Gravitational form factors and mechanical properties of a quark at one loop in light-front Hamiltonian QCD

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1. Introduction

A major interest in hadron physics and QCD is to understand the mechanical properties like mass, angular momentum, pressure distribution inside the nucleon in terms of quarks and gluons.

- These mechanical properties are encoded in gravitaional form factors (GFFs). They are functions of square of the momentum transfer (q^2) in the process.
- GFFs are related to generalized parton distributions (GPDs), can be accessed in exclusive electron-proton scattering process, e.g. Deeply virtual compton scattering (DVCS).
- $A(q^2)$, $B(q^2)$, $C(q^2)$ and $\bar{C}(q^2)$ are four GFFs of proton.

2. Gravitational Form Factors

Mass appears in physics as:

- Inertial mass: Newtonian mechanics, Quantum Mechanics.
- Measurement of source strength of a gravitational field.

$\langle p'|j_\mu(x)|p\rangle$: Charge and magnetic structure
$j_\mu(x)$: Total current operator, source of $A_\mu(x)$

$\langle p'|\theta_{\mu\nu}(x)|p\rangle$: Response to gravitational field $g_{\mu\nu}$
$\theta_{\mu\nu}$: Symmetric energy-momentum tensor (EMT)

The standard parametrization in QCD:

$$\langle P',S'|\theta_i^{\mu\nu}(0)|P,S\rangle = \overline{U}(P',S')\Big[-B_i(q^2)\frac{\overline{P}^\mu\,\overline{P}^\nu}{M} + (A_i(q^2)+B_i(q^2))\frac{1}{2}(\gamma^\mu\overline{P}^\nu+\gamma^\nu\overline{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \overline{C}_i(q^2)M\,g^{\mu\nu}\Big]U(P,S)$$

where, $\overline{P}^\mu = \frac{1}{2}(P'+P)^\mu$, $\overline{U}(P',S')$, $U(P,S)$ are the Dirac spinors and M is the mass of the target state, $i\equiv(Q,G)$.
Poincare invariance puts the following constraints,
$A(0)=1$, related to the mass of the system.
Ji's sum rule: $A(x)+B(x)=\frac{1}{2}$, $B(0)=0$, $J(0)=\frac{1}{2}$
$\partial_\mu\theta^{\mu\nu}=0 \rightarrow \overline{C}=0$

3. D-term and Mechanical Properties

- The C form factor also known as D-term is unconstrained at zero momentum transfer as it is not related to any Poincare generators.
- This D-term is related to the pressure $p(\boldsymbol{b}^\perp)$ and shear $s(\boldsymbol{b}^\perp)$ distribution inside the nucleon as

$$p(\boldsymbol{b}^\perp) = \frac{1}{2M\boldsymbol{b}}\frac{d}{d\boldsymbol{b}^\perp}\left[\boldsymbol{b}^\perp\frac{d}{d\boldsymbol{b}^\perp}D_Q(\boldsymbol{b}^\perp)\right] - M\,\overline{C}_Q(\boldsymbol{b}^\perp),$$

$$s(\boldsymbol{b}^\perp) = -\frac{\boldsymbol{b}^\perp}{M}\frac{d}{d\boldsymbol{b}^\perp}\left[\frac{1}{\boldsymbol{b}^\perp}\frac{d}{d\boldsymbol{b}^\perp}D_Q(\boldsymbol{b}^\perp)\right]$$

where,

$$F(\boldsymbol{b}^\perp) = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{q}^\perp\, e^{-i\boldsymbol{q}^\perp\boldsymbol{b}^\perp}\mathscr{F}(q^2) = \frac{1}{2\pi}\int_0^\infty d\boldsymbol{q}^{\perp 2}J_0\left(\boldsymbol{q}^\perp\boldsymbol{b}^\perp\right)\mathscr{F}(q^2),$$

where, $\mathscr{F}=(A,B,C,\overline{C})$, J_0 : Bessel function of zeroth order, $\boldsymbol{b}^\perp$: impact parameter, M : mass of the dressed quark state.

