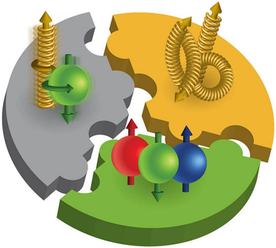
Orbital Angular Momentum at Small-x

Brandon Manley
with Yuri Kovchegov
Based on (1901.07453)



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- Results extracted from experiment (RHIC): (1212.1701)

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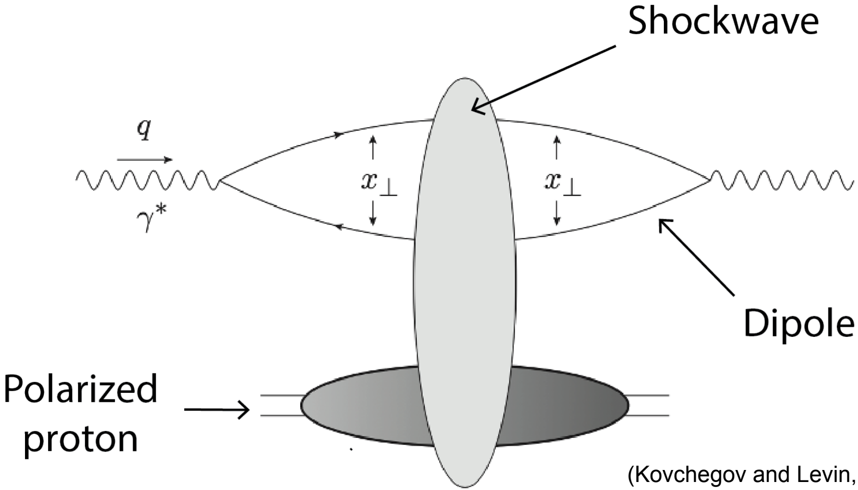
- Missing spin either at $x \leq x_{min}$ or/and L
- **Good theoretical understanding of L at small x is crucial!**

Physical picture



(Kovchegov and Levin, 2012, Cambridge University Press)

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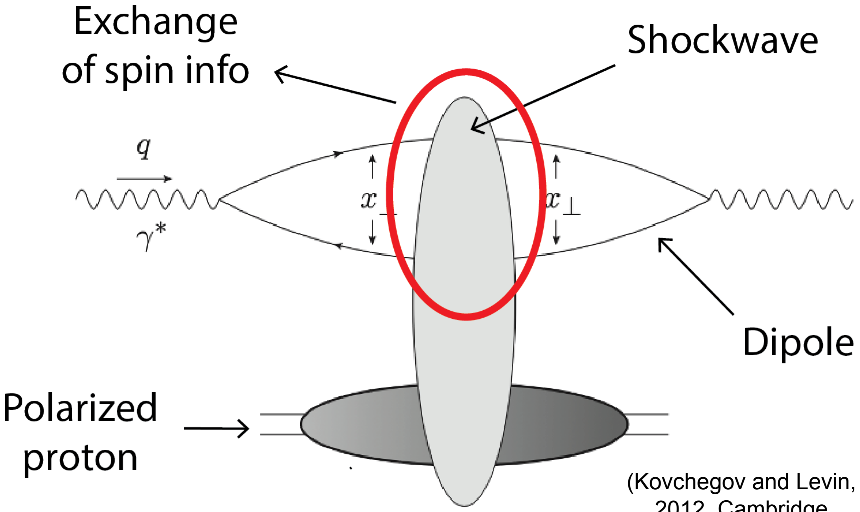

