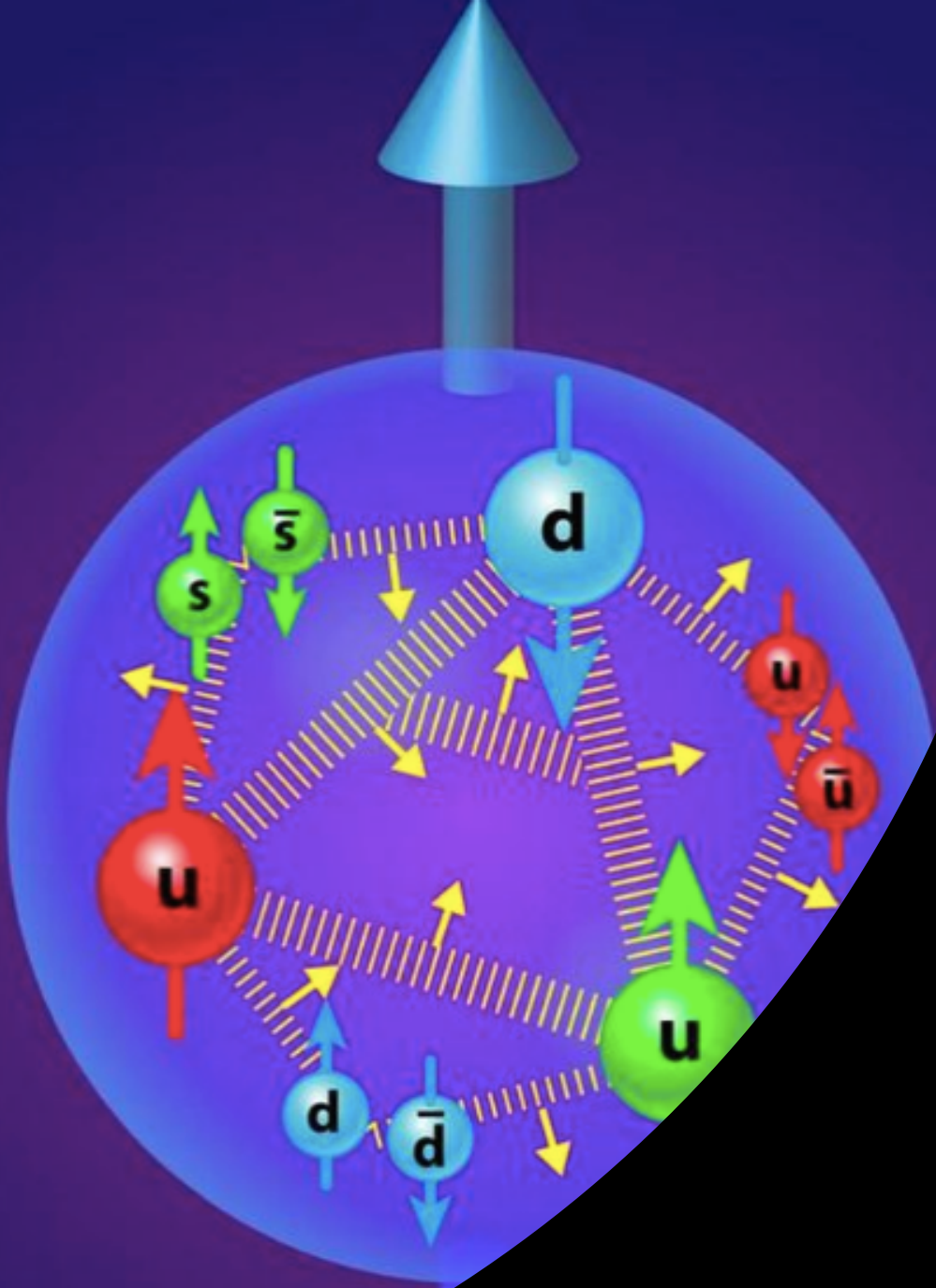


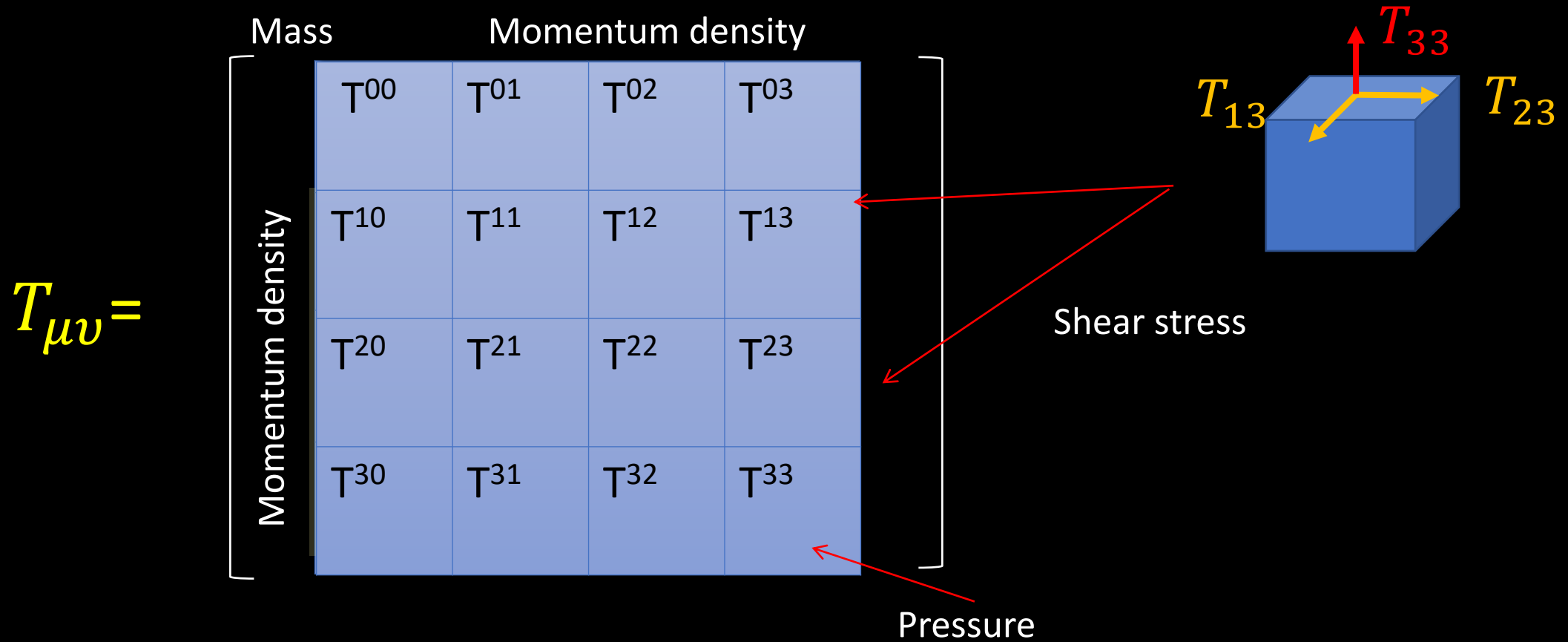
SIMONETTA LIUTI
UNIVERSITY OF VIRGINIA

J-FUTURE
Messina, Italy, March 28th 2002



How does the proton/neutron get its mass and spin and how do we test this dynamics?

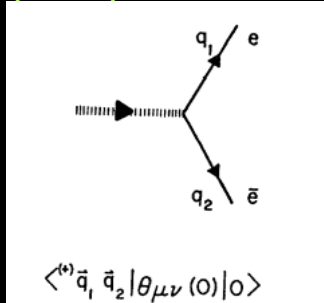
The QCD Energy Momentum Tensor



- The mechanical properties of the proton are described by the EMT: but how do we probe this?

EMT matrix elements directly through gravitational field coupling

Pagels, Phys.Rev. 144(1966)



$$\begin{aligned} \langle \mathbf{p}_1 | \theta_{\mu\nu}(0) | \mathbf{p}_2 \rangle &= (m^2/p_0^1 p_0^2)^{1/2} [\bar{u}(p_1, s_1)/4m] [G_1(q^2)(l_\mu \gamma_\nu + l_\nu \gamma_\mu) \\ &+ G_2(q^2) l_\mu l_\nu / m + G_3(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu) / m] u(p_2, s_2), \end{aligned}$$

- The matrix elements describe the response of the system to a probing gravitational field, $g_{\mu\nu}$
- The couplings are too small to ever be measured!

