Gluon PDFs from Lattice QCD using twisted-mass fermions

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Outline

Why Lattice QCD?

- QCD Lagrangian: easy to write, difficult to solve

$$\mathcal{L}_{QCD} = \sum_f \bar{\Psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \Psi_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

  - Can't be solved analytically because of non-linearity
  - Perturbation theory fails to fully detail low- and mid-range energy behaviors
  - Infinite degrees of freedom $\rightarrow$ Quantum Effective Theory
- Lattice QCD is non-perturbative and starts from first-principles (no model or approximation)

Building Lattice QCD

- Start from Feynman's path integral formalism
- Wick rotate to Euclidean time

$$e^{iS_{QCD}} \Rightarrow e^{-S_{QCD}}$$

- Discretize continuum theory on a 4-dimensional lattice
- Lattice parameters regularize theory
  - UV cutoff from inverse lattice spacing (a^{-1})
  - IR cutoff from inverse lattice size ($V^{-1/4}$)
- Parameters of the lattice regularization same as QCD Lagrangian
  - Quark masses
  - Lattice spacing (related to coupling constant)
  - Lattice size ($L^3 \times T$)

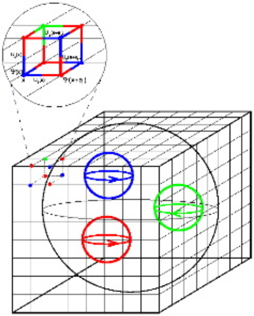


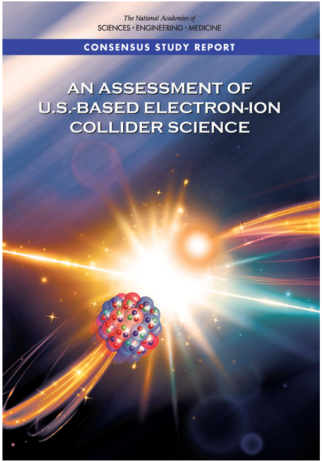
Figure: USQCD proton on the lattice

Hadron Structure from LQCD

- Fermions and gluons on the lattice
  - basic units for building hadrons
  - gluonic action discretized multiple ways (not unique)
  - fermionic action difficult to discretize $\Rightarrow$ several methods to do so (Wilson, Clover, Twisted Mass, etc)
    - each has positive and negative aspects
- Hadron states created and annihilated on lattice (source and sink)
  - excited state investigations
- Current insertion gives different observables

- Difficulties
  - All quantities must be gauge invariant
  - Fermion doubling
  - PDFs and GPDs have light-cone nature $\Rightarrow$ cannot be studied directly on the lattice (Euclidean metric)
  - Computational costs (billions of degrees of freedom)
  - Many sources of uncertainty (systematic and statistical)

Lattice and the EIC



- Lattice provides input for EIC measurements from first principle calculations
  - proton mass
  - spin decomposition
- EIC seeks to:
  - illuminate structure and interactions of gluon-dominated matter
  - probe sea quark region
  - perform parton imaging with high statistics and polarization from small- to moderate-x
- Lattice QCD today can:
  - study gluon observables
  - simulate QCD at physical values of the quark masses
  - calculate unpolarized, polarized, and transversity distributions from first principles

Theoretical Setup

- Limited studies of gluon contributions to quantities related to hadron structure (e.g., $\langle x \rangle_g$)
- Difficult to control statistical uncertainties
- x-dependence of gluon PDFs even more challenging (only a few calculations available)

Calculation of unpolarized gluon PDF for the proton

- Matrix elements of **non-local operators** and **momentum-boosted proton states**

$$\langle M^{\mu\nu}(z,P)\rangle \equiv \langle N(P)|\frac{1}{2}\sum_i F^{ti}(z)W(z,0)F_{3i}(0)|P\rangle$$

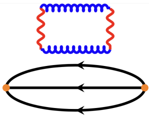


Figure: The red wavy (blue wiggling) line represents a Wilson line of length z (the field strength tensor).

- This operator avoids finite mixing under renormalization
  - must subtract vacuum expectation value
  - unpolarized gluon PDF mixes with unpolarized singlet quark PDF in the continuum and the lattice

Lattice Parameters and Statistics

- Ensemble of twisted-mass clover fermions and Iwasaki improved gluons
  - $N_f = 2 + 1 + 1$
  - $m_\pi = 260$ MeV
  - $a = 0.09471(39)$ fm
  - $L^3 \times T = 32^3 \times 64$
  - $L = 3.0$ fm
  - $Lm_\pi = 4$
- Statistics
  - Average over proton and neutron
  - Average over all 6 spatial directions of Wilson line / momentum ($\pm x$, $\pm y$, $\pm z$)
  - Statistics much higher than quark PDFs

