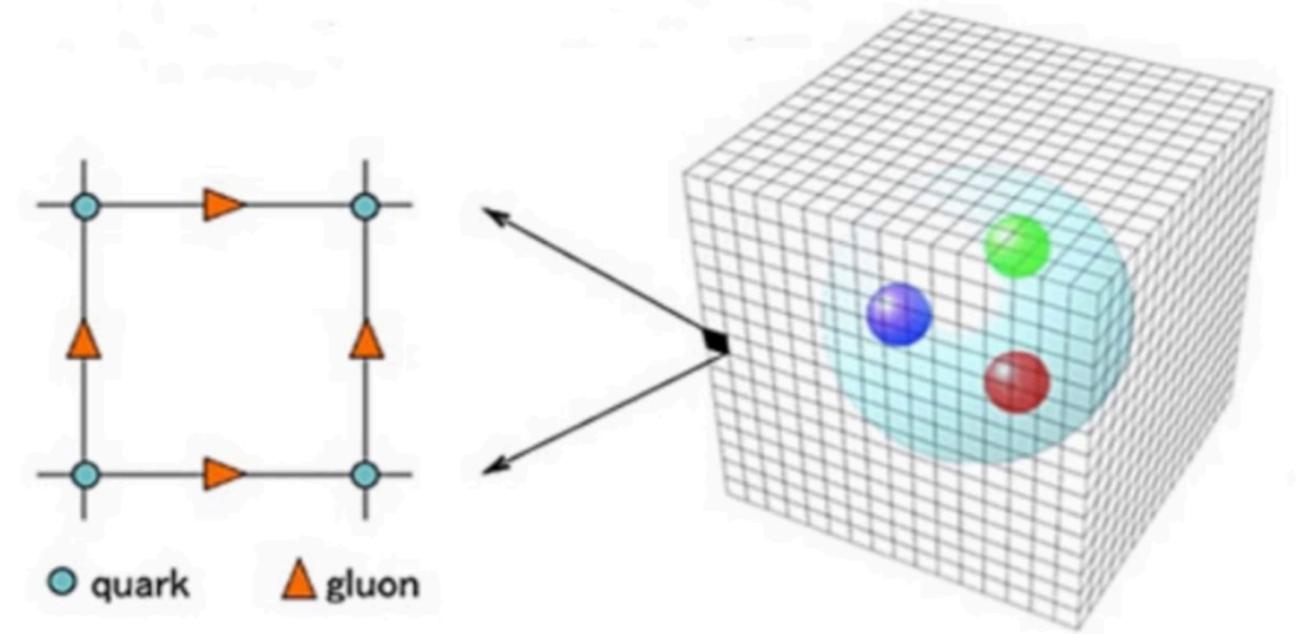
Lightest scalar mesons



Questions??

Lattice QCD

Numerical approach to QCD



Good

1. Non-perturbative
2. Systematically improvable

1. Discretize space/time
2. Change to imaginary time
3. Compute quarks
4. Build hadrons

Lattice QCD

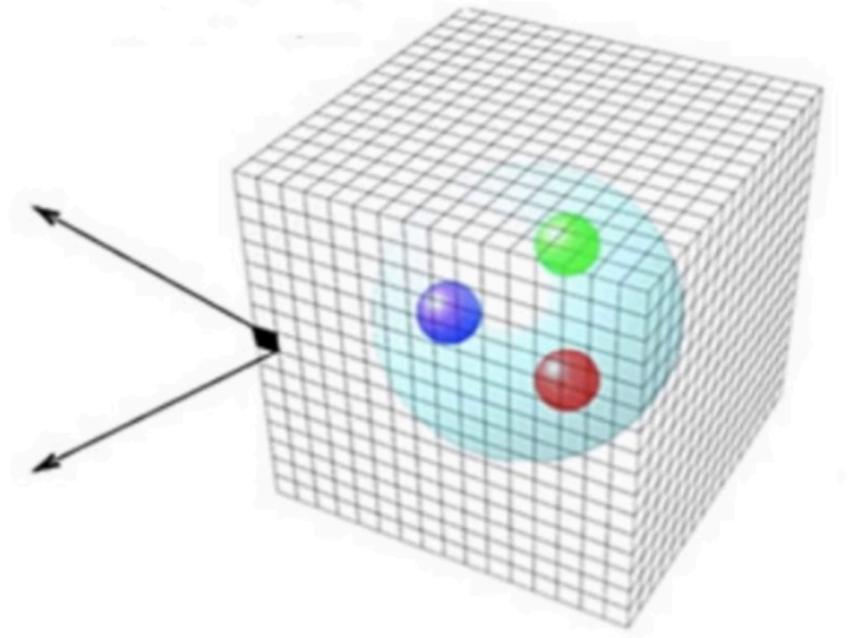
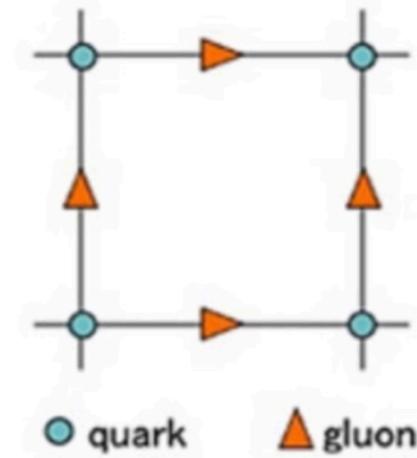
Compute correlation functions

$$\langle 0 | \hat{\varphi}(y) \hat{\varphi}(z) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}\varphi(x) \varphi(y) \varphi(z) e^{-iS[\varphi(x)]}$$

Sum over all paths

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

Discretized fields



Lattice QCD

Compute correlation functions

$$\langle 0 | \hat{\varphi}(y) \hat{\varphi}(z) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}\varphi(x) \varphi(y) \varphi(z) e^{-iS[\varphi(x)]}$$

Sum over all paths

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

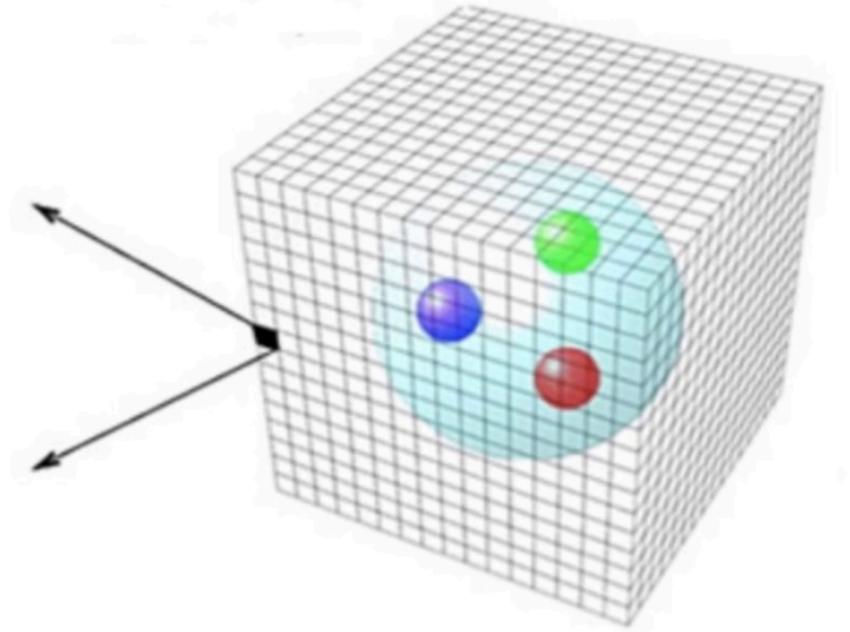
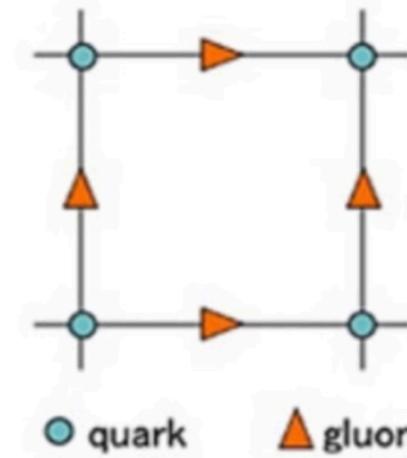
Discretized fields