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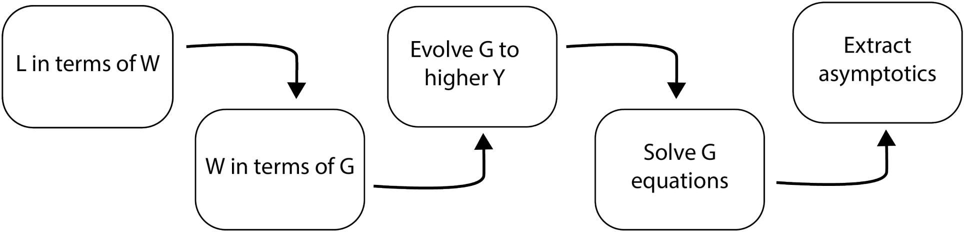
Roadmap


L in terms of W
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Evolve G to higher Y
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Quark OAM: Definitions

- OAM operator definition:

$$L_z = \int \frac{d^2 b_\perp \, db^- \, d^2 k_\perp \, dk^+}{(2\pi)^3} \, (\underline{b} \times \underline{k})_z \, W(k, b)$$

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- Expect contributions from *polarized* proton due to cross product in definition!

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- Expect contributions from *polarized* proton due to cross product in definition!
- Quark Wigner function (e.g. for SIDIS)

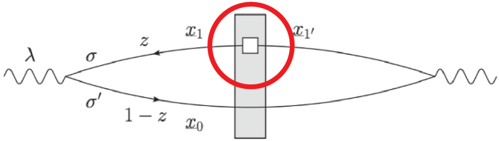
$$W^q(k,b) = 2\sum_X \int d^2 r \, dr^- e^{ik\cdot r} \left\langle \overline{\psi}_\alpha \left(b - \frac{1}{2} r\right) V_- |X\rangle \left(\frac{1}{2}\gamma^+\right)_{\alpha\beta} \langle X| V_+ \psi_\beta \left(b + \frac{1}{2} r\right) \right\rangle$$

where the Wilson lines are

$$V_\pm \equiv \mathcal{P} \exp \left[\pm ig \int_{b^- \pm \frac{1}{2} r^-}^{\infty} dx^- A^+ \left(x^+ = 0, x^-, \underline{b} \pm \frac{1}{2} \underline{r}\right) \right]$$

Quark OAM: Small x simplification

- Polarized contribution enters through *sub-eikonal* vertex in shockwave-antiquark interaction



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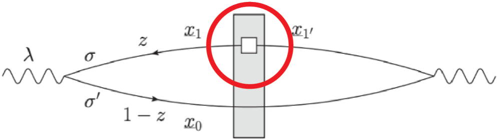

- Define polarized dipole amplitude (cf. $S \sim \frac{1}{N_C} \left\langle \mathrm{tr} \left[V_{\underline{x}_1} V^\dagger_{\underline{x}_0} \right] \right\rangle$) (1511.06737)

$$G_{10}(zs) = \frac{zs}{2N_c} \mathrm{Re} \left\langle \mathrm{T}\ \mathrm{tr} \left[V_{\underline{x}_0} V^{\dagger\ \mathrm{pol}}_{\underline{x}_1} \right] + \mathrm{T}\ \mathrm{tr} \left[V^{\mathrm{pol}}_{\underline{x}_1} V^\dagger_{\underline{x}_0} \right] \right\rangle$$

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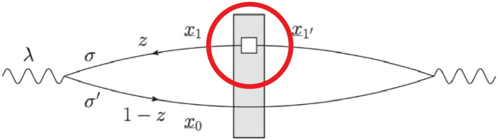

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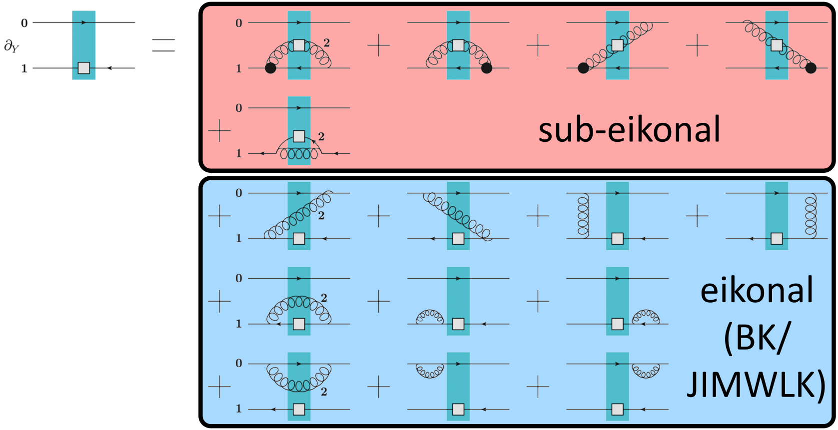
- Write $L_{q+\bar{q}}$ in terms of G_{10}:

