

Double inclusive gluon production in ultra-peripheral collisions

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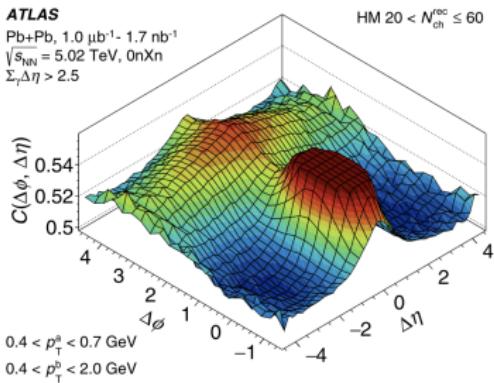
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In preparation, with Alex Kovner and Vladi Skokov

HUGS, 2022

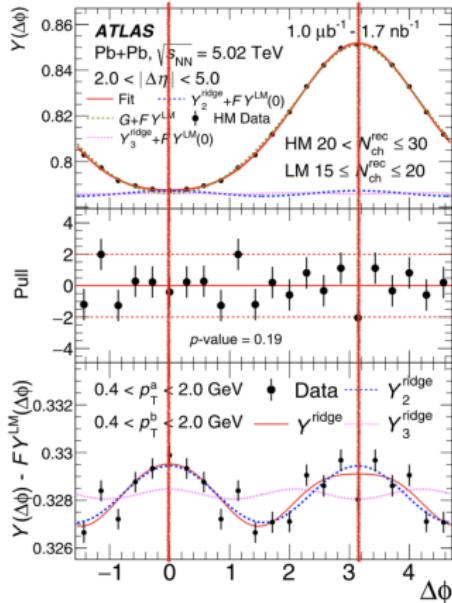
Supported by DOE

What's new?

Two particle angular correlation observed in UPC measurement at LHC

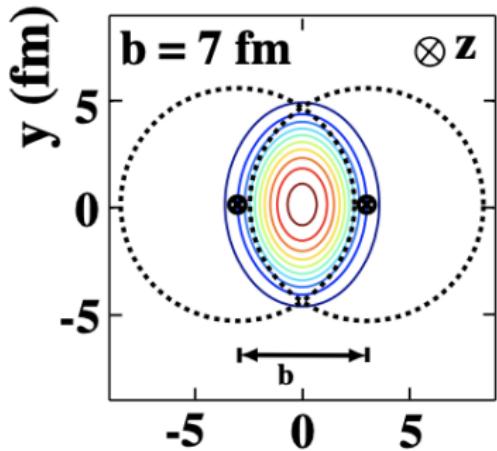


(a) PHYSICAL REVIEW C 104,
014903 (2021), ATLAS

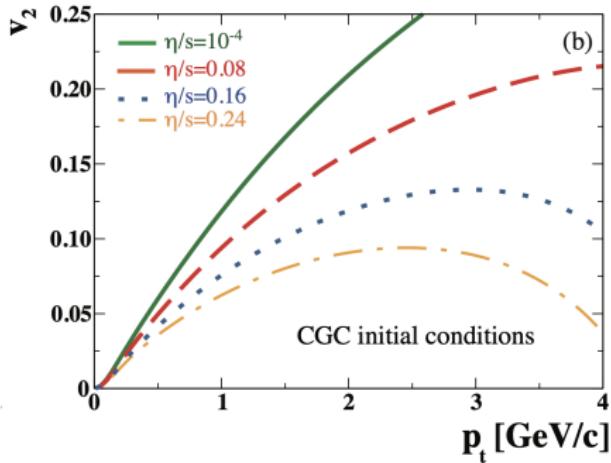


(b) Backgrounds & signals

Elliptic flow



(a) Peripheral collision for AA



(b) $v_2 \rightarrow$ viscosity

Small viscosity η/s leads to higher v_2 . Signal of QGP as nearly perfect fluid.

