Pion parton distribution functions within a light-front wave functions approach

Contribution for “Frontieers and Careers in Nuclear and Hadronic Physics”

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HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

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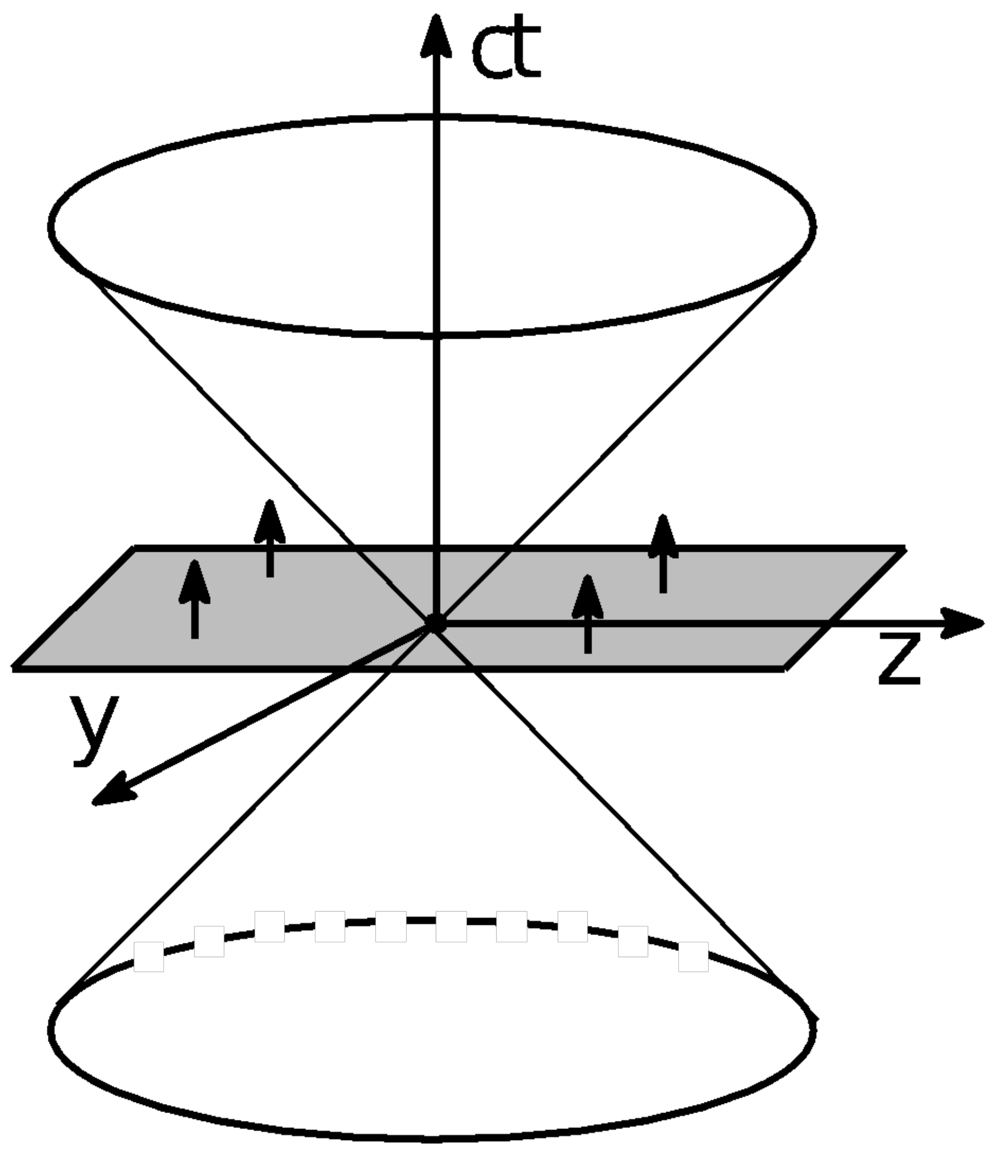
Outline

- Light-front formalism
- Parton distribuiton functions
- Our model
- Results and Comments

Light-front formalism

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x^\mu = (x^0, x^1, x^2, x^3)$$



Canonical instant-form quantization

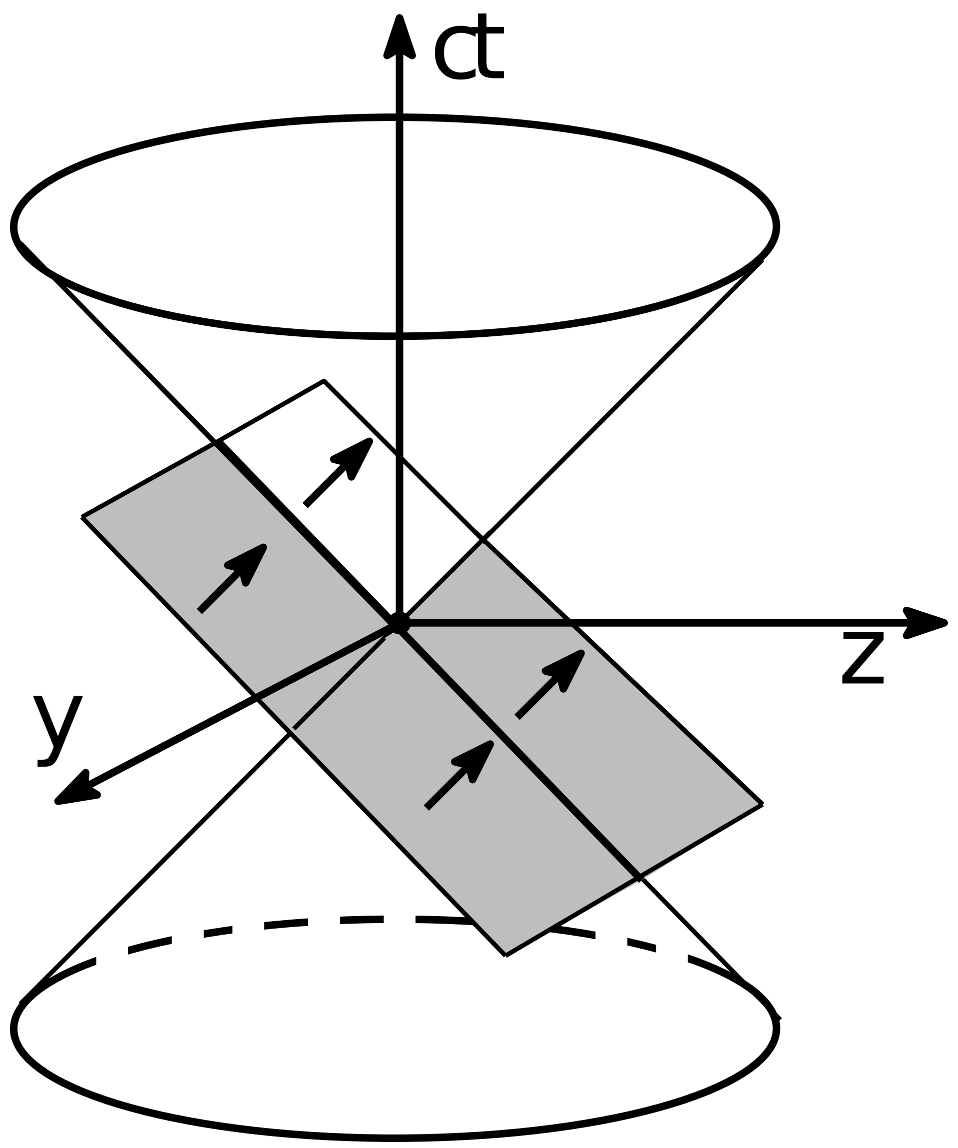
$$g_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x'^\mu(x^\mu) = (x^+, x^-, \vec{x}_\perp)$$

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$$

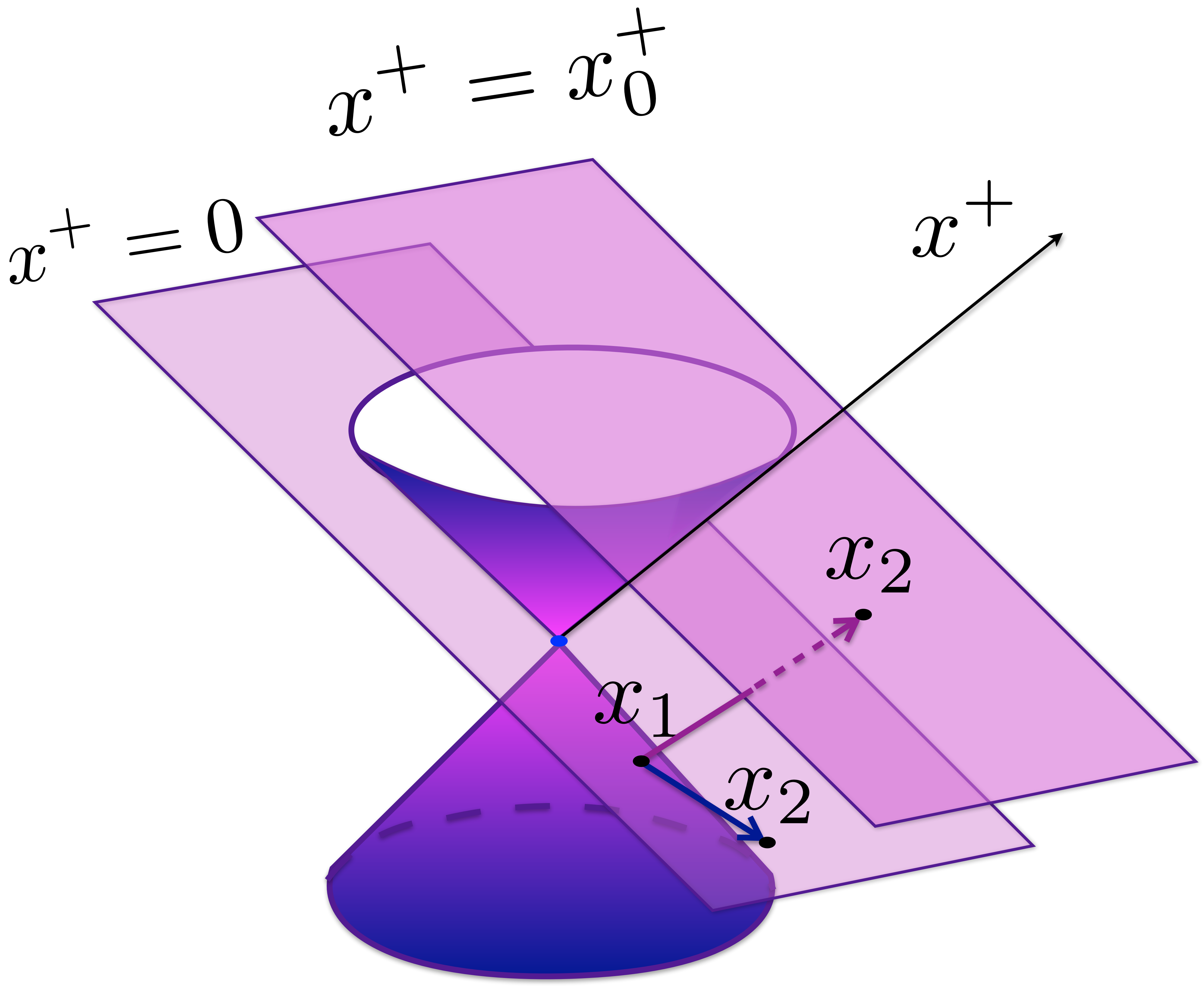
$$\vec{x}_\perp = (x^1, x^2)$$

Light-cone garage: $A^+ = 0$



Light-front quantization

Light-front formalism



Kinematic operators

$$P^+, \quad J_z, \quad \vec{B}_\perp, \quad \vec{P}_\perp$$

Dynamic operators

$$P^-$$

$$|\psi\rangle \equiv |P^+, \vec{P}_\perp, \Lambda\rangle$$

$$|\psi\rangle \leftrightarrow |n, w_1^{c_1}, w_2^{c_2}, \ldots, w_n^{c_n}\rangle$$