Donoghue et al. PLB529 (2002): quantum corrections for different metrics

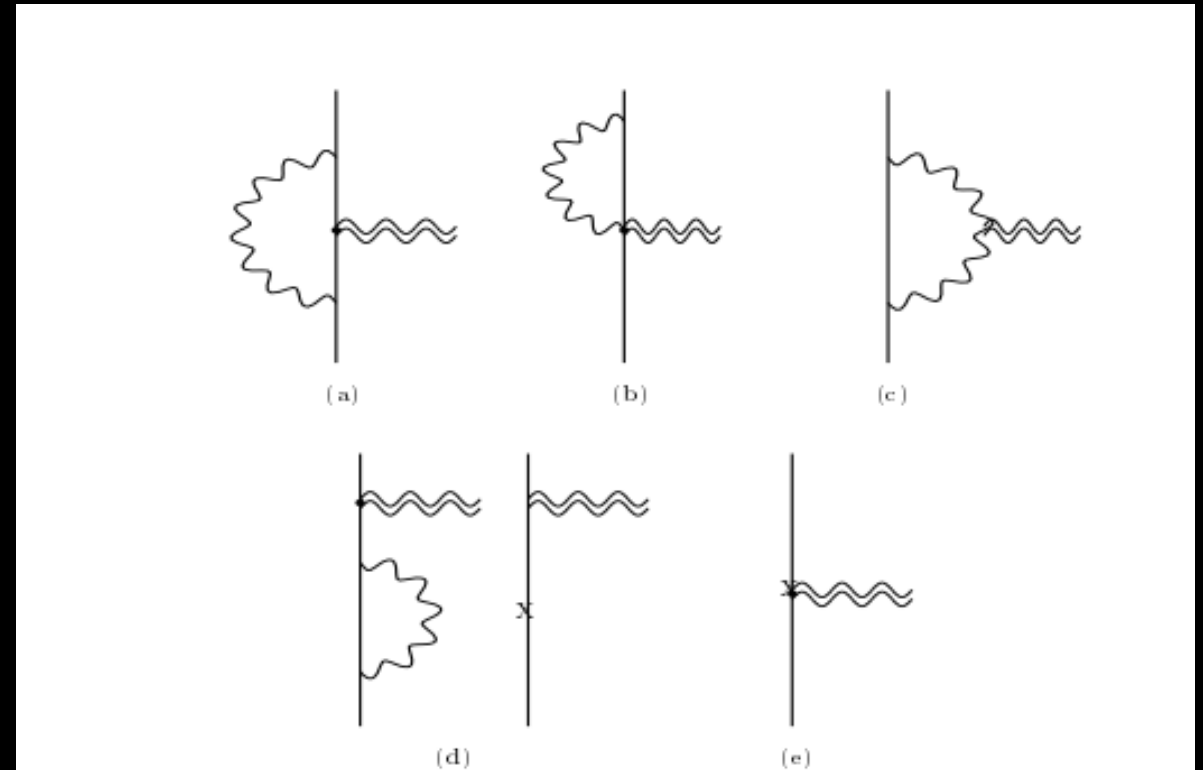
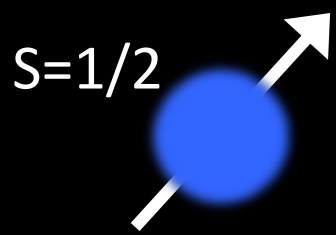


Figure 2: Feynman diagrams for spin 1/2 radiative corrections to $T_{\mu\nu}$.



