

HUGS 2022

Thermodynamic approach to Quark-Gluon Plasma

Rutik Manikandhan

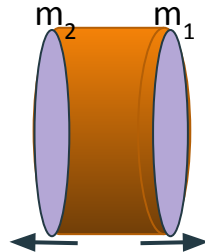
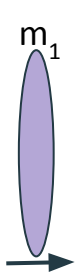
Advisor: Prof. Rene Bellwied

06/16/2022

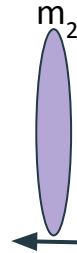


Relativistic Heavy-Ion Collisions

- ❖ Heavy Atoms like Pb/Au/Cu.
- ❖ Stripped off of their electrons \longrightarrow Ions (Heavy-Ion).
- ❖ Accelerated to speed of light.
- ❖ Collide them with each other to form “Little Bangs”.
- ❖ Study the aftermath by specialized detector systems around the collision.

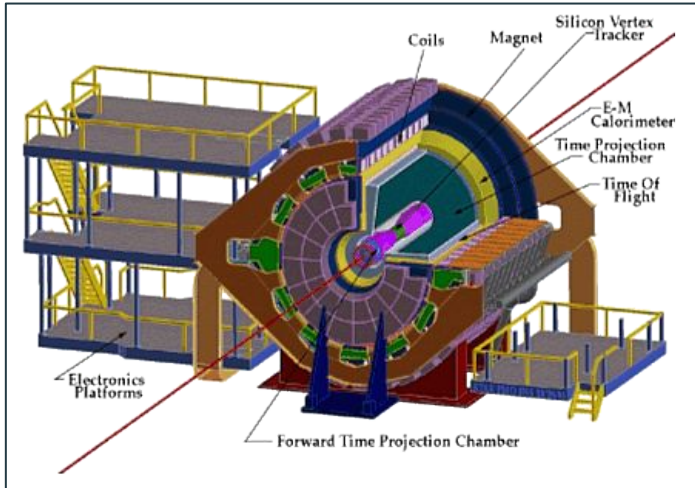


$t = 10^{-20}$ s !!!

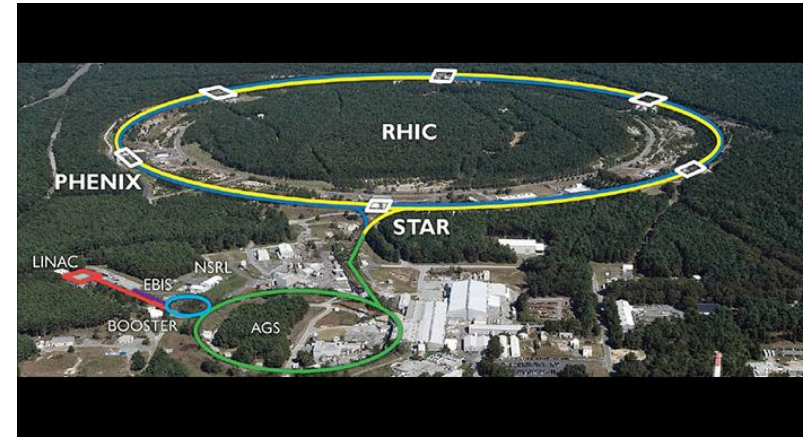


RHIC: STAR Experiment

- ❖ **STAR: Solenoidal Tracker At RHIC.**
- ❖ Heavy ion collisions of Au, Cu and d.
- ❖ Energy range from 3 GeV - 200 GeV ($\sqrt{s_{NN}}$).



