EQUATION OF STATE FOR NEUTRON STAR MATTER INCLUDING HYPERONS

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Jun.16, 2022.

MOTIVATION

- Hyperons may exist near center of the neutron stars.

- It is known that predicted mass of the star from equation of state including hyperons is not consistent with the observations (Hyperon Puzzle).

- Construct the equation of state with Flavor SU(3) Nambu-Jona-Lasinio model to seek the way to solve the problem on the quark level.

OUTLINE

- Introduction of Neutron Stars
- Our model
- Numerical Results
- Summary

WHAT ARE NEUTRON STARS?

- One of the high-density compact stars in universe!
- Forms after supernova explosion of a massive star, only when it has specific mass.

Heavier: Black Hole

(https://chandra.harvard.edu/photo/2009/crab/more.html)

WHAT ARE NEUTRON STARS?

- Structure

Outer Core: Neutron, Proton, Electron

Inner Core: Quarks?, Hyperon?, Pion?, etc…

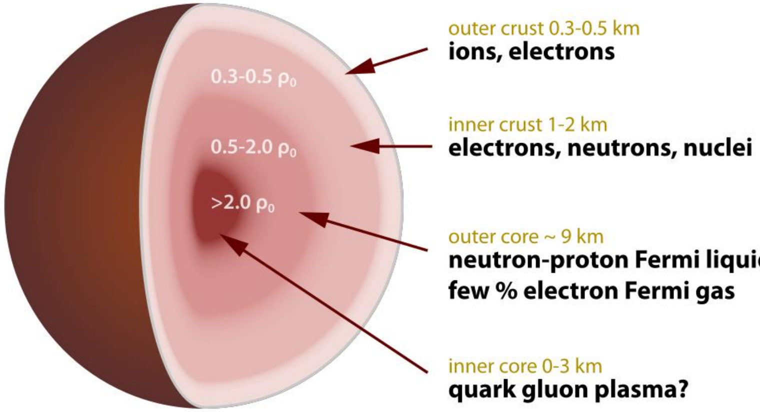
- Properties

Maximum Mass: about 2.1 $M_\odot$

$M_\odot$: solar mass = sun's mass

Radius: about 10km

10^5 times smaller than sun



Hyperon?
Deconfined quarks?

(https://en.wikipedia.org/wiki/Neutron_star)

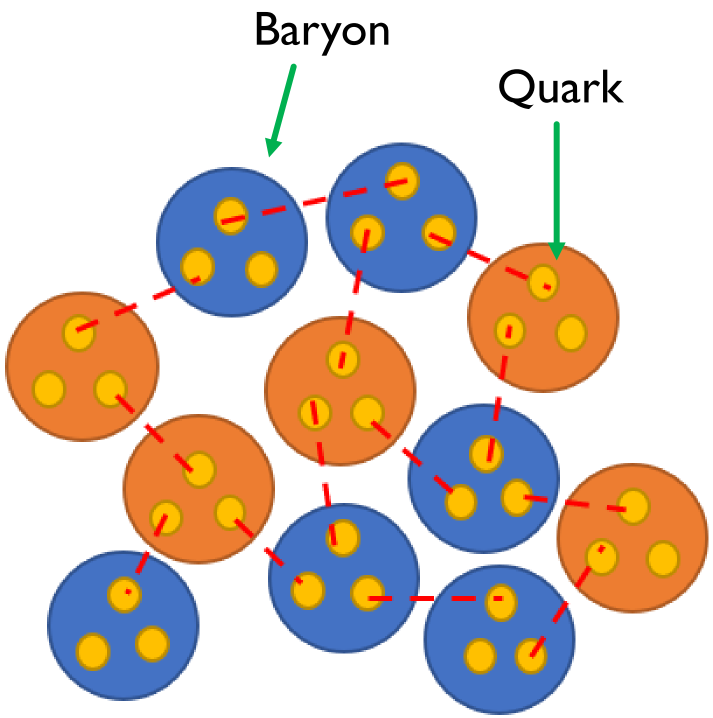
OUR MODEL (EQUATION OF STATE)

Mean field approximation:

→ Based on mean field description of baryons interacting via quark-quark interaction.

