How the 20+ GeV SIDIS/TMD can encounter lattice QCD?

The Next Generation of 3D Imaging
Jefferson Lab, Jul. 7-8, 2022

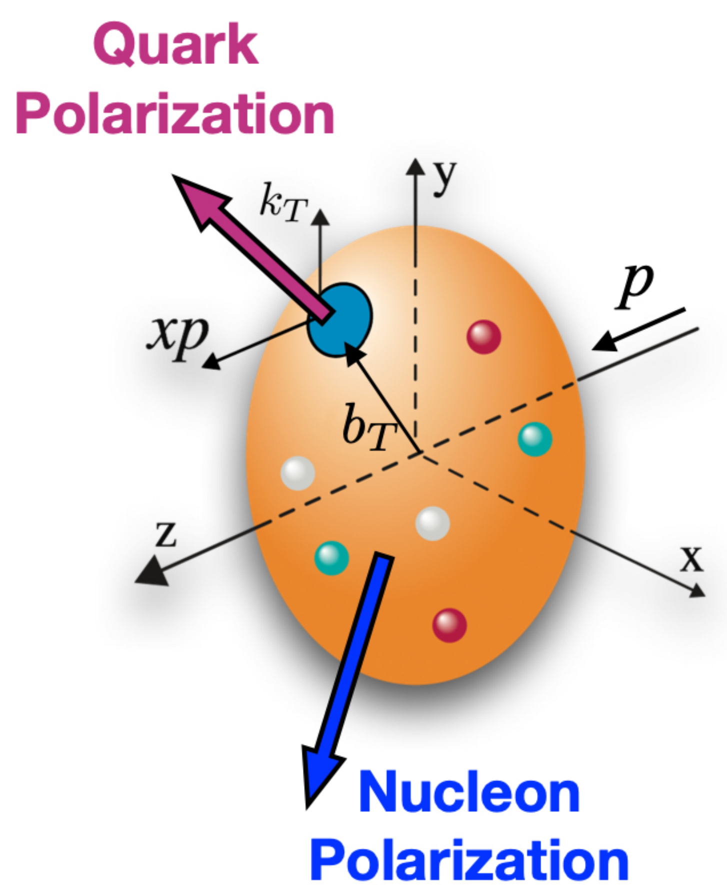
YONG ZHAO

JUL 8, 2022

Argonne NATIONAL LABORATORY

What can lattice QCD calculate?

