WHY 20+ GEV FOR SIDIS/TMDS?

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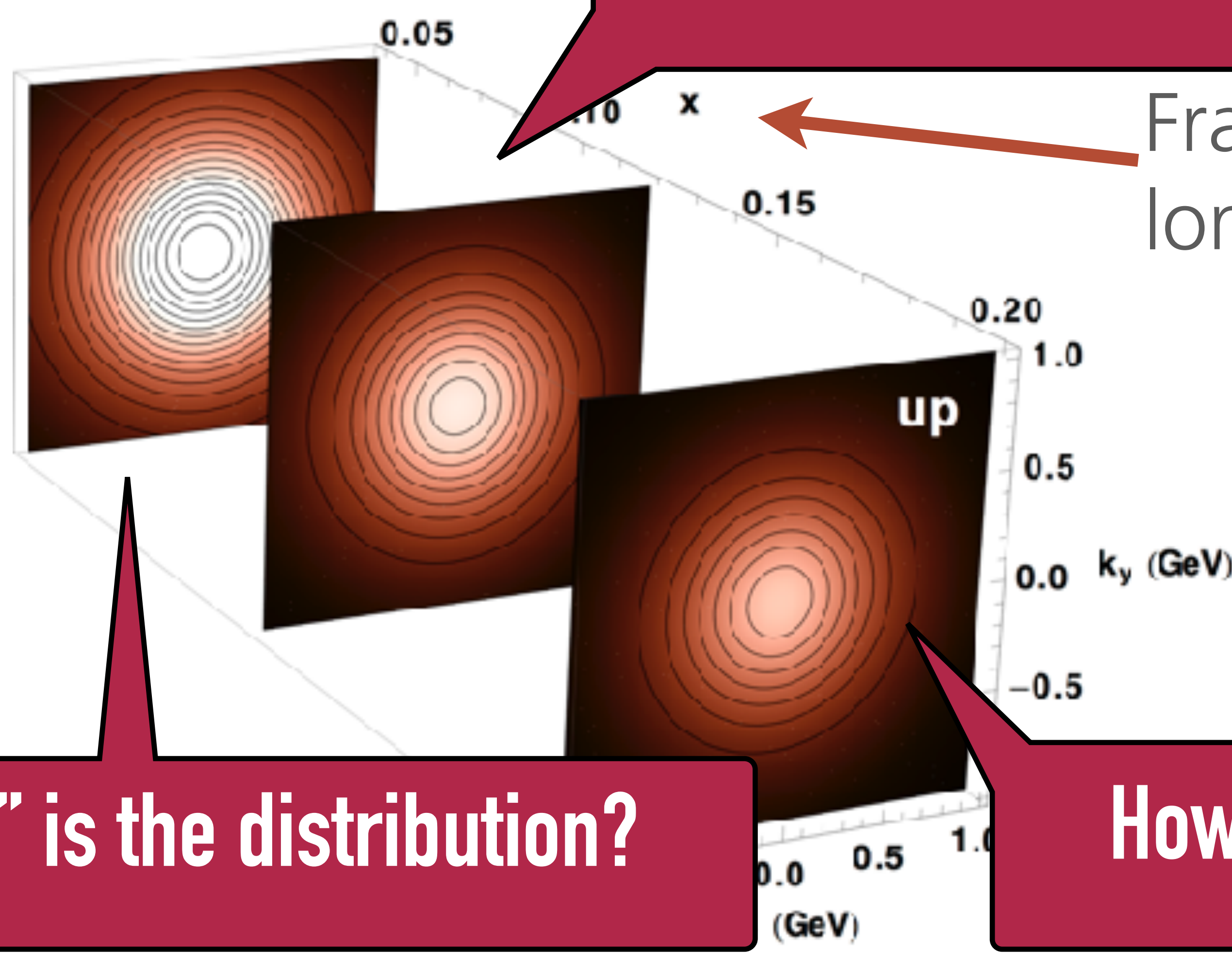
TMDS

TRANSVERSE MOMENTUM DISTRIBUTIONS

TMDs describe the distribution of partons in three dimensions in momentum space. They also have to be extracted through global fits.



Is there a difference between flavors?

Fraction of longitudinal momentum

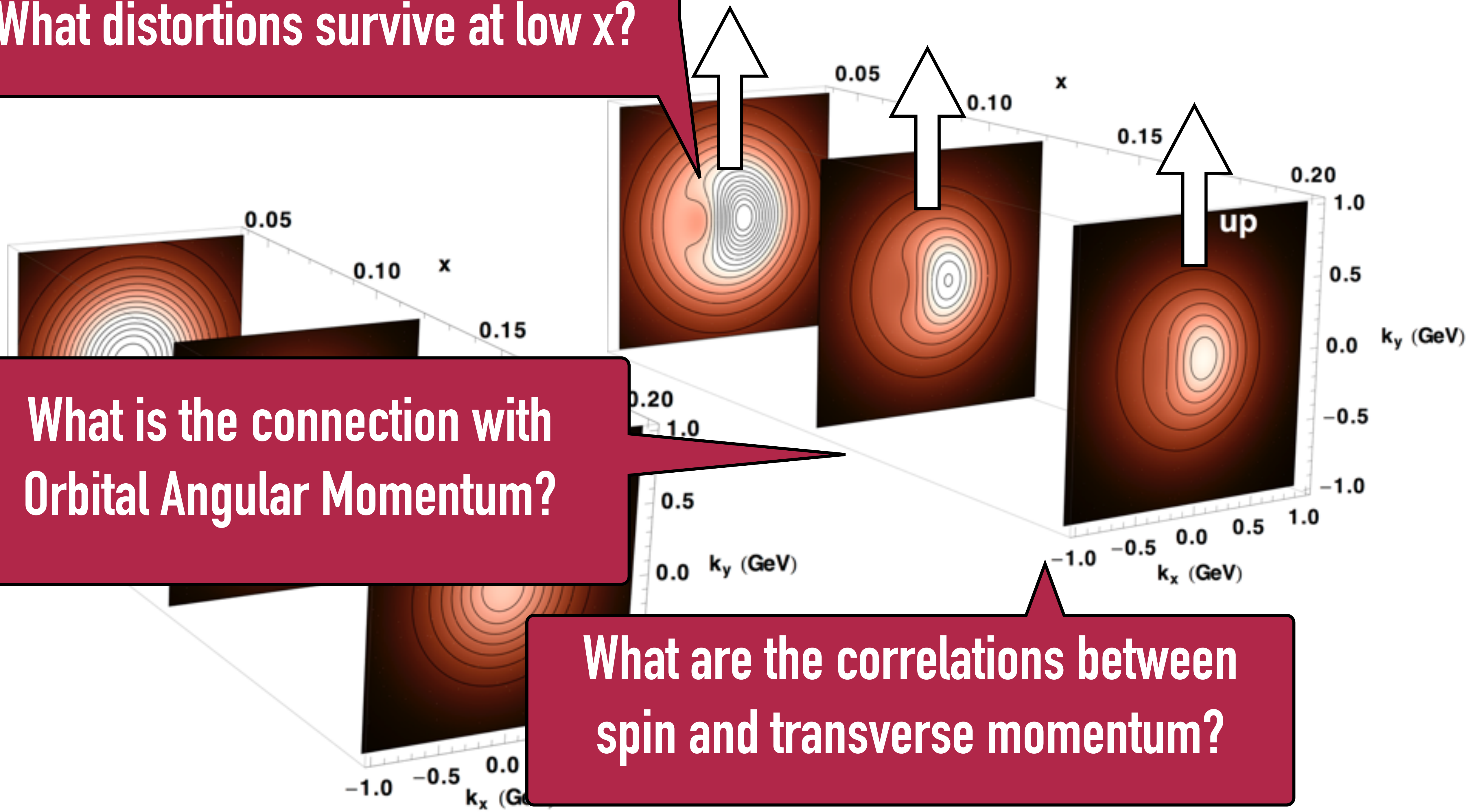
How "wide" is the distribution?

How does it change with x?

Transverse momentum

WITH SPIN

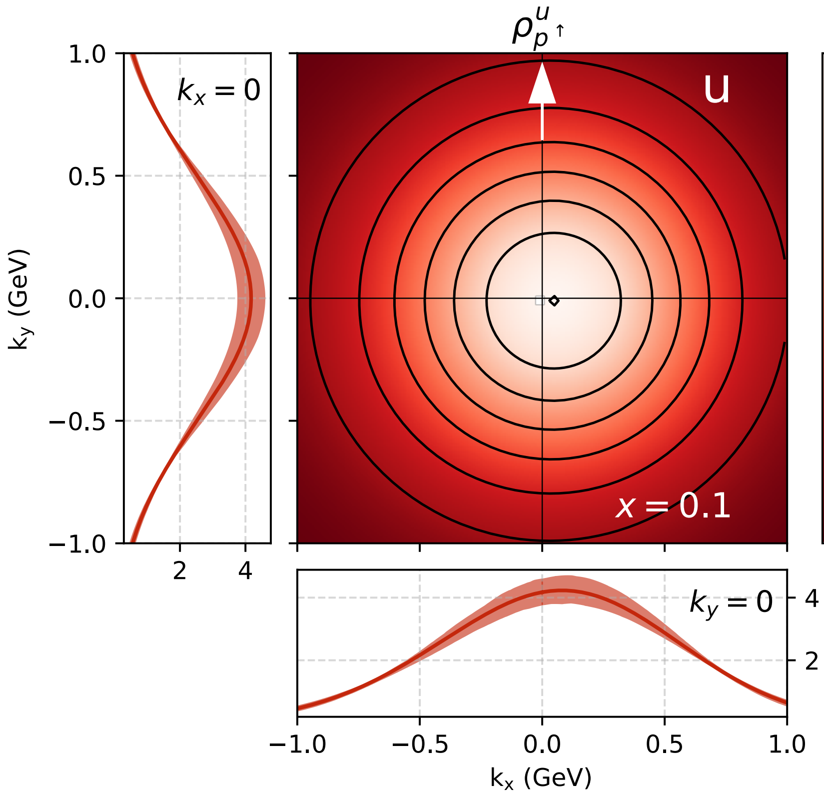
What distortions survive at low x?

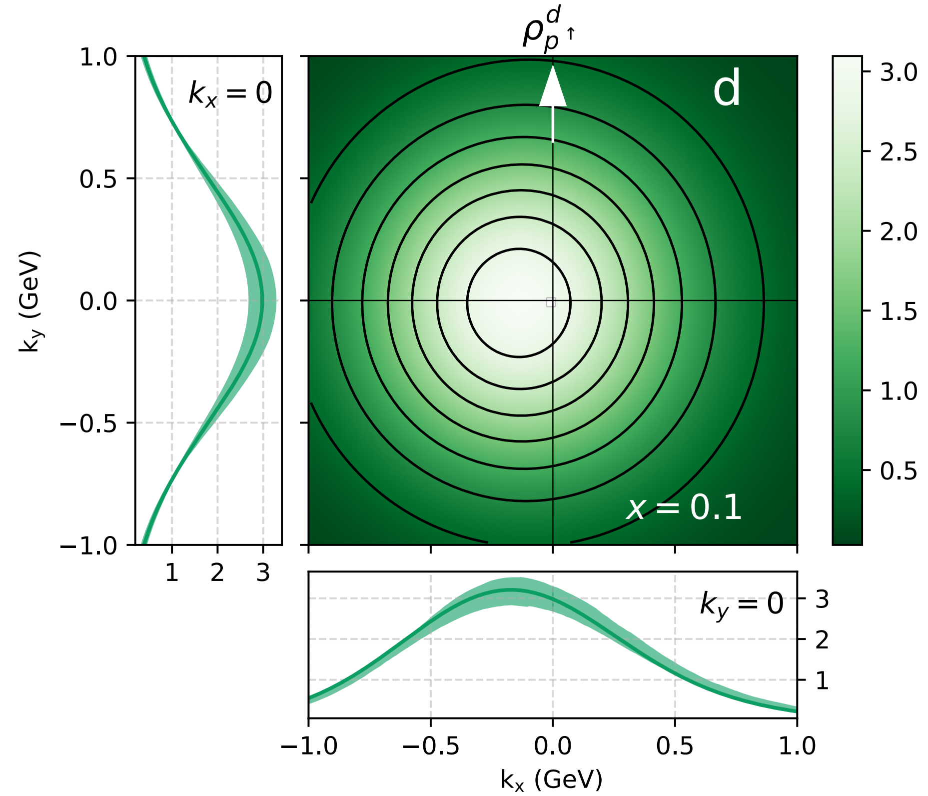


What is the connection with Orbital Angular Momentum?

What are the correlations between spin and transverse momentum?