- D-term has been extracted from the Jlab data and it is found to be negative.
- $p(\boldsymbol{b}^\perp)$ calculated from the data is found to be repulsive at the core and attractive towards the periphery.
- Theoretical models on $p(\boldsymbol{b}^\perp)$ and $s(\boldsymbol{b}^\perp)$ distributions: Bag model, chiral quark soliton model, AdS/QCD motivated quark-diquark model, multipole model. But these are phenomenological models and do not incorporate any gluonic degree of freedom. Lattice results are also there.
- Total quark + gluon EMT:

$$\theta^{ij}(r) = \left(\frac{r^i r^j}{r^2}-\frac{1}{3}\delta^{ij}\right)s(r)+\delta^{ij}p(r)$$

$s(r)$: Shear force, $p(r)$: Pressure

4. Dressed Quark Model (DQM)

- A simple relativistic spin-1/2 state, like a quark dressed with a gluon at one loop in QCD.
- This model employs a gluonic degree of freedom.
- The dressed quark state can be expanded in Fock space in terms of multiparton light-front wavefunctions (LFWFs), which can be calculated using light-front Hamiltonian.
- LFWFs can be written in terms of relative momenta that are frame independent. Thus, LFWFs are boost invariant.

$$|P,\lambda\rangle = \psi_1(P,\lambda)b_\lambda^\dagger(P)|0\rangle + \sum_{\lambda_1,\lambda_2}\int[k_1][k_2]\sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2) \times\psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)b_{\lambda_1}^\dagger(k_1)a_{\lambda_2}^\dagger(k_2)|0\rangle,$$

$$\text{where, } [k] = \frac{dk^+d^2\boldsymbol{k}^\perp}{\sqrt{2(2\pi)^3k^+}},\ \psi_1 \text{ :Normalization}$$

Jacobi Transformation,
$k_i^+ = x_iP^+$, $\boldsymbol{k}_i^\perp = \boldsymbol{\kappa}_i^\perp + x_i\boldsymbol{P}^\perp$, $x_1+x_2=1$, $\boldsymbol{\kappa}_1^\perp+\boldsymbol{\kappa}_2^\perp=0$.

$$\phi_{\lambda_1,\lambda_2}^{\lambda a}(x_i,\boldsymbol{\kappa}_i^\perp) = \left[\frac{x(1-x)}{\boldsymbol{\kappa}^{\perp 2}+m^2(1-x)^2}\right]\frac{g}{\sqrt{2(2\pi)^3}} \times\frac{T^a}{\sqrt{1-x}}\chi_{\lambda_1}^\dagger\Big[\frac{-2(\boldsymbol{\kappa}^\perp\cdot\boldsymbol{\varepsilon}_{\lambda_2}^{\perp *})}{1-x} - \frac{1}{x}(\tilde{\sigma}^\perp\cdot\boldsymbol{\kappa}^\perp)(\tilde{\sigma}^\perp\cdot\boldsymbol{\varepsilon}_{\lambda_2}^{\perp *}) + im(\tilde{\sigma}^\perp\cdot\boldsymbol{\varepsilon}_{\lambda_2}^{\perp *})\frac{1-x}{x}\Big]\chi_\lambda\psi_1^\lambda$$

where, $\phi_{\lambda_1,\lambda_2}^{\lambda a}(x_i,\boldsymbol{\kappa}_i^\perp) = \sqrt{P^+}\psi_2(P,\lambda|k_1,\lambda_1;k_2,\lambda_2)$, g : quark-gluon coupling, T^a : colour SU(3) matrices, $\varepsilon_{\lambda_2}^\perp$: polarization vector of gluon, m : quark mass, χ_λ : two-component spinor for the quark respectively, $\lambda=1,2$: helicity up/down, $\tilde{\sigma}_1=\sigma_2$, $\tilde{\sigma}_2=-\sigma_1$