Short answer: almost everything for leading-power TMDs ☺

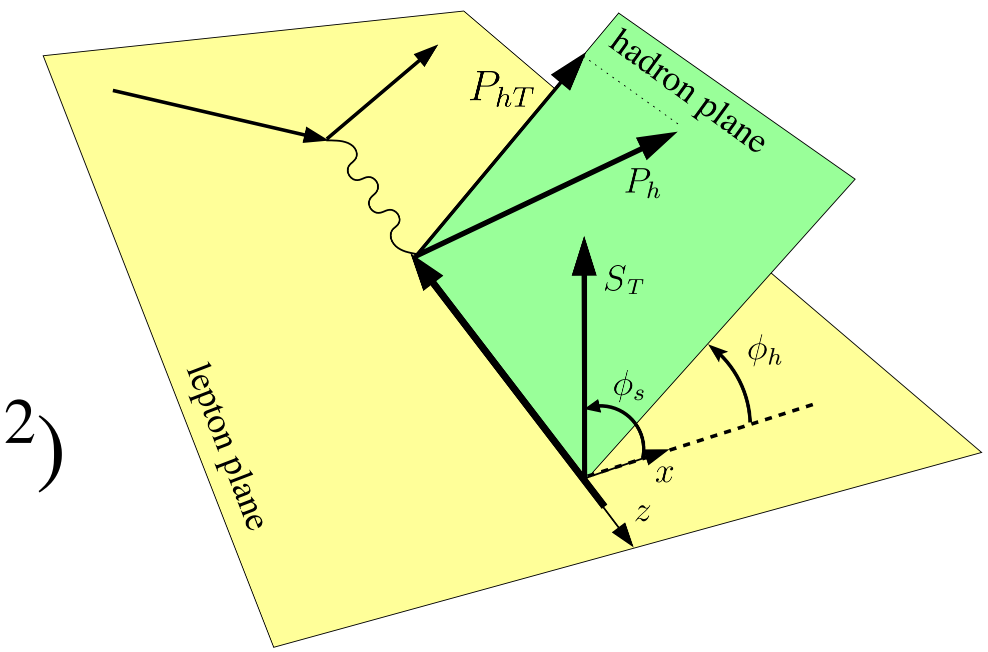


TMDs from global analyses

Semi-inclusive deep inelastic scattering: $l + p \longrightarrow l + h(P_h) + X$

$$\frac{d\sigma^W}{dxdydz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T \, e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2) \; D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, Phys. Rev. D 93, 014009 (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f_{i/p}^{\text{pert}}(x, b*(b_T), \mu, \zeta)$$

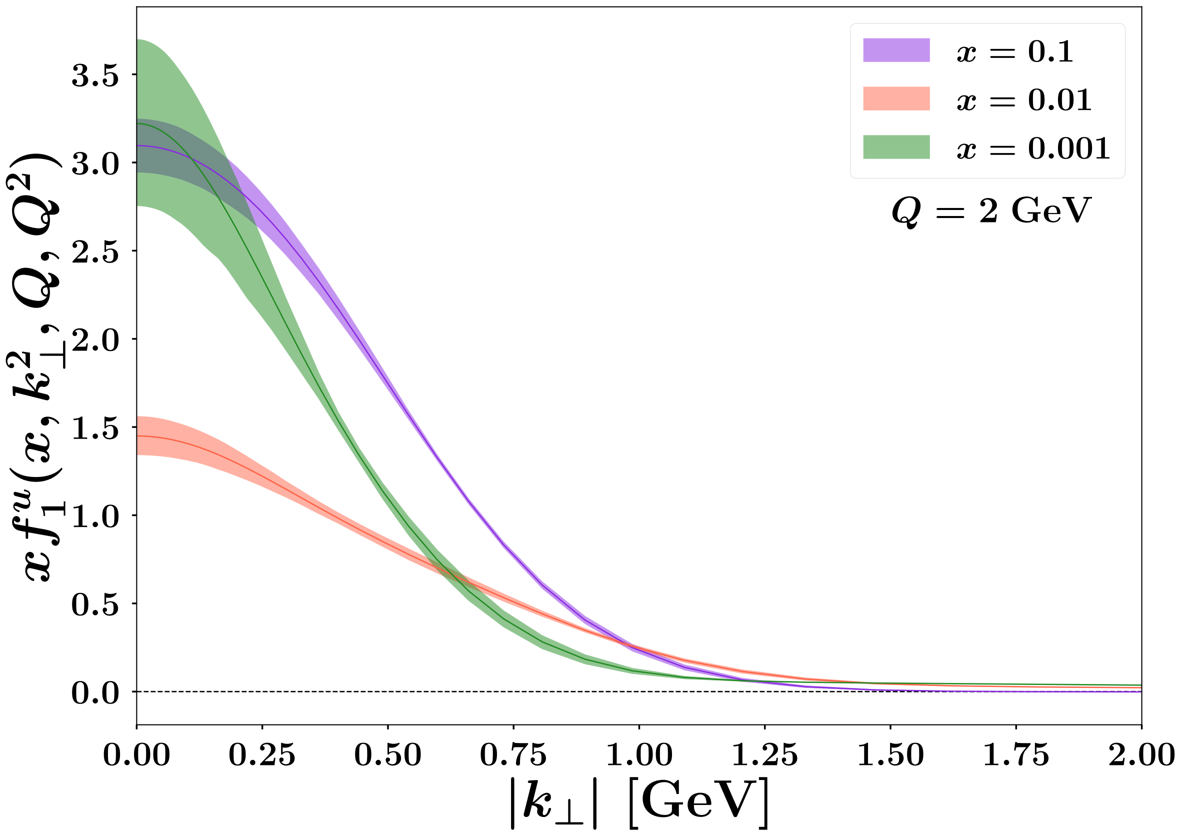
$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2} \quad f_{i/p}^{\text{NP}}(x, b_T)$$