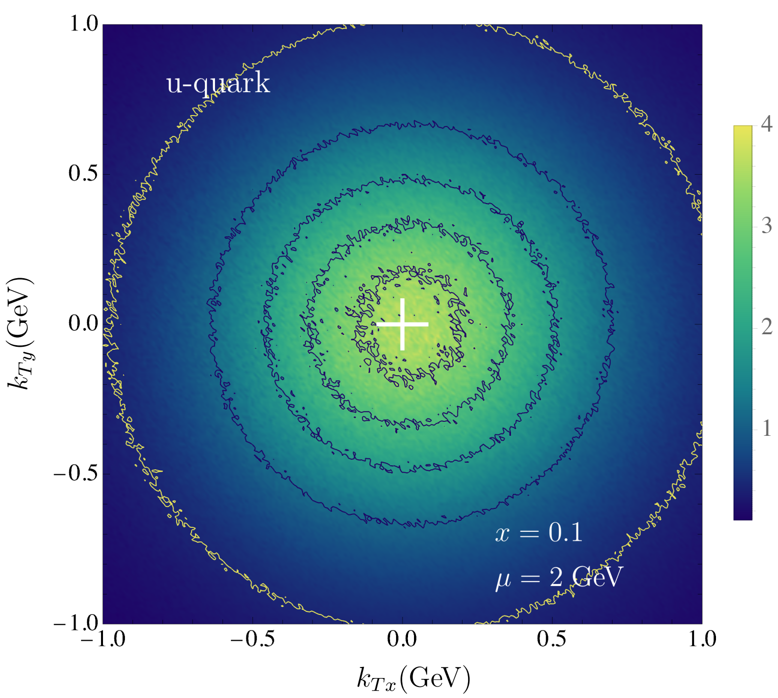
3D MOMENTUM DISTRIBUTIONS FROM DATA



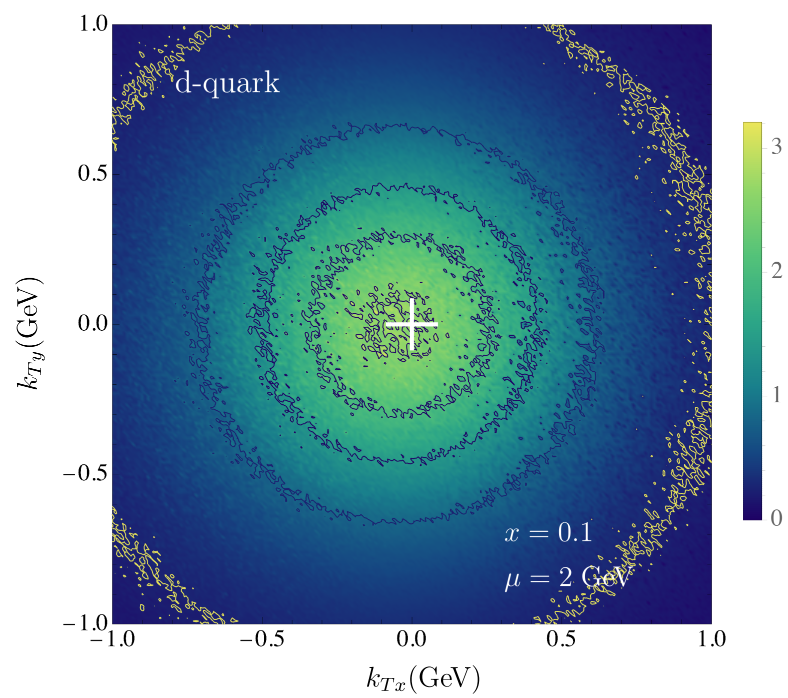


Q= 2GeV

Bacchetta, Delcarro, Pisano, Radici, arXiv:2004.14278



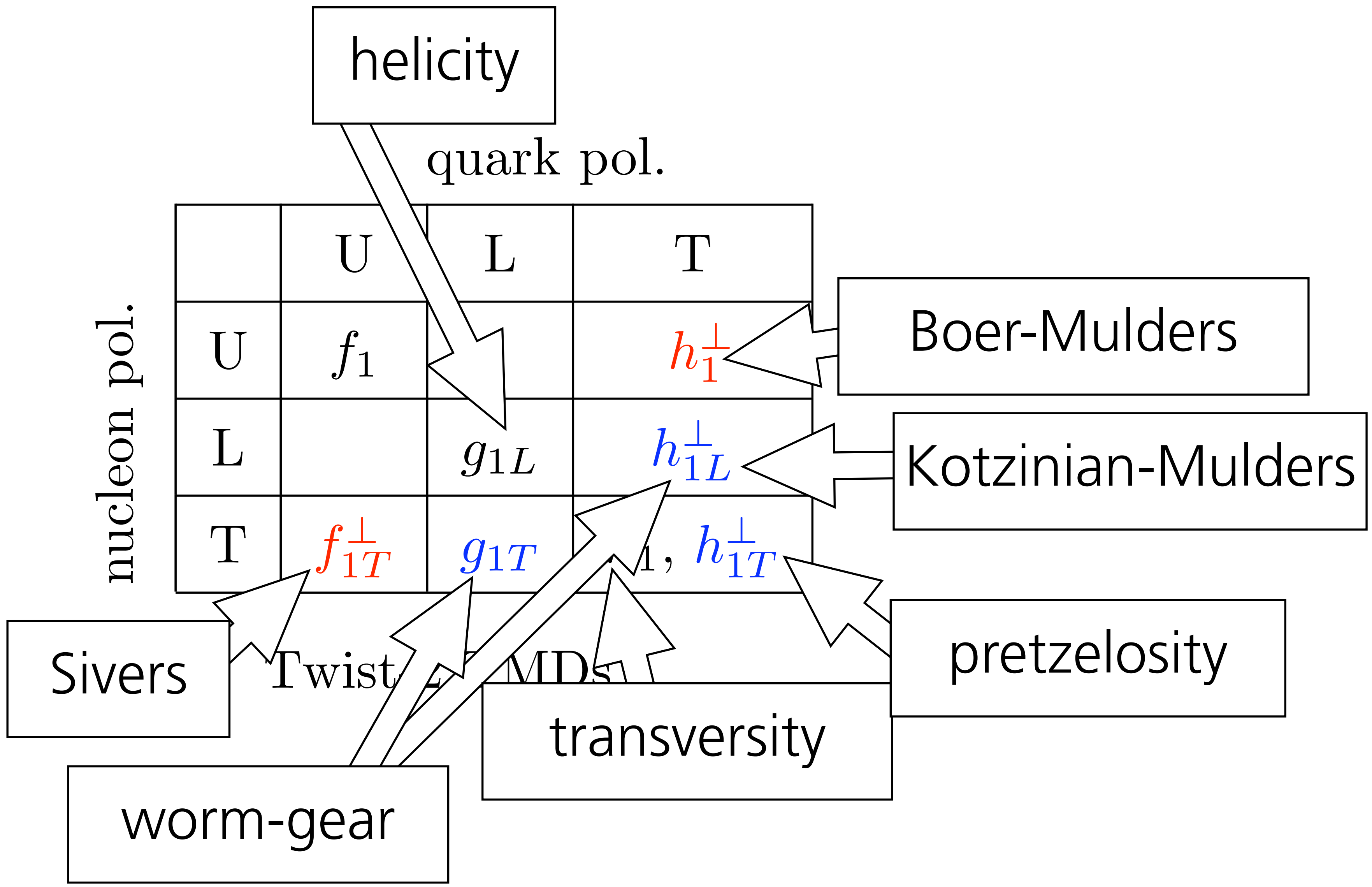
(a)



(b)

Bury, Prokudin, Vladimirov, arXiv:2103.03270

TMD TABLE



TMDs in black survive integration over transverse momentum

TMDs in red are time-reversal odd

Mulders-Tangerman, NPB 461 (96)

Boer-Mulders, PRD 57 (98)

TMD TABLE: TWIST 3

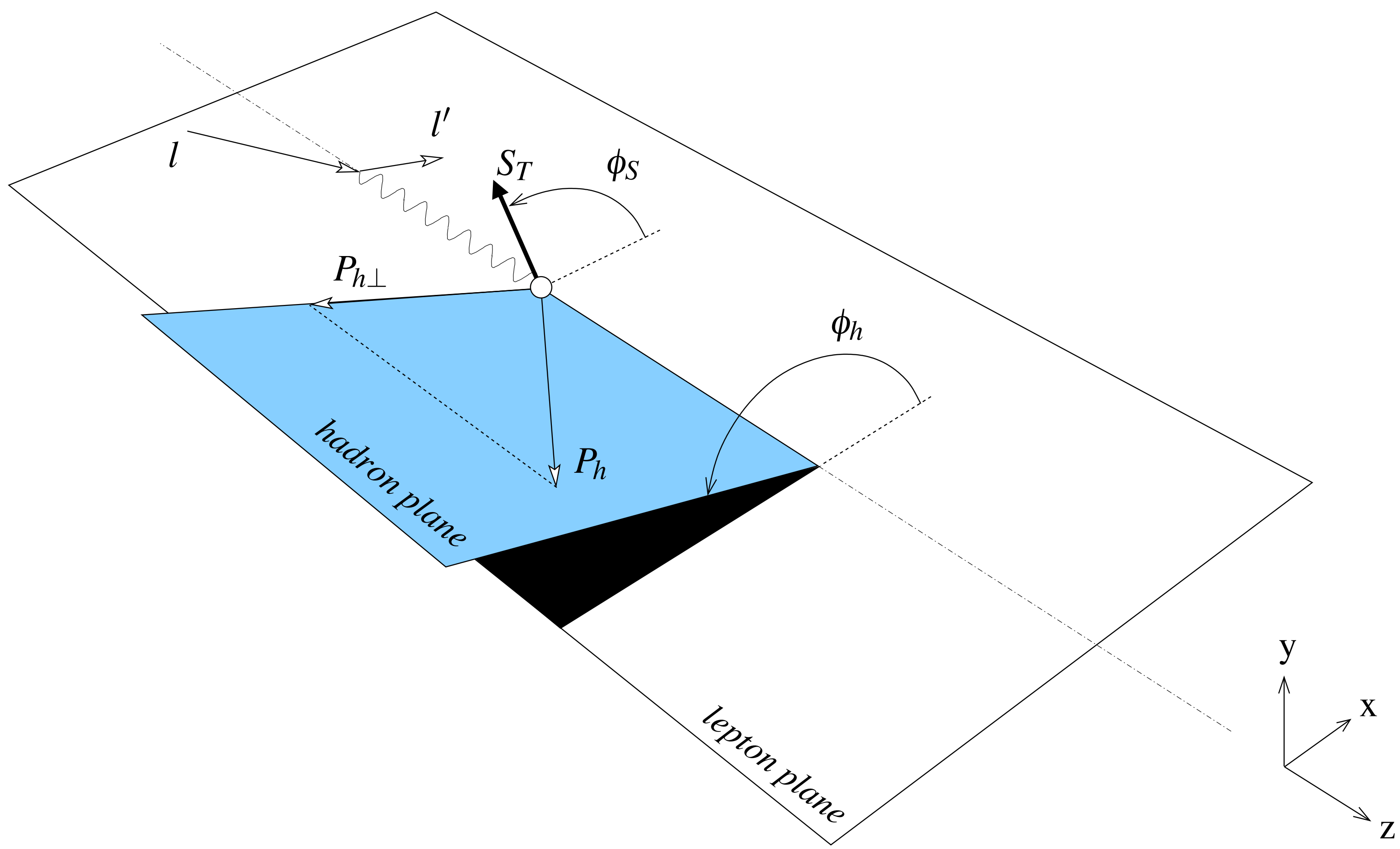
quark pol.

nucleon pol.	U	L	T
U	$f^{\perp}$	$g^{\perp}$	h, e
L	$f_L^{\perp}$	$g_L^{\perp}$	h_L, e_L
T	f_T, $f_T^{\perp}$	g_T, $g_T^{\perp}$	h_T, $h_T^{\perp}$, e_T, $e_T^{\perp}$

Twist-3 TMDs

Mulders-Tangerman, NPB 461 (96)
Boer-Mulders, PRD 57 (98)
Bacchetta, Mulders, Pijlman, hep-ph/0405154
Goeke, Metz, Schlegel, hep-ph/0504130

SEMI-INCLUSIVE DIS



Q = photon virtuality

M = hadron mass

$P_{h\perp}$ = hadron transverse momentum = P_{hT}

$$q_T^2 \approx P_{h\perp}^2 / z^2$$

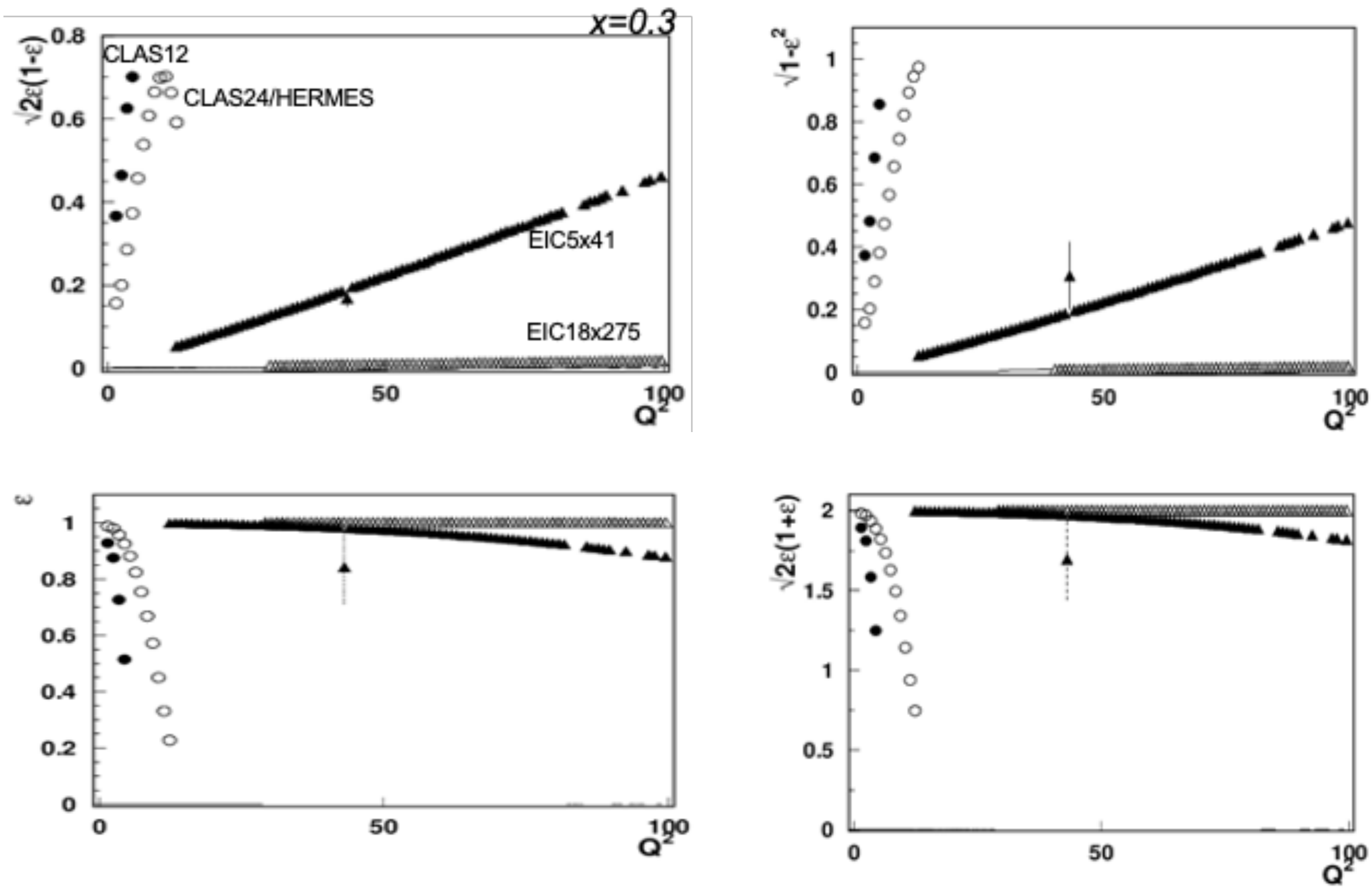
STRUCTURE FUNCTIONS

AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

STRUCTURE FUNCTIONS' REFACTORS

Harut Avakian, talk at Transversity 2022



LIST OF STRUCTURE FUNCTIONS

	observable	twist
"SIDIS F_T"	$F_{UU,T}$	2
"SIDIS F_L"	$F_{UU,L}$	4
"Cahn" - $f^{\perp}$	$F_{UU}^{\cos\phi_h}$	3
"Boer-Mulders"	$F_{UU}^{\cos 2\phi_h}$	2
e, $g^{\perp}$ and friends	$F_{LU}^{\sin\phi_h}$	3
	$F_{UL}^{\sin\phi_h}$	3
"Kotzinian-Mulders"	$F_{UL}^{\sin 2\phi_h}$	2
"SIDIS g_1"	F_{LL}	2
	$F_{LL}^{\cos\phi_h}$	3
"Sivers"	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2
	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	4
"Collins"	$F_{UT}^{\sin(\phi_h+\phi_S)}$	2
"Pretzelosity"	$F_{UT}^{\sin(3\phi_h-\phi_S)}$	2
f_T and friends	$F_{UT}^{\sin\phi_S}$	3
	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3
"Worm gear"	$F_{LT}^{\cos(\phi_h-\phi_S)}$	2
"SIDIS g_2" - g_T	$F_{LT}^{\cos\phi_S}$	3
	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3