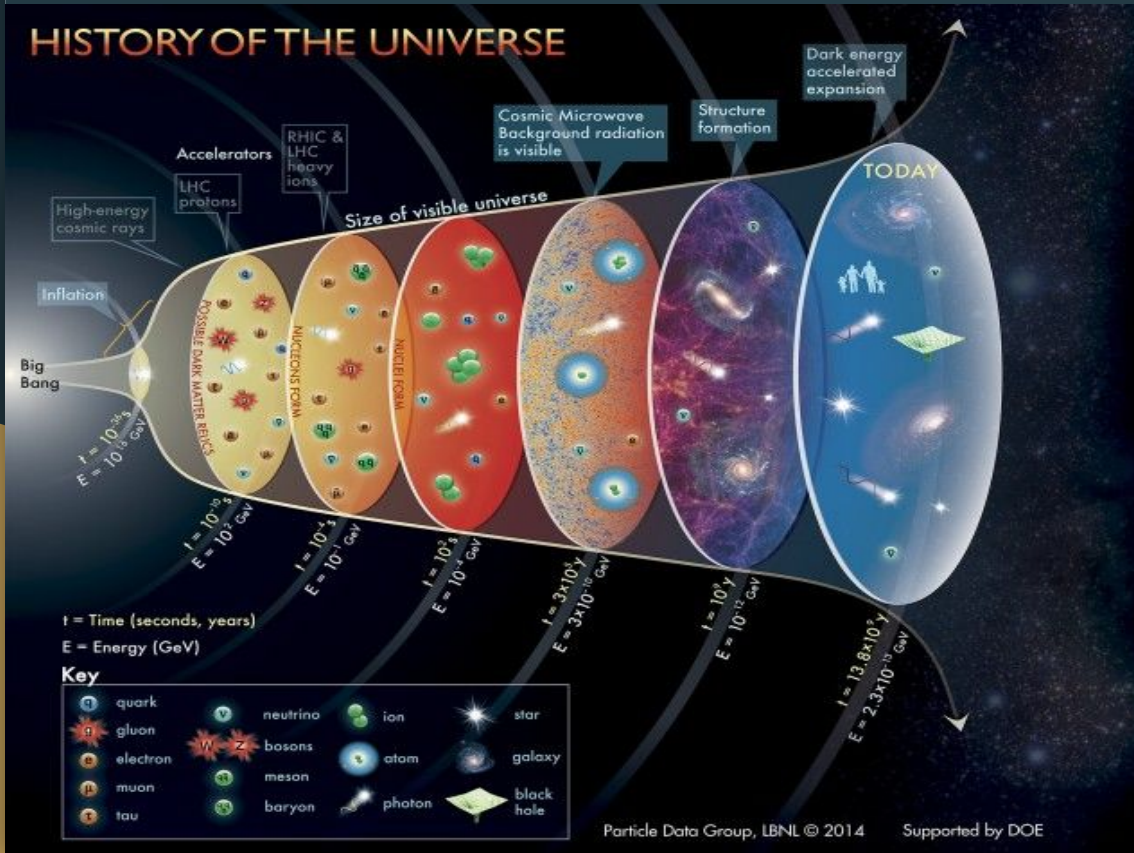
<https://www.star.bnl.gov>



<https://www.star.bnl.gov>

- ❖ Located at Brookhaven National Laboratory (BNL).
- ❖ Long Island, New York, USA.

What is our Mission ?

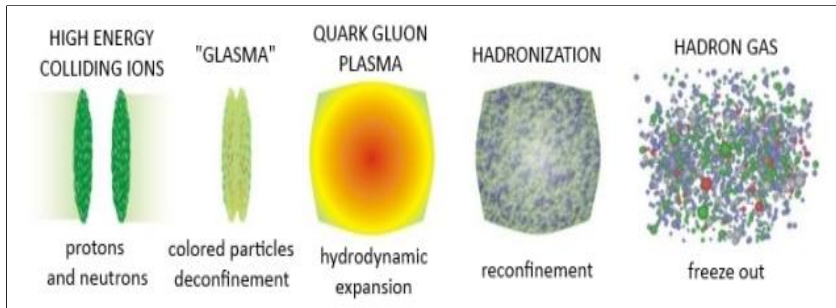


- ❖ Probe to look back into the past.
- ❖ What happened micro seconds after the Big Bang?
- ❖ Constraints on the Standard Model.
- ❖ Establish the existence of a phase (state) of deconfined and chirally symmetric matter.
- ❖ Determine state variables.
- ❖ Predict new Particles?

Confinement and Deconfinement of Hadrons

- ❖ Hadrons are composite objects made of quarks and gluons. q-q interactions become weaker as the inter-quark distance becomes shorter (asymptotic freedom). The system behaves like free quarks and gluons.

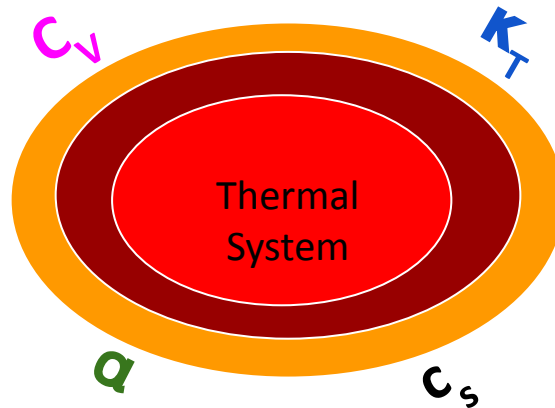
- ❖ The deconfined state of hadronic matter is called Quark-Gluon Plasma (QGP).
- ❖ QGP a **thermalised**, or near to thermalised state of quarks and gluons, where quarks and gluons are free to move over a nuclear volume.
- ❖ With QCD the only strongly interacting theory in the Standard Model, mapping the transition region of its **phase diagram** is a scientific goal of the highest order.



[arXiv:1207.7028](https://arxiv.org/abs/1207.7028)

QGP as a Thermal System

- ❖ With the advent of Heavy-Ion Experiments, each collision produced thousands of particles. So a single collision can be approximated to a thermodynamic state.
- ❖ In particular we can establish a **well defined temperature** and study its fluctuations.
- ❖ Hence if the collision is to be identified as a thermal system we can experimentally determine important state variables of the system.