$$\frac{dN}{dq_1^2 dq_2^2} \propto 1 + \sum_n 2v_n^2 \cos(n\Delta\theta)$$

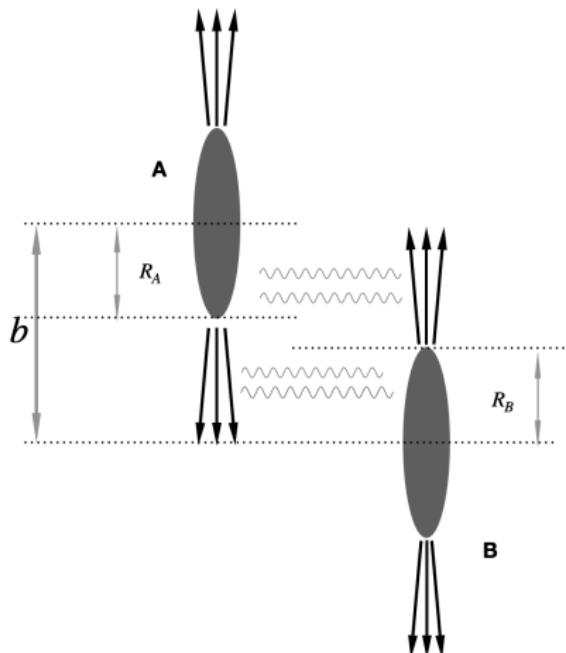
Flow signal in small systems

- If flow signal indicates Quark-gluon plasma, what is the smallest collision system to create QGP?
 - High multiplicity p+p (2010), p+Pb (2012) at LHC
 - p+Au, d+Au, ^3He +Au at RHIC (2013-2020)
- Is there alternative origin of the flow signal without creating QGP?

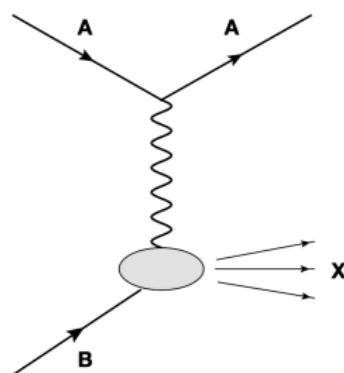
Flow signal in small systems

- If flow signal indicates Quark-gluon plasma, what is the smallest collision system to create QGP?
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- Is there alternative origin of the flow signal without creating QGP?
- The smallest projectile is photon! But we don't have EIC yet.

Ultra-peripheral collisions



- $b > R_A + R_B$
- equivalent photon approximation
- photon-nuclear interaction



Origins of the angular correlation

- Highly non-equilibrium right after the scattering
- Hydrodynamics assumed instant local equilibrium which leads to superluminal modes.



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The unreasonable effectiveness of hydrodynamics in heavy ion collisions

Jacquelyn Noronha-Hostler ^{a, b}, Jorge Noronha ^c, Miklos Gyulassy ^a

-
- Quantum correlations
 - Bose-Einstein statistics
 - HBT(Hanbury Brown and Twiss) effect
 - Both are presented

Bose enhancement

- When produced two gluons are identical, there is a quantum correction to two particle distribution.

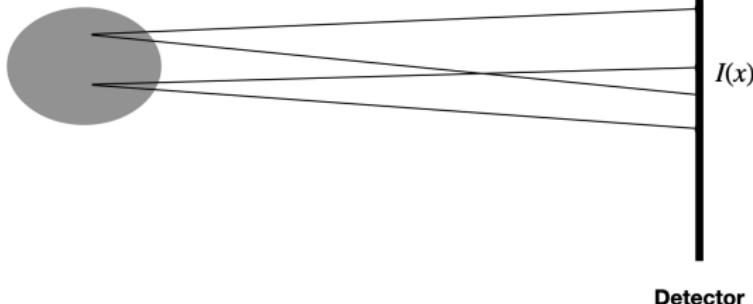
$$\begin{aligned} & \langle a_m^\dagger(\mathbf{k}) a_m(\mathbf{k}) a_n^\dagger(\mathbf{p}) a_n(\mathbf{p}) \rangle \\ &= n(\mathbf{k})n(\mathbf{p}) + \frac{(2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{p})}{(N_c^2 - 1)} n(\mathbf{k})n(\mathbf{p}) \end{aligned}$$

- Due to the symmetry of 2-d kinematics, at $\mathbf{k} = -\mathbf{p}$, we have

$$\frac{(2\pi)^2 \delta^{(2)}(\mathbf{k} + \mathbf{p})}{(N_c^2 - 1)} n(\mathbf{k})n(\mathbf{p})$$

- In practice, δ function smeared and leads to peaks at $\theta = 0, \pi, 2\pi$

Some extended source of radiation



- Collecting data for the intensity of the signal $I(x)$
- $I(x) \Rightarrow I(\mathbf{k}) = \bar{I}(\mathbf{k}) + \Delta_{\mathbf{k}}$
- If $\Delta_{\mathbf{k}}$ satisfy Gaussian distribution

$$\langle \Delta_{\mathbf{k}} \Delta_{\mathbf{p}} \rangle \propto \delta^{(2)}(\mathbf{k} - \mathbf{p})$$

