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Christopher E. Thomas,[1,§] and David J. Wilson[1,‖]

decays of an exotic 1^{-+} hybrid meson in QCD

Jozef Dudek

hadspec
hadron spectrum collaboration
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Jefferson Lab
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hybrid mesons

one hypothesis to go beyond the $q\bar{q}$ picture of mesons

– add an **excitation of the gluonic field** $q\bar{q}G$

– can give rise to J^{PC} not allowed for $q\bar{q}$

e.g. $0^{+-}, 1^{-+}, 2^{+-} \ldots$

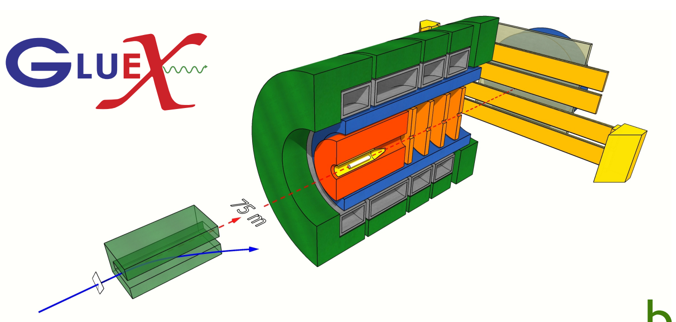
long history of study within QCD-motivated models

– constituent gluon
– bag model
– flux-tube model

⋮

all have exotic J^{PC} mesons, but spectra differ

a strong motivation for the GlueX experiment



but how do they decay ?

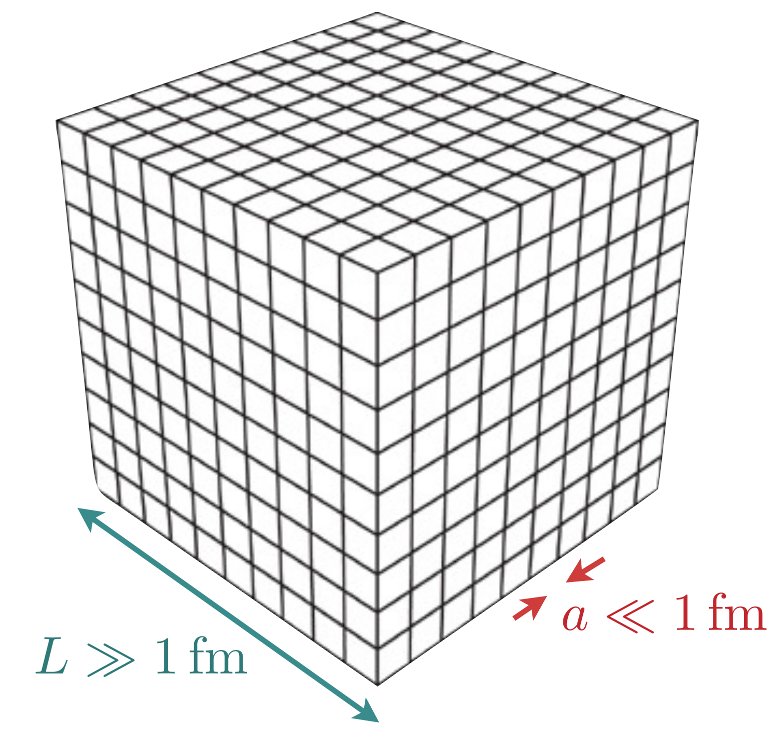
more recently studied in (incomplete) **lattice QCD calculations** …

QCD on a spacetime lattice

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, f(\psi, \bar{\psi}, A_\mu) \, e^{i\int d^4x \, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

sum over quark/gluon field configurations

in Euclidean spacetime, probability for a field configuration



generate field configurations → compute **correlation functions**

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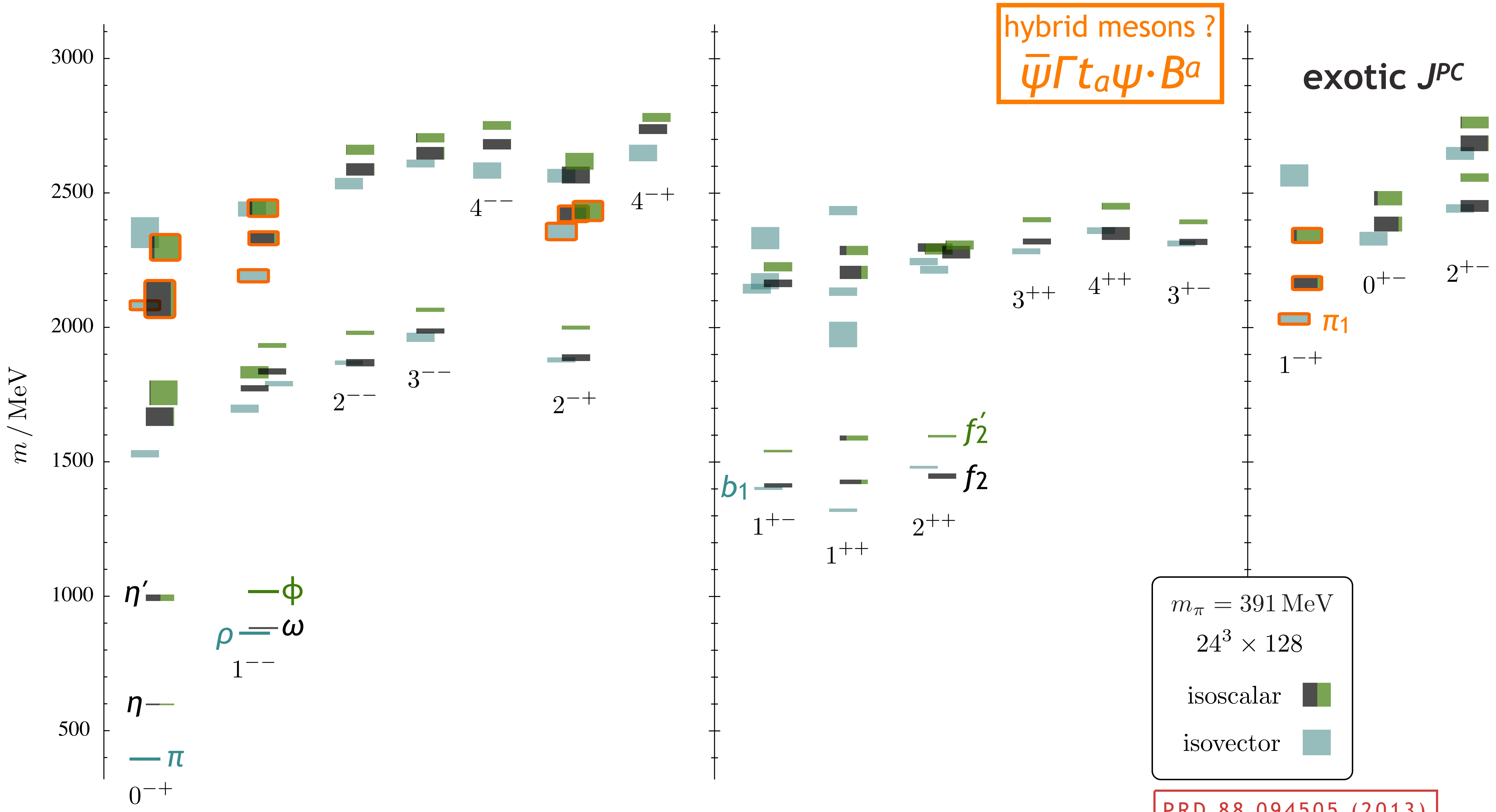
lattice QCD **is QCD** under controlled approximations

discretisation choice / finite lattice spacing ⇝ *relatively unimportant here*

choice of quark mass value ⇝ *a tool to explore QCD*

finite spacetime volume ⇝ *how we access scattering*

(incomplete) lattice QCD spectrum of mesons



PRD 88 094505 (2013)

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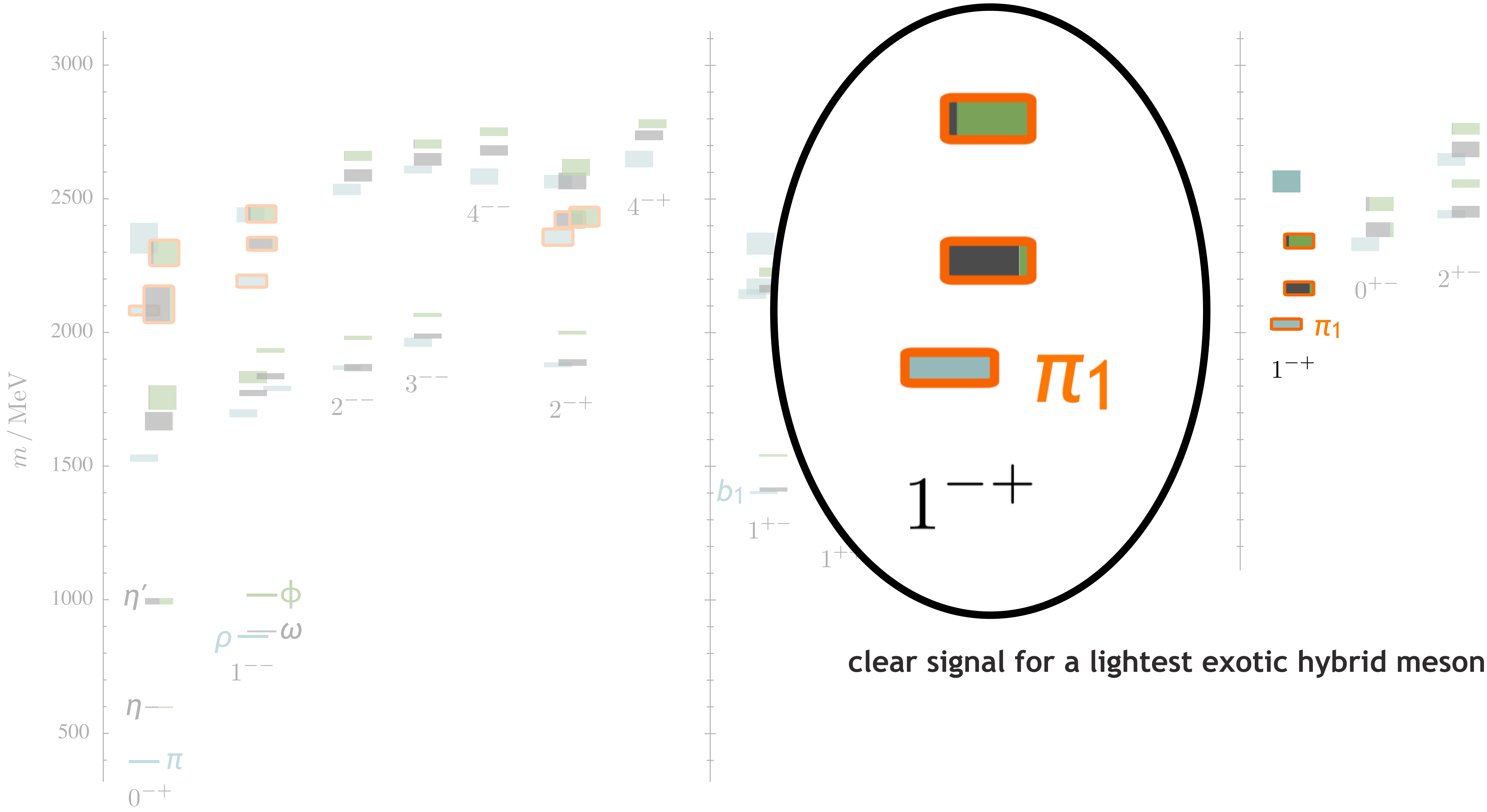


clear signal for a lightest exotic hybrid meson

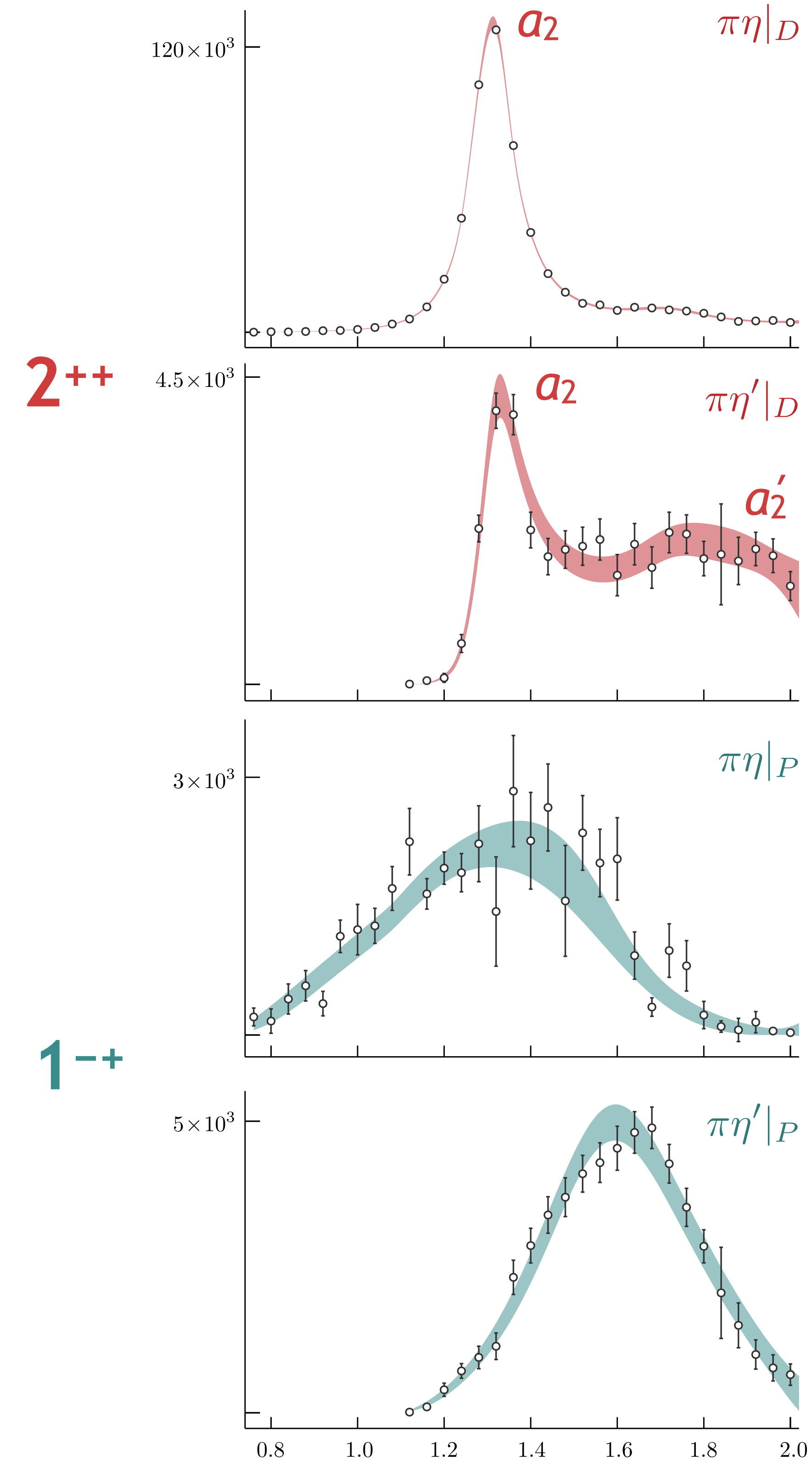
experimental situation

a recent **JPAC** analysis of **COMPASS** data on $\pi p \to \pi\eta\, p$, $\pi p \to \pi\eta'\, p$

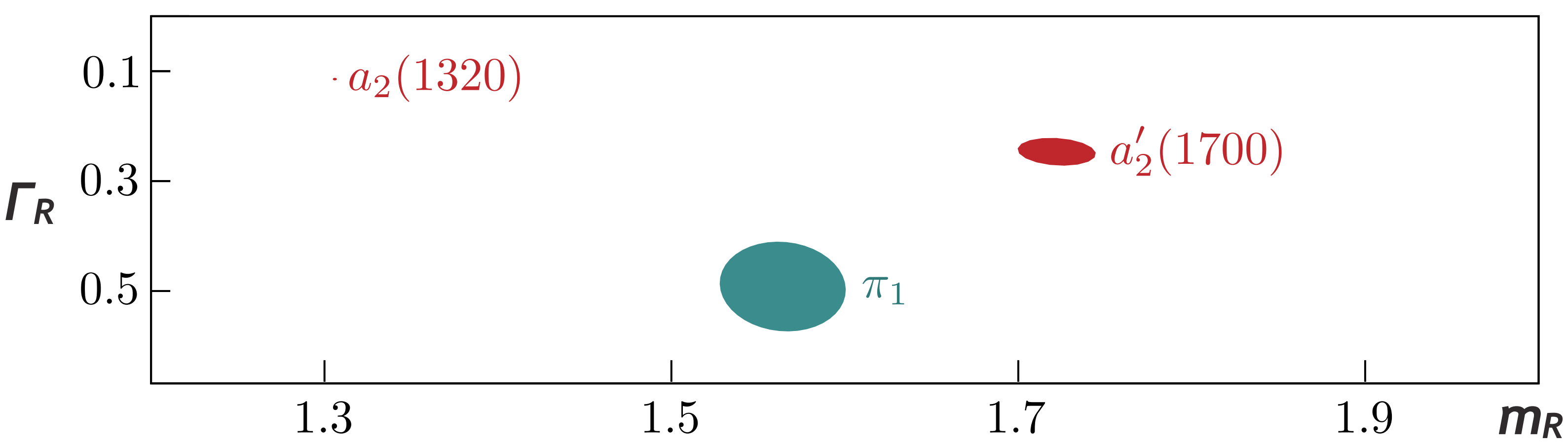
Determination of the Pole Position of the Lightest Hybrid Meson Candidate

A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2]
M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

(Joint Physics Analysis Center)



pole singularity of a π_1 resonance



m_R=1564(89) MeV, Γ_R=492(115) MeV

a rather broad resonance

a resonance in QCD ?

how would an unstable resonance appear in lattice QCD ?

the lattice has a **finite-volume** $\Rightarrow$ spectrum is **discrete**

but the mapping **discrete-spectrum** $\longleftrightarrow$ **scattering matrix** is known

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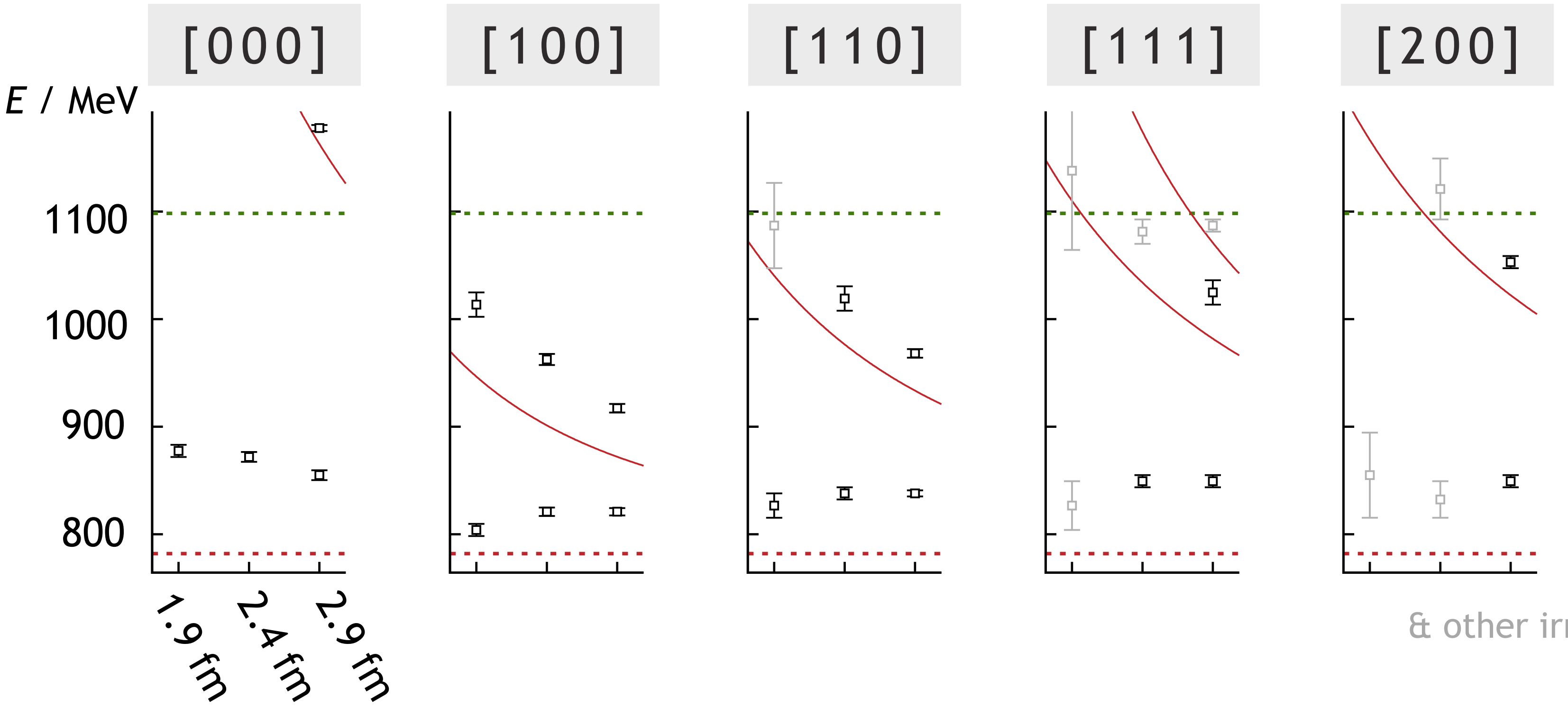
ππ I=1 J^P=1^−

PRD87 034505 (2013)

m_π ~ 391 MeV



& other irreps

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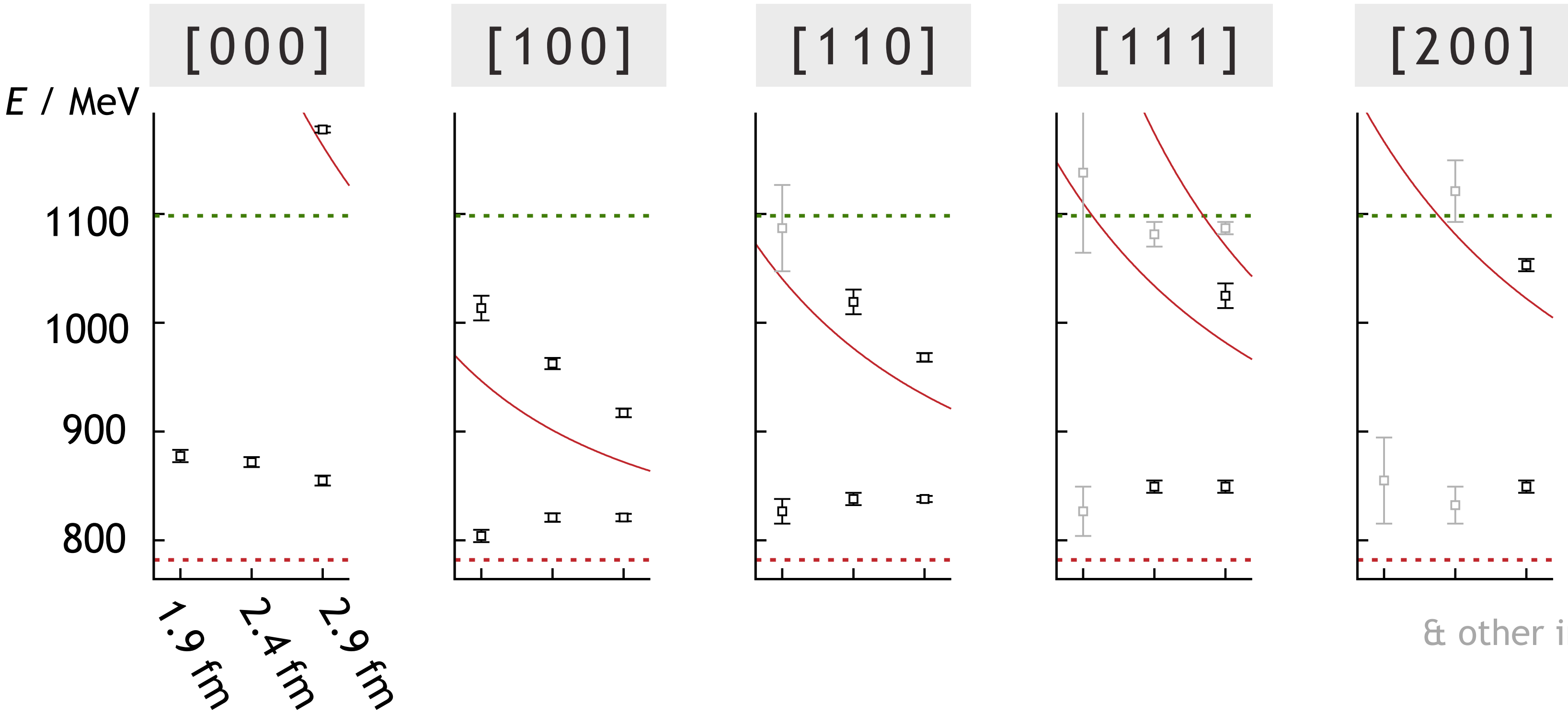
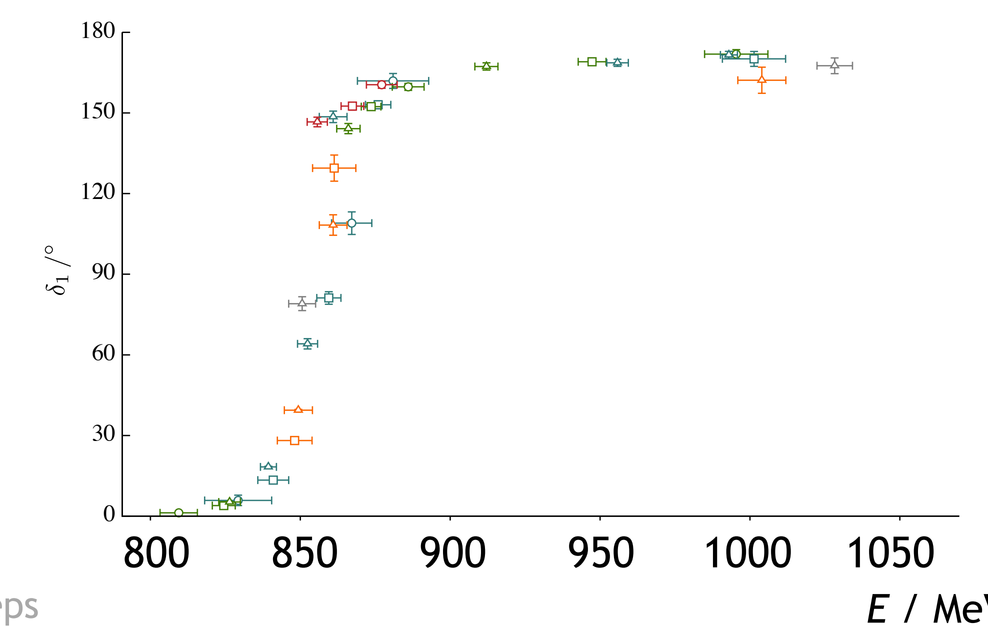
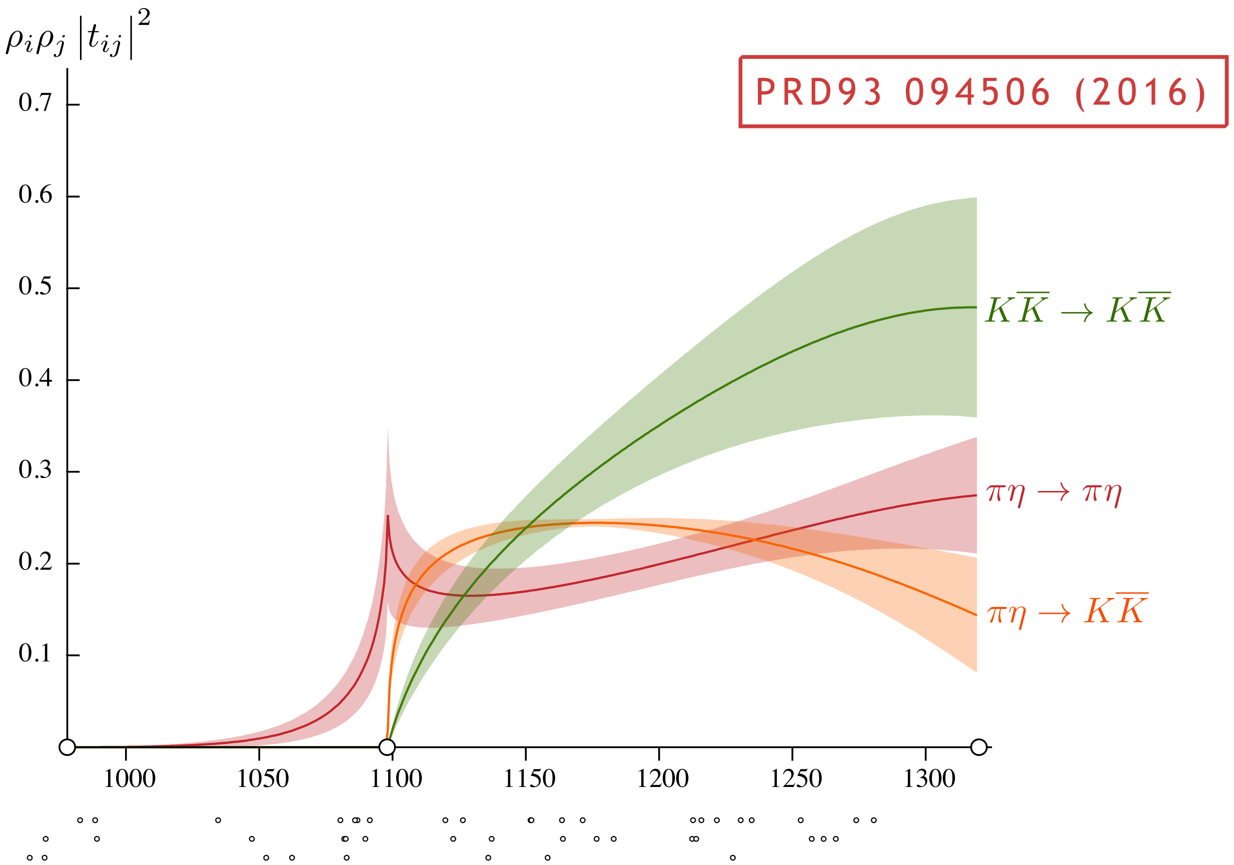


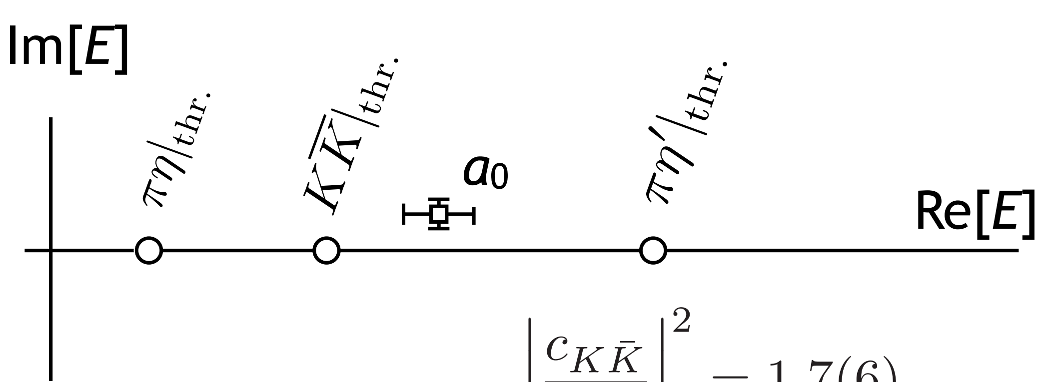
& other irreps

scattering phase-shift



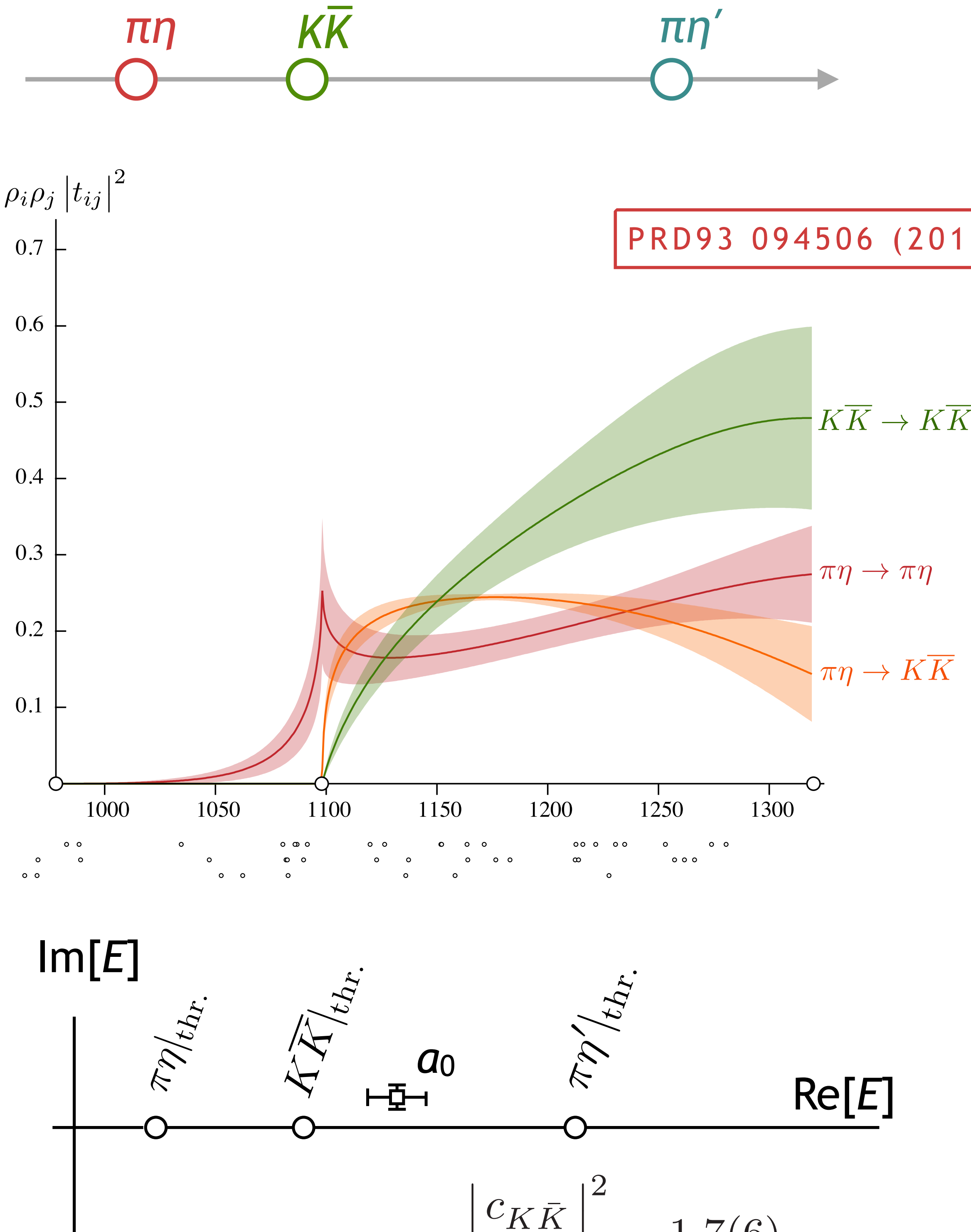
isospin=1 πη, K$\bar{K}$





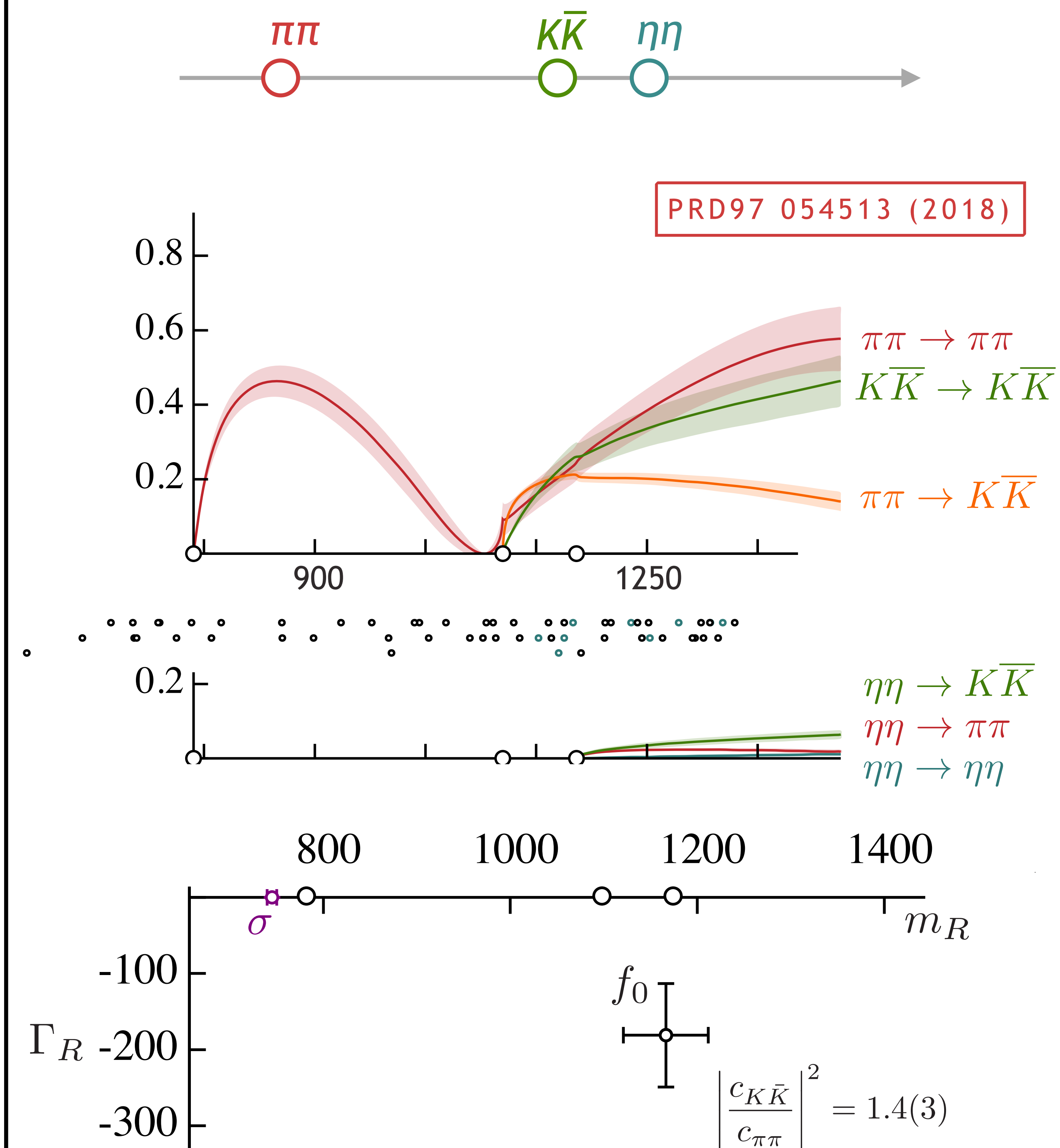
$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right|^2 = 1.7(6)$$

isospin=1 πη, $K\bar{K}$



$$\left|\frac{c_{K\bar{K}}}{c_{\pi\eta}}\right|^2 = 1.7(6)$$

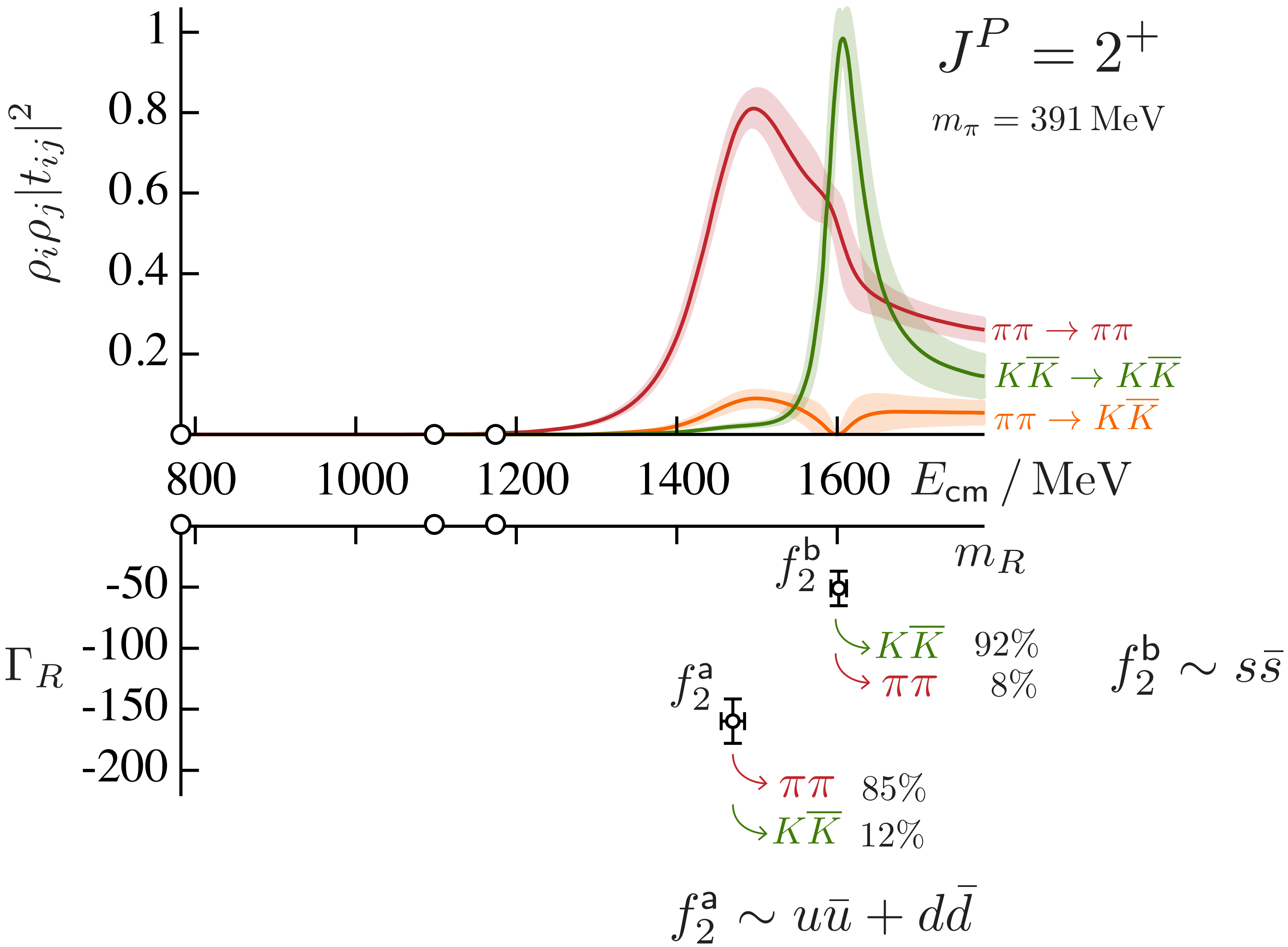
isospin=0 ππ, $K\bar{K}$, ηη



$$\left|\frac{c_{K\bar{K}}}{c_{\pi\pi}}\right|^2 = 1.4(3)$$

f_2 resonances

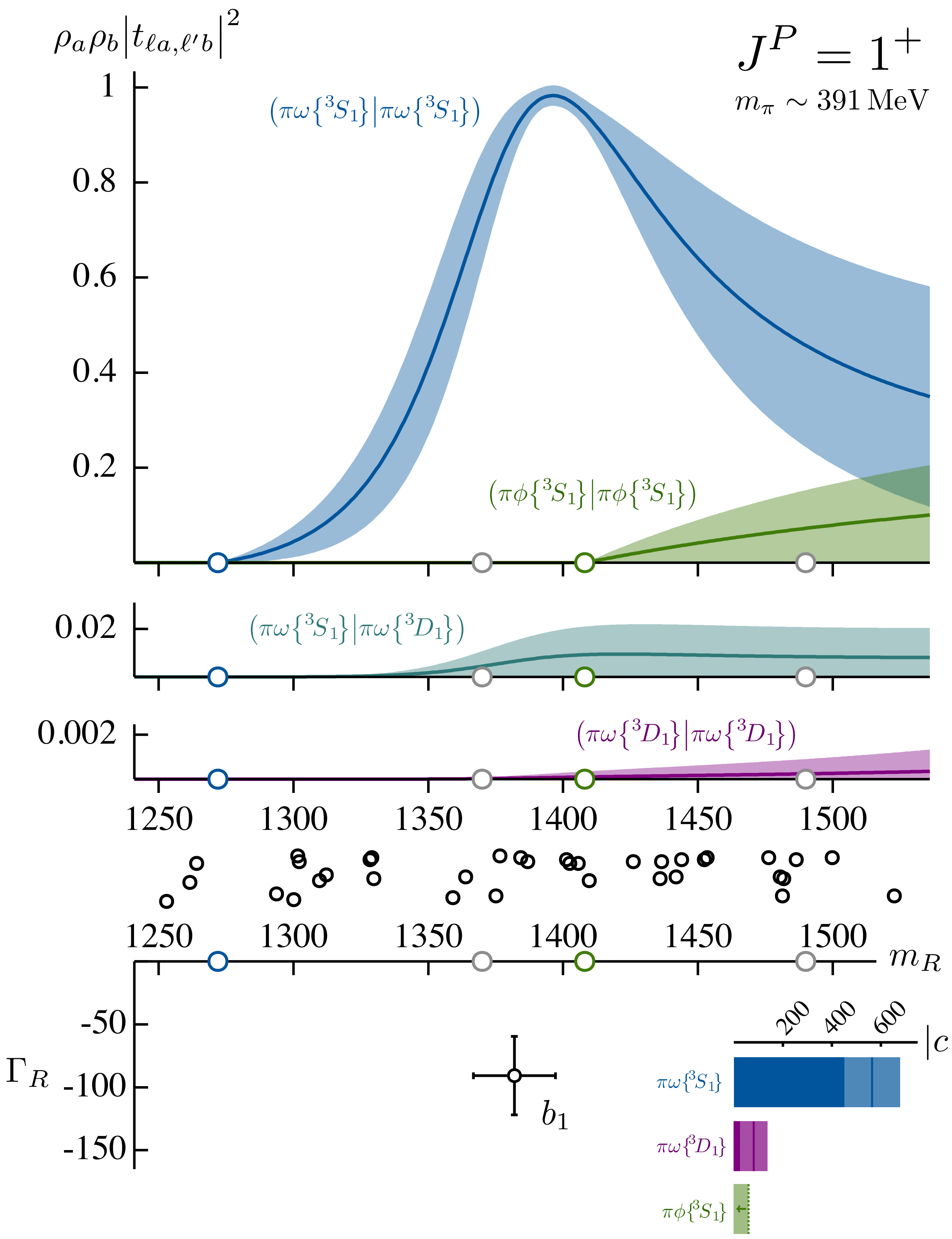
m_π ~ 391 MeV PRD97 054513 (2018)