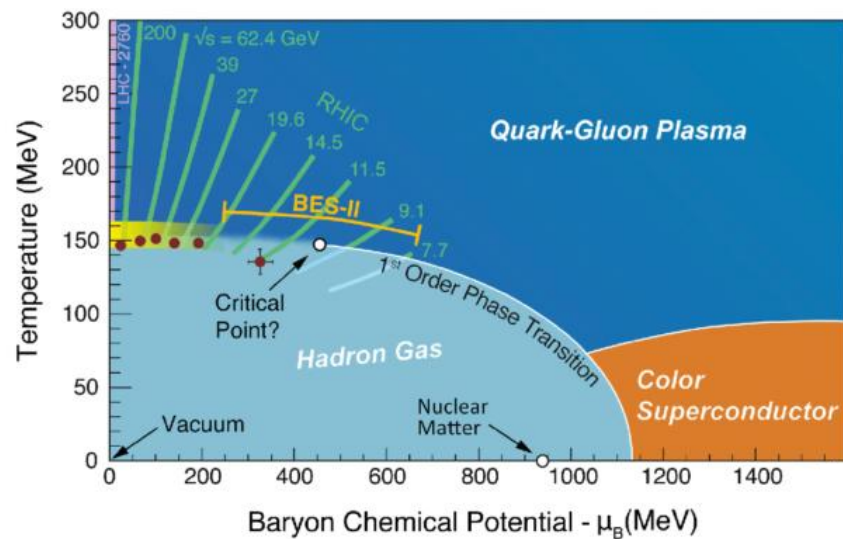


Statistical Hadronization Models

- ❖ The Hadron gas can be approximated as non-interacting ground state hadrons and their heavier resonances.
- ❖ We can calculate a partition function involving all known species and calculate macroscopic quantities from it .
- ❖ Such a model can describe **quantitatively** the yield of most particles, including multi-strange ones, has in fact been indicated by fits to average particle abundances at AGS, SPS and RHIC energies.
- ❖ Models give **no information on how and when** the system reaches equilibrium.
- ❖ Model makes no predictions of dynamical quantities.
- ❖ A completely phenomenological description.
- ❖ Hence we need **Quantum Chromodynamics**, a fundamental theory on first principles and Lattice QCD, a non-perturbative approach to QCD.

Objective

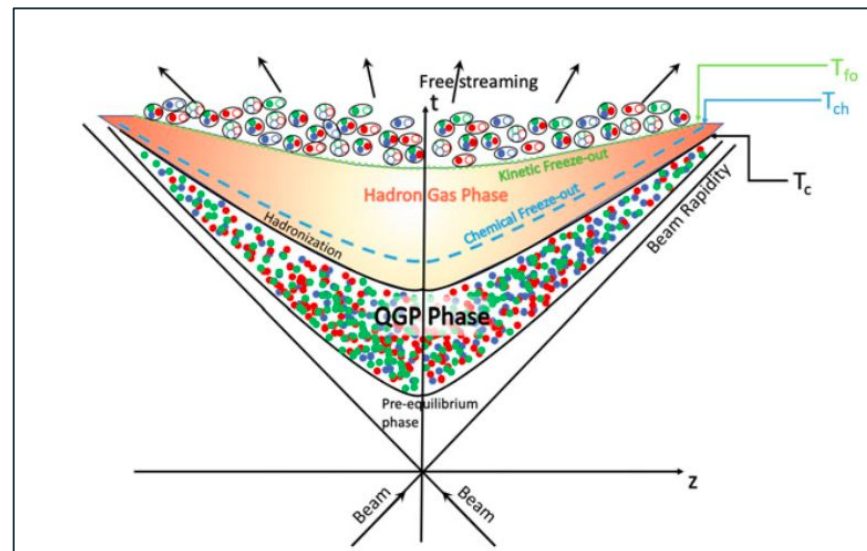
- ❖ Lattice QCD calculations show that the QCD transition is an **analytical crossover** at $\mu_B \approx 0$, involves a rapid change as opposed to a discontinuous jump.
- ❖ On the other hand QCD inspired phenomenological models predict a first order phase transition at high μ_B .
- ❖ The point where the doping of matter over antimatter becomes large enough to instigate a **first order phase transition** is referred to as the QCD critical point.
- ❖ This suggests the possible existence of a QCD critical point where the first order transition **terminates**.



[arXiv:2108.13867](https://arxiv.org/abs/2108.13867)

What do we measure in these Experiments?

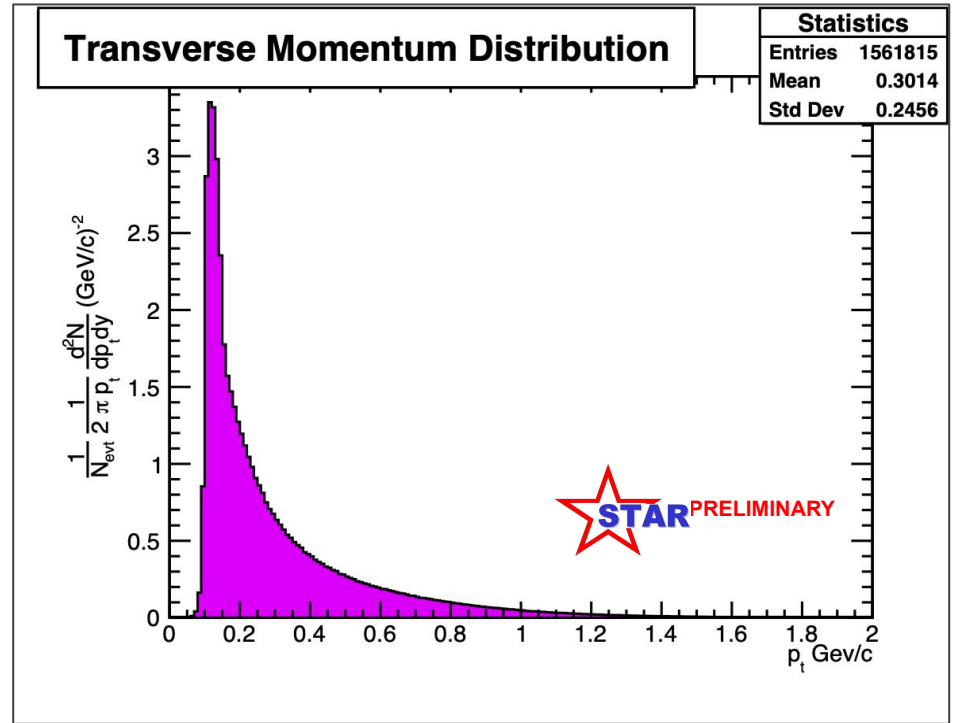
- ❖ **Spectra** : Distribution of particle momenta.
- ❖ **Yield**: Count of particle numbers.
(Multiplicity)
- ❖ After the chemical freeze-out (T_{ch}), the yield is fixed.
- ❖ After the kinetic freeze-out (T_{kin}), the spectra are fixed.