Not all of them are easy to access at EIC due to: x-range, twist, evolution, prefactors

Examples:

probably high-x effect, probably suppressed by evolution, "bad" prefactor

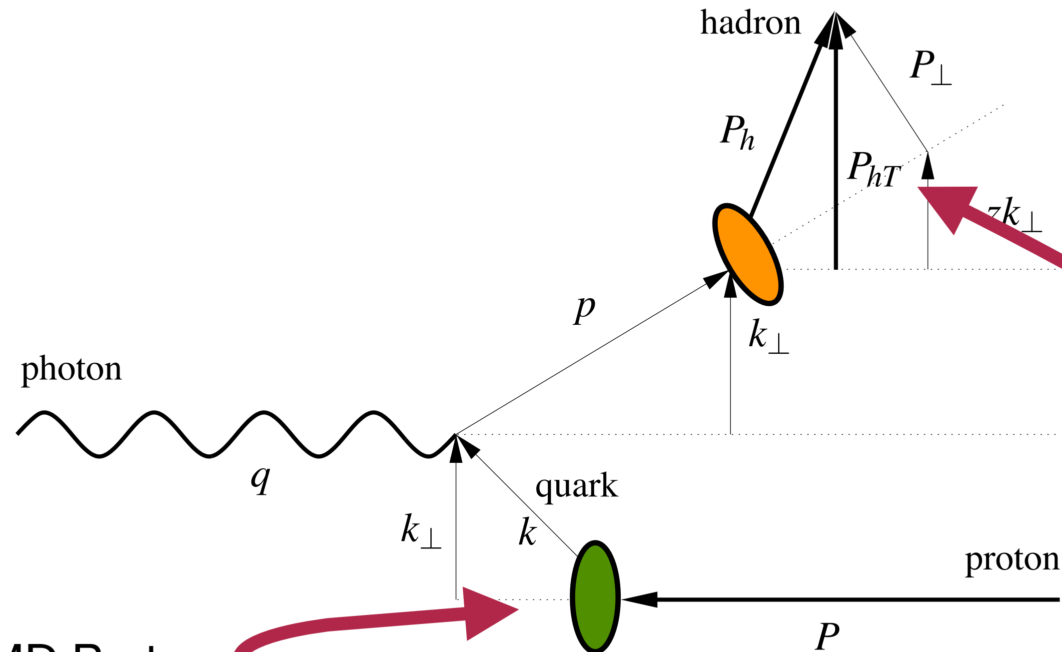
probably high-x effect, twist-3, "bad" prefactor

GENERAL CONSIDERATIONS

- Some functions won't be easily accessible at EIC and JLab might be a unique opportunity to look at them
- Even for the functions that might be accessible at the EIC, if we don't measure them BEFORE the EIC starts, most probably will be neglected at the EIC (lack of expertise, lower priority…)

DETAILS ABOUT TMD ANALYSIS

TMDS IN SEMI-INCLUSIVE DIS



TMD Parton
Distribution Functions

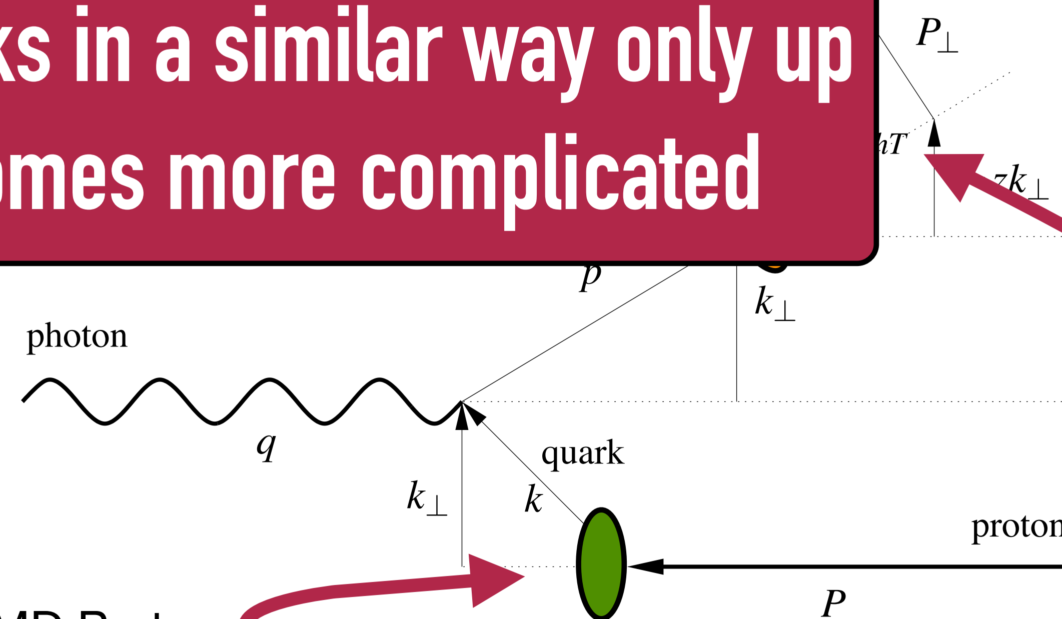
TMD Parton
Fragmentation Functions

$$
\begin{aligned}
&F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) \\
&= x\sum_q \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int d^2\boldsymbol{k}_\perp\, d^2\boldsymbol{P}_\perp\, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2)\, D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;\mu^2)\,\delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) \\
&= x\sum_a \mathcal{H}^q_{UU,T}(Q^2,\mu^2)\int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|)\hat{f}_1^q(x,z^2b_\perp^2;\mu^2)\,\hat{D}_1^{a\to h}(z,b_\perp^2;\mu^2) \\
&\qquad + Y_{UU,T}(Q^2,\boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)
\end{aligned}
$$

hard factor

TMDS IN SEMI-INCLUSIVE DIS

Subleading twist works in a similar way only up to NLL, then it becomes more complicated



TMD Parton Distribution Functions

TMD Parton Fragmentation Functions

$$
\begin{aligned}
F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)
&= x \sum_q \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) \\
&= x \sum_a \mathcal{H}^q_{UU,T}(Q^2, \mu^2) \int db_T b_T J_0(b_T|\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2) \\
&\quad + Y_{UU,T}(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)
\end{aligned}
$$

hard factor

TMD STRUCTURE

$$\hat{f}_1^a(x, |\boldsymbol{b}_T|; \mu, \zeta) = \int d^2\boldsymbol{k}_\perp \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu, \zeta)$$

perturbative Sudakov form factor

$$\hat{f}_1^a(x, b_T^2; \mu_f, \zeta_f) = [C \otimes f_1](x, \mu_{b_*}) \; e^{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left(\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_f}}{\mu}\right)} \left(\frac{\sqrt{\zeta_f}}{\mu_{b_*}}\right)^{K_{\rm resum} + g_K} f_{1\,NP}(x, b_T^2; \zeta_f, Q_0)$$

$$\mu_b = \frac{2e^{-\gamma_E}}{b_*}$$

matching coefficients (perturbative)

collinear PDF

Collins-Soper kernel (perturbative and nonperturbative)

nonperturbative part of TMD

see, e.g.,
Collins, "Foundations of Perturbative QCD" (11)

TMD GLOBAL FITS

	Accuracy	HERMES	COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✔	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✔	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✔	2031	1.06

MAP22 FUNCTIONAL FORM

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_T^2}{g1}} + \lambda^2 k_T^2 e^{-\frac{k_T^2}{g1B}} + \lambda_2^2 e^{-\frac{k_T^2}{g1C}} \right)$$

$$g_1(x) = N_1 \, \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

$$g_K(b_T^2) = -\frac{g_2^2}{2} b_T^2$$

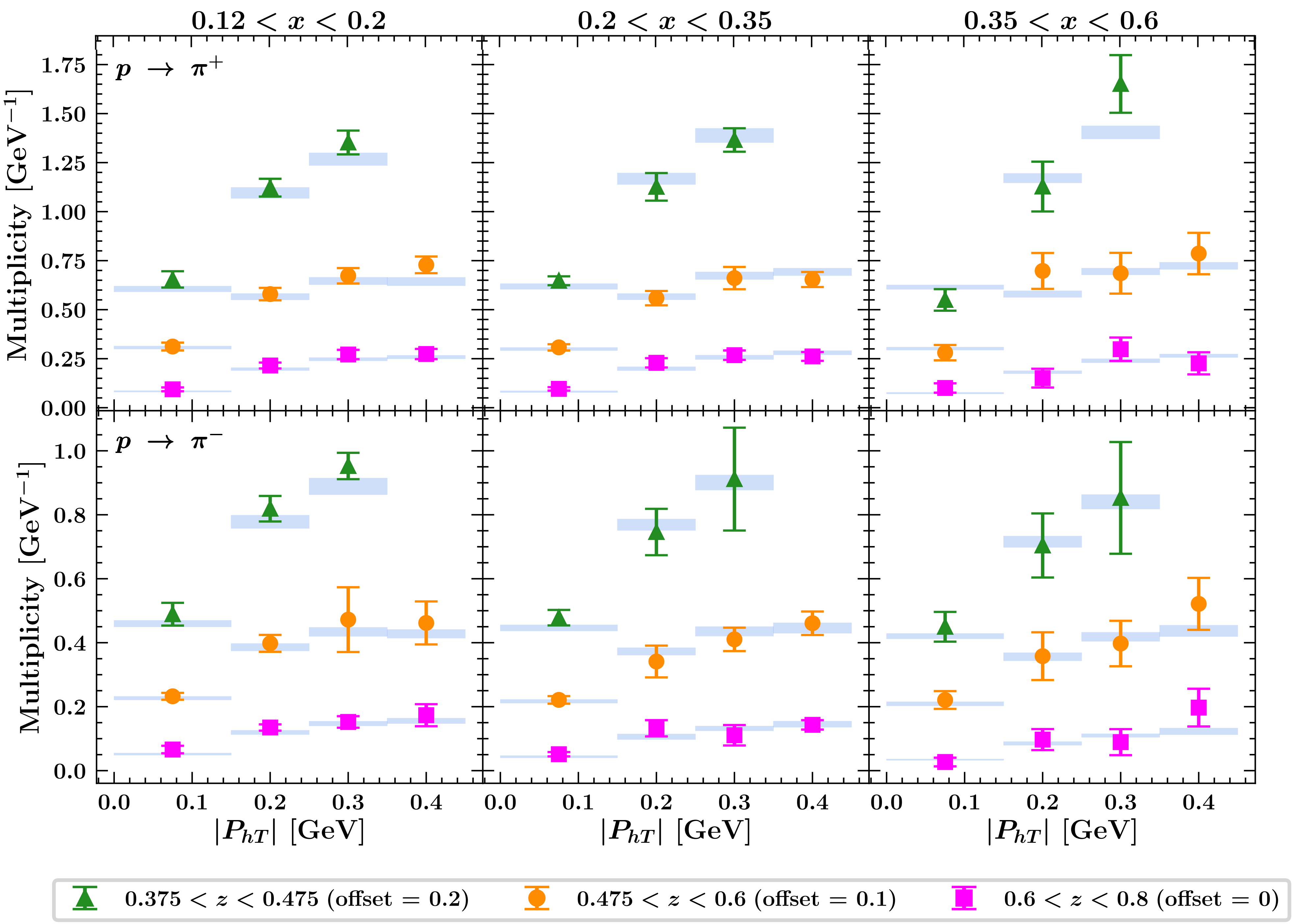
11 parameters for TMD PDF
\+ 1 for NP evolution +9 for FF
= 21 free parameters

MAP22 GOODNESS OF FIT

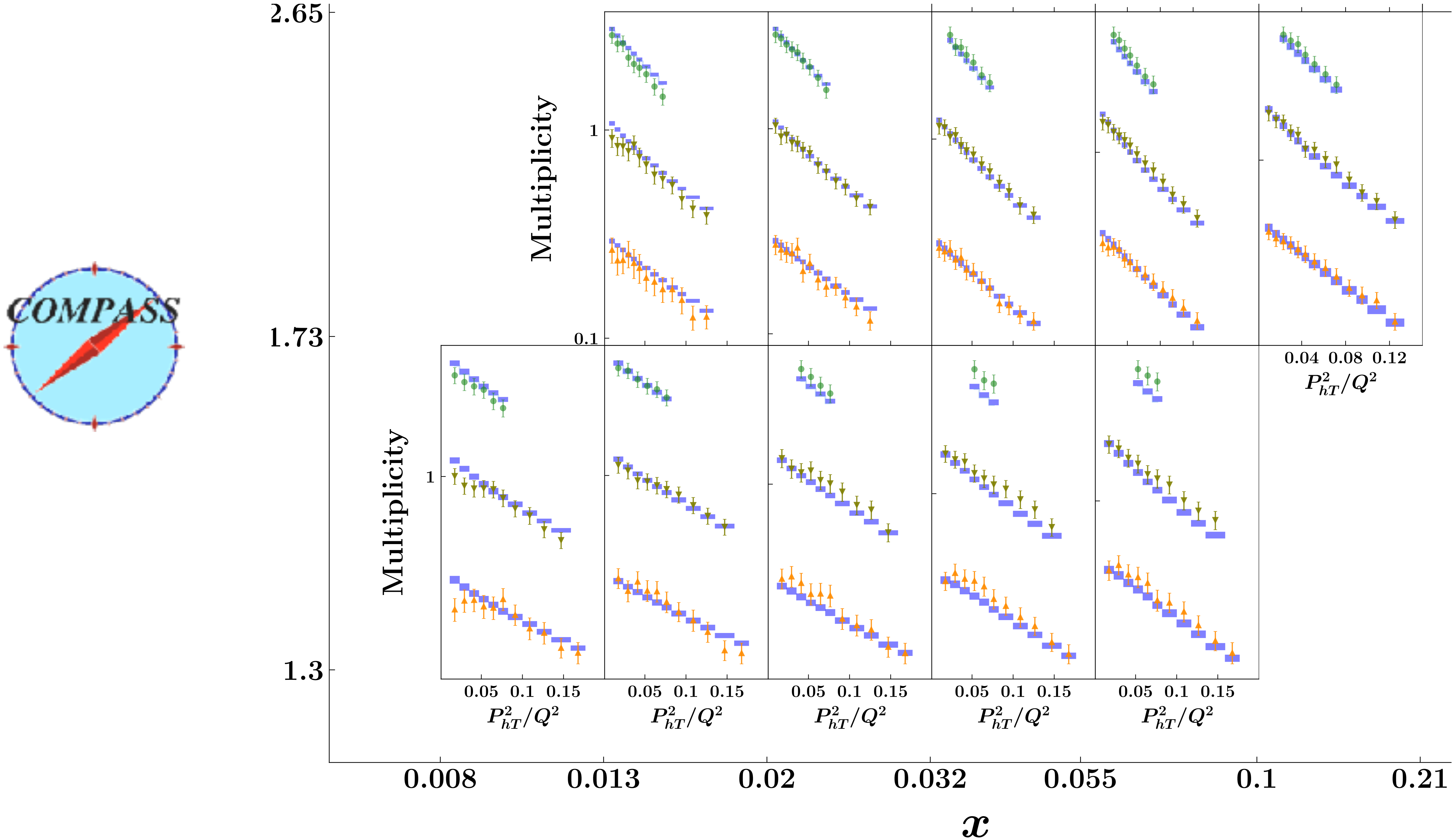
Data set	N_{dat}	$\chi_D^2/N_{\mathrm{dat}}$	$\chi_\lambda^2/N_{\mathrm{dat}}$	$\chi_0^2/N_{\mathrm{dat}}$
Tevatron total	71	0.87	0.06	0.93
LHCb total	21	1.15	0.3	1.45
ATLAS total	72	4.56	0.48	5.05
CMS total	78	0.53	0.02	0.55
PHENIX 200	2	2.21	0.88	3.08
STAR 510	7	1.05	0.10	1.15
DY collider total	251			
DY fixed-target total	233			
HERMES total	344	0.48	0.23	0.71
COMPASS total	1203	0.62	0.3	0.92
SIDIS total	1547	0.59	0.28	0.87
Total	**2031**	**0.77**	**0.29**	**1.06**