Model for quark-quark interaction:

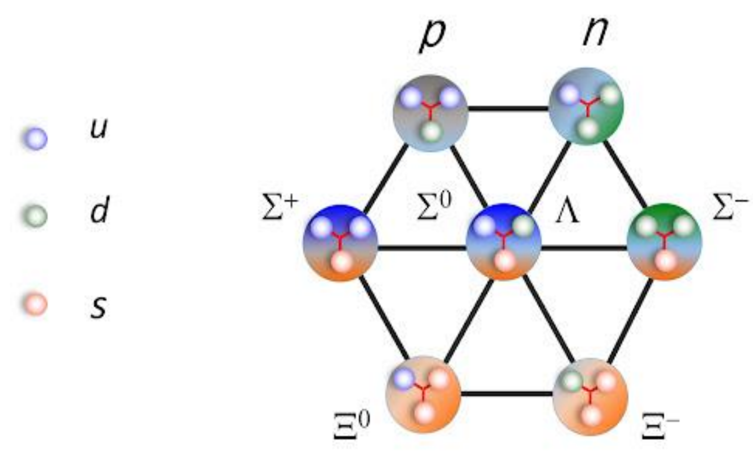
→ Nambu-Jona-Lasinio (NJL) model



WHAT ARE HYPERONS?

- Baryons with strange quark .
- In our study, we included 8 different baryons.

p, n, $\underbrace{\Sigma^+, \Sigma^0, \Sigma^-, \Lambda, \Xi^0, \Xi^-}_{\text{hyperons}}$



(http://kakudan.rcnp.osaka-u.ac.jp/jp/overview/world/Flavor.html)

OUR MODEL (EQUATION OF STATE)

Flavor SU(3) NJL Model Lagrangian:

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - \hat{m})\Psi + G_\pi\left[(\bar{\Psi}\lambda_a\Psi)^2 - (\bar{\Psi}\lambda_a\gamma_5\Psi)^2\right]$$ ··· Lorentz scalar $\bar{q}q$ channel (2nd term)

$$-G_\omega\left[(\bar{\Psi}\lambda_a\gamma^\mu\Psi)^2 + (\bar{\Psi}\lambda_a\gamma^\mu\gamma_5\Psi)^2\right]$$ ··· Lorentz vector $\bar{q}q$ channel

$$+G_S\left[\bar{\Psi}\gamma_5 C\lambda_a^{(f)}\lambda_A^{(c)}\bar{\Psi}^T\right]\left[\Psi^T C^{-1}\gamma_5\lambda_a^{(f)}\lambda_A^{(c)}\Psi\right]$$ ··· Scalar diquark channel

$$+G_A\left[\bar{\Psi}\gamma_\mu C\lambda_a^{(f)}\lambda_A^{(c)}\bar{\Psi}^T\right]\left[\Psi^T C^{-1}\gamma_\mu\lambda_a^{(f)}\lambda_A^{(c)}\Psi\right]$$ ··· Axial-vector diquark channel

Table. 1 Values of coupling constants $[\mathrm{GeV}^{-2}]$

G_π	G_ω	G_S	G_A
19.04	6.030	5.839	4.907

OUR MODEL (EQUATION OF STATE)

Determining the Four Coupling Constants :

- G_π: Solving the gap and Bethe-Salpeter equations, reproducing pion decay constant $\mathrm{f_\pi = 93[MeV]}$ and the pion mass $\mathrm{m_\pi = 140[MeV]}$.

- G_ω: Binding energy per-nucleon in symmetric nuclear matter $\mathrm{E_B/A = 16[MeV]}$ with the saturation density of $\rho_{\mathrm{B_0}} = 0.15[\mathrm{fm}^{-3}]$.