<https://doi.org/10.1007/978-3-030-67235-5>

Spectra

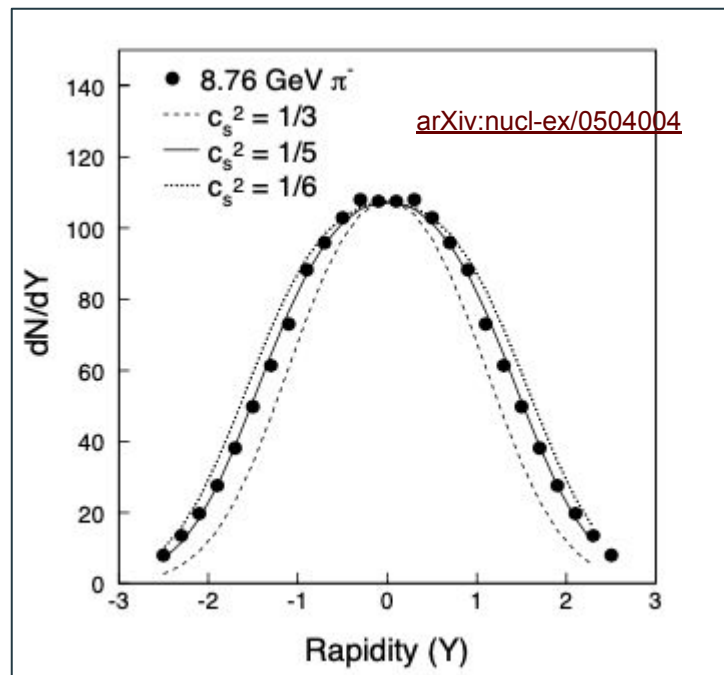
- ❖ Transverse Momentum p_T is defined as $\sqrt{(p_x^2 + p_y^2)}$.
- ❖ Fluctuations in **temperature** are obtained from the fluctuations in transverse momentum.
- ❖ Temperature is found by applying fits to the spectra under different formalisms.



❖ $\sqrt{s_{NN}} = 3 \text{ GeV Au+Au collisions.}$

Yields

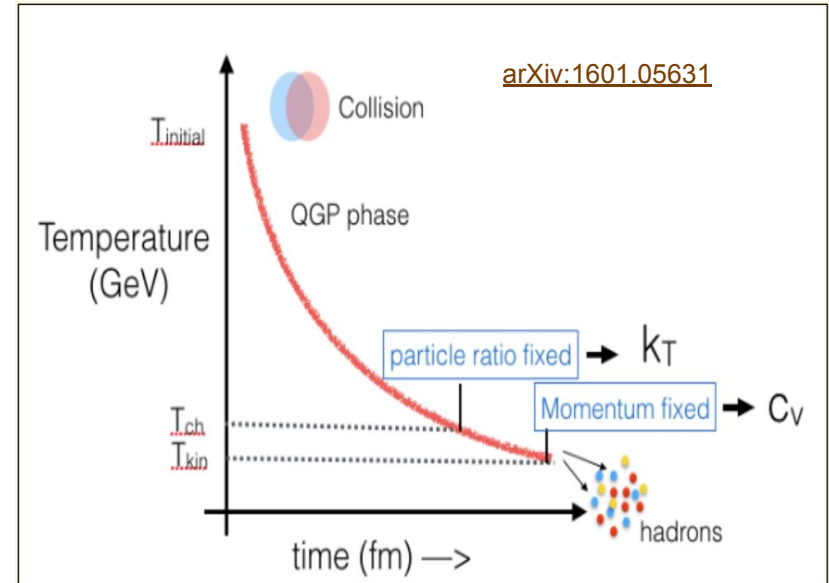
- ❖ Multiplicity: Particle yields.
- ❖ Fluctuations can be studied for identified particles or charged particles.
- ❖ Multiplicity fluctuations are associated to **conserved charges** :
 - Electric Charge
 - Baryon Number
 - Strangeness