The SIDIS errors should be reduced

EXAMPLE OF AGREEMENT WITH DATA: HERMES



EXAMPLE OF AGREEMENT WITH DATA: SIDIS



RESULTING TMDS

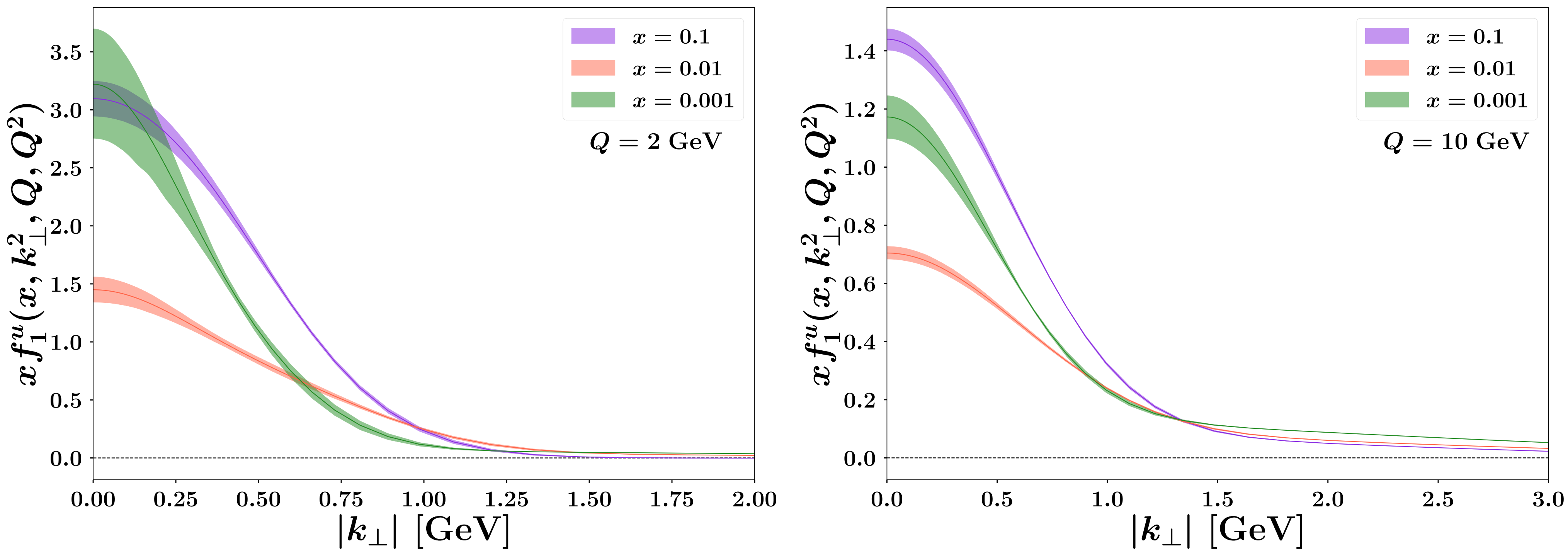
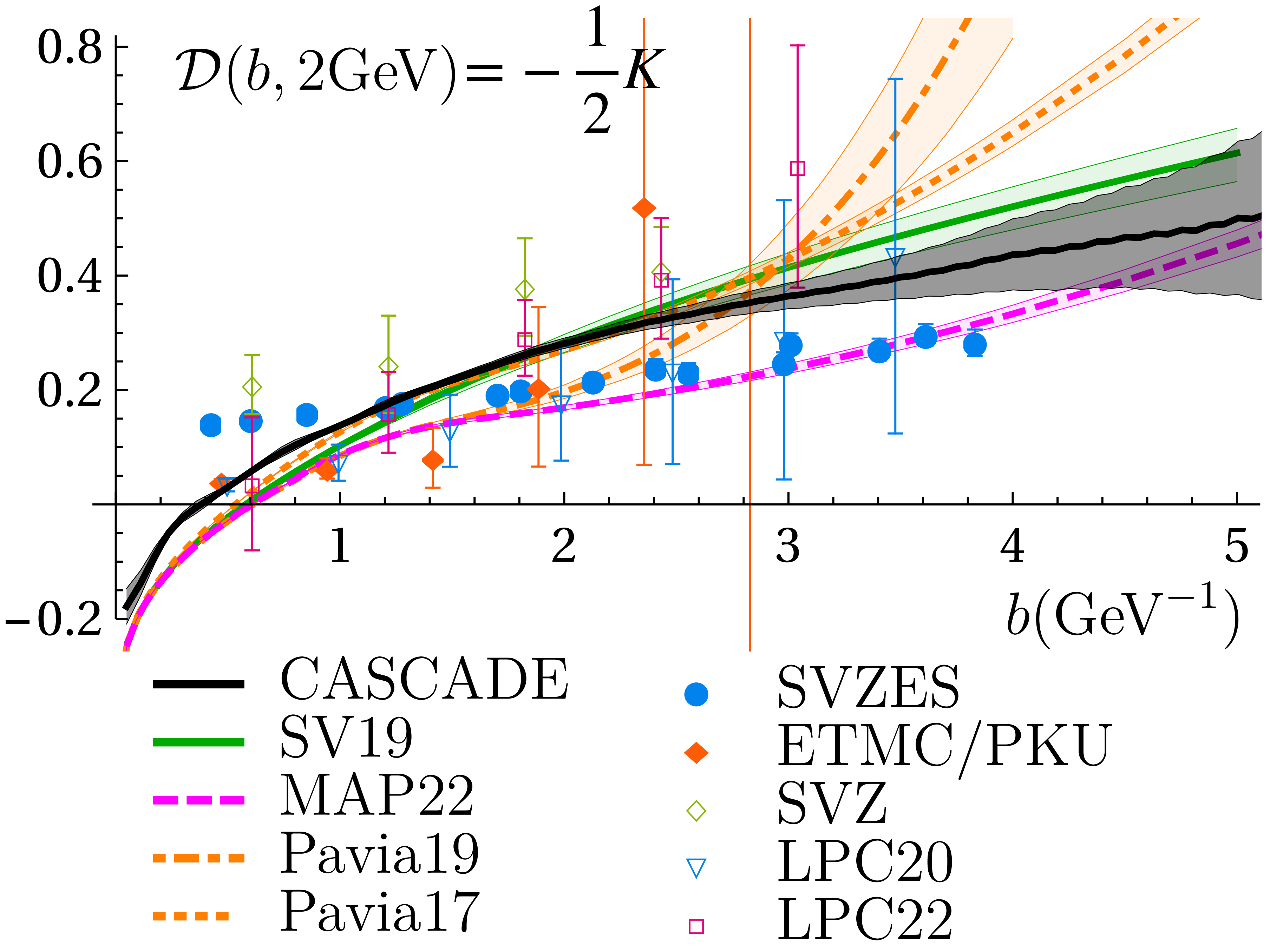


FIG. 13: The TMD PDF of the up quark in a proton at $\mu = \sqrt{\zeta} = Q = 2$ GeV (left panel) and 10 GeV (right panel) as a function of the partonic transverse momentum $|\boldsymbol{k}_\perp|$ for $x = 0.001$, 0.01 and 0.1. The uncertainty bands represent the 68% CL.

As usual, the rigidity of the functional form plays a role and probably leads to underestimated bands

RESULTING COLLINS-SOPER KERNEL

Bermudez Martinez, Vladimirov, arXiv:2206.01105



AVAILABLE TOOLS: NANGA PARBAT

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

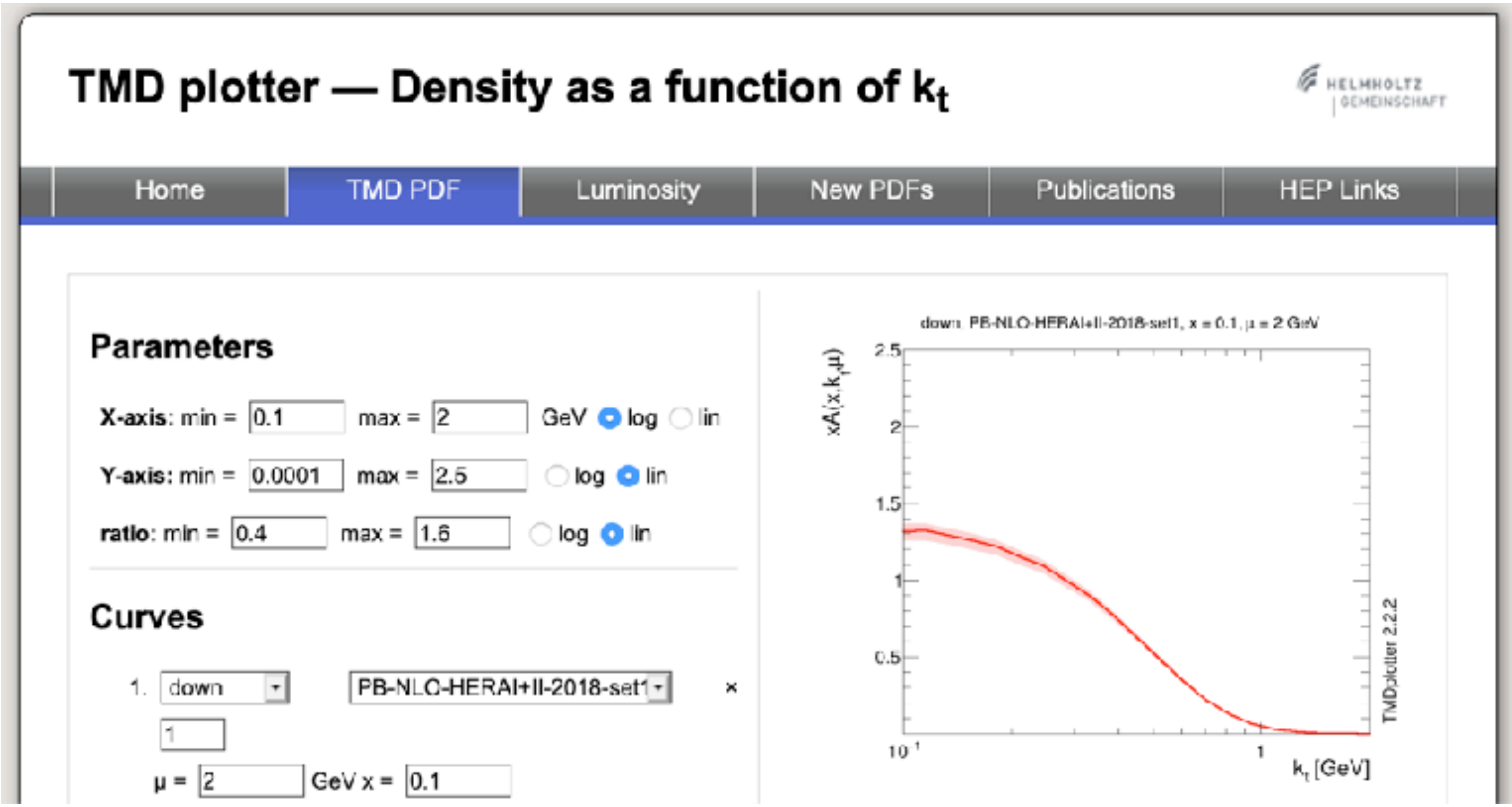
https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