- G_S, G_A: From the T matrix of the Faddeev equation to reproduce the masses in vacuum of the nucleons as 940 MeV and the Δ particle as 1232 MeV.

OUR MODEL (EQUATION OF STATE)

Masses of the Baryons:

Determined by the Faddeev equations.

Table 2. Mass of the baryons [MeV]

	p	n	Σ^+	Σ^0	Σ^+	Λ	Ξ^0	Ξ^-
Calc.	940.0	940.0	1168.5	1168.5	1168.5	1124.6	1318.7	1318.7
Obs.	938.3	939.6	1189.4	1192.6	1197.7	1115.7	1314.9	1321.7

OUR MODEL (EQUATION OF STATE)

Equation of State for Neutron Star Matter (T=0):

$$\begin{cases} \mathcal{E} = V + \sum\limits_{\alpha=a,i} \mu_\alpha \rho_\alpha \\ P = -V \end{cases}$$

where

- $\mathcal{E}$: Energy density
- V: Effective potential
- P: Pressure

- α: Baryons and leptons(e, μ)
- μ_α: Chemical potentials
- ρ_α: Density for each particle

OUR MODEL (EQUATION OF STATE)

Effective Potential (in mean field approximation) V:

$$V = V_{vac} + V_B + V_l - \frac{\omega^2 + \rho^2}{4G_\omega} - \frac{\phi^2}{8G_\omega}$$

where

- V_{vac}:Vacuum term of constituent quarks (u, d, s)
- V_B: Baryon kinetic term (Baryons moving in mean scalar and vector fields)
- V_l: Lepton kinetic terms
- ω, ρ, ϕ: Mean vector fields

OUR MODEL (EQUATION OF STATE)

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OUR MODEL (EQUATION OF STATE)

- Consider for example the vector couplings

$$\mathcal{L}_{I,v} = -G_\omega \sum_{a=0,3,8} (\bar{\Psi}\lambda_a \gamma^\mu \Psi)^2$$
$$= -G_\omega[(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\tau_3\gamma^\mu\psi)^2 + 2(\bar{s}\gamma^\mu s)^2]$$

This is equivalent to the Yukawa Couplings,

$$\mathcal{L}_{I,v} = -\bar{\psi}\gamma^\mu(\omega_\mu + \tau_3\rho_\mu)\psi - \bar{s}\gamma^\mu\phi_\mu s$$
$$\boxed{+\frac{\omega_\mu^2 + \rho_\mu^2}{4G_\omega} + \frac{\phi_\mu^2}{8G_\omega}}$$

where

$$\omega_\mu = 2G_\omega\langle\bar{\psi}\gamma_\mu\psi\rangle, \rho_\mu = 2G_\omega\langle\bar{\psi}\tau_3\gamma_\mu\psi\rangle, \phi_\mu = 4G_\omega\langle\bar{s}\gamma_\mu s\rangle$$

OUR MODEL (EQUATION OF STATE)

Two Conditions for Neutron Stars:

(1) Chemical Equilibrium

$$\mu_{\Sigma^+} = \mu_p = \mu_n - \mu_e$$

$$\mu_{\Sigma^0} = \mu_\Lambda = \mu_{\Xi^0} = \mu_n$$

$$\mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e$$

$$\mu_e = \mu_\mu$$

(2) Charge Neutrality

$$\rho_Q = \sum_\alpha Q_\alpha \rho_\alpha - \rho_e - \rho_\mu = 0$$

OUR MODEL (STAR STRUCTURE)

Tolmann-Oppenheimer-Volkoff (TOV) equation:

→ Based on Einstein's general theory of relativity, constrains the structure of a spherically symmetric body which is in static gravitational equilibrium.