$$w_i^{c_i} = \left(p_i^+, \vec{p}_{\perp i}, \mu_i, c_i\right)$$

Light-front formalism

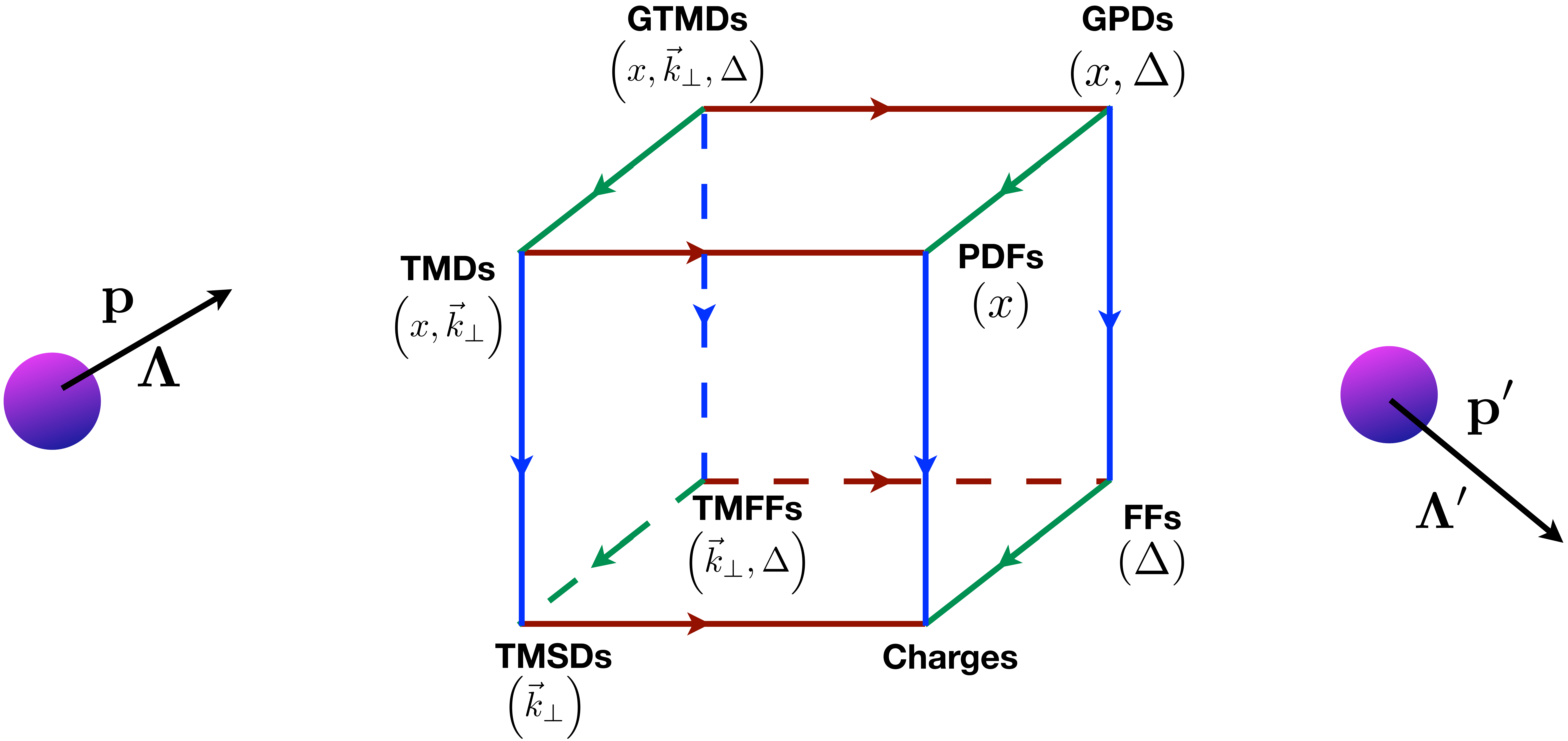
$$|n, w_1^{c_1}, w_2^{c_2}, ..., w_n^{c_n}\rangle = \prod_{j=1}^{n_q} b^\dagger_{q_j^c}(w_j) \prod_{l=1}^{n_{\bar{q}}} d^\dagger_{q_l^{\bar{c}}}(w_l) \prod_{m=1}^{n_{gl}} a^\dagger_A(w_m)\,|0\rangle$$

$$|\psi\rangle = \sum_n \sum_{c_i} \sum_{\mu_i} \int [Dx]_n \, \psi_n^\Lambda(r) |n, w_1^{c_1}, w_2^{c_2}, ..., w_n^{c_n}\rangle$$

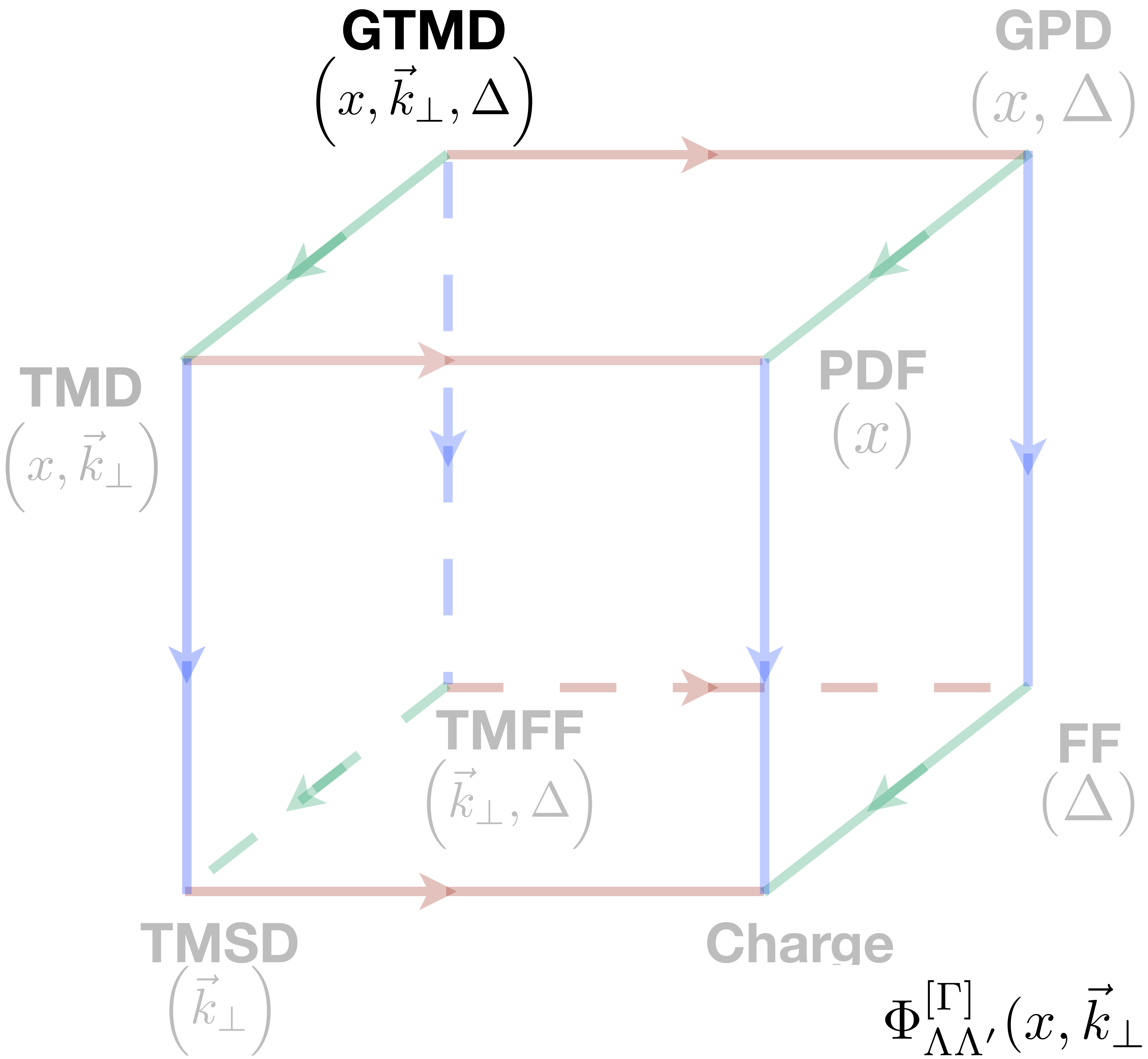
Light-Front Wave Functions

$$\psi_n^\Lambda(r) = \langle \psi | n, w_1^{c_1}, w_2^{c_2}, \ldots, w_n^{c_n}\rangle$$