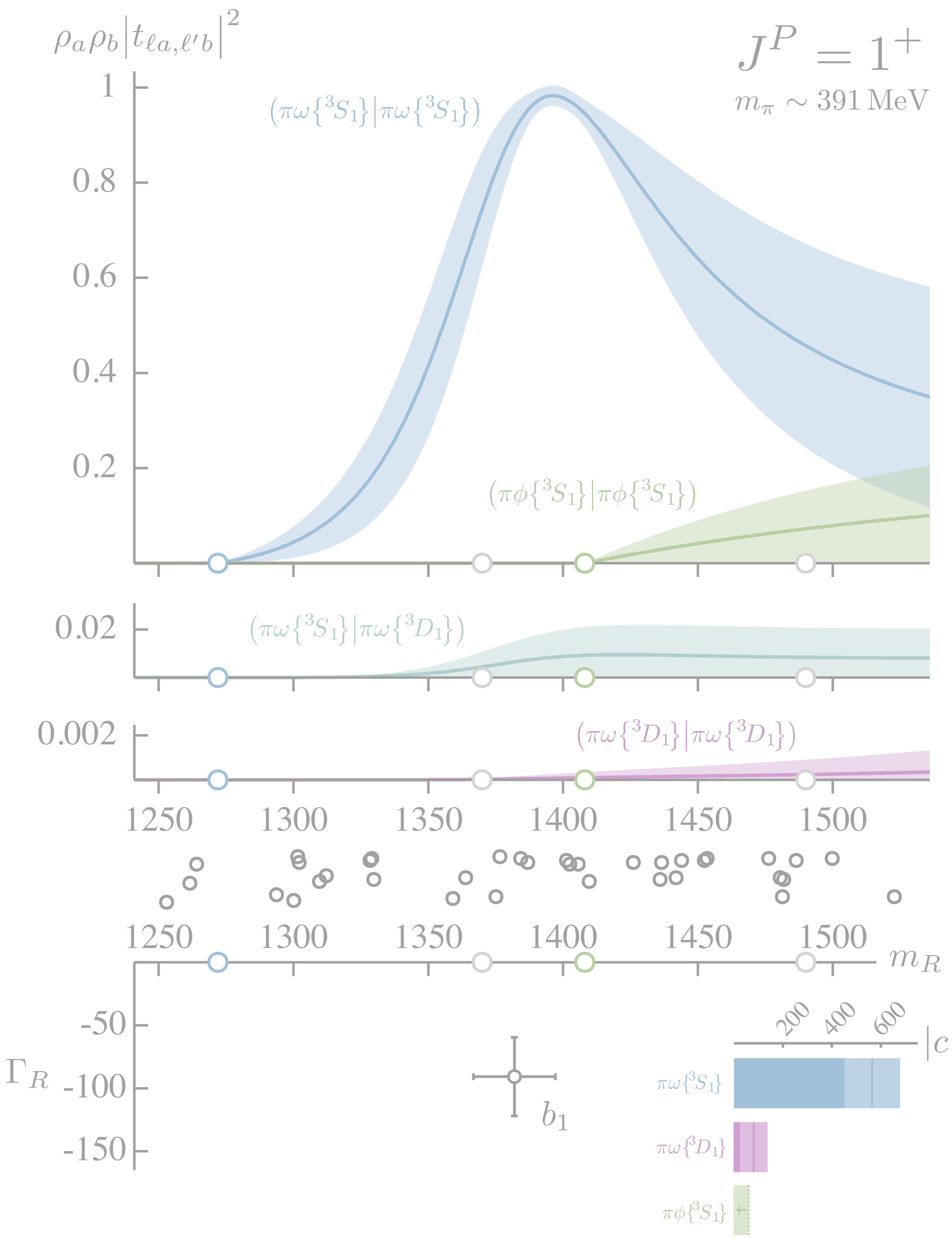


thus the 'OZI rule' emerges ...

resonances decaying to hadrons with spin – $b_1 \to \omega\pi$

ω is stable at m_π~391 MeV



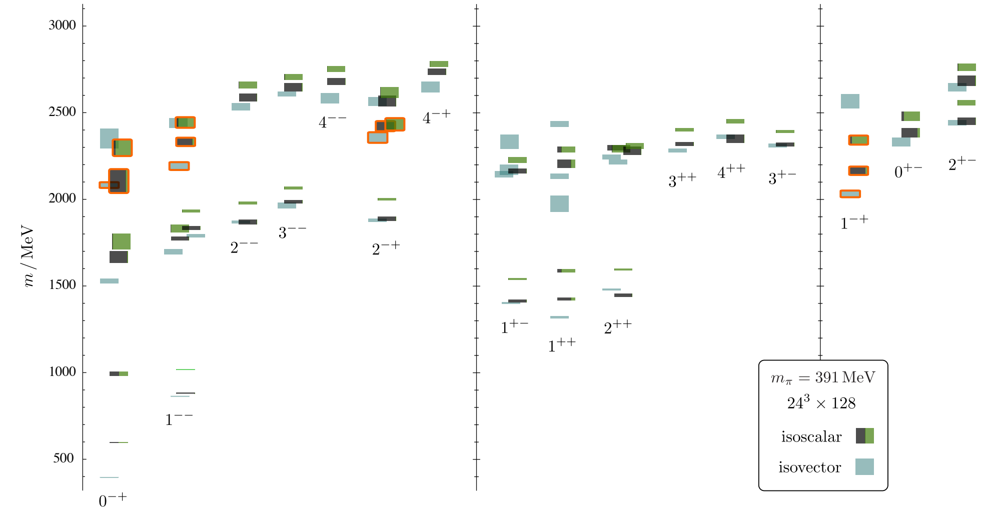


several successful calculations with m_π~391 MeV

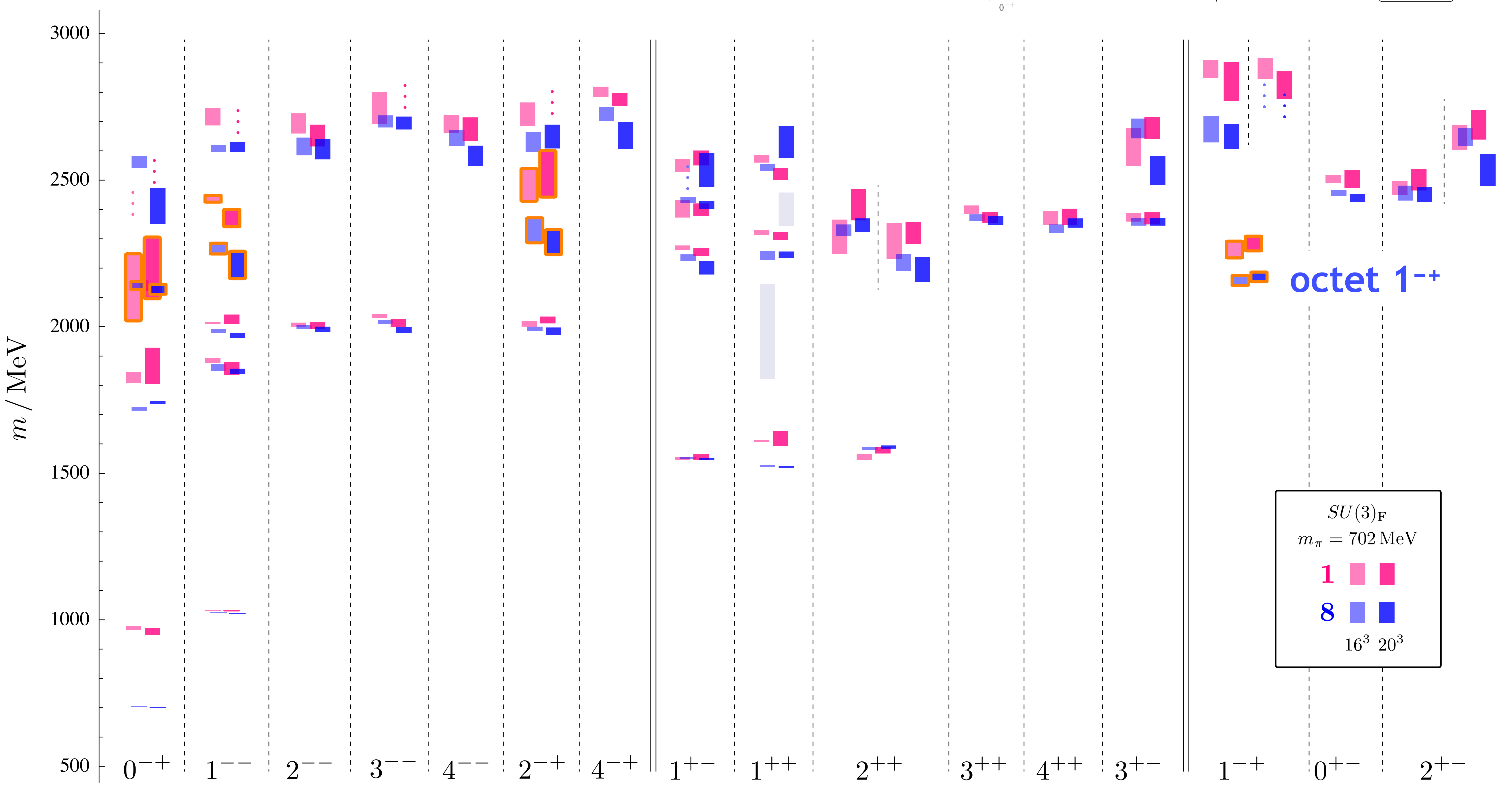
but a π_1 resonance potentially has a very large set of decay modes …

$m_u = m_d = m_s$ SU(3)$_F$ point

increase the light quark mass to the strange quark mass …

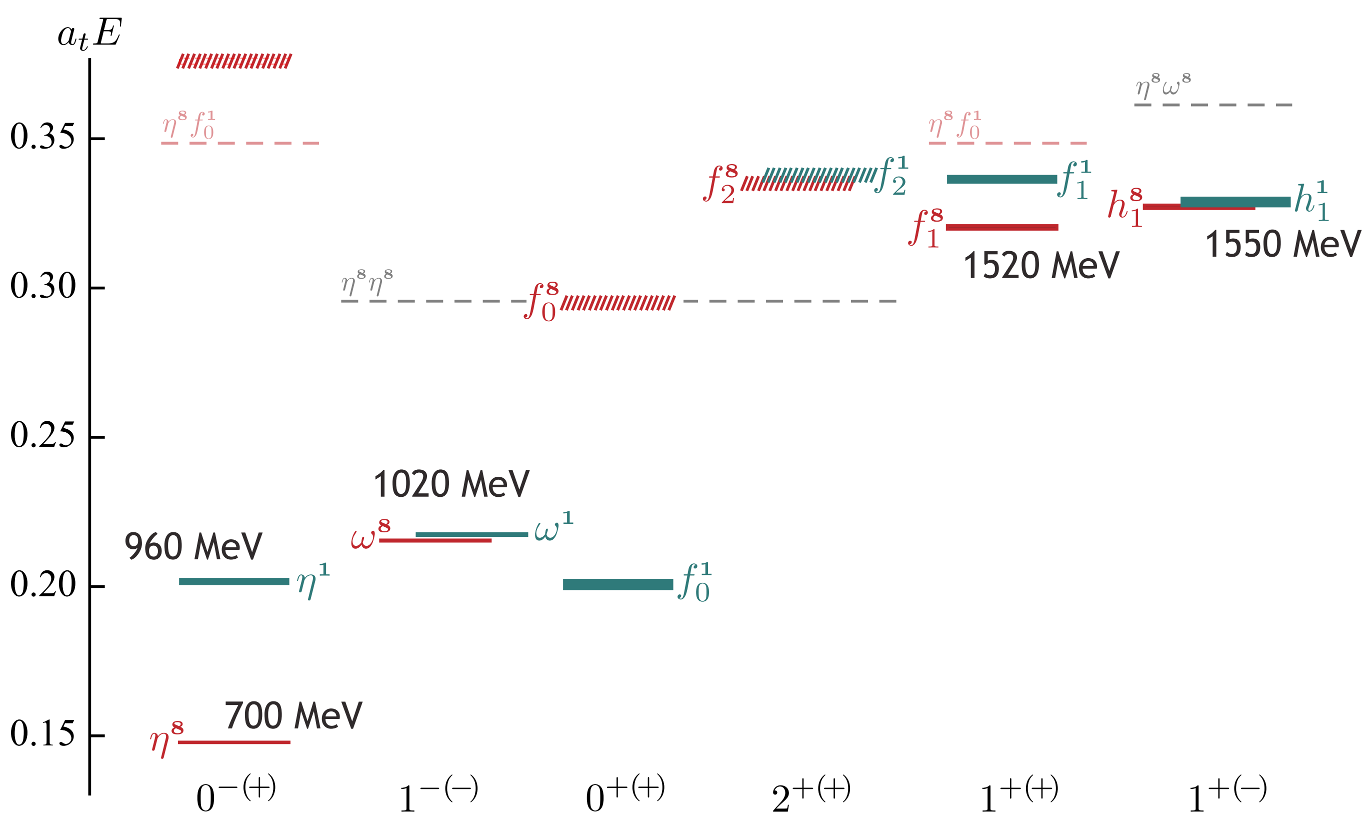


(incomplete) **lattice spectrum calculation** PRD 88 094505 (2013)



$m_u=m_d=m_s$ SU(3)$_F$ point

several stable mesons:



please forgive the obscure lattice units, will convert at the end ...

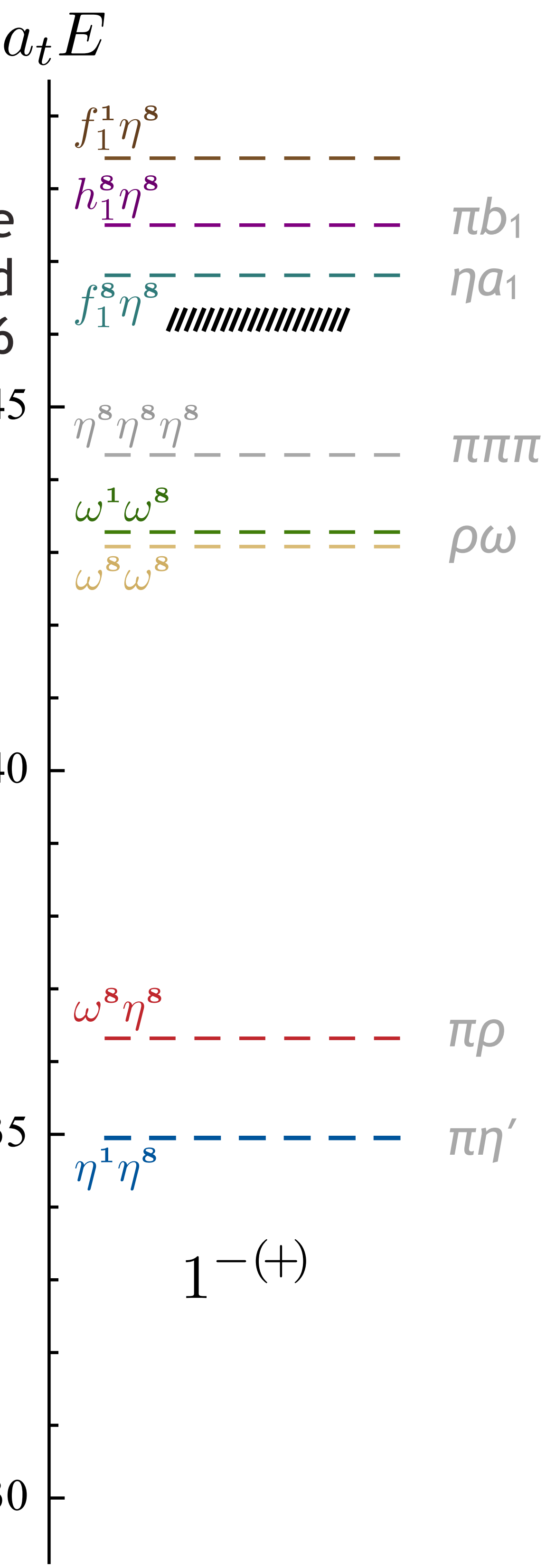
0^{-+} $\eta^8 \rightsquigarrow \pi,K,\eta$ $\eta^1 \rightsquigarrow \eta'$

1^{--} $\omega^8,\omega^1 \rightsquigarrow \rho,K^*,(\omega,\varphi)$

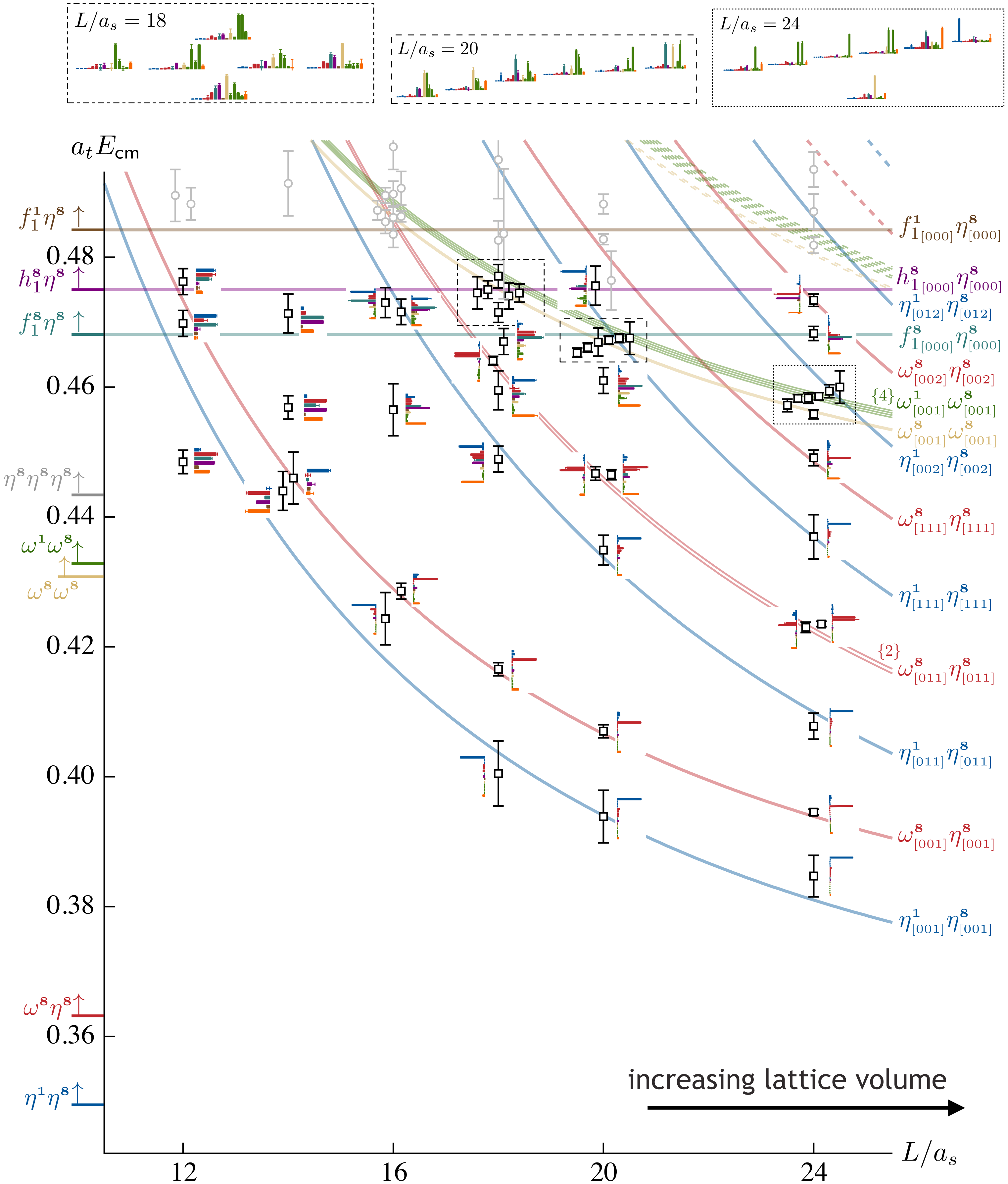
1^{+-} $h_1{}^8,h_1{}^1 \rightsquigarrow b_1,K_1,(h_1,h_1')$

1^{++} $f_1{}^8,f_1{}^1 \rightsquigarrow a_1,K_1,(f_1,f_1')$

resonance expected near 0.46

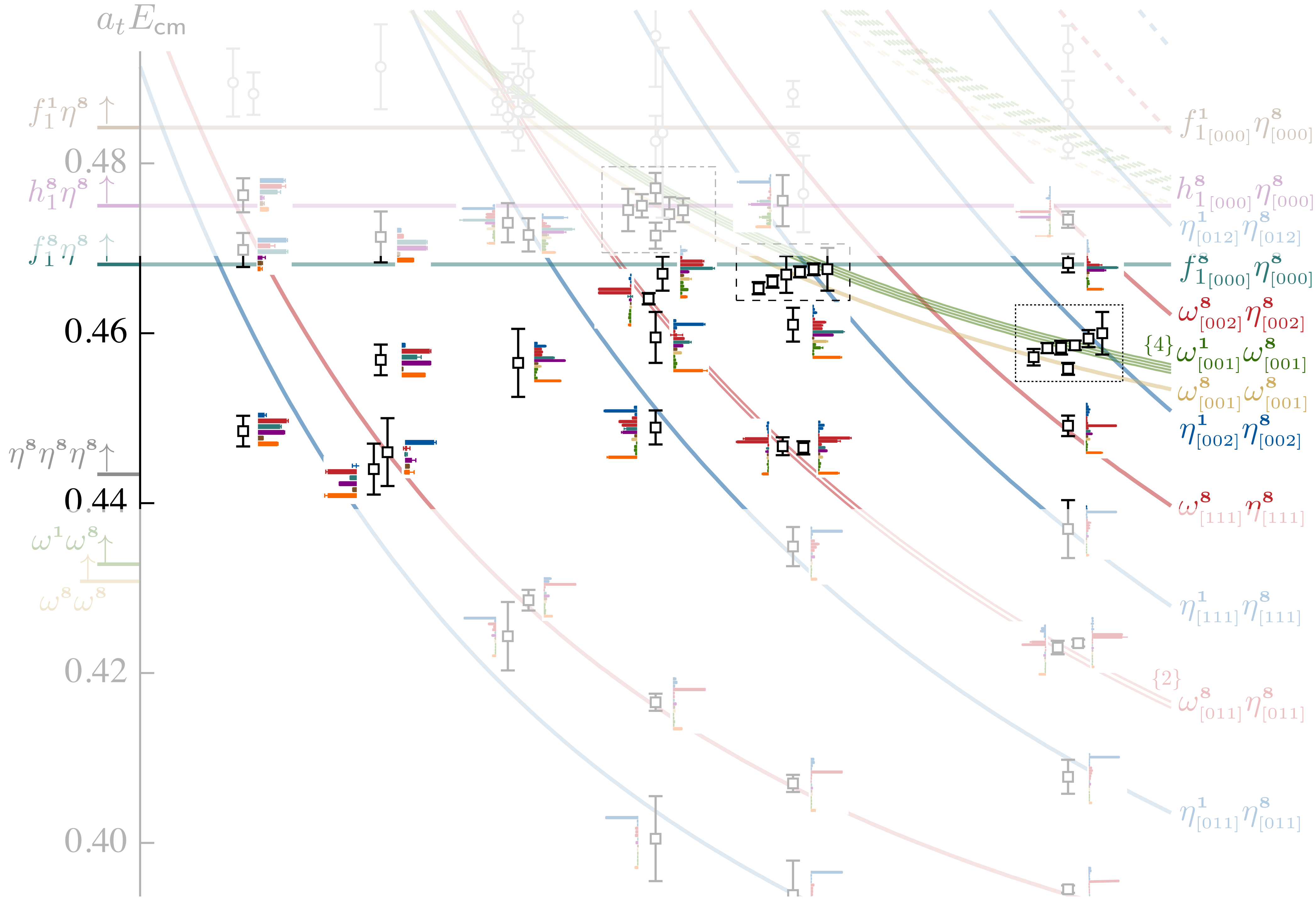


lattice QCD spectrum computed in 6 volumes



53 energy levels to constrain 'eight' channel scattering

lattice QCD spectrum computed in 6 volumes

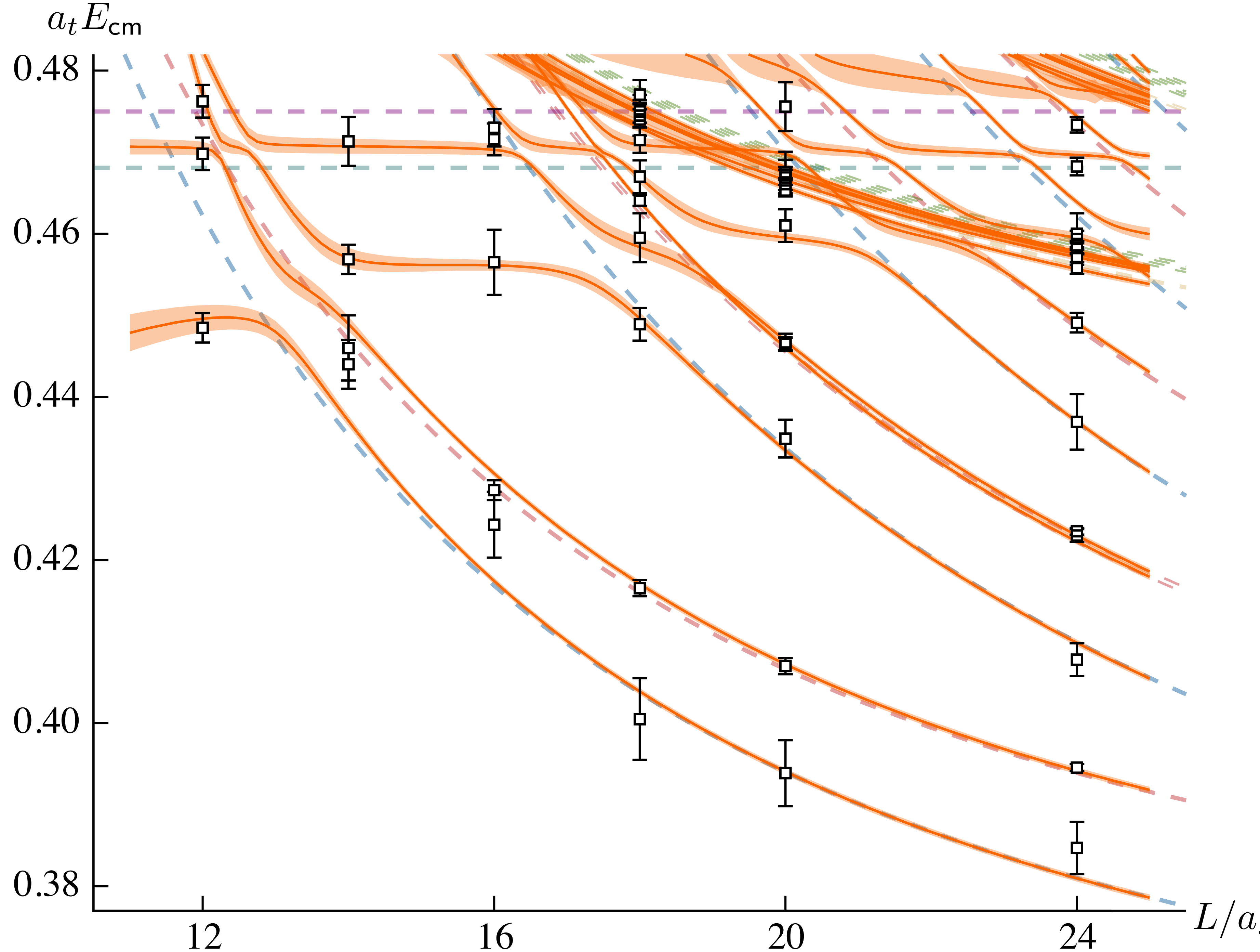


states have overlap with

$\bar{\psi}\Gamma t_a \psi \cdot B^a$

an ‘eight’ channel scattering amplitude

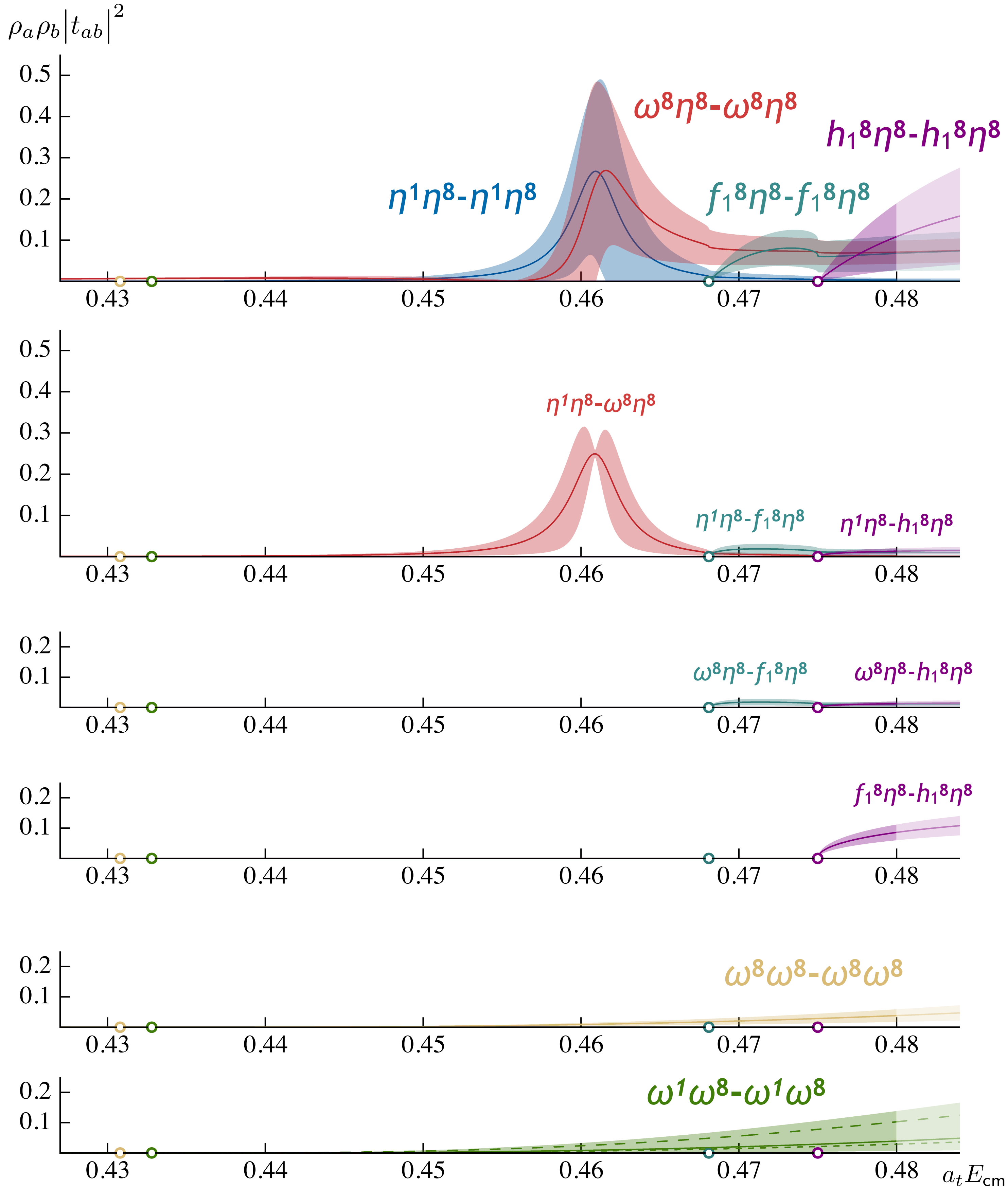
describe scattering by a unitarity-preserving *K*-matrix featuring a pole
(11 free parameters)



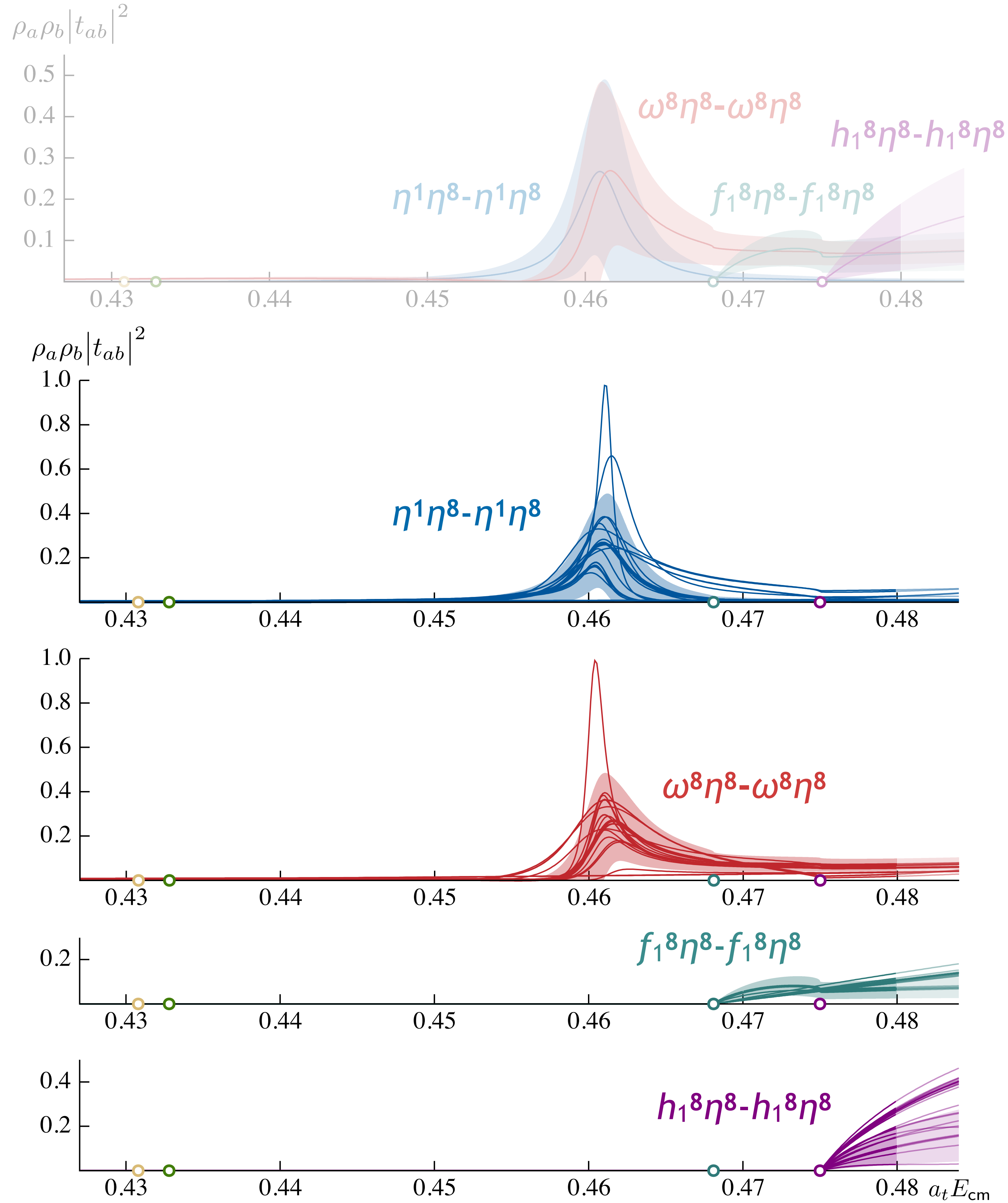
$\chi^2/N_{dof} = \frac{43.6}{53-11} = 1.04$

a good description of the spectrum …

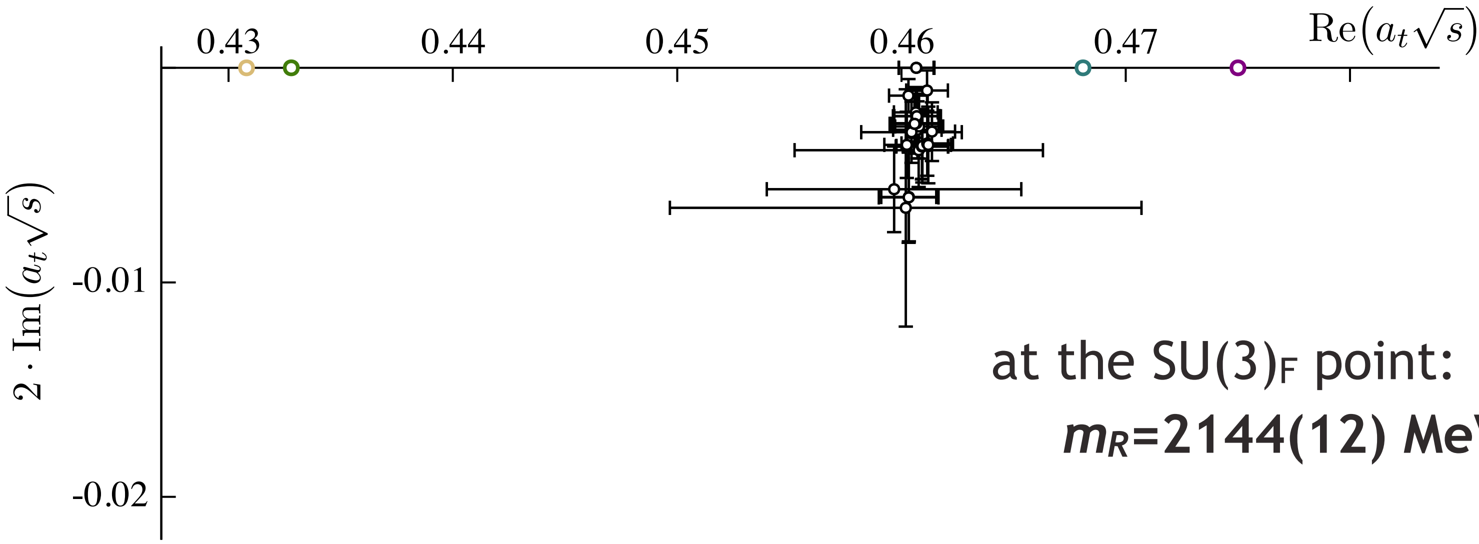
an ‘eight’ channel scattering amplitude



‘eight’ channel scattering amplitudes — varying parameterization



octet 1^{-+} resonance pole & couplings



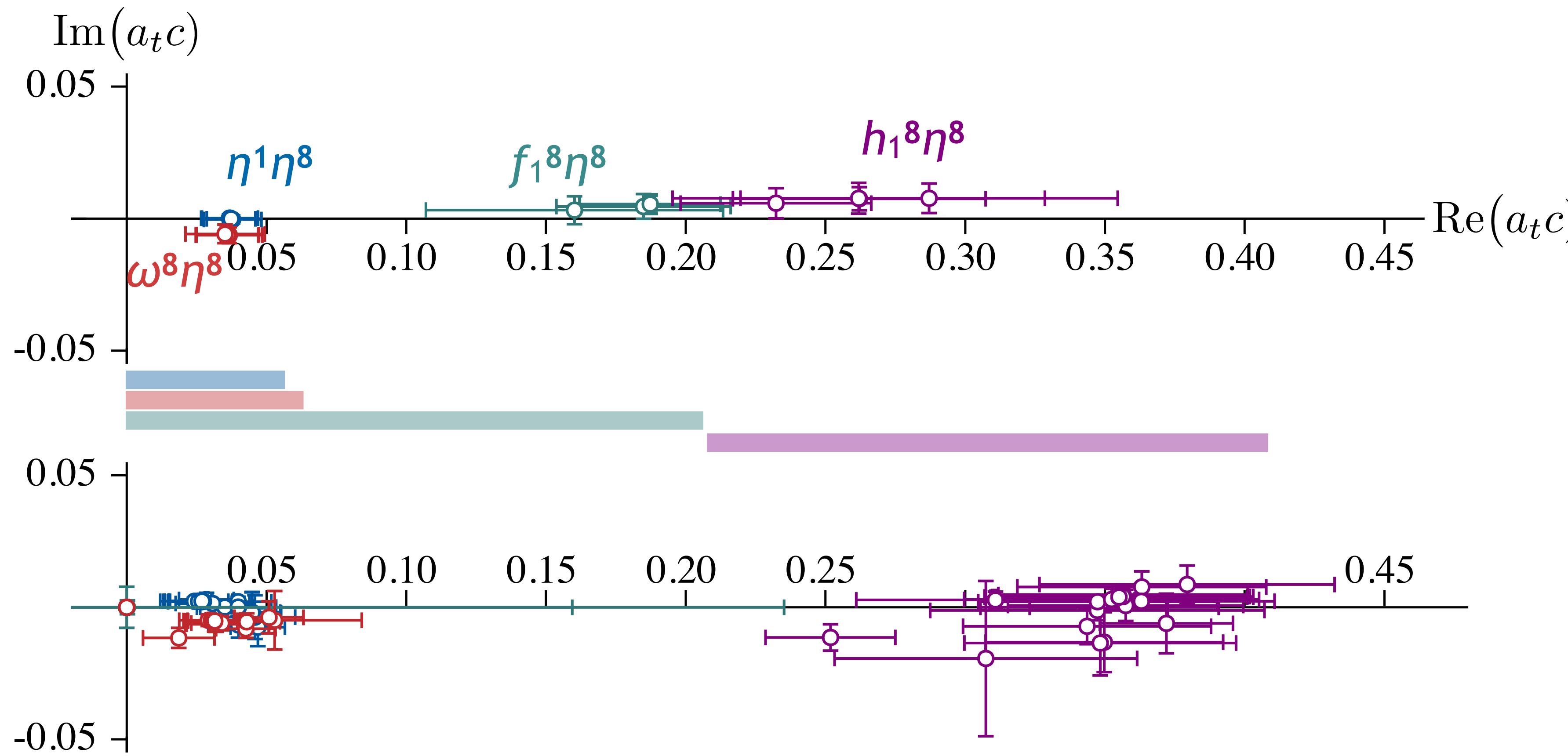
$$t_{ab}(s) \sim \frac{c_a c_b}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\tfrac{1}{2}\Gamma_R$$

at the SU(3)$_F$ point:

***m_R*=2144(12) MeV, *Γ_R*=21(21) MeV** (a narrow resonance)

resonance couplings:



resonance below $h_1{}^8\eta^8$ threshold, but with a large coupling

a crude extrapolation to physical point

core assumption: couplings scale only with the relevant barrier factor k^ℓ

use PDG masses & COMPASS/JPAC π_1 mass

generates for a π_1 at 1564 MeV:

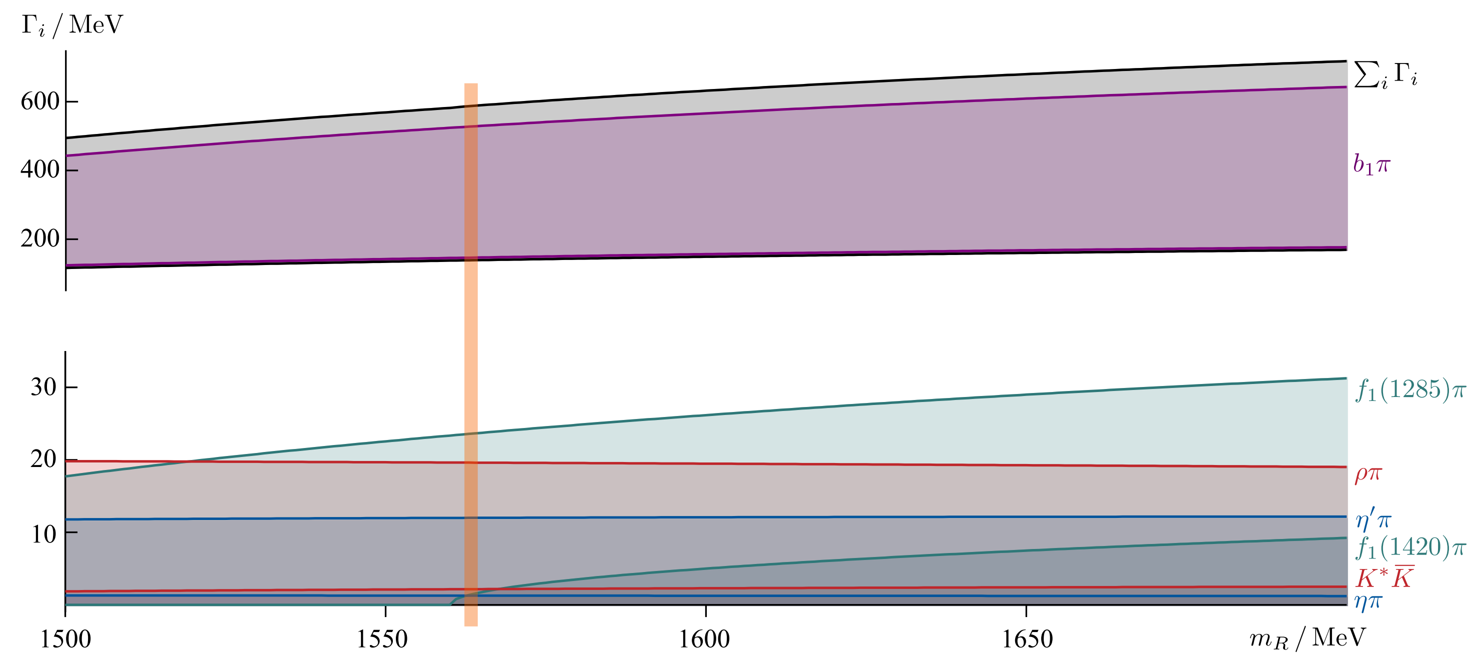
Γ_{TOT} ~ 140-600 MeV

$\Gamma(\pi\eta) \lesssim 1$ MeV

$\Gamma(\pi\eta') \lesssim 12$ MeV

$\Gamma(\pi\rho) \lesssim 20$ MeV

$\Gamma(\pi b_1)$ ~ 140-530 MeV



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JPAC/COMPASS analysis

Γ_{TOT} ~ 492(115) MeV

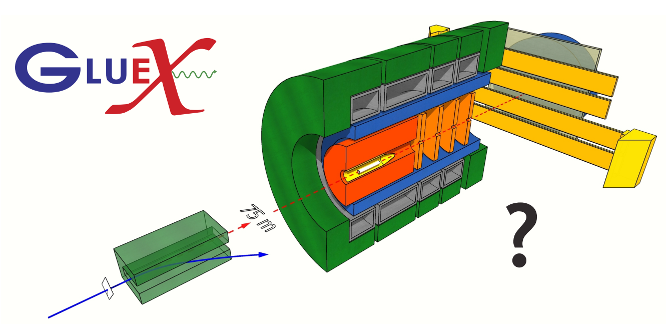
Kopf et al analysis
COMPASS + Crystal Barrel

Γ_{TOT} ~ 455(200) MeV

$$\frac{\Gamma(\pi\eta')}{\Gamma(\pi\eta)} = 5.5^{+2.1}_{-1.2}$$

if correct, suggests prior observations in $\pi\eta$, $\pi\eta'$, $\pi\rho$
are in heavily suppressed decay channels

$\pi b_1 \to \pi\pi\omega \to \pi\pi\pi\pi\pi$



summary & outlook

first ever calculation of an **exotic hybrid meson** as a **resonance** in QCD

simplified scattering system using exact $SU(3)_F$ and m_π~700 MeV

flavor octet 1^{-+} state appears as a narrow resonance

crude extrapolation to physical kinematics
suggests a **potentially broad resonance**

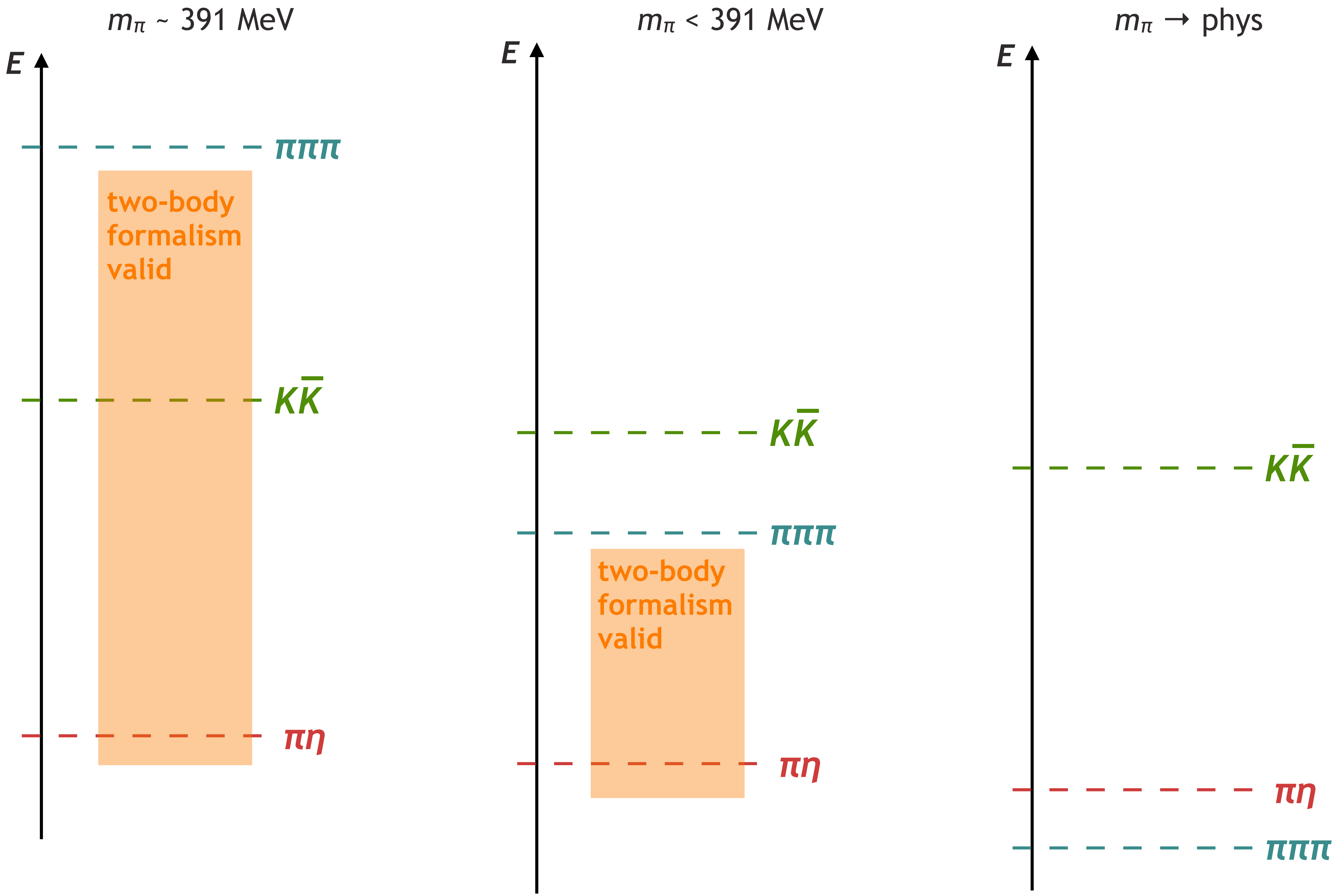
what about other exotic J^{PC} ?

can we build a phenomenology of hybrid decays starting from QCD ?

challenge of **reducing quark mass** really the challenge of **including three-meson decays**

summary & outlook

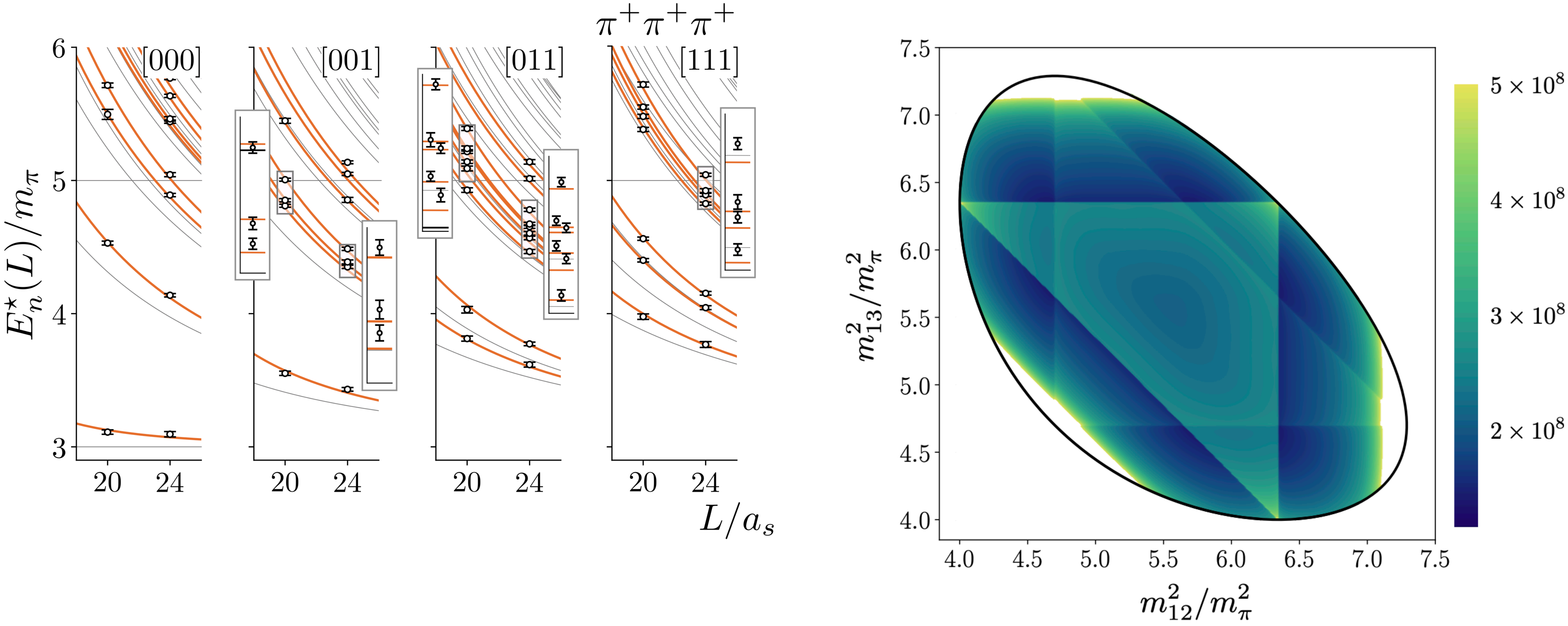
challenge of **reducing quark mass** really the challenge of **including three-meson scattering**



including three-meson scattering is starting to become practical

The energy-dependent $\pi^+\pi^+\pi^+$ scattering amplitude from QCD

Maxwell T. Hansen,[1,*] Raul A. Briceño,[2,3,†] Robert G. Edwards,[2,‡] Christopher E. Thomas,[4,§] and David J. Wilson[4,¶]
(for the Hadron Spectrum Collaboration)



$$|c|^{\text{phys}} = \left| \frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)} \right|^{\ell} |c|.$$

$$\Gamma(R \to i) = \frac{|c_i^{\text{phys}}|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}}).$$

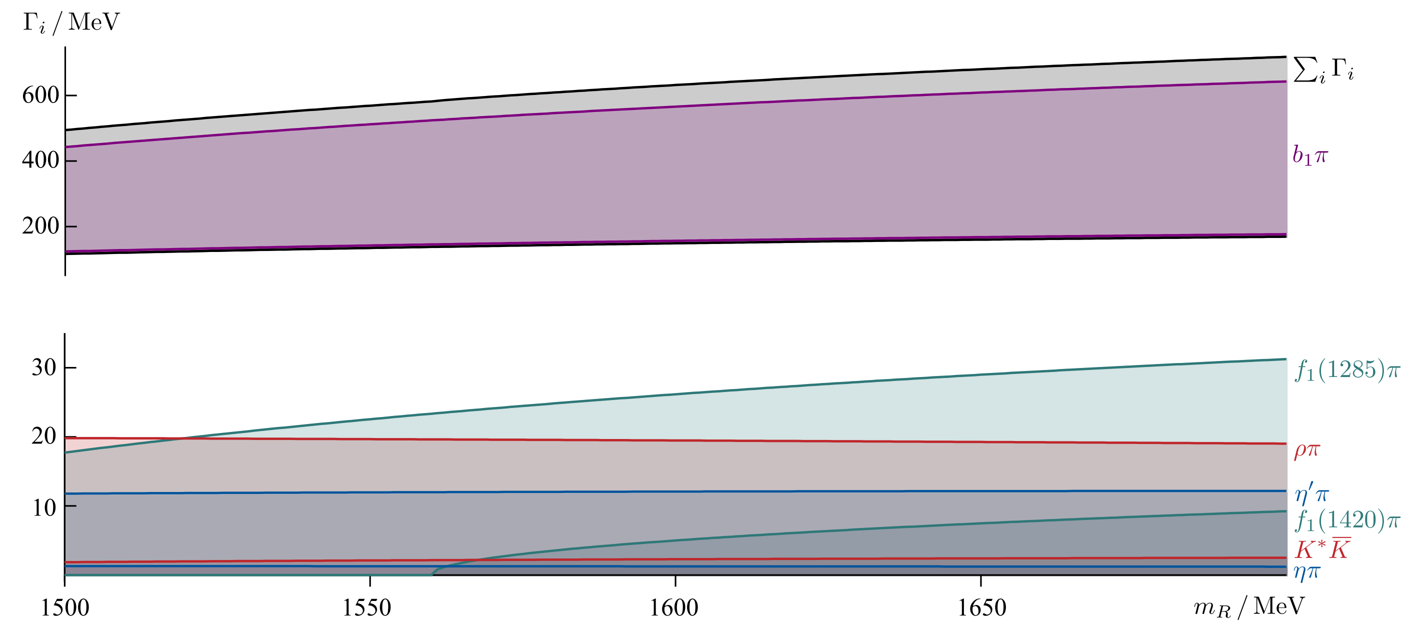
example ‘success’ – f_2,f_2' calculated at m_π~400 MeV

	Scaled	PDG
$\lvert c(f_2 \to \pi\pi)\rvert$	488(28)	453^{+9}_{-4},
$\lvert c(f_2 \to K\bar{K})\rvert$	139(27)	132(7),
$\lvert c(f_2' \to \pi\pi)\rvert$	103(32)	33(4),
$\lvert c(f_2' \to K\bar{K})\rvert$	321(50)	389(12),

$$\frac{1}{\sqrt{3}}(\pi^+\rho^0 - \pi^0\rho^+) + \frac{1}{\sqrt{6}}(K^+\bar{K}^{*0} - \bar{K}^0 K^{*+}),$$

$$-\sqrt{\frac{3}{10}}(K_{1A}^+\bar{K}^0 + \bar{K}_{1A}^0 K^+) + \frac{1}{\sqrt{5}}(a_1^+\eta_8 + (f_1)_8\pi^+),$$

$$\frac{1}{\sqrt{6}}(K_{1B}^+\bar{K}^0 - \bar{K}_{1B}^0 K^+) + \frac{1}{\sqrt{3}}(b_1^+\pi^0 - b_1^0\pi^+),$$



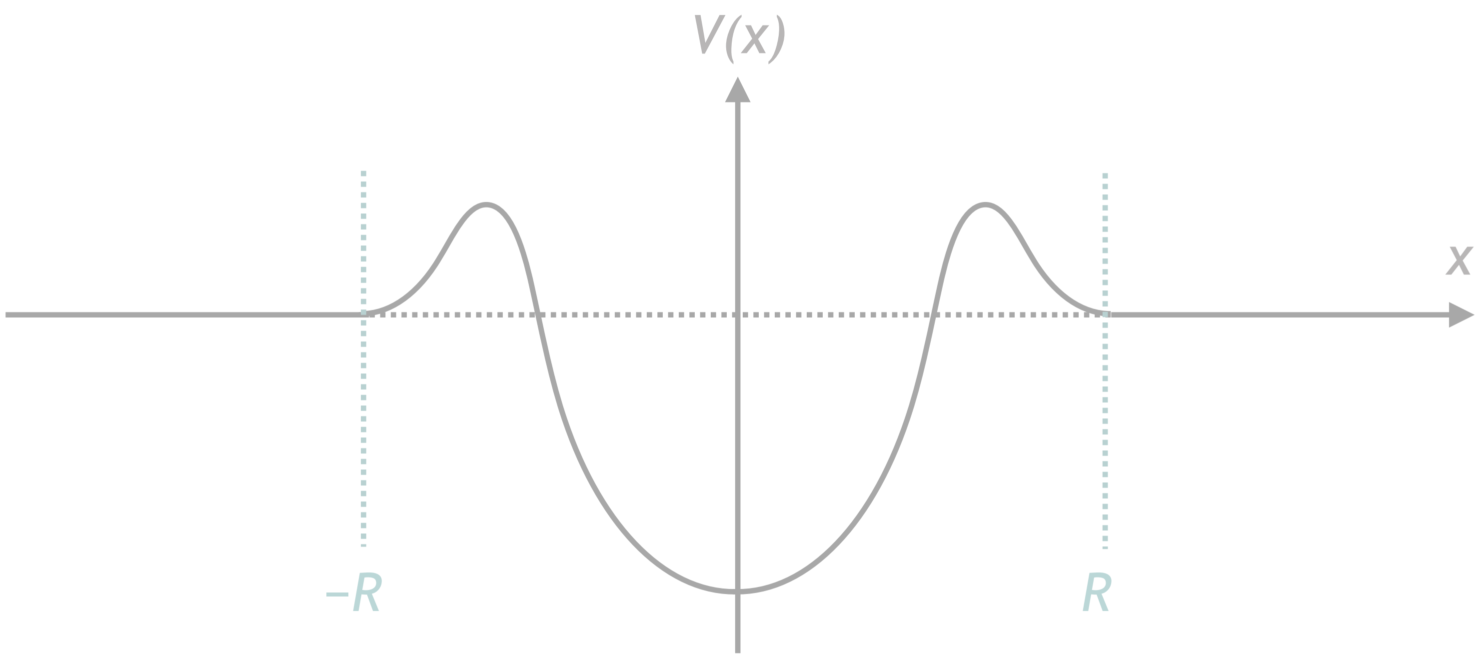
illustrative K-matrix form

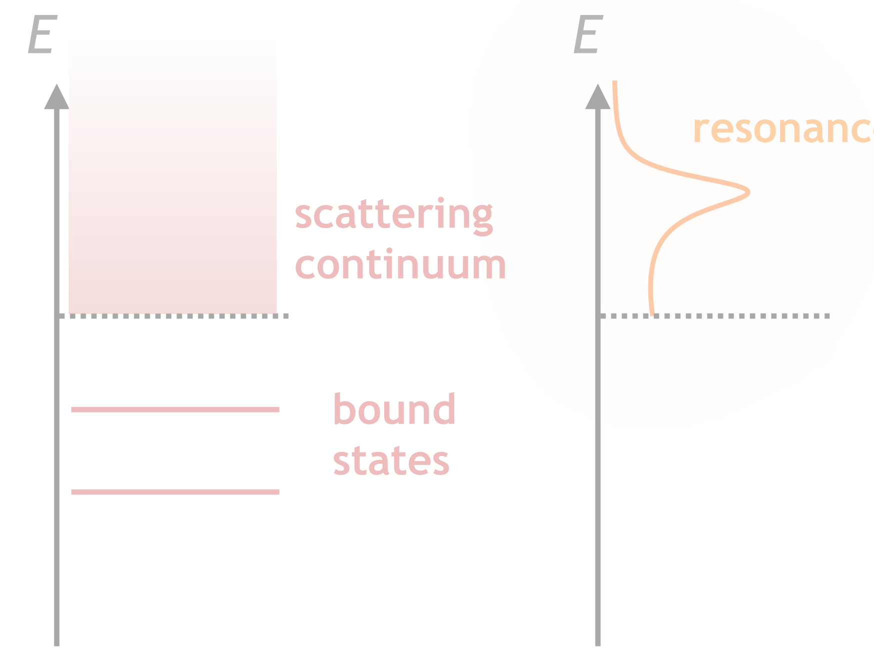
$$
\boldsymbol{K}_{\cancel{VV}}(s) = \frac{\boldsymbol{g}\boldsymbol{g}^T}{m^2 - s} + \begin{bmatrix} \gamma_{\eta^1\eta^8\{^1P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^8\eta^8\{^3P_1\}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\boldsymbol{g} = \left(g_{\eta^1\eta^8\{^1P_1\}}, g_{\omega^8\eta^8\{^3P_1\}}, g_{f_1^8\eta^8\{^3S_1\}}, g_{h_1^8\eta^8\{^3S_1\}}\right),
$$

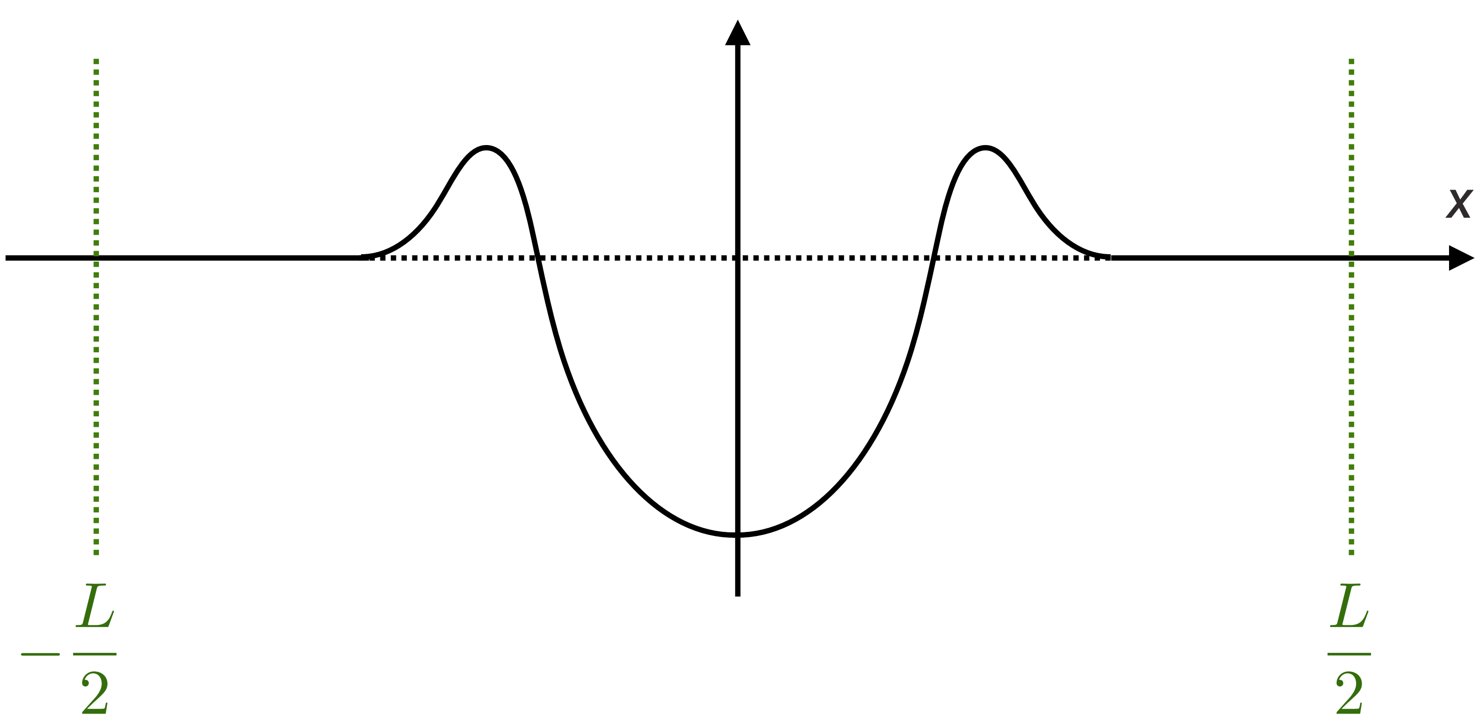
$$
\boldsymbol{K}_{VV}(s) = \begin{bmatrix} \gamma_{\omega^8\omega^8\{^3P_1\}} & 0 & 0 & 0 \\ 0 & \gamma_{\omega^1\omega^8\{^1P_1\}} & 0 & 0 \\ 0 & 0 & \gamma_{\omega^1\omega^8\{^3P_1\}} & 0 \\ 0 & 0 & 0 & \gamma_{\omega^1\omega^8\{^5P_1\}} \end{bmatrix}.
$$

resonances in a finite volume ?

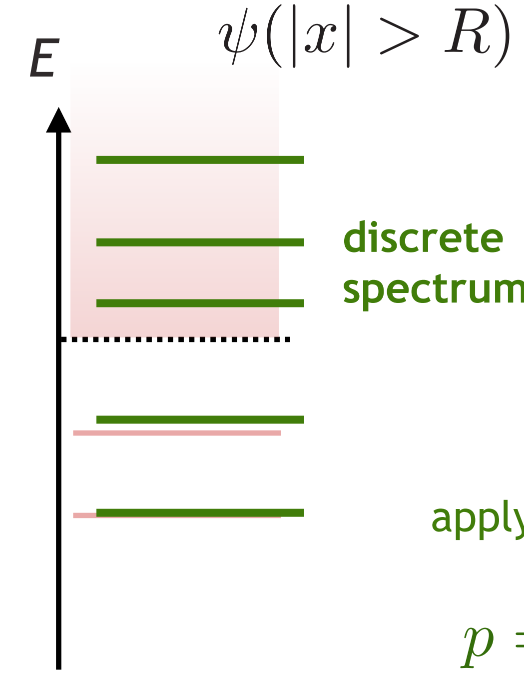




but in a periodic volume …



$$\psi(|x| > R) \sim \cos\big(p|x| + \delta(p)\big)$$



applying the boundary conditions

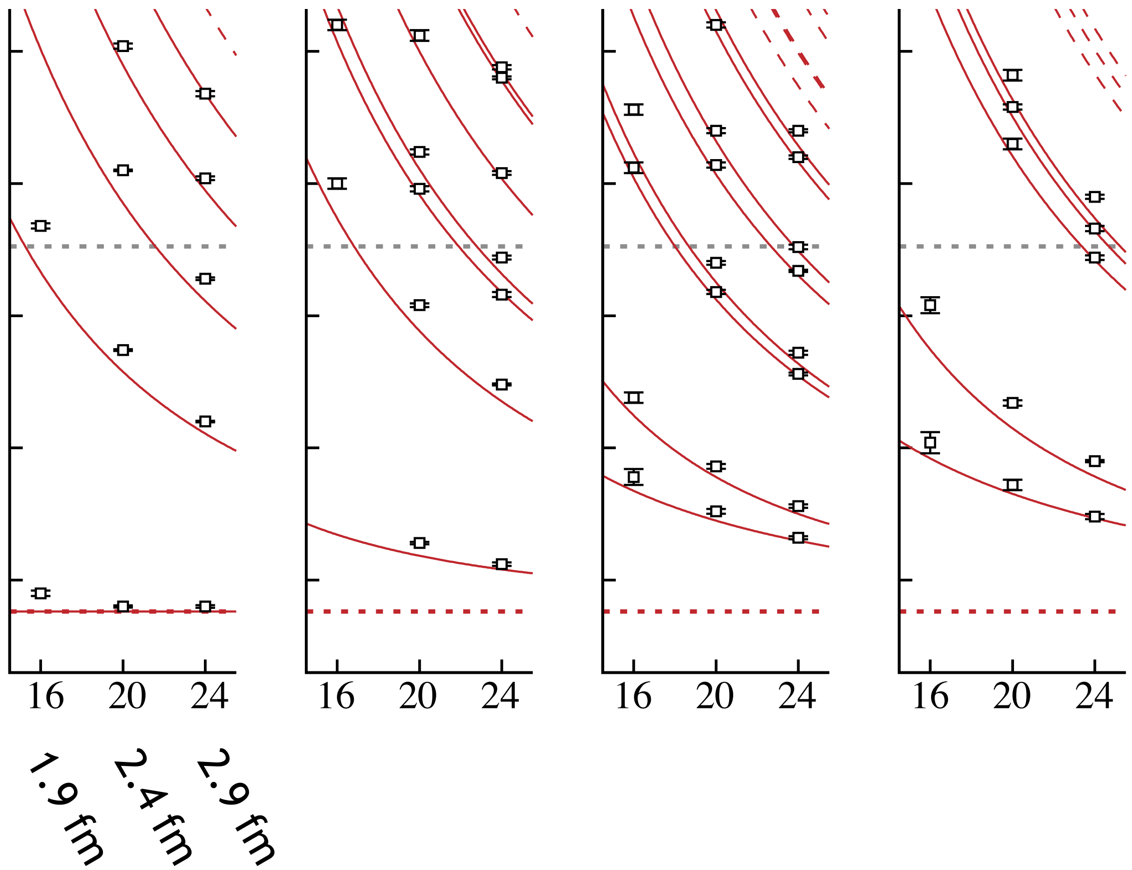
$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

solved by **discrete** $p_n(L)$

non-resonant elastic scattering — $\pi^+\pi^+$ (isospin=2)

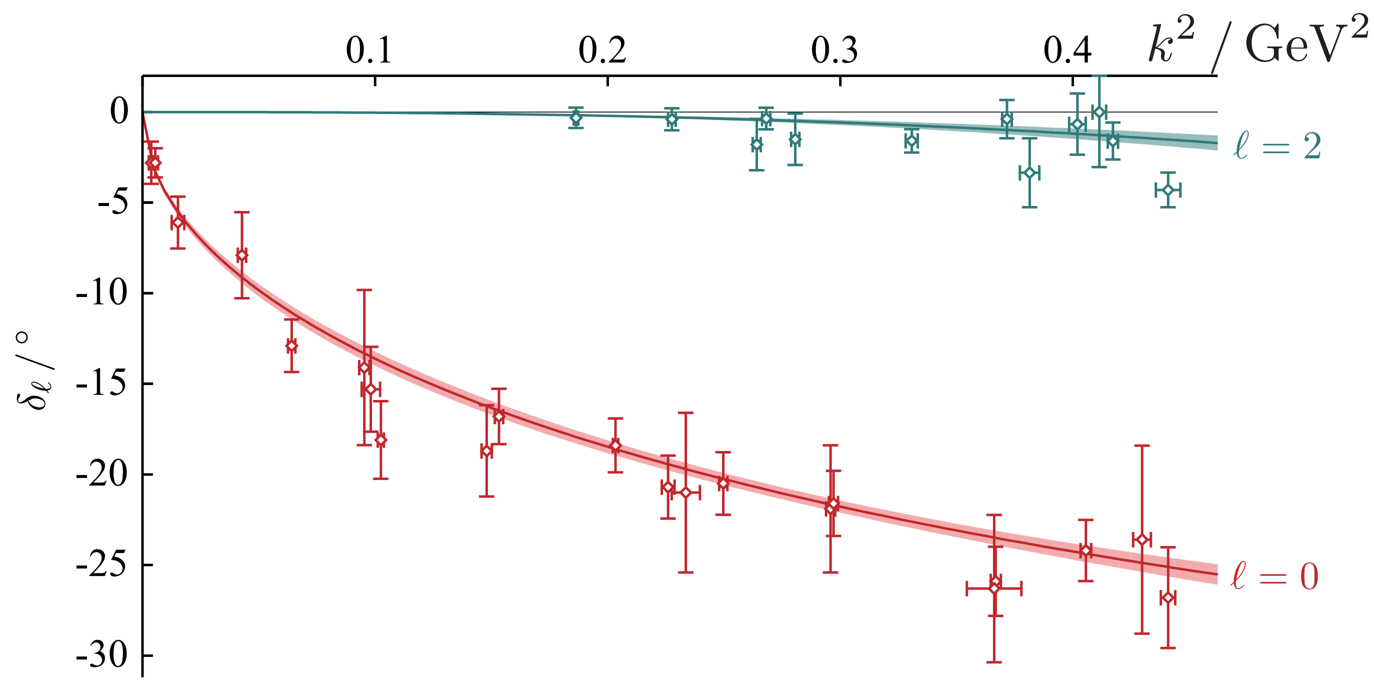
PRD86 034031 (2012)

m_π ~ 391 MeV

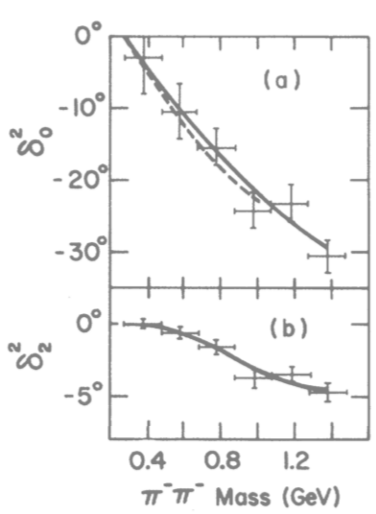


$m_\pi L$ 4 5 6

increasing total momentum



repulsive scattering, no resonances

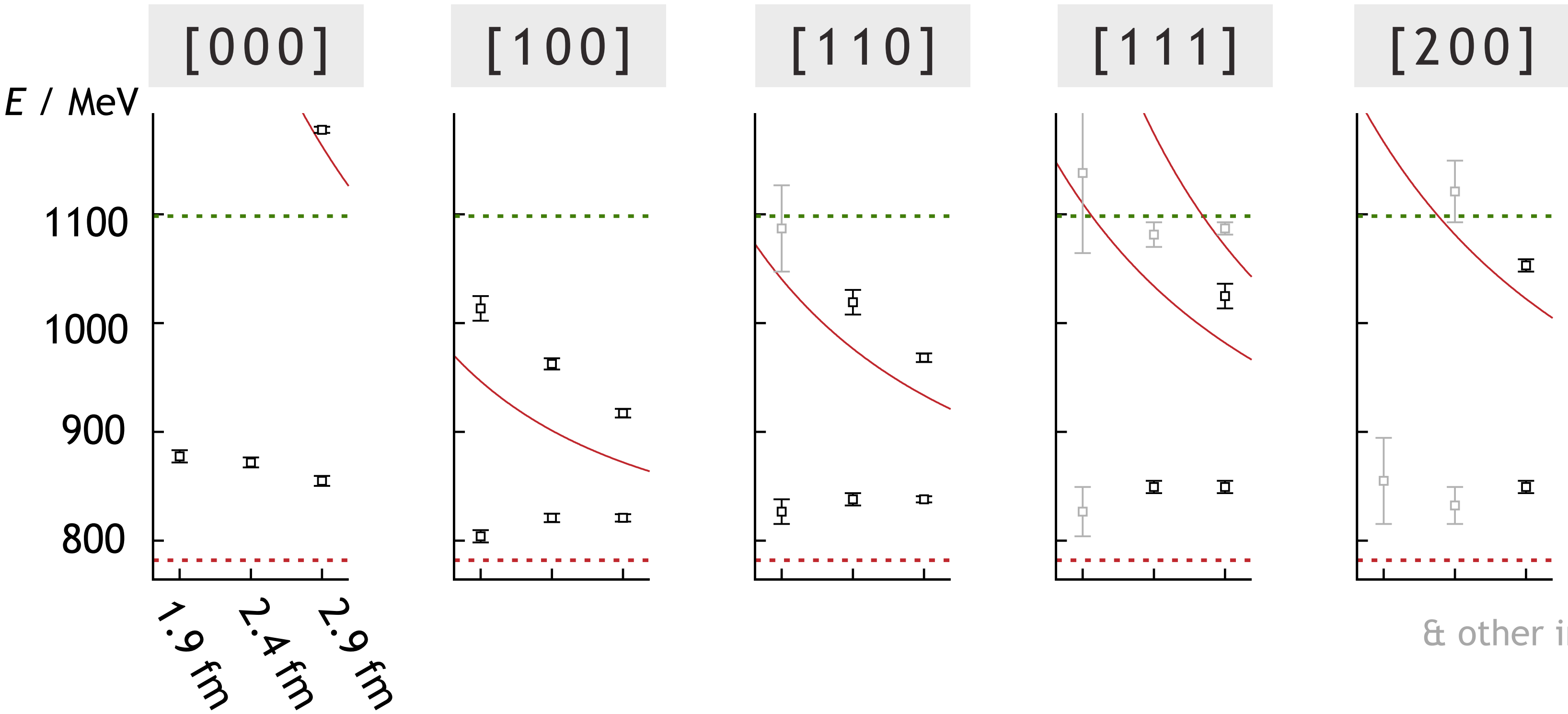


Cohen 1972

an elastic resonance — the ρ in $\pi\pi$ (isospin=1)