- 2-d symmetry leads to another peak at $\theta = \pi$

Non-perturbative photon

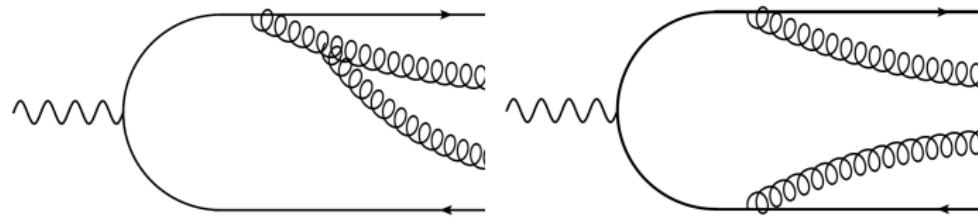
- Photon emitted by the nucleus coherently
- Resolution bounded by nucleus size

$$\frac{1}{Q} \gtrsim R_A$$

- For $A > 16$

$$Q^2 \lesssim (60\text{Mev})^2$$

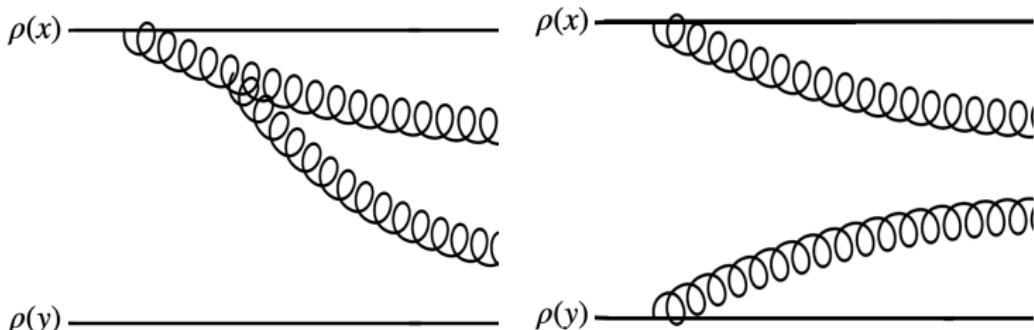
Dipole model ($|Q| < \Lambda_{QCD}$)



- Dipole model to approximate the photon
Small Q^2 suppresses the longitudinal polarization

$$\Psi_{\lambda}^T(z, \mathbf{r}, s_1) = -i \frac{2ee_f}{2\pi} \delta_{s_1, -s_2} (2z - 1 + 2\lambda s_1) \sqrt{z(1-z)} \frac{\mathbf{r} \cdot \boldsymbol{\epsilon}_{\lambda}}{|\mathbf{r}|} \varepsilon_f K_1(\varepsilon_f |\mathbf{r}|)$$

MV model



- Inspired by Vector Meson Dominance Model
- Due to the existence of the high energy fixed point,
 ρ -meson at asymptotically high energy \equiv nucleus
- Valence degrees of freedom $\rho_a(\mathbf{x})$ follow the distribution defined by McLerran-Venugopalan (MV) model

$$W(\rho_a) = \exp \left\{ - \int_{\mathbf{x}} \frac{\rho_a(\mathbf{x}) \rho_a(\mathbf{x})}{2\mu^2} \right\}$$

Angular correlation from the cross section

From the cross section of the two gluon production

$$\Sigma = \frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2}$$

one can extract the angular correlation function

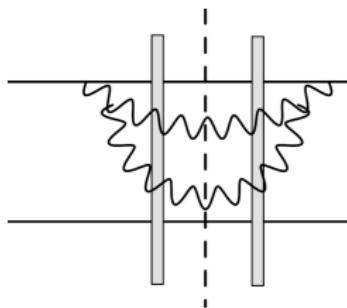
$$C(q, \theta) = \frac{\Sigma(q, \theta)}{\frac{1}{2\pi} \int_0^{2\pi} \Sigma(q, \theta) d\theta}$$

set $|q_1| = |q_2| = q$, and θ is the angle between the two particles

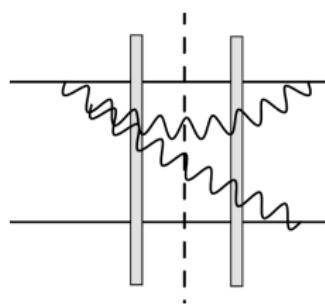
Organize the cross section

Organize the cross section Σ according to the order of ρ

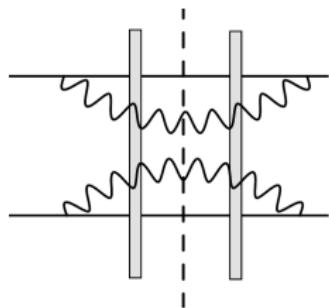
$$\Sigma = \Sigma_2 + \Sigma_3 + \Sigma_4$$



(a) $\Sigma_2(\rho^2)$



(b) $\Sigma_3(\rho^3)$



(c) $\Sigma_4(\rho^4)$

Isolating the signal

- Symmetrization of $\hat{\rho}_s$ (MV model)

$$\begin{aligned} 2\hat{\rho}_a(\mathbf{x})\hat{\rho}_b(\mathbf{y}) &= \{\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})\} + [\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})] \\ &= \rho_a(\mathbf{x})\rho_b(\mathbf{y}) - \delta^{(2)}(x - y)T_{ab}^c\rho_c(\mathbf{x}) \end{aligned}$$