Bad

1. Oscillatory

$$e^{-iS[\varphi(x)]}$$

2. Millions of integrals

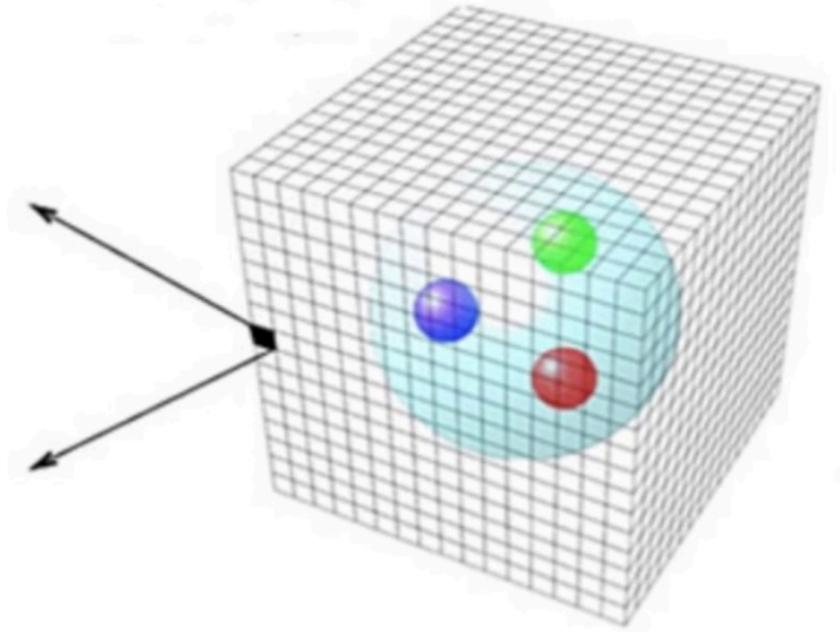
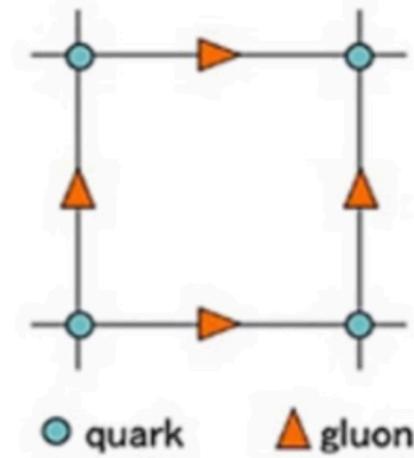


Lattice QCD

Euclidean action

$$t \rightarrow -it$$

$$-iS = -i \int d^3x dt \mathcal{L} \rightarrow - \int d^3x dt \mathcal{L}_E = -S_E$$



Fermion integral

$$0 < \det D[U] < 1$$

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U e^{-S_E[\psi, \bar{\psi}, U]} = \int \mathcal{D}U \det D[U] e^{-S_E^g[U]}$$

Probability like

$$Z = \int \mathcal{D}U \det D[U] e^{-S_E^g[U]}$$

Calculating integrals as averages

If we sample points uniformly

$$\rho_u(x_n) = 1/(b-a) \longrightarrow \frac{1}{b-a} \int_a^b dx f(x) = \langle f \rangle_{\rho_u} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N f(x_n)$$

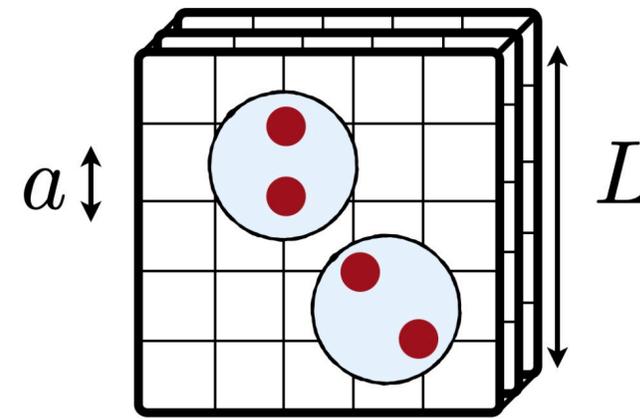
We compute our observables as averages

$$\langle O \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N O[U_n]$$

$$dP(U) = \frac{e^{-S[U]} \mathcal{D}[U]}{\int \mathcal{D}[U] e^{-S[U]}}$$

We have to sample like this

Lattice QCD



$$D_\mu = \left(\begin{array}{c} \\ \\ \end{array} \right) \updownarrow (L/a)^3 \times (T/a)$$

Quark propagator

$$\langle 0 | \psi_x^{i\alpha} \bar{\psi}_y^{j\beta} | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \psi_x^{i\alpha} \bar{\psi}_y^{j\beta} e^{-S_E[\psi, \bar{\psi}, U]}$$

Definition

Splitting the fermions

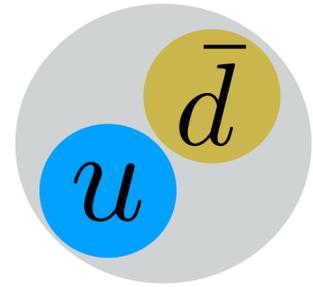
$$= \int \mathcal{D}U e^{-S_E^g[U]} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \psi_x^{i\alpha} \bar{\psi}_y^{j\beta} e^{-\bar{\psi} D[U] \psi}$$

Algebraic solution

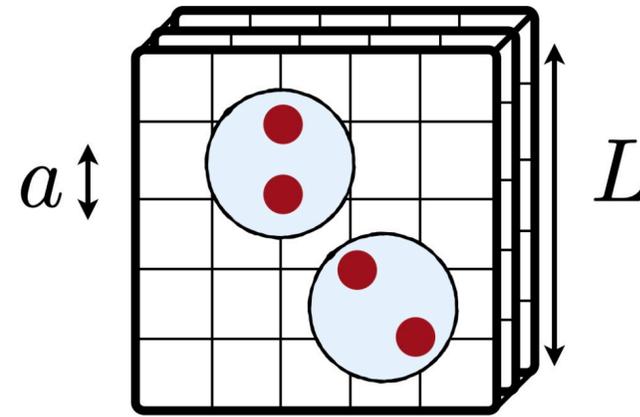
$$= \int \mathcal{D}U [D^{-1}[U]]_{x,y}^{i\alpha,j\beta} \det D[U] e^{-S_E^g[U]}$$

$$= \sum_{\{U\}} [D^{-1}[U]]_{x,y}^{i\alpha,j\beta}$$

Lattice QCD

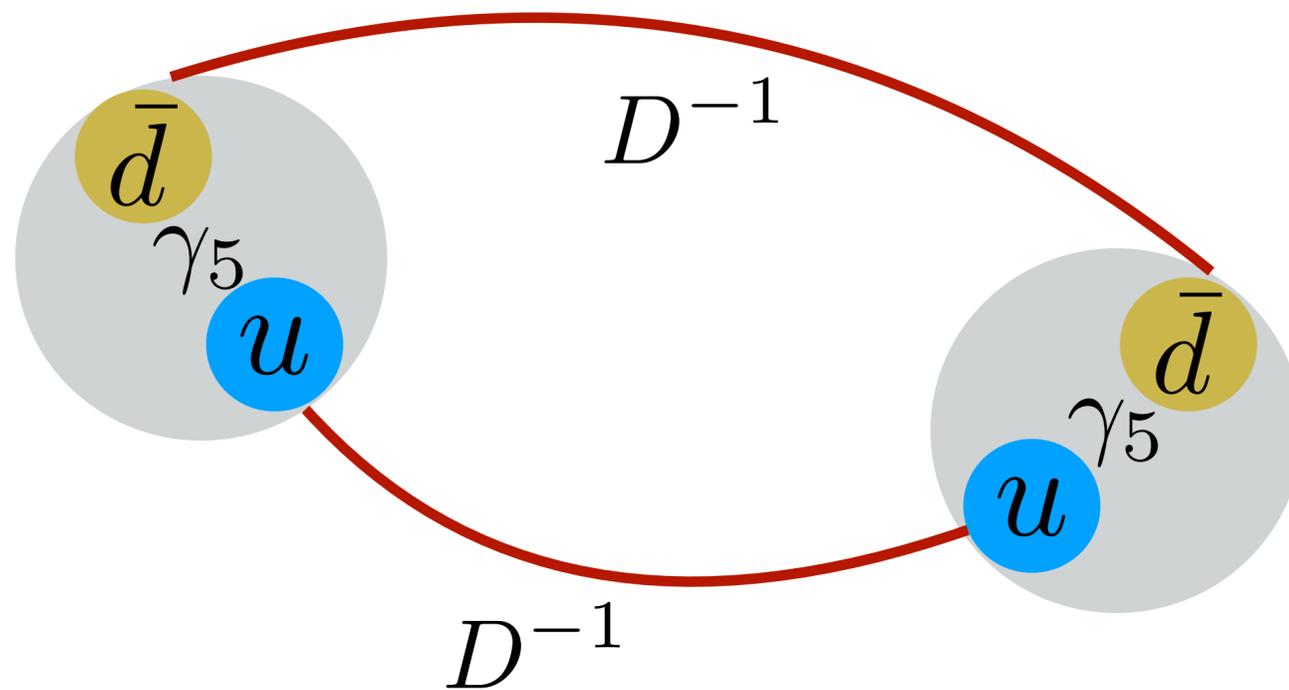


We build our π



$$D_\mu = \left(\begin{array}{c} \\ \\ \end{array} \right) \updownarrow (L/a)^3 \times (T/a)$$

$$\langle 0 | (\bar{\psi} \gamma_5 \psi)_{x,t} (\bar{\psi} \gamma_5 \psi)_{0,0} | 0 \rangle = -\text{tr} \left([D^{-1}[U]]_{00,xt} \gamma_5 [D^{-1}[U]]_{xt,00} \gamma_5 \right)$$