- ❖ Yield distributions

Fluctuations

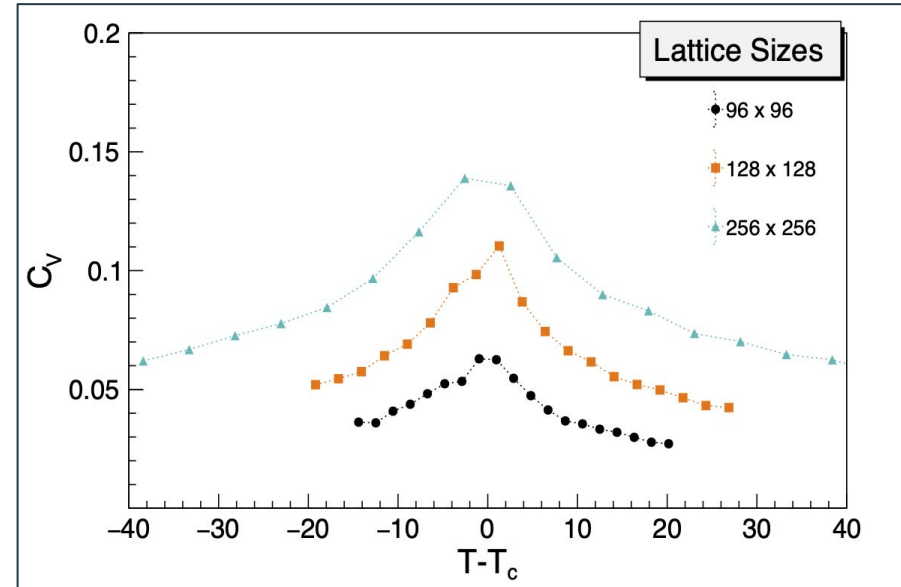
- ❖ Fluctuations quantify proximity to critical point.
- ❖ Yield fluctuations \longrightarrow Conserved quantum numbers.
- ❖ Spectra fluctuations \longrightarrow Temperature
- ❖ Ratios of cumulants, thermodynamic functions are calculated to quantify these fluctuations.



- ❖ Temperature Vs time

Why Specific Heat C_V ?

- ❖ For a system undergoing phase transition, C_V is expected to **diverge** at the critical point.
- ❖ Thus the variation of thermal fluctuations with temperature can be effectively used to **probe** the critical point.
- ❖ In the 2D Ising Model we can see the divergence of C_V as we change the temperature of the system.



- ❖ Simulations from 2D Ising Model for C_V .

Why Specific Heat C_V ?

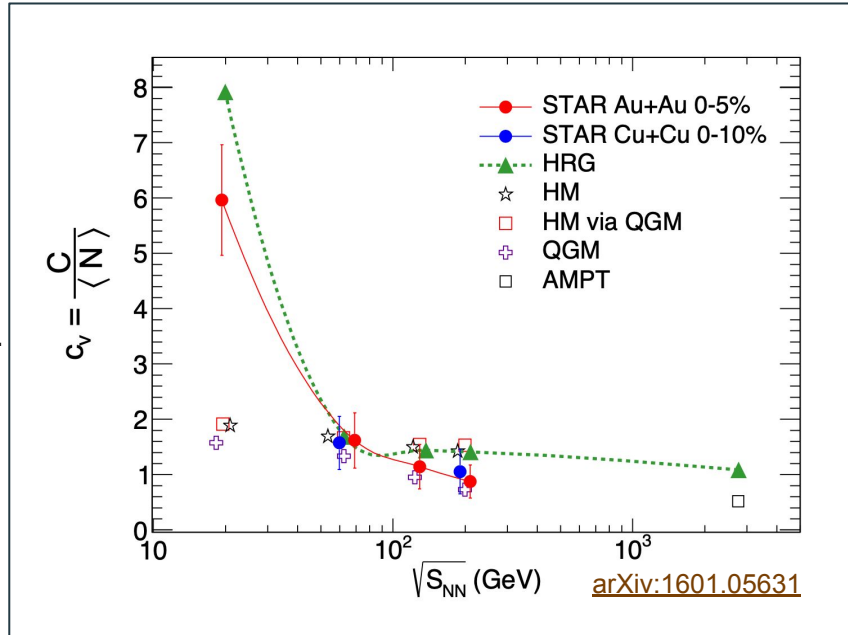
- ❖ Specific heat is a thermodynamic quantity characterizing the **equation of state** of the system.
- ❖ $C_V = \left(\frac{\partial E}{\partial T} \right)_V$ Definition from Thermodynamics
- ❖ Heat Capacity, C of a system in thermal equilibrium connected to a bath at T can be computed from the event- by-event fluctuations of temperature:

$$\frac{1}{C} = \frac{\langle T_{kin}^2 \rangle - \langle T_{kin} \rangle^2}{\langle T_{kin} \rangle^2}$$
- ❖ Temperature fluctuation of the system provides an estimation of C_V .
- ❖ Thus, with the measurement of T on an event-by-event basis, it is possible to extract the C_V of the hot and dense strongly interacting matter produced in heavy-ion collisions.
- ❖ Assuming complete thermal equilibrium up to the surface of last scattering which is the kinetic freeze-out surface, C_V is then expected to reveal the thermodynamic state of the matter at the moment of **kinetic freeze-out**.