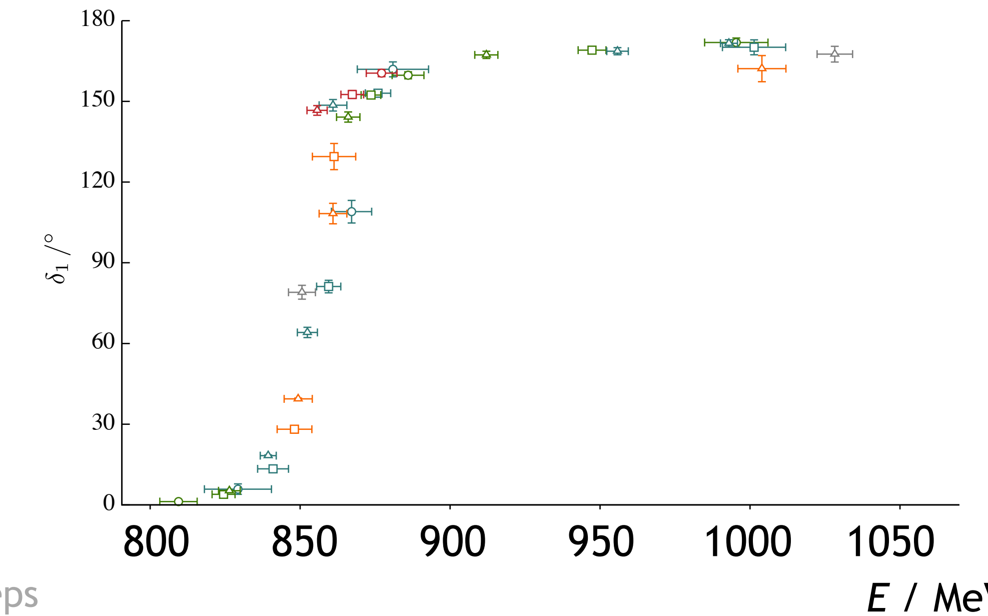
PRD87 034505 (2013)

$m_\pi \sim 391$ MeV



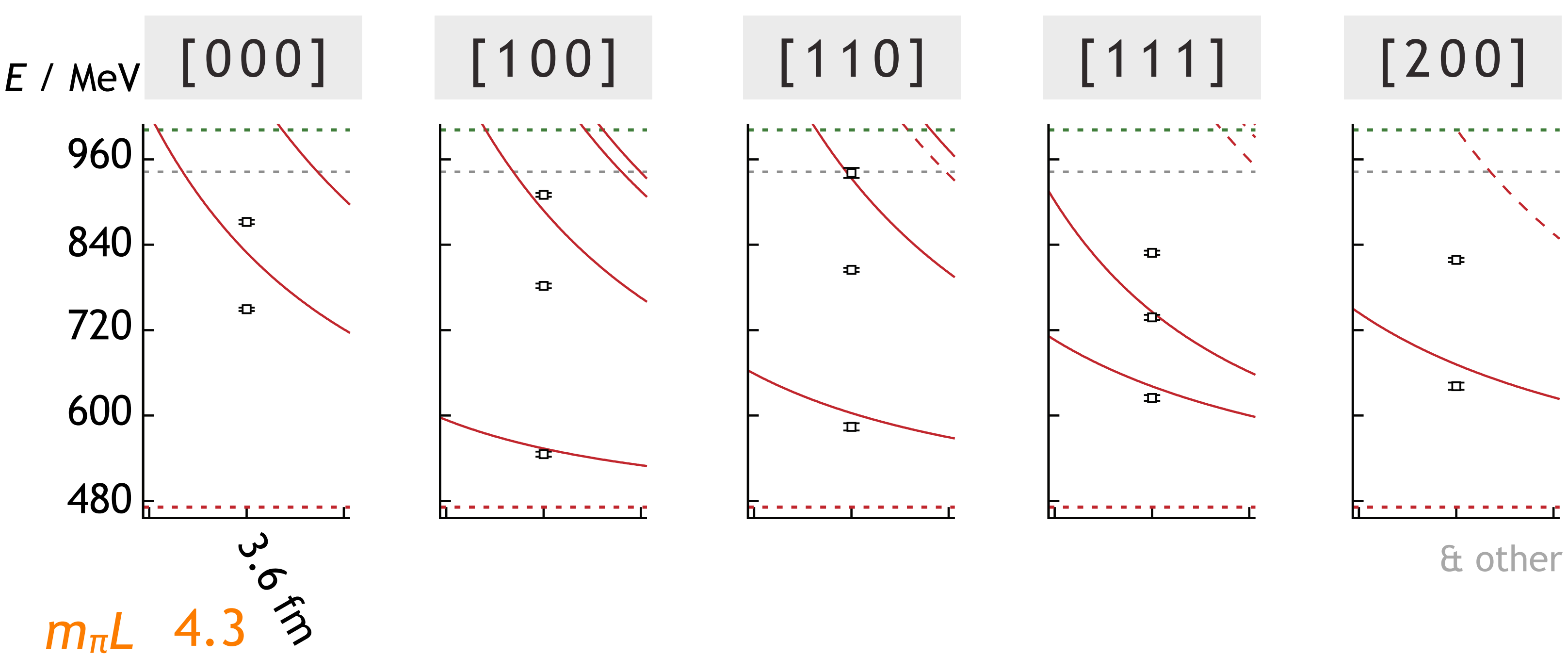
& other irreps

scattering phase-shift



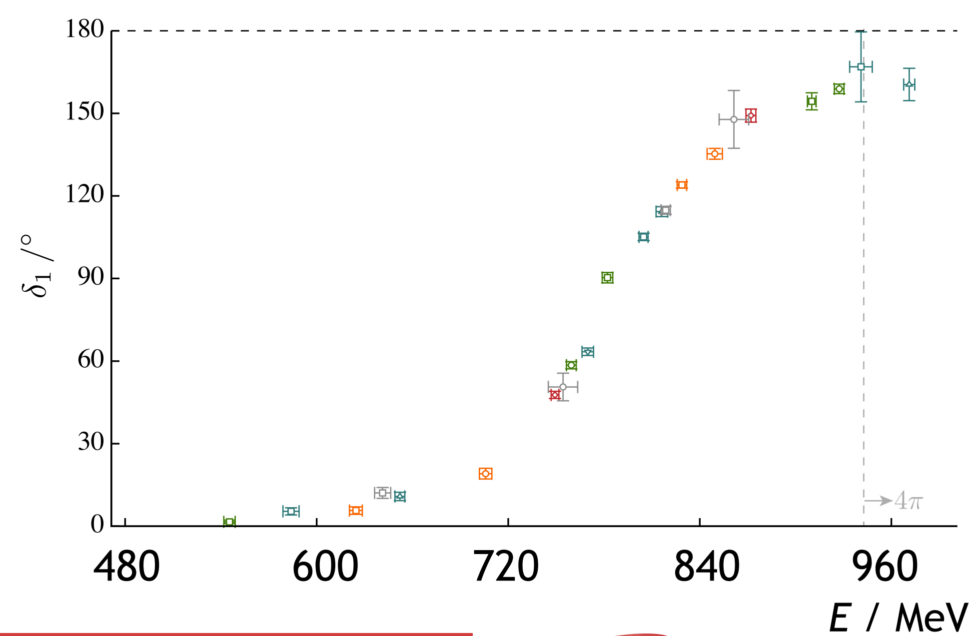
PRD92 094502 (2015)

$m_\pi \sim 236$ MeV



& other irreps

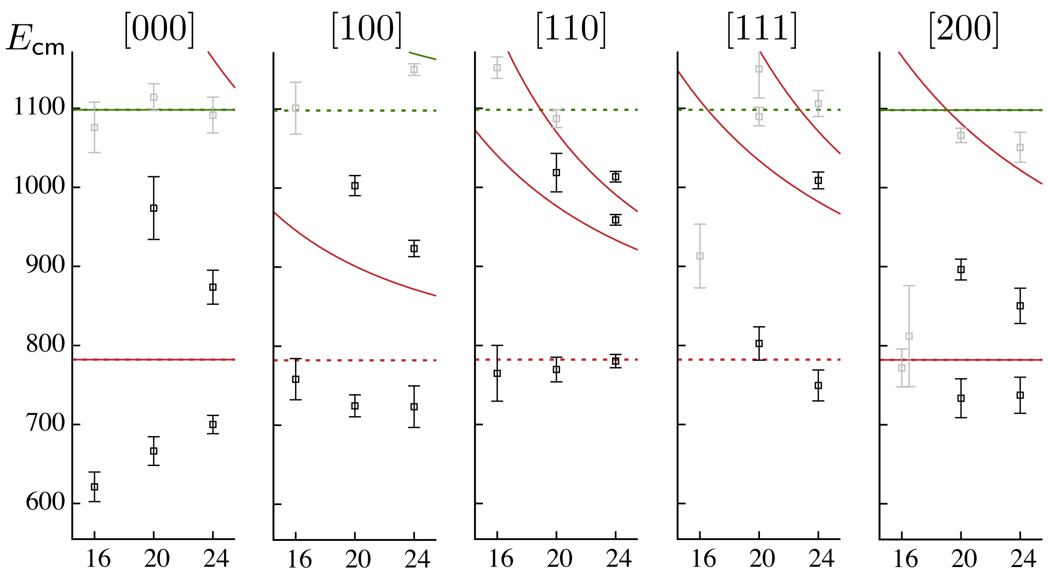
scattering phase-shift



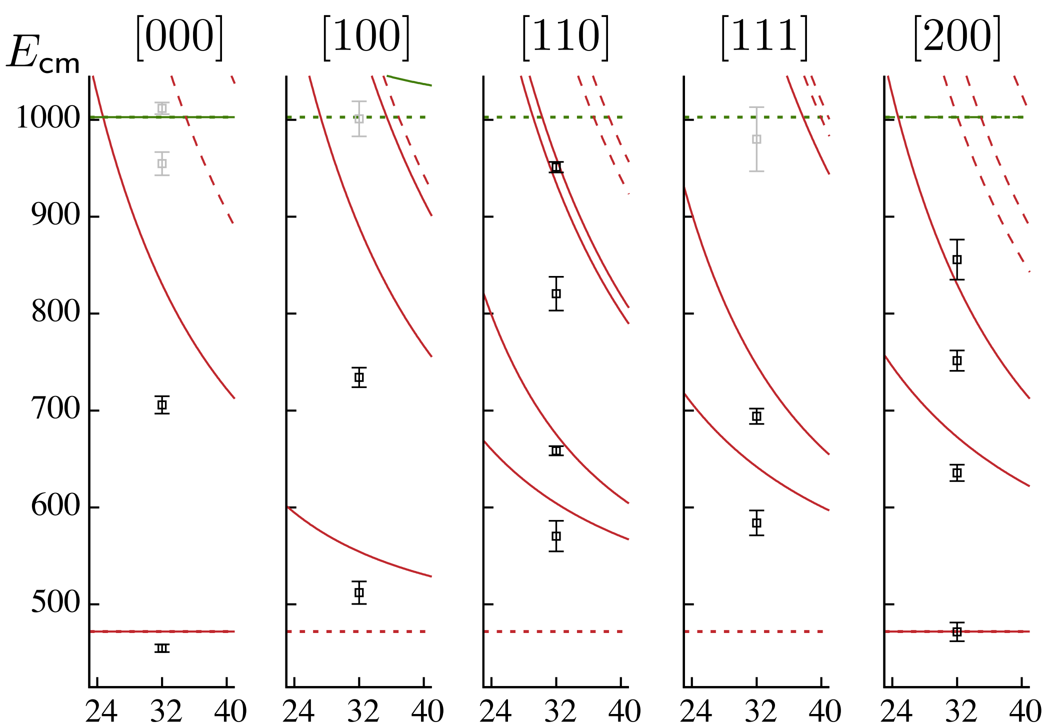
$m_\pi L$ 4.3

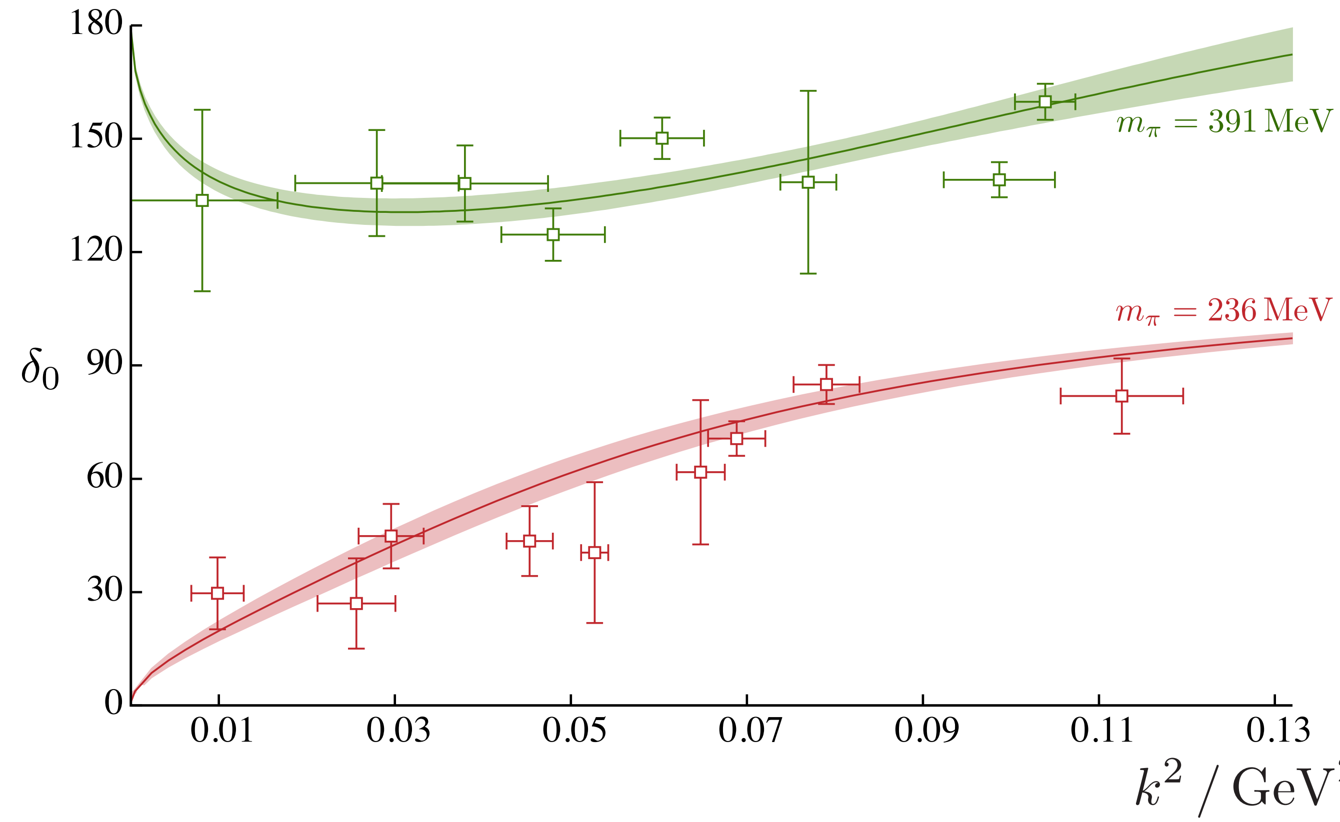
$\pi\pi$ isospin=0

m_π~391 MeV



m_π~236 MeV





heavier quark mass – **a bound-state**

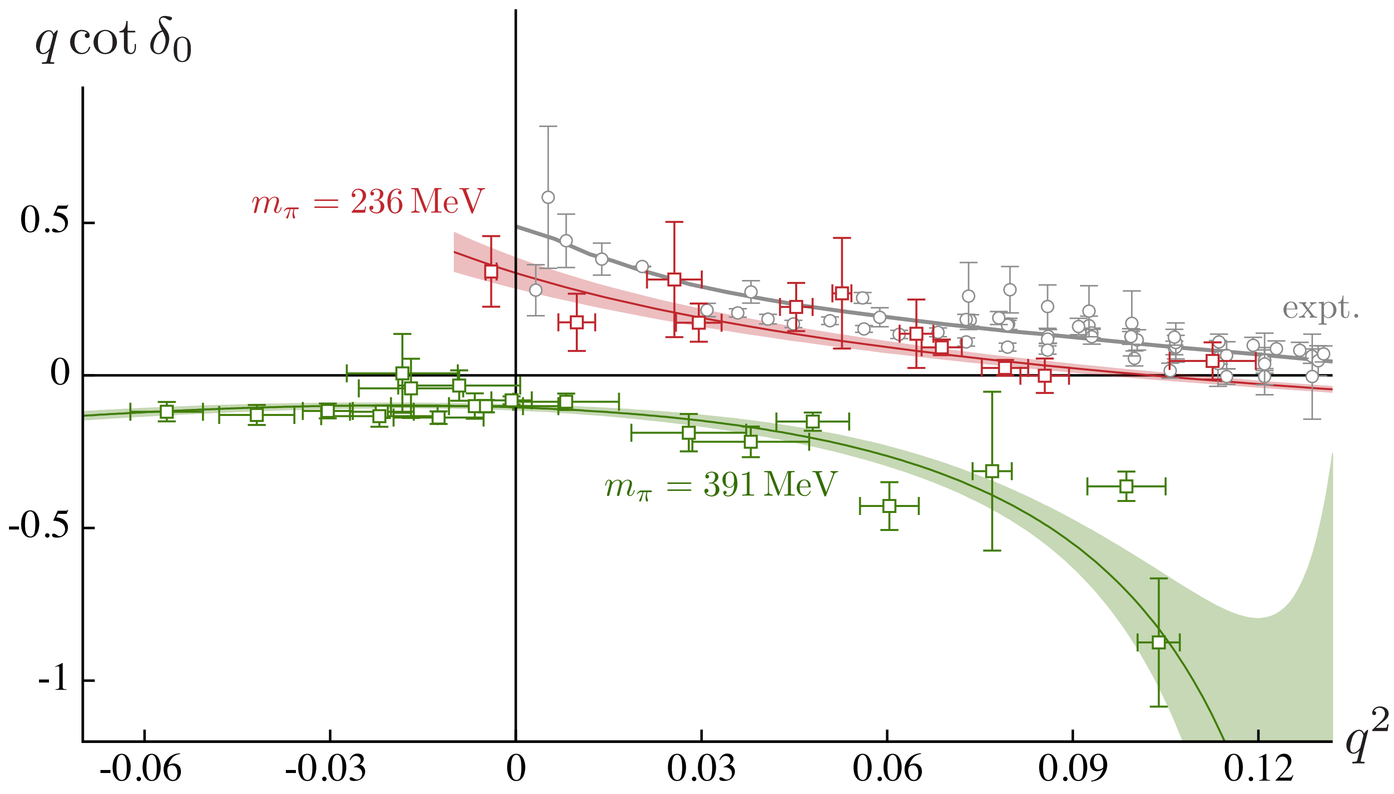
lighter quark mass – attraction, maybe a **broad resonance** ?

c.f. the experimental σ resonance ...

quark mass dependence of σ meson

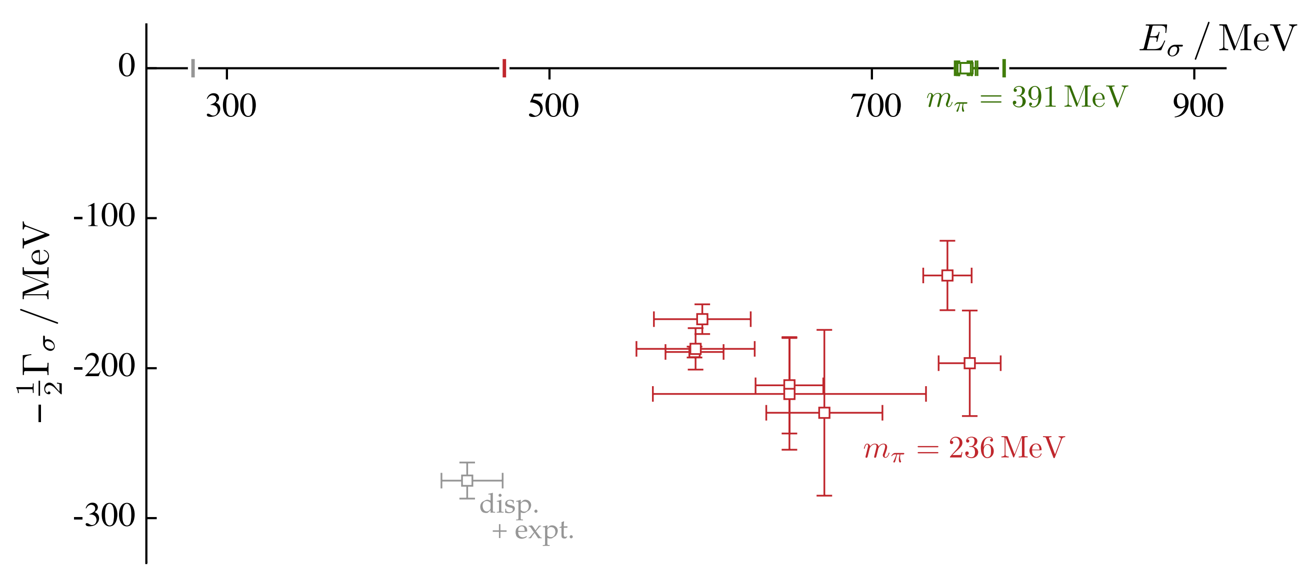
elastic part of the calculation also done on a lighter quark mass

ππ isospin=0 elastic scattering

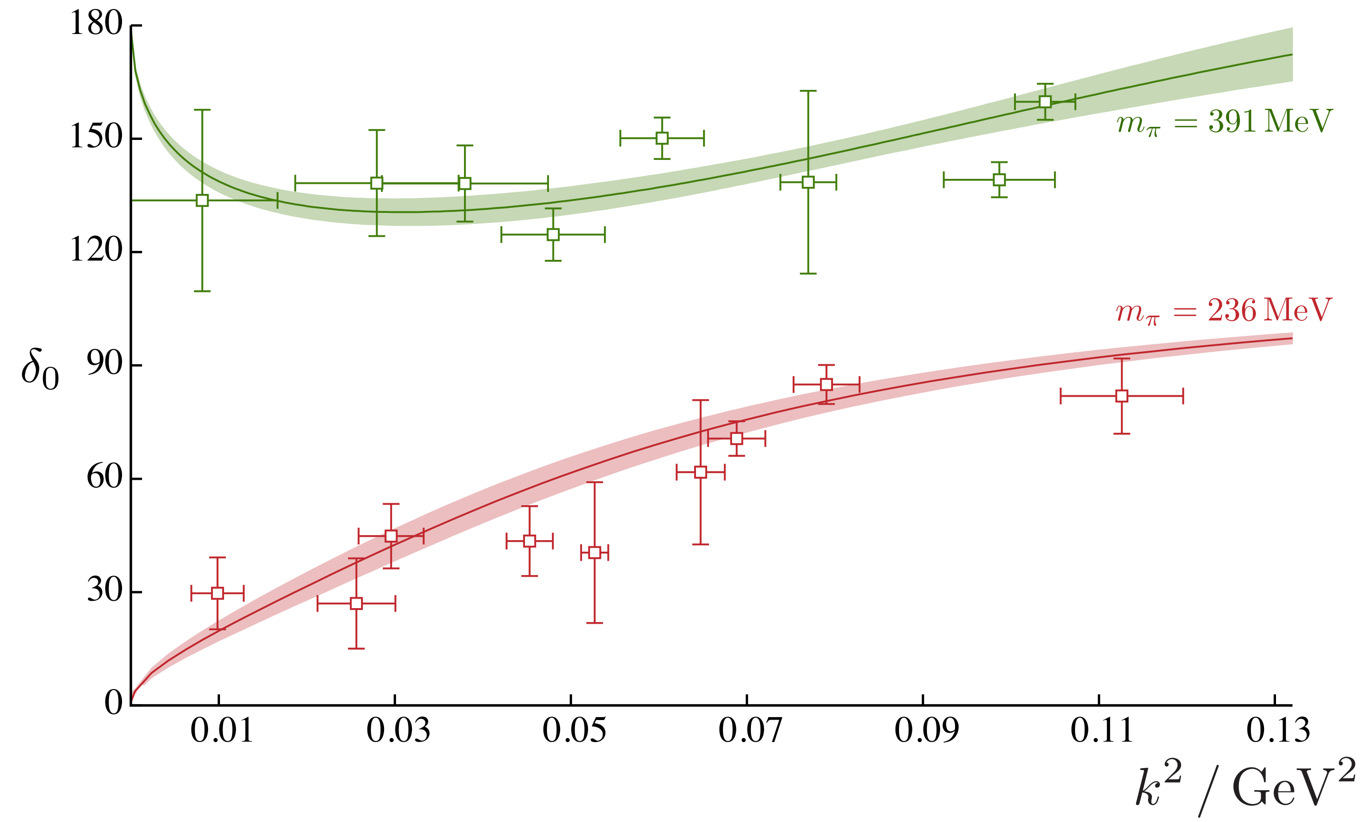


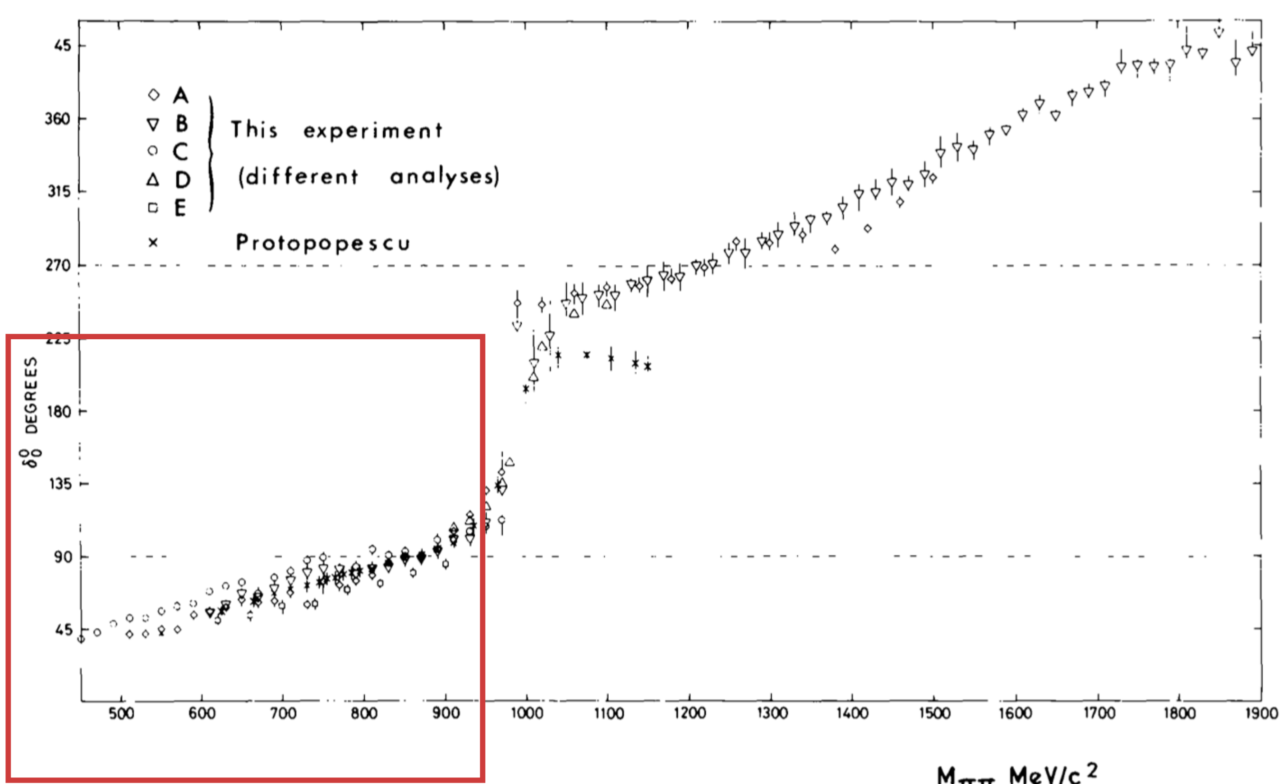
σ pole singularity

evolves from a bound-state to a broad resonance as the pion mass decreases

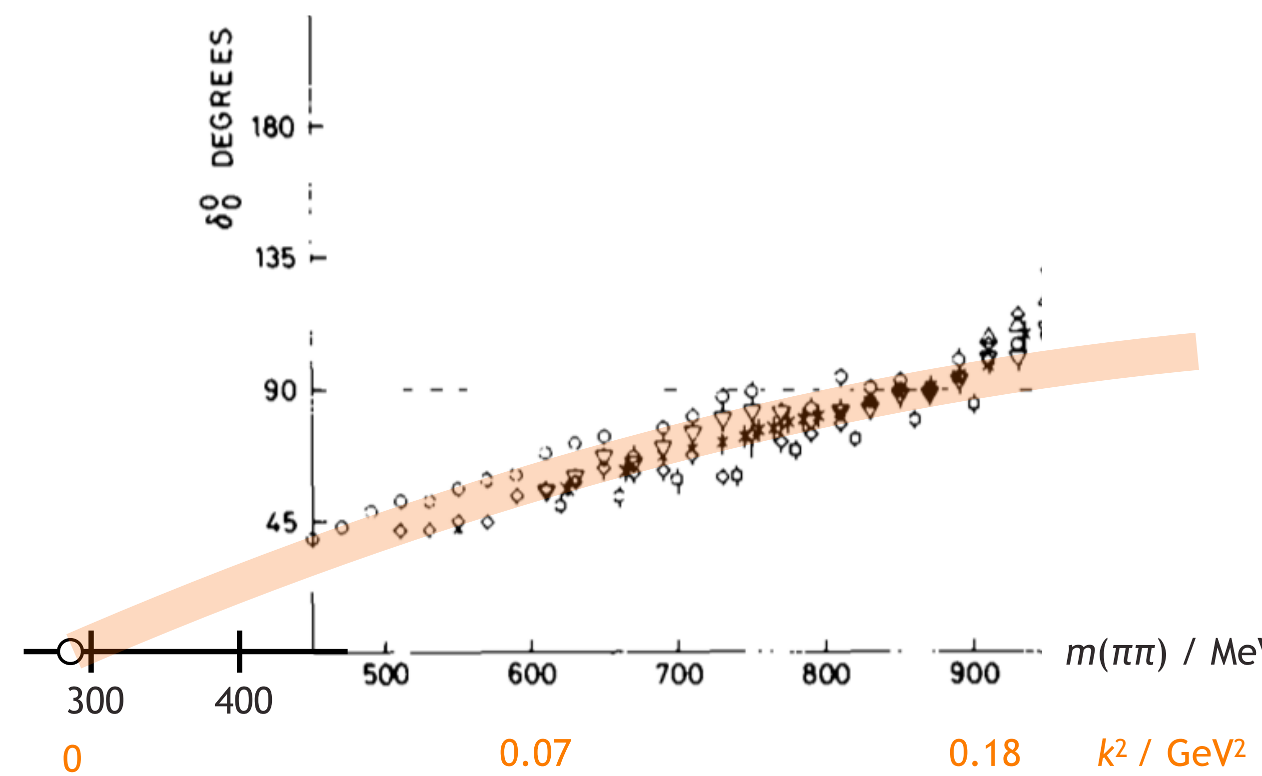


$\pi\pi$ isospin=0



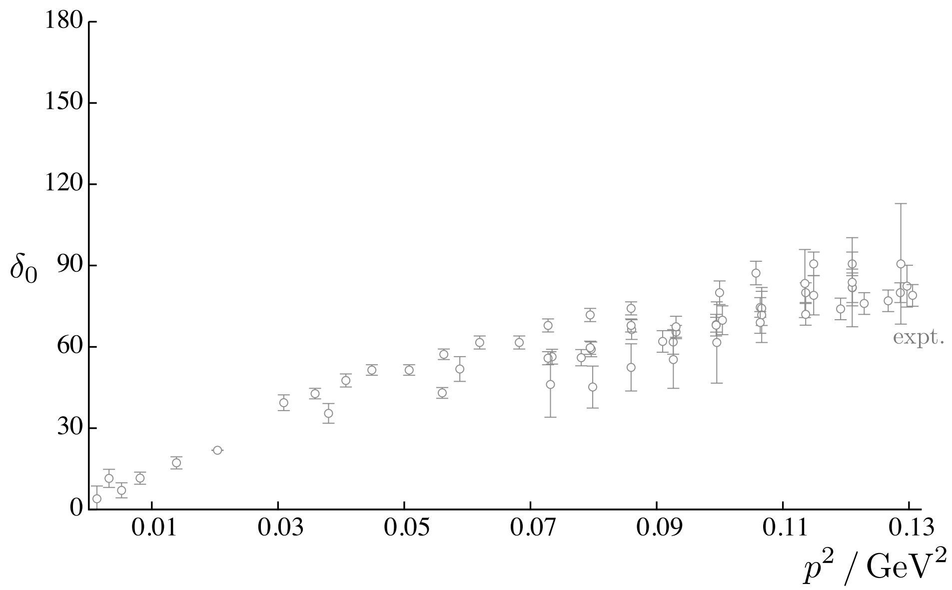


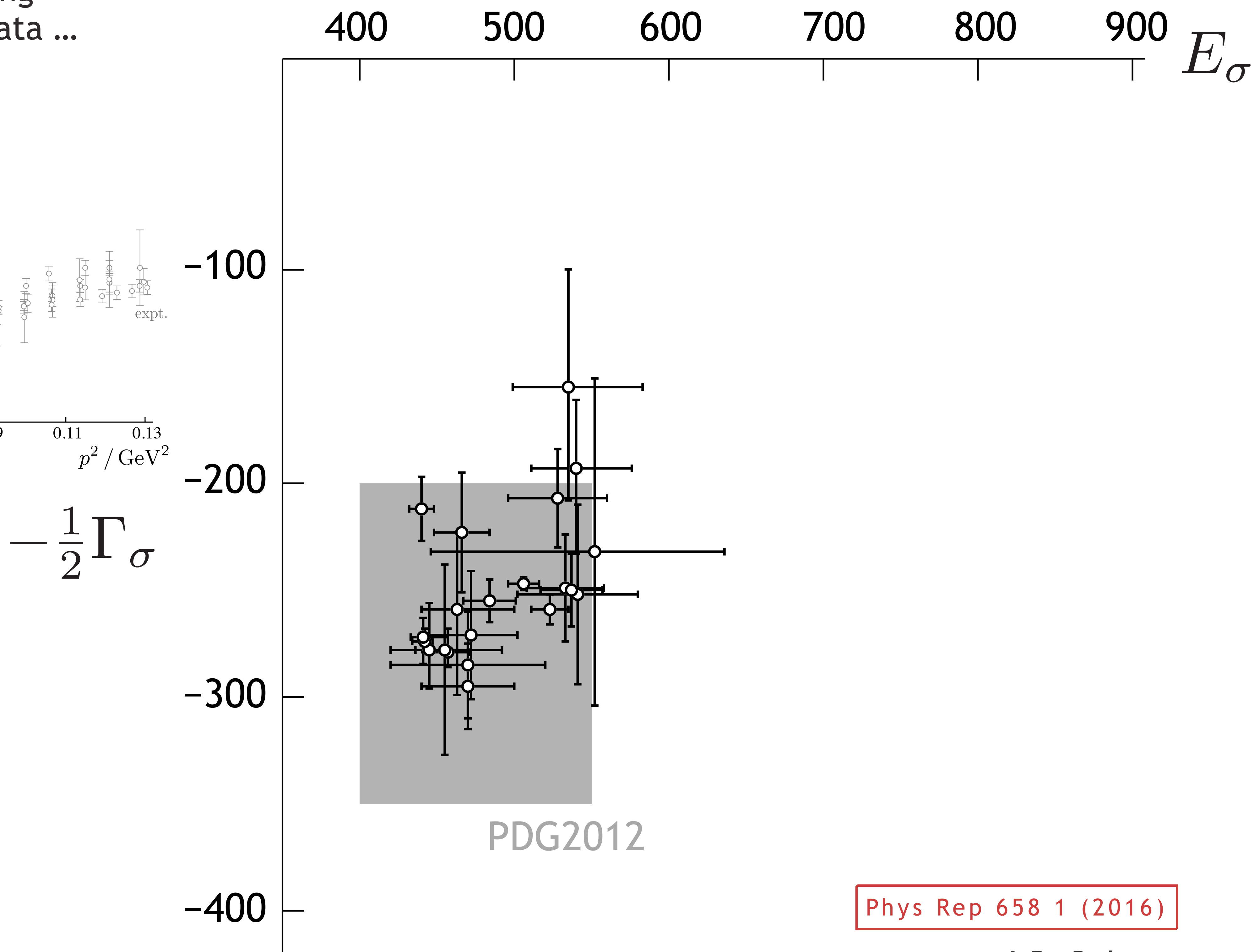
Grayer 1974



σ pole scatter from experimental phase-shift data

actually same problem using experimental scattering data ...