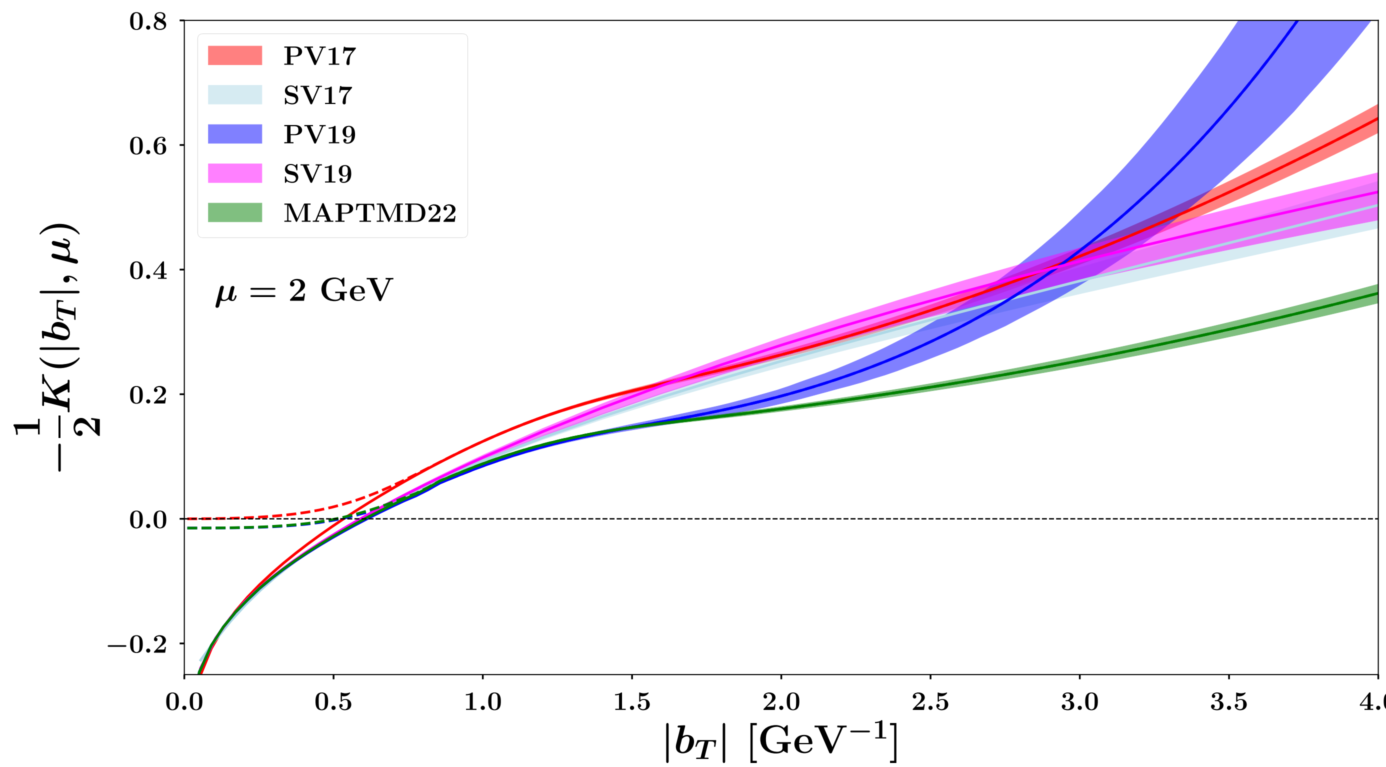
$g_K(b_T)/2$ ⟶ **Collins-Soper kernel (NP part)**

$f_{i/p}^{\text{NP}}(x, b_T)$ ⟶ **Intrinsic TMD**

TMDs from global analyses

Results of the TMDs and Collins-Soper kernel:

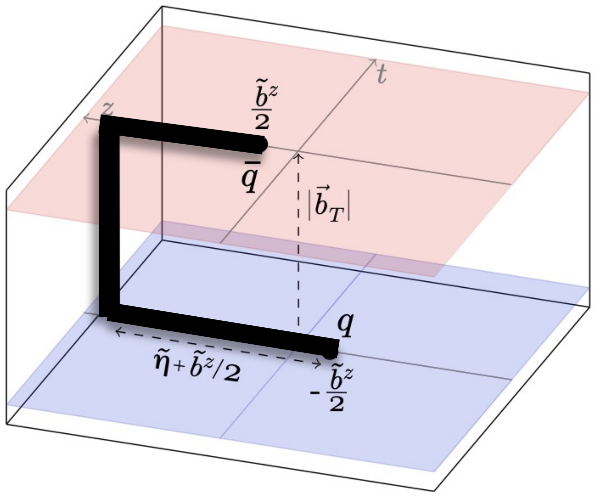




Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

TMDs from lattice QCD

Quasi-TMDs in Large-Momentum Effective Theory:



(Naive) Quasi-TMD ✓

NLO Perturbative matching ✓

$$\frac{\tilde f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \tilde P^z)}{\sqrt{S_r^q(b_T, \mu)}} = C\left(\mu, x\tilde P^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde P^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde P^z b_T)^2}, \frac{\Lambda_{\rm QCD}^2}{(x\tilde P^z)^2}\right]\right\}$$

See Ebert, Schindler, Stewart and YZ, JHEP 04 (2022) 178 and references therein.

Reduced soft function ✓

Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020).

The Lorentz invariant approach by Musch, Hägler, Engelhardt, Negele and Schäfer has mainly been applied to calculate the ratios of TMD moments.

TMDs from lattice QCD

$$\frac{\tilde{f}^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S^q_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f^{[s]}_{i/p}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(x\tilde{P}^z)^2}\right]\right\}$$

Matching coefficient:

- Independent of spin;
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

TMDs from lattice QCD

$$\frac{\tilde{f}_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r^q(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \ \exp\left[\frac{1}{2}K(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- ✳ **Collins-Soper kernel;**

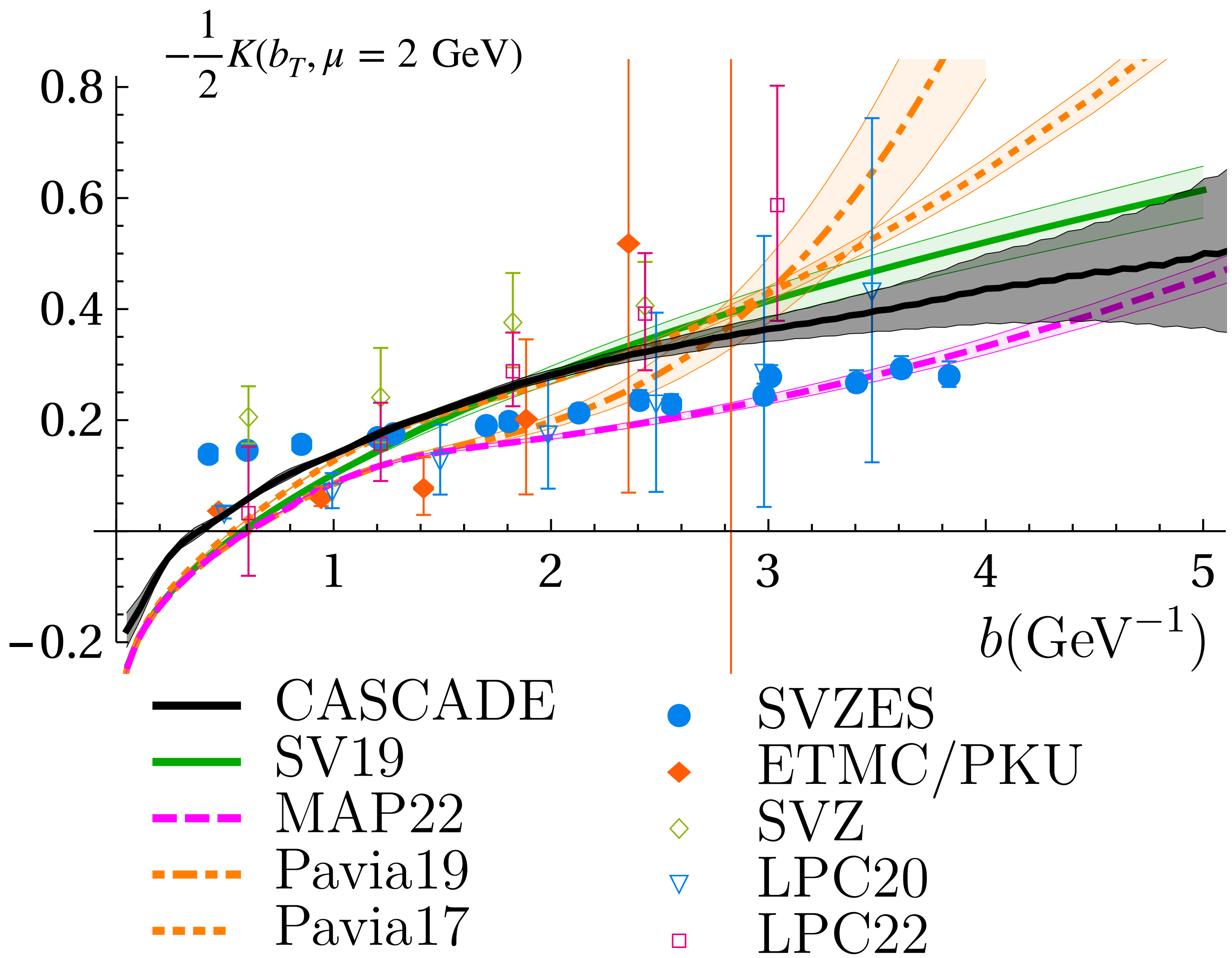
$$K(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$$