5. Matrix Elements of EMT & Extraction of GFFs

- Two component formulation of light-front QCD, with $A^+=0$.
- Drell-Yan frame:

$$P^\mu = (P^+,\boldsymbol{P}^\perp,P^-) = \left(P^+,\boldsymbol{0}^\perp,\frac{M^2}{P^+}\right),$$

$$P'^\mu = \left(P^+,\ \boldsymbol{q}^\perp,\frac{\boldsymbol{q}^{\perp 2}+M^2}{P^+}\right),$$

$$q^\mu = (P'-P)^\mu = \left(0,\boldsymbol{q}^\perp,\frac{\boldsymbol{q}^{\perp 2}}{P^+}\right).$$

- Fermionic part of QCD EMT:

$$\theta_Q^{\mu\nu} = \frac{1}{2}\overline{\psi}i\left[\gamma^\mu D^\nu+\gamma^\nu D^\mu\right]\psi$$

Results

$$A_Q(q^2) = 1+\frac{g^2\,C_F}{2\pi^2}\left[\frac{11}{10}-\frac{4}{5}\left(1+\frac{2m^2}{q^2}\right)\frac{f_2}{f_1}-\frac{1}{3}\log\left(\frac{\Lambda^2}{m^2}\right)\right],$$

$$B_Q(q^2) = \frac{g^2C_F}{12\pi^2}\frac{m^2}{q^2}\frac{f_2}{f_1},$$

$$D_Q(q^2) = \frac{5g^2C_F}{6\pi^2}\frac{m^2}{q^2}\left(1-f_1f_2\right) = 4\ C_Q(q^2),$$

$$\overline{C}_Q(q^2) = \frac{g^2C_F}{72\pi^2}\left(29-30\ f_1\ f_2+3\ \log\left(\frac{\Lambda^2}{m^2}\right)\right),$$

where, $f_1=\frac{1}{2}\sqrt{1+\frac{4m^2}{q^2}}$, $f_2=\log\left(1+\frac{q^2(1+2f_1)}{2m^2}\right)$.

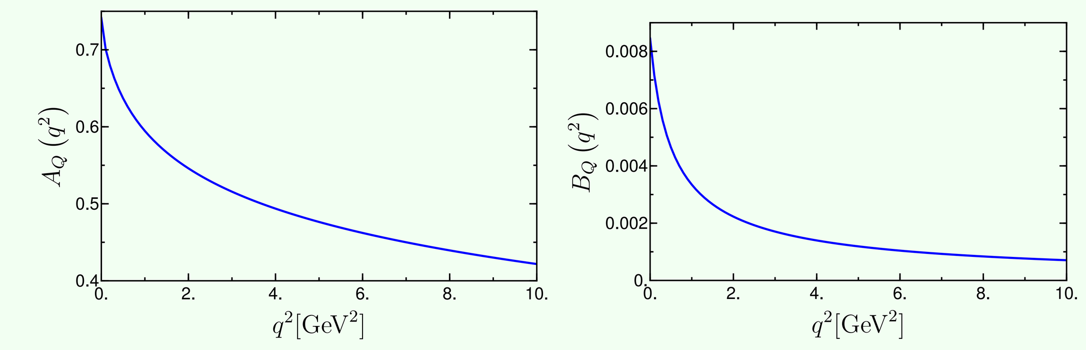


Figure: GFF $A_Q(q^2)$ & $B_Q(q^2)$ as a function of q^2.