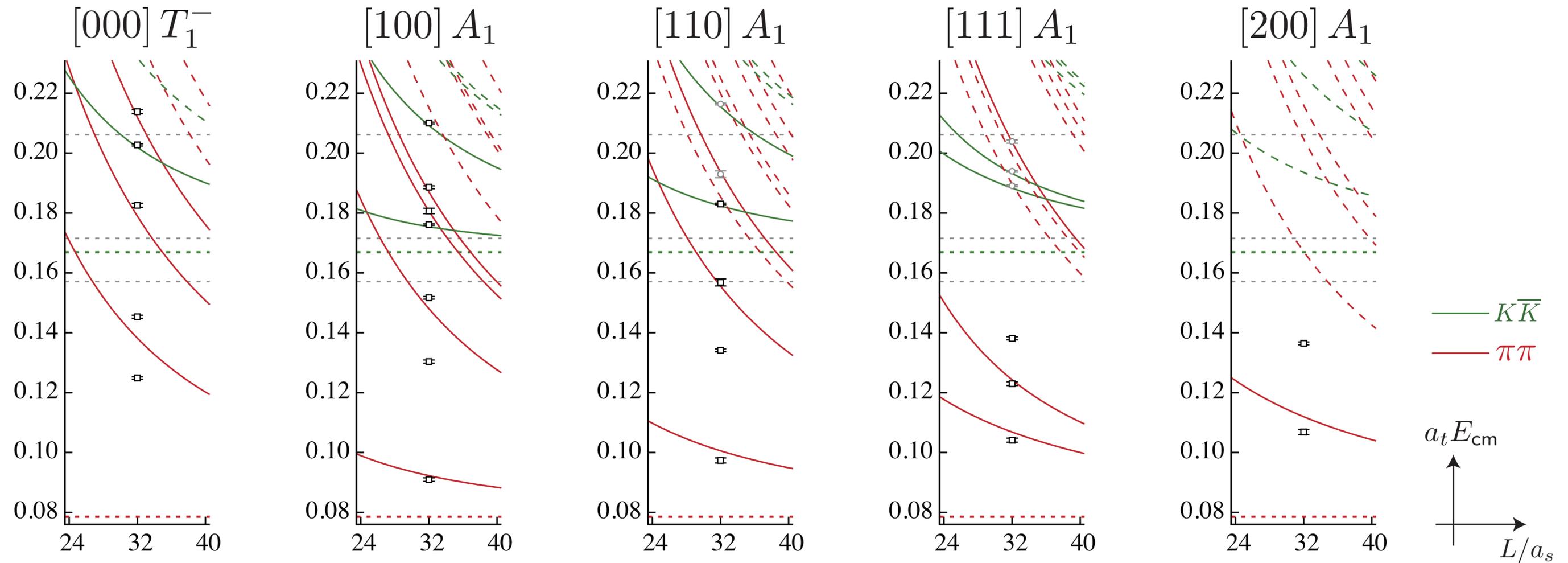
Lattice QCD

Hamiltonian evolution

$$e^{-iHt} \xrightarrow{t \rightarrow -it} \sum_n e^{-E_n t} \langle 0 | O_f(0) | n \rangle \langle n | O_i^\dagger(0) | 0 \rangle$$