[arXiv:1601.05631](https://arxiv.org/abs/1601.05631)

Progress so far

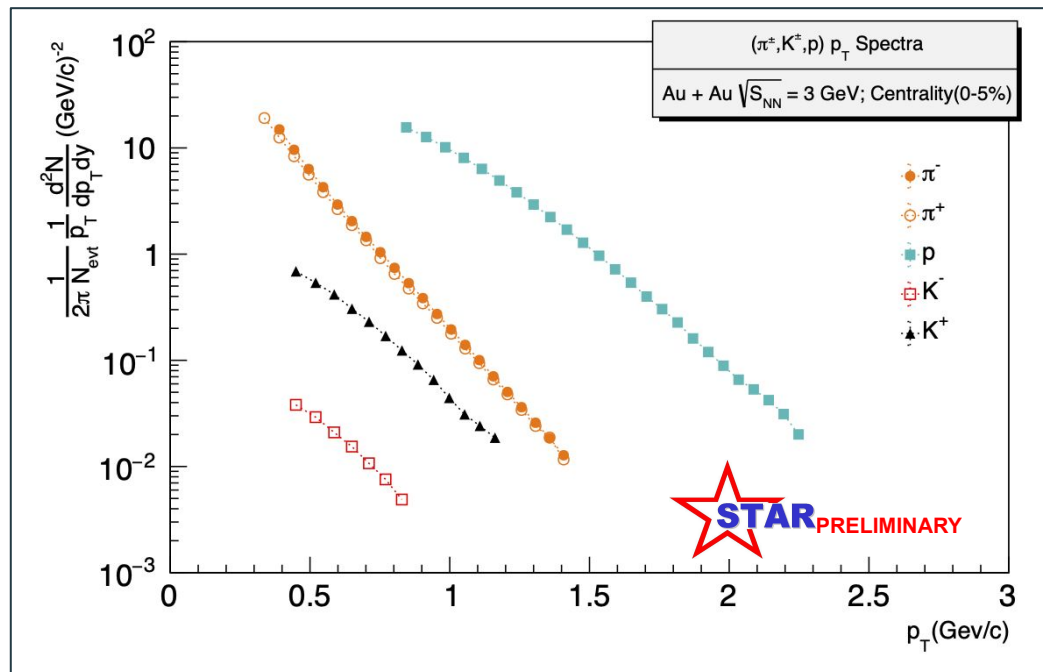
- ❖ STAR has calculated C_v at energies 19.6, 62.4, 130 and 200 GeV.
- ❖ The experimental values match the prediction from the **HRG Model**.
- ❖ Monotonic increase coming into effect and the analysis at 3 GeV is critical to understand this trend.
- ❖ PACIAE (Event Generator).
- ❖ HM : Hadronic Matter.
- ❖ QGM : Quark-Gluonic Matter.



❖ Specific Heat Vs $\sqrt{s_{NN}}$

Outlook

- ❖ Calculation of collective parameters.
- ❖ Fluctuations in temperature.
- ❖ Baseline calculation from SHM.
- ❖ Perform a BES for criticality location.



- ❖ Transverse Momentum Distribution at 3 GeV

Acknowledgement

- ❖ Prof. Rene Bellwied, Dr. Tapan Nayak and my fellow group members for their valuable inputs.
- ❖ Dr. Alberto Accardi and the School Managers.



References

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Questions???

Backup Slides

Why Isothermal Compressibility κ_T ?

- ❖ κ_T describes the relative variation of the volume of a system due to a change in the pressure at constant temperature.

$$\kappa_T = \frac{-1}{V} \left(\frac{\partial V}{\partial P} \right) \Big|_{T, \langle N \rangle}$$

- ❖ κ_T is linked to density fluctuations and can be expressed in terms of the second derivative of the free energy with respect to the pressure.

Under the Grand-Canonical Ensemble Formalism (G.C.E), we have:

- ❖ In a second order phase transition κ_T is expected to show a **singularity**.

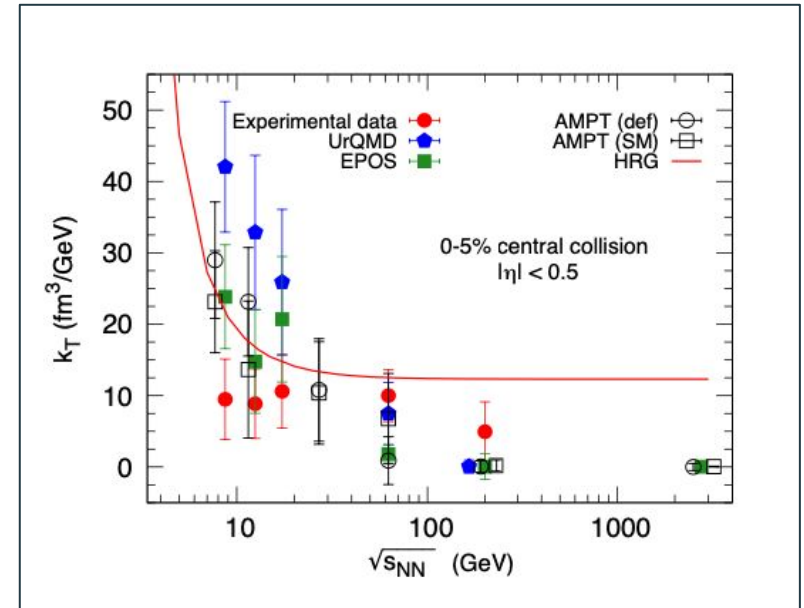
- ❖
$$\sigma^2 = \frac{k_B T \langle N \rangle^2}{V} \kappa_T$$

