Twist-3 GPDs from Lattice QCD

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Abstract

Calculating the x-dependence of PDFs and GPDs from lattice QCD has become feasible in the last years due to novel approaches. In this work, we employ the quasi-distributions method, which relies on matrix elements of non-local operators, matched to the light-cone distributions using Large Momentum Effective Theory (LaMET). In this presentation, we focus on results for the first-ever lattice QCD calculation of twist-3 GPDs of the axial operator. The calculation is performed using one ensemble of two degenerate light, a strange and a charm quark (Nf=2+1+1) of maximally twisted mass fermions with a clover term, reproducing a pion mass of 260 MeV.

Why Twist-3 GPDs?

GPDs provide information of Spatial Distribution of Partons inside the hadron, as well as it's mechanical properties. These are experimentally accessed via DVCS and DVMP. We are especially interested in twist-3 GPDs, which are the sub-leading twist contributions:

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q_o} + \frac{f_i^{(2)}}{Q_o^2}$$

Here the term that is zero-order in $\frac{1}{Q_o}$ is known as the twist-2 contribution and the first order in $\frac{1}{Q_o}$ is the twist-3 contribution. Twist-3 are not negligible for the energy scales explored experimentally.

Higher-twist distributions:

- lack density interpretation, but can still be sizable.
- are sensitive to soft dynamics.
- are needed for proton tomography.
- are related to certain spin-orbit correlations. [1]
- estimate the power corrections in hard exclusive processes (DVCS).

However, higher twist distributions are difficult to probe experimentally, since they are difficult to isolate from the leading twist contributions.

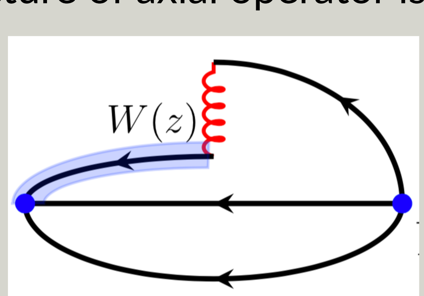
Access of GPDs on Euclidean Lattice

The quasi-distributions approach relies on Large Momentum Effective theory (LaMET), which relates the lattice data to the physical GPDs [2] [3].

We compute matrix elements of **spatial operators** with fast moving hadrons.

$$\tilde{q}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{ixP_3z} \langle N(P_f)|\,\bar{\Psi}(z)\gamma^j\gamma^5\mathcal{W}(z,0)\Psi(0)\,|N(P_i)\rangle_\mu , \qquad j=1,2$$

In the twist-3 case, the Dirac structure of axial operator is transverse to the boost direction.



where z is the length of the Wilson line, P_3 is the nucleon momentum, $t \equiv -Q^2$ is the momentum transfer squared, and ξ is the skewness. There are several challenges of the calculation:

1. Increased statistical uncertainties due to momentum transfer
2. Need for multiple matrix elements to disentangle GPDs
3. Frame dependence (GPDs defined in Breit frame)
4. Matching for nonzero skewness is non-trivial

Quasi Distribution Approach

1. **Hadronic Matrix Elements**

$$C^{\rm 2pt} = \langle N(P)|\,|N(P)\rangle , \qquad C^{\rm 3pt} = \langle N(P+\frac{Q}{2})|\,\bar{\psi}\gamma^j\gamma^5\mathcal{W}(z,0)\psi(0)\,|N(P-\frac{Q}{2})\rangle$$

2. **Identification of Ground State** ($\mathcal{P}_0$, $\mathcal{P}_k$: parity projectors)

$$R_\Gamma(\mathcal{P}_k,P_f,P_i;t,\tau) = \frac{C^{\rm 3pt}(\mathcal{P}_k,P_f,P_i;t,\tau)}{C^{\rm 2pt}(\mathcal{P}_\prime,P_f;t)}\sqrt{\frac{C^{\rm 2pt}(\mathcal{P}_0,P_i,t-\tau)C^{\rm 2pt}(\mathcal{P}_0,P_f,\tau)C^{\rm 2pt}(\mathcal{P}_0,P_f,t)}{C^{\rm 2pt}(\mathcal{P}_0,P_f,t-\tau)C^{\rm 2pt}(\mathcal{P}_0,P_i,\tau)C^{\rm 2pt}(\mathcal{P}_0,P_i,t)}} \to h_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3)$$

3. **Renormalization**
$h^R_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3,\mu) = Z_\mathcal{O}(z,\mu)h_{\mathcal{O},\mathcal{P}}(z,t,\xi,P_3)$ (See, e.g., Ref. [4])