$$L_{q+\bar{q}} = \frac{8N_c}{(2\pi)^5} \int d^2k_\perp d^2x_{10} d^2x_1 e^{i\underline{k}\cdot \underline{x}_{10}} \frac{\underline{x}_{10}}{x_{10}^2} \times \frac{\underline{k}}{\underline{k}^2}\ \underline{x}_1 \times \underline{k} \int_{\frac{\Lambda^2}{s}}^1 \frac{dz}{z} G_{10}(zs) - \underbrace{\sum_f (\Delta q + \Delta \bar{q})}_{\equiv \Delta\Sigma}$$

Quark OAM: Small x simplification

- G_{10} is evolved through soft gluon or quark emission: (1511.06737)



"Helicity BFKL" evolution

Quark OAM: Small x asymptotics

- Leading high-energy asymptotics for both terms:

$$\int d^2k_\perp d^2x_{10} d^2x_1 e^{i\underline{k}\cdot\underline{x}_{10}} \frac{\underline{x}_{10}}{x_{10}^2} \times \frac{\underline{k}}{\underline{k}^2}\, \underline{x}_1 \times \underline{k} \int_{\frac{\Lambda^2}{s}}^{1} \frac{dz}{z} G_{10}(zs) \sim \left(\frac{1}{x}\right)^{2\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

$$\Delta\Sigma \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \equiv \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

(1610.06197)

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(1610.06197)

- $\Delta\Sigma$ dominates over "moment term" at small x! Small x asymptotics of quark OAM follow:

$$L_{q+\bar{q}} \approx -\Delta\Sigma \sim \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Gluon OAM: Small x simplification

- Gluon OAM constructed via gluon dipole Wigner function

$$W^{G\ \mathrm{dip}}(k,b) = \frac{4}{xP^+} \int d\xi^- d^2\xi_\perp e^{ixP^+\xi^- - i\underline{k}\cdot\underline{\xi}} \times \left\langle \mathrm{tr}\left[F^{+i}\left(b - \frac{1}{2}\xi\right)\mathcal{U}^{[+]} F^{+i}\left(b + \frac{1}{2}\xi\right)\mathcal{U}^{[-]}\right]\right\rangle$$

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- Define "gluonic" polarized dipole amplitude (1706.04236)

$$G^i_{10}(zs) \equiv \frac{1}{2N_c}\left\langle \mathrm{tr}\left[V_{\underline{0}}\left(V^{\mathrm{pol}\dagger}_{\underline{1}}\right)^i_\perp\right] + \mathrm{c.c.}\right\rangle(zs)$$

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- Rewrite gluon OAM (to lowest order in $A^i_\perp$):

$$L_G = -\frac{8iN_c}{g^2(2\pi)^3}\int d^2x_1 d^2x_{10} d^2k_\perp e^{i\underline{k}\cdot\underline{x}_{10}}\left(\underline{x}_1\times\underline{k}\right)\nabla^i_{10}G^i_{10}\left(zs=\frac{Q^2}{x}\right)$$