- ❖
$$\omega_{ch} = \frac{\langle N_{ch}^2 \rangle - \langle N_{ch} \rangle^2}{\langle N_{ch} \rangle}$$

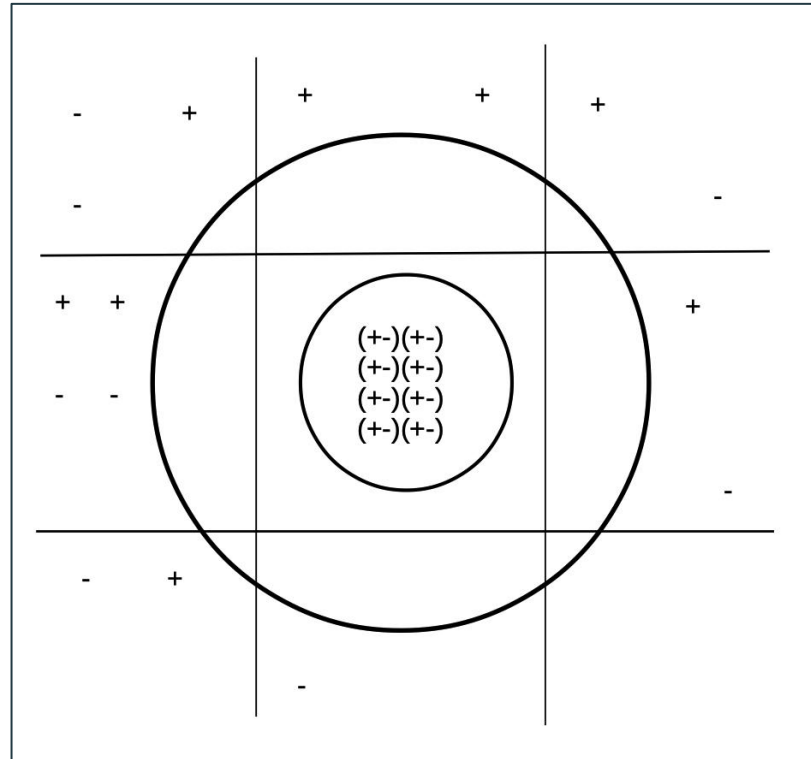
- ❖ ω_{ch} Is the scaled variance of the **Charged particle yields**.

Progress so far

- ❖ Calculations carried out at 7.7, 11.5, 19.6, 62.4 and 200 GeV.
- ❖ A higher value of κ_T at low energies compared to higher energies indicates that the collision system is more compressible at the lower energies.
- ❖ AMPT, UrQMD and EPOS (Event Generators).



- ❖ Isothermal Compressibility Vs $\sqrt{s_{NN}}$



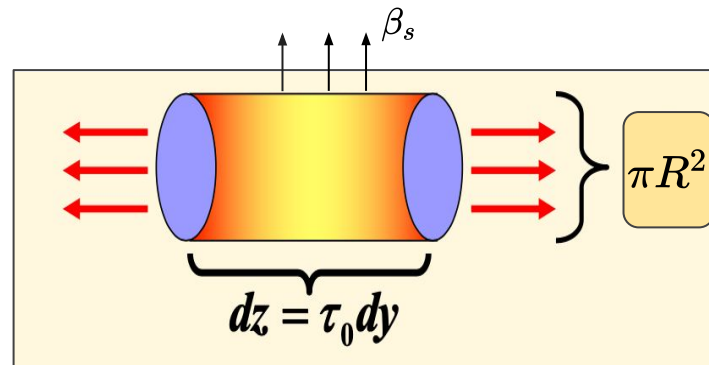
Calculating T_{kin} from the Blast Wave

❖ Blast-Wave formalism:

- ❖ The measured p_T spectral shape flattens significantly with increasing particle mass in central Au+Au collisions. This suggests the presence of a **collective transverse radial flow** field (cylindrical symmetry).

- ❖ This p_T distribution is found from the **Cooper-Frye equation**:

$$E \frac{d^3 N}{dp^3} = \int f(x, p) p^\mu d\sigma_\mu$$

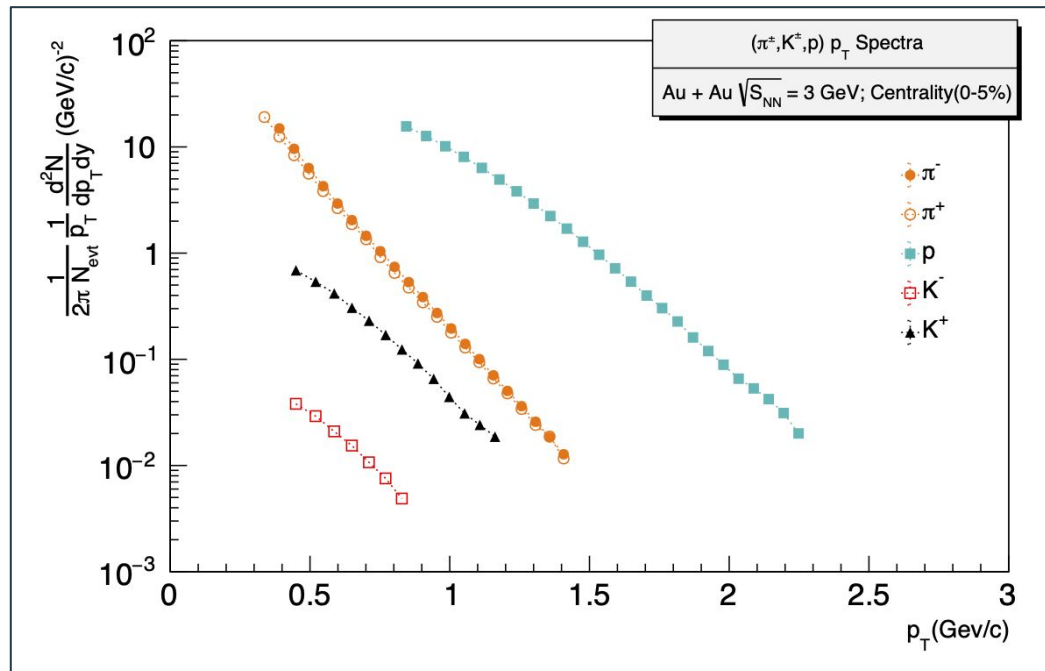


- ❖ $E \frac{d^3 N}{dp^3} = \frac{1}{2\pi p_T} \frac{d^2 N}{dp_T dy}$ (Particle Spectra)
- ❖ $f(x, p)$ is the distribution function of the phase space.
- ❖ p^μ is the momentum 4-vector
- ❖ $d\sigma_\mu$ is the hypersurface from which the particles are emitted.

Kinetic Freeze-out



$$\frac{\chi^2}{N.D.F.}$$



Combined Blast-Wave Fit

Collision Energy ($\sqrt{s_{NN}}$)

- ❖ \sqrt{s} is the total energy in the Center of Mass Frame (CM), invariant mass of the CM.
- ❖ Natural units, $\hbar = c = 1$.
- ❖ $1\text{eV} = 1.60218 \times 10^{-19} \text{ J}$.
- ❖ The Collision Energy enables us to reach the desired energy density and temperatures for the system.
- ❖ Energy of the collision expressed in terms of nucleon- nucleon center of mass energy.
- ❖ In the nucleon-nucleon CM frame, two nuclei approach each other with the same velocity.
- ❖ The nucleon-nucleon CM energy is denoted by $\sqrt{s_{NN}}$ and is related to the total CM energy by:

$$s = E_{cm}^2 = m_1^2 + m_2^2 + 2(E_1 E_2 + |p_1||p_2|)$$

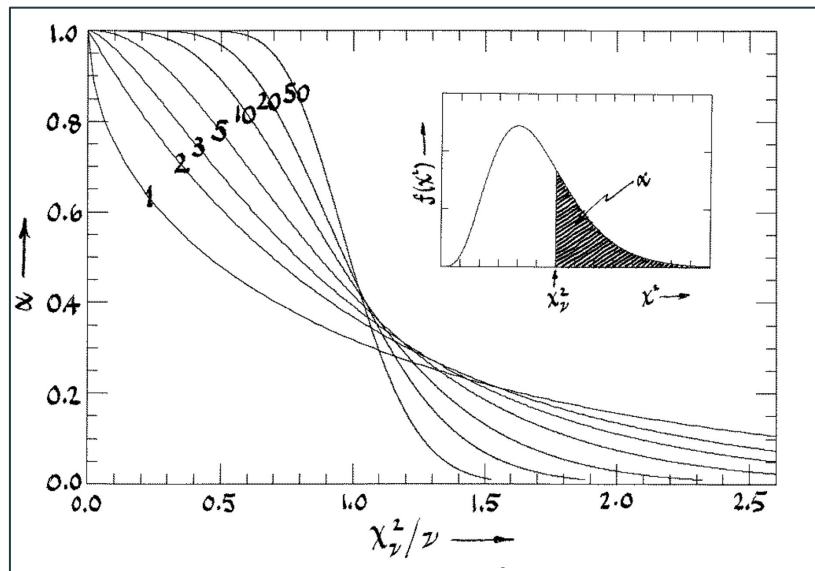
$$\sqrt{s_{NN}} = \frac{\sqrt{s}}{A}$$

- ❖ A : Number of Nucleons in the Heavy-Ion

Backup Slides

χ^2 Test for Fitting:

- ❖ $\chi^2 = \sum_{i=1}^{\nu} \frac{(x_i - \mu_i)^2}{\sigma_i^2}$
- ❖ ν : Number of degrees of freedom (N-r)
- ❖ $\chi^2/\nu \approx 1$ (For a good fit).



```

QQMD version:      30400  1000  30400  output_file 13
projectile: (mass, char) 208 82  target: (mass, char) 208 82
transformation betas (NN,lab,pro)  0.0000000  0.9953427 -0.9953427
impact_parameter_real/min/max(fm):  1.25  0.00  3.50  total_cross_section(mbarn):      384.85
equation_of_state:  0  E_lab(GeV/u): 0.2000E+03  sqrt(s)(GeV): 0.1946E+02  p_lab(GeV/u): 0.2009E+03
event#      1 random seed: 1605073721 (auto)  total_time(fm/c):      50  Delta(t)_0(fm/c):      50.000
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op 0  0  0  1  1  0  0  0  0  0  0  0  0  1  0
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