4. **"Form Factor" disentanglement** ($\widetilde{H}$, $\widetilde{E}$: twist-2, $\widetilde{G}_i$: twist-3)

$$h^R_{\gamma^j\gamma^\nabla,\mathcal{P}}(z,t,\xi,P_3,\mu) = P^\mu\frac{\widetilde{h}^+}{P^+}F_{\widetilde{H}} + P^\mu\frac{\widetilde{e}^+}{P^+}F_{\widetilde{E}} + \Delta^\mu_\perp\frac{\widetilde{b}}{2m}F_{(\widetilde{E}+\widetilde{G}_1)} + h^\mu_\perp F_{(\widetilde{H}+\widetilde{G}_2)} + \Delta^\mu_\perp\frac{\widetilde{h}^+}{P^+}F_{(\widetilde{G}_3)} + \Delta^\mu_\perp\frac{\widetilde{h}^+}{P^+}F_{(\widetilde{G}_4)}$$

5. **x-dependence reconstruction**

$$\widetilde{qG_i}(x,t,\xi,\mu,P_3) = \int\frac{dz}{4\pi}e^{ixp_3z}F_{G_i}(z,P_3,t,\xi,\mu)$$

6. **Matching to light-cone GPDs**

$$\widetilde{G}_i(x,t,\xi,\mu_0,(\mu_0)_3,P_3) = \int_{-1}^{1}\frac{dy}{|y|}C_{\widetilde{G}_i}(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{yP_3},\frac{(\mu_0)_3}{yP_3},r)\widetilde{qG_i}(y,t,\xi,\mu) + \mathcal{O}(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda_QCD^2}{x^2P_3^2})$$

Paramaters of calculation (u-d flavor combination)

- Pion Mass: 260 MeV
- Lattice spacing: 0.093 fm
- Volume $32^2 \times 64$
- Spatial Extent 3 fm

Excited States: $T_{\rm sink} \approx 1$ fm

P_3 [GeV]	$q\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm meas}$
0.83	(2,0,0)	0.69	0	4288
1.25	(2,0,0)	0.69	0	4288
1.25	(2,2,0)	1.39	0	4288
1.67	(2,0,0)	0.69	0	4288

Decomposition of Matrix Elements

We decompose the matrix elements that contain twist-3 GPDs (see, the Quasi Distribution Approach section).

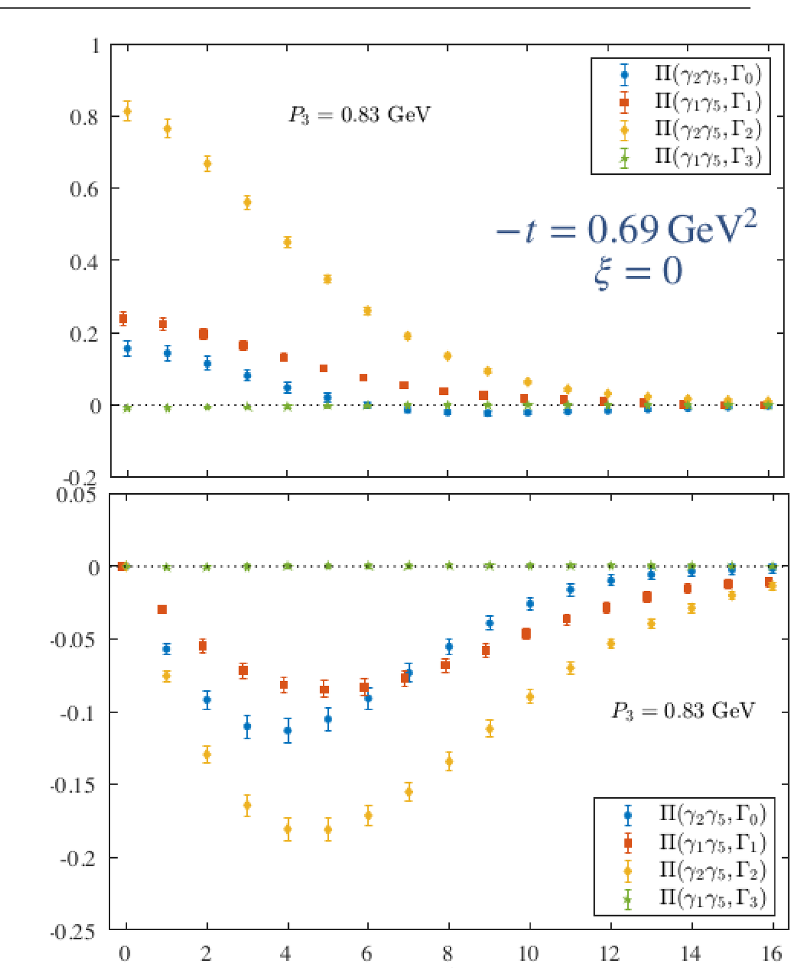
We have 4 GPDs for the $\gamma^j\gamma^5$ operator at twist-3 level. The following matrix elements are required to disentangle them:

- $\Pi(\gamma^2\gamma^5,\Gamma_0)$: $\widetilde{H}+\widetilde{G}_2,\widetilde{G}_4$
- $\Pi(\gamma^2\gamma^5,\Gamma_2)$: $\widetilde{H}+\widetilde{G}_2,\widetilde{G}_4$
- $\Pi(\gamma^1\gamma^5,\Gamma_1)$: $\widetilde{H}+\widetilde{G}_2,\widetilde{E}+\widetilde{G}_4$
- $\Pi(\gamma^1\gamma^5,\Gamma_3)$: $\widetilde{G}_3$

where the Γ are parity projectors:

- $\Gamma_0 = \frac{1}{4}(1+\gamma^0)$
- $\Gamma_\kappa = \frac{1}{4}(1+\gamma^0)\gamma^5\gamma^\kappa$

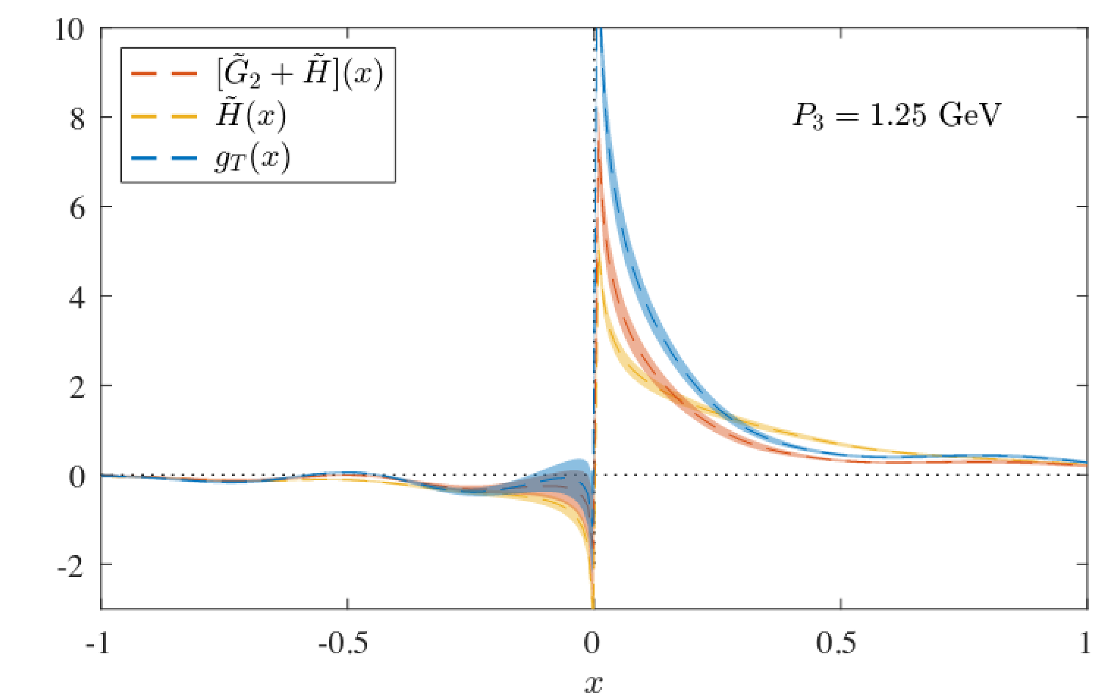
We find a very good signal, with $\Pi(\gamma^2\gamma^5,\Gamma_0)$ being dominant. $\Pi(\gamma^1\gamma^5,\Gamma_3)$ is suppressed and compatible with zero.



Matrix elements contributing to the twist-3 axial GPDs

x-dependence of GPDs

- We reconstruct the x-dependence from the decomposed matrix elements and apply the matching kernel.
- Here we present results for the $\widetilde{H}+\widetilde{G}_2$ GPD. The setup corresponds to $P_3 = 1.25$ GeV^2, $-t = 0.69$ GeV^2, and $\xi = 0$.
- The combination $\widetilde{H}+\widetilde{G}_2$ appears in the sub-leading term of the expansion in terms of the energy scale of the process $(1/Q_o)$
- Its forward limit is the twist-3 PDF g_T, which we calculated in a separate work [5].
- The twist-2 counterpart, $\widetilde{H}$ is also calculated separately [6]



- $g_T(x)$ is the dominant distribution
- $\widetilde{H}+\widetilde{G}_2$ is similar in magnitude to $\widetilde{H}$
- Thus we can conclude that $\widetilde{G}_2$ is expected to be small

Concluding Remarks

- The multi-dimensionality of the GPDs poses computational challenges
- At twist-3, there are 2-parton correlations as well as 3-parton correlations (e.g. quark-gluon-quark)
- The $0^{\rm th}$ moment of twist-3 GPDs is zero
- $1^{\rm st}$ moments have zero qgq contribution
- Extraction of twist-3 GPDs is promising with several interesting invetigations (WW-approximation, sum rules)
- Nonzero skewness of particular interest: the twist-3 GPDs (in models) exhibit discontinuities at $x = \pm\xi$

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