$$
\begin{cases}
\dfrac{dP}{dr} = -\dfrac{G\left(\rho + \dfrac{P}{c^2}\right)\left(4\pi r^3 \dfrac{P}{c^2} + M\right)}{r^2\left(1 - \dfrac{2GM}{c^2 r}\right)} \\[2em]
\dfrac{dM}{dr} = 4\pi r^2 \rho
\end{cases}
$$

RESULTS: (1) EQUATION OF STATE

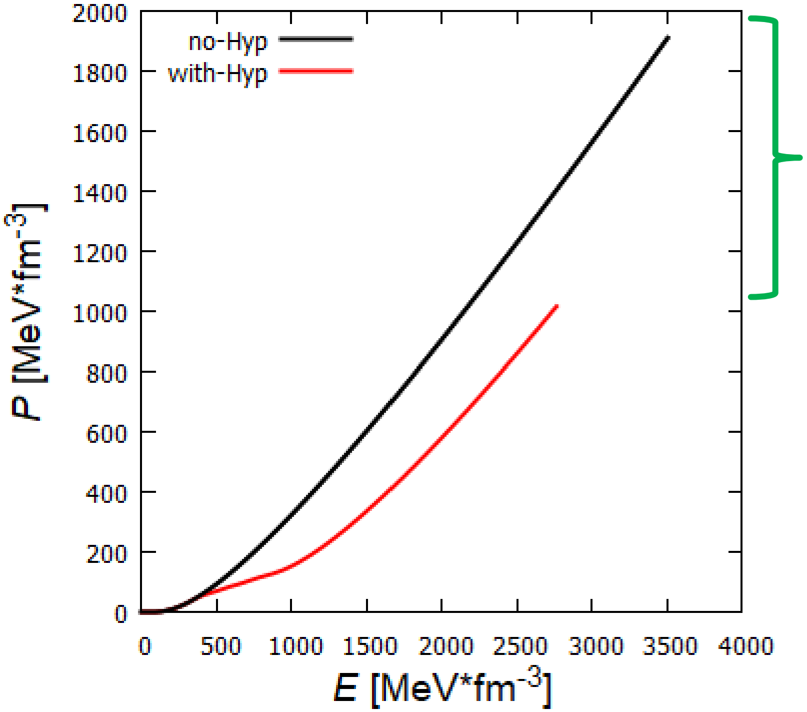
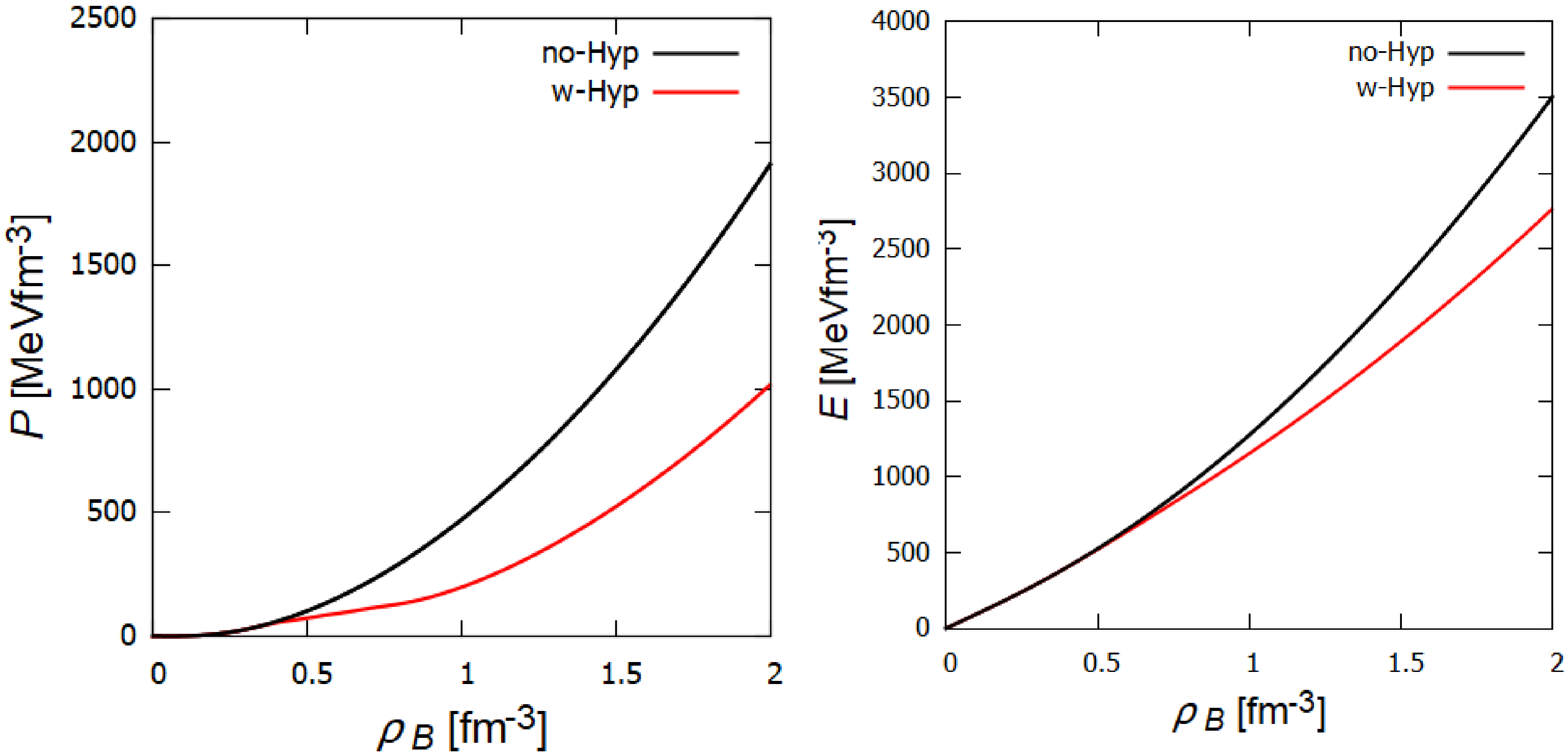