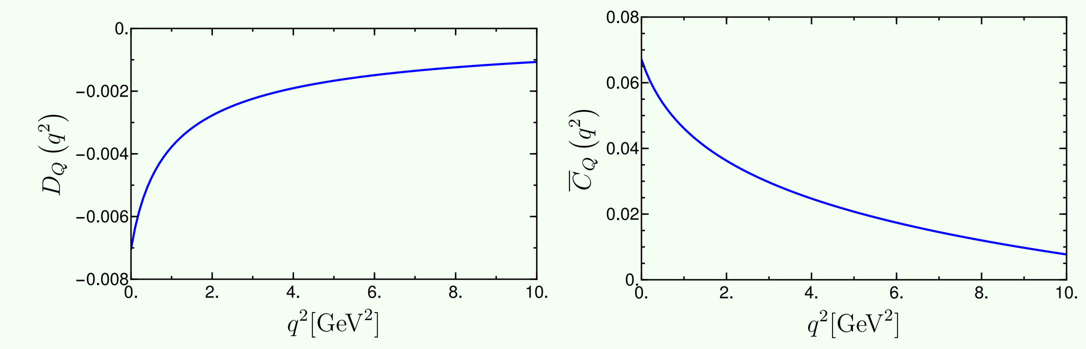
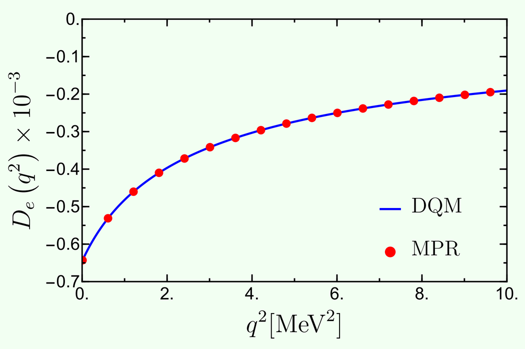


Figure: GFF $D_Q(q^2)$ & $\overline{C}_Q(q^2)$ as a function of q^2.



MPR: Metz *et al.* (Phys.Lett.B 820 (2021), 136501)

Figure: Electron D-term as a function of q^2.

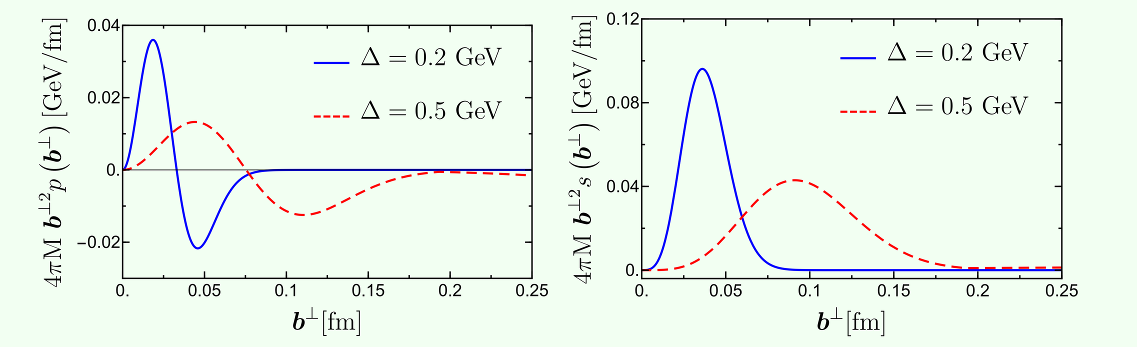


Figure: $4\pi M b^{\perp 2}p(b^\perp)$ & $4\pi M b^{\perp 2}s(b^\perp)$ as a function of $b^\perp$.

$$\frac{1}{16\pi^3}\int\frac{d^2\boldsymbol{p}^\perp dp^+}{p^+}\phi(p)\mid p^+,\boldsymbol{p}^\perp,\lambda\rangle,$$

with $\phi(p) = p^+\ \delta(p^+-p_0^+)\ e^{-\frac{p^{\perp 2}}{2\Delta^2}}$

7. Conclusions

- We have studied the four Gravitational Form Factors in a composite spin-1/2 system, a quark dressed with a gluon at one loop level in QCD.
- We have also analysed the pressure and shear distributions in this model.

8. Acknowledgements

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9. Reference

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