- Symmetrization of color factors (Dipole model)

$$2t^a t^b = \left\{ t^a, t^b \right\} + i f_{ab}^c t^c$$

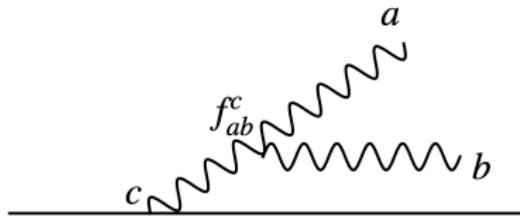
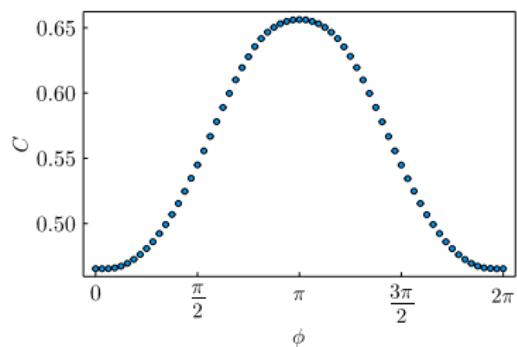
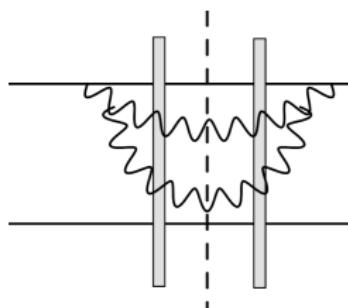
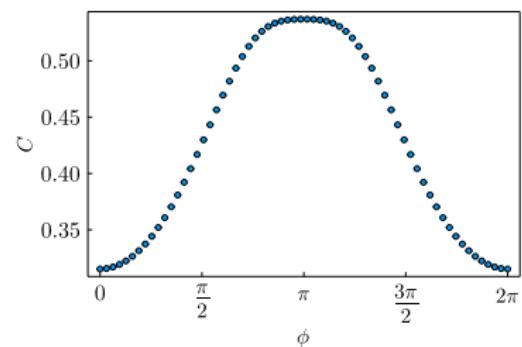


Figure: The diagram for the correction term

Σ_2 , $q = Q_s$

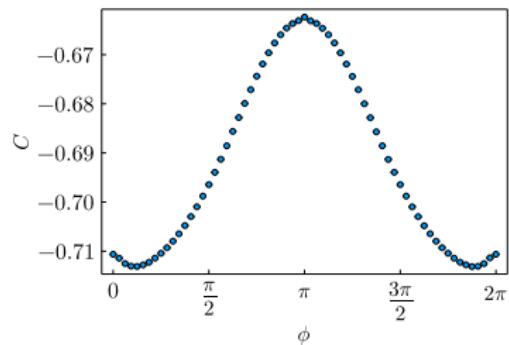
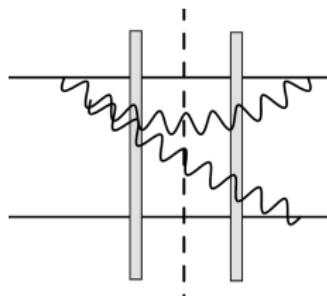