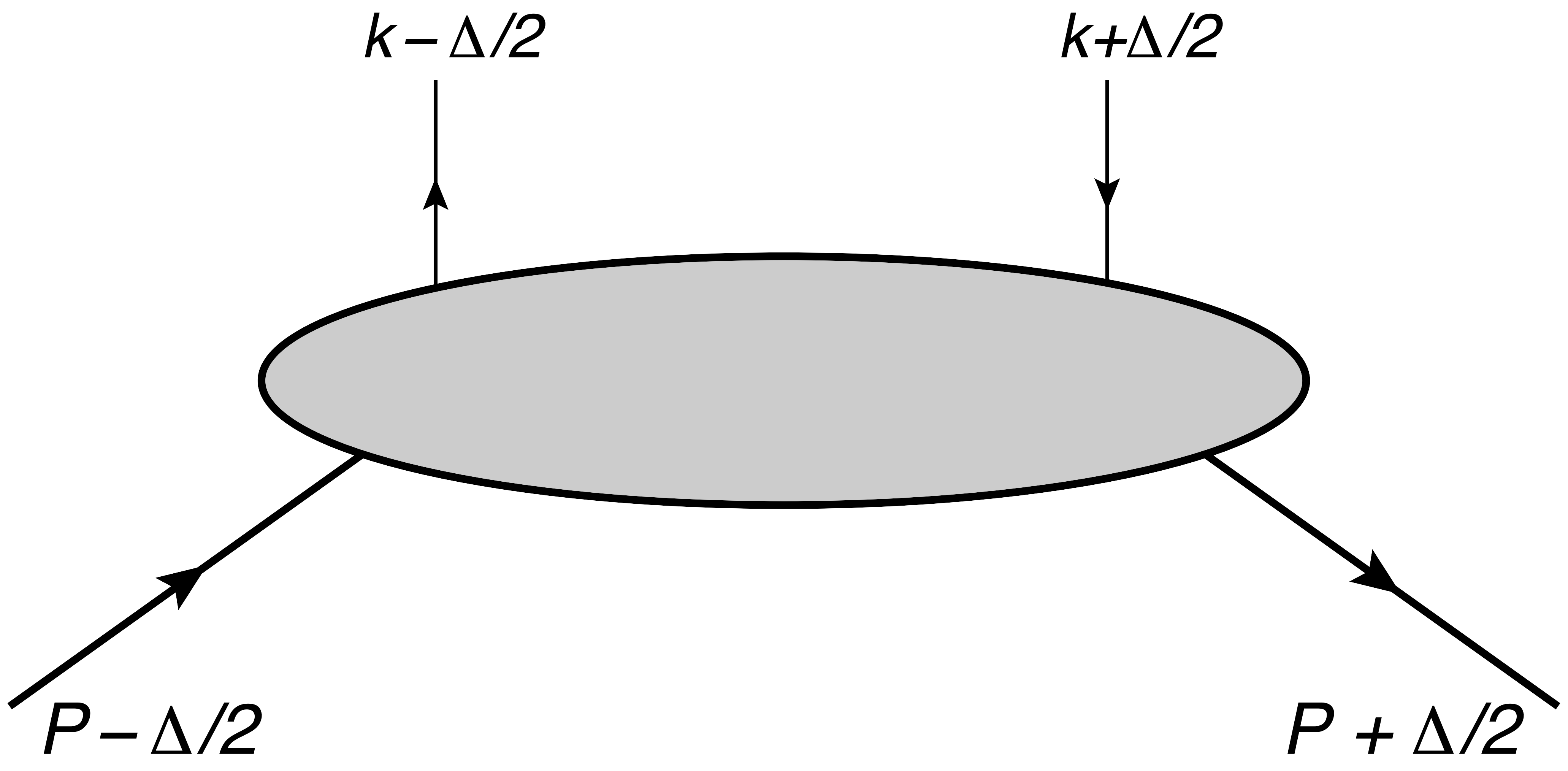
Parton distribution functions



Parton distribution functions



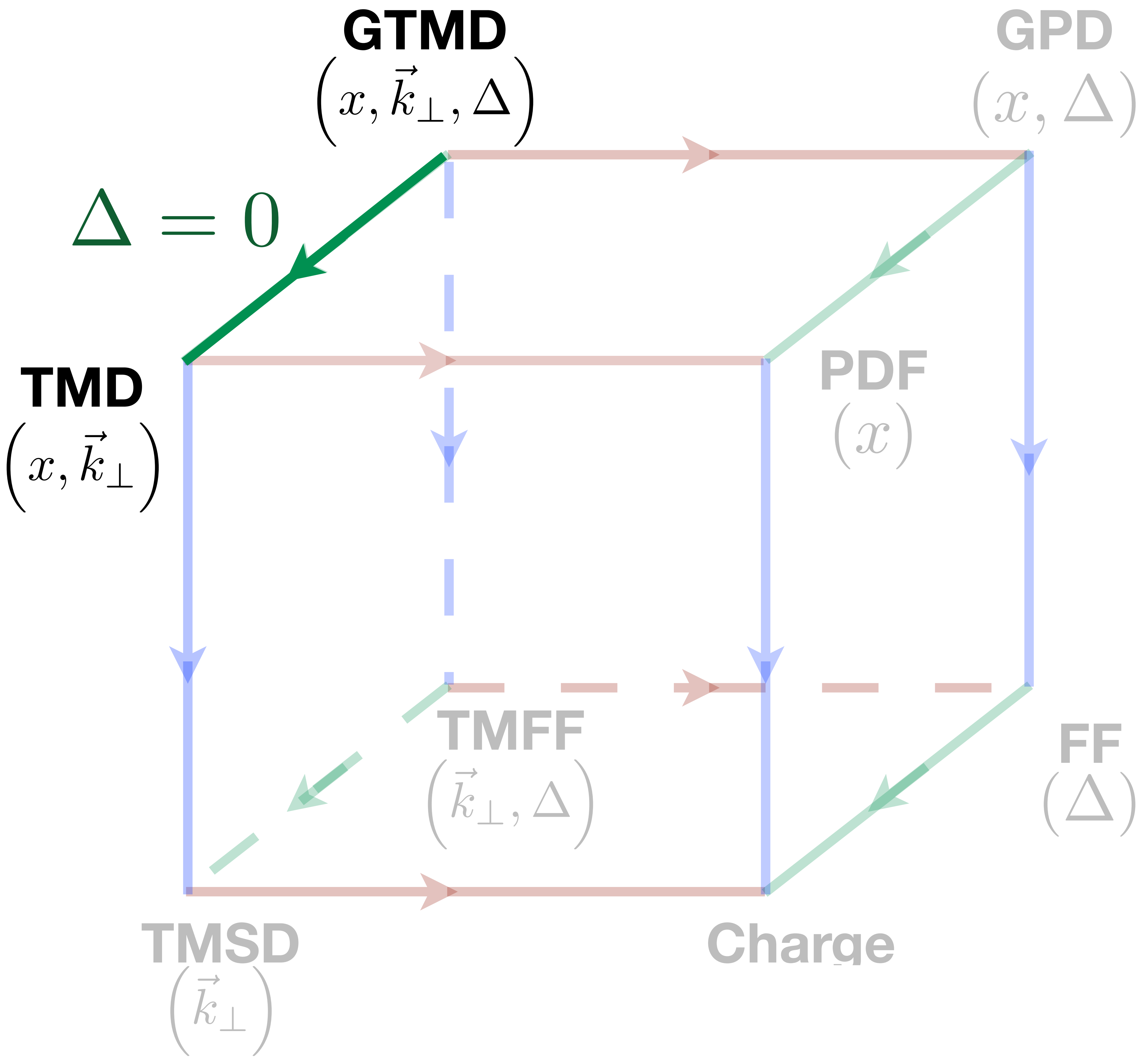
Generalized parton correlator



$$\Phi^{[\Gamma]}_{\Lambda\Lambda'}(x, \vec{k}_\perp, \Delta; P) = \frac{1}{2}\int \frac{dz^- d^2\vec{z}_\perp}{(2\pi)^3} e^{i(k^+z^- - \vec{z}_\perp\cdot\vec{k}_\perp)} \langle p', \Lambda'|\bar{q}(0)\mathcal{U}_{(0,z)}\Gamma q(z)|p, \Lambda\rangle\Big|_{z^+=0}$$

$$\mathcal{D} = \{\mathbb{1}, \gamma^\mu, \gamma_5, \gamma^\mu\gamma_5, i\sigma^{\mu\nu}\}$$

Parton distribution functions



$$\Phi^{[\Gamma]}_{\Lambda\Lambda'}(x, \vec{k}_\perp, \Delta; P)$$

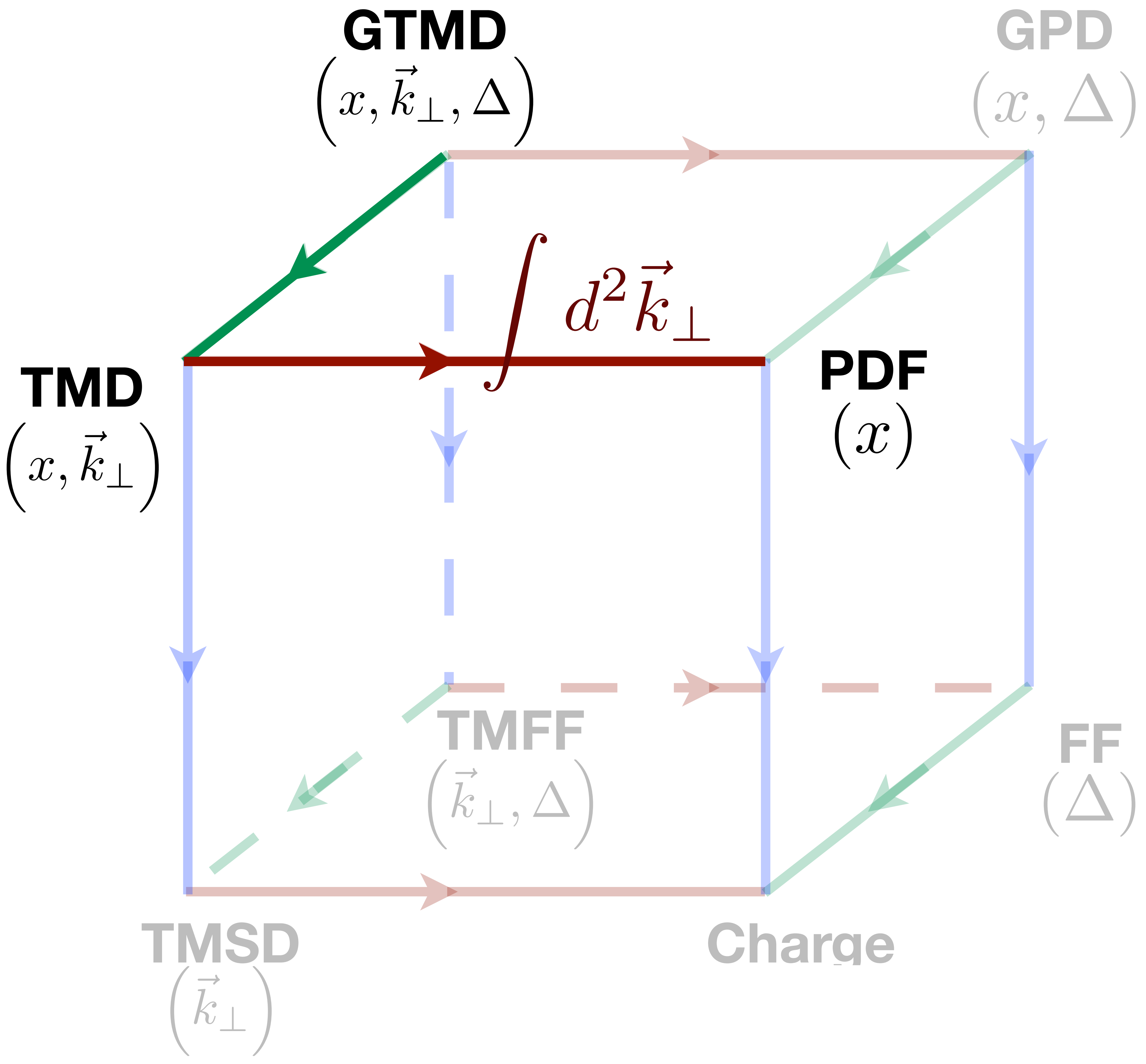
$\Delta = 0$

$$\Phi^{[\Gamma]}\left(x, \vec{k}_\perp; P\right) \equiv \Phi^{[\Gamma]}\left(x, \vec{k}_\perp, \Delta = 0; P\right)$$