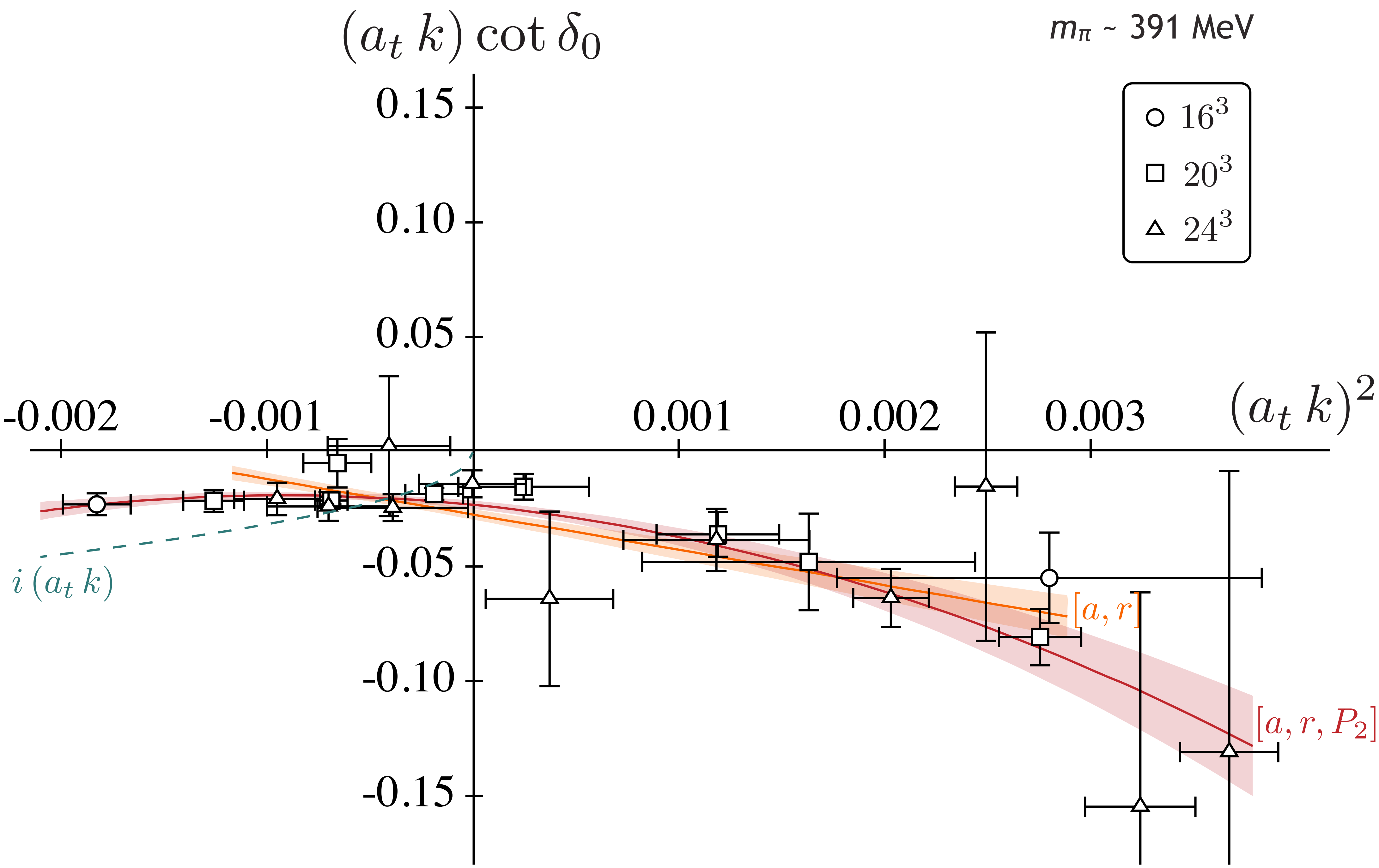




Phys Rep 658 1 (2016)

J.R. Pelaez

σ bound state – elastic and subthreshold region



some technical stuff — rotational symmetry

a finite cubic lattice has a smaller rotational symmetry group than an infinite continuum

simpler example of the problem: a rotationally symmetric two-dim system $\psi(r,\theta) = R_m(r)\, e^{im\theta}$

now considered on a square grid — minimum rotation is by $\pi/2$

m and *m*+4*n* transform the same !

back in 3D — **irreducible representations** of the reduced symmetry group contain multiple spins

cubic symmetry

$\Lambda(\text{dim})$	$A_1(1)$	$T_1(3)$	$T_2(3)$	$E(2)$	$A_2(1)$
J	$\mathbf{0}, 4\ldots$	$\mathbf{1}, 3, 4\ldots$	$\mathbf{2}, 3, 4\ldots$	$\mathbf{2}, 4\ldots$	$\mathbf{3}\ldots$

subduction $\big|\Lambda,\rho\big\rangle = \sum_m S^{\Lambda,\rho}_{J,m}\, \big|J,m\big\rangle$

for non-zero momentum it's even worse
— in continuum have **little group**, those rotations which don't change ***p***

⇒ label by **helicity**

can subduce helicity states into irreps of the reduced cubic symmetry

PRD85 014507 (2012)

some technical stuff — rotational symmetry

reduction of rotational symmetry is an important feature of the quantization condition too

for elastic scattering, what we previously presented as $\cot\delta_\ell(E) = \mathcal{M}_\ell\big(E(L), L\big)$

should actually be $0 = \det\Big[\cot\delta_\ell \; \delta_{\ell,\ell'}\, \delta_{m,m'} - \mathcal{M}_{\ell m;\ell' m'}\Big]$

which when subduced becomes $0 = \det\Big[\cot\delta_\ell \; \delta_{\ell,\ell'}\, \delta_{n,n'} - \mathcal{M}^{\Lambda}_{\ell n;\ell' n}\Big]$

features **all ℓ** subduced into irrep Λ

n = embedding of ℓ into Λ

what allows us to make progress is that $\delta_\ell(E) \sim k^{2\ell+1}$ at energies not too far from threshold

so higher angular momenta are naturally suppressed

in practice, truncate at some ℓ_{max} ...

the quantization condition generalizes to

$$0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}\right)\right]$$

coupled-channel scattering in a finite-volume

the quantization condition generalizes to

$$0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}\right)\right]$$

e.g. in A_1^+ irrep (ℓ = 0, 4 …)

$$\mathbf{t} = \begin{pmatrix} \begin{pmatrix} t^{(0)}_{11} & t^{(0)}_{12} \\ t^{(0)}_{12} & t^{(0)}_{22} \end{pmatrix} & \mathbf{0} & \cdots \\ \mathbf{0} & \begin{pmatrix} t^{(4)}_{11} & t^{(4)}_{12} \\ t^{(4)}_{12} & t^{(4)}_{22} \end{pmatrix} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

dense in channel space
– infinite-volume dynamics mixes channels

diagonal in angular momentum space
– ℓ good q.n. in infinite-volume

coupled-channel scattering in a finite-volume

the quantization condition generalizes to

$$0 = \det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}\right)\right]$$

e.g. in $A_1{}^+$ irrep (ℓ = 0, 4 …)

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dense in channel space
— infinite-volume dynamics mixes channels

diagonal in angular momentum space
— ℓ good q.n. in infinite-volume

$$\boldsymbol{\mathcal{M}} = \begin{pmatrix} \begin{pmatrix} \mathcal{M}^{A_1^+}_{00}(k_1) & 0 \\ 0 & \mathcal{M}^{A_1^+}_{00}(k_2) \end{pmatrix} & \begin{pmatrix} \mathcal{M}^{A_1^+}_{04}(k_1) & 0 \\ 0 & \mathcal{M}^{A_1^+}_{04}(k_2) \end{pmatrix} & \cdots \\ \begin{pmatrix} \mathcal{M}^{A_1^+}_{40}(k_1) & 0 \\ 0 & \mathcal{M}^{A_1^+}_{40}(k_2) \end{pmatrix} & \begin{pmatrix} \mathcal{M}^{A_1^+}_{44}(k_1) & 0 \\ 0 & \mathcal{M}^{A_1^+}_{44}(k_2) \end{pmatrix} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

diagonal in channel space
— no dynamics

dense in angular momentum
— cubic symmetry lives here

$$k_1 = \tfrac{1}{2}\sqrt{E^2 - 4m_1^2}$$

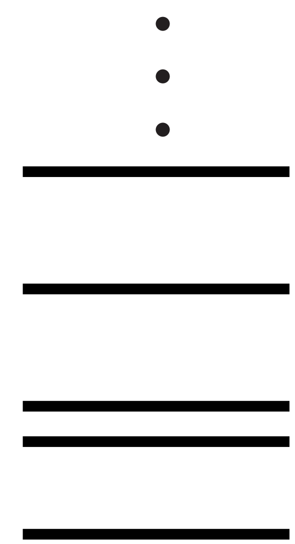
$$k_2 = \tfrac{1}{2}\sqrt{E^2 - 4m_2^2}$$

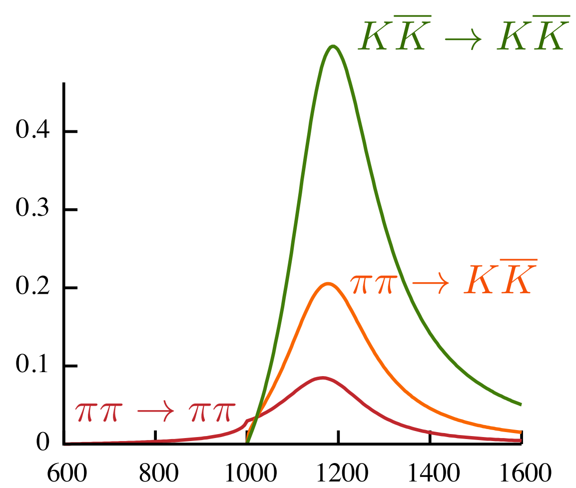
excited states as unstable resonances

in the case of **coupled-channel scattering** it's more challenging …

e.g. some energy region where $\pi\pi$, $K\bar{K}$ accessible

$$\mathbf{S}(E) = \begin{bmatrix} S_{\pi\pi,\pi\pi}(E) & S_{\pi\pi,K\bar{K}}(E) \\ S_{\pi\pi,K\bar{K}}(E) & S_{K\bar{K},K\bar{K}}(E) \end{bmatrix} \xrightarrow{L\times L\times L} E_n(L)$$





you want to know this

?

lattice QCD gives you this

an approach:

parameterize the energy dependence of the scattering matrix

an important observation
— not like experiment
— can't study 'channel-by-channel'
— all channels contribute, have to solve the 'whole problem'

what do you calculate

calculate correlation functions

e.g. $\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle$ where the operators are constructed from quark and gluon fields and have the quantum numbers of the hadronic system you want to study

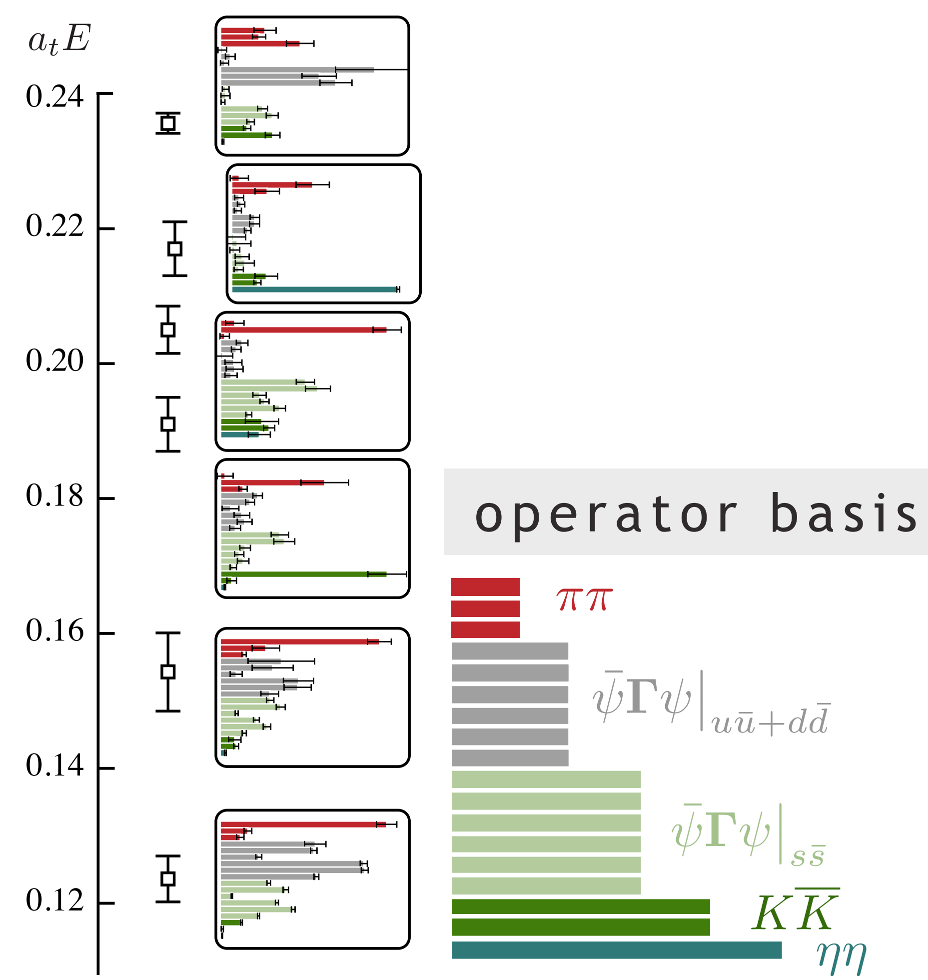
$$\langle 0 | \mathcal{O}_i(t) \mathcal{O}_j(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | 0 \rangle \, e^{-E_n t}$$

a superposition of the (finite-volume) eigenstates of QCD

powerful approach:

- use a large basis of operators*
- form a matrix of correlation functions
- diagonalize this matrix

e.g. [000] A_1^+ 24^3



* could give a whole interesting talk on the construction of these operators

operator basis — I=0 $\pi\pi$, $K\bar{K}$, $\eta\eta$

operator basis: 'single-meson' + 'meson-meson'

$$\bar{\psi}\mathbf{\Gamma}\psi$$

(& if you like, tetraquark & …)

$$\sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

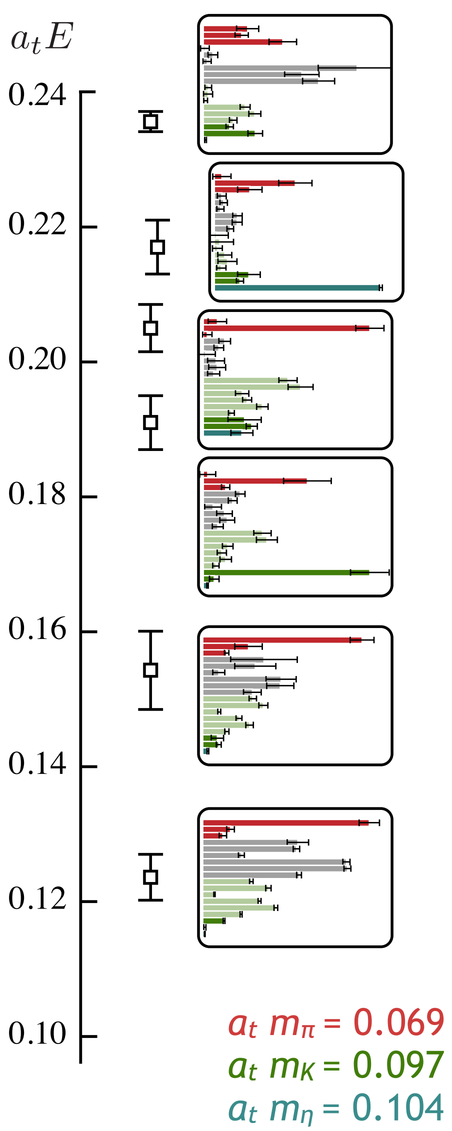
$$\mathbf{p} = \frac{2\pi}{L}[n_x, n_y, n_z]$$

maximum momentum guided by non-interacting energies

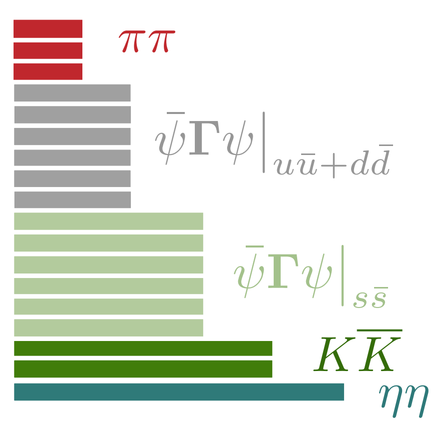
$$\sqrt{m_1^2 + \mathbf{p}_1^2} + \sqrt{m_2^2 + \mathbf{p}_2^2}$$

solutions of the det equation when **t** = **0**

[000] A_1^+ 24^3



operator basis

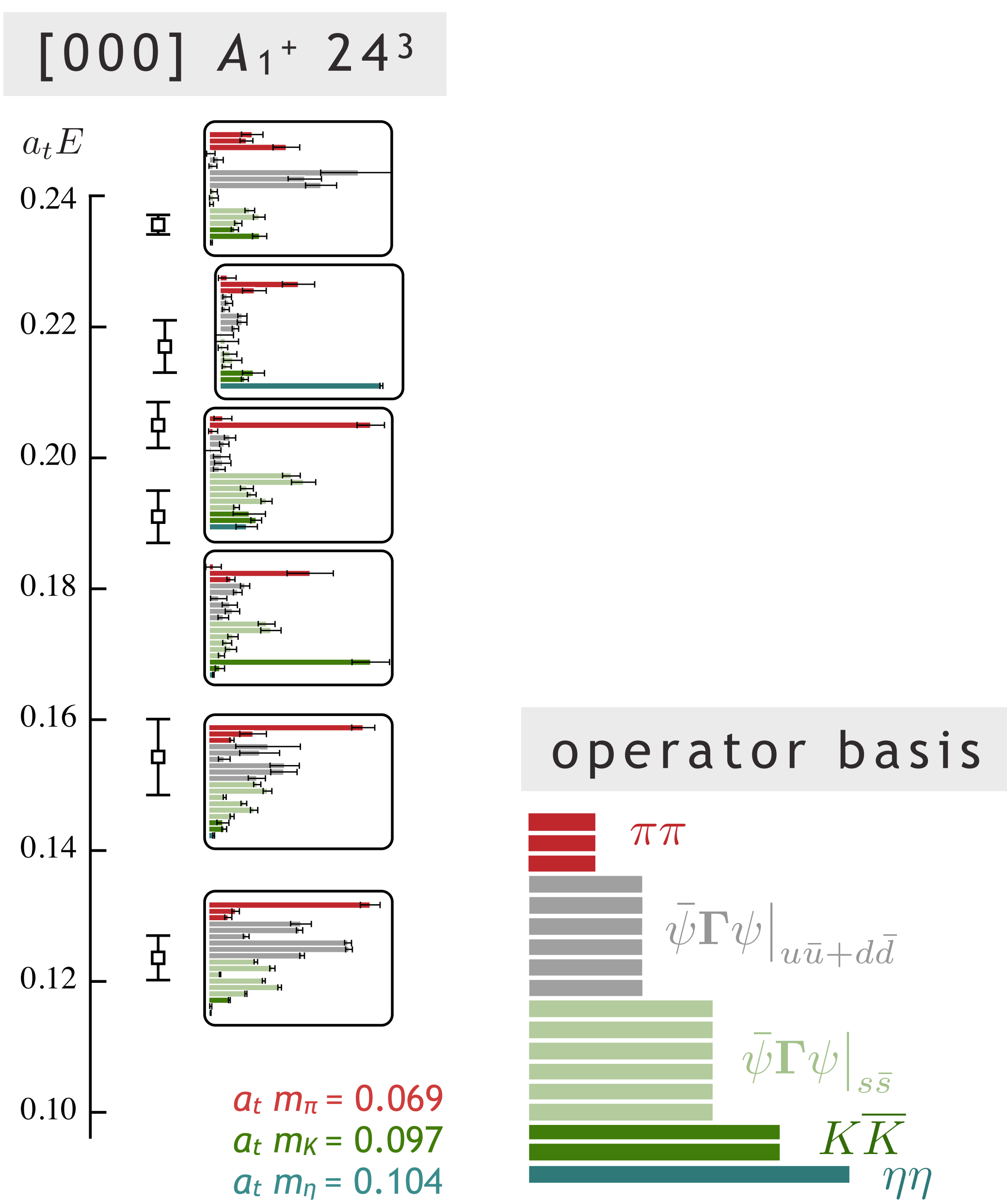


operator basis: ‘single-meson’ + ‘meson-meson’

$$\bar{\psi}\mathbf{\Gamma}\psi \qquad + \qquad \sum_{\hat{\mathbf{p}}_1,\hat{\mathbf{p}}_2} C(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p})\, M_1(\mathbf{p}_1)\, M_2(\mathbf{p}_2)$$

$$\sum_{\mathbf{x}} e^{i\mathbf{p}_1\cdot\mathbf{x}}\, \bar{\psi}_{\mathbf{x}}\mathbf{\Gamma}\psi_{\mathbf{x}} \sum_{\mathbf{y}} e^{i\mathbf{p}_2\cdot\mathbf{y}}\, \bar{\psi}_{\mathbf{y}}\mathbf{\Gamma}'\psi_{\mathbf{y}}$$

sampling the whole lattice volume

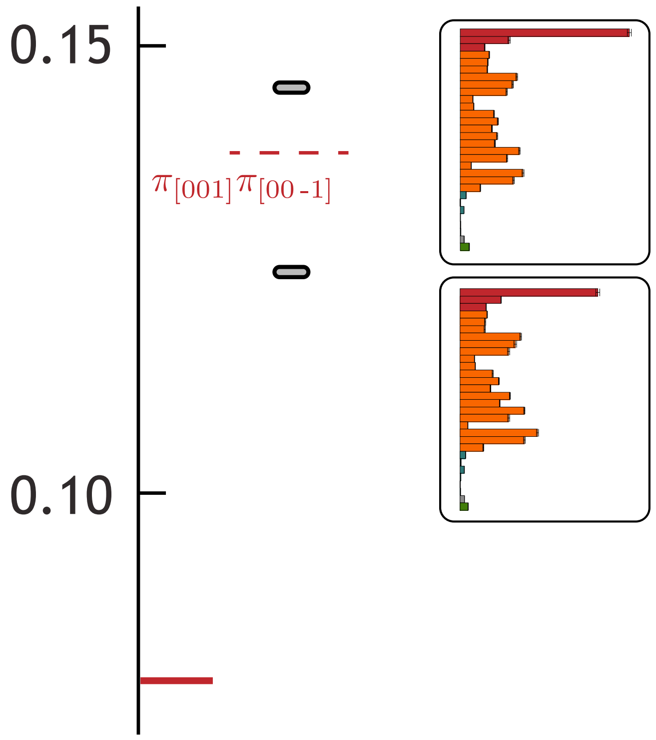


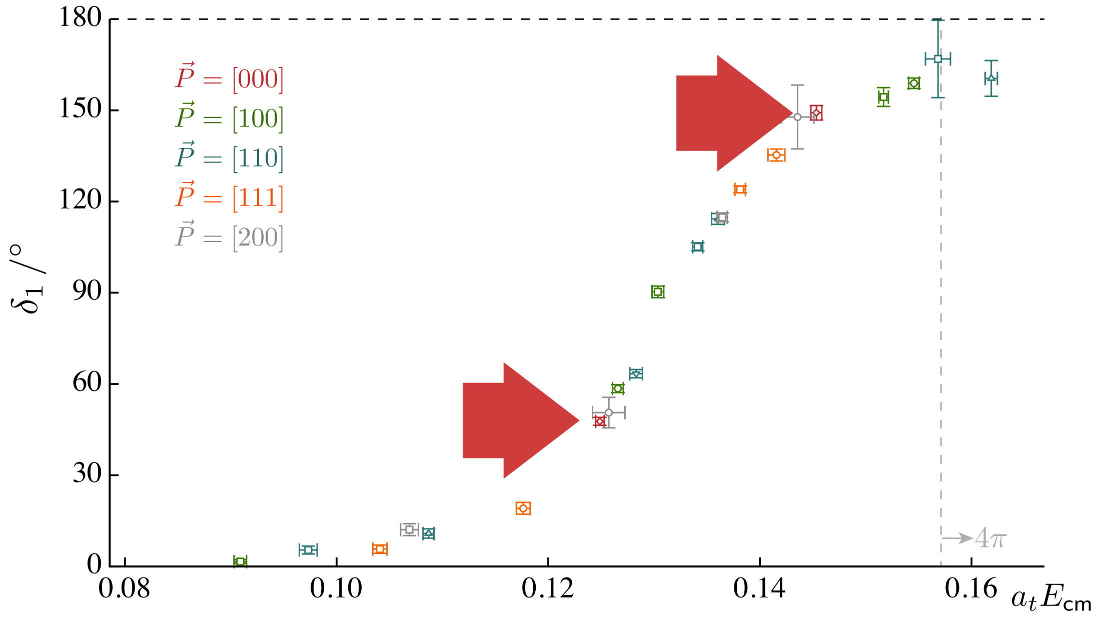
prefer to use
optimized single-meson operators …

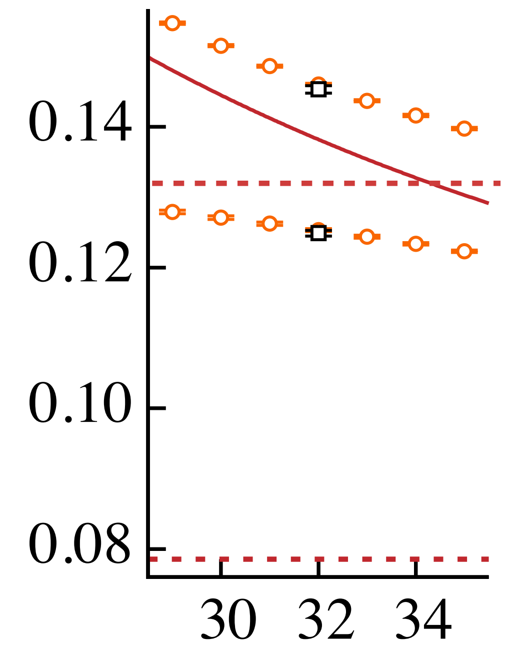
$m_\pi = 0.039$
$m_K = 0.083$
$L \sim 3.8$ fm

what's happening here ?

focus on the lowest two states





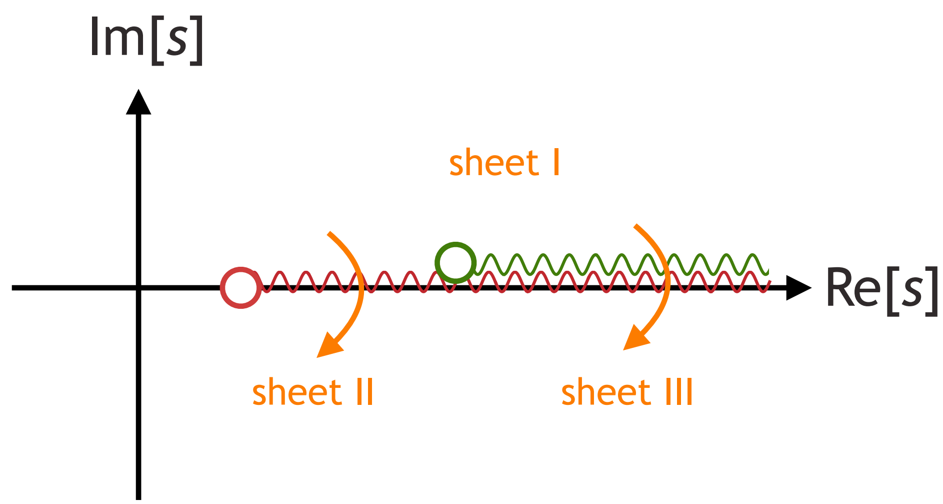


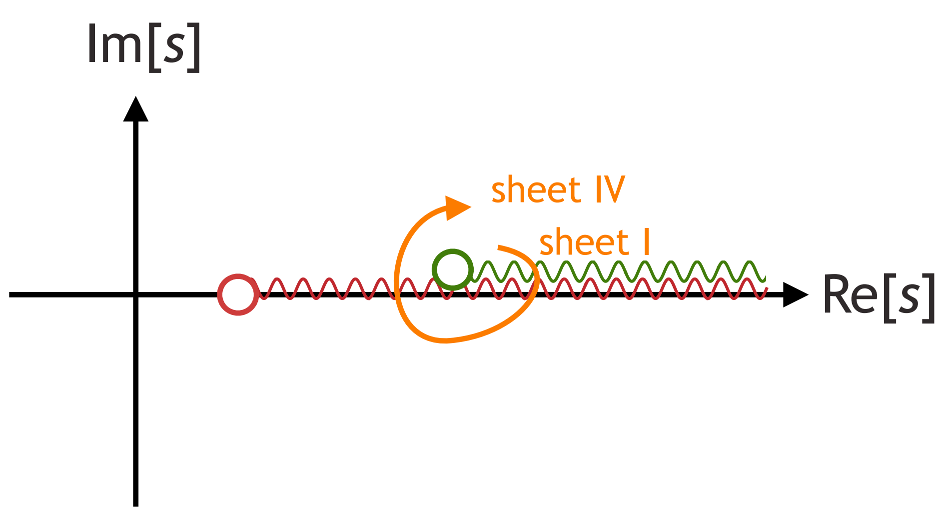
an avoided level crossing

coupled-channel Riemann sheet structure

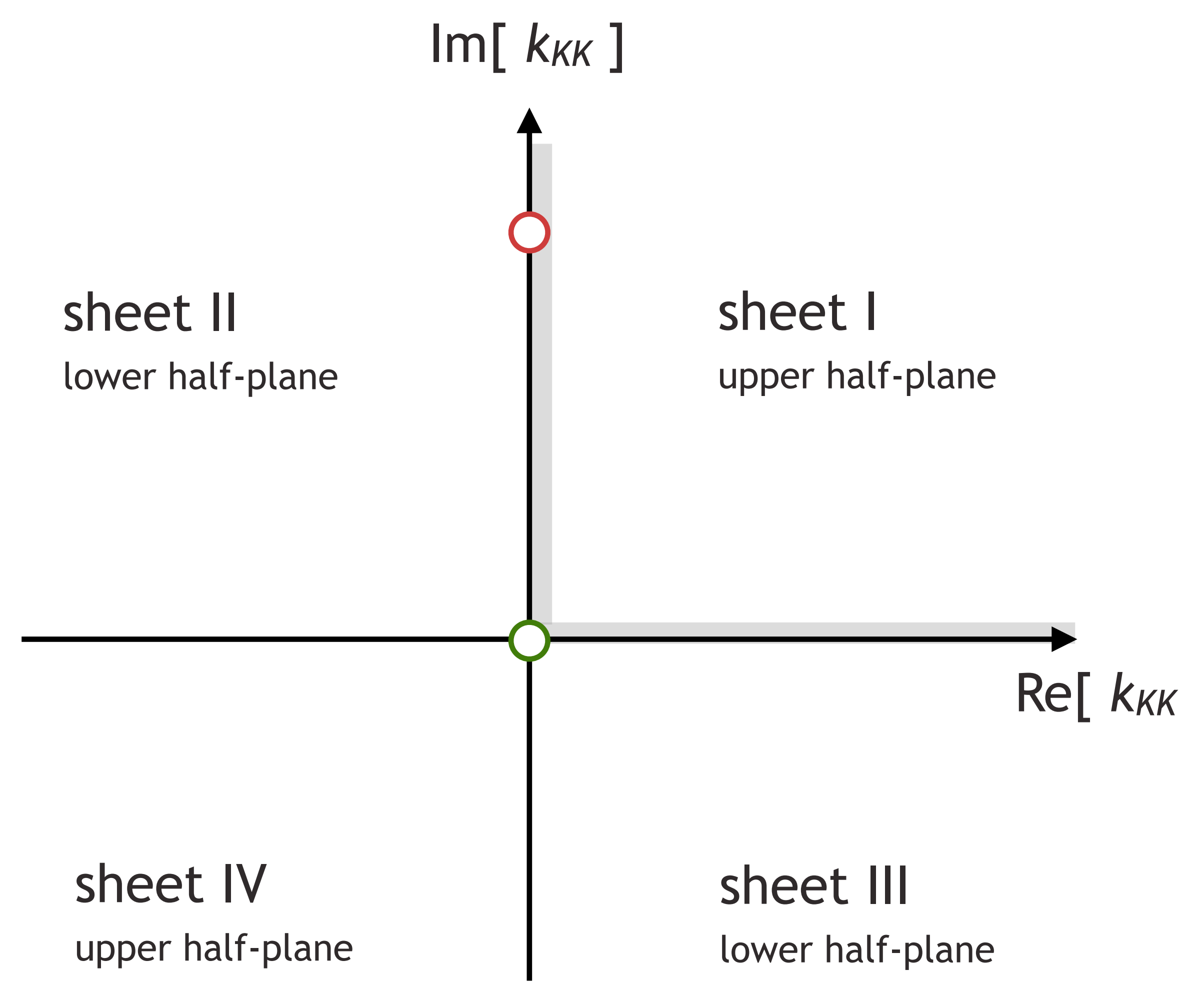
for each new channel, each sheet splits in two ⇒ 2^N sheets for N channels

e.g. two channels

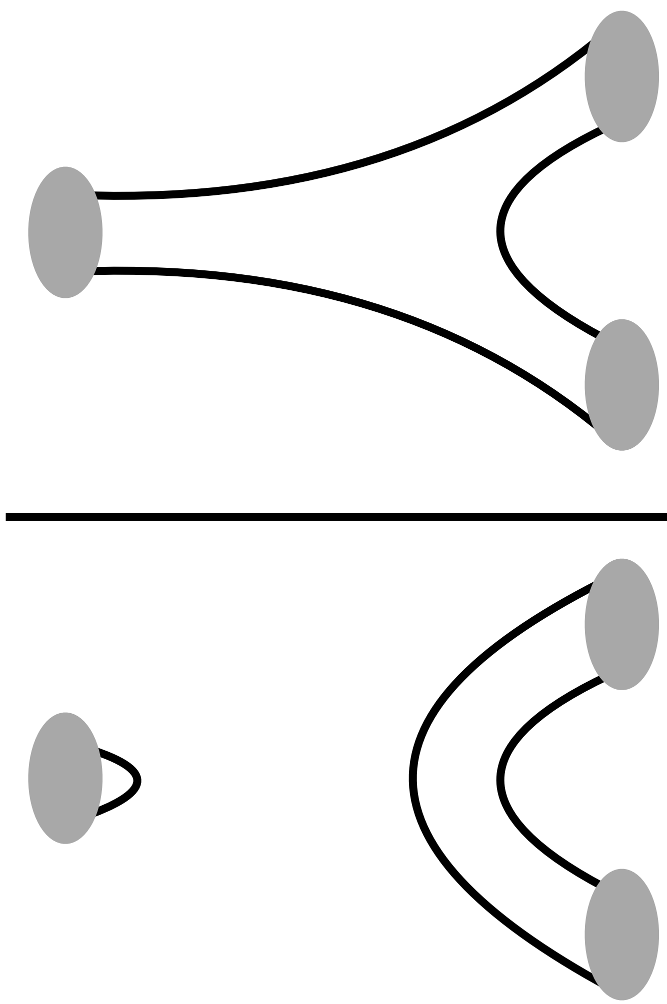


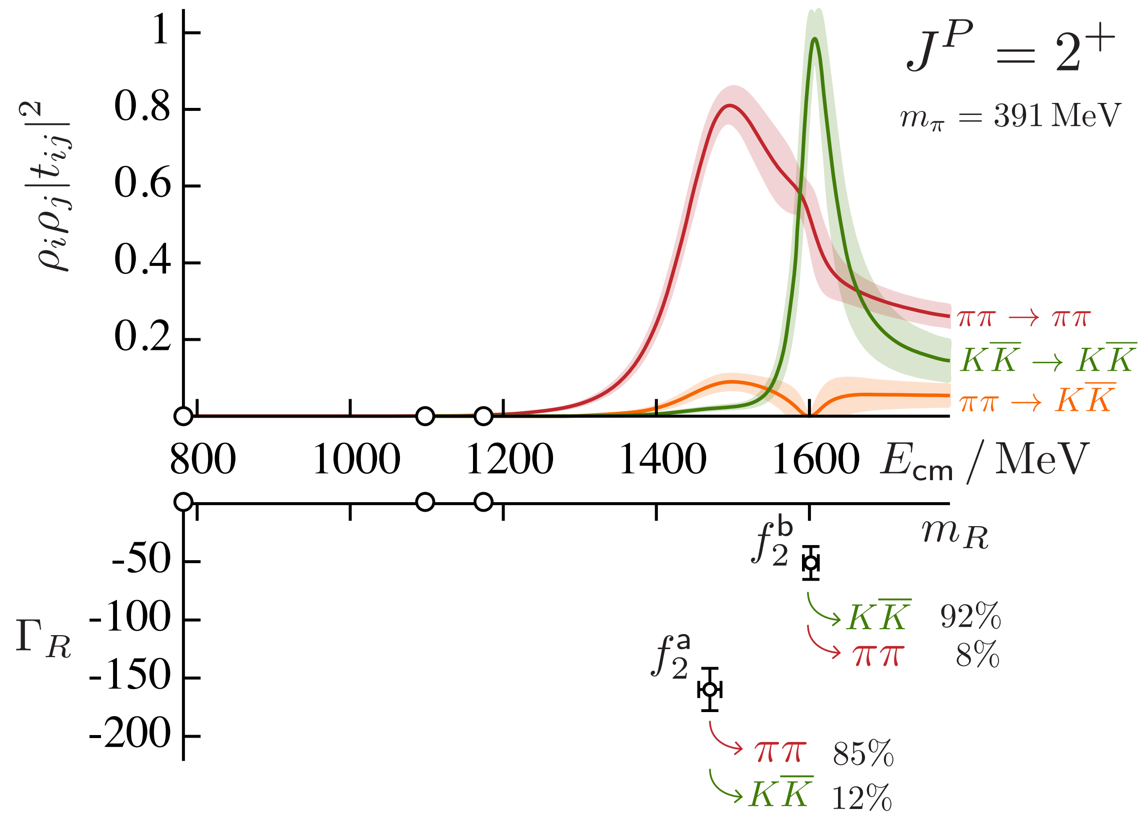


	$\text{Im}[\ k_{\pi\pi}\ ]$	$\text{Im}[\ k_{KK}\ ]$
sheet I	+	+
sheet II	−	+
sheet III	−	−
sheet IV	+	−



f_2 resonances — decay couplings & OZI





$f_2^a \sim u\bar{u} + d\bar{d}$ $f_2^b \sim s\bar{s}$

couplings from pole residue

	$\frac{a_t \lvert c_{\pi\pi} \rvert}{(a_t k_{\pi\pi})^2}$	$\frac{a_t \lvert c_{K\bar{K}} \rvert}{(a_t k_{K\bar{K}})^2}$
f_2^a	7.1(4)	4.8(9)
f_2^b	1.0(3)	5.5(8)

zero in 'OZI' limit
— requires $s\bar{s}$ annihilation

Luescher finite-volume functions

$$0 = \det\Big[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\big(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\big)\Big]$$

$$\overline{\mathcal{M}}_{\ell J m,\ell' J' m'} = \sum_{m_\ell, m'_\ell, m_S} \langle \ell m_\ell; 1 m_S | J m\rangle \langle \ell' m'_\ell; 1 m_S | J' m'\rangle$$

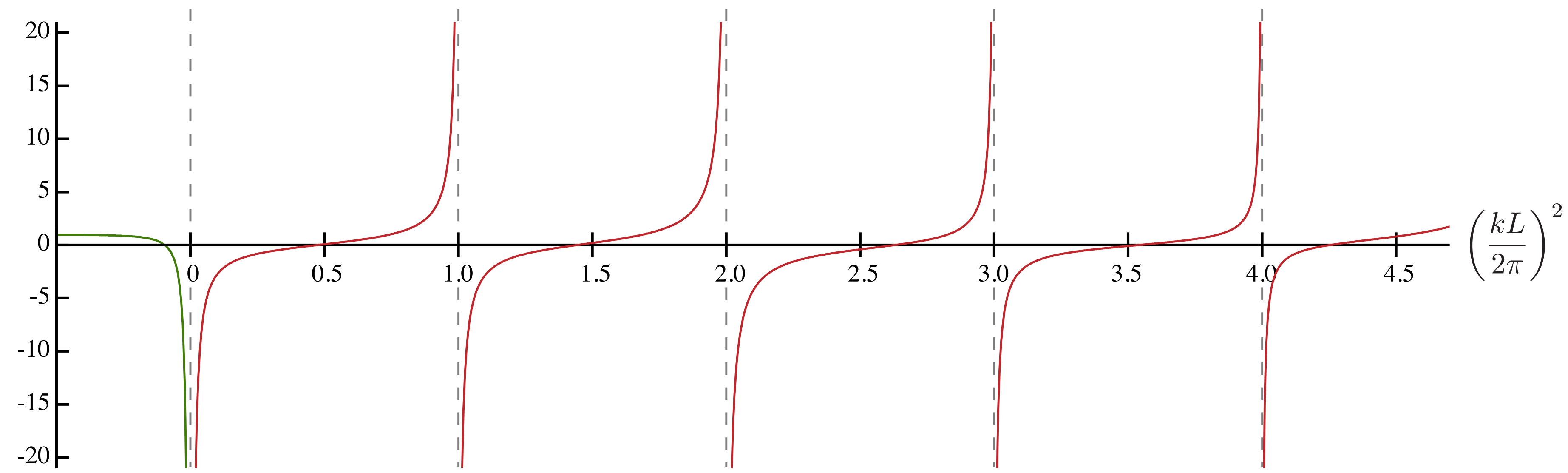
$$\times \sum_{\bar{\ell},\bar{m}_\ell} \frac{(4\pi)^{3/2}}{k_{\mathsf{cm}}^{\bar{\ell}+1}} c^{\vec{n}}_{\bar{\ell},\bar{m}_\ell}(k^2_{\mathsf{cm}};L) \int d\Omega\, Y^*_{\ell m_\ell} Y^*_{\bar{\ell}\bar{m}_\ell} Y_{\ell' m'_\ell}$$

"spinless" Luescher functions

to respect the lattice symmetries,
need to subduce into irreducible representations

$$\overline{\mathcal{M}}^{\vec{n},\Lambda}_{\ell J n,\ell' J' n'}\,\delta_{\Lambda,\Lambda'}\delta_{\mu,\mu'} = \sum_{\substack{m,\lambda \\ m',\lambda'}} \mathcal{S}^{J\lambda *}_{\Lambda\mu n}\, D^{(J)*}_{m\lambda}(R)\ \overline{\mathcal{M}}^{\vec{n}}_{\ell J m,\ell' J' m'}\ \mathcal{S}^{J'\lambda'}_{\Lambda'\mu' n'}\, D^{(J')}_{m'\lambda'}(R)$$