Desired energies

Our π



Lattice QCD

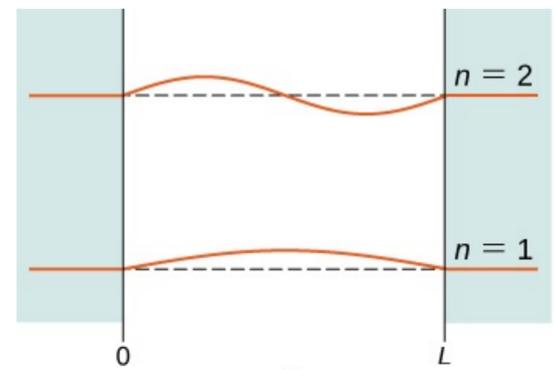
In a 1D box

$$R \ll L$$

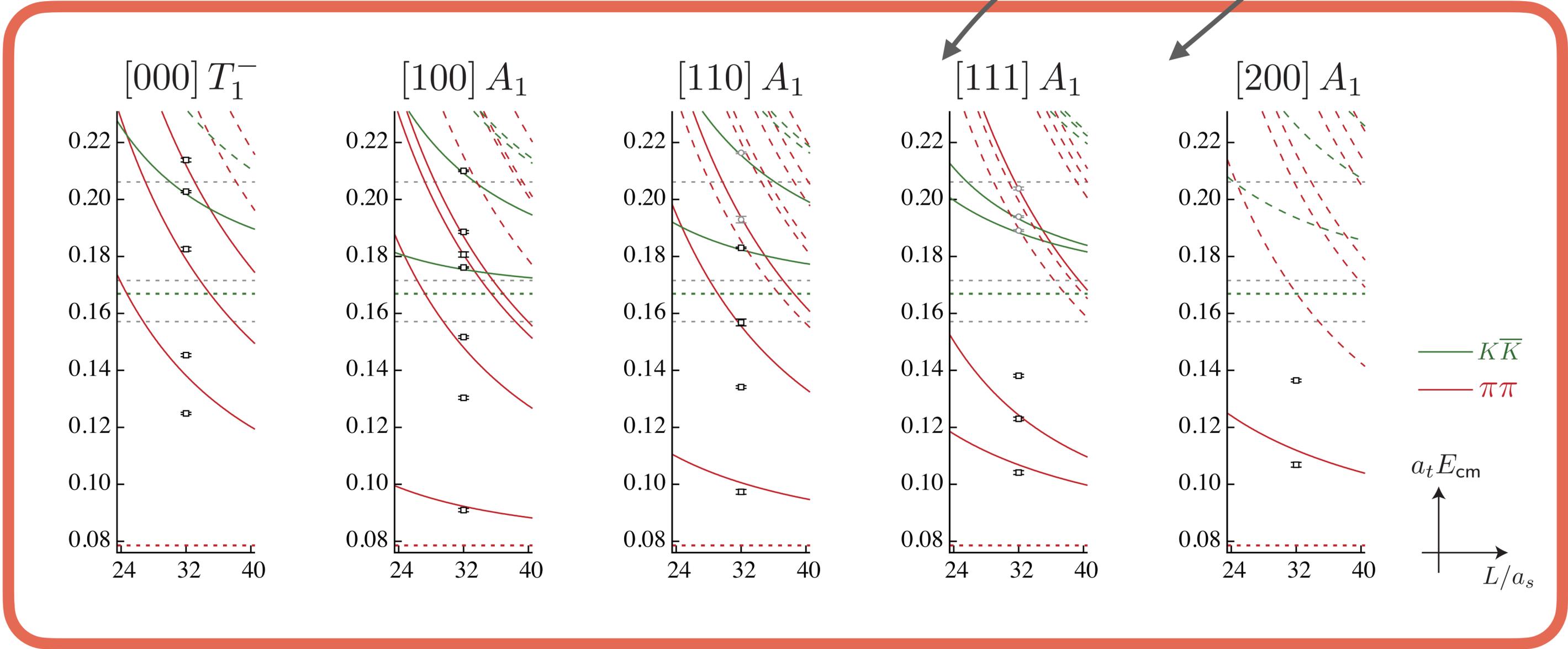
In

Out

Boundary



$$e^{ikL} + 2i\delta(k) = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$



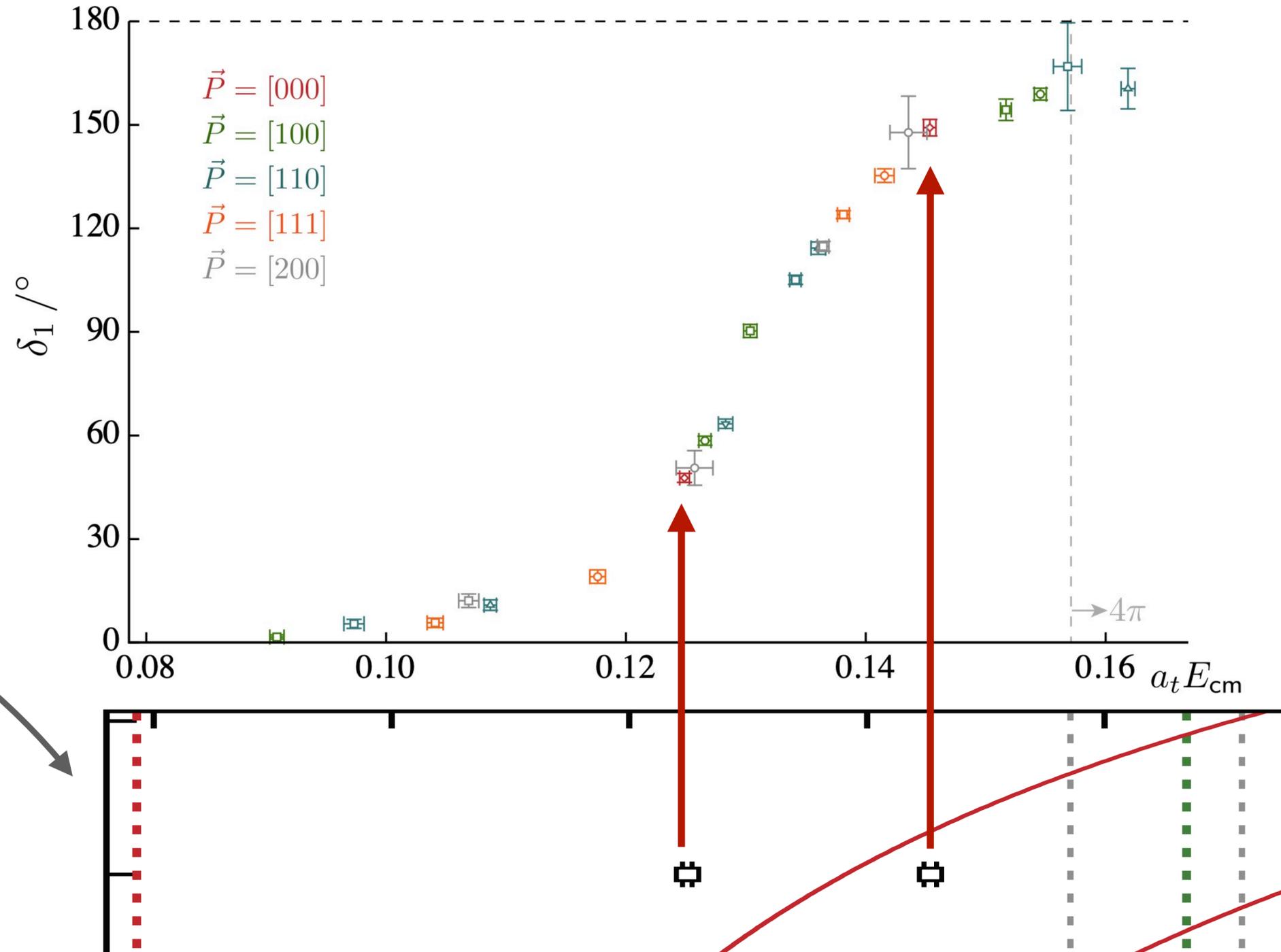
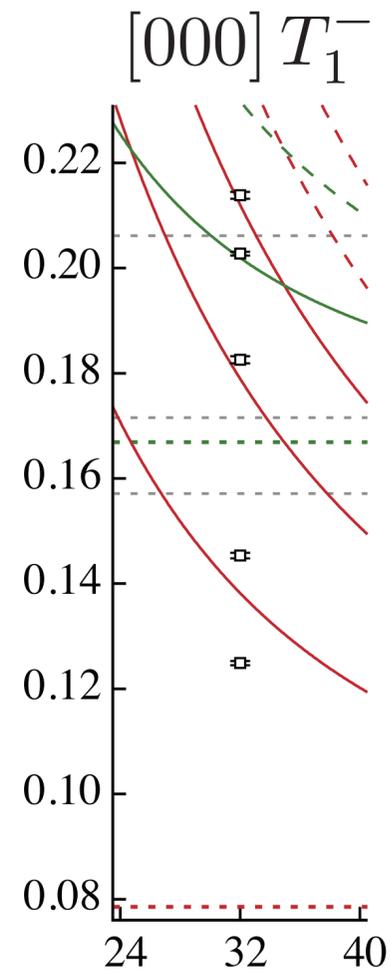
— $K\bar{K}$
— $\pi\pi$

$a_t E_{\text{cm}}$
 L/a_s

$$e^{ikL+2i\delta(k)} = e^{ik0} = 1 \rightarrow k_n L + 2\delta(k_n) = 2n\pi$$

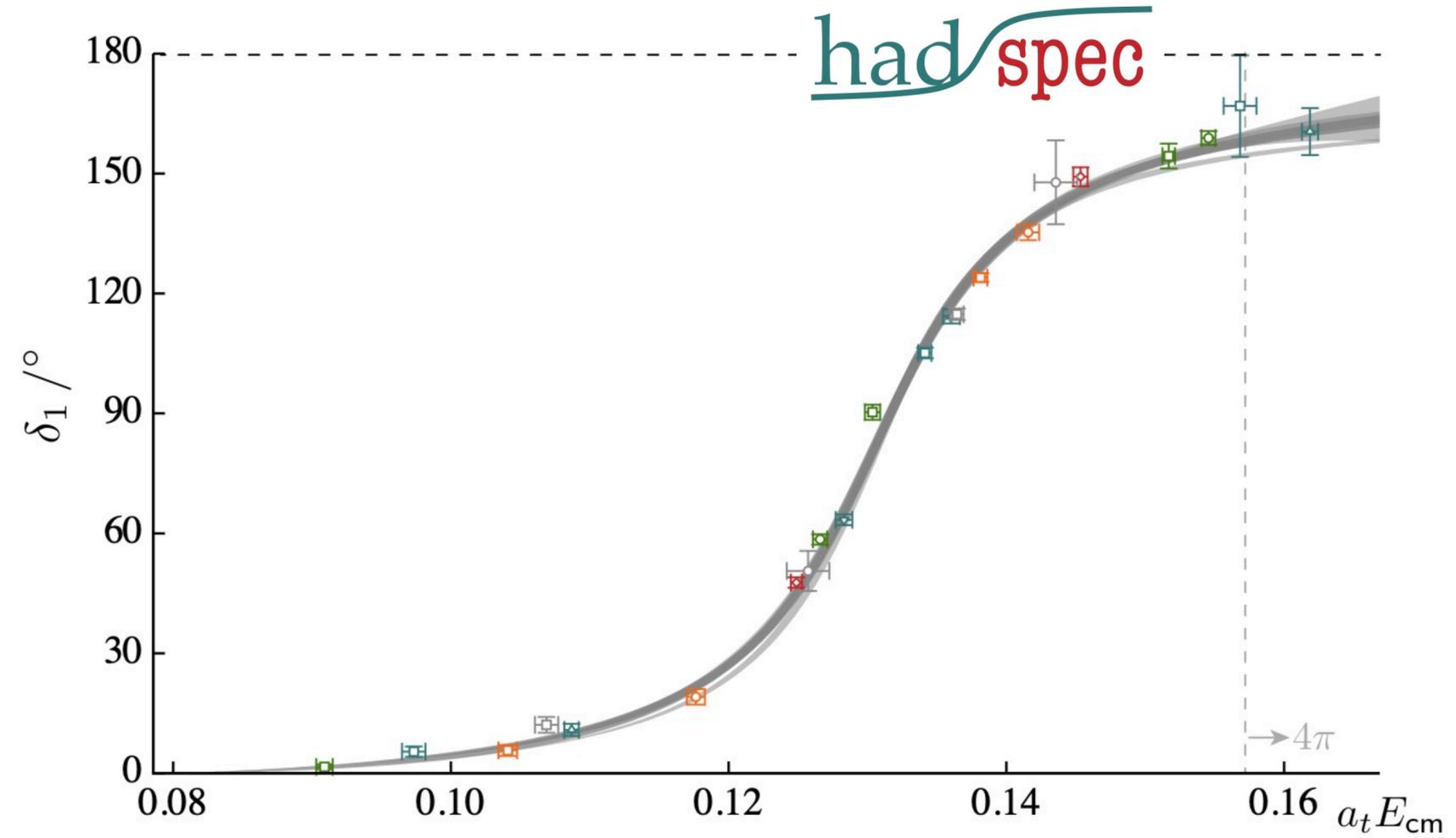
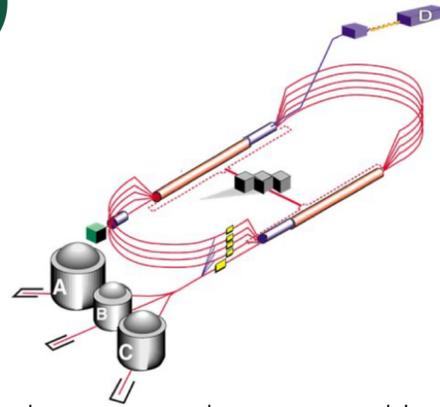
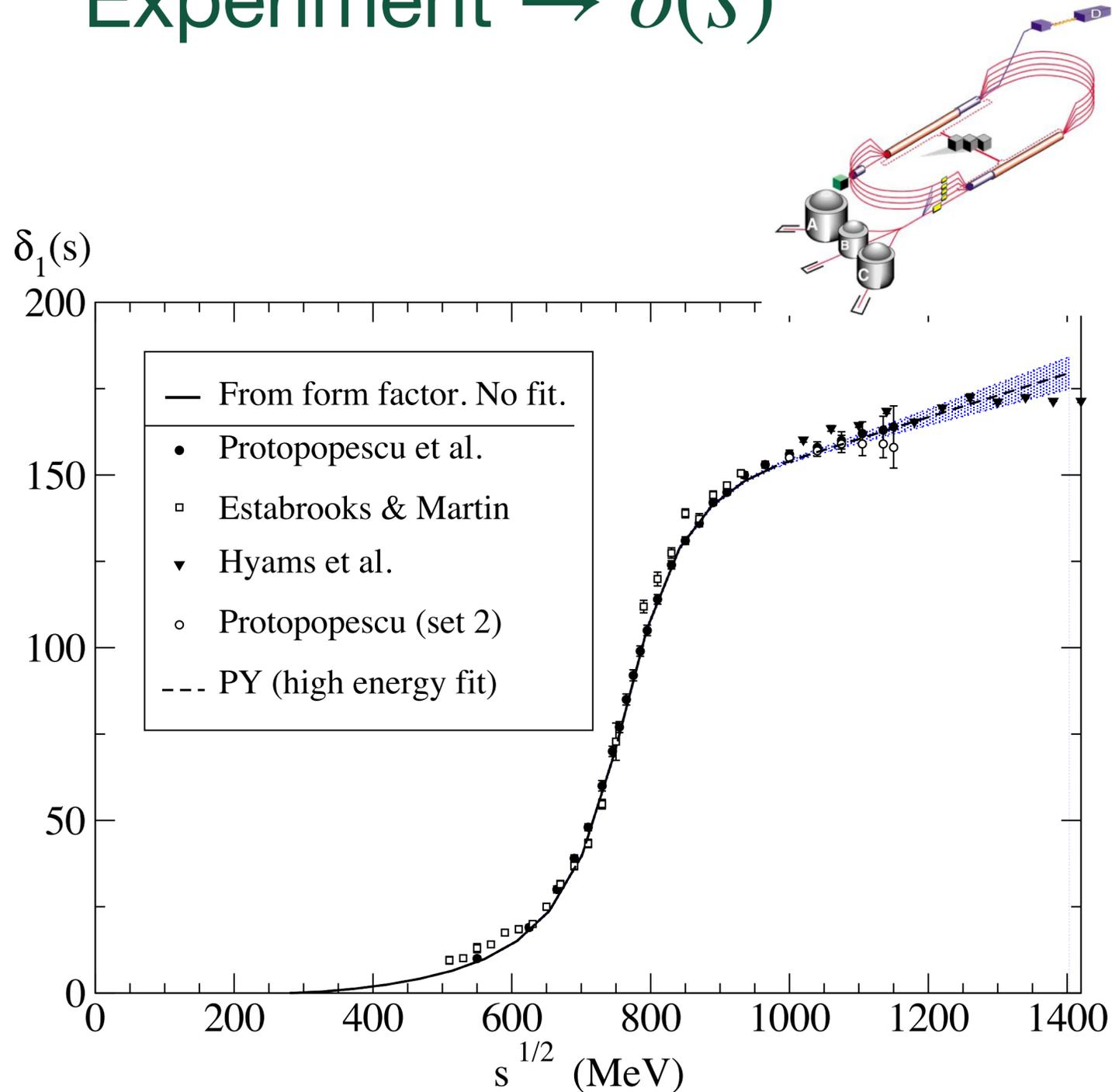
We recover $\delta(s)$