$$\Gamma = \gamma^+ \qquad f_{1,\pi}\left(x, \vec{k}_\perp\right)$$

Transverse momentum dependent parton distribution function

Parton distribution functions



$$\Phi^{[\Gamma]}_{\Lambda\Lambda'}(x, \vec{k}_\perp, \Delta; P)$$

$$\Phi^{[\Gamma]}\left(x, \vec{k}_\perp; P\right) \equiv \Phi^{[\Gamma]}\left(x, \vec{k}_\perp, \Delta = 0; P\right)$$

$$\Gamma = \gamma^+$$

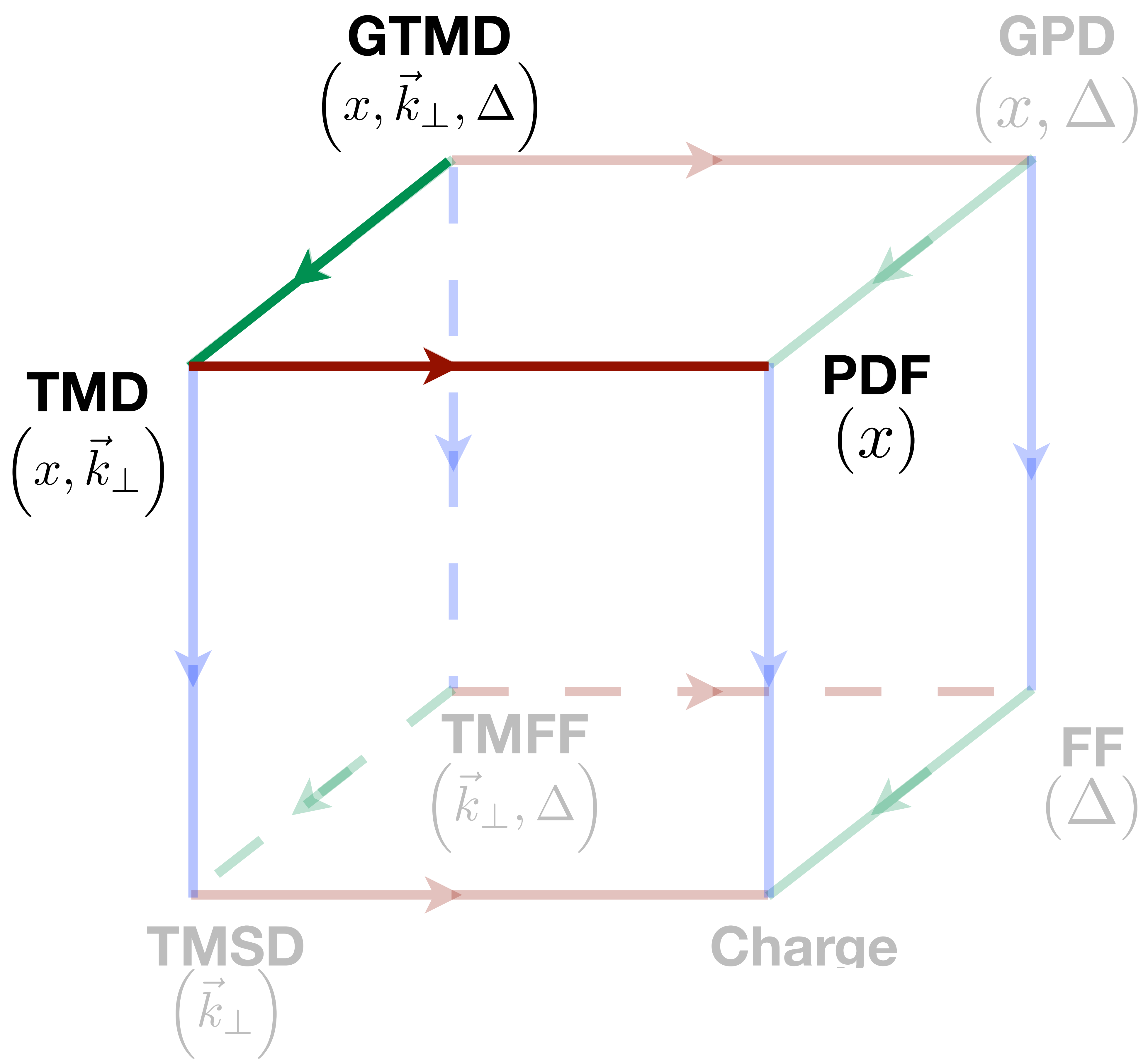
$$f_{1,\pi}\left(x, \vec{k}_\perp\right)$$