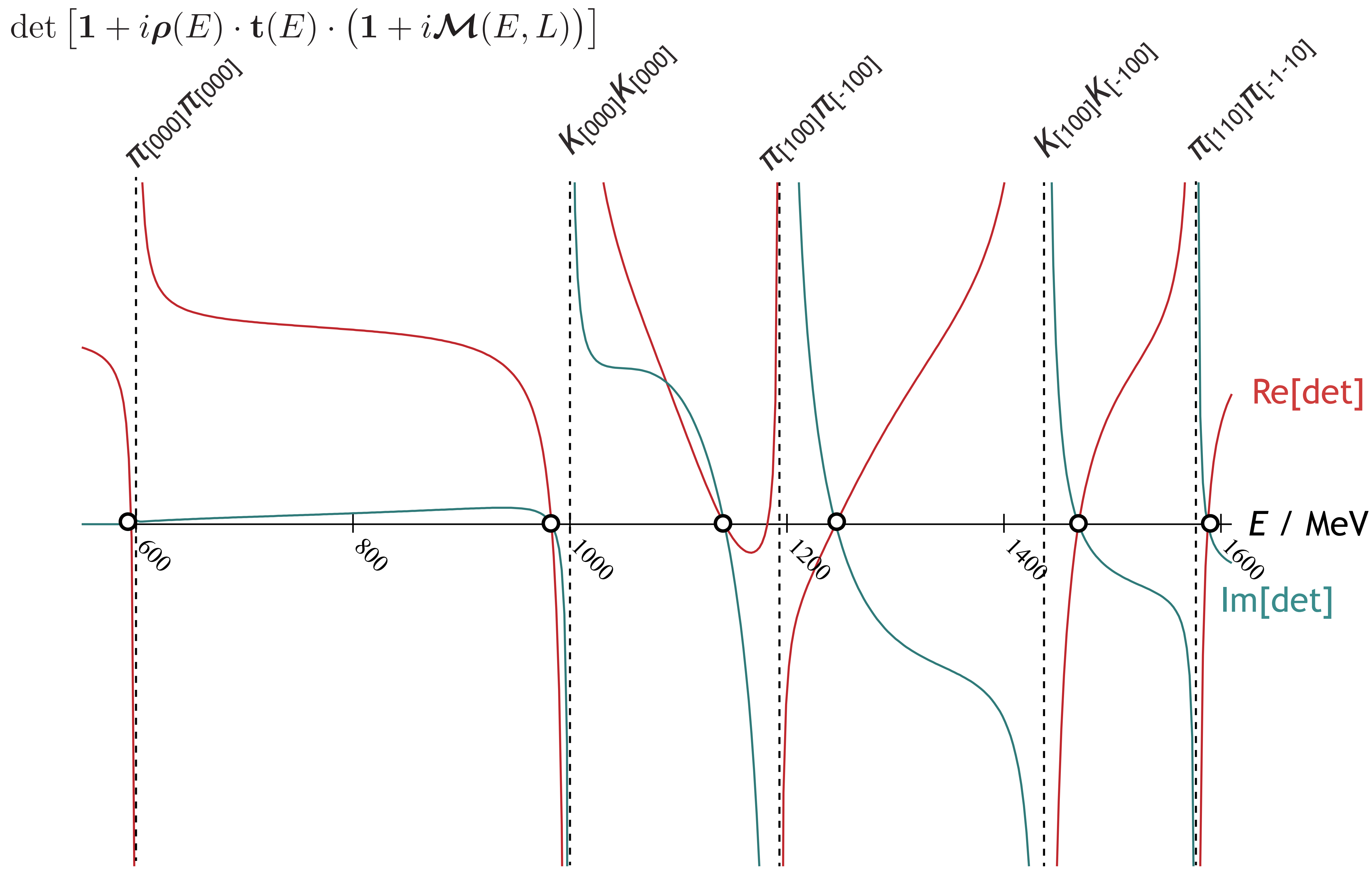
e.g. $\mathcal{M}_0$



zeroes of the determinant

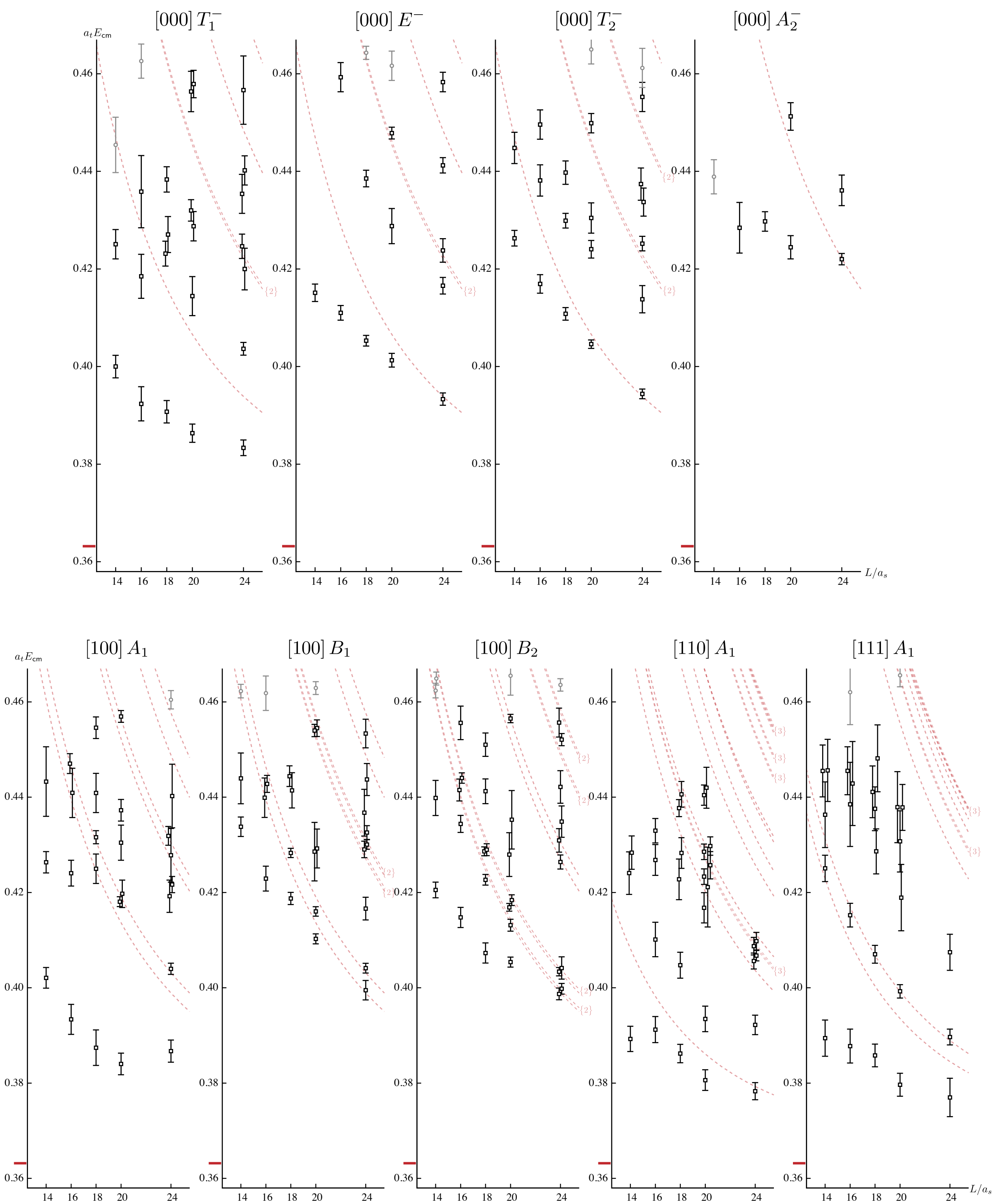
e.g. a two-channel Flatté form — [000] A_1^+ irrep in L=2.4 fm box

m_π = 300 MeV
m_K = 500 MeV

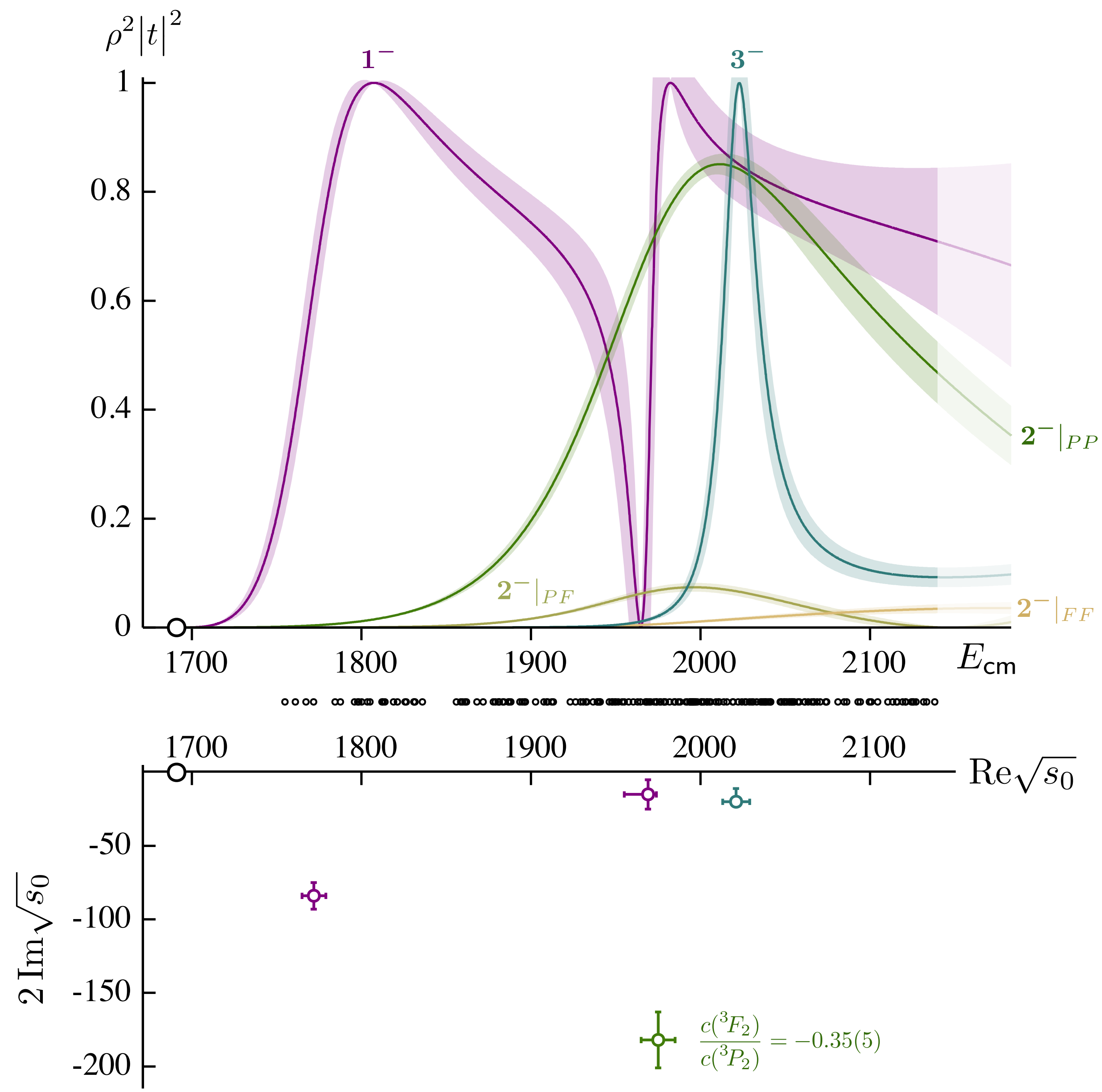


excited J^{--} meson resonances — m_π~700 MeV

exact SU(3) flavor symmetry $\omega_J^{\mathbf{1}} \rightarrow \eta^{\mathbf{8}}\omega^{\mathbf{8}}$

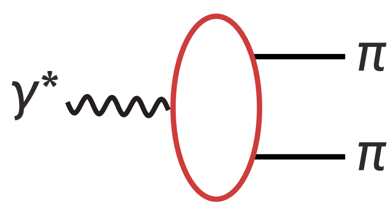


unprecedented number of energy levels



coupling resonances to currents

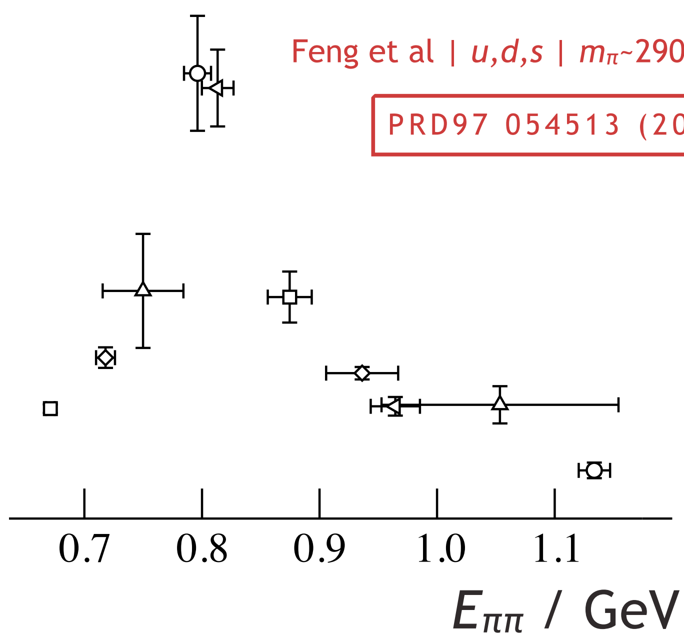
$\gamma^* \to \pi\pi$



$F_\pi(E_{\pi\pi})$

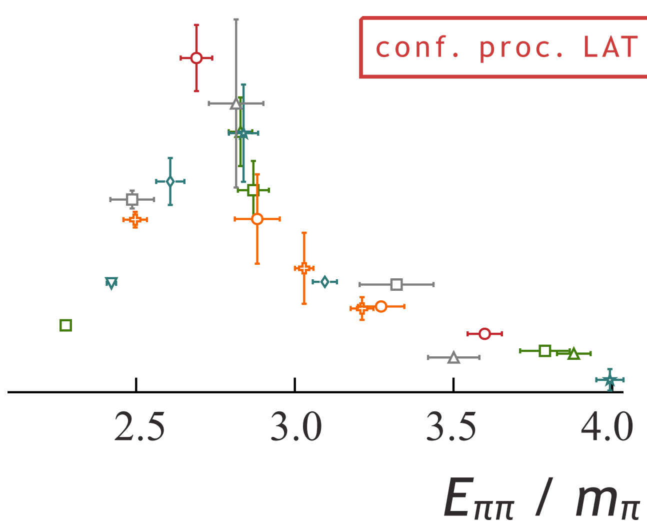
Feng et al | *u,d,s* | m_π~290 MeV

PRD97 054513 (2018)

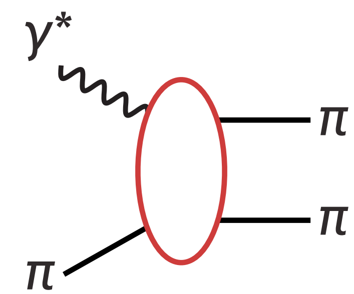


Bulava et al | *u,d,s* | m_π~280 MeV

conf. proc. LAT '15



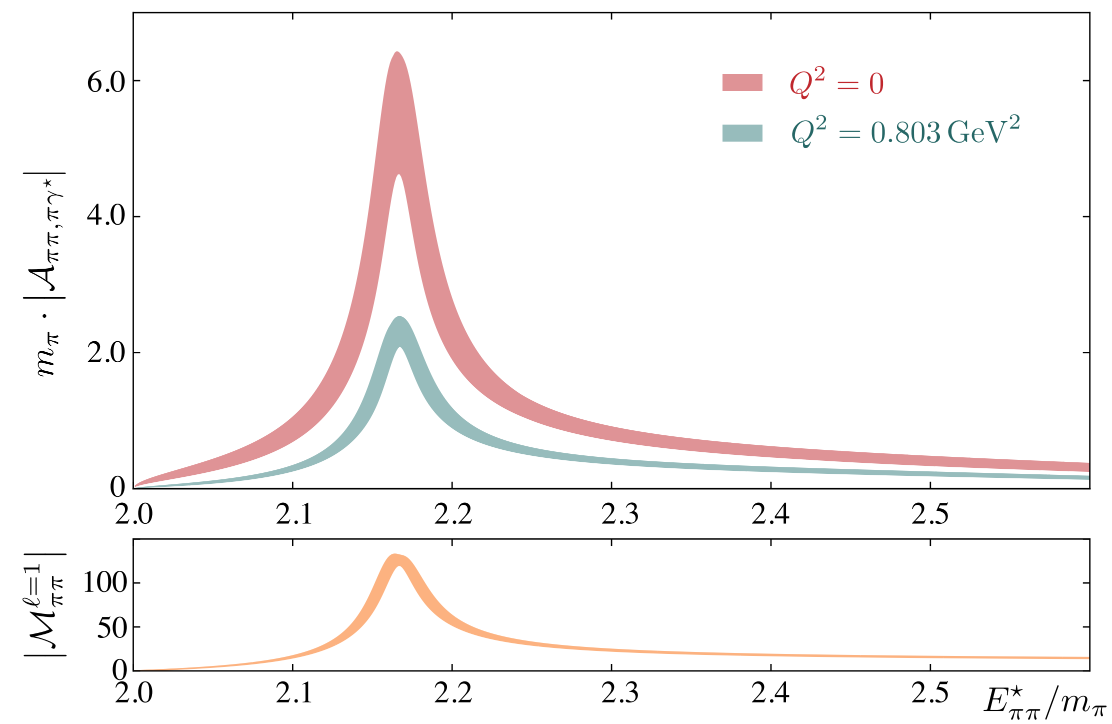
$\gamma^*\pi \to \pi\pi$

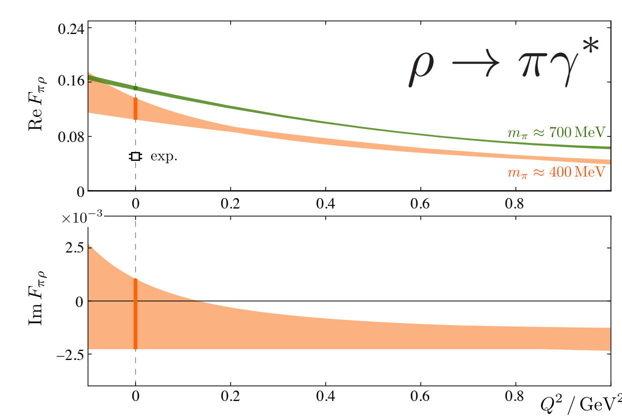


$\mathcal{A}(E_{\pi\pi}, Q^2)$

Briceno et al | *u,d,s* | m_π~391 MeV

PRL115 242001 (2015)





coupling scattering systems to external currents

e.g. consider the process in which
a pion absorbs a photon* to become two pions

$\gamma\pi \to \pi\pi$

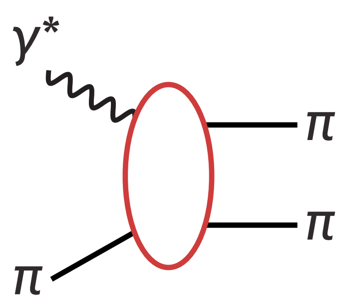
* could be virtual

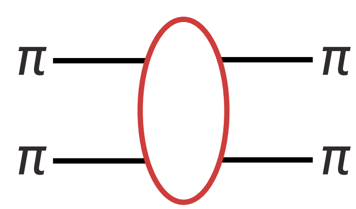
in infinite volume, described by a matrix element $\langle \pi\pi(E_{\mathsf{cm}}, \mathbf{P}) | j^\mu(0) | \pi(\mathbf{p}) \rangle$

$\propto F(E_{\mathsf{cm}}, Q^2)$

$\pi\pi$ state can be projected into a partial wave, e.g. ℓ=1

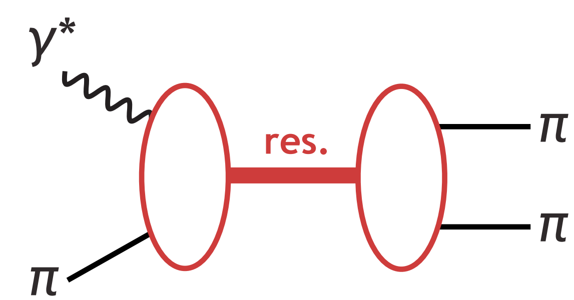
after the current produces $\pi\pi$ … $\pi\pi$ will rescatter strongly





$\Rightarrow$ the matrix element is proportional to $t_\ell(E_{\mathsf{cm}})$

if there's a resonance $t_\ell(s \sim s_0) \sim \dfrac{c^2}{s_0 - s}$ and $F(s \sim s_0, Q^2) \sim \dfrac{c\, f(Q^2)}{s_0 - s}$



resonance transition form-factor $f(Q^2)$
rigorously defined at the complex pole position

e.g. $\rho \to \pi\gamma$

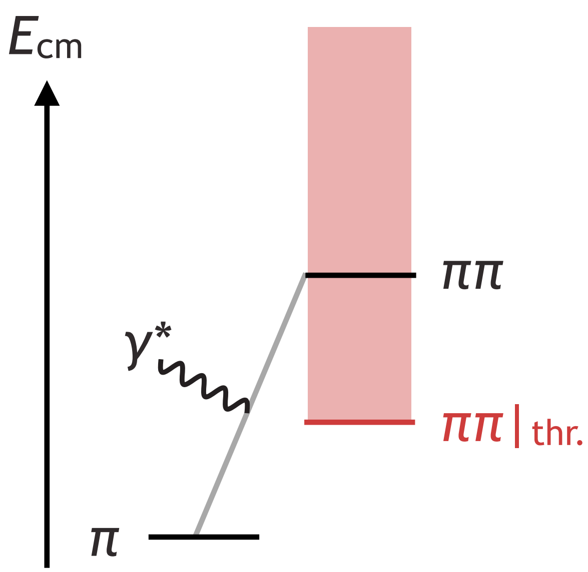
but what changes in a finite volume … ?

coupling scattering systems to external currents — fin. vol.

e.g. consider the process in which
a pion absorbs a photon to become two pions

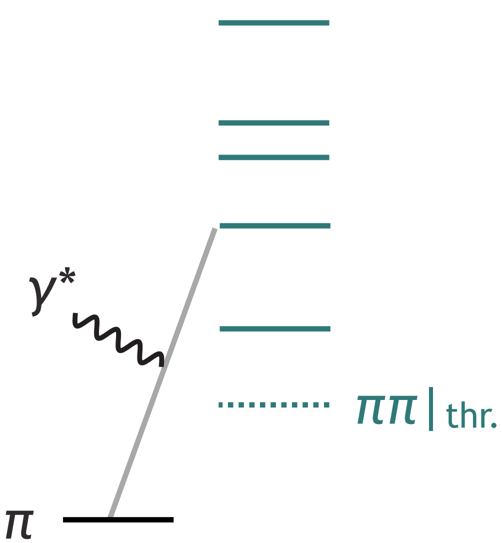
$$\gamma\pi \to \pi\pi$$

infinite volume



can transition
to any energy
in the $\pi\pi$ continuum

finite volume



can only transition
to one of the discrete
f.v. eigenstates

finite-volume matrix element

$${}_L\langle \pi\pi(E_n(L),\mathbf{P}) | j^\mu(0) | \pi(\mathbf{p}) \rangle_L$$

single hadron state

$$|\pi(\mathbf{p})\rangle_L = |\pi(\mathbf{p})\rangle_\infty + O(e^{-m_\pi L})$$

hadron-hadron state

$$|\pi\pi(E_n(L),\mathbf{P})\rangle_L \sim \sqrt{\mathcal{R}_n}\, |\pi\pi(E_{\mathsf{cm}} = E_n(L),\mathbf{P})\rangle_\infty$$

effective f.v.
normalization

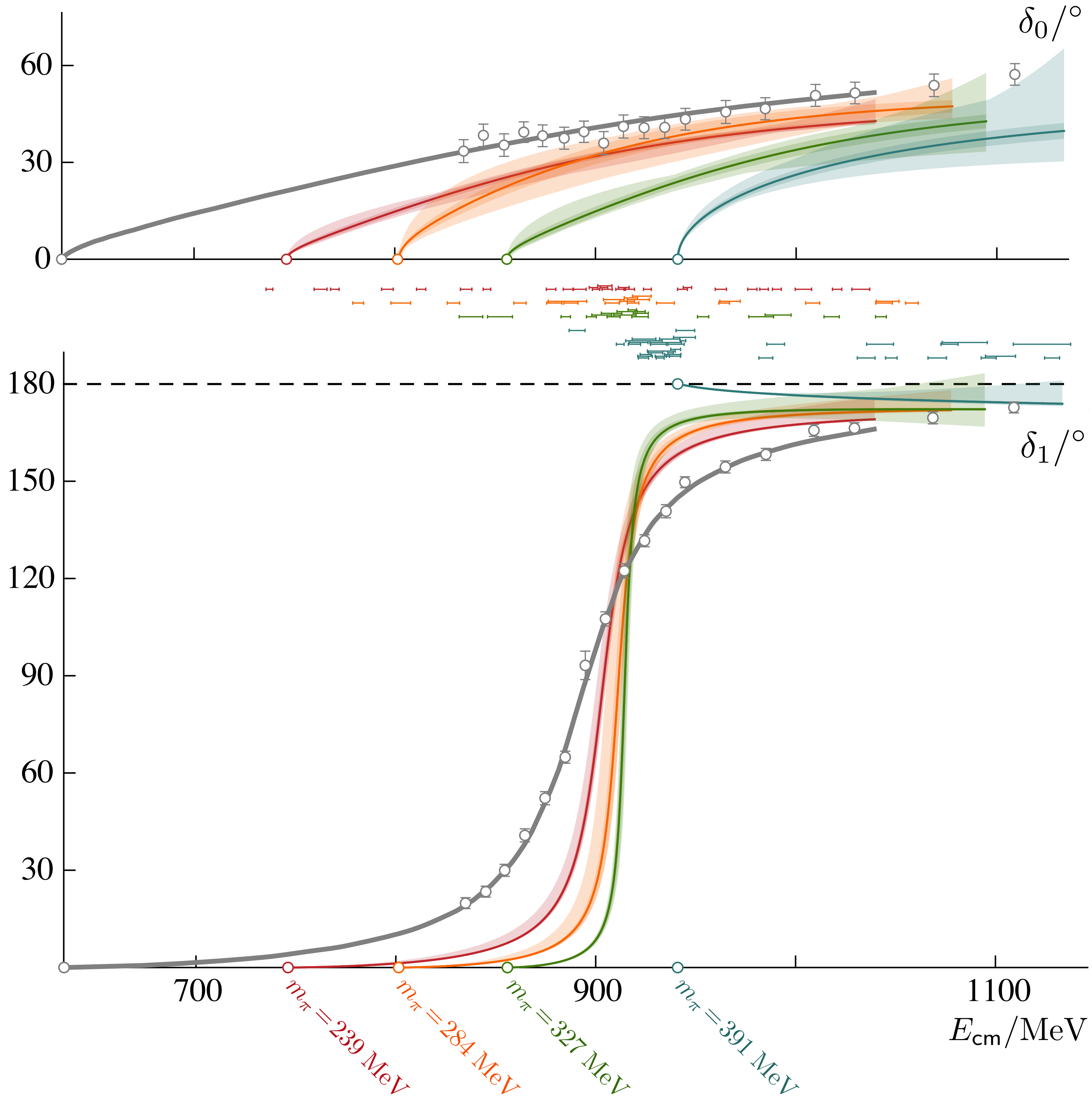
$$\mathcal{R}_n = 2E_n \lim_{E\to E_n} (E - E_n)\Big(F^{-1}(E,\mathbf{P};L) + M(E)\Big)^{-1}$$

$$F = \tfrac{1}{16\pi}\, i\rho\,(1 + i\mathcal{M})$$

$$M = 16\pi\, t$$

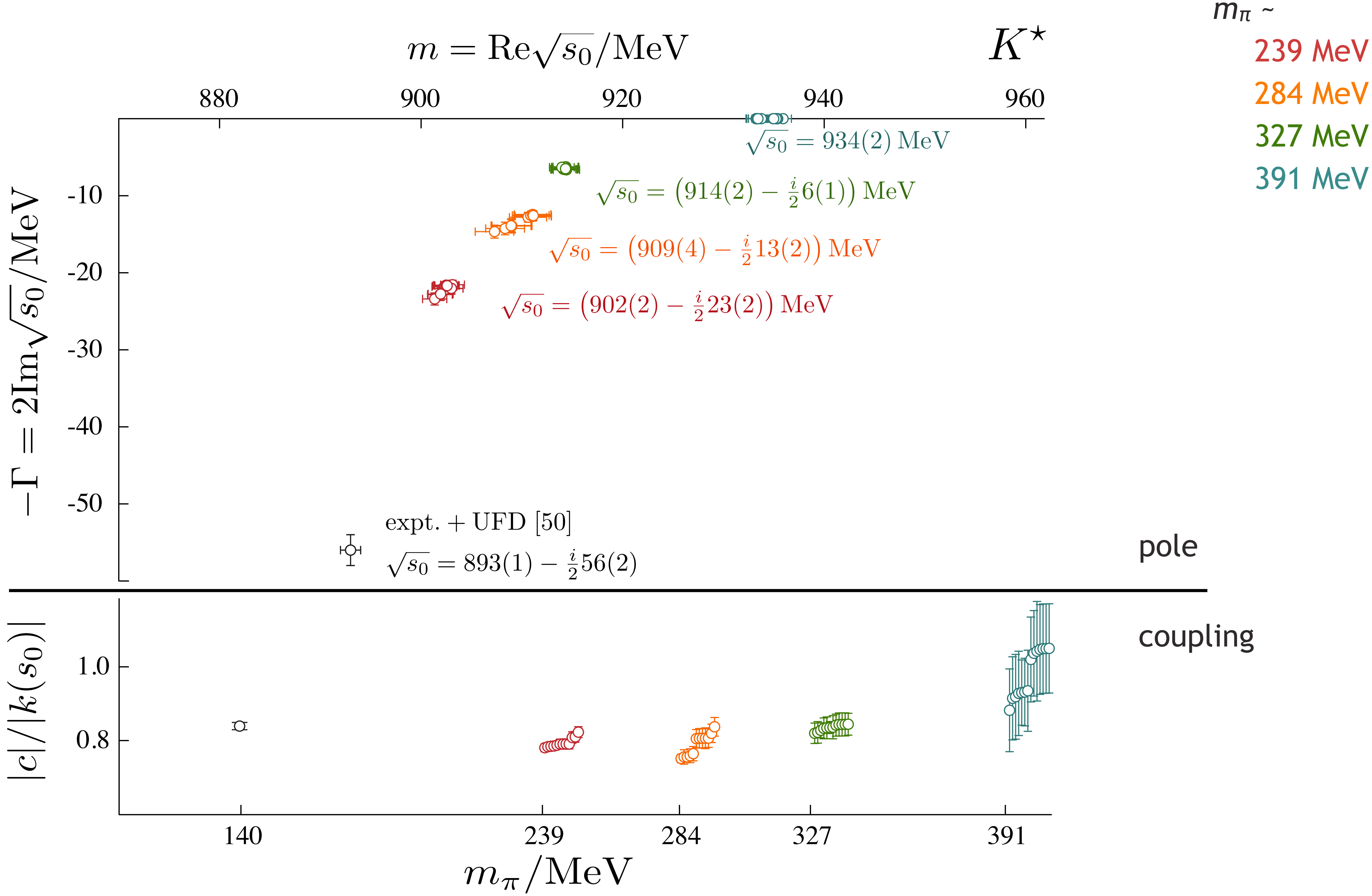
effective f.v. normalization depends
on the hadron-hadron scattering amplitude

πK elastic scattering at four light quark masses

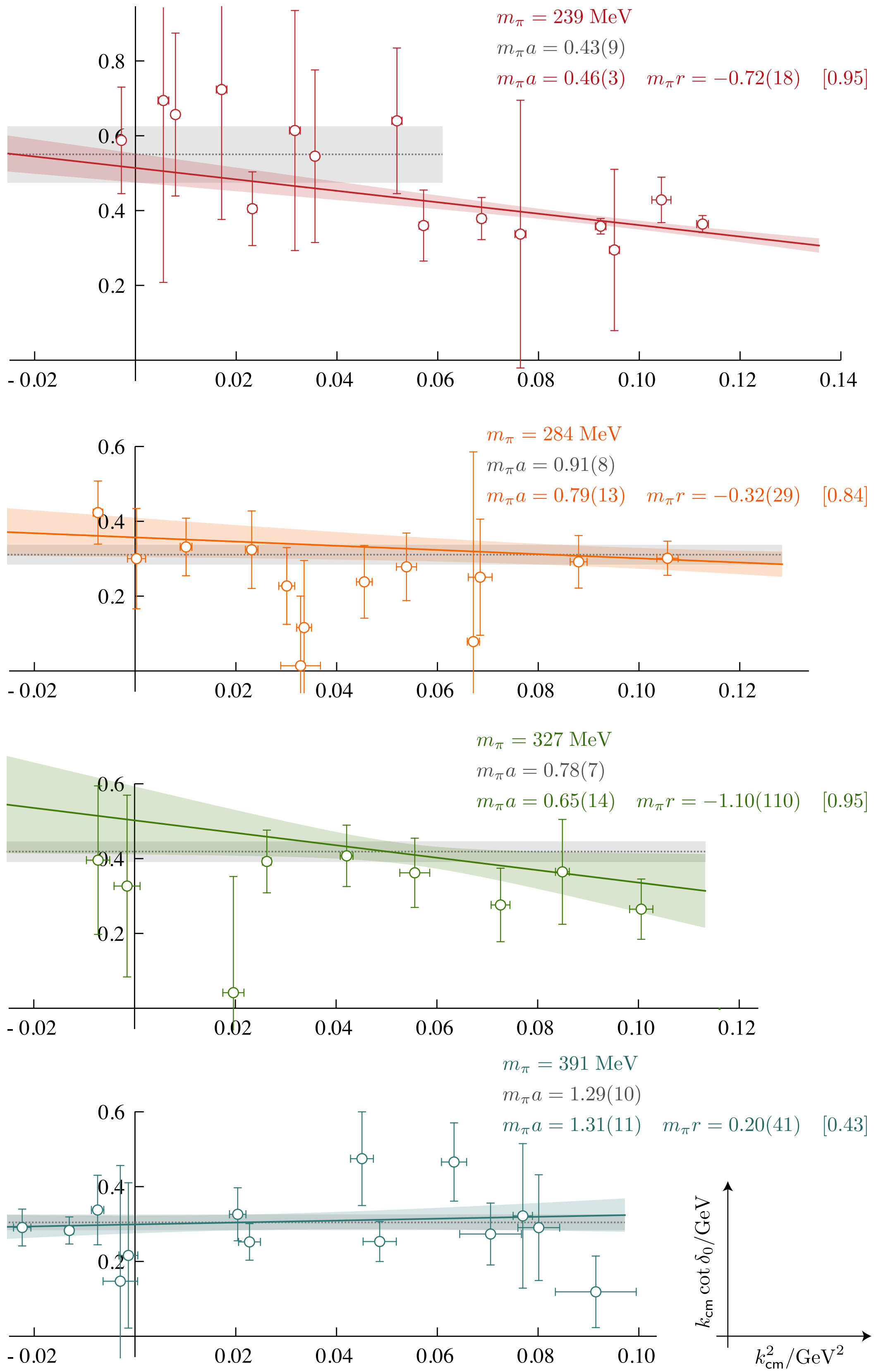


m_π ~
239 MeV
284 MeV
327 MeV
391 MeV

πK elastic scattering at four light quark masses



πK elastic scattering at four light quark masses



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239 MeV
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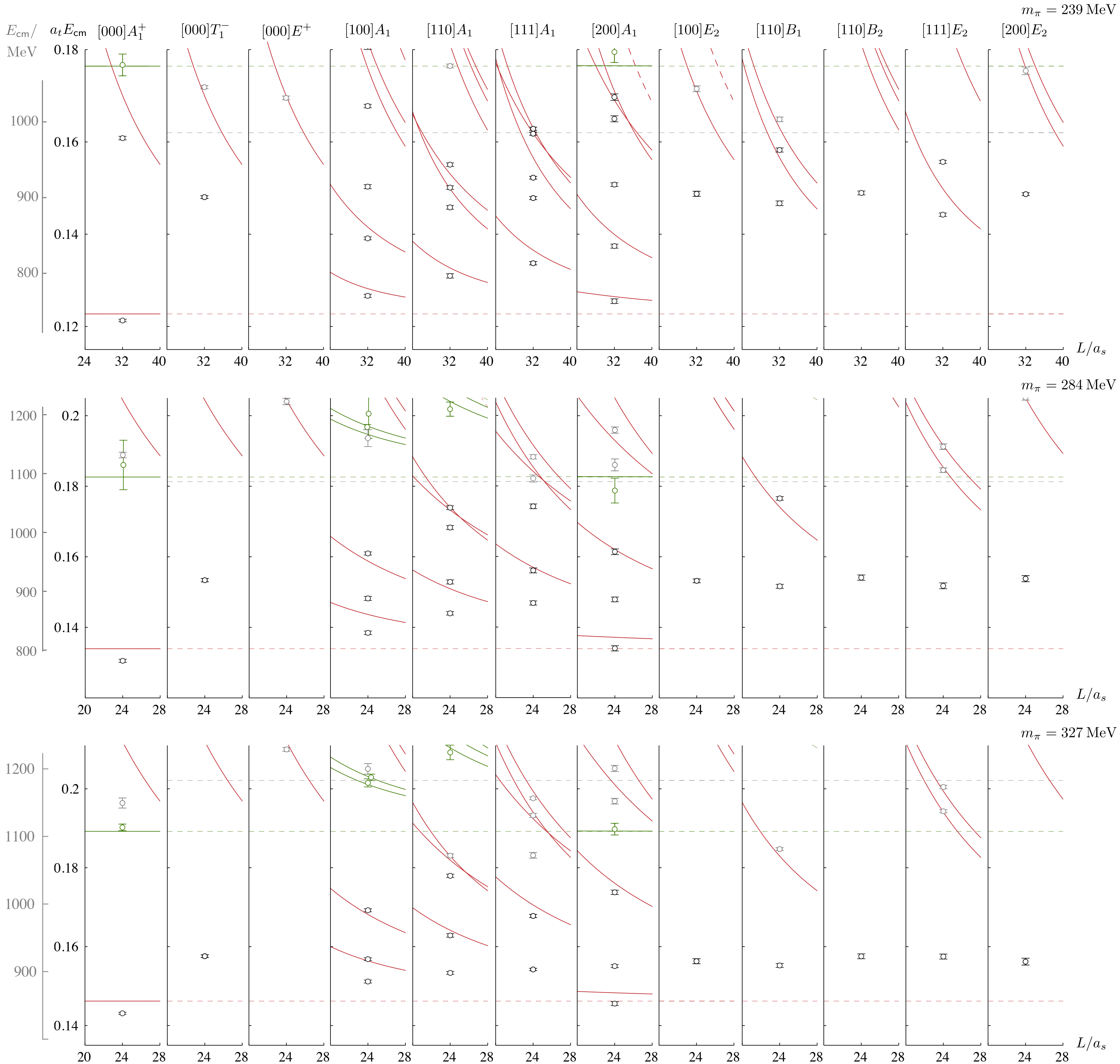
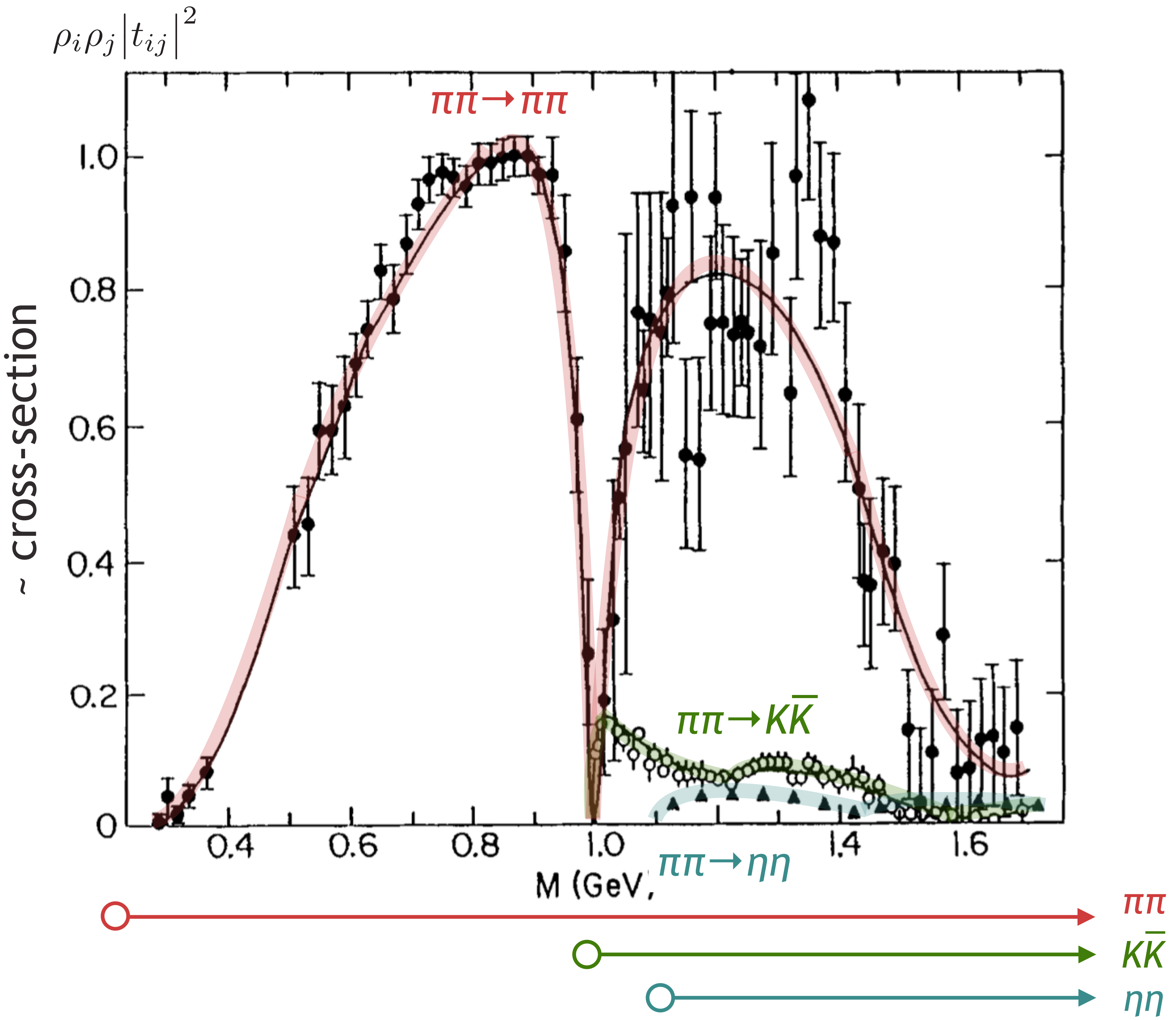
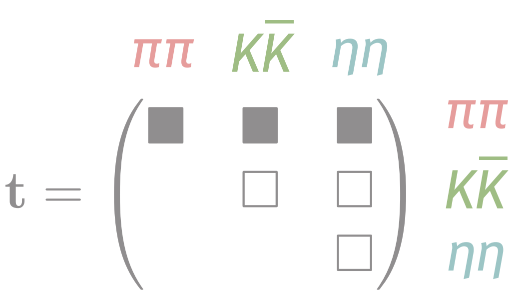
πK elastic scattering at four light quark masses



resonances aren't always bumps



experimentally
quite difficult to fill out
the whole t-matrix

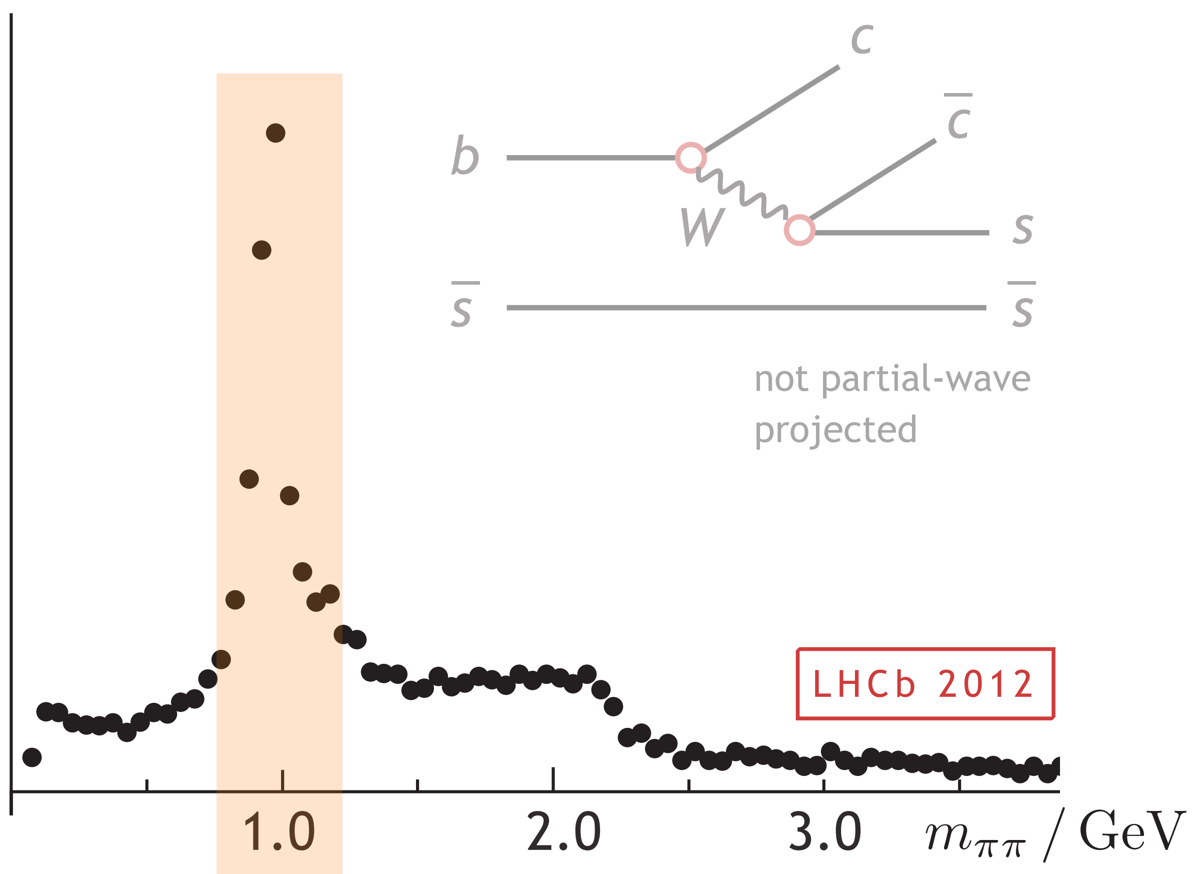


normalization of $\pi\pi \to KK$ slightly uncertain …

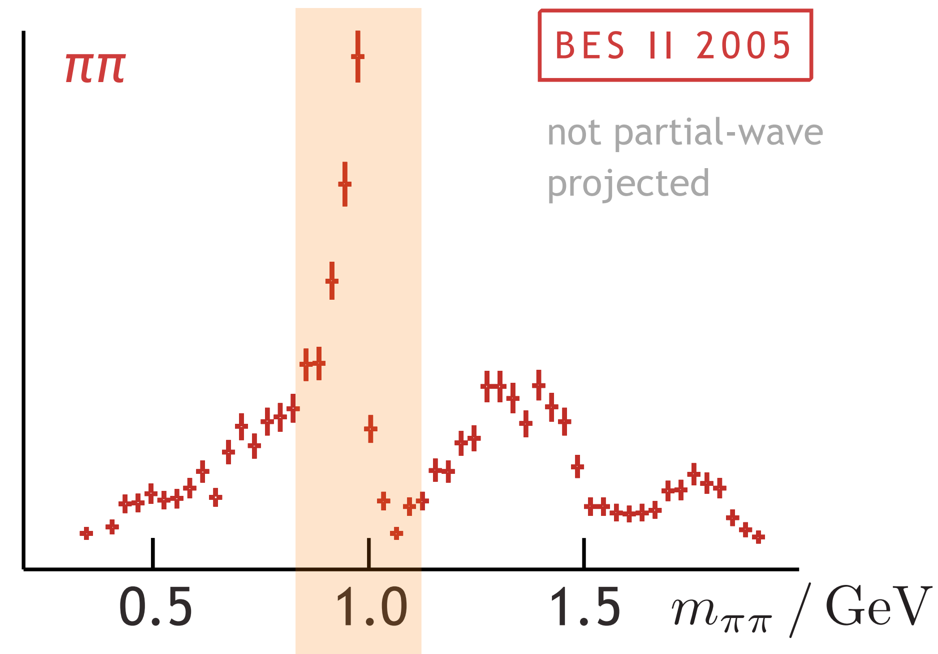
isolating kaon exchange hard & η beams don't exist