(a) Dipole

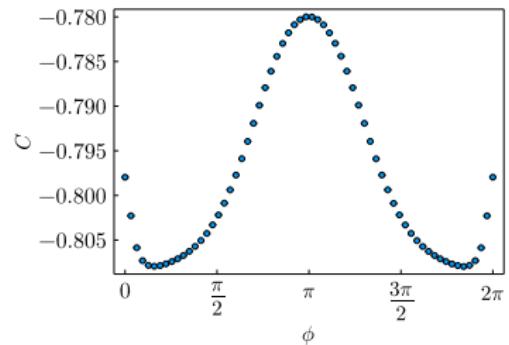


(b) MV

Σ_3 , $q = Q_s$

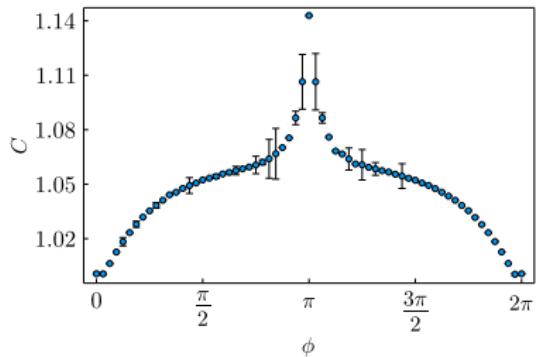


(a) Dipole

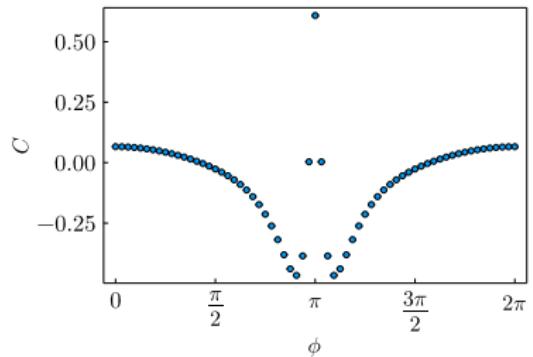


(b) MV

Σ_4^{nsym} , non-symmetric part, $q = Q_s$



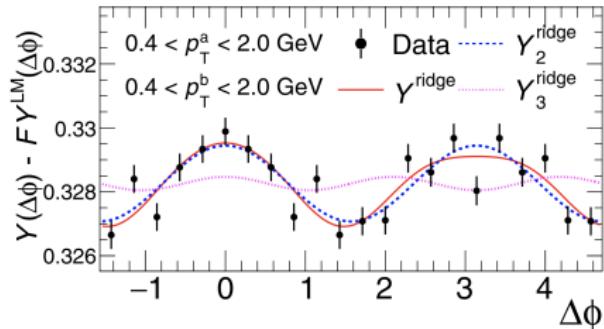
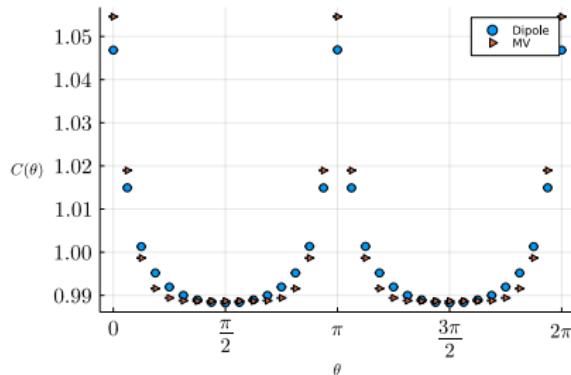
(a) Dipole



(b) MV

Also gives us back-to-back correlation.

Σ_4^{sym} , symmetric part, $q = Q_s$

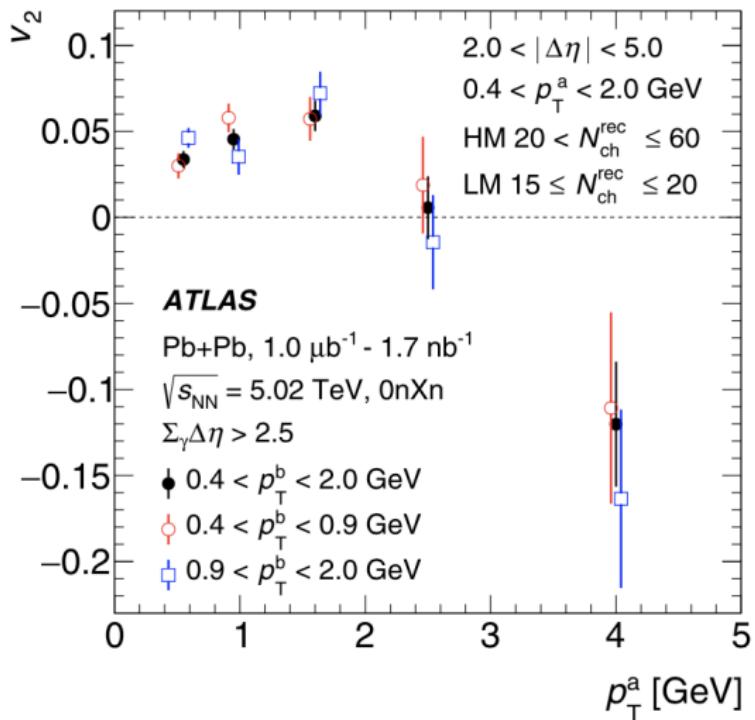


As what was done in experimental analysis, we subtract backgrounds and normalize the signal. The preliminary results shows similar correlations in CGC calculation.