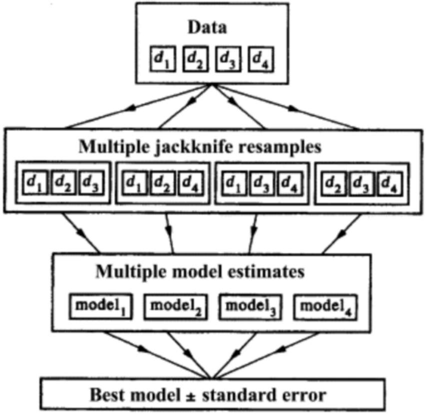
$\|\mathbf{P}\|$	t_s/a	N_{confs}	N_{src}	N_{dir}	Total statistics	Approx. Core Hours
0	9	1,134	200	1	226,800	174,636
1	9	1,134	200	6	1,360,800	1,047,816
2	9	1,134	200	6	1,360,800	1,047,816
3	9	1,134	200	6	1,360,800	1,047,816
4	9	1,134	200	6	1,360,800	1,047,816

A Statistical Aside

- Statistical resampling is a common tool in lattice to estimate bias and uncertainty
- Jackknife method
  - create bins by omitting one (or more) observation(s)
  - take the mean of the measurements in the bins
  - take average of the bins
  - calculate jackknife error

$$\sqrt{\frac{N_{bin}}{N_{bin}-1}}\sqrt{\sum_i (D_i - \bar{D})^2}$$

  - Statistical error scales as $1/\sqrt{N}$:
  - to reduce error by 50% one needs 4 times the statistics → computational expensive



Pseudo-PDF Approach

- Calculate correlators (2pt and 3pt functions)
- Take the ratio to calculate the matrix elements

$$\frac{C^{3pt}(t,\tau,0,\vec{P})}{C^{2pt}(t,0,\vec{P})} \overset{0<<\tau<<t}{\equiv} M(\nu, z^2)$$

- Form the ratio

$$\mathcal{M}(\nu, z^2) \equiv \frac{M(P, z^2)}{M(0, z^2)}$$

- Evolve the ITD

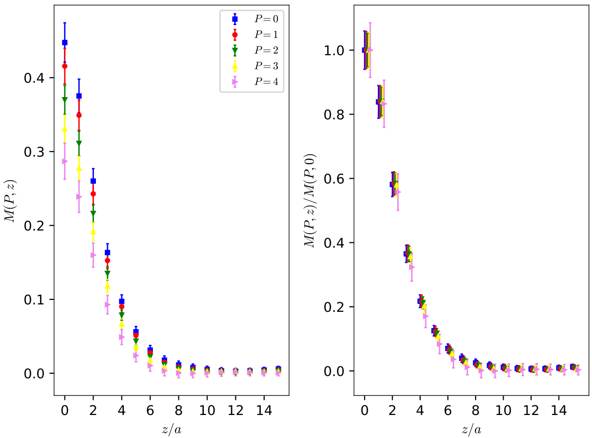
$$\tilde{\mathcal{M}}(\nu, z^2, \mu^2) = \mathcal{M} + \frac{\alpha_s N_c}{2\pi} \int_0^1 du \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B(u)\mathcal{M}(u\nu, z^2)$$

- Form the matched-ITD

$$\mathcal{Q}(\nu, z^2, \mu^2) = \tilde{\mathcal{M}}(\nu, z^2, \mu^2) + \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, L(u)\mathcal{M}(u\nu, z^2)$$

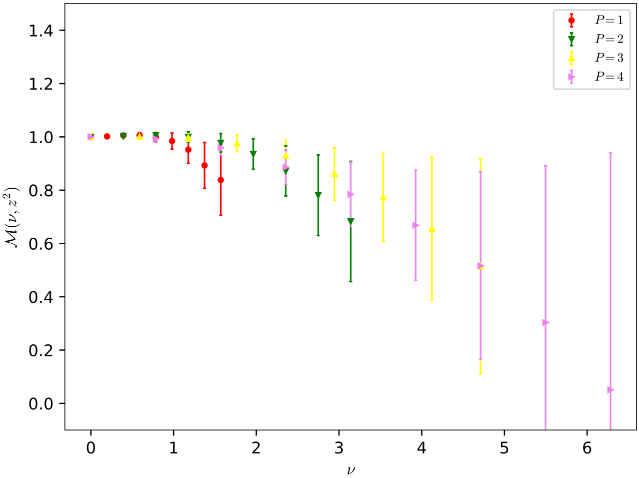
- Fourier transform to get PDF

Matrix Elements

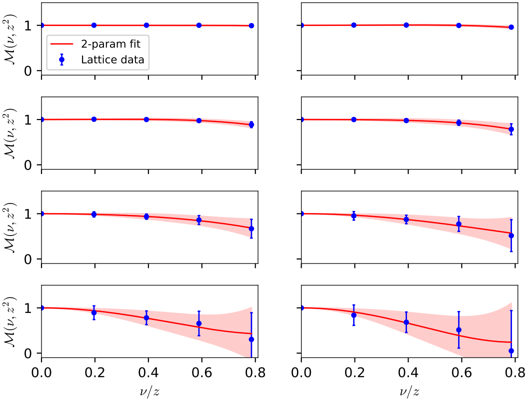


- Statistical errors increase with momentum boost
- MEs have expected behavior (higher boosts go to 0 faster)