Our E_n become the x axis



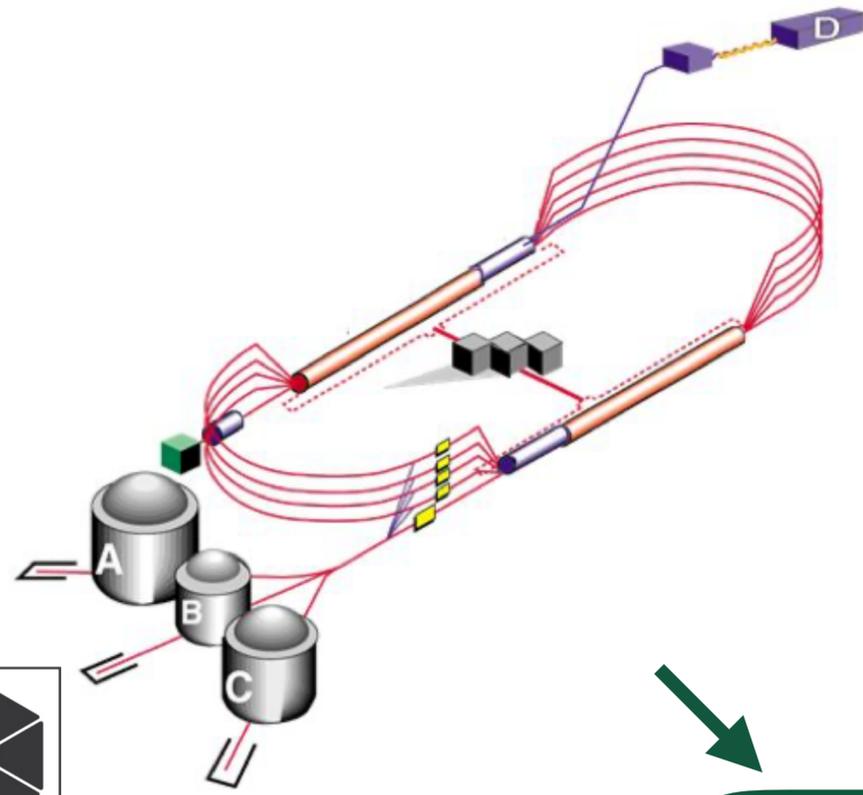
Decays

Experiment $\rightarrow \delta(s)$

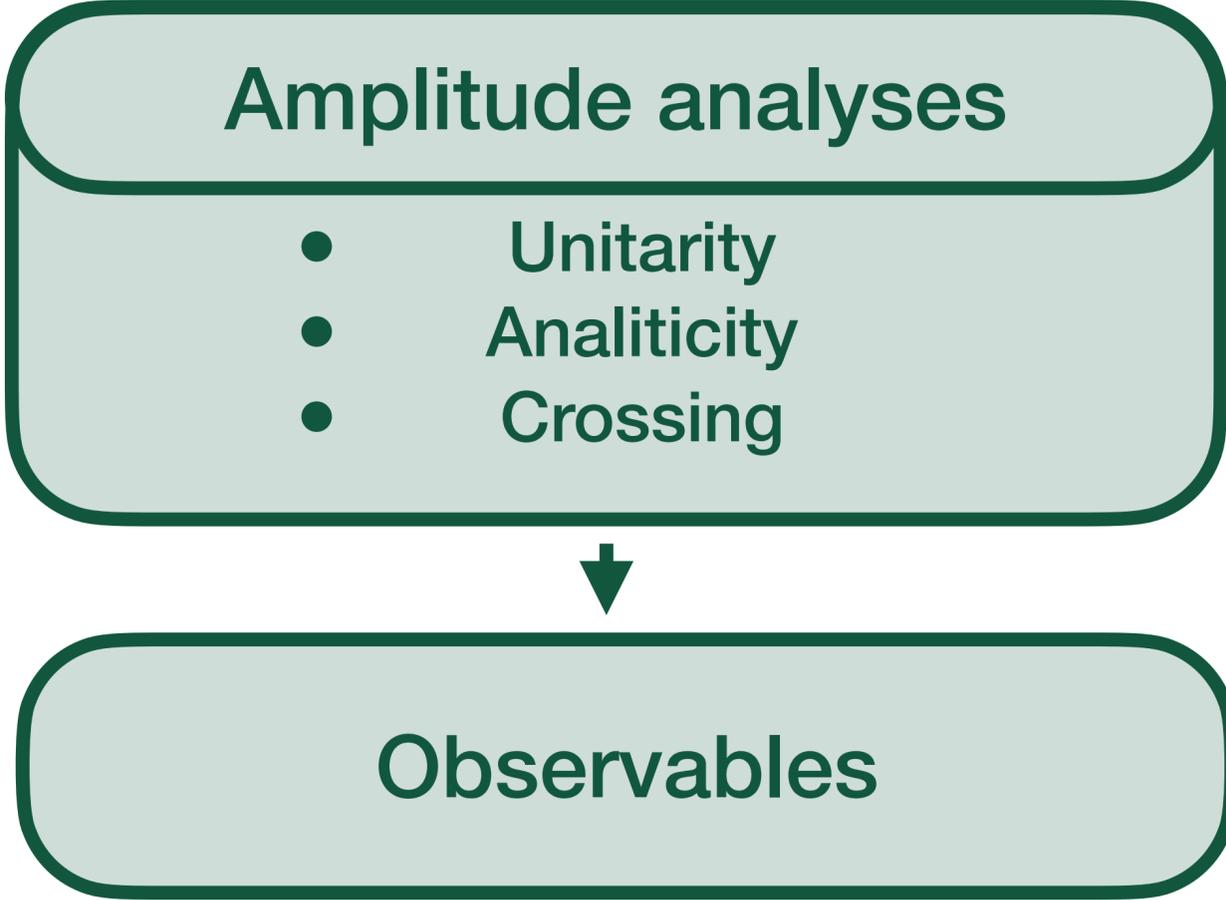
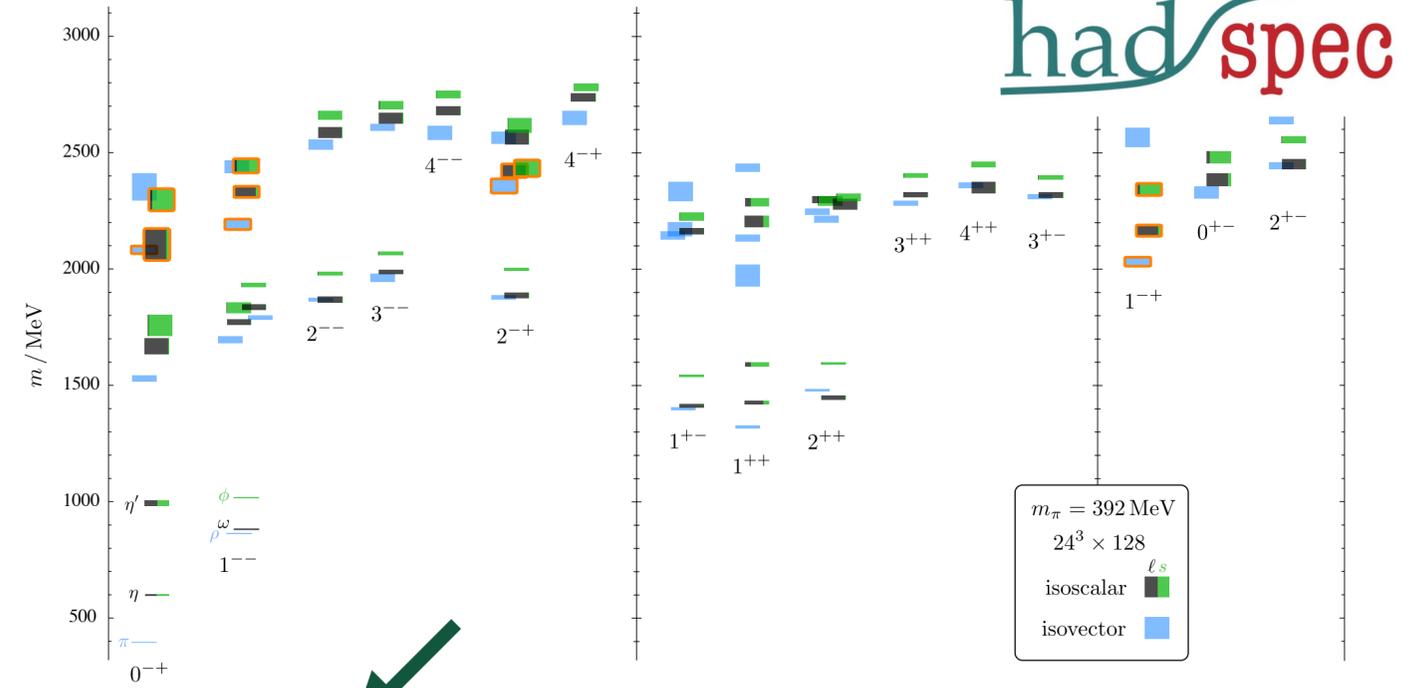