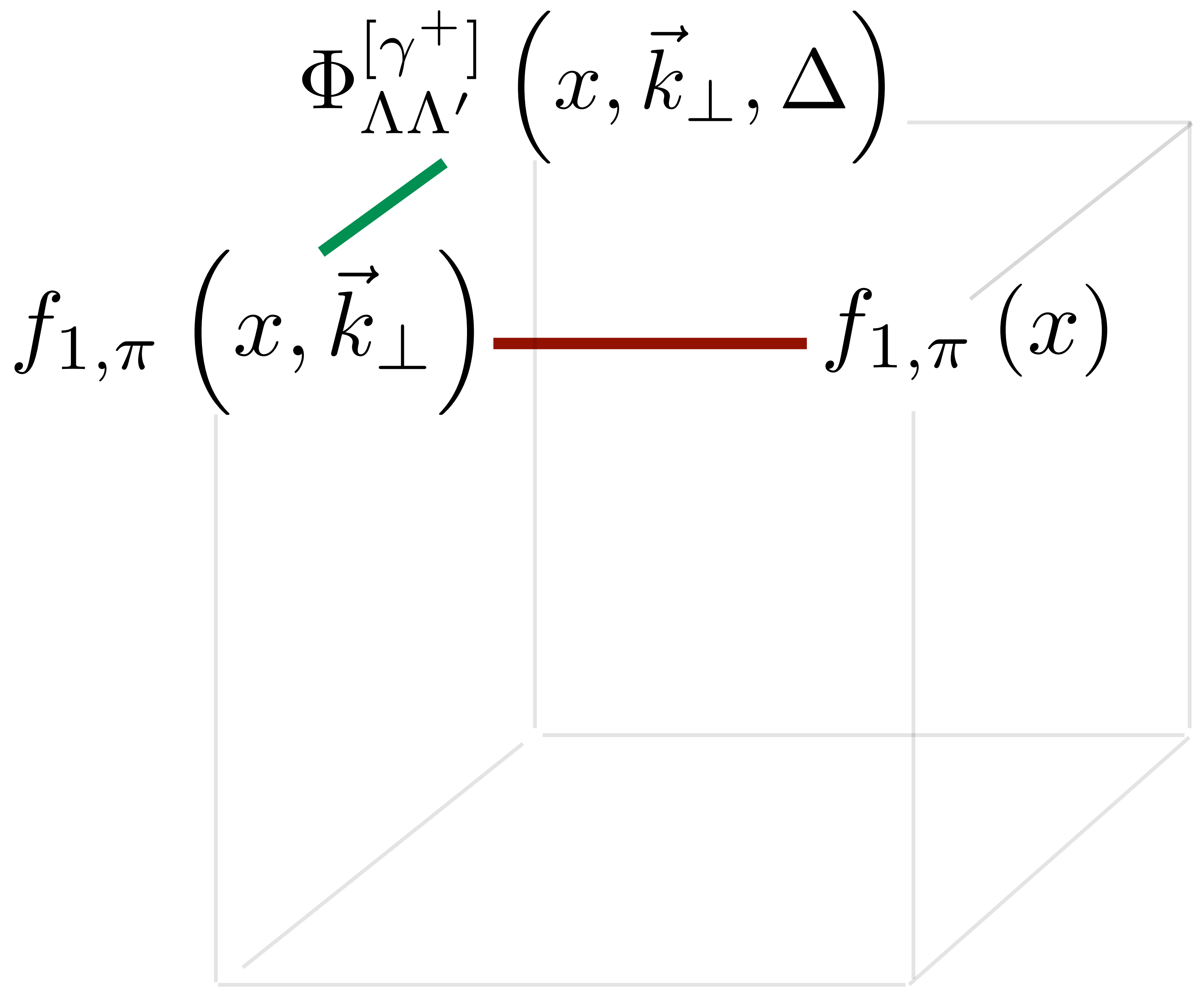
$$\int d^2\vec{k}_\perp$$

$$f_{1,\pi}(x)$$

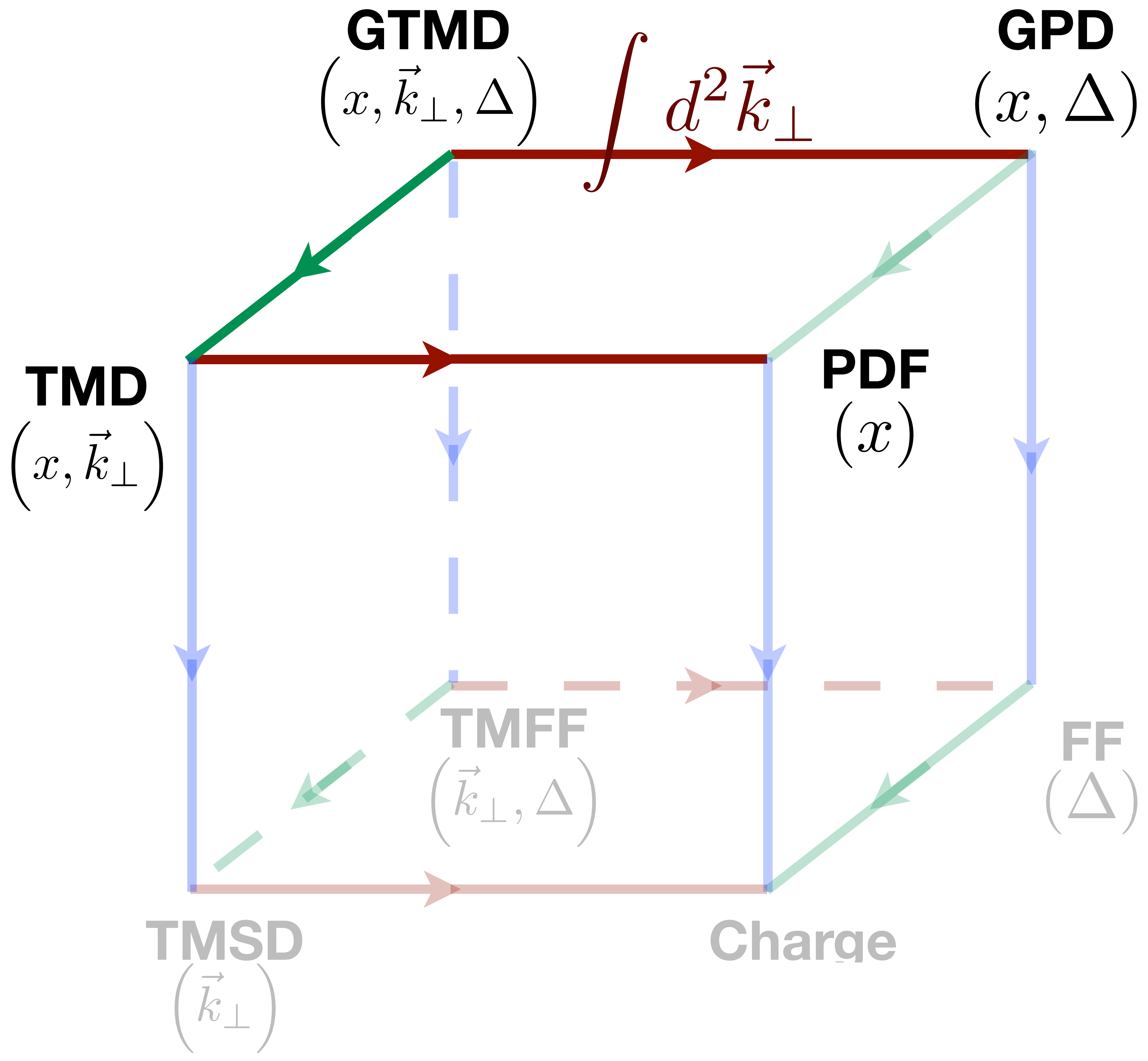
Collinear parton distribution function

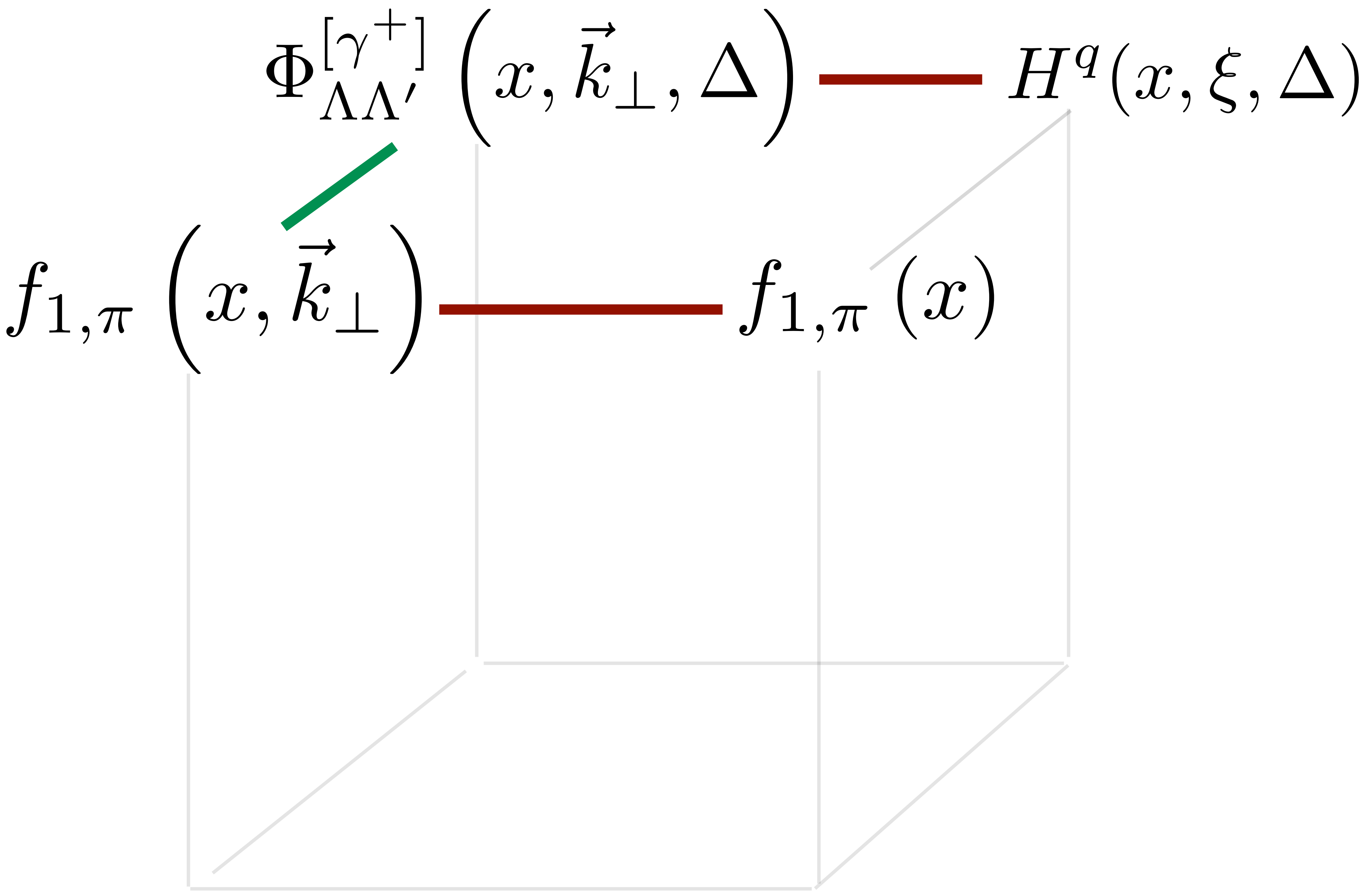
Parton distribution functions



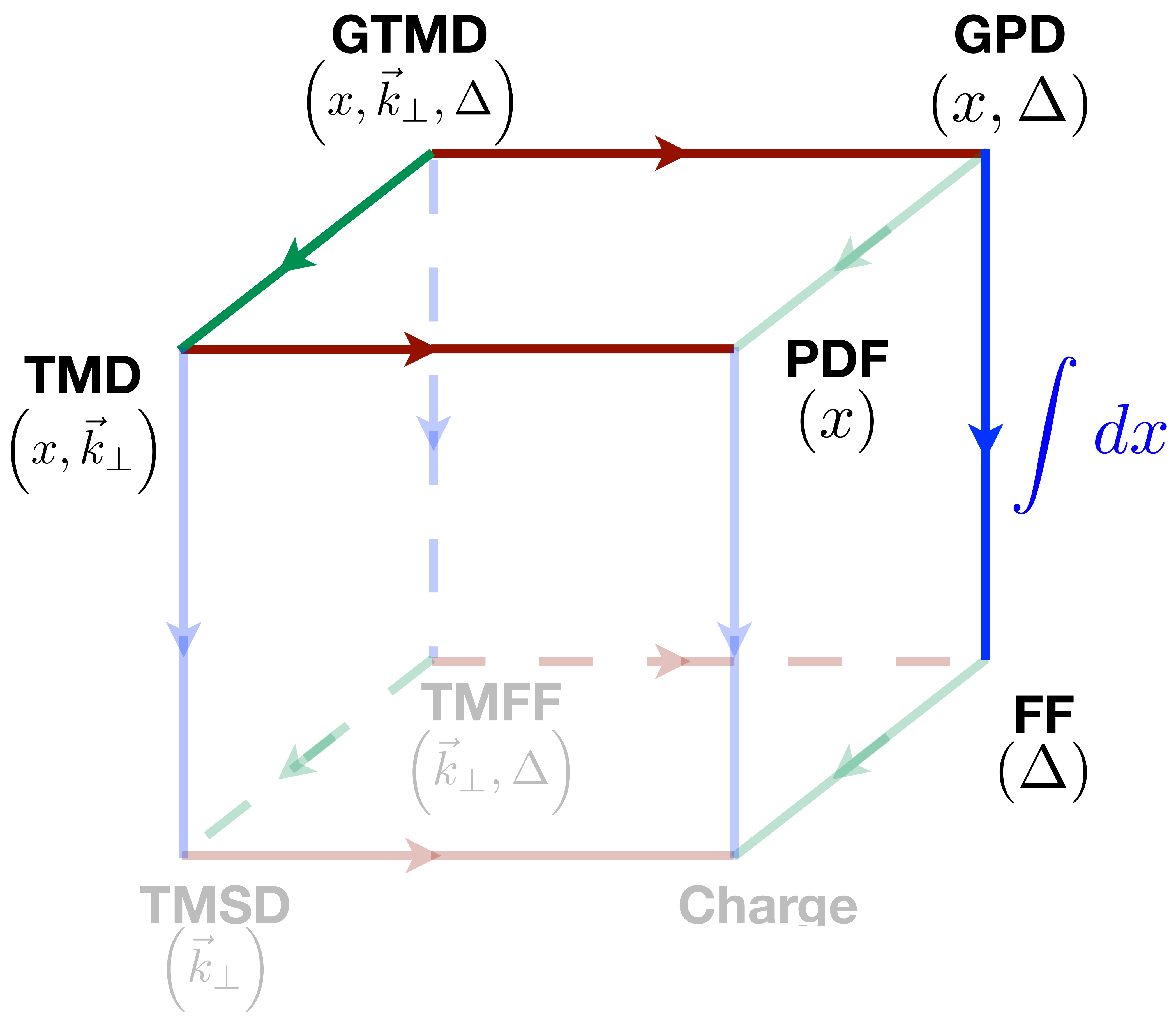


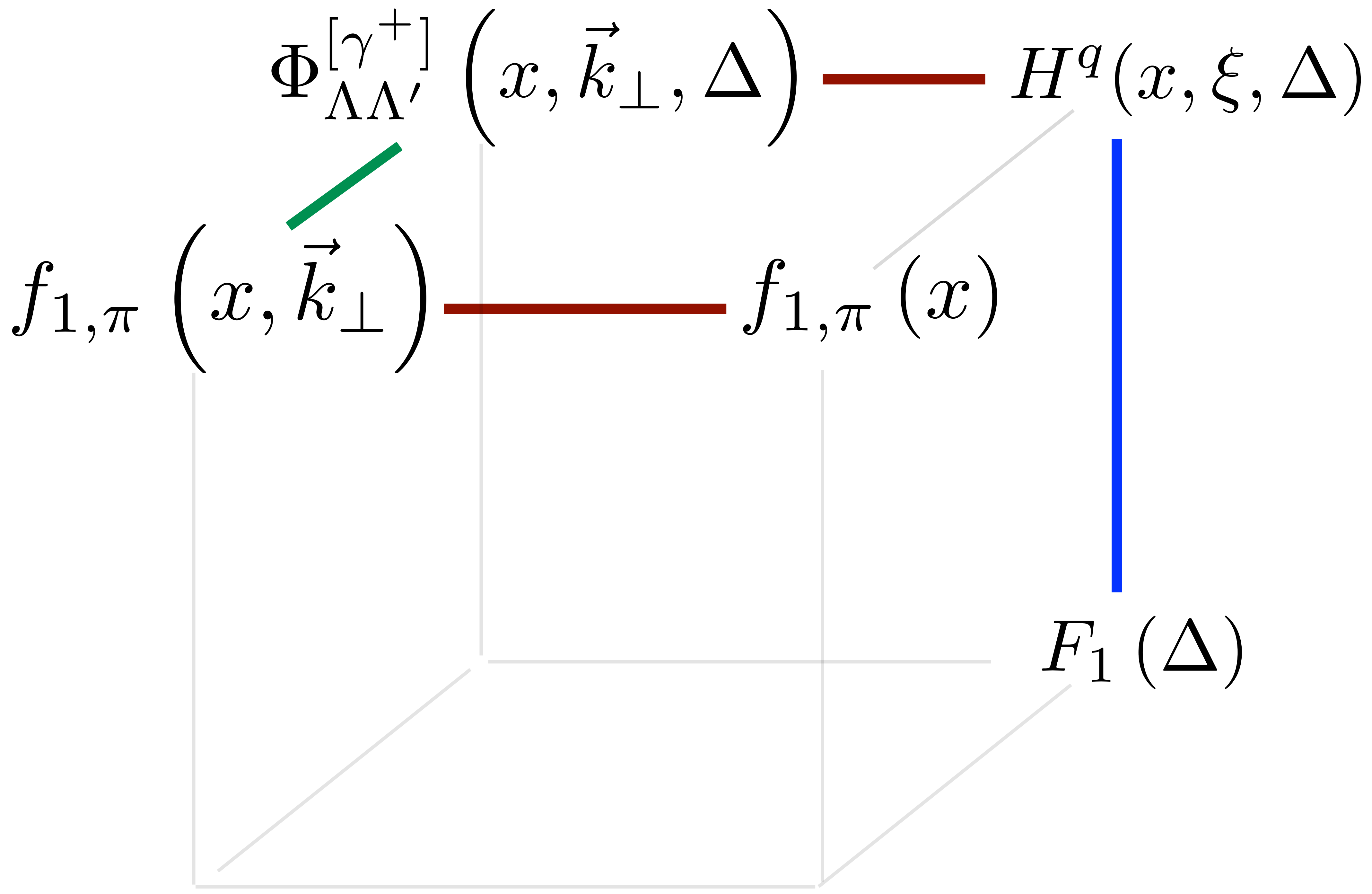
Parton distribution functions





Parton distribution functions

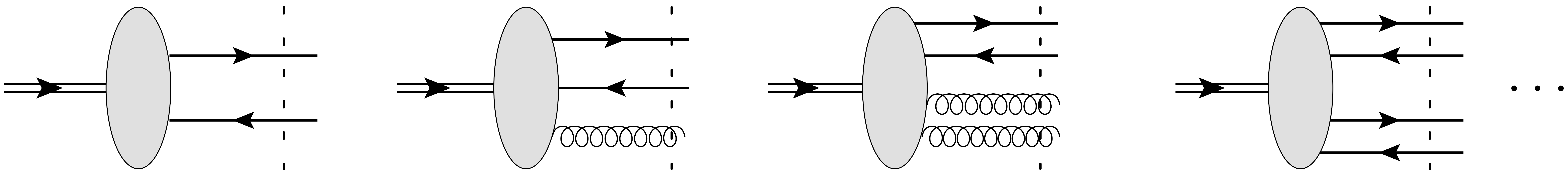




Our model

$$\Phi^{[\gamma^+]}_{\Lambda\Lambda'}\left(x,\vec{k}_\perp,\Delta;P\right)=\frac{1}{2}\int\frac{dz^-d^2\vec{z}_\perp}{(2\pi)^3}e^{i\left(k^+z^--\vec{z}_\perp\cdot\vec{k}_\perp\right)}\langle\pi(p',\Lambda')|\bar{q}(0)\mathcal{U}_{(0,z)}\gamma^+q(z)|\pi(p,\Lambda)\rangle\Big|_{z^+=0}$$

$$|\pi(p,\Lambda)\rangle=\psi^\Lambda_{q\bar{q}}|\pi(p,\Lambda)_{q\bar{q}}\rangle+\psi^\Lambda_{q\bar{q}g}|\pi(p,\Lambda)_{q\bar{q}g}\rangle+\psi^\Lambda_{q\bar{q}gg}|\pi(p,\Lambda)_{q\bar{q}gg}\rangle+\psi^\Lambda_{q\bar{q}s\bar{s}}|\pi(p,\Lambda)_{q\bar{q}s\bar{s}}\rangle+\ldots$$



Our model

$$\psi^{(1)}_{q\bar{q}}(1,2) = \phi^{(1)}_{q\bar{q}}(x_1,x_2)\,\Omega_2\left(x_1,x_2,\vec{k}_{\perp 1},\vec{k}_{\perp 2}\right)$$

$$\psi^{(1)}_{q\bar{q}g}(1,2,3) = \phi^{(1)}_{q\bar{q}g}(x_1,x_2,x_3)\,\Omega_3\left(x_1,x_2,x_3,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3}\right)$$