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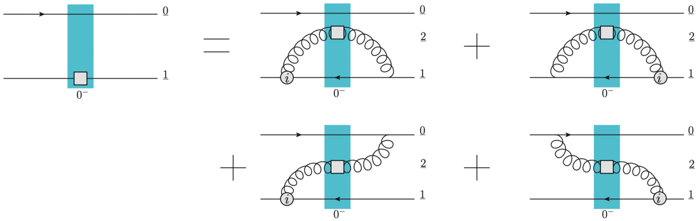
- Only "moment" contributes, no term proportional to ΔG (at lowest order)

Gluon OAM: Small x asymptotics

- Evolve G^i_{10} via soft gluon or quark emission: (1706.04236)



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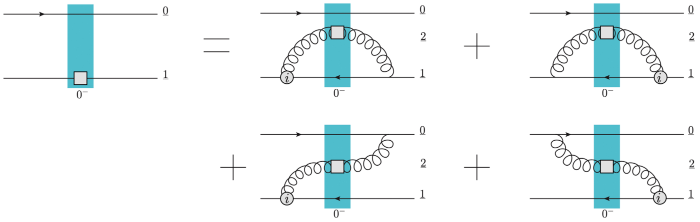

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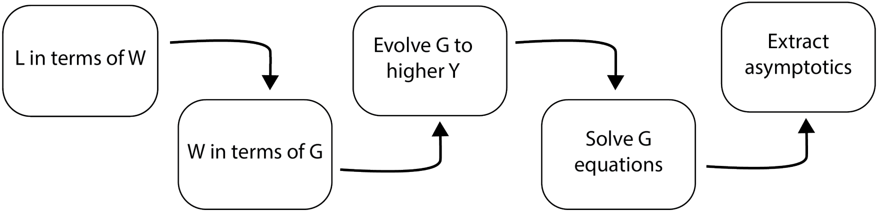
- Gluon OAM asymptotics follows from evolution equations for $G^i_{10}(zs)$:

$$L_G(x, Q^2) \sim \Delta G \sim \left(\frac{1}{x}\right)^{1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Summary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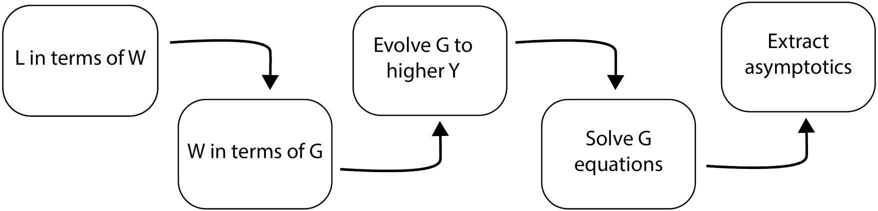

- Asymptotics for OAM distributions (to be updated!):

$$L_{q+\bar{q}}(x, Q^2) \sim \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}} \qquad L_G(x, Q^2) \sim \left(\frac{1}{x}\right)^{1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

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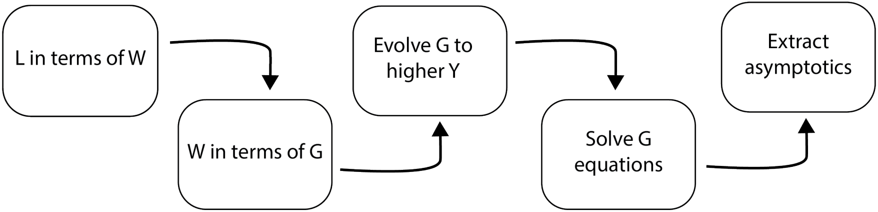

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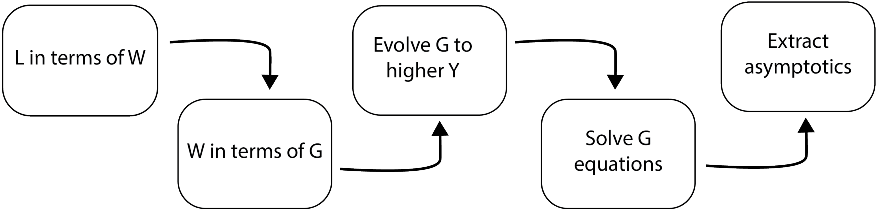