Lattice QCD $\rightarrow \delta(s)$





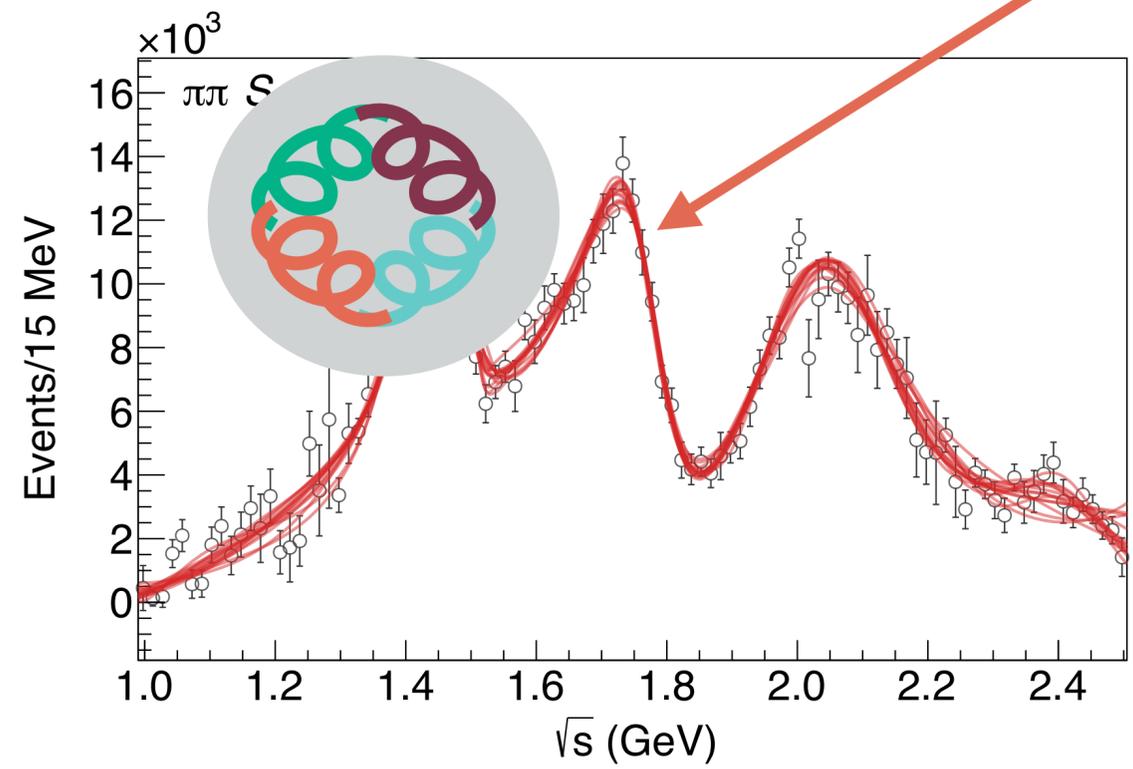
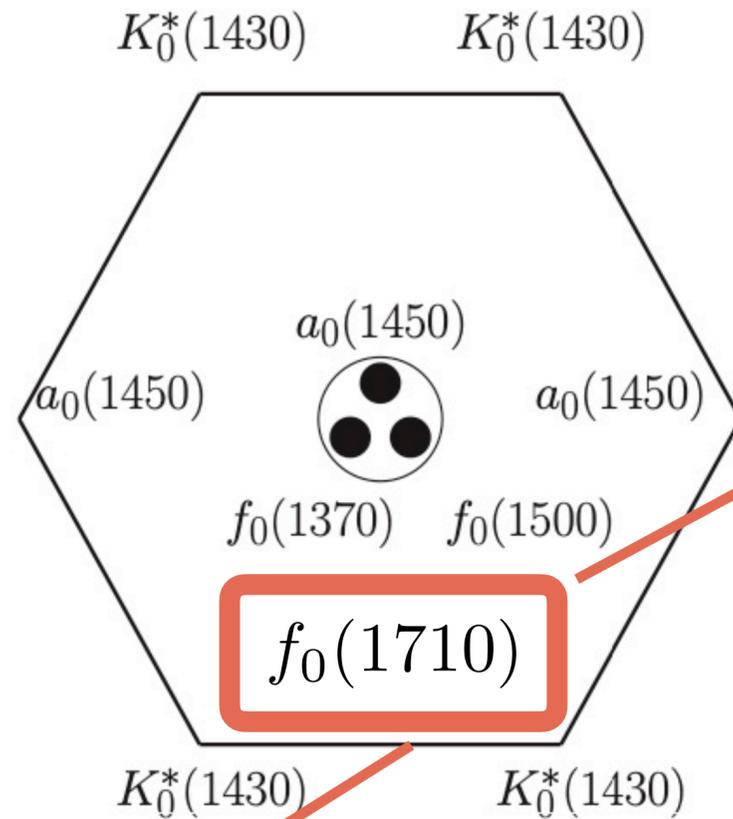
GLUEX



Questions??