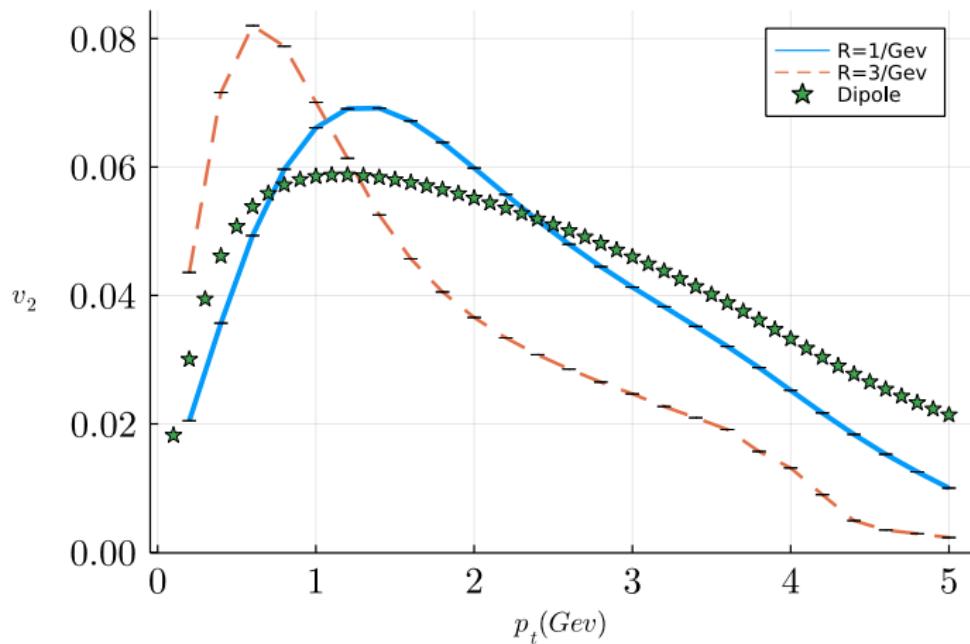
Summary and outlook

- We analytically derived inclusive two gluon production in UPC at mid-rapidity.
- To estimate systematic uncertainty originated from the poor knowledge of the real photon wave function, we studied two limiting cases.
- Both models result in qualitatively similar correlation.
Quantitatively, the amplitude of azimuthal anisotropy for MV model is about two times the dipole model.
- Our results show similar correlation as experimental data.
- Further developments
 - To compare with experimental data
 - To incorporate rapidity difference
 - To extend to EIC physics (large Q^2)
 - To use diffraction to remove backgrounds.

v_2 data



v_2 preliminary numerical results



Gluon production

Create gluons within initial states

One account for the emission of the gluons using coherent operators

$$C = \mathcal{P} e^{i\sqrt{2} \int d^2x d\xi \hat{b}_a^i(\xi, \mathbf{x}) [a_{i,a}^\dagger(\xi, \mathbf{x}) + a_{i,a}(\xi, \mathbf{x})]}$$

with the background field

$$\hat{b}_a^i(\xi, \mathbf{x}) = \frac{g}{2\pi} \int d^2y \frac{(\mathbf{x} - \mathbf{y})^i}{|\mathbf{x} - \mathbf{y}|^2} \hat{\rho}_P^a(\xi, \mathbf{y})$$

- MV model source ρ_a
- $\hat{\rho}_D^a(\mathbf{x}) = b_{\alpha\sigma}^\dagger(\mathbf{x}_1) t_{\alpha\beta}^a b_{\beta\sigma}(\mathbf{x}_1) \delta^{(2)}(\mathbf{x} - \mathbf{x}_1) - d_{\alpha\sigma}^\dagger(\mathbf{x}_2) t_{\beta\alpha}^a d_{\beta\sigma}(\mathbf{x}_2) \delta^{(2)}(\mathbf{x} - \mathbf{x}_2)$
- $\hat{\rho}_g^a(\zeta, \mathbf{x}) = a_b^{i\dagger}(\eta, \mathbf{x}) T_{bc}^a a_c(\eta, \mathbf{x})$



The cross section

$$\frac{d\mathcal{N}}{d\eta dq_1^2 d\xi dq_2^2} = \frac{1}{(2\pi)^4} \int d^2 u_1 d^2 u_2 d^2 \bar{u}_1 d^2 \bar{u}_2 e^{-i\mathbf{q}_1(\mathbf{u}_1 - \bar{\mathbf{u}}_1)} e^{-i\mathbf{q}_2(\mathbf{u}_2 - \bar{\mathbf{u}}_2)} \\ \times \langle \gamma^* | C^\dagger \hat{S}^\dagger \textcolor{teal}{C} a_{i,a}^\dagger(\eta, \mathbf{u}_1) a_{j,b}^\dagger(\xi, \mathbf{u}_2) a_{i,a}(\eta, \bar{\mathbf{u}}_1) a_{j,b}(\xi, \bar{\mathbf{u}}_2) C^\dagger \hat{S} \textcolor{red}{C} | \gamma^* \rangle$$

where $C = C_\xi C_\eta$, and $\eta \gg \xi$,

$$C_\eta \simeq 1 + i\sqrt{2} \int d^2 v_1 \hat{b}_{Da}^i(\mathbf{v}_1) \left[a_a^{i\dagger}(\eta, \mathbf{v}_1) + a_a^i(\eta, \mathbf{v}_1) \right]$$

$$C_\xi \simeq 1 + i\sqrt{2} \int d^2 v_2 \left(\hat{b}_{Db}^j(\mathbf{v}_2) + \delta \hat{b}_b^j(\eta, \mathbf{v}_2) \right) \left[a_b^{j\dagger}(\xi, \mathbf{v}_2) + a_b^j(\xi, \mathbf{v}_2) \right]$$