X. Ji, 1997: The off-forward matrix elements can be accessed in deeply virtual Compton scattering



$$\langle p', \Lambda | T^{\mu\nu} | p, \Lambda \rangle = A(t) \bar{U}(p', \Lambda') [\gamma^\mu P^\nu + \gamma^\nu P^\mu] U(p, \Lambda) + B(t) \bar{U}(p', \Lambda') i \frac{\sigma^{\mu(\nu} \Delta^{\nu)}}{2M} U(p, \Lambda) + C(t) [\Delta^2 g^{\mu\nu} - \Delta^{\mu\nu}] \bar{U}(p', \Lambda') U(p, \Lambda) + \tilde{C}(t) g^{\mu\nu} \bar{U}(p', \Lambda') U(p, \Lambda)$$

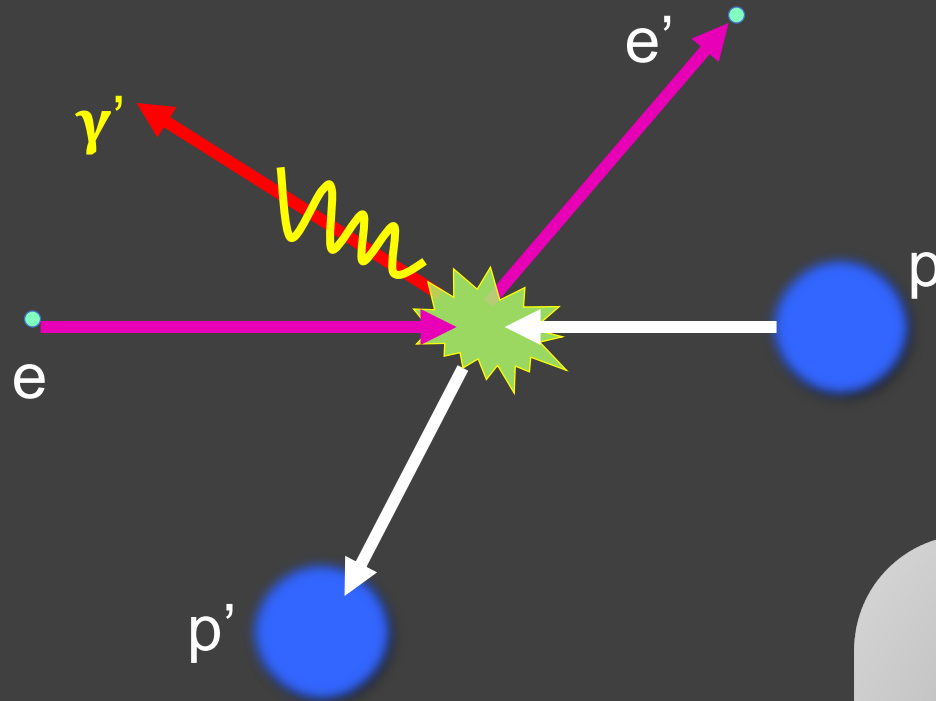
off-forward

q and g not separately conserved

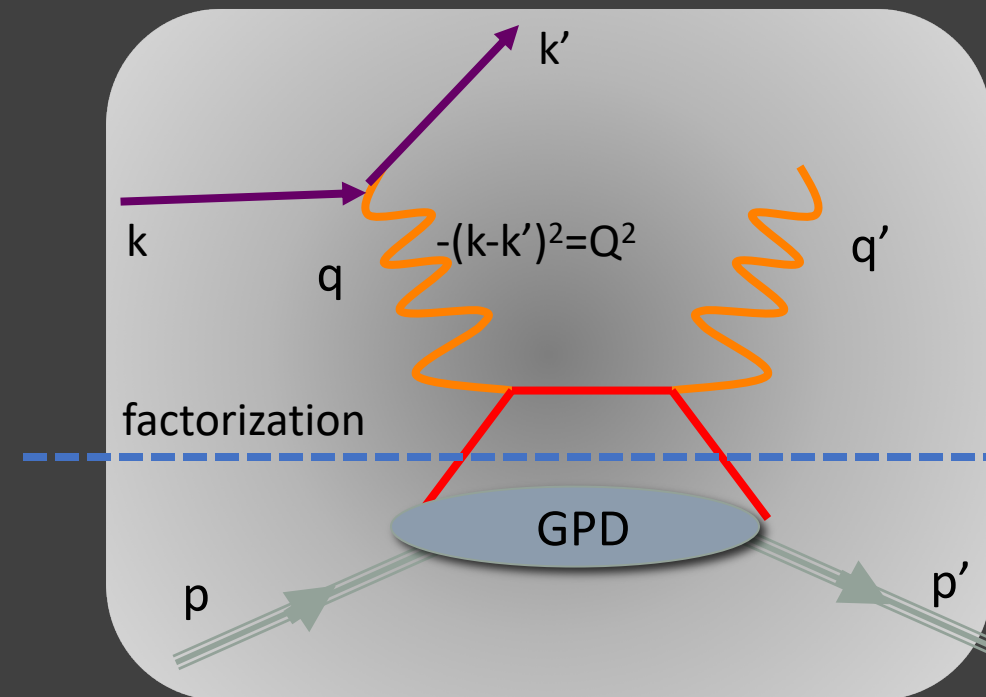
$$\left\{ \begin{array}{l} P = \frac{p+p'}{2} \\ \Delta = p' - p = q - q' \\ t = (p-p')^2 = \Delta^2 \end{array} \right.$$