Exotics

• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

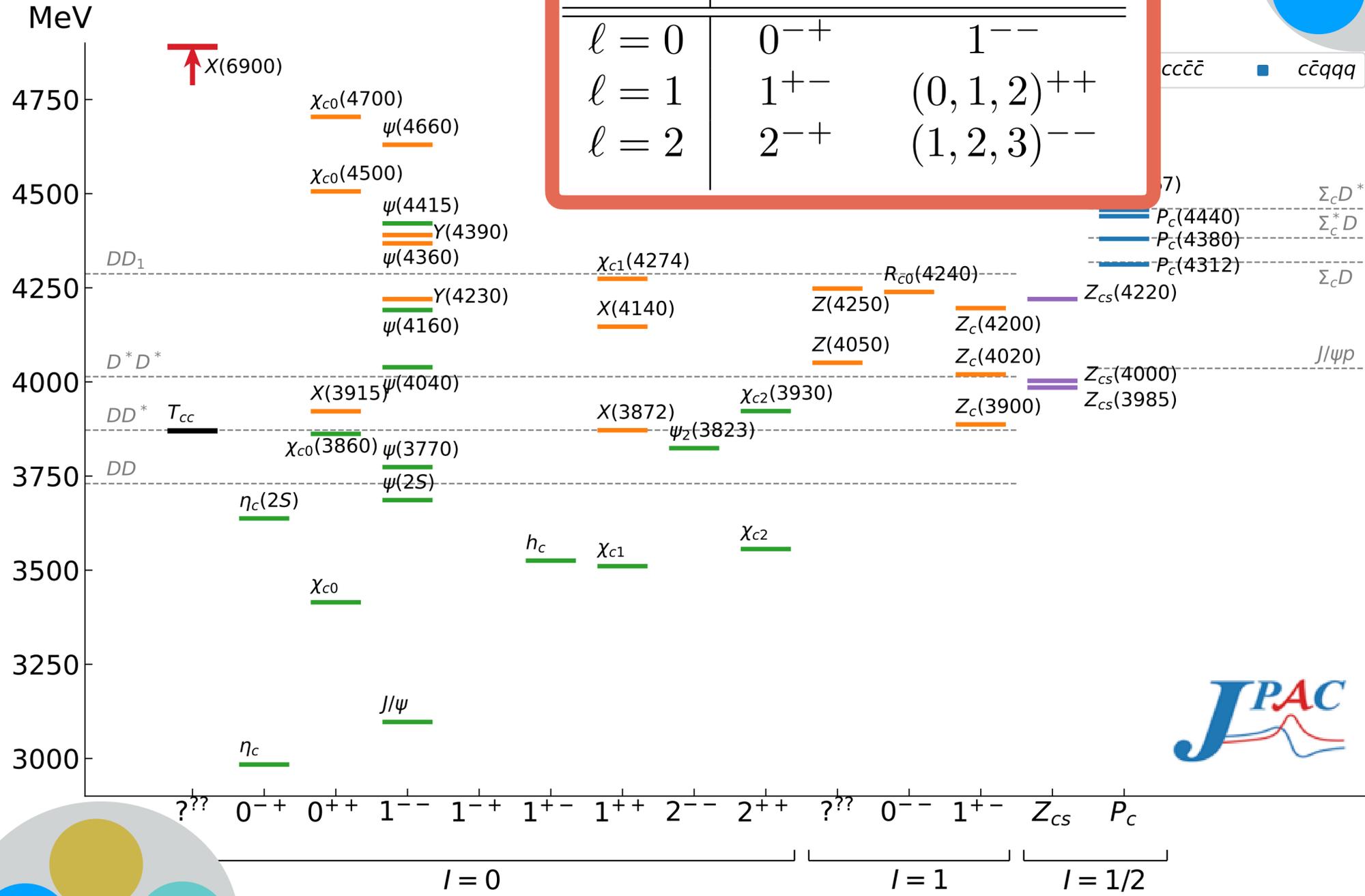


LIGHT UNFLAVORED (S = C = B = 0)		STRANGE (S = ±1, C = B = 0)	CHARMED, STRANGE (C = S = ±1)	c \bar{c} continued I ^G (J ^{PC})		
I ^G (J ^{PC})	I ^G (J ^{PC})	I(J ^P)	I(J ^P)	I ^G (J ^{PC})		
• π [±] 1 ⁻ (0 ⁻)	• π ₂ (1670) 1 ⁻ (2 ⁻ +) • φ(1680) 0 ⁻ (1 ⁻ -)	• K [±] 1/2(0 ⁻)	• D _s [±] 0(0 ⁻)	• ψ(3770) 0 ⁻ (1 ⁻ -)		
• π ⁰ 1 ⁻ (0 ⁻ +) • η 0 ⁺ (0 ⁻ +) • f ₀ (500) 0 ⁺ (0 ⁺ +) • ρ(770) 1 ⁺ (1 ⁻ -) • ω(782) 0 ⁻ (1 ⁻ -) • η'(958) 0 ⁺ (0 ⁻ +) • f ₀ (980) 0 ⁺ (0 ⁺ +) • a ₀ (980) 1 ⁻ (0 ⁺ +) • φ(1020) 0 ⁻ (1 ⁻ -) • h ₁ (1170) 0 ⁻ (1 ⁺ -) • b ₁ (1235) 1 ⁺ (1 ⁺ -) • a ₁ (1260) 1 ⁻ (1 ⁺ +) • f ₂ (1270) 0 ⁺ (2 ⁺ +) • f ₁ (1285) 0 ⁺ (1 ⁺ +) • η(1295) 0 ⁺ (0 ⁻ +) • π(1300) 1 ⁻ (0 ⁻ +) • a ₂ (1320) 1 ⁻ (2 ⁺ +) • f ₀ (1370) 0 ⁺ (0 ⁺ +) • π ₁ (1400) 1 ⁻ (1 ⁺ -) • η(1405) 0 ⁺ (0 ⁻ +) • h ₁ (1415) 0 ⁻ (1 ⁺ -) • a ₁ (1420) 1 ⁻ (1 ⁺ +) • f ₁ (1420) 0 ⁺ (1 ⁺ +) • ω(1420) 0 ⁻ (1 ⁻ -) • f ₂ (1430) 0 ⁺ (2 ⁺ +) • a ₀ (1450) 1 ⁻ (0 ⁺ +) • ρ(1450) 1 ⁺ (1 ⁻ -) • η(1475) 0 ⁺ (0 ⁻ +) • f ₀ (1500) 0 ⁺ (0 ⁺ +) • f ₁ (1510) 0 ⁺ (1 ⁺ +) • f ₂ '(1525) 0 ⁺ (2 ⁺ +) • f ₂ (1565) 0 ⁺ (2 ⁺ +) • ρ(1570) 1 ⁺ (1 ⁻ -) • h ₁ (1595) 0 ⁻ (1 ⁺ -) • π ₁ (1600) 1 ⁻ (1 ⁺ -) • a ₁ (1640) 1 ⁻ (1 ⁺ +) • f ₂ (1640) 0 ⁺ (2 ⁺ +) • η ₂ (1645) 0 ⁺ (2 ⁻ +) • ω(1650) 0 ⁻ (1 ⁻ -) • ω ₃ (1670) 0 ⁻ (3 ⁻ -)	• K ⁰ 1/2(0 ⁻) • K _s ⁰ 1/2(0 ⁻) • K _L ⁰ 1/2(0 ⁻) • K ₀ [*] (700) 1/2(0 ⁺) • K [*] (892) 1/2(1 ⁻) • K ₁ (1270) 1/2(1 ⁺) • K ₁ (1400) 1/2(1 ⁺) • K [*] (1410) 1/2(1 ⁻) • K ₀ [*] (1430) 1/2(0 ⁺) • K ₂ [*] (1430) 1/2(2 ⁺) • K(1460) 1/2(0 ⁻) • K ₂ (1580) 1/2(2 ⁻) • K(1630) 1/2(2 ⁻) • K ₁ (1650) 1/2(1 ⁺) • K [*] (1680) 1/2(1 ⁻) • K ₂ (1770) 1/2(2 ⁻) • K ₃ [*] (1780) 1/2(3 ⁻) • K ₂ (1820) 1/2(2 ⁻) • K(1830) 1/2(0 ⁻) • K ₀ [*] (1950) 1/2(0 ⁺) • K ₂ [*] (1980) 1/2(2 ⁺) • K ₄ [*] (2045) 1/2(4 ⁺) • K ₂ (2250) 1/2(2 ⁻) • K ₃ (2320) 1/2(3 ⁺) • K ₅ [*] (2380) 1/2(5 ⁻) • K ₄ (2500) 1/2(4 ⁻) • K(3100) 1/2(3 ⁻)	• D _s [±] 0(0 ⁻) • D _s ^{*±} 0(2 ⁻) • D _{s0} [*] (2317) [±] 0(0 ⁺) • D _{s1} (2460) [±] 0(1 ⁺) • D _{s1} (2536) [±] 0(1 ⁺) • D _{s2} [*] (2573) 0(2 ⁺) • D _{s1} [*] (2700) [±] 0(1 ⁻) • D _{s1} [*] (2860) [±] 0(1 ⁻) • D _{s3} [*] (2860) [±] 0(3 ⁻) • D _{s,J} (3040) [±] 0(2 ⁻)	• ψ(3770) 0 ⁻ (1 ⁻ -) • ψ ₂ (3823) 0 ⁻ (2 ⁻ -) • ψ ₃ (3842) 0 ⁻ (3 ⁻ -) • χ _{co} (3860) 0 ⁺ (0 ⁺ +) • χ _{c1} (3872) 0 ⁺ (1 ⁺ +) • Z _c (3900) 1 ⁺ (1 ⁺ +) • X(3915) 0 ⁺ (0/2 ⁺ +) • χ _{c2} (3930) 0 ⁺ (2 ⁺ +) • X(3940) 0 ⁺ (2 ⁺ +) • X(4020) [±] 1 ⁺ (2 ⁻ -) • ψ(4040) 0 ⁻ (1 ⁻ -) • X(4050) [±] 1 ⁻ (2 ⁻ +) • X(4055) [±] 1 ⁺ (2 ⁻ -) • X(4100) [±] 1 ⁻ (2 ⁻ -) • χ _{c1} (4140) 0 ⁺ (1 ⁺ +) • ψ(4160) 0 ⁻ (1 ⁻ -) • X(4160) 0 ⁺ (2 ⁻ -) • Z _c (4200) 1 ⁺ (1 ⁺ -) • ψ(4230) 0 ⁻ (1 ⁻ -) • R _{co} (4240) 1 ⁺ (0 ⁺ -) • X(4250) [±] 1 ⁻ (2 ⁻ +) • ψ(4260) 0 ⁻ (1 ⁻ -) • χ _{c1} (4274) 0 ⁺ (1 ⁺ +) • X(4350) 0 ⁺ (2 ⁻ +) • ψ(4360) 0 ⁻ (1 ⁻ -) • ψ(4390) 0 ⁻ (1 ⁻ -) • ψ(4415) 0 ⁻ (1 ⁻ -) • Z _c (4430) 1 ⁺ (1 ⁺ -) • χ _{co} (4500) 0 ⁺ (0 ⁺ +) • ψ(4660) 0 ⁻ (1 ⁻ -) • χ _{co} (4700) 0 ⁺ (0 ⁺ +) • η _b (1S) 0 ⁺ (0 ⁻ +) • γ(1S) 0 ⁻ (1 ⁻ -) • χ _{bo} (1P) 0 ⁺ (0 ⁺ +) • χ _{b1} (1P) 0 ⁺ (1 ⁺ +) • h _b (1P) 0 ⁻ (1 ⁺ -) • χ _{b2} (1P) 0 ⁺ (2 ⁺ +) • η _b (2S) 0 ⁺ (0 ⁻ +) • γ(2S) 0 ⁻ (1 ⁻ -) • γ ₂ (1D) 0 ⁻ (2 ⁻ -) • χ _{bo} (2P) 0 ⁺ (0 ⁺ +) • χ _{b1} (2P) 0 ⁺ (1 ⁺ +) • h _b (2P) 0 ⁻ (1 ⁺ -) • χ _{b2} (2P) 0 ⁺ (2 ⁺ +) • γ(3S) 0 ⁻ (1 ⁻ -) • χ _{b1} (3P) 0 ⁺ (1 ⁺ +) • χ _{b2} (3P) 0 ⁺ (2 ⁺ +) • γ(4S) 0 ⁻ (1 ⁻ -) • Z _b (10610) 1 ⁺ (1 ⁺ -) • Z _b (10650) 1 ⁺ (1 ⁺ -) • γ(10753) 0 ⁻ (1 ⁻ -) • γ(10860) 0 ⁻ (1 ⁻ -) • γ(11020) 0 ⁻ (1 ⁻ -)			