$$\psi^{(1)}_{q\bar{q}gg}(1,2,3,4) = \phi^{(1)}_{q\bar{q}gg}(x_1,x_2,x_3,x_4)\,\Omega_4\left(x_1,x_2,x_3,x_4,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3},\vec{k}_{\perp 4}\right)$$

$$\psi^{(1)}_{q\bar{q}s\bar{s}}(1,2,3,4) = \phi^{(1)}_{q\bar{q}s\bar{s}}(x_1,x_2,x_3,x_4)\,\Omega_4\left(x_1,x_2,x_3,x_4,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3},\vec{k}_{\perp 4}\right)$$

$$\Omega_n \equiv \Omega_n\left(x_i,\vec{k}_{\perp i};\tau_n\right)_{i=1,\ldots,n}$$

$$\phi \equiv \phi\left(x_i;\lambda_j,\tau_n\right)_{i=1,\ldots,n} = C\left(x_i;\lambda_j\right)T\left(\tau_n\right)_{i=1,\ldots,n}$$

$$\int\left[d^2\vec{k}_\perp\right]_n \Omega_n = 1$$

(Link between LFWFs and DAs)

$$\int\left[d^2\vec{k}_\perp\right]_n \Omega_n^2 = \frac{1}{T^2\prod_{i=1}^{n} x_i}$$

(Normalization of pion state)

$$f_{1,\pi}(x) = \sum_n \int [dx]_n \left[d^2\vec{k}_\perp\right]_n \left|\psi_\pi^{LF}\left(\{x_i\},\left\{\vec{k}_{\perp i}\right\}\right)\right|^2 \delta(x-x_i)\delta_{iq}$$

Our model

“State of art”

1) The **collinear** parton distribution function can be parametrized only by the set $\{\lambda_i\}$.

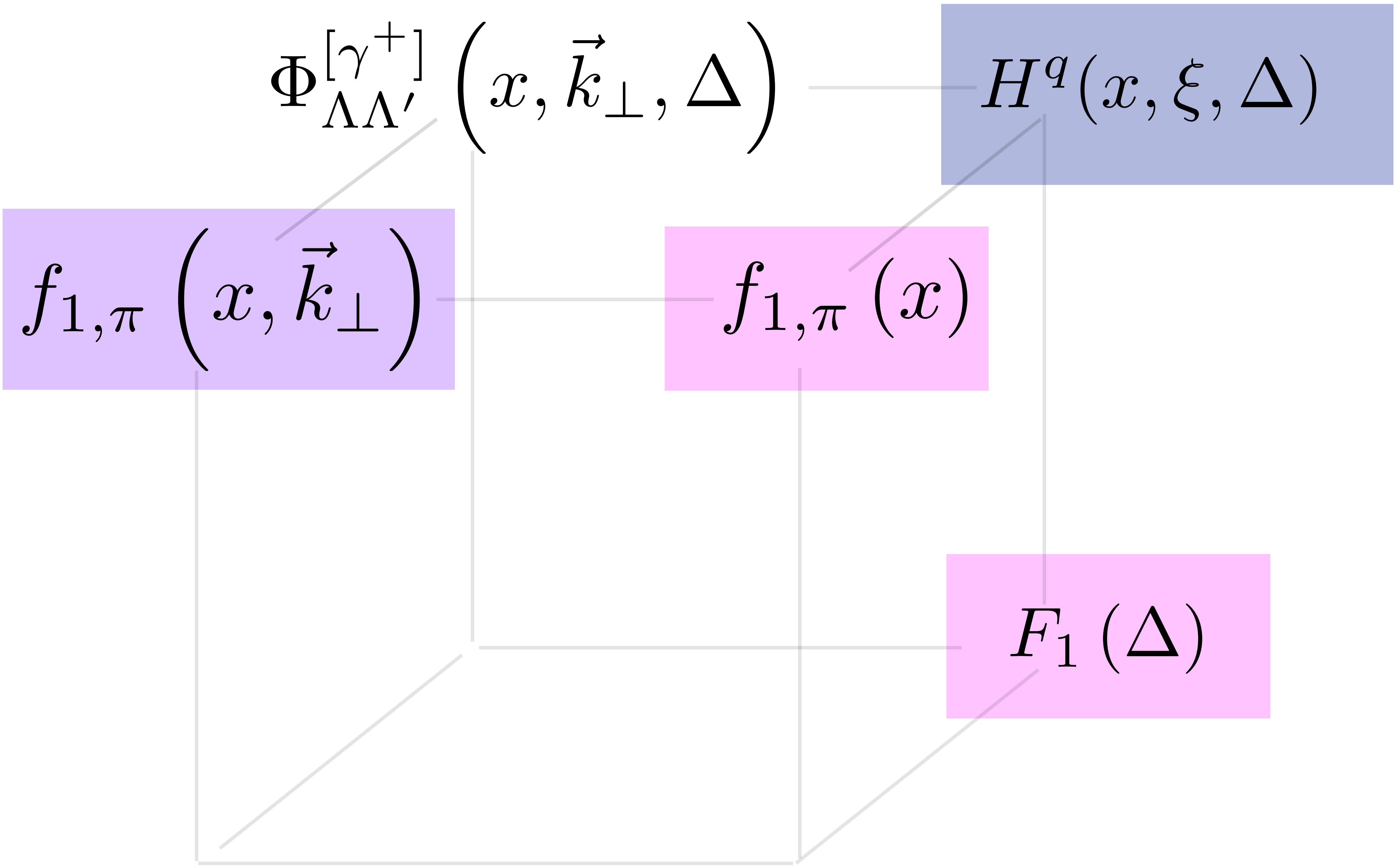
Preliminary Results for collinear parton distribution function fit

2) In the **transverse** direction, we have other “free” parameters, the set $\{\tau_i\}$, that can be used to fit other distribution functions.