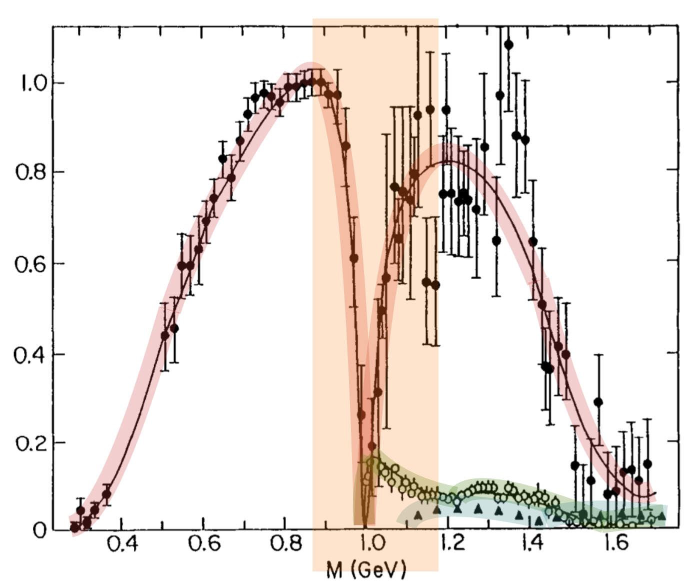
"production" of $\pi\pi$ (as opposed to scattering)

$\bar{B}_s^0 \to J/\psi\ \pi^+\pi^-$



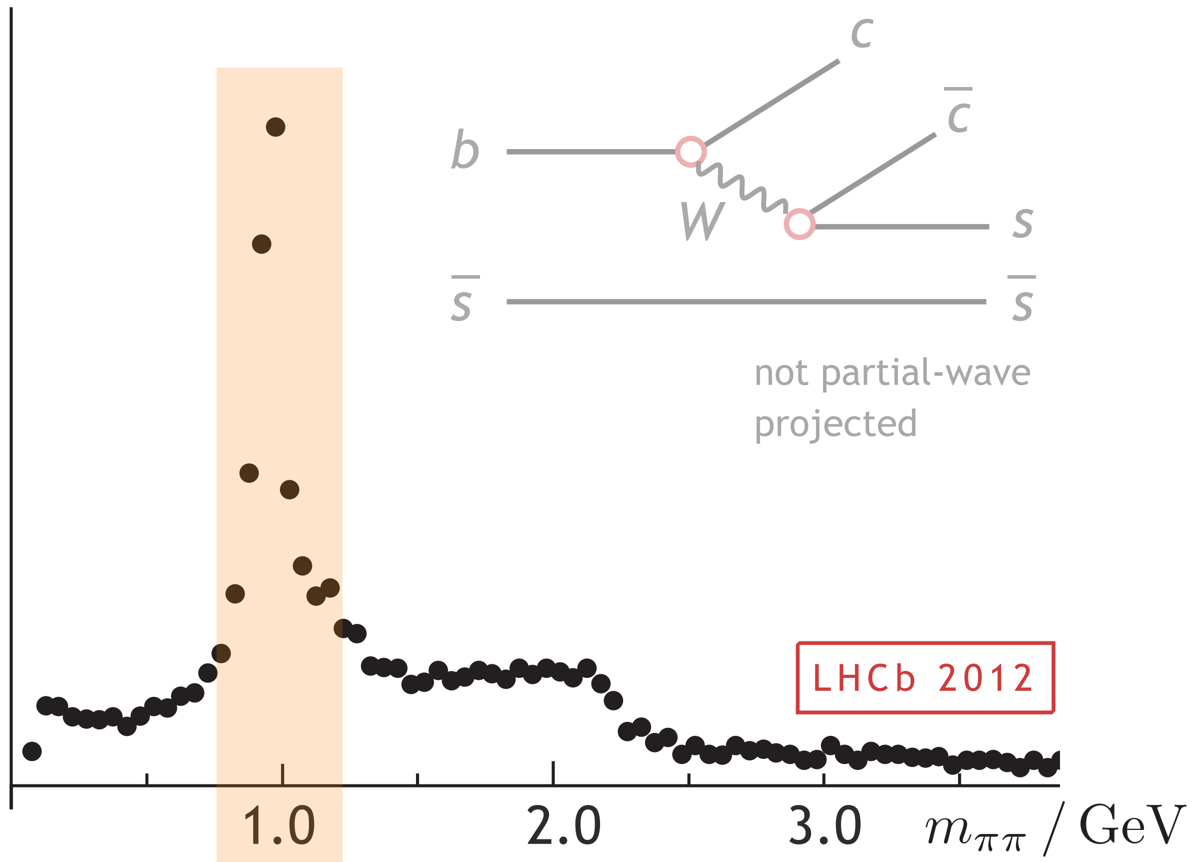
$J/\psi \to \varphi\ \pi\pi$



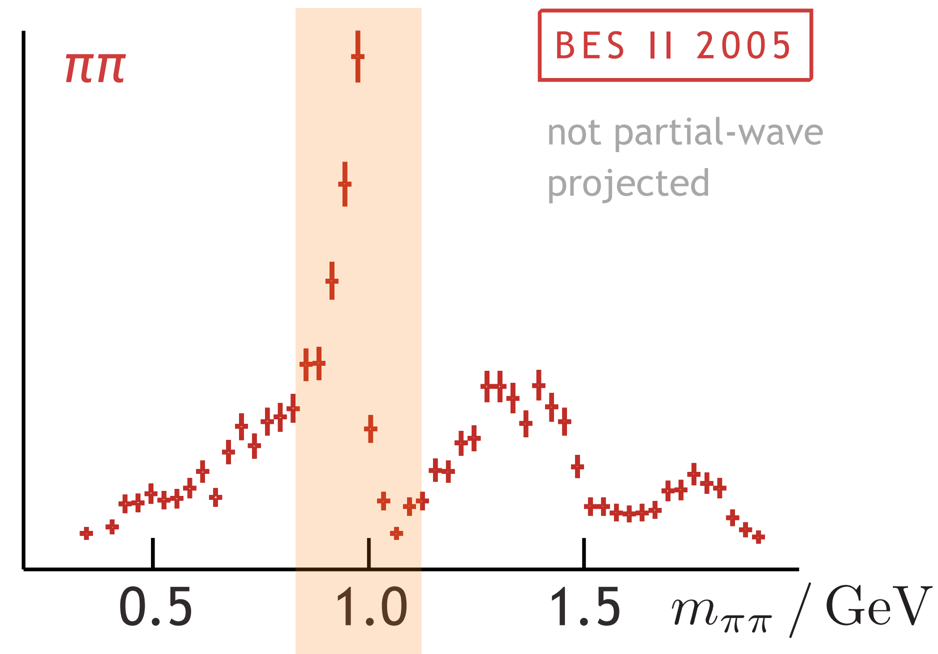


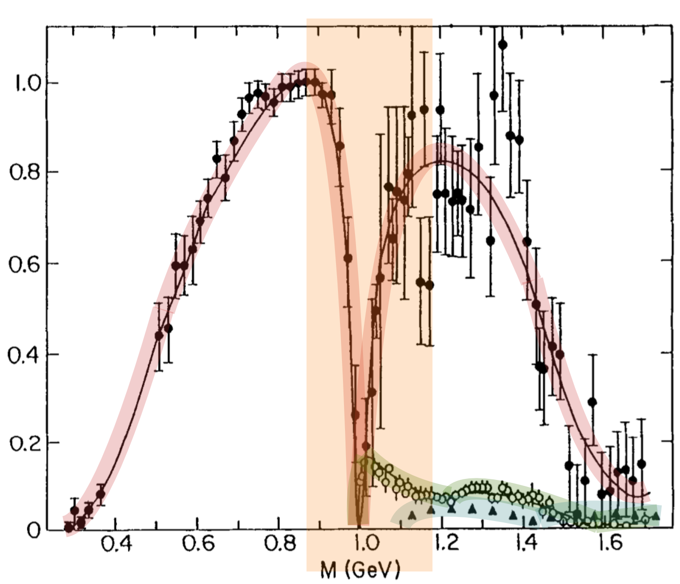
can 'look' drastically different to scattering !

$\bar{B}^0_s \to J/\psi\ \pi^+\pi^-$

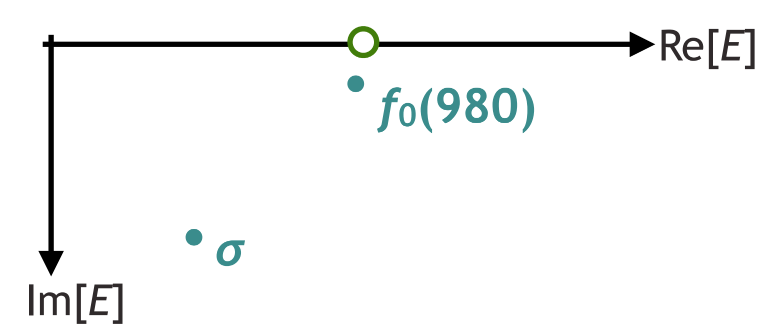


$J/\psi \to \varphi\ \pi\pi$

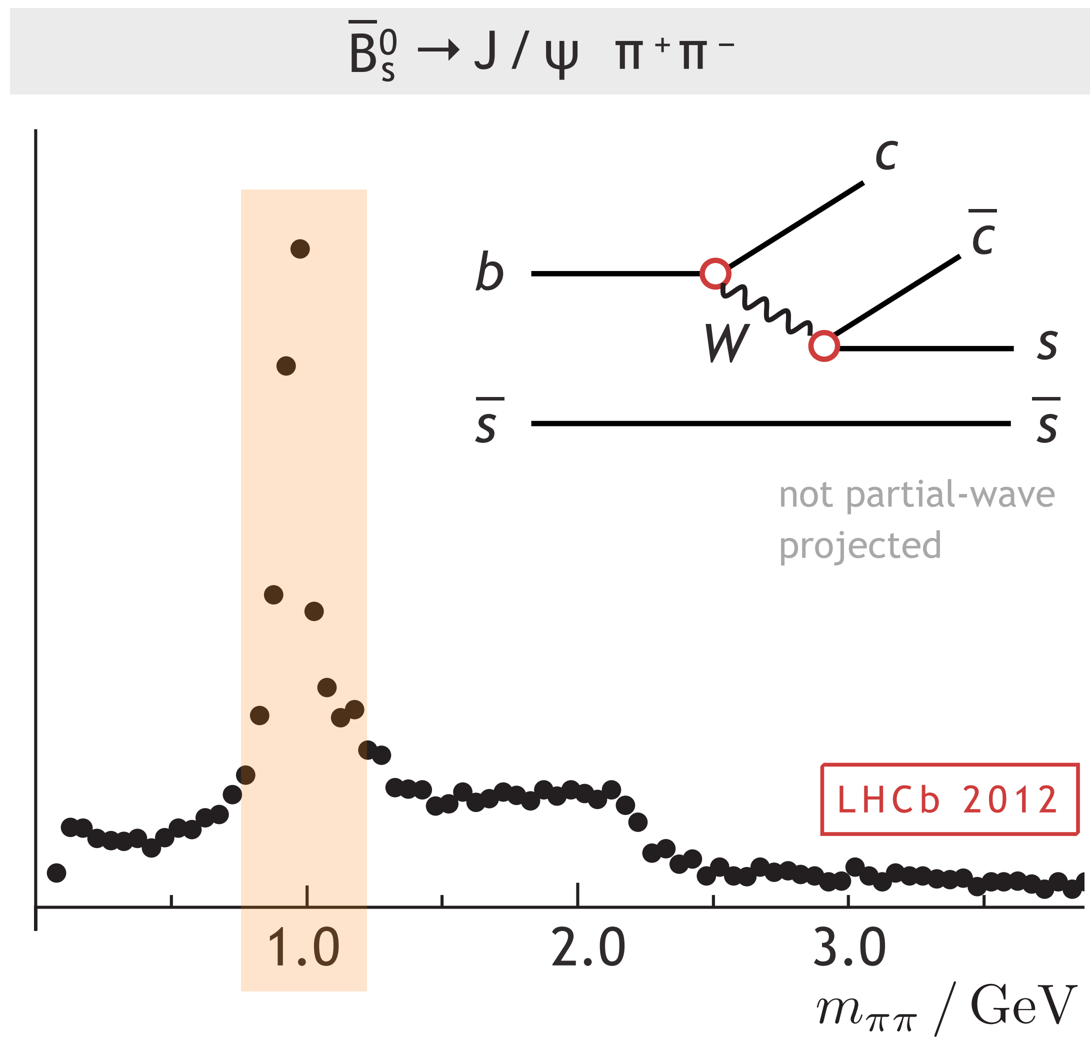


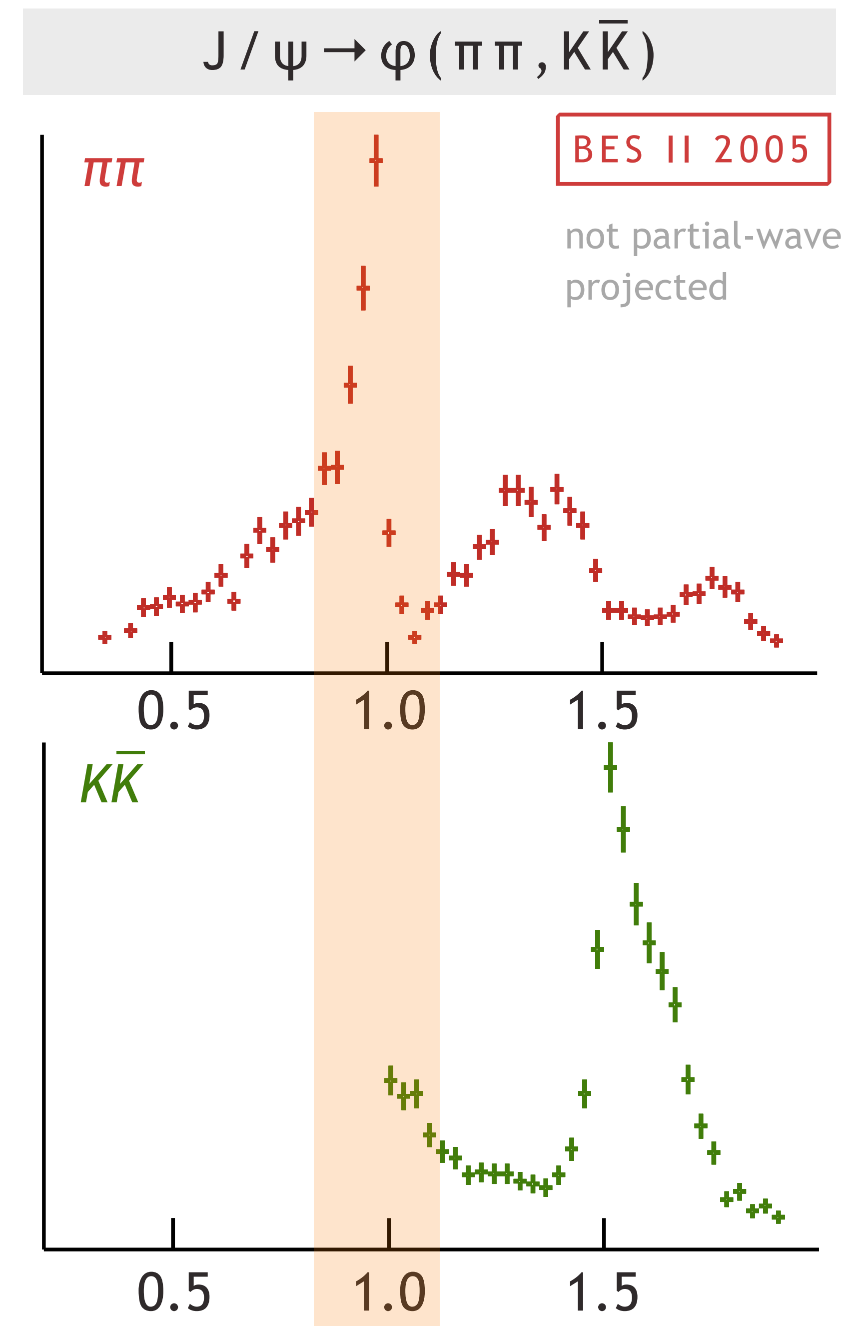


… same poles (σ, $f_0(980)$) — different couplings …



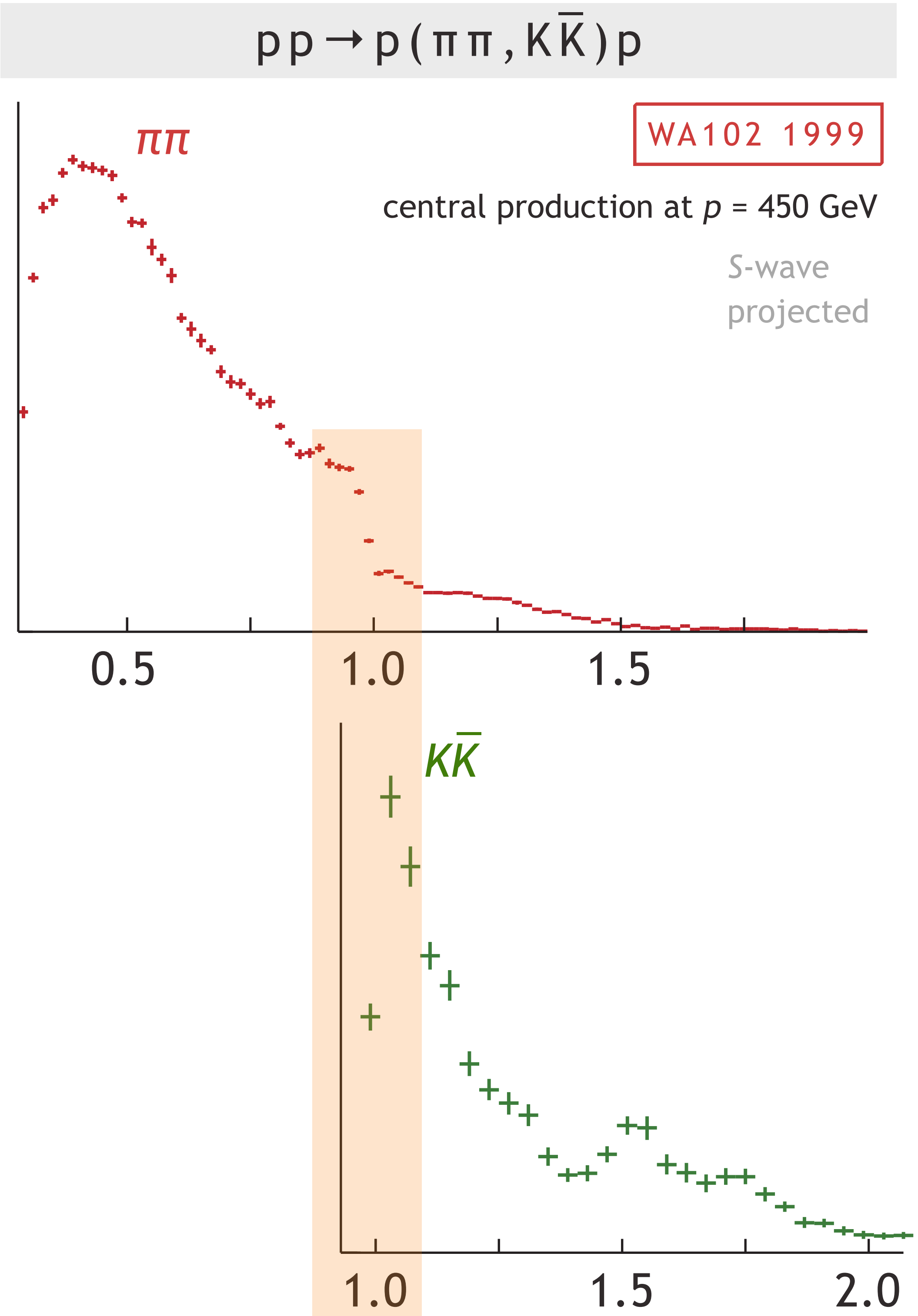
$f_0(980)$ as a peak in "$s\bar{s}$" production



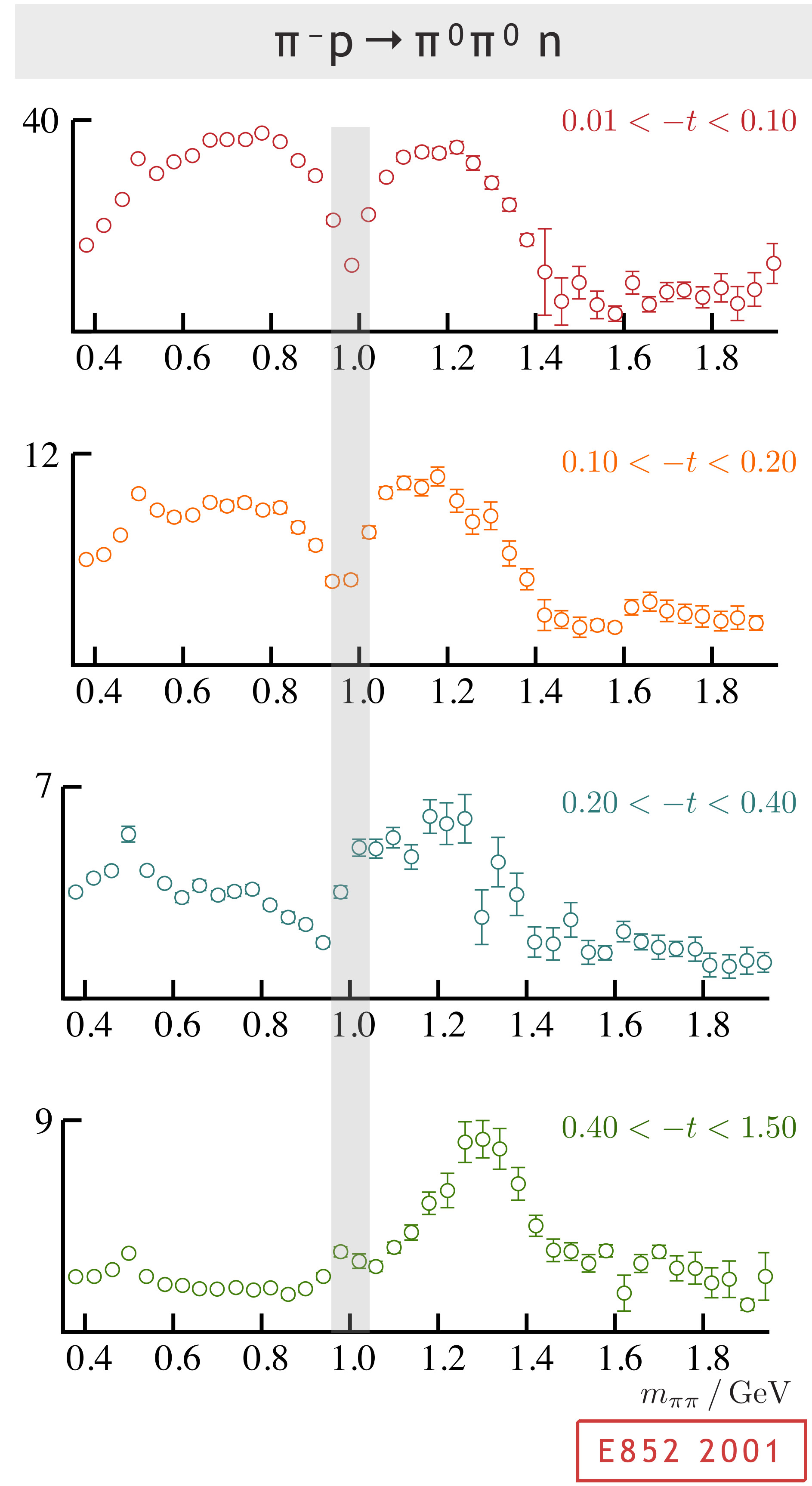


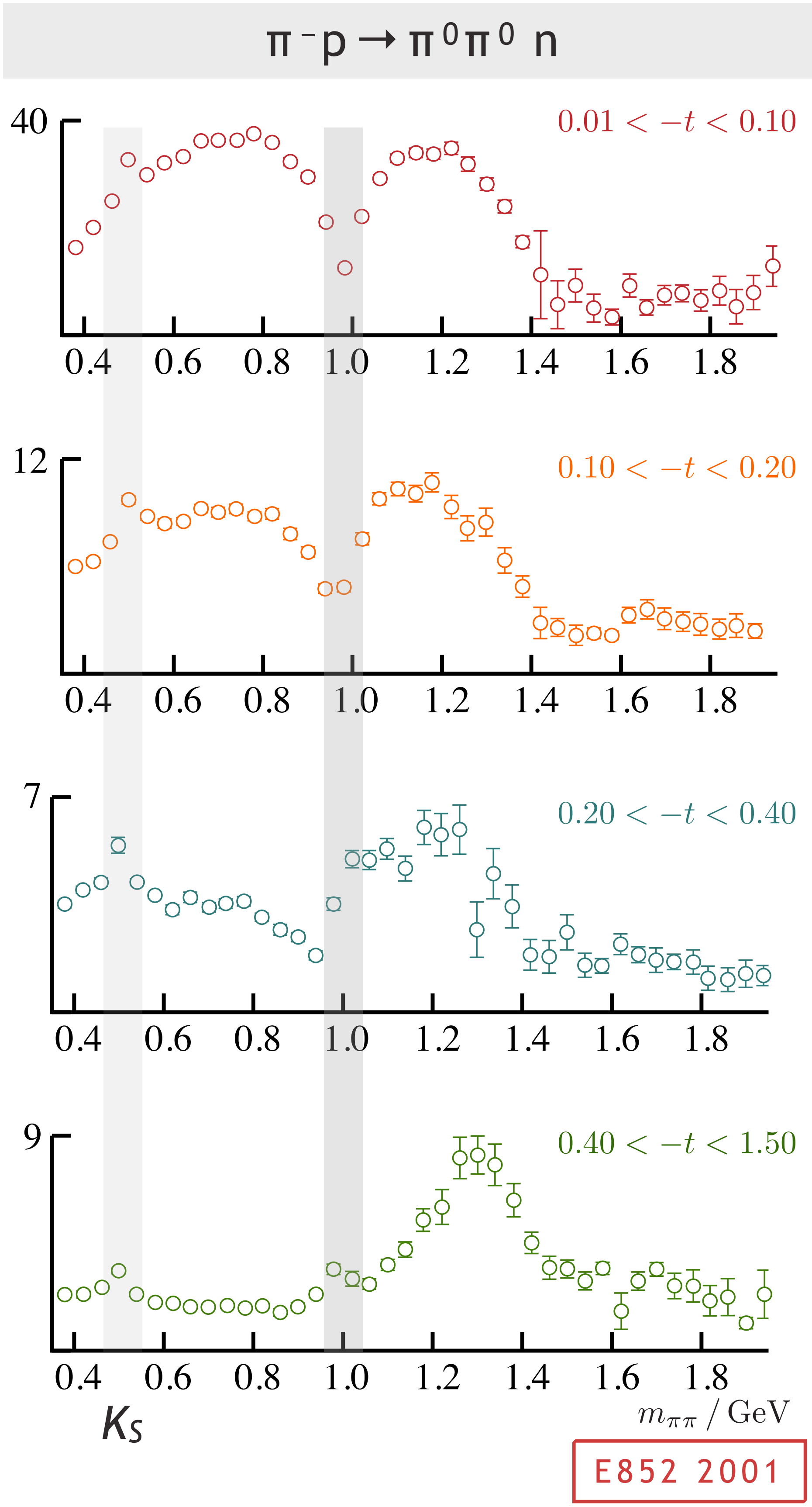
note the rapid turn-on of $K\bar{K}$ at threshold

$f_0(980)$ as ?

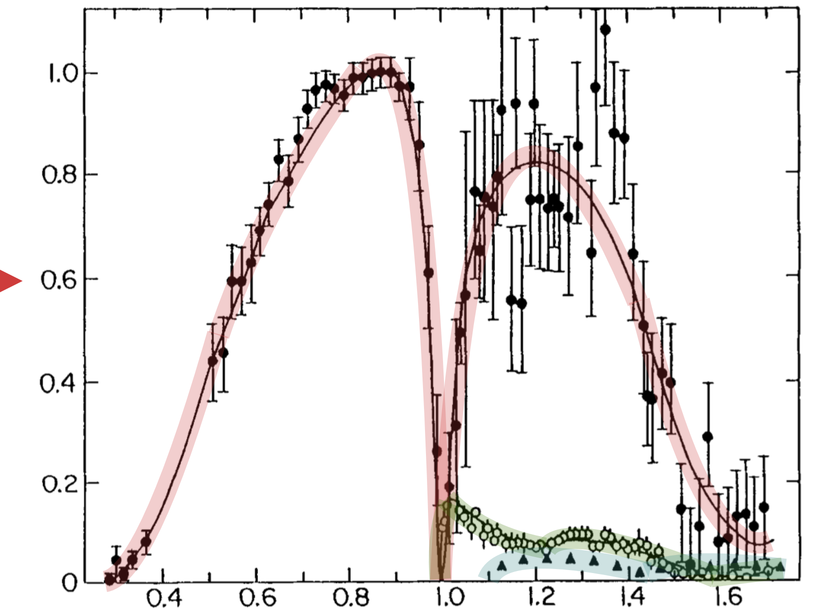


$f_0(980)$ as a shoulder on
a large σ 'background'





dominated by π exchange
— looks like the the 1970s
elastic phase-shift data

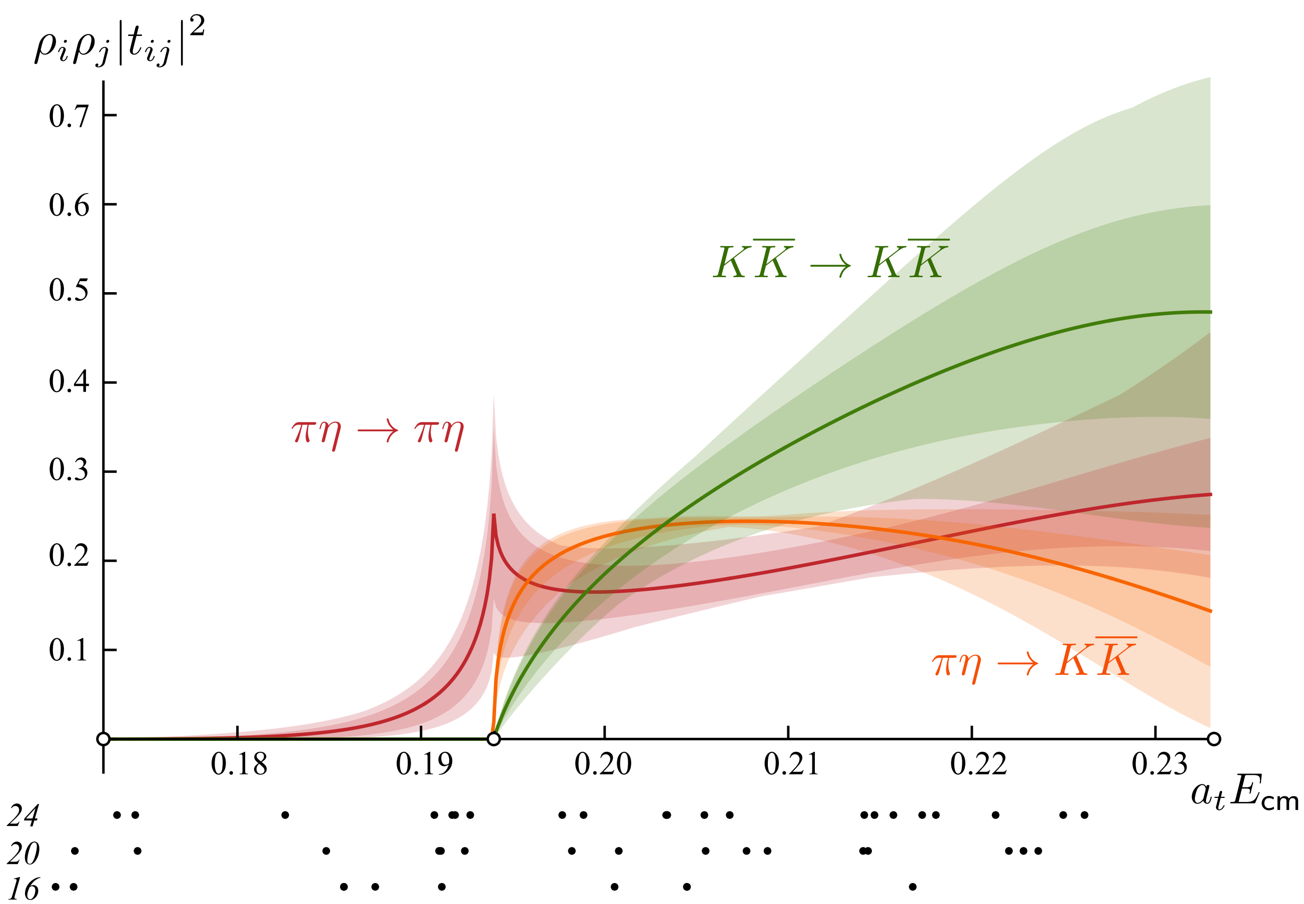


other (non-π) exchanges becoming significant, $f_0(980)$ dip less pronounced

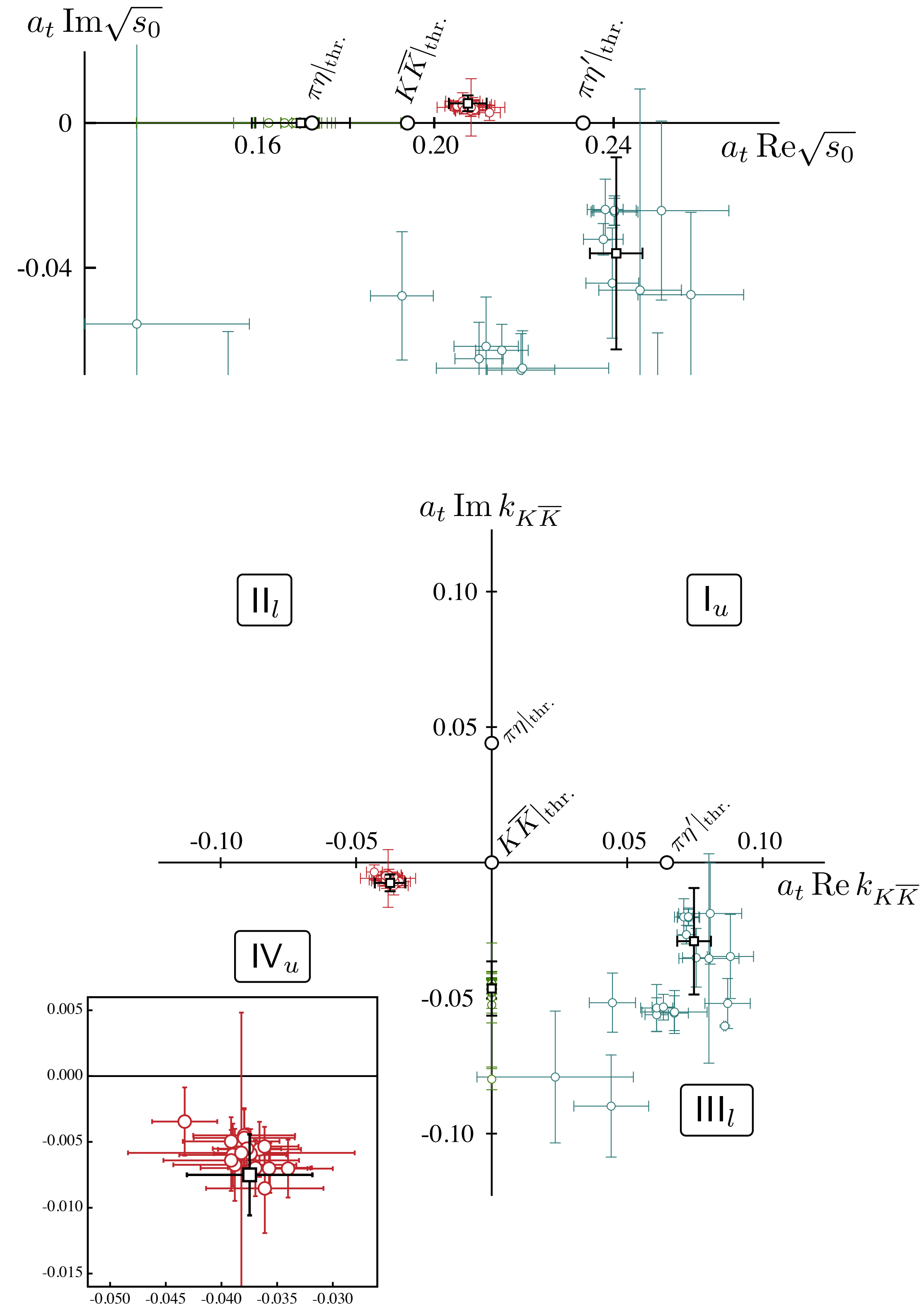
σ no longer large, $f_0(980)$ starting to be a peak ?