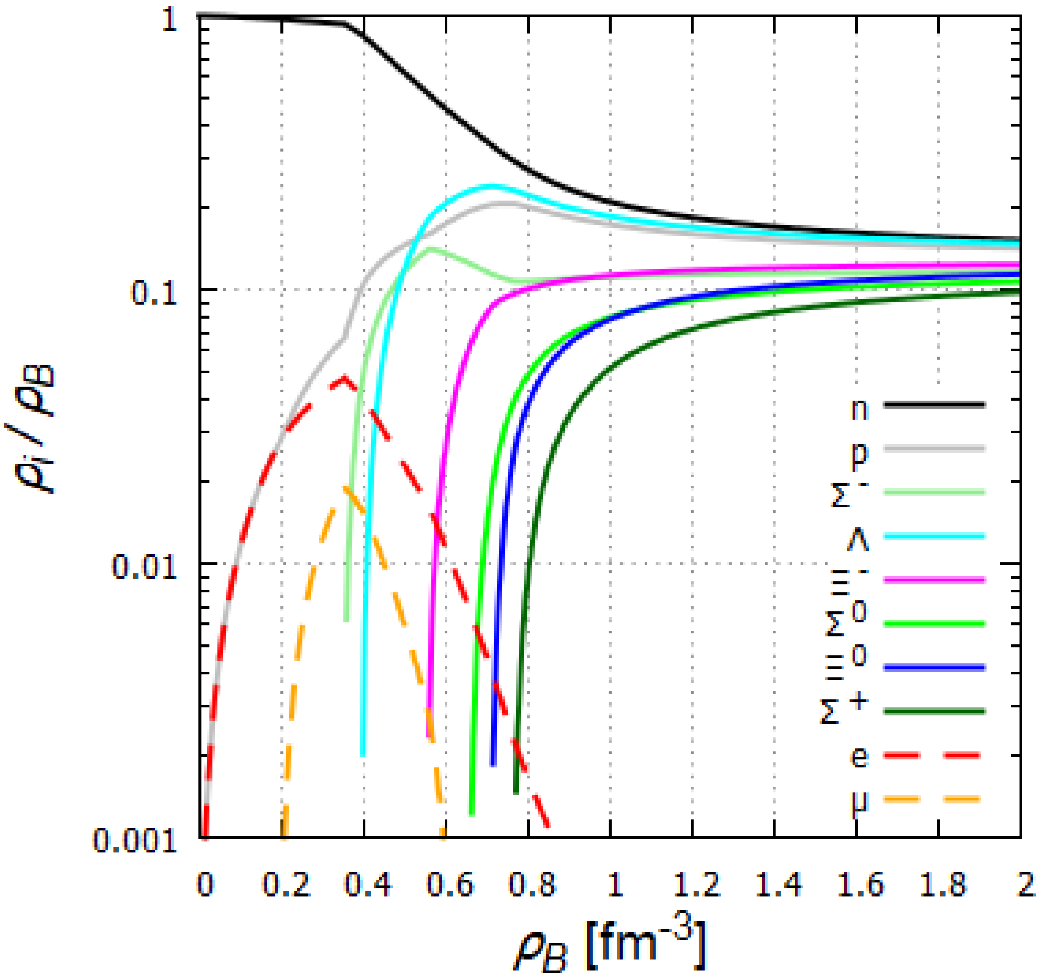
Fig 1. Relation between energy and pressure.