Preliminary Results for pion form factor fit.

Work in Progress for transverse momentum distribution fit.

For future: investigate the pion generalized parton distribution.



Results and Comments

Pion collinear parton distribution function

260 experimental data have been included in the fit: the observables come from Drell Yan (**NA10, E615**) and prompt photon production experiments (**WA70**). From the original dataset, we excluded the data in the rigions of hadronic resonances and the Drell Yan data with $x_F < 0$.

For the fit of pion PDF, we made use of the free software developed by the xFitter collaboration arXiv:1410.4412 [hep-ex].

Like in the xFitter model, in our model the following definition of "valence", "sea" and "gluon" collinear parton distribution functions are intended:

$$
\begin{aligned}
v &= d_v - u_v = \left(d - \bar{d}\right) - \left(u - \bar{u}\right) = 2\left(d - u\right) = 2d_v \\
S &= 2u + 2\bar{d} + s + \bar{s} = 6u \\
g &= g
\end{aligned}
$$

$$
\int_0^1 v(x)dx = 2
$$

$$
\int_0^1 x\left(v(x) + g(x) + S(x)\right)dx = 1
$$

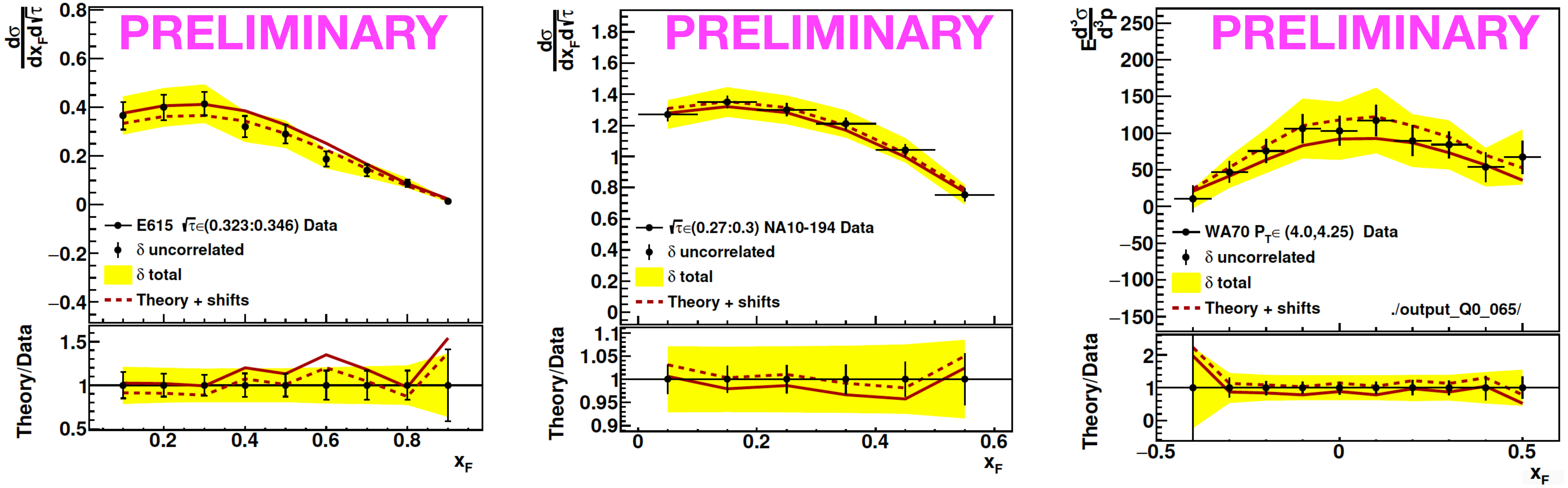
6 parameters for the **valence** collinear parton distribution function;
4 (of the 6) **parameters** for the **gluon** distribution;
3 (of the 6) **parameters** for the **sea** distribution.

https://doi.org/10.1103/PhysRevD.102.014040

Results and Comments

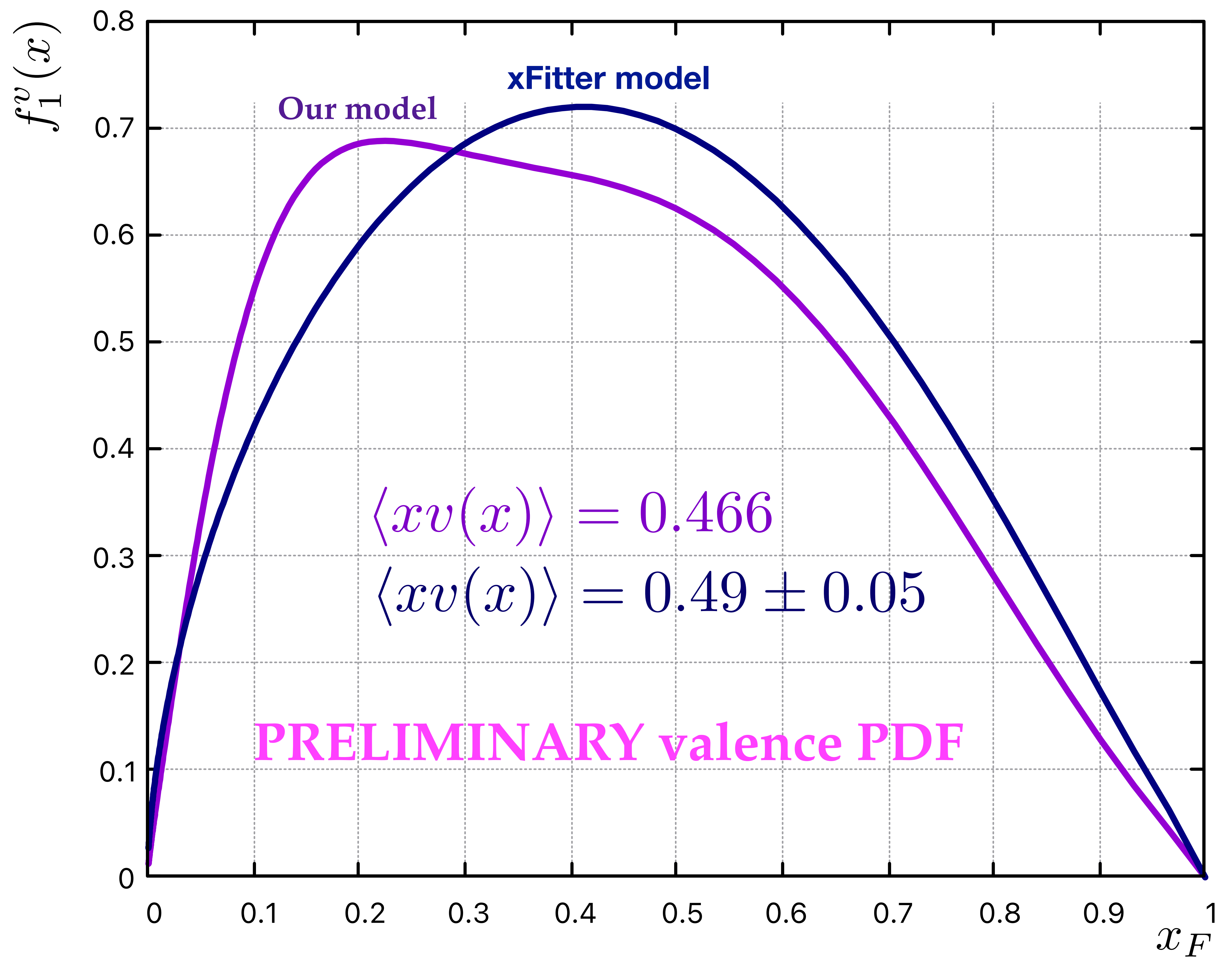
Pion collinear parton distribution function