- ✳ **Flavor separation;**

$$\frac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \frac{\tilde{f}_{i/p}^{[s]}(x, \mathbf{b}_T)}{\tilde{f}_{j/p}^{[s']}(x, \mathbf{b}_T)}$$

- ✳ **Spin-dependence, e.g., Sivers function;**
- ✳ **Full TMD kinematic dependence.**
- ✳ **Twist-3 PDFs from small b_T expansion of TMDs.**

Collins Soper kernel

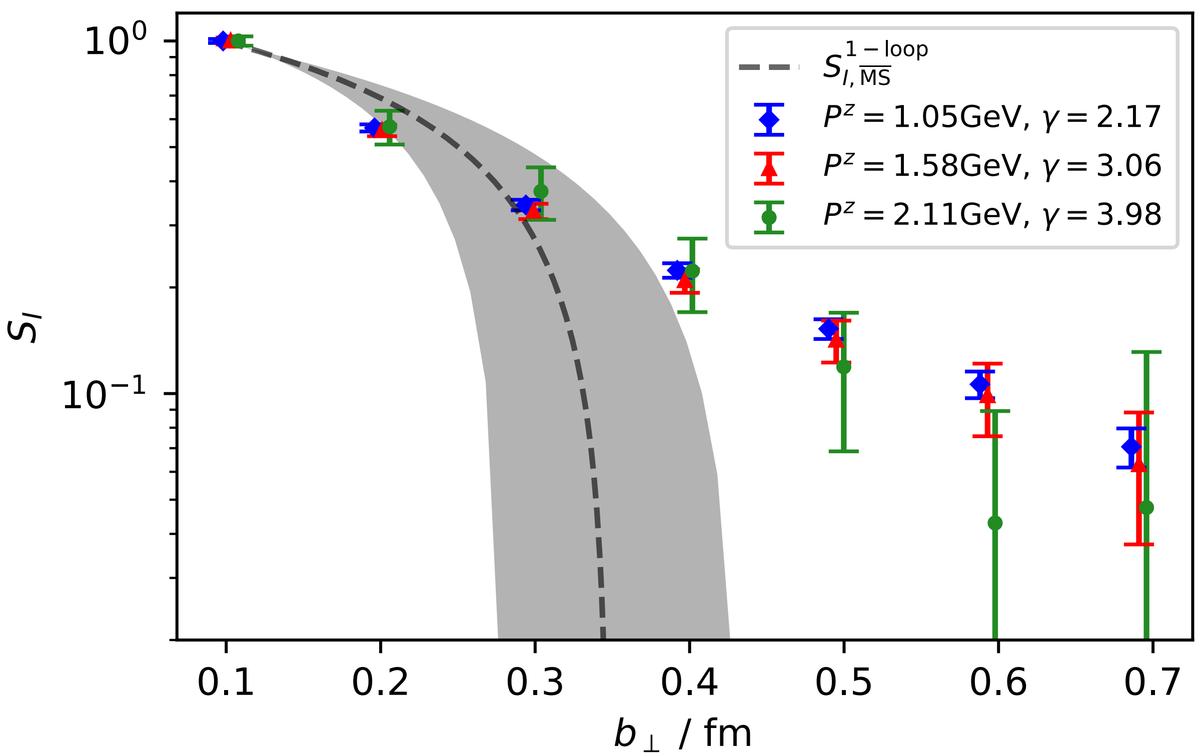


Approach	Collaboration
Quasi-Beam Functions	**SWZ 21** PRD 104 (2021)
Quasi-TMD Wavefunctions	**LPC 20** PRL 125 (2020)