In high density region, pressure and energy density with hyperon decreases.

RESULTS (1) : EQUATION OF STATE



RESULTS (I) : EQUATION OF STATE



RESULTS: (2) STAR MASS AND RADII

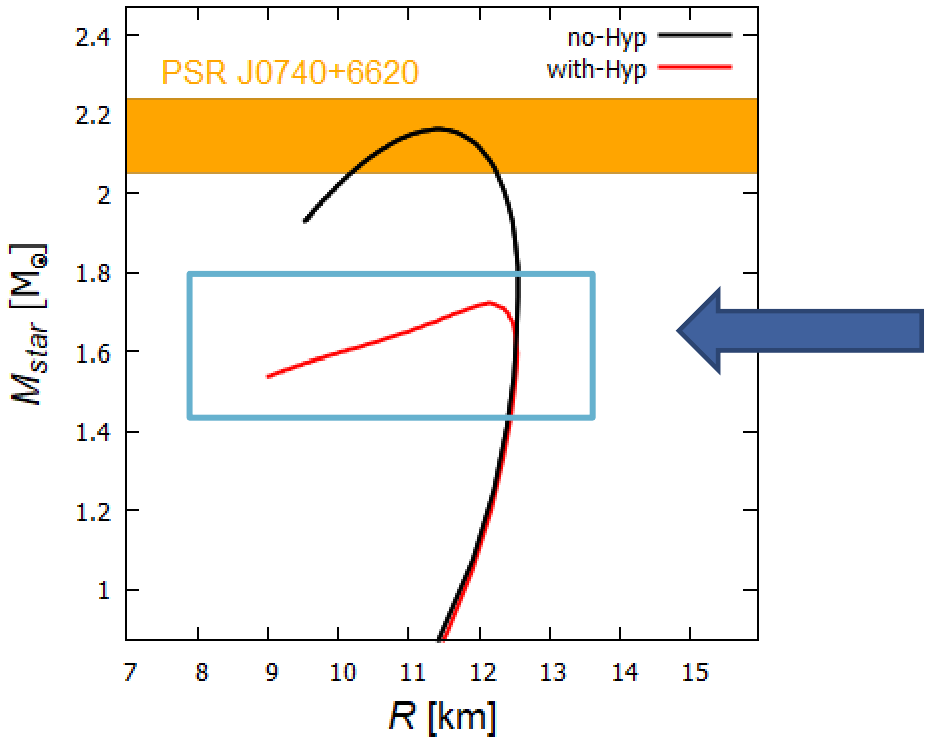


Fig 2. Relation between star mass and radii.