$$\chi^2 = 0.875$$



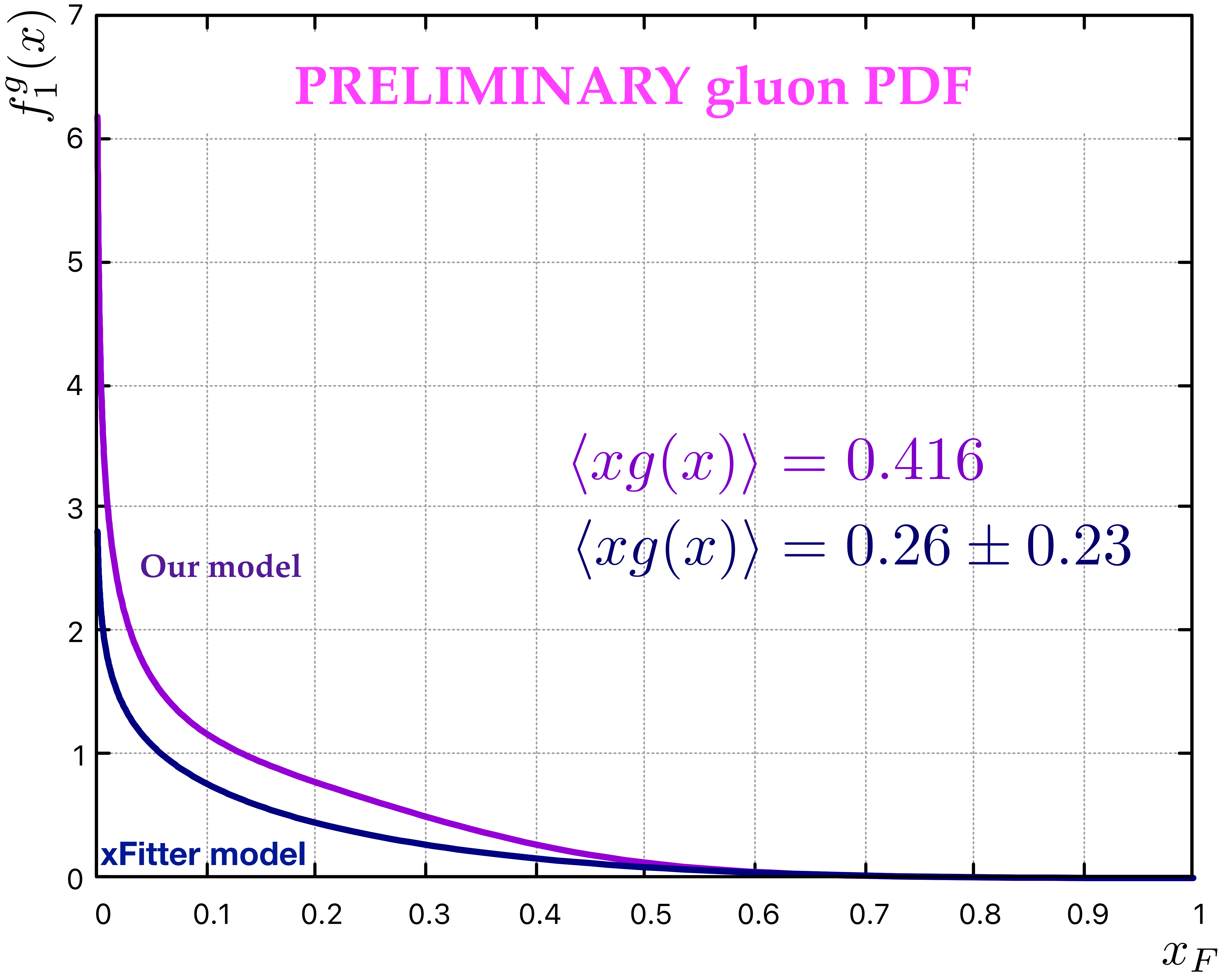
Results and Comments

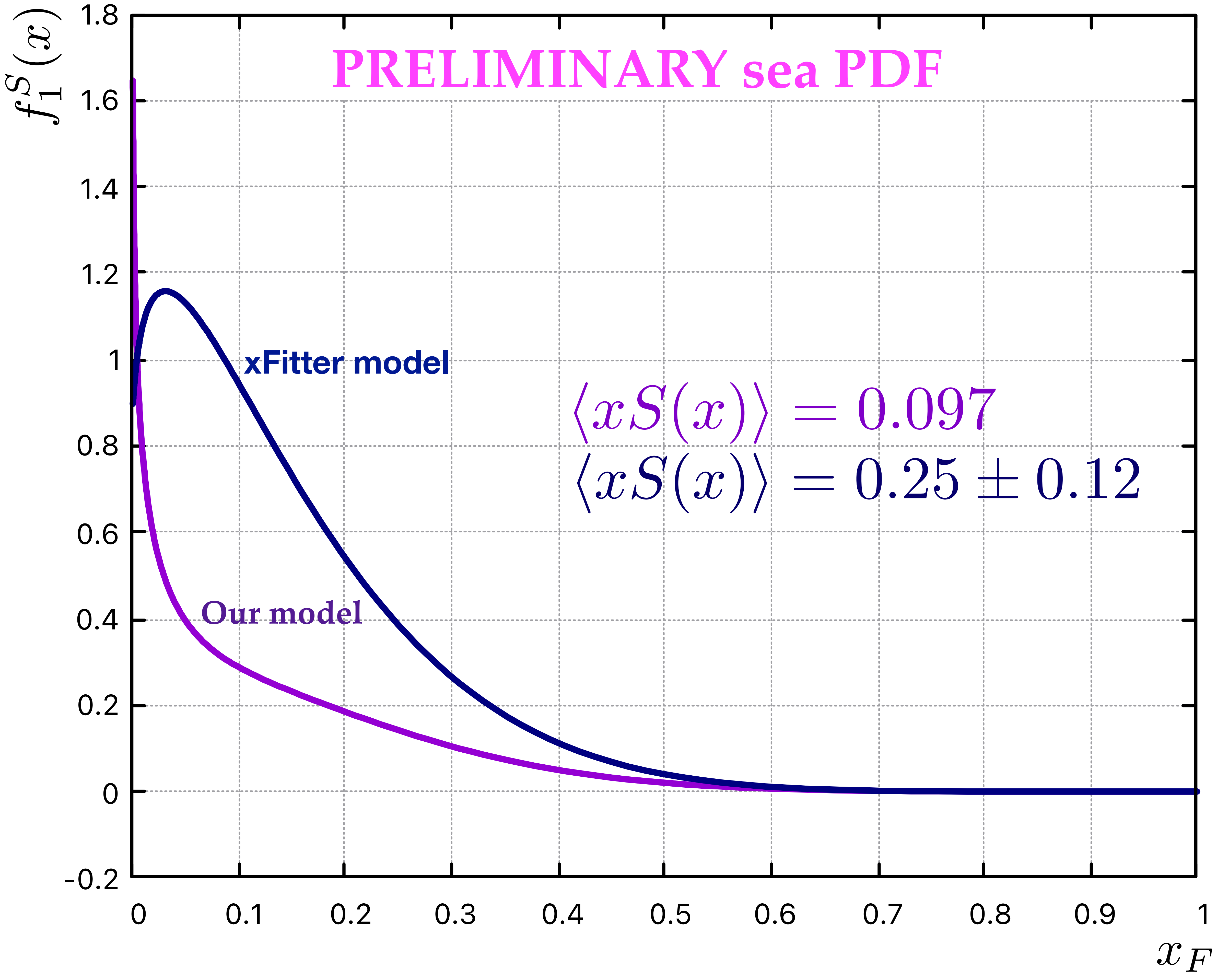
Pion collinear parton distribution function



Results and Comments

Pion collinear parton distribution function





Results and Comments

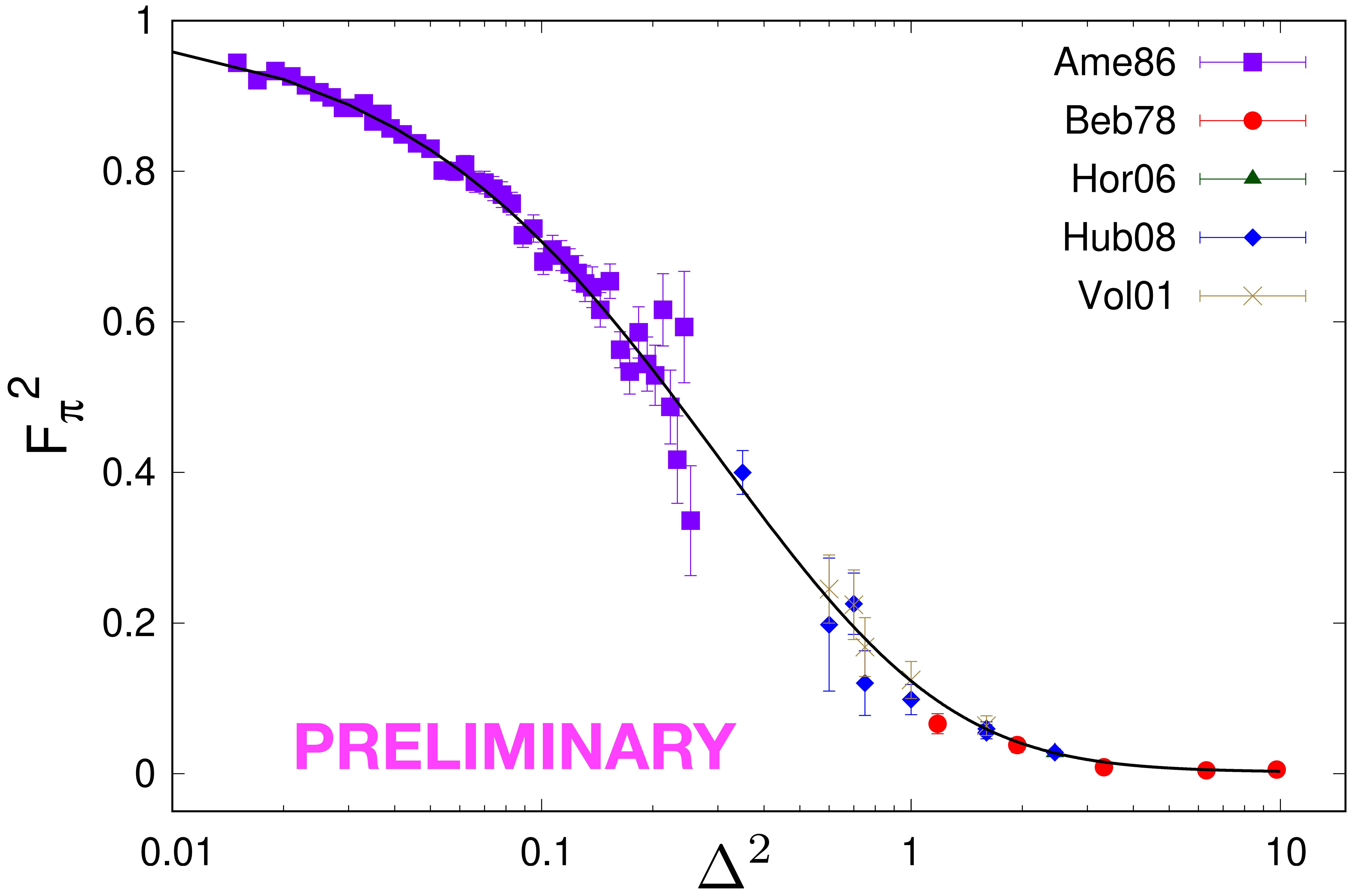
Pion Form Factor

4 parameters for the fit of the pion form factor, starting from the parameters given by the best fit of PDFs.

$$\Omega_n \equiv \Omega_n\left(x_1,\ldots,x_n,\vec{k}_{\perp 1},\ldots,\vec{k}_{\perp n}\right) = \frac{16\pi^2 a_n^2}{\prod_{i=1}^{n} x_i} \exp\left[-a_n^2\left(\sum_{i=1}^{n}\frac{\vec{k}_{\perp i}^2}{x_i}\right)\right]$$

65 experimental data included in the fit.

Chi-squared of the best-fit: $\chi^2 = 0.938$.



Results and Comments

For future:

- Error bands on PDF.
- Error bands on FFs
- Investigate other distributions, like TMDs and GPDs.