- $\textcolor{red}{C}|\gamma^*\rangle$ Initial state
- \hat{S} S-matrix
- $\textcolor{teal}{C} a_{j,b}(\xi, \bar{\mathbf{u}}_2) C^\dagger$ dressed gluons in the final state

Continue the calculation of Σ

Use Σ_2 as example, in coordinate space,

$$\Sigma_2 = 4 \int d^2\mathbf{x} \int d^2\bar{\mathbf{x}} \textcolor{blue}{f^i(\bar{u}_1 - \mathbf{x})} f^i(u_1 - \bar{\mathbf{x}}) f^j(\bar{u}_2 - \bar{u}_1) f^j(u_2 - u_1) \langle \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) \rangle_P \\ \left\langle \left[[U^\dagger(u_1) T^a U(u_1)] [U^\dagger(u_2) - U^\dagger(u_1)] [U(\bar{u}_2) - U(\bar{u}_1)] [U^\dagger(\bar{u}_1) T^a U(\bar{u}_1)] \right]_{d'd'} \right\rangle_T$$

where $f^i(\mathbf{x}) = \frac{g}{(2\pi)^2} \frac{x_i}{x^2}$.

- Kinematic factors (Eikonal emission vertices)
- Projectile (photon)
- Target (nucleus)

Expectation values for projectile and target

Dipole expectation values

- Expectation values for $q\bar{q}$

$$\langle q\bar{q} | \hat{\rho}_{d'}(\bar{\mathbf{x}}) \hat{\rho}_d(\mathbf{x}) | q\bar{q} \rangle = \frac{\delta^{dd'}}{2} (\delta^2(\bar{\mathbf{x}} - z_1) - \delta^2(\bar{\mathbf{x}} - z_2)) (\delta^2(\mathbf{x} - z_1) - \delta^2(\mathbf{x} - z_2))$$

$$\begin{aligned} & \langle q\bar{q} | \hat{\rho}^a(\mathbf{x}_1) \hat{\rho}^b(\mathbf{x}_2) \hat{\rho}^c(\mathbf{x}_3) | q\bar{q} \rangle \\ &= \frac{if_{abc}}{4} \left(\delta^{(2)}(\mathbf{x}_2 - \mathbf{z}_1) + \delta^{(2)}(\mathbf{x}_2 - \mathbf{z}_2) \right) \prod_{i=1,3} \left(\delta^{(2)}(\mathbf{x}_i - \mathbf{z}_1) - \delta^{(2)}(\mathbf{x}_i - \mathbf{z}_2) \right) \end{aligned}$$

z_1, z_2 are the transverse coordinates of quark and anti-quark.

- Average over different dipole size $\mathbf{r} = \mathbf{z}_1 - \mathbf{z}_2$

$$\langle \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) \rangle_P \approx \sum_{s_1} \int_z \int d^2 \mathbf{r} \Psi_\lambda^{T*}(z, r, s_1) \Psi_\lambda^T(z, r, s_1) \langle q\bar{q} | \rho_{d'}(\bar{\mathbf{x}}) \rho_d(\mathbf{x}) | q\bar{q} \rangle$$

MV model projectile average

- MV model describes the distribution of classical color source not quantum operators.

$$W(\rho_a) = \exp \left\{ - \int_{\mathbf{x}} \frac{\rho_a(\mathbf{x}) \rho_a(\mathbf{x})}{2\mu^2} \right\}$$

- Two and three point correlators

$$\langle \hat{\rho}_a(\mathbf{x}) \hat{\rho}_b(\mathbf{y}) \rangle_{\text{MV}} = \langle \rho_a(\mathbf{x}) \rho_b(\mathbf{y}) \rangle_{\text{MV}} = \mu^2 \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta_{ab}$$

$$\langle \hat{\rho}_a(\mathbf{x}) \hat{\rho}_b(\mathbf{y}) \hat{\rho}_c(\mathbf{z}) \rangle_{\text{MV}} = -\frac{1}{2} \delta^{(2)}(\mathbf{x} - \mathbf{y}) \delta^{(2)}(\mathbf{y} - \mathbf{z}) T_{bc}^a \mu^2$$