Star mass with hyperons is too low compared to the observation of heavy star (PSR).
→ "Hyperon Puzzle"

SUMMARY

Hyperons may exist near center of the neutron star.

Equation of State with hyperons is not consistent with the observations.

We are now seeking the way to solve the Hyperon Puzzle on the quark level.

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BACKUP SLIDES

OUR MODEL (EQUATION OF STATE)

- Chemical Potentials for each baryons

$$\mu_p = \mu_p^* + 3\omega + \rho, \qquad \mu_n = \mu_n^* + 3\omega - \rho$$

$$\mu_{\Sigma^+} = \mu_{\Sigma^+}^* + 2\omega + 2\rho + \phi, \qquad \mu_{\Sigma^0} = \mu_{\Sigma^0}^* + 2\omega + \phi, \qquad \mu_{\Sigma^-} = \mu_{\Sigma^-}^* + 2\omega - 2\rho + \phi$$

$$\mu_\Lambda = \mu_\Lambda^* + 2\omega + \phi$$

$$\mu_{\Xi^0} = \mu_{\Xi^0}^* + \omega + \rho + 2\phi, \qquad \mu_{\Xi^-} = \mu_{\Xi^-}^* + \omega - \rho + 2\phi$$

where $\mu_\alpha^* = \sqrt{k_\alpha^2 + M_\alpha^2}$

OUR MODEL (YUKAWA COUPLINGS)

- Yukawa Couplings

$$G(\bar{q}\Gamma q)^2 = (\bar{q}\Gamma q)\varphi - \frac{\varphi^2}{4G}$$

define the auxiliary fields

$$\varphi = 2\mathrm{G}\langle\bar{q}\Gamma q\rangle$$

OUR MODEL (YUKAWA COUPLINGS)

- Consider for example the scalar couplings

$$\mathcal{L}_S = G_\pi \sum_{a=0,3,8} (\bar{\Psi}\lambda_a \Psi)^2$$

$$= G_\pi[(\bar{\psi}\psi)^2 + \left(\bar{\psi}\tau_3\, \psi\right)^2 + 2(\bar{s}s)^2]$$

This is also equivalent to Yukawa Couplings

$$\mathcal{L}_S = \bar{\psi}(\sigma + \tau_3\delta)\psi + \bar{s}\sigma_s s - \frac{\sigma^2 + \delta^2}{4G_\pi} - \frac{\sigma_s^2}{8G_\pi}$$

where

$$\sigma = 2G_\pi\langle\bar{\psi}\psi\rangle, \delta = 2G_\pi\langle\bar{\psi}\tau_3\psi\rangle, \sigma_s = 4G_\pi\langle\bar{s}s\rangle$$

WEAK PROCESSES (EXAMPLE 1)

- The decays

$$\mu^- \rightarrow e^- \bar{\upsilon}_e \upsilon_\mu$$

$$n \rightarrow p e^- \bar{\upsilon}_e$$

and the reverse processes, give the relations

$$\mu_\mu = \mu_e$$

$$\mu_n = \mu_p + \mu_e$$

WEAK PROCESSES (EXAMPLE 2)

- The Λ has decays via 2 modes

$$\Lambda \to p\pi^- \to p\mu^-\bar{\nu}_\mu$$

$$\Lambda \to n\pi^0 \to n\gamma\gamma$$

This, and the reverse processes, gives the relation

$$\mu_\Lambda = \mu_n$$