	ETMC/PKU 21 PRL 128 (2022)
	LPC 22 2204.00200
Mellin Moments of Quasi-TMDs	**SVZES 21** JHEP 08 (2021)

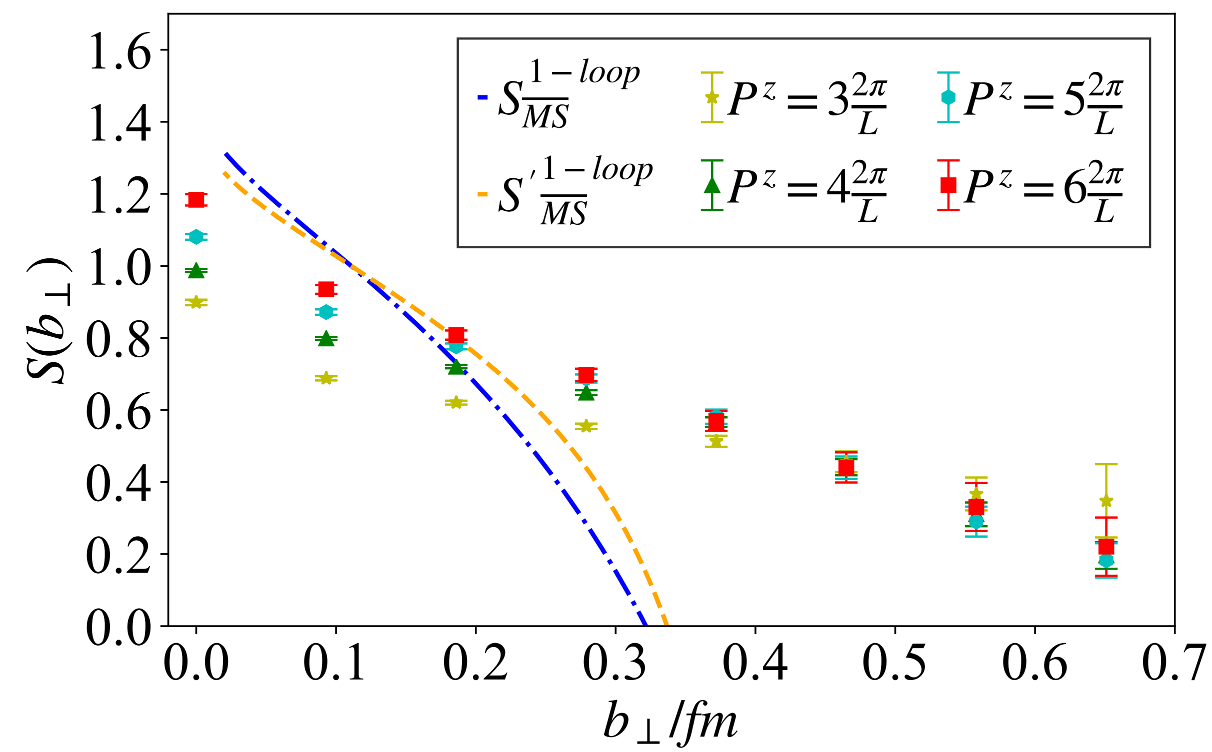
MAP22: Bacchetta, Bertone, Bissolotti, et al., 2206.07598
SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020) 137
Pavia19: A. Bacchetta et al., JHEP 07 (2020) 117
Pavia 17: A. Bacchetta et al., JHEP 06 (2017) 081
CASCADE: Martinez and Vladimirov, 2206.01105

Important systematics:
Lattice renormalization, perturbative matching, Fourier transform, power corrections.