AVAILABLE TOOLS: TMDLIB AND TMDPLOTTER

https://tmdlib.hepforge.org/



MAP22 TMDs soon available

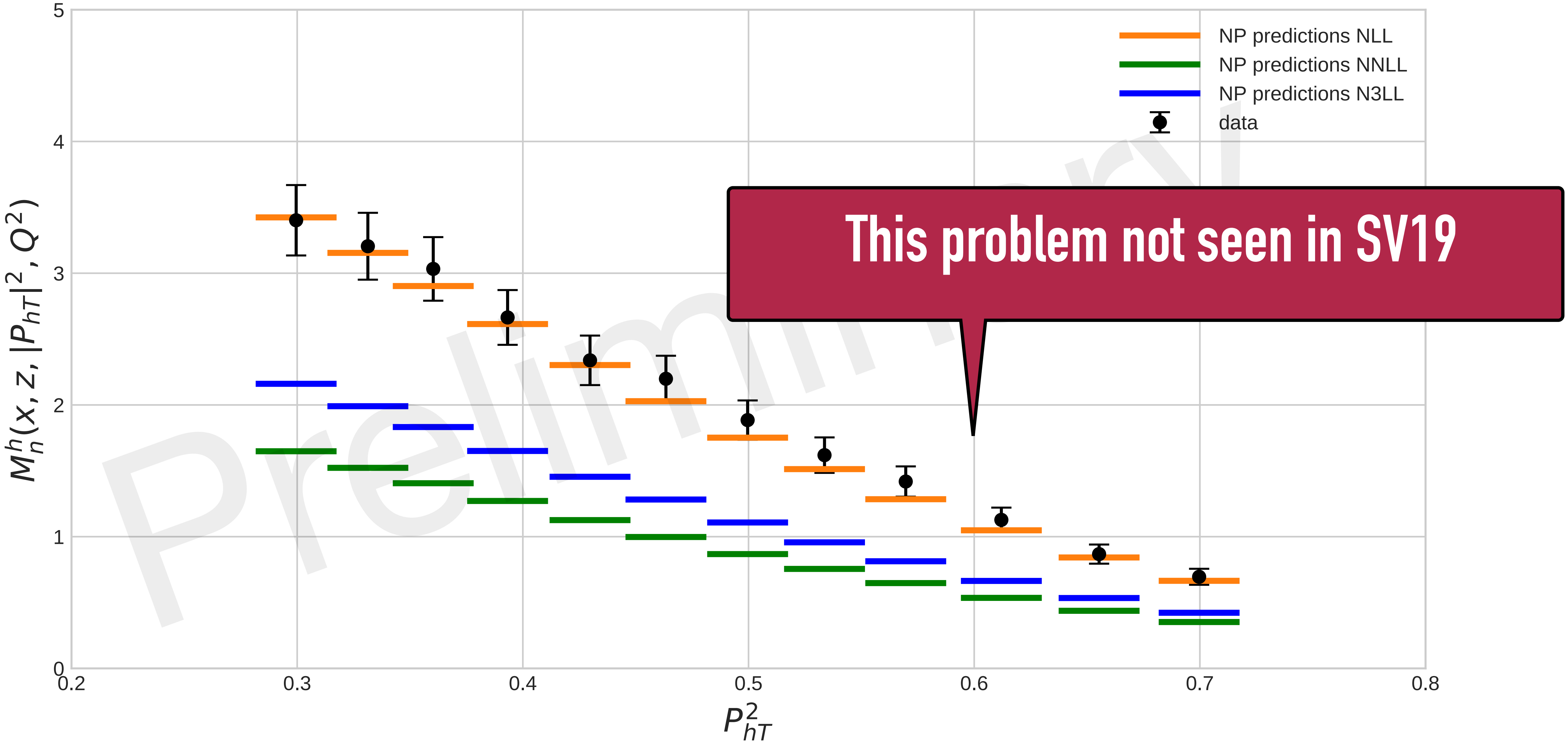
WARNING

PROCEED WITH CAUTION

OPEN PROBLEM:
SIDIS NORMALIZATION

COMPARISON OF DIFFERENT ORDER SIDIS

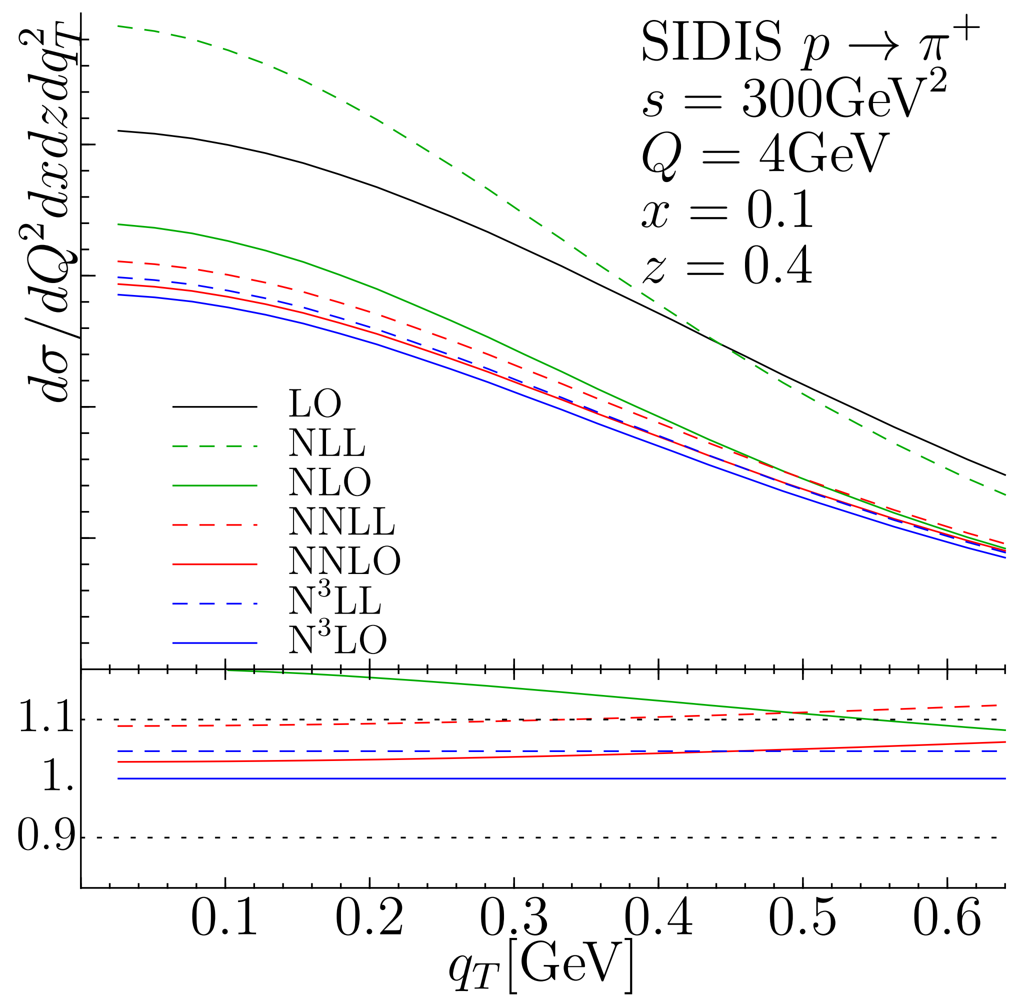
COMPASS multiplicities (one of many bins)



The description considerably worsens at higher orders

COMPARISON OF DIFFERENT ORDERS

Scimemi, Vladimirov, arXiv:1912.06532



We agree that the predictions beyond NLL are significantly smaller, but according to SV the lowest orders should overestimate data

MAP22 TENTATIVE SOLUTION

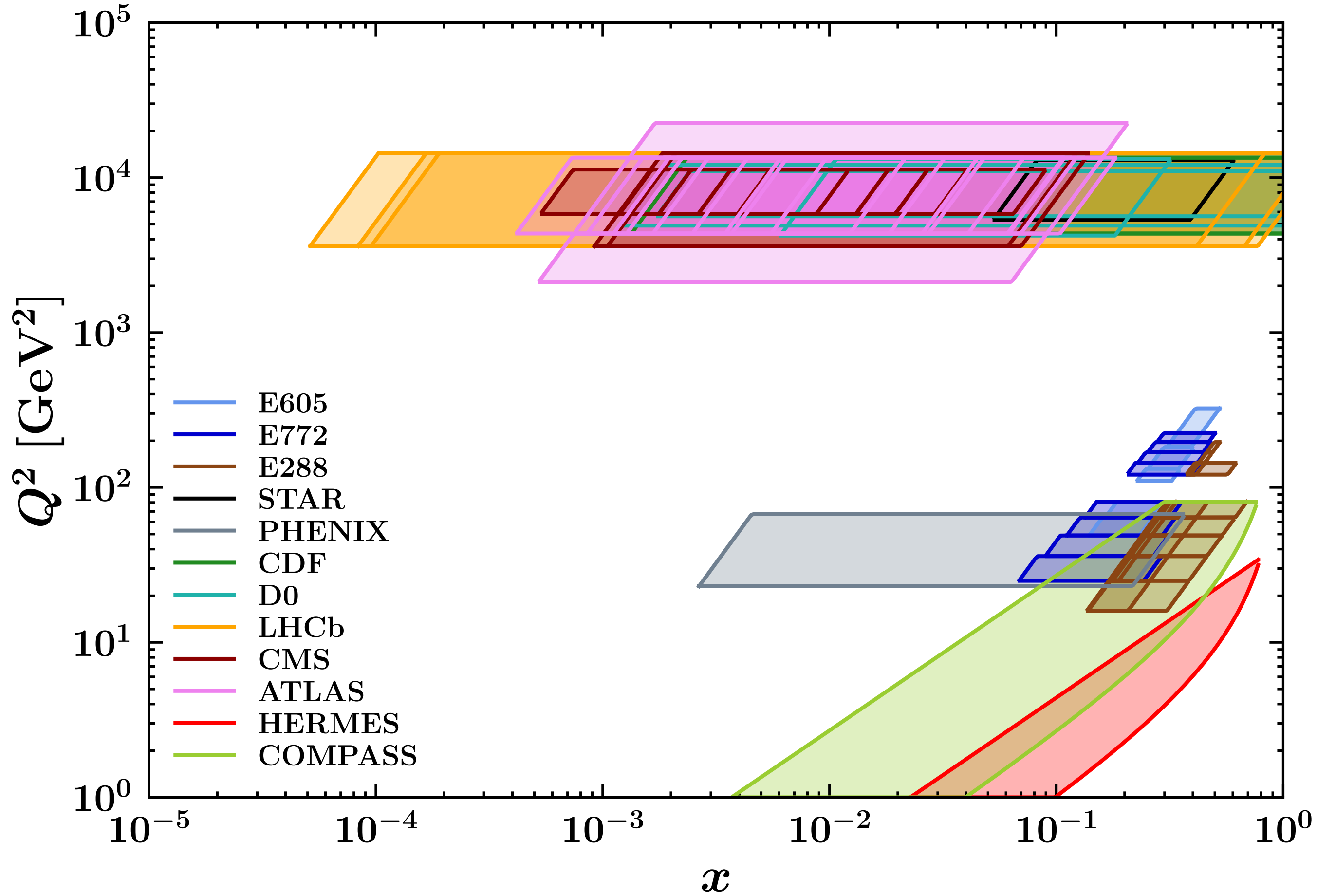
ENHANCEMENT PREFACTOR $= \dfrac{\left.\dfrac{d\sigma}{dx\,dz\,dQ^2}\right|_{\text{nonmix.}}}{\int \text{TMD}\; d^2 P_{hT}}$

The prefactor is independent of the fitting parameters

Higher-order corrections decrease the role of the TMD region.
We need to enhance it with a prefactor.

OPEN PROBLEM:
TMD REGION IN SIDIS

MAP22 TMD DATA SELECTION



$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$q_T < 0.2\,Q \quad \text{(DY)}$

$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\mathrm{GeV}, zQ\right] \quad \text{(SIDIS)}$