Double Ratio (Reduced ITD)

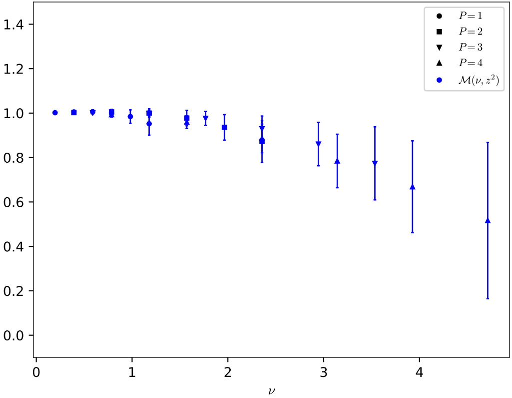


Interpolation of Double Ratio



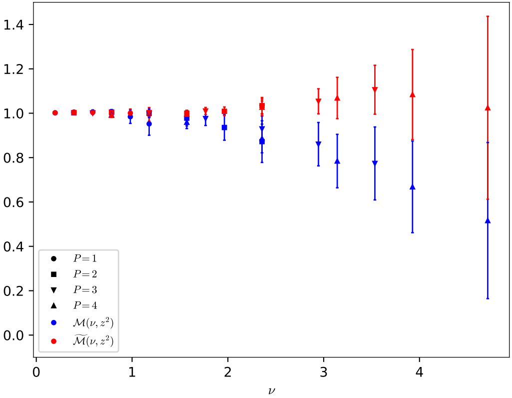
- Interpolate the double-ratio at each z-step to get a continuous function for the integration

ITD Development



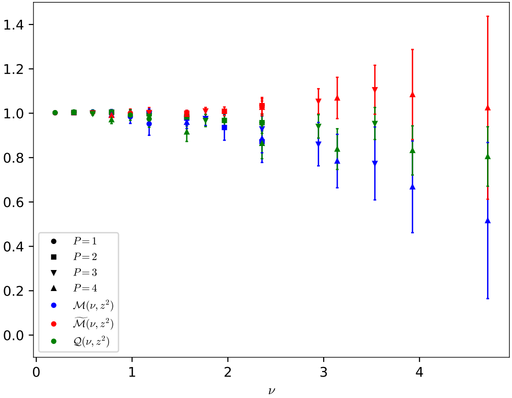
- Calculate the reduced-ITD for each boost

ITD Development



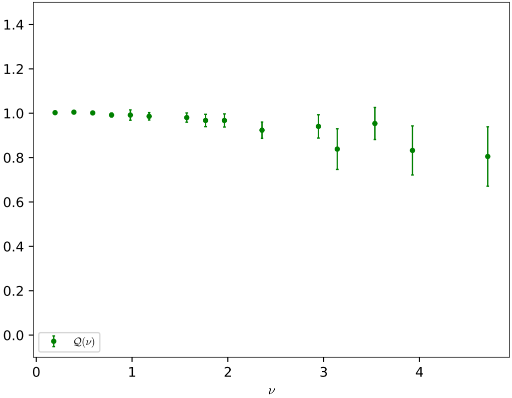
- Add first integral to get evolved-ITD

ITD Development



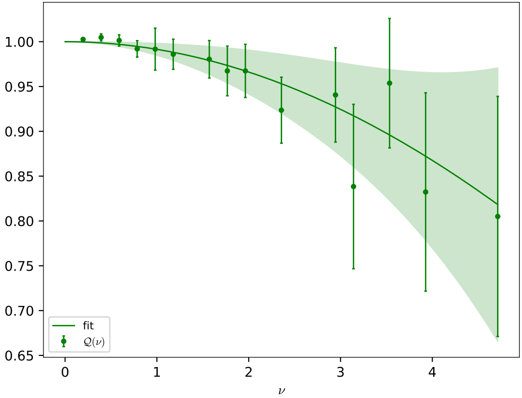
- Add second integral to get matched-ITD

ITD Development



- Average over common values on ν

ITD Development



- Fit $\mathcal{Q}$ to get function for FT

Future Work

- Fourier transform pseudo-ITD to get PDF
- Perform with additional lattice parameters
  - physical pion mass
  - larger lattice
- Compare to experiment and other lattice

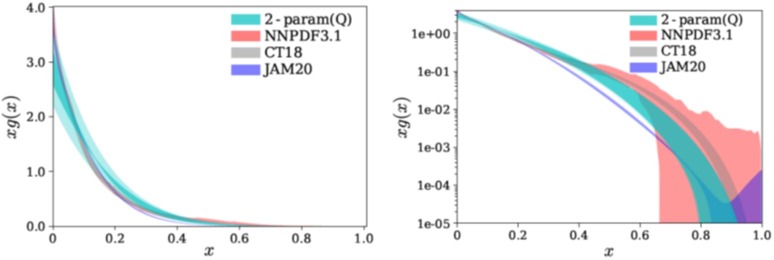


Figure: Khan et al. Phys. Rev. D 104, 094516

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