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- Quarks more important than gluons at small x
- **Significant portion of nucleon spin at small x could be found in the quark and gluon OAM!**

Next steps

- Recently realized spin isn't the whole story

$$\bar{v}_\sigma(k_1)\left(\hat{V}^\dagger_{\underline{w}}\right)v_{\sigma'}(k_2) \sim \delta_{\sigma\sigma'}\left(V^\dagger_{\underline{w}} + \sigma V^{\mathrm{pol}[1]\,\dagger}_{\underline{w}} + V^{\mathrm{pol}[2]\,\dagger}_{\underline{w}} + \ldots\right)$$

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(2204.11898)

Quark and Gluon Helicity Evolution at Small x: Revised and Updated

Florian Cougoulic,[1,*] Yuri V. Kovchegov,[2,†] Andrey Tarasov,[2,3,‡] and Yossathorn Tawabutr[2,§]

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oint BNL-SBU Center for Frontiers in Nuclear Science (CFNS) at Stony Brook University, Stony Brook, NY 11794, US

We revisit the problem of small Bjorken-x evolution of the gluon and flavor-singlet quark helicity distributions in the shock wave (s-channel) formalism. Earlier works on the subject in the same framework [1–3] resulted in an evolution equation for the gluon field-strength F^{12} and quark "axial current" $\bar{\psi}\gamma^+\gamma^5\psi$ operators (sandwiched between the appropriate light-cone Wilson lines) in the

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- Thank you!! Questions?

Backup

Wigner distributions

- Wigner distributions encode the average position and average momentum of a distribution
- Can be negative, only give a probabilistic interpretation when integrated over

$$W(k,b) = \int \frac{d^2\Delta_\perp d\Delta^+}{2(2\pi)^3 k^+} \langle k + \frac{1}{2}\Delta|\psi_p\rangle\langle\psi_p|k - \frac{1}{2}\Delta\rangle e^{i\Delta\cdot b}$$

$$\langle\psi_p|\hat{\mathcal{O}}|\psi_p\rangle = \int \frac{dpdb}{2\pi} W(p,b) O(p,b)$$

CGC Averaging

- Expectation value of an operator $\hat{\mathcal{O}}$ in proton state is related to Color Glass Condensate averaged operator:

$$\left\langle \hat{O}(b,r) \right\rangle = \frac{1}{2P^+} \int \frac{d^2\Delta d\Delta^+}{(2\pi)^3} \left\langle P + \frac{\Delta}{2} \middle| \hat{\mathcal{O}}(0,r) \middle| P - \frac{\Delta}{2} \right\rangle$$

Polarized Wilson lines: Explicit operators

(2204.11898)

$$V_{\underline{x}}^{\mathrm{G}[1]} = \frac{i\, g\, P^+}{s} \int\limits_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-]\, F^{12}(x^-, \underline{x})\ V_{\underline{x}}[x^-, -\infty],$$

$$V_{\underline{x}}^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-]\, t^b\, \psi_\beta(x_2^-, \underline{x})\, U_{\underline{x}}^{ba}[x_2^-, x_1^-]\ \left[\gamma^+ \gamma^5\right]_{\alpha\beta}\ \bar{\psi}_\alpha(x_1^-, \underline{x})\, t^a\, V_{\underline{x}}[x_1^-, -\infty],$$

$$V_{\underline{x},\underline{y}}^{\mathrm{G}[2]} = -\frac{i\, P^+}{s} \int\limits_{-\infty}^{\infty} dz^- d^2 z\ V_{\underline{x}}[\infty, z^-]\, \delta^2(\underline{x} - \underline{z})\, \overleftarrow{D}^i(z^-, \underline{z})\, D^i(z^-, \underline{z})\, V_{\underline{y}}[z^-, -\infty]\, \delta^2(\underline{y} - \underline{z}),$$

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