$\pi\eta$, $K\bar{K}$ scattering with m_π~391 MeV

S-wave amplitudes

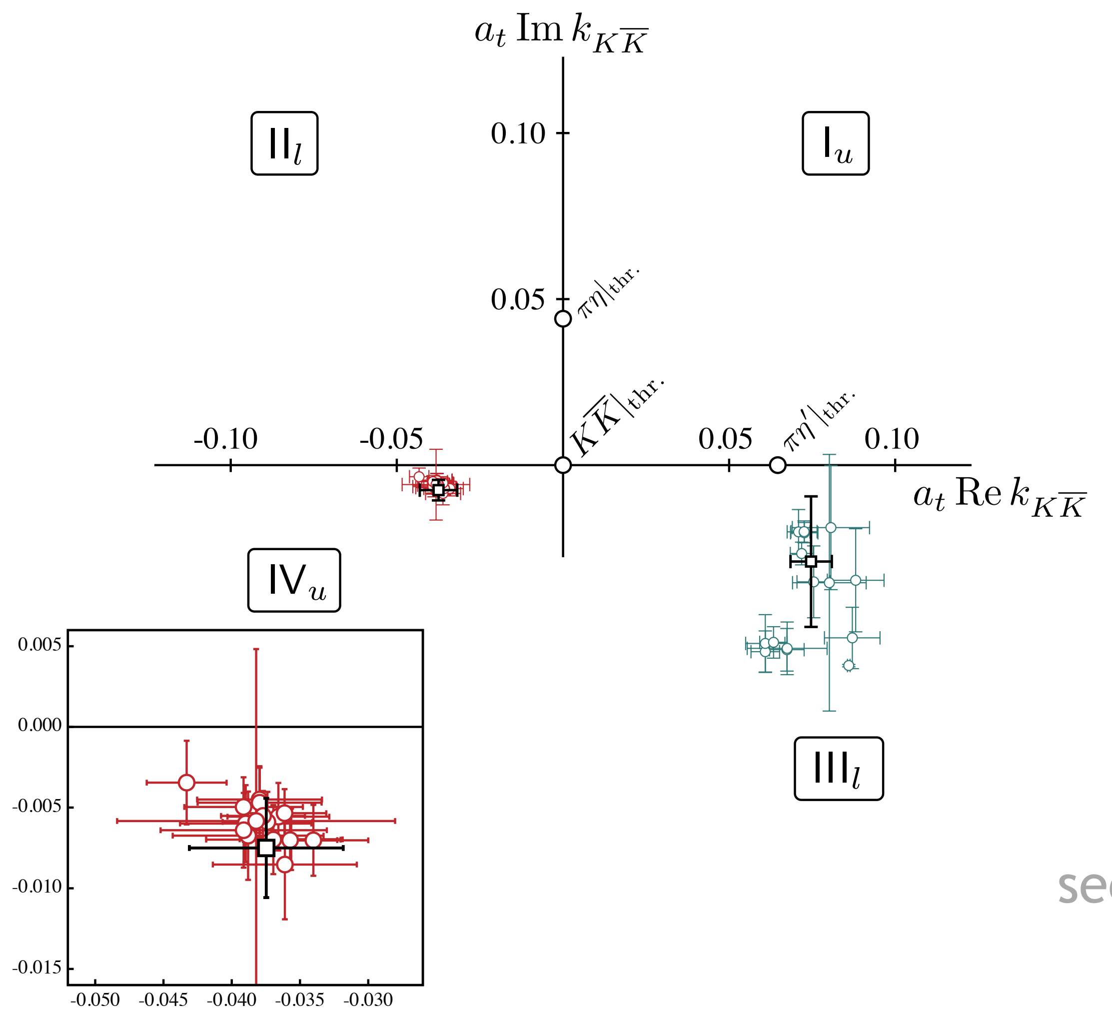


pole singularities

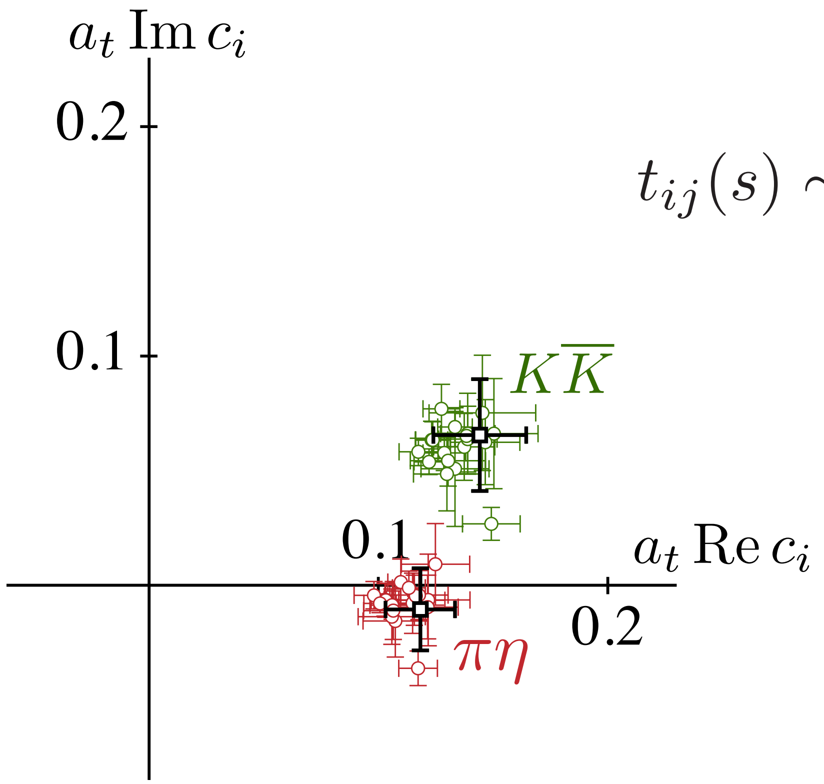


a_0 resonance at m_π~391 MeV

Riemann sheet structure



pole couplings



$$t_{ij}(s) \sim \frac{c_i \, c_j}{s_0 - s}$$

$$\left| \frac{c_{K\bar{K}}}{c_{\pi\eta}} \right| = 1.3(4)$$

see also

PHYSICAL REVIEW D **95**, 054004 (2017)

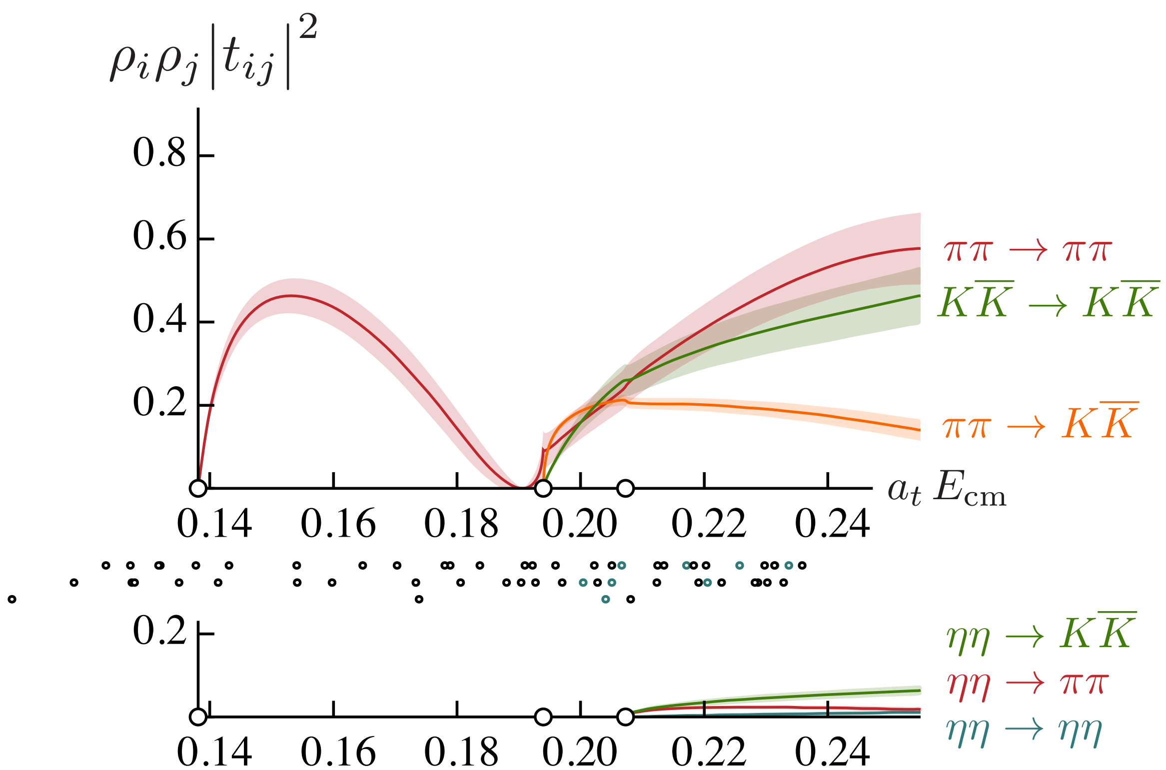
Chiral study of the $a_0(980)$ resonance and $\pi\eta$ scattering phase shifts in light of a recent lattice simulation

Zhi-Hui Guo,[1,2] Liuming Liu,[2] Ulf-G. Meißner,[2,3] J. A. Oller,[4] and A. Rusetsky[2]

$\pi\pi$, $K\bar{K}$, $\eta\eta$ scattering with m_π~391 MeV

PRD 97 054513 (2018)

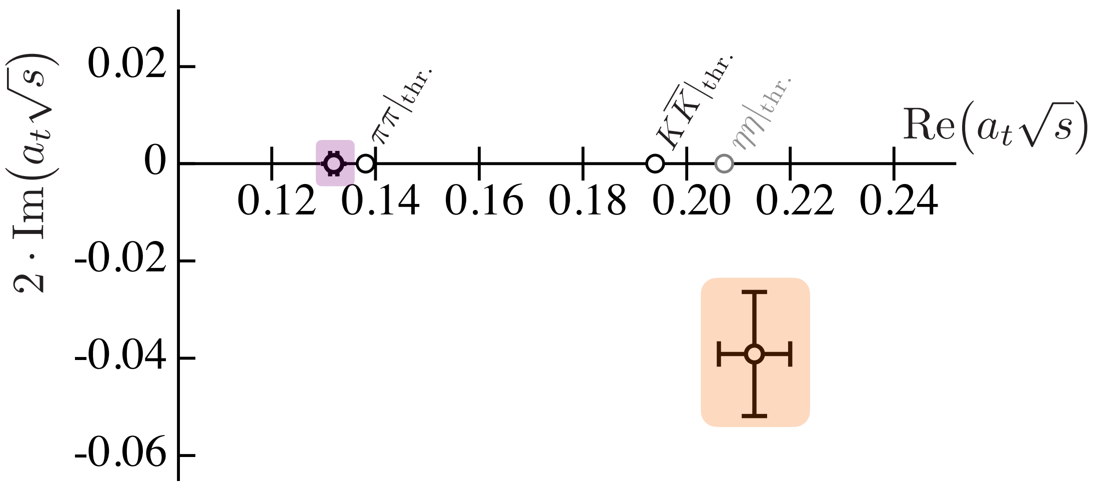
S-wave amplitudes

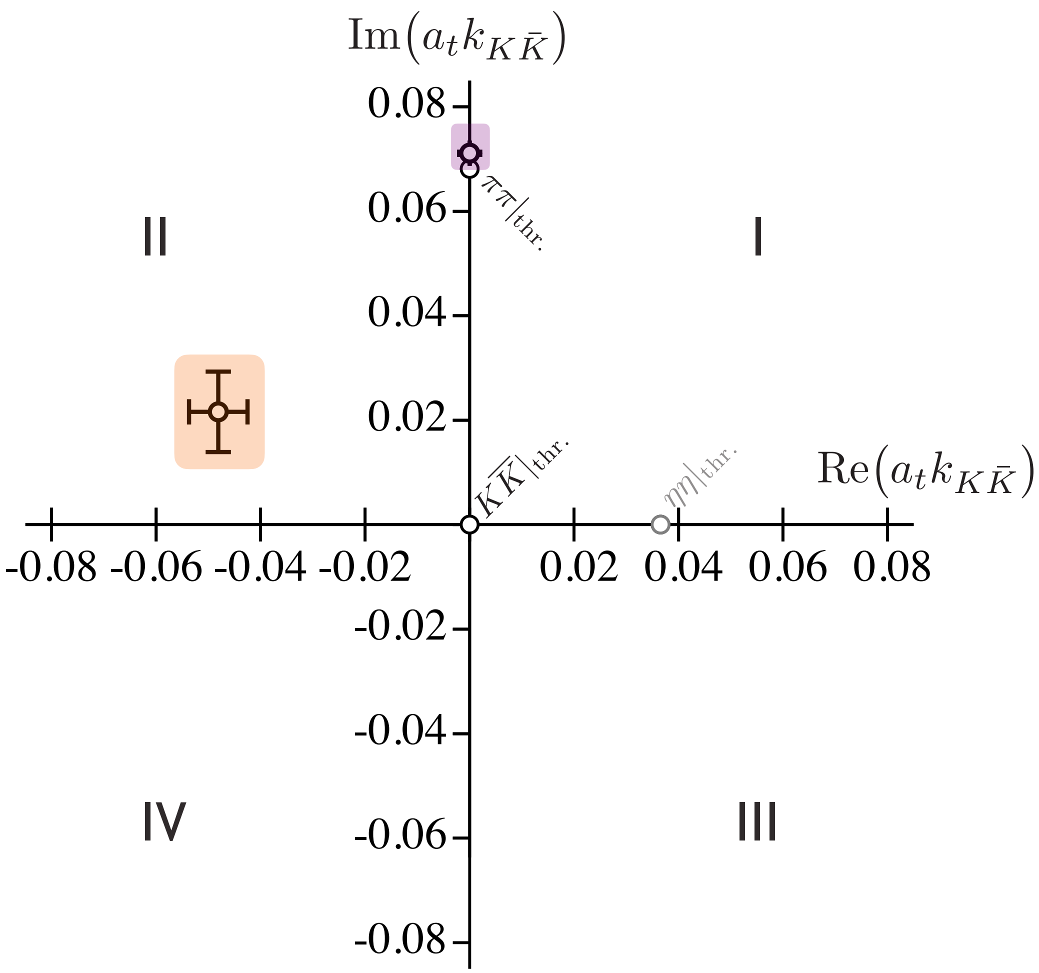


$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a + b\,s & c + d\,s & e \\ c + d\,s & f & g \\ e & g & h \end{pmatrix}$$

with Chew-Mandelstam phase-space

pole singularities

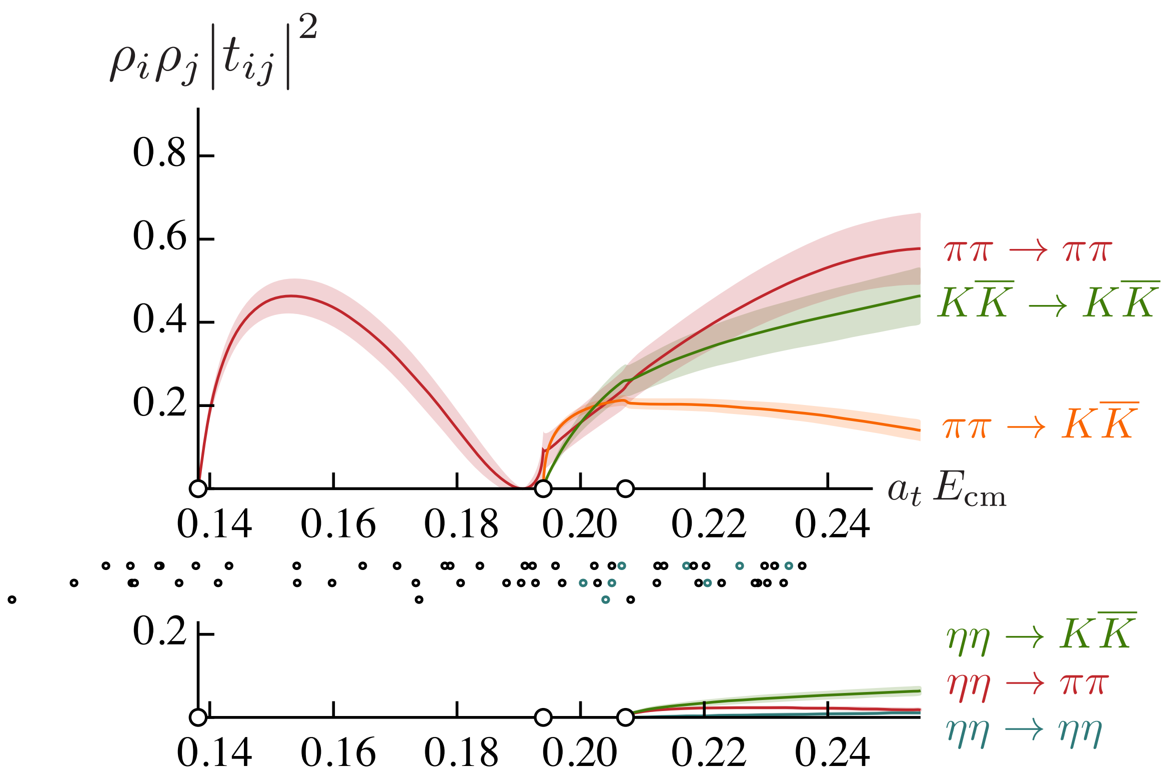




$\pi\pi$, $K\bar{K}$, $\eta\eta$ scattering with m_π~391 MeV

PRD 97 054513 (2018)

S-wave amplitudes



$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a + b\,s & c + d\,s & e \\ c + d\,s & f & g \\ e & g & h \end{pmatrix}$$

with Chew-Mandelstam phase-space

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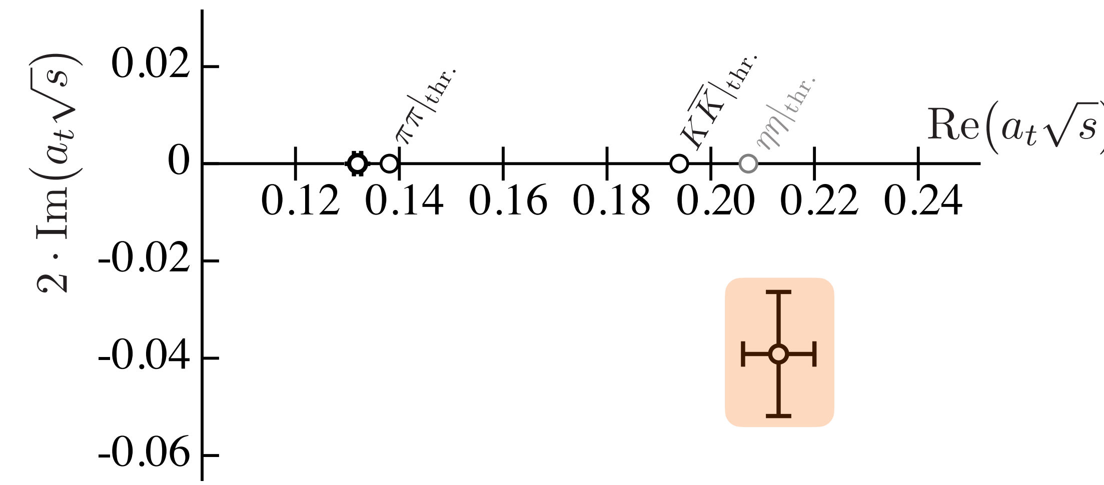
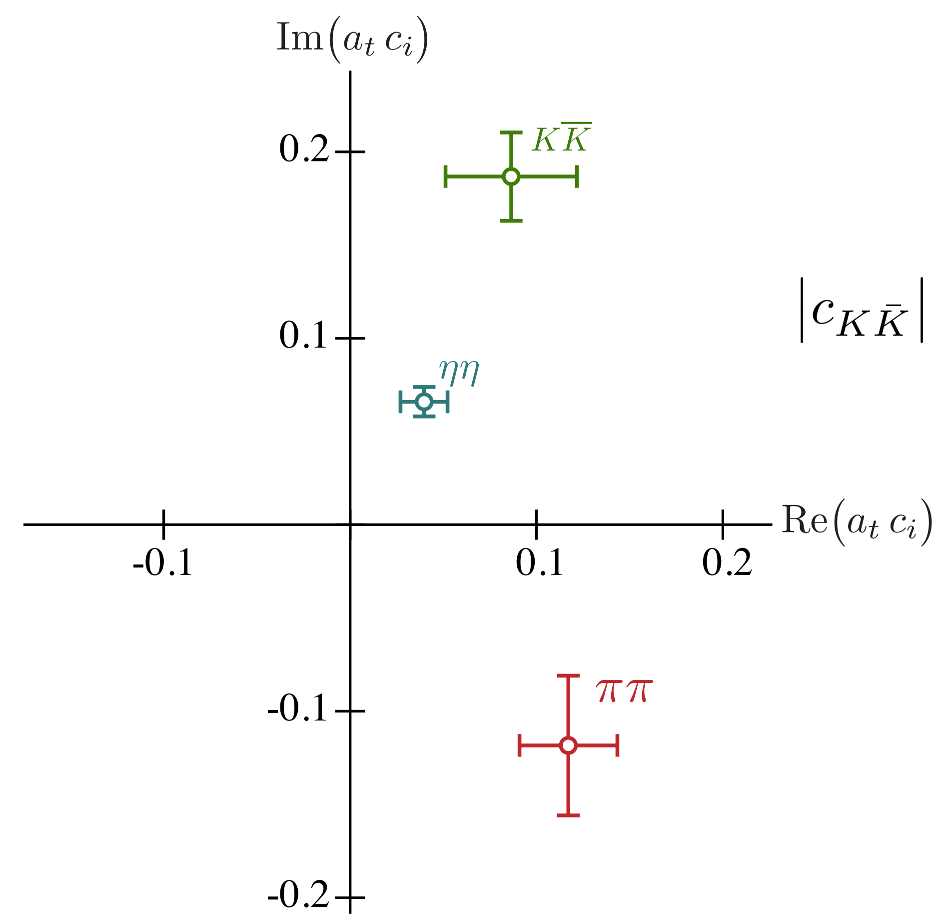


sheet II pole couplings

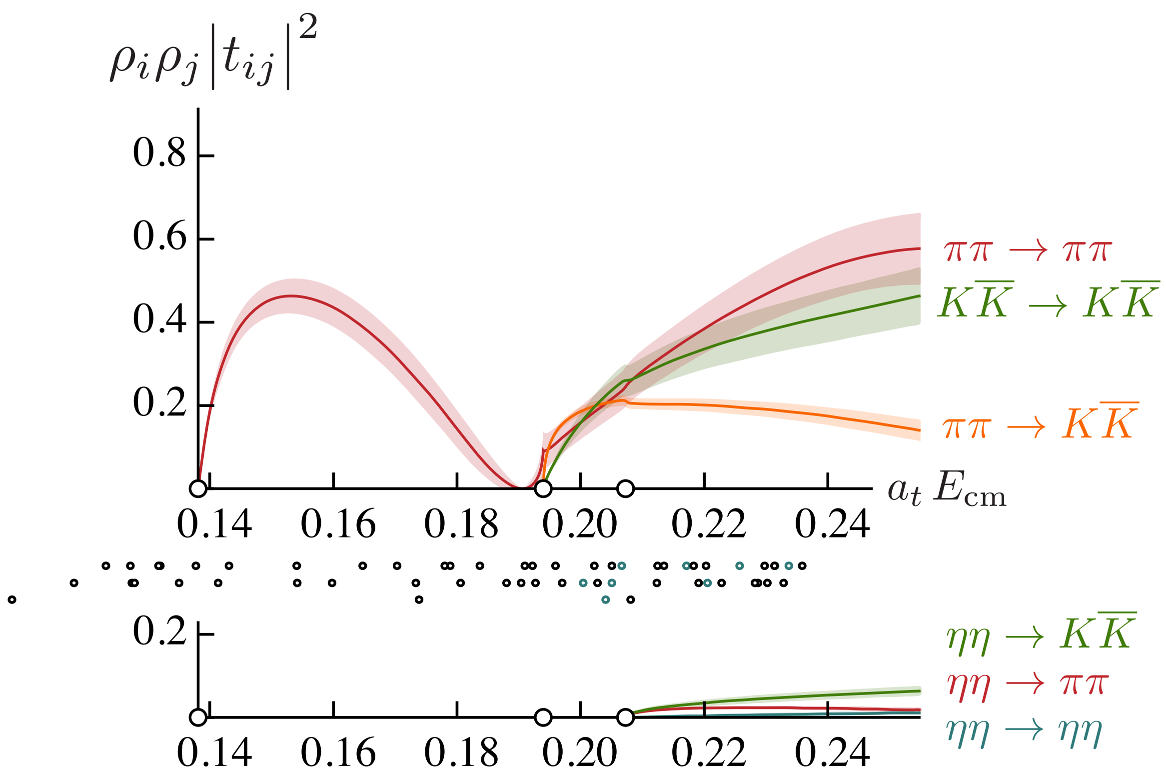


$$|c_{K\bar{K}}| \approx |c_{\pi\pi}|$$

$\pi\pi$, $K\bar{K}$, $\eta\eta$ scattering with m_π~391 MeV

PRD 97 054513 (2018)

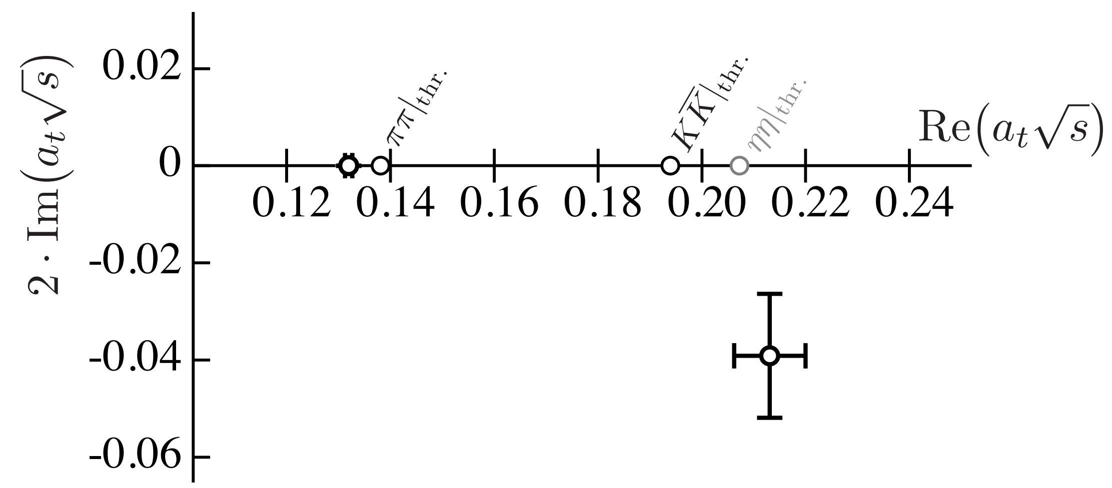
S-wave amplitudes

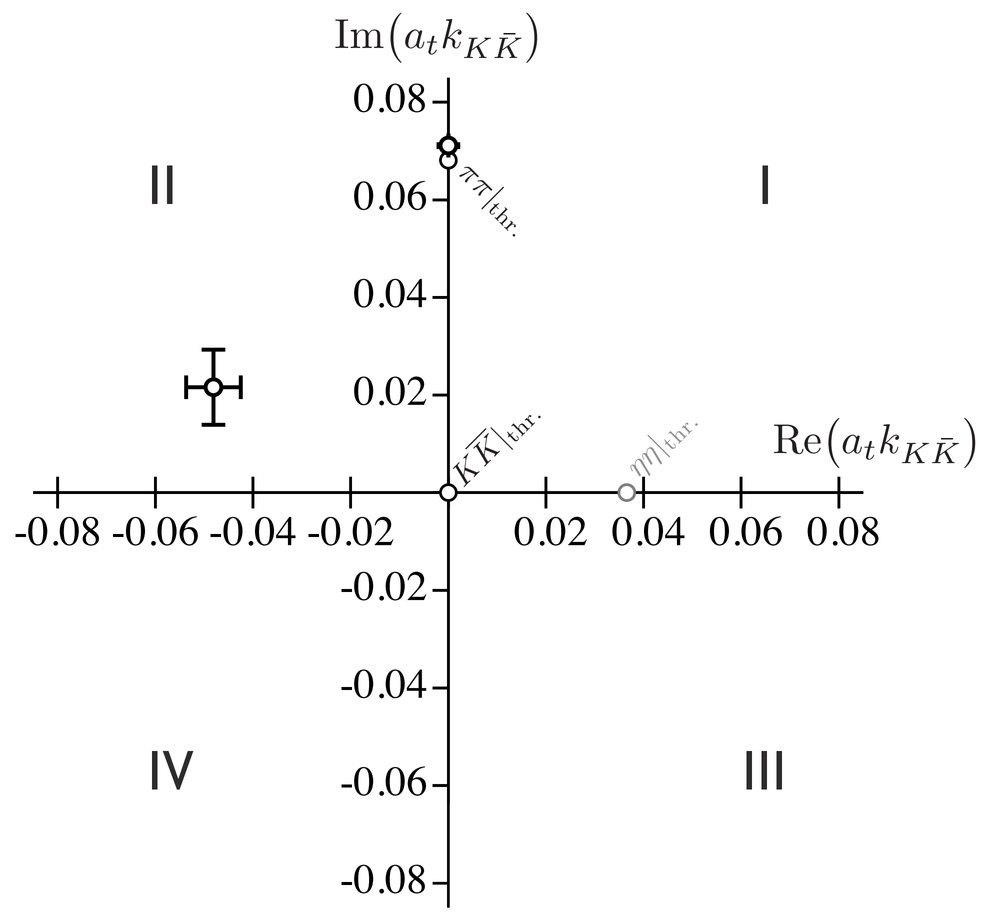


$$\mathbf{K}^{-1}(s) = \begin{pmatrix} a + b\,s & c + d\,s & e \\ c + d\,s & f & g \\ e & g & h \end{pmatrix}$$

with Chew-Mandelstam phase-space

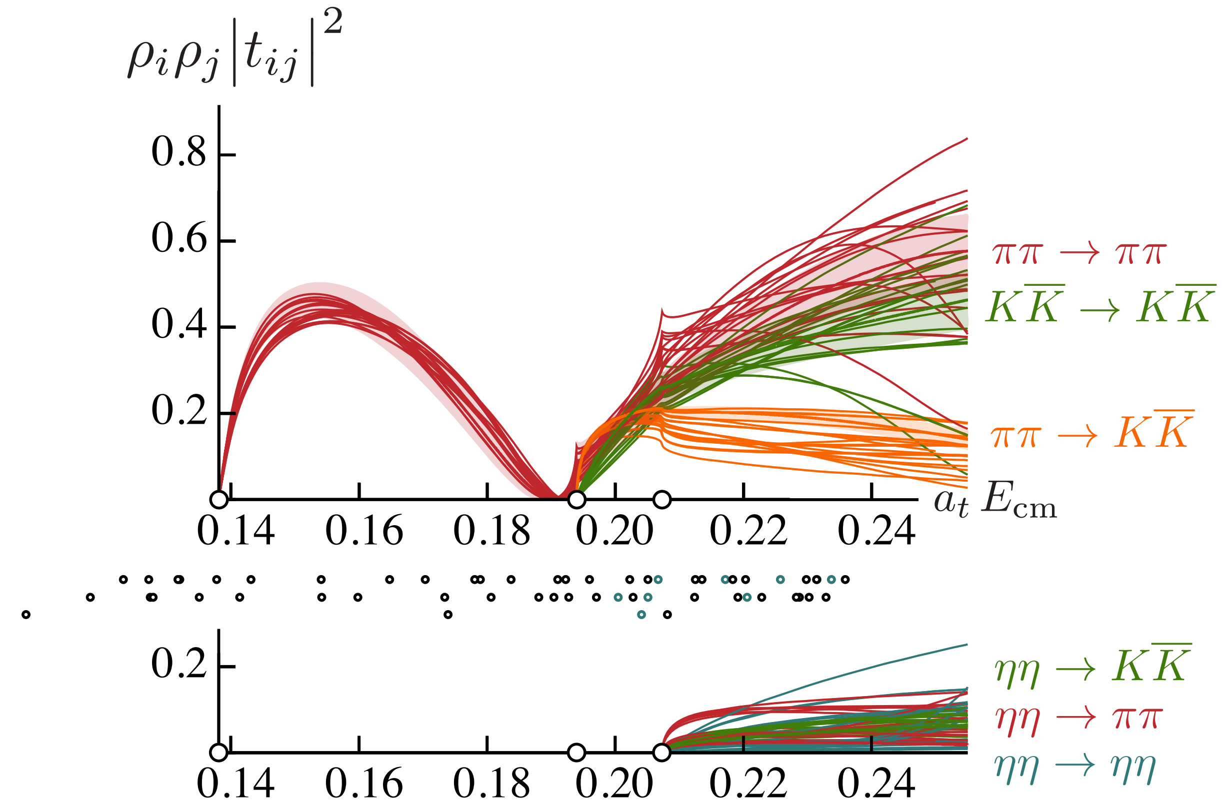
pole singularities



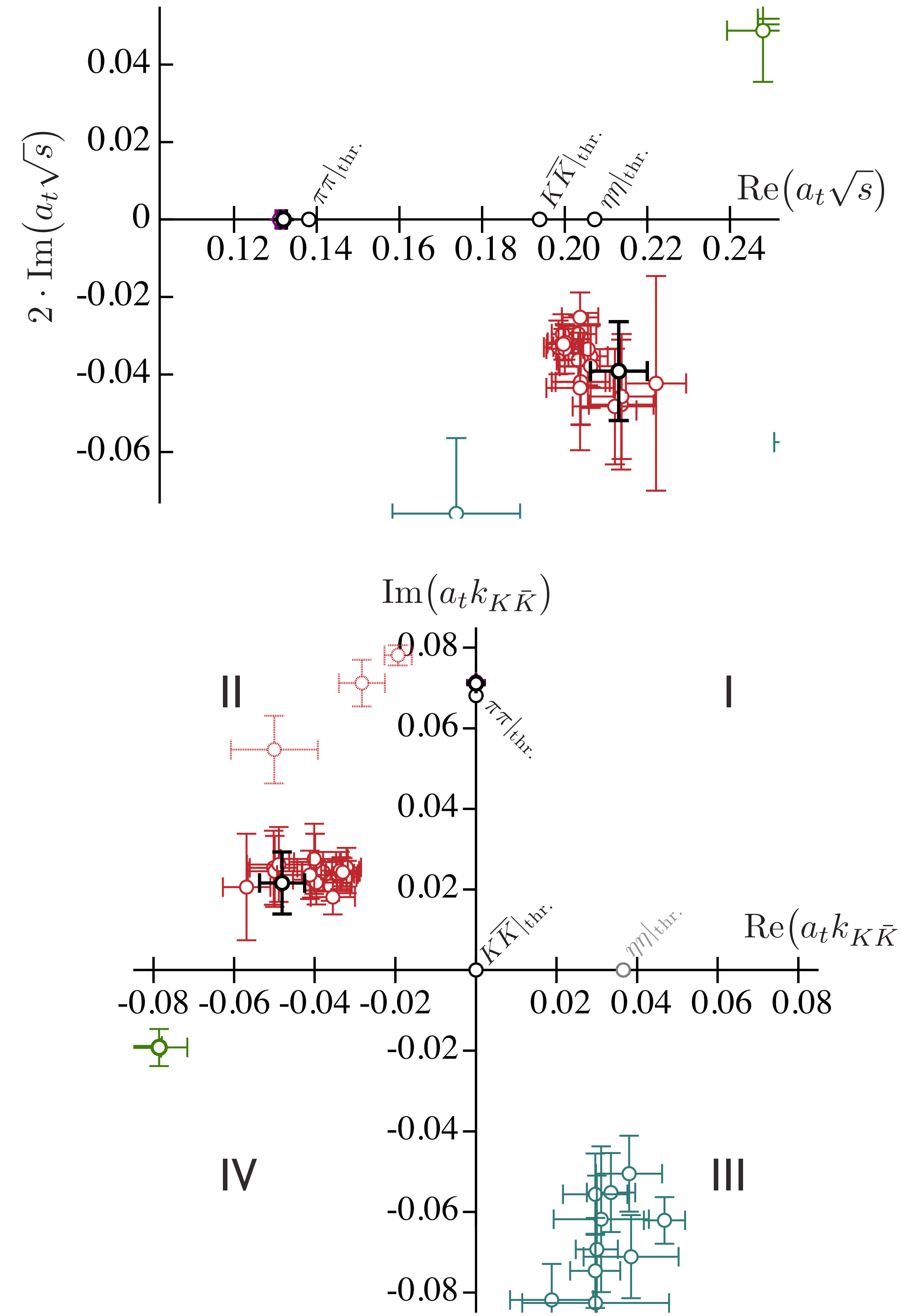


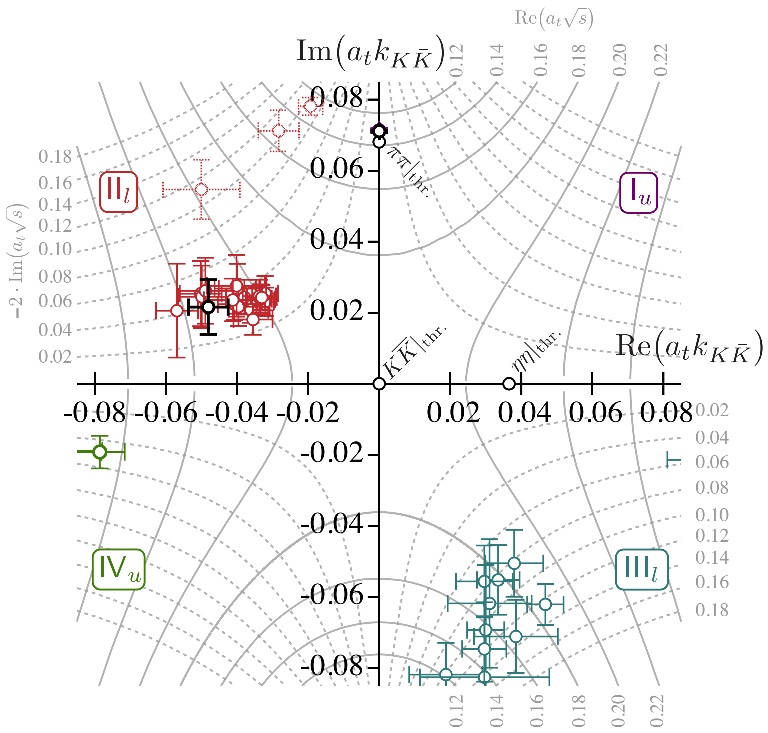
$\pi\pi$, $K\bar{K}$, $\eta\eta$ scattering with m_π~391 MeV

S-wave amplitudes



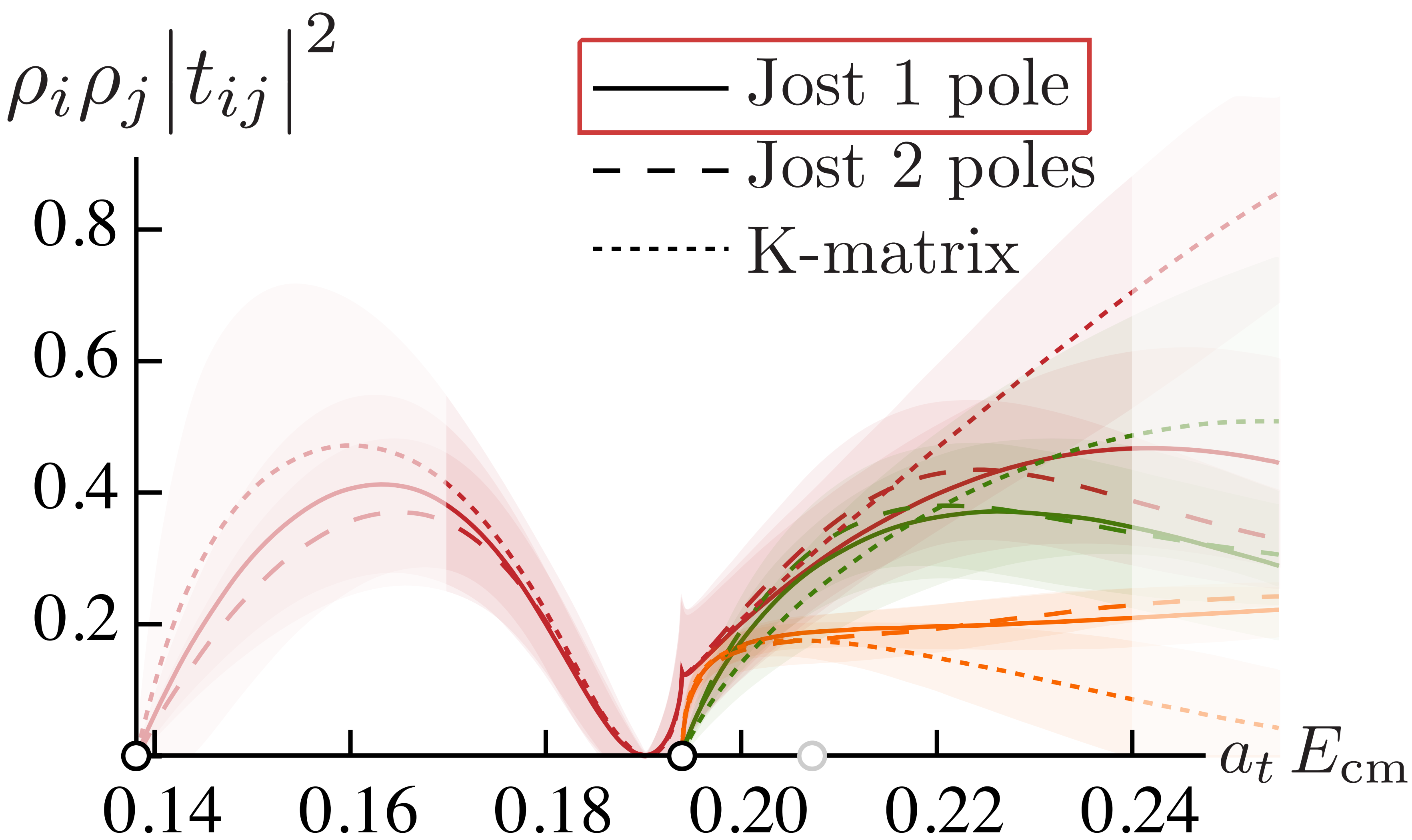
pole singularities





do we really need the **sheet III** pole ?

Jost functions allow us to directly control the pole distribution …



f_0, a_0 similarities ?

masses similar

$$m_R(f_0) = 1166(45)\,\text{MeV},$$
$$m_R(a_0) = 1177(27)\,\text{MeV},$$

widths a little different

$$\Gamma_R(f_0) = 181(68)\,\text{MeV},$$
$$\Gamma_R(a_0) = 49(33)\,\text{MeV}.$$

but channel couplings quite similar ?

$$\big|c(a_0 \to K\overline{K})\big| \approx \big|c(f_0 \to K\overline{K})\big| \quad \sim 850\,\text{MeV}$$
$$\big|c(a_0 \to \pi\eta)\big| \approx \big|c(f_0 \to \pi\pi)\big| \quad \sim 700\,\text{MeV}.$$

main difference is the **larger phase-space for $\pi\pi$ compared to $\pi\eta$**

can explore the effect using the simple Flatté amplitude

Flatté denominator $D(s) = m_0^2 - s - i g_1^2\, \rho_1(s) - i g_2^2\, \rho_2(s)$

has zeros at

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\,\rho_2}{m_0}\left[\left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} - 1\right] \quad \text{on sheet II, if} \quad \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} > 1, \text{ or,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\,\rho_2}{m_0}\left[1 - \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2}\right] \quad \text{on sheet IV, if} \quad \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} < 1, \text{ and,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2\,\rho_2}{m_0}\left[1 + \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2}\right] \quad \text{on sheet III, in all cases,}$$