			BOTTOM (B = ±1)			
			• B [±] 1/2(0 ⁻) • B ⁰ 1/2(0 ⁻) • B [±] /B ⁰ ADMIXTURE • B [±] /B ⁰ /B _s ⁰ /b-baryon ADMIXTURE • V _{cb} and V _{ub} CKM Matrix Elements • B [*] 1/2(1 ⁻) • B ₁ (5721) ⁺ 1/2(1 ⁺) • B ₁ (5721) ⁰ 1/2(1 ⁺) • B _J [*] (5732) 0(2 ⁻) • B ₂ [*] (5747) ⁺ 1/2(2 ⁺) • B ₂ [*] (5747) ⁰ 1/2(2 ⁺) • B _J (5840) ⁺ 1/2(2 ⁻) • B _J (5840) ⁰ 1/2(2 ⁻) • B _J (5970) ⁺ 1/2(2 ⁻) • B _J (5970) ⁰ 1/2(2 ⁻)			
			CHARMED (C = ±1)			
			• D [±] 1/2(0 ⁻) • D ⁰ 1/2(0 ⁻) • D [*] (2007) ⁰ 1/2(1 ⁻) • D [*] (2010) [±] 1/2(1 ⁻) • D _s ⁰ (2300) ⁰ 1/2(0 ⁺) • D _s [±] (2300) [±] 1/2(0 ⁺) • D ₁ (2420) ⁰ 1/2(1 ⁺) • D ₁ (2420) [±] 1/2(2 ⁻) • D ₁ (2430) ⁰ 1/2(1 ⁺) • D ₂ [*] (2460) ⁰ 1/2(2 ⁺) • D ₂ [*] (2460) [±] 1/2(2 ⁺) • D(2550) ⁰ 1/2(2 ⁻) • D _J [*] (2600) 1/2(2 ⁻) • D [*] (2640) [±] 1/2(2 ⁻) • D(2740) ⁰ 1/2(2 ⁻) • D ₃ [*] (2750) 1/2(3 ⁻) • D(3000) ⁰ 1/2(2 ⁻)		BOTTOM, STRANGE (B = ±1, S = ∓1)	
			• B _s ⁰ 0(0 ⁻) • B _s [*] 0(1 ⁻) • X(5568) [±] 0(2 ⁻) • B _{s1} (5830) ⁰ 0(1 ⁺) • B _{s2} [*] (5840) ⁰ 0(2 ⁺) • B _{s,J} [*] (5850) 0(2 ⁺)		b\bar{b} (+ possibly non-q\bar{q} states)	
			BOTTOM, CHARMED (B = C = ±1)			
			• B _c ⁺ 0(0 ⁻) • B _c (2S) [±] 0(0 ⁻)			
			c\bar{c} (+ possibly non-q\bar{q} states)			
			• η _c (1S) 0 ⁺ (0 ⁻ +) • J/ψ(1S) 0 ⁻ (1 ⁻ -) • χ _{co} (1P) 0 ⁺ (0 ⁺ +) • χ _{c1} (1P) 0 ⁺ (1 ⁺ +) • h _c (1P) 0 ⁻ (1 ⁺ -) • χ _{c2} (1P) 0 ⁺ (2 ⁺ +) • η _c (2S) 0 ⁺ (0 ⁻ +) • ψ(2S) 0 ⁻ (1 ⁻ -)			
		OTHER LIGHT				
		Further States				

Exotics

• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

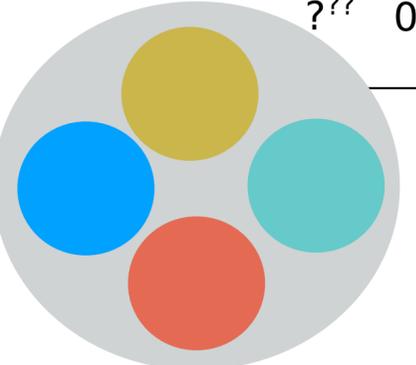
	J^{PC}	
	singlet	triplet
$l = 0$	0^{-+}	1^{--}
$l = 1$	1^{+-}	$(0, 1, 2)^{++}$
$l = 2$	2^{-+}	$(1, 2, 3)^{--}$



$ccc\bar{c}$ ■ $c\bar{c}qqq$



J^{PC}	RED	STRANGE ($S = \pm 1, C = B = 0$)	CHARMED, STRANGE ($C = S = \pm 1$)	$c\bar{c}$ continued J^{PC}
	$I(J^{PC})$	$I(J^P)$	$I(J^P)$	$I(J^{PC})$
OTHER LIGHT	π^\pm	K^\pm	D_s^\pm	$\psi(3770)$
	π^0	K^0	$D_s^{*\pm}$	$\psi_2(3823)$
		K_S^0	$D_{s0}^*(2317)^\pm$	$\psi_3(3842)$
		K_L^0	$D_{s1}^*(2460)^\pm$	$\chi_{c0}(3860)$
		$K_0^*(700)$	$D_{s1}^*(2536)^\pm$	$\chi_{c1}(3872)$
		$K^*(892)$	$D_{s2}^*(2573)$	$Z_c(3900)$
		$K_1(1270)$	$D_{s1}^*(2700)^\pm$	$X(3915)$
		$K_1(1400)$	$D_{s1}^*(2860)^\pm$	$\chi_{c2}(3930)$
		$K^*(1410)$	$D_{s1}^*(2860)^\pm$	$X(3940)$
		$K_0^*(1430)$	$D_{s3}^*(2860)^\pm$	$X(4020)^\pm$
		$K_2^*(1430)$	$D_{sJ}^*(3040)^\pm$	$\psi(4040)$
		$\eta(1870)$		$X(4050)^\pm$
		$\rho(1900)$		$X(4055)^\pm$
		$f_2(1910)$		$X(4100)^\pm$
	CHARMED ($C = \pm 1$)			
BOTTOM ($B = \pm 1$)				
	BOTTOM, STRANGE ($B = \pm 1, S = \mp 1$)			
BOTTOM, CHARMED ($B = C = \pm 1$)				
	BOTTOM, STRANGE ($B = \pm 1, S = \mp 1$)			
BOTTOM, CHARMED ($B = C = \pm 1$)				
	OTHER LIGHT			
Further States				



Questions??