Deeply Virtual Compton Scattering (X. Ji, 1997)

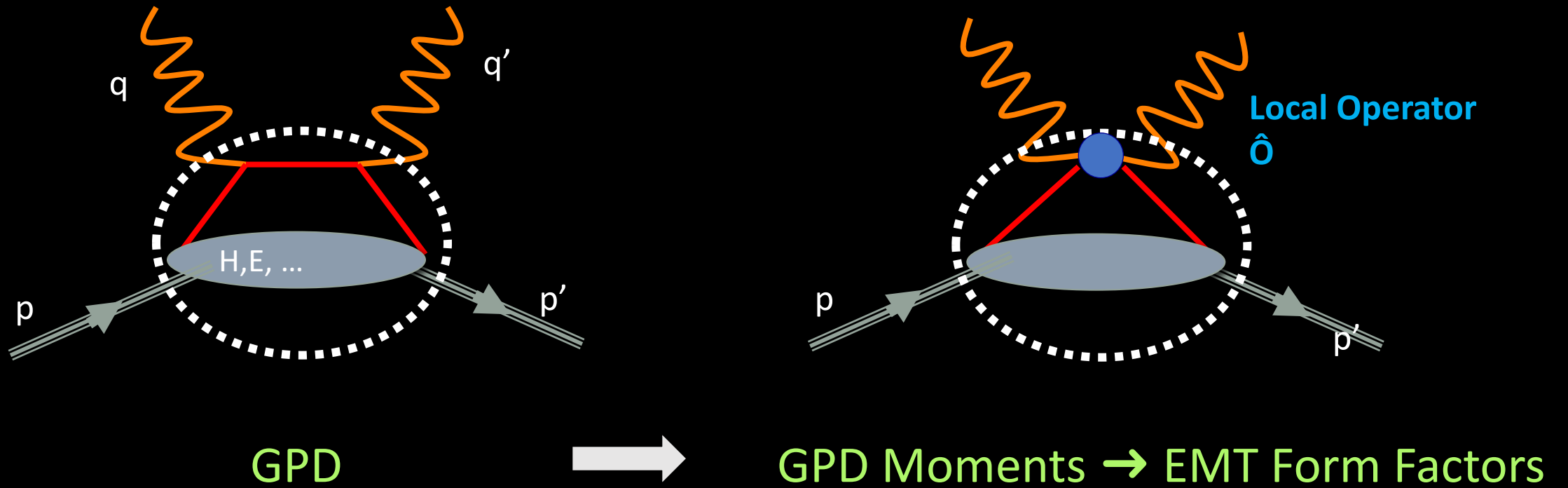
$$ep \rightarrow e' \gamma' p'$$



- Large momentum transfer $Q^2 \gg M^2 \rightarrow$ “deep”
- Large Invariant Mass $W^2 \gg M^2 \rightarrow$ equivalent to an “inelastic” process



From Generalized Parton Distributions Moments to EMT matrix elements



2nd Mellin moment