Soft function



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).



Y. Li et al., PRL 128 (2022).

Important systematics:

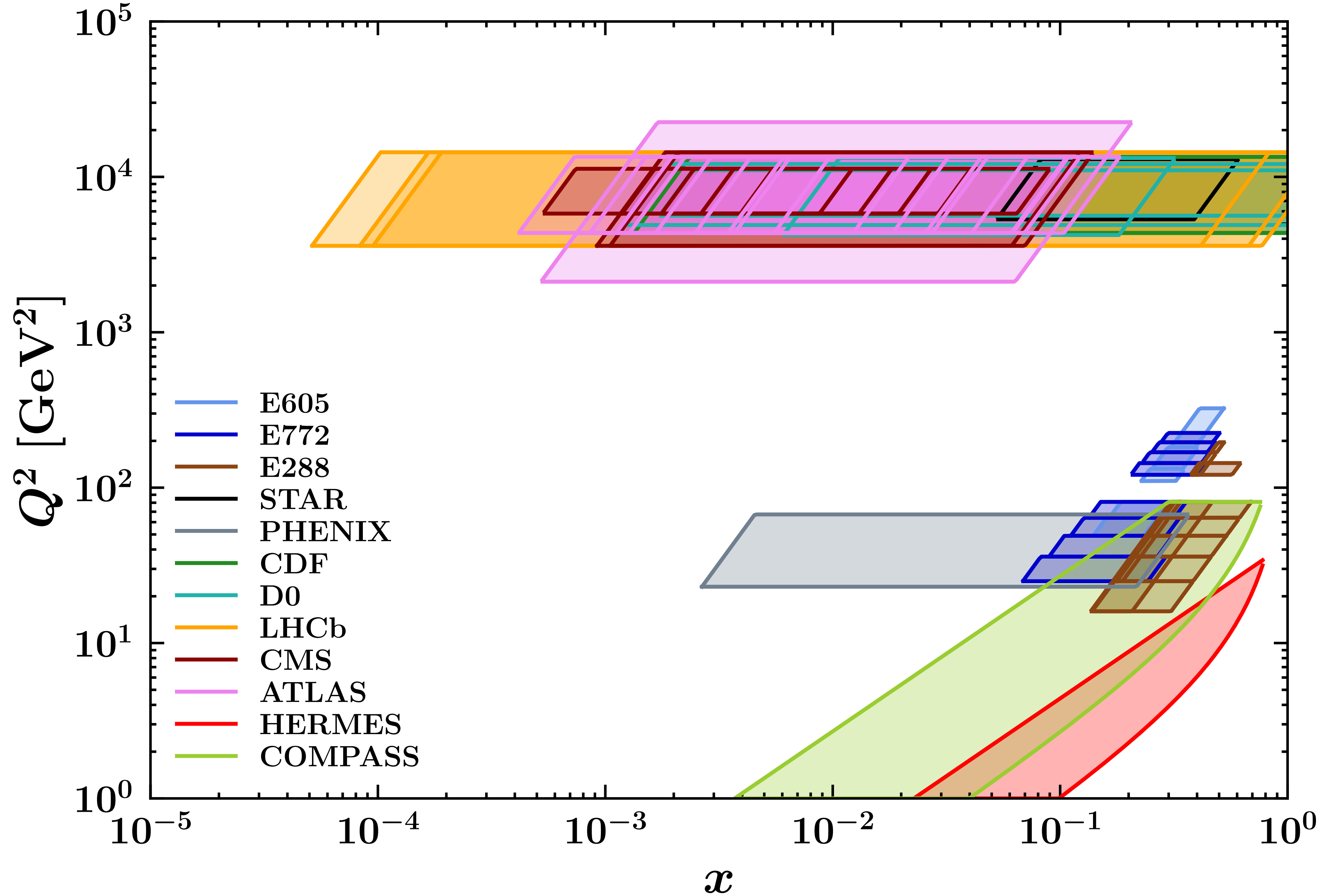
Matching beyond leading order, Fourier transform, power corrections at finite P^z.