Number of points: 2031

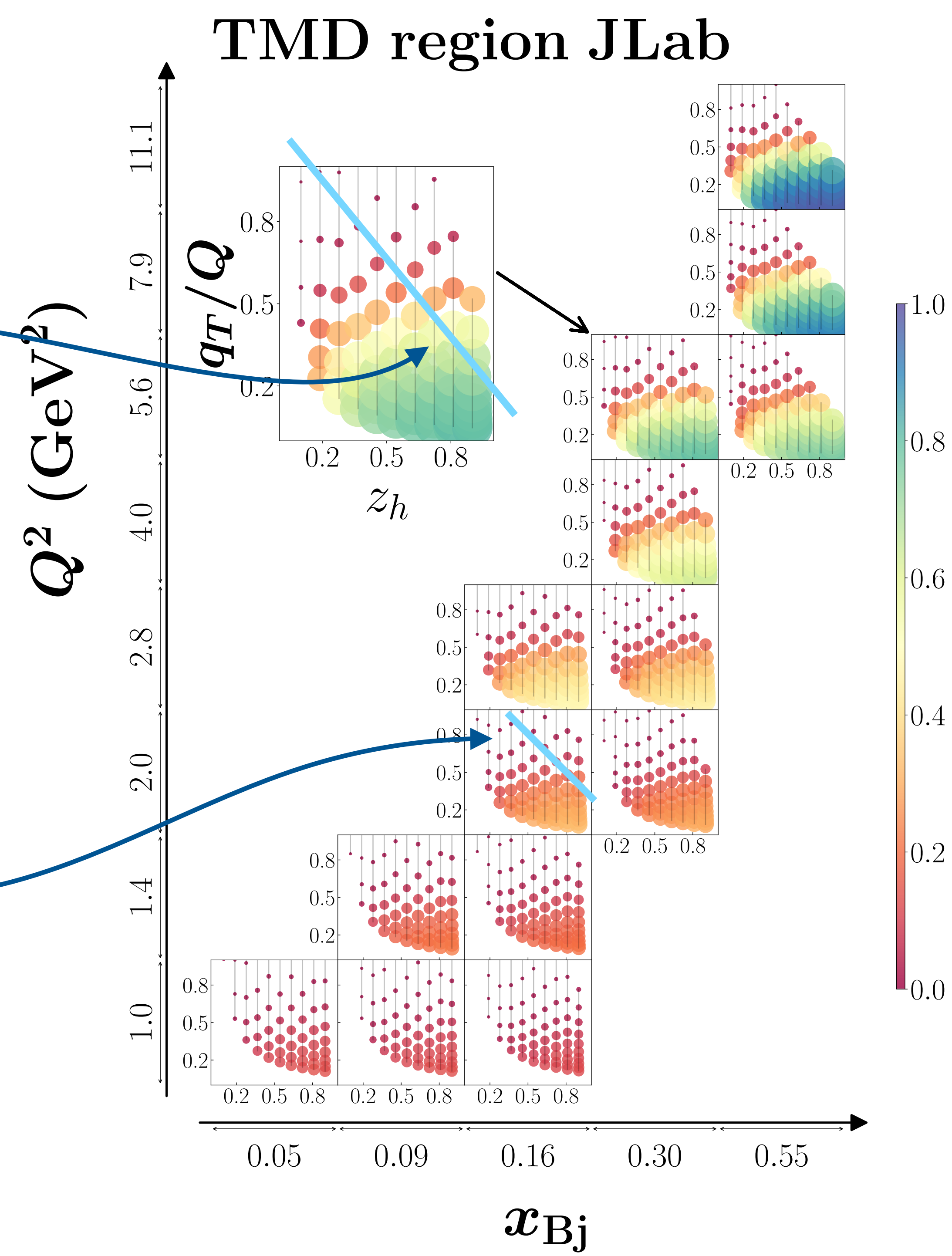
REGION OF VALIDITY OF TMD FORMALISM

Boglione, Diefenthaler, Dolan, Gamberg, Melnitchouk, arXiv:2201.12197

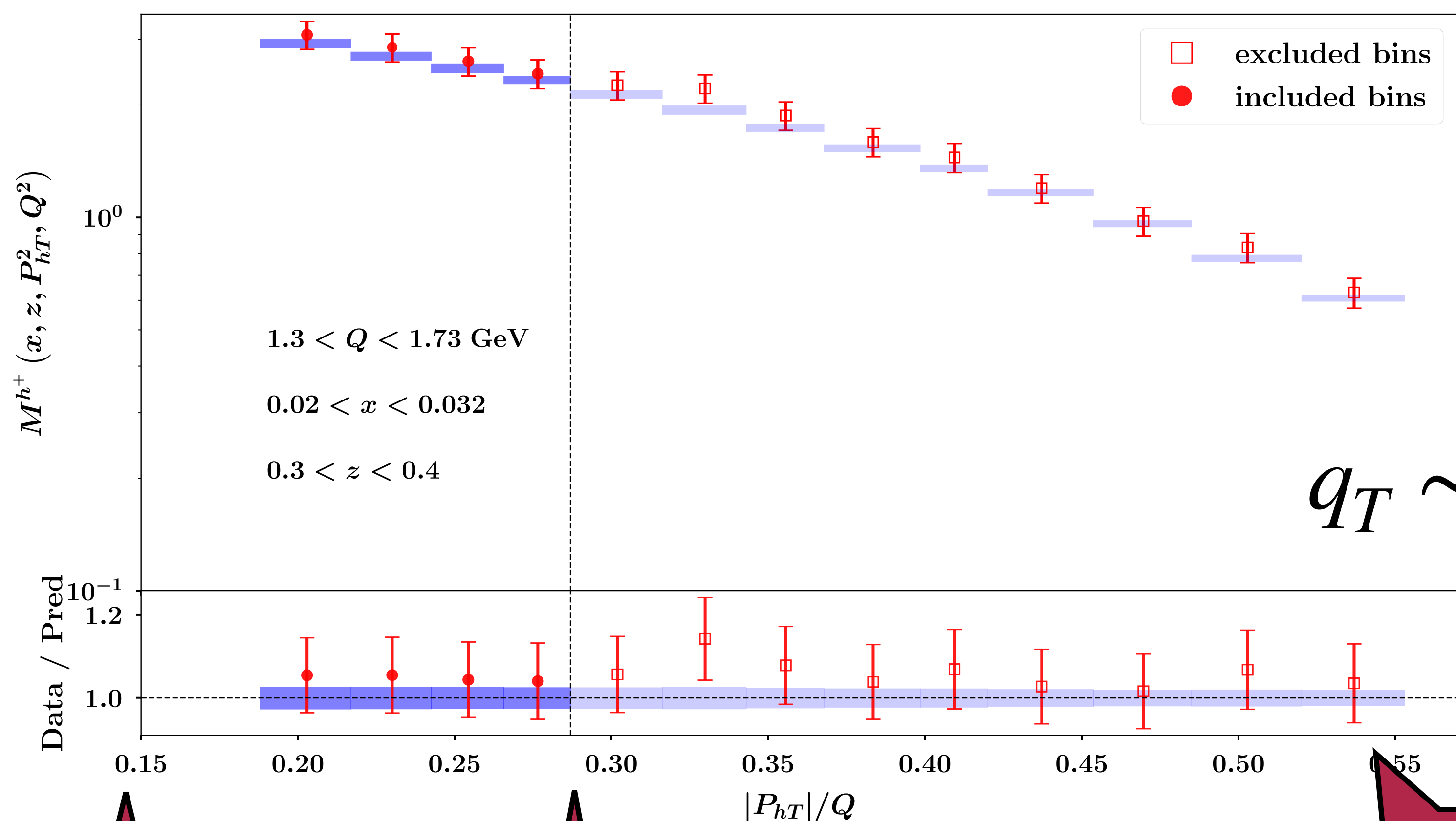
$$|q_T| = |P_{hT}|/z \ll Q$$

?

Approximate region corresponding to MAP22 cuts



REGION OF VALIDITY OF TMD FORMALISM



$q_T \sim 1.5\ Q$!

$q_T < 0.2\,Q$

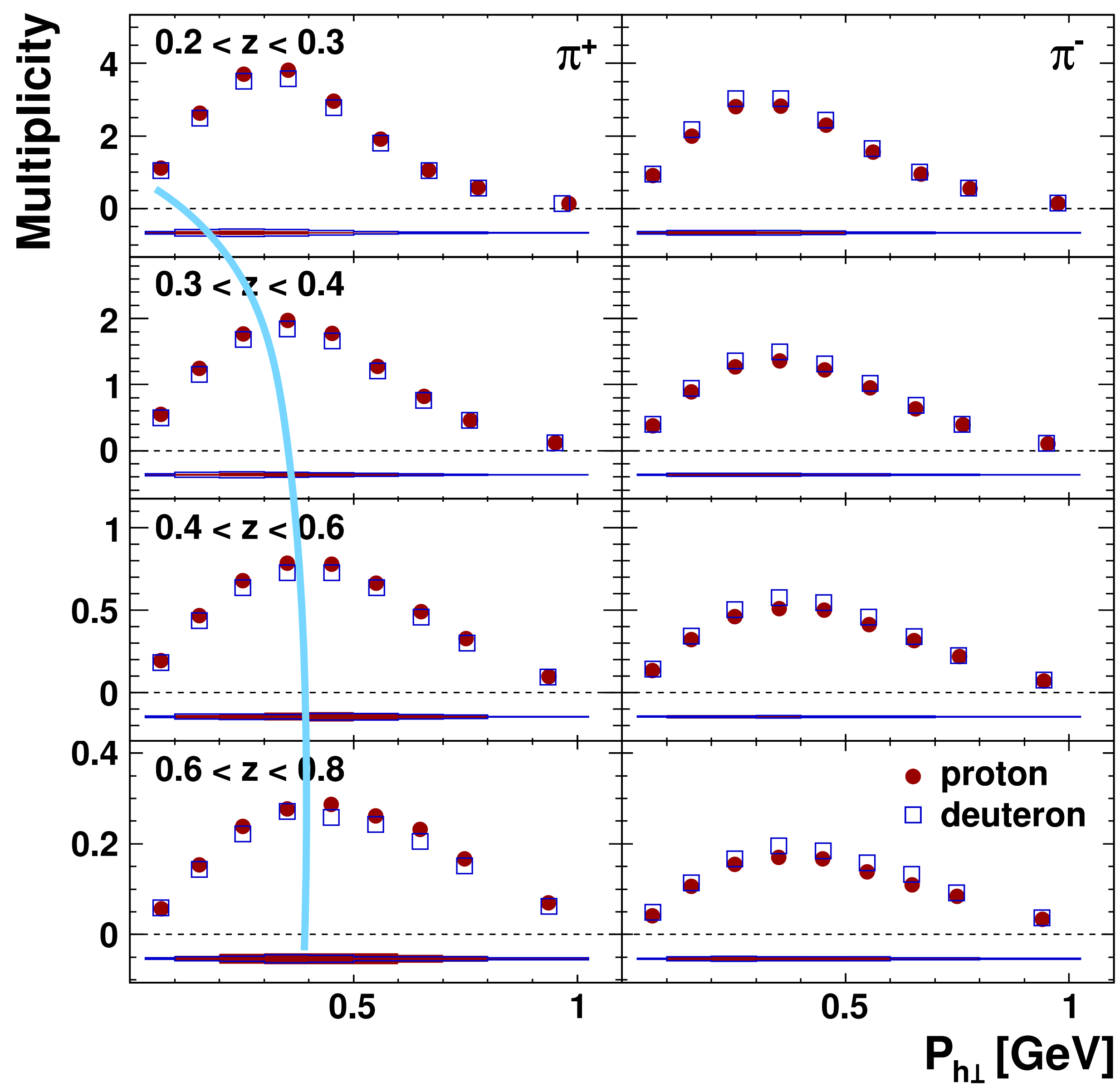
MAP22 cut

MAP22 extrapolation

The MAP22 cut is already considered to be “generous”, but the physics seems to be the same for a much wider P_T

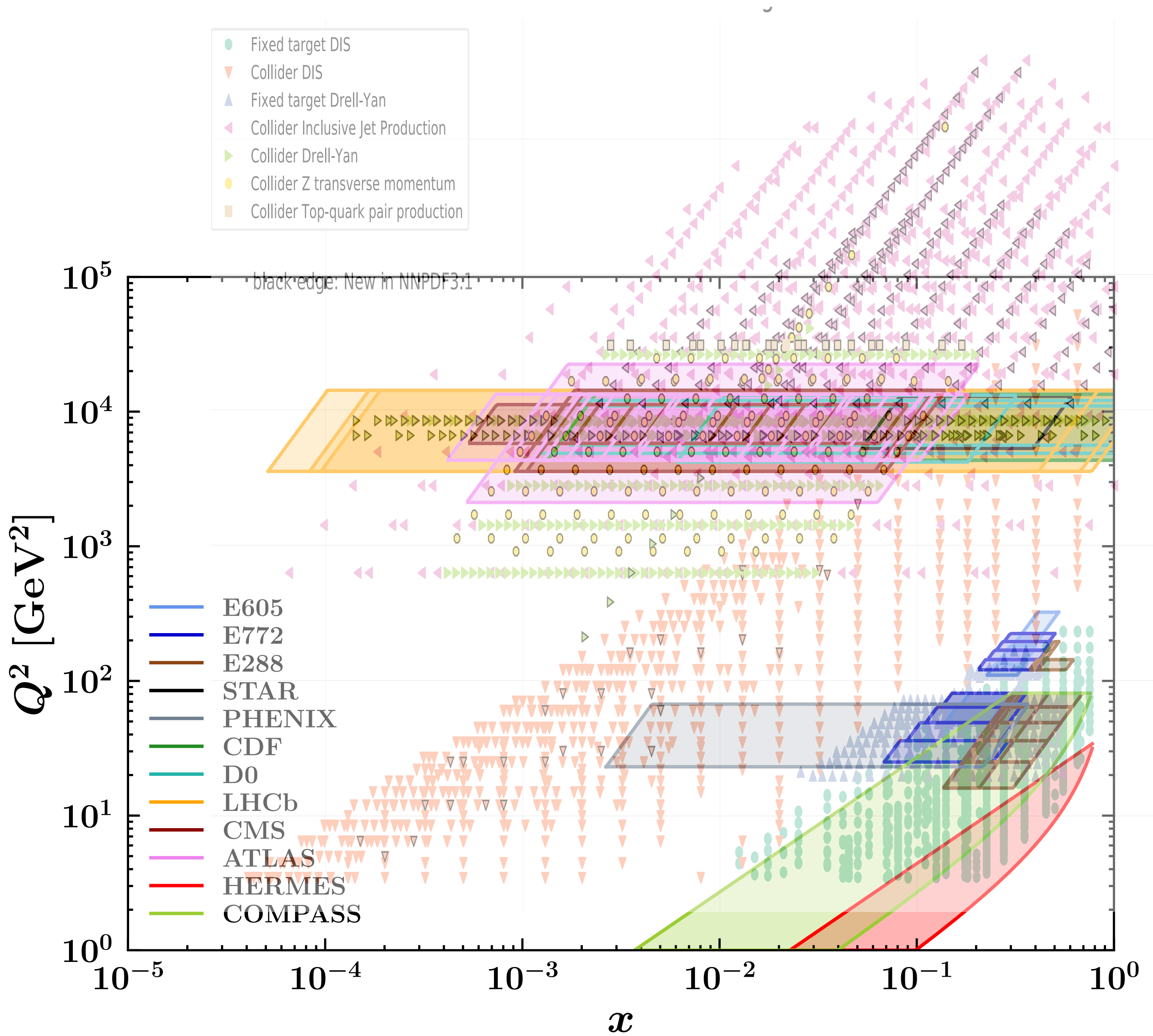
REGION OF VALIDITY OF TMD FORMALISM

With our cuts, we use about 45 points per channel.
With the cuts of SV, $\langle q_T \rangle < 0.25 \langle Q \rangle$, they reduce to 24



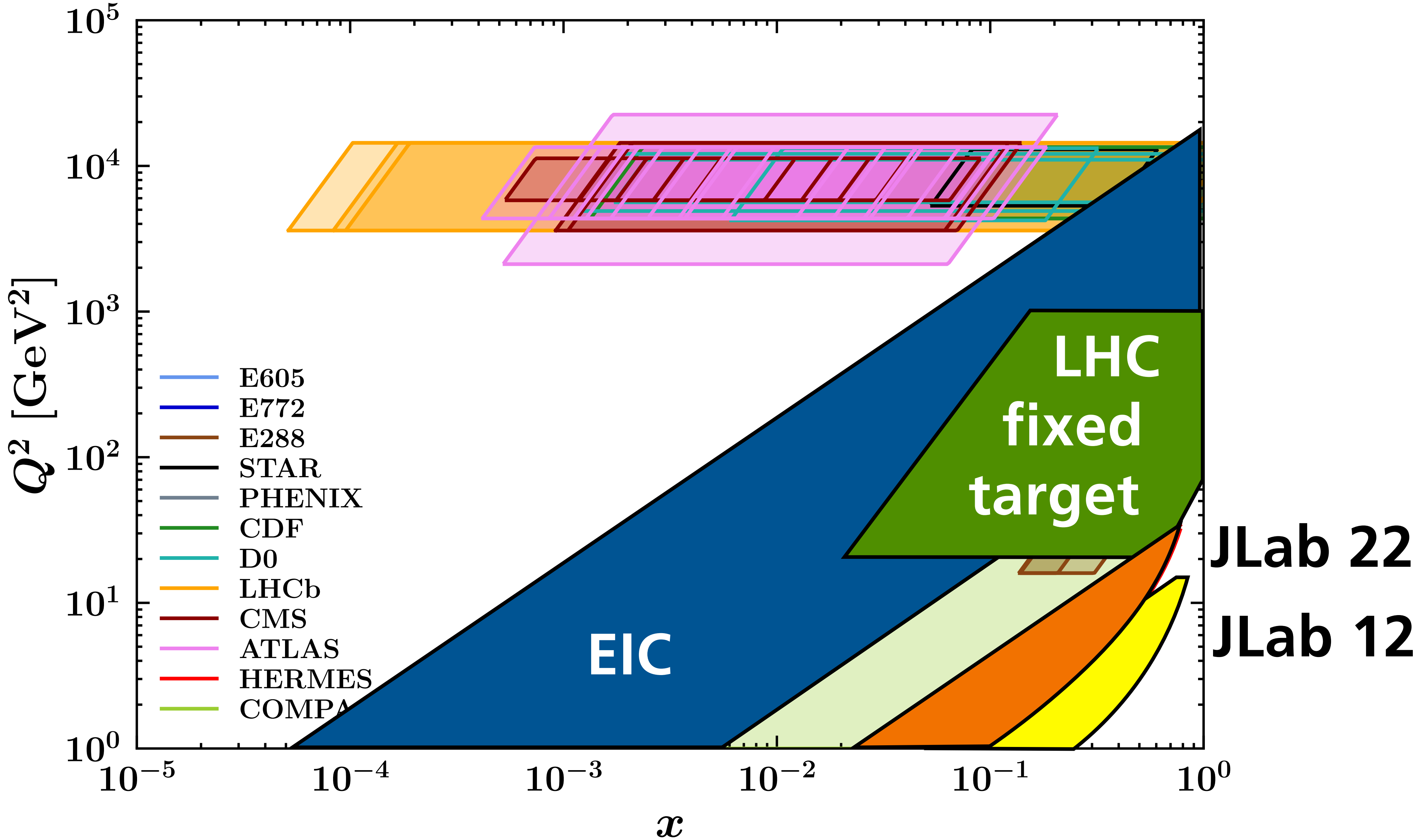
Personal opinion: regions where nonperturbative TMD components are dominating must be included in the TMD description

x–Q^2 COVERAGE

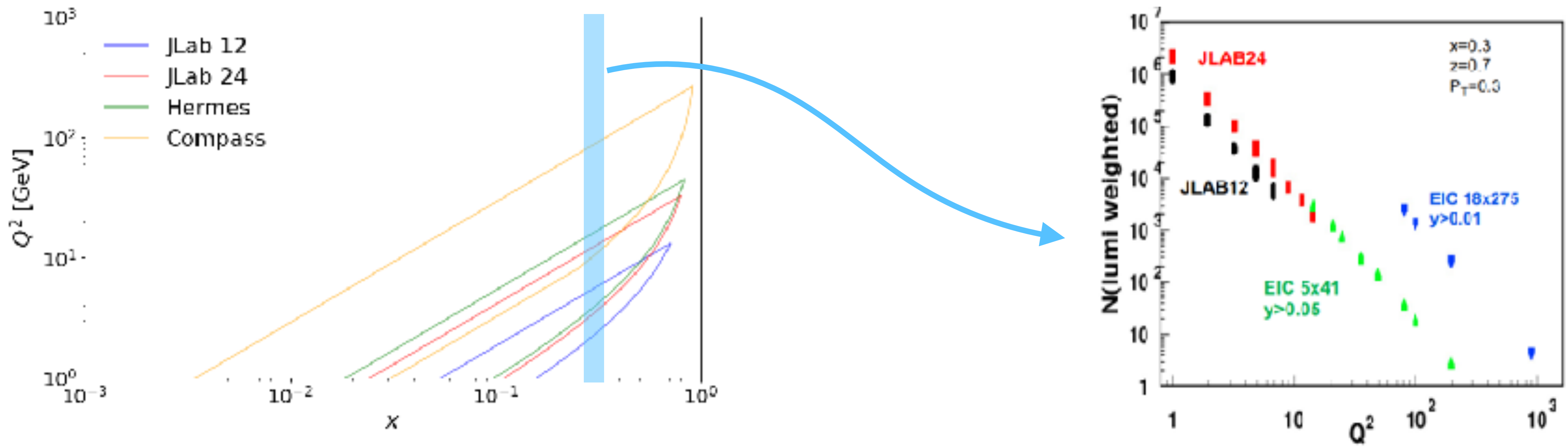


MAP Collaboration
Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti,
Piacenza, Radici, Signori, arXiv:2206.07598