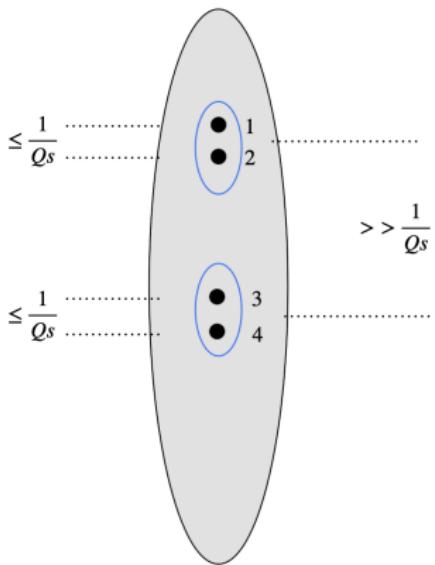
- Symmetrization of $\hat{\rho}$ s

$$\begin{aligned} \hat{\rho}_a(x) \hat{\rho}_b(y) &= \{\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})\} + [\hat{\rho}_a(\mathbf{x}), \hat{\rho}_b(\mathbf{y})] \\ &= \rho_a(\mathbf{x}) \rho_b(\mathbf{y}) - \frac{1}{2} \delta^{(2)}(x - y) T_{ab}^c \rho_c(\mathbf{x}) \end{aligned}$$

Target average(I)

- “Dipole” approximation

Phys. Rev. D 96, 074018, Kovner, Rezaeian



- Dense target \rightarrow Saturated
- $\frac{1}{Q_s}$ serves the role of correlation length in transverse plane

- For the example configuration

$$\text{Tr} [U(x_1)U^\dagger(x_2)U(x_3)U^\dagger(x_4)]$$

\approx

$$\frac{1}{N_c^2 - 1} \text{Tr} [U(x_1)U^\dagger(x_2)] \text{Tr} [U(x_3)U^\dagger(x_4)] +$$

...

Target average (II)

We only have one type of Wilson line correlator in momentum space

$$\begin{aligned} & \left\langle \text{Tr} \left[U(k_1) T^a U^\dagger(k_2) U(k_3) T^a U^\dagger(k_4) \right] \right\rangle_T \\ &= T_{bc}^a T_{de}^a \left\langle \left[U^{fb}(k_1) U^{\dagger cg}(k_2) U^{gd}(k_3) U^{\dagger ef}(k_4) \right] \right\rangle_T \\ &\approx T_{bc}^a T_{de}^a \left(\frac{(2\pi)^2}{N_c^2 - 1} \right)^2 \left\{ (N_c^2 - 1) \delta^{bc} \delta^{de} \delta^{(2)}(k_1 - k_2) D(k_1) \delta^{(2)}(k_3 - k_4) D(k_3) \right. \\ &\quad + (N_c^2 - 1) \delta^{bd} \delta^{ce} \delta^{(2)}(k_1 + k_3) D(k_1) \delta^{(2)}(k_2 + k_4) D(-k_2) \\ &\quad \left. + (N_c^2 - 1)^2 \delta^{be} \delta^{cd} \delta^{(2)}(k_1 - k_4) D(k_1) \delta^{(2)}(k_2 - k_3) D(-k_2) \right\} \end{aligned}$$

here the dipole $D(p)$ is defined as

$$D(p) = \frac{1}{N_c^2 - 1} \int dx^2 e^{ipx} \langle \text{Tr} \left(U^\dagger(x) U(0) \right) \rangle_T$$

The structure of analytic results(Σ_2)

- Dipole model

$$\frac{dN^{(2)}}{d\eta dq_1^2 d\xi dq_2^2} \propto \int_{k, \bar{k}} S_\perp \mathcal{I}(\varepsilon_f, |q_1 + q_2 + k + \bar{k}|) D(-\bar{k}) D(-k)$$
$$\frac{(N_c^2 - 1)\Gamma(q_2, q_2 + k, q_2 + k) - \Gamma(q_2, k + q_2, \bar{k} + q_2)}{(q_1 + q_2 + k + \bar{k})^2}$$

- S_\perp -Transverse area
- $\mathcal{I}(\varepsilon_f, |\mathbf{P}|) = \int dr r \varepsilon_f^2 K_1^2(\varepsilon_f r) (1 - J_0(|r||\mathbf{P}|))$

- MV model

$$\frac{dN_2}{dq_1^2 dq_2^2 d\eta d\xi} \propto \int d^2x \mu^2(x) \int_{k_1, k_2} D(k_1 + q_1) D(-k_2 + q_2)$$
$$\left\{ (N_c^2 - 1) \left(\frac{(k_2 - k_1) \cdot L(q_2, k_2)}{(k_2 - k_1)^2} \right)^2 \right.$$
$$\left. - \frac{(k_2 - k_1) \cdot L(q_2, k_2)}{(k_2 - k_1)^2} \frac{(k_2 - k_1) \cdot L(q_2, q_2 - k_2 - k_1)}{(k_2 - k_1)^2} \right\}$$

- $L^i(\mathbf{p}, \mathbf{q}) = \frac{\mathbf{p}^i}{\mathbf{p}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2}$ (Lipatov vertex)
- $\Gamma(\mathbf{p}, \mathbf{k}, \mathbf{q}) = L(\mathbf{p}, \mathbf{k}) \cdot L(\mathbf{p}, \mathbf{q})$