Conclusion

- The method for calculating all the leading-power TMDs is complete;
- Lattice results for the Collins-Soper kernel and soft function are promising, but systematics need to be under control;
- What to expect in the short term?
  - Improved Collins-Soper kernel and soft function results;
  - Ratios of spin-dependent TMDs;
  - Ratios of quark TMDs of different flavors (challenge in the calculation of disconnected lattice diagrams);
- What to expect in the long term?
  - x and b_T dependence of intrinsic TMDs;
  - Twist-3 PDFs.

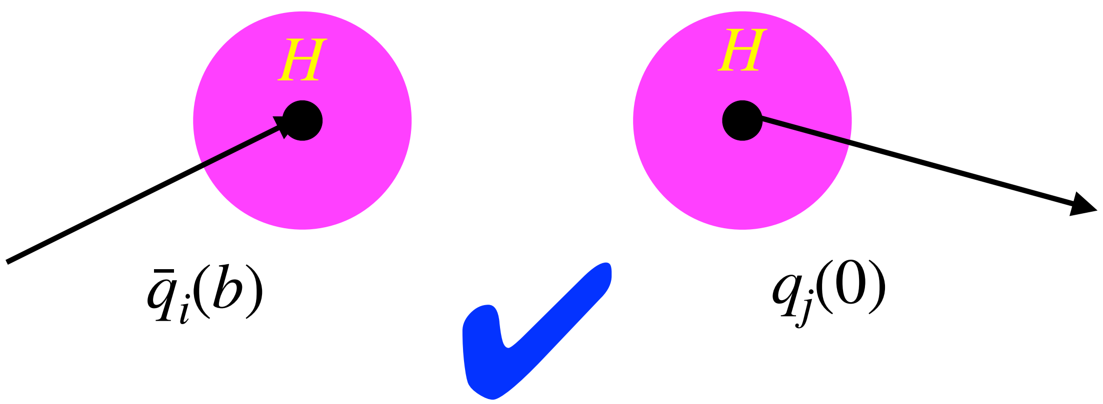
Backup slides

Data used by the MAP collaboration in 2206.07598

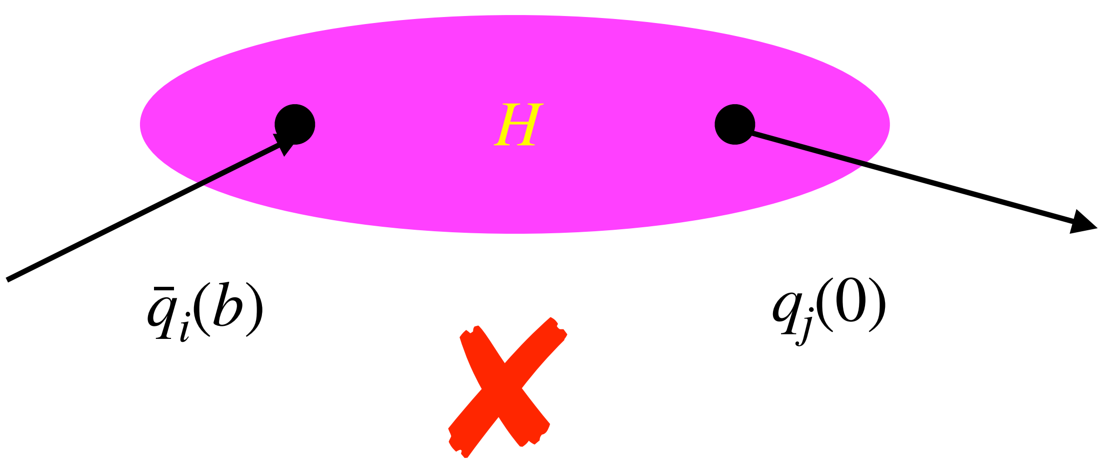


Bacchetta, Bertone, Bissolotti, et al., MAP Collaboration, 2206.07598

Backup slides



i, j **(including spinor indices) remain intact**



$\propto \delta_{ij}$ **Can mix with singlet channel and with gluons**

$$b^2 = -b_z^2 - b_T^2 < b_T^2 \sim 1/\Lambda_{\rm QCD}^2$$

Hard particles cannot propagate that far!