EXPECTED EXTENSIONS OF DATA RANGE



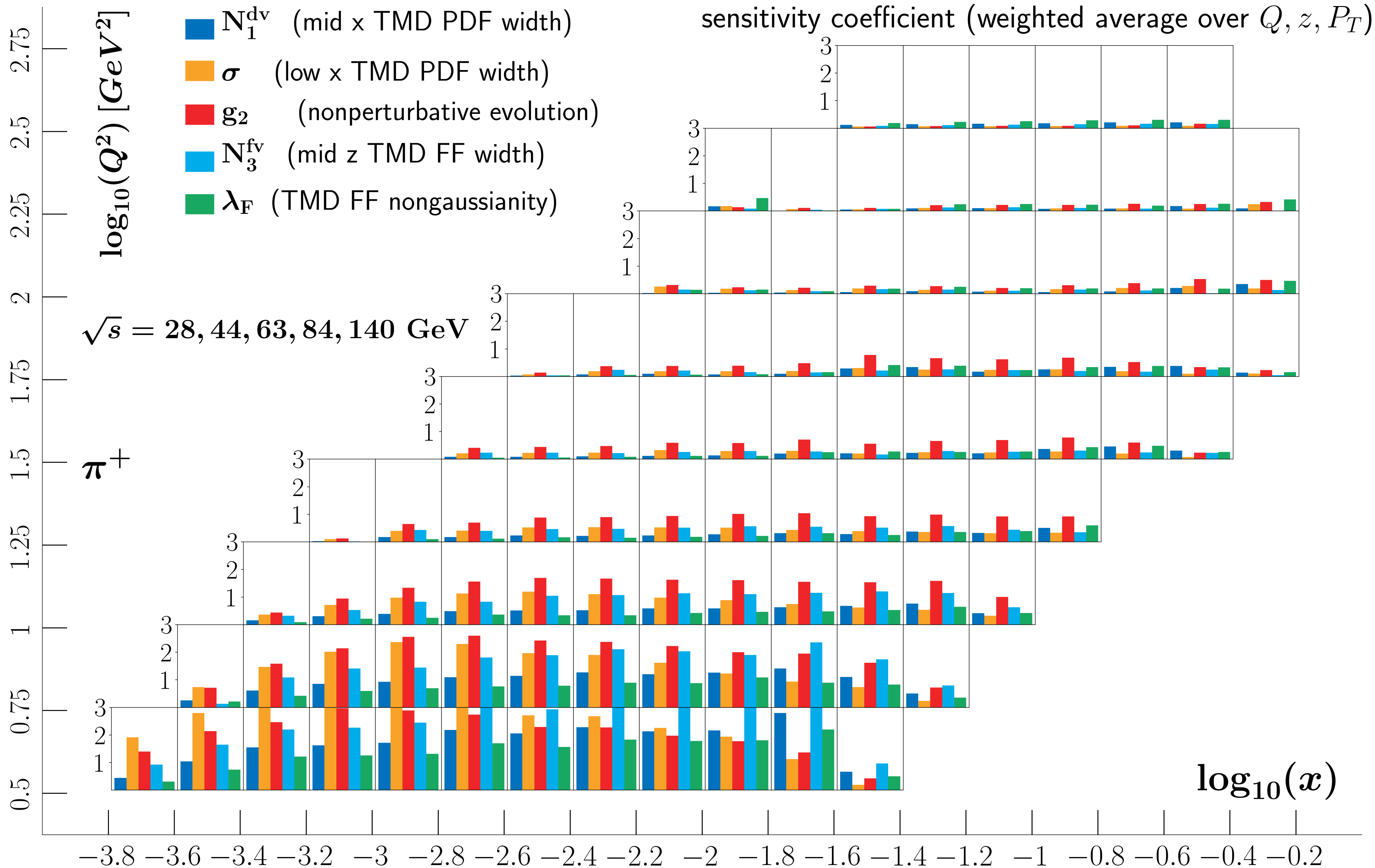
COVERAGE AND LUMINOSITY



Covering a large range but with limited precision may be useless

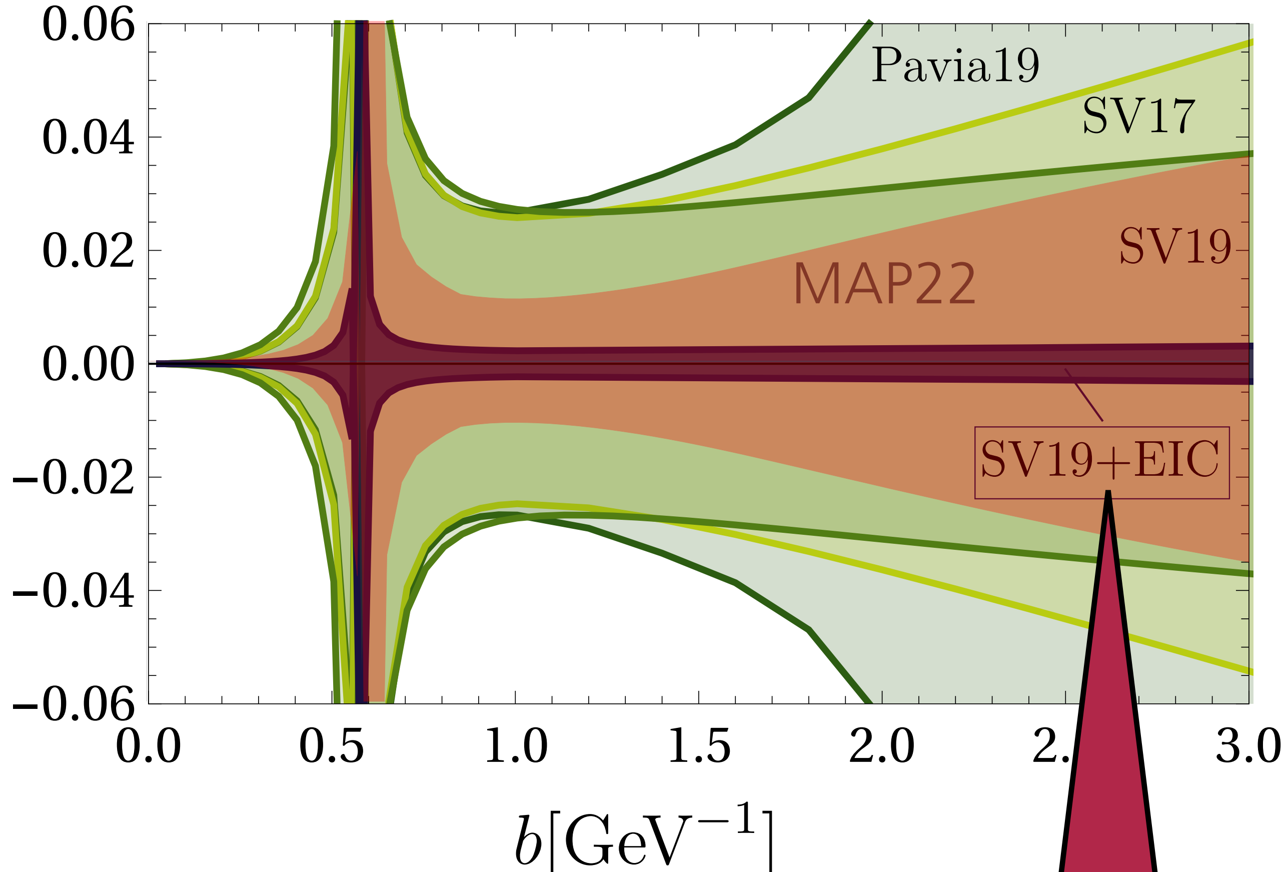
EIC SENSITIVITY COEFFICIENTS

EIC Yellow Report, arXiv:2103.05419



EIC IMPACT: REDUCTION OF ERRORS

EIC Yellow Report, arXiv:2103.05419



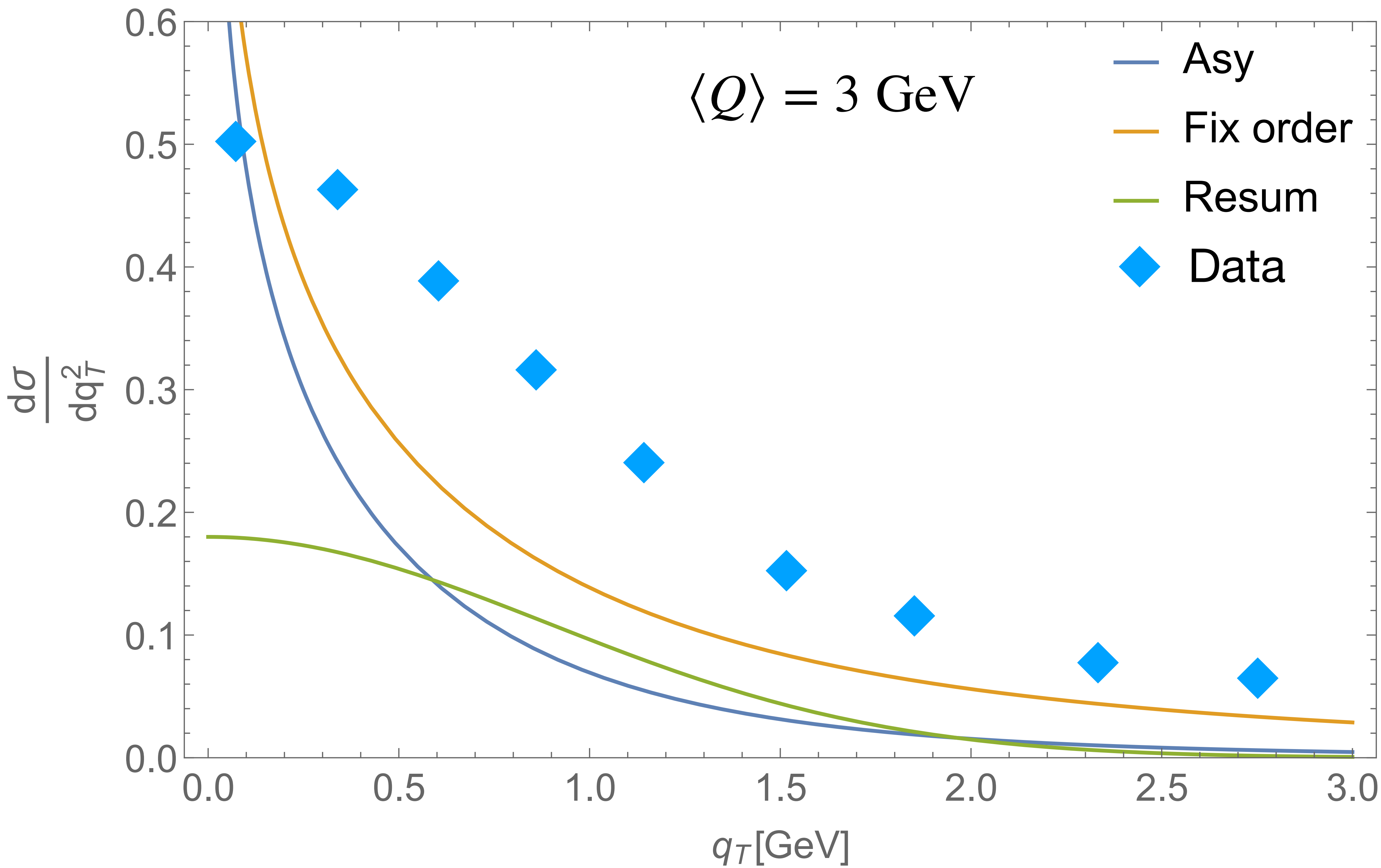
Big impact of EIC on TMD evolution

CONCLUSIONS

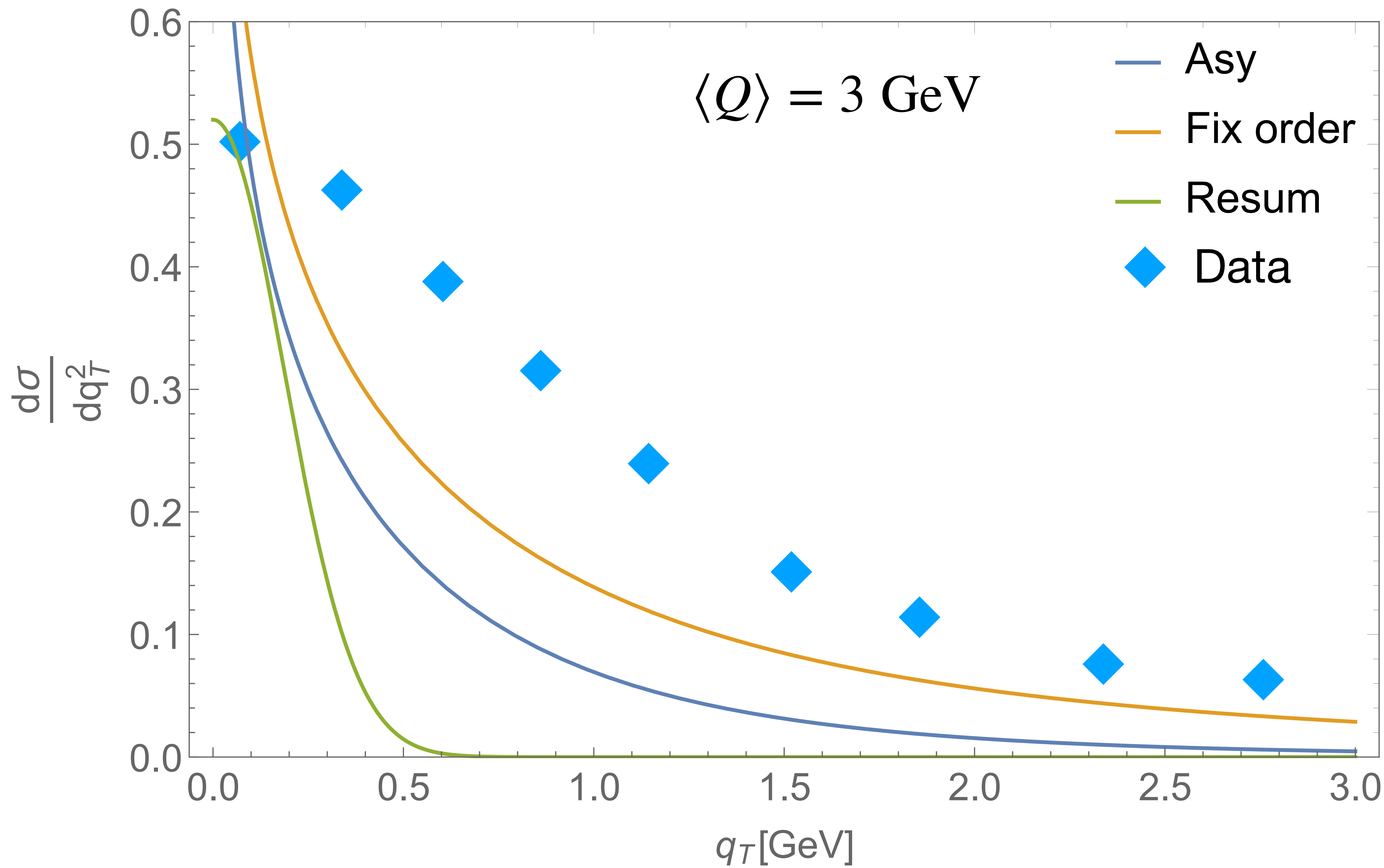
- The formalism works well, but some problems need still to be understood
- Low-Q data are dominated by nonperturbative physics and are invaluable to understand QCD.
- Certain effects (certain structure functions, certain details of TMDs…) may be uniquely accessible at JLab 20+
- Impact studies should be performed to understand the potential of JLab 20+. For unpolarized TMDs, it is feasible. We need to know the foreseen binning and estimated relative errors.

BACKUP SLIDES

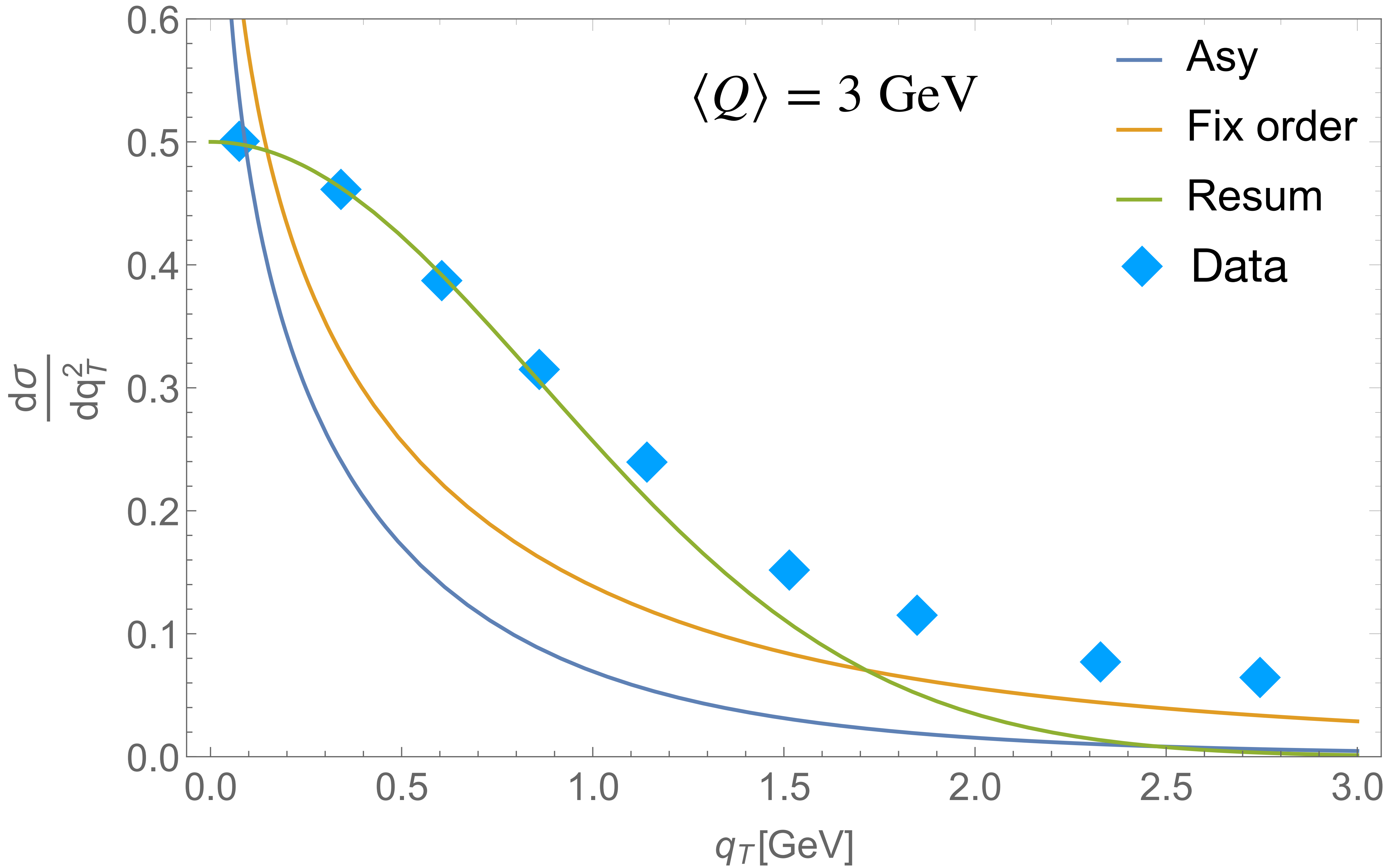
SOME JUSTIFICATION: INITIAL SITUATION



SOLUTION 1: RESTRICT TMD REGION



SOLUTION 2: ENHANCE TMD CONTRIBUTIONS

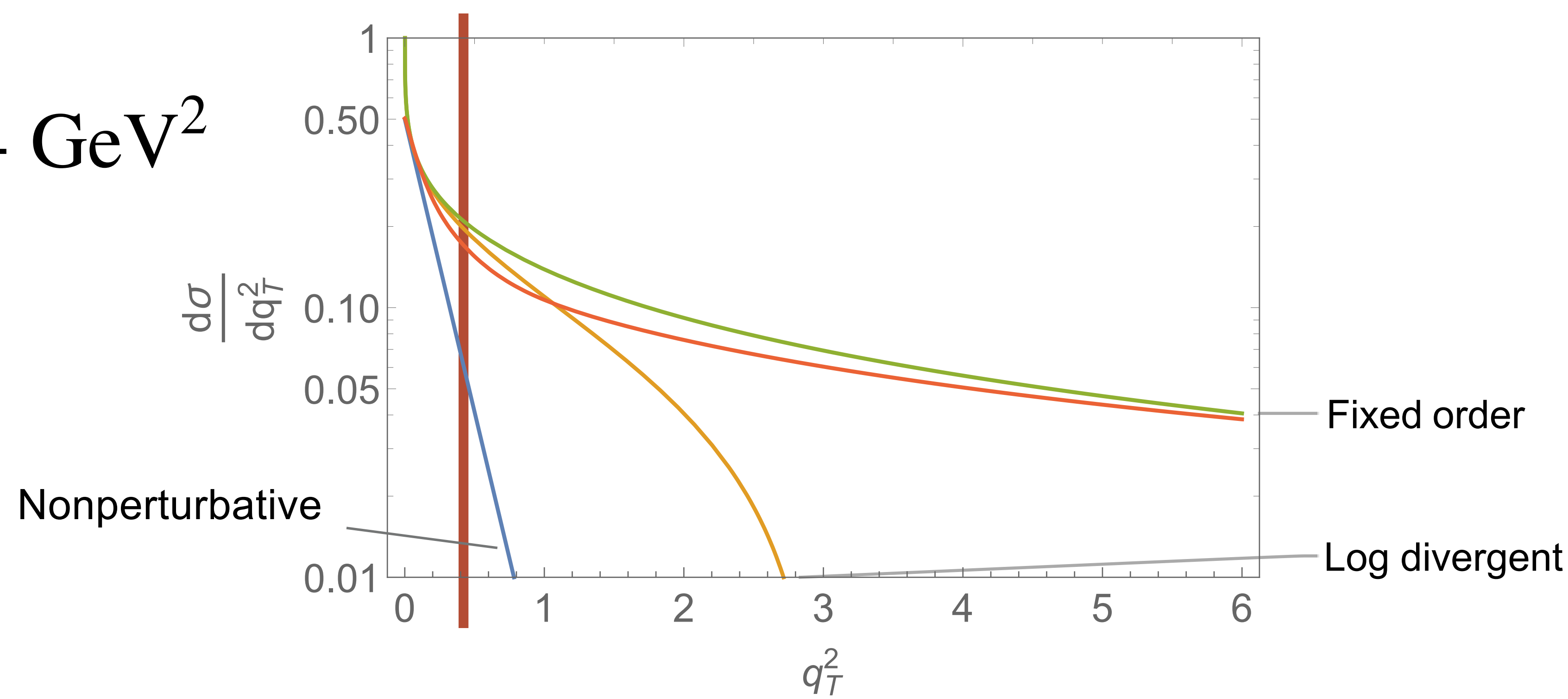


TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

Perturbative approach:
TMD region = where the log divergence of the fixed-order calculation dominates (resummation is required)

Nonperturbative approach:
TMD region = where either the log divergence OR the nonperturbative contributions dominate

$$\langle Q^2 \rangle = 4\ \mathrm{GeV}^2$$



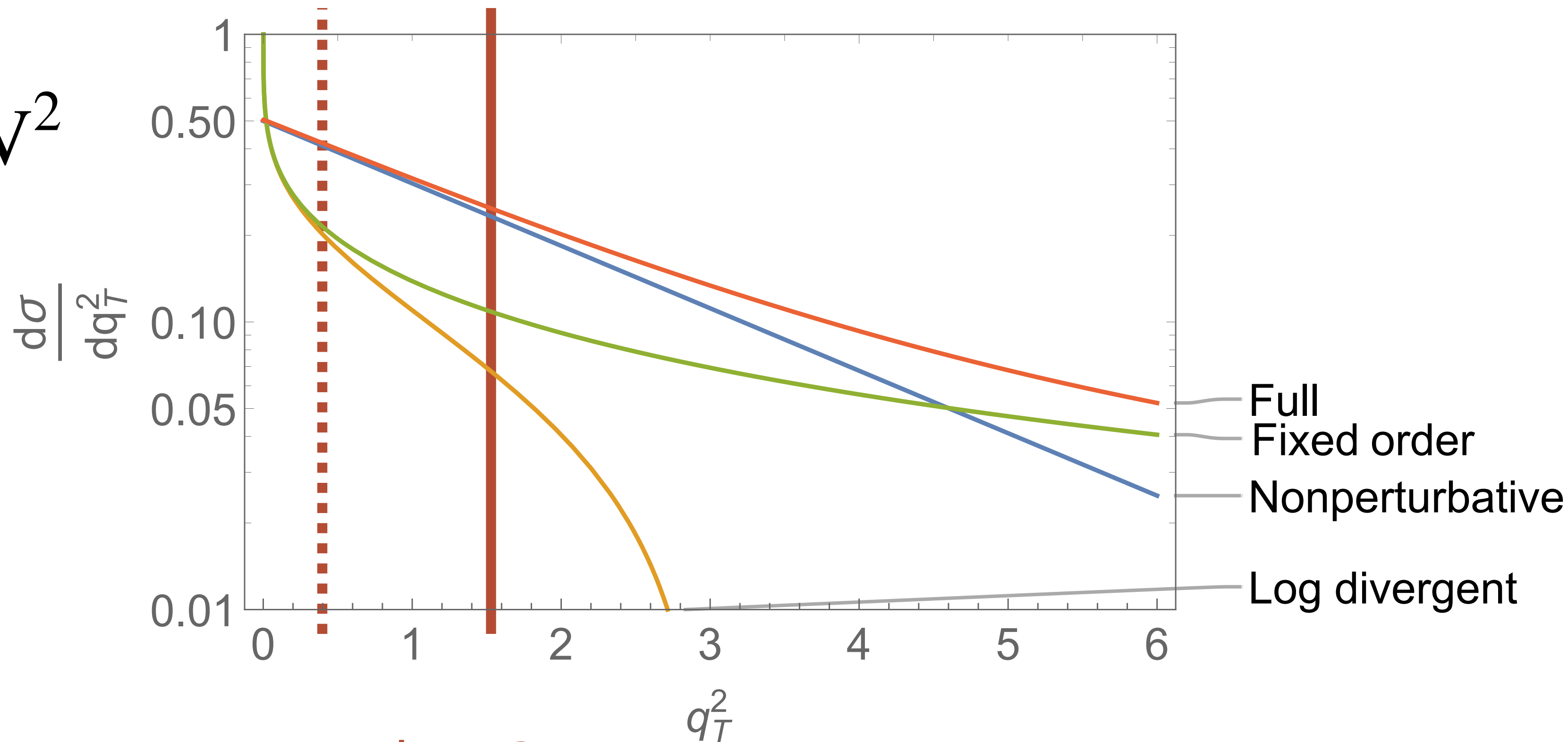
TMD region (ideal situation)

TMD REGIONS: PERTURBATIVE VS. NONPERTURBATIVE

Perturbative approach:
TMD region = where the log divergence of the fixed-order calculation dominates (resummation is required)

Nonperturbative approach:
TMD region = where either the log divergence OR the nonperturbative contributions dominate

$$\langle Q^2 \rangle = 4 \text{ GeV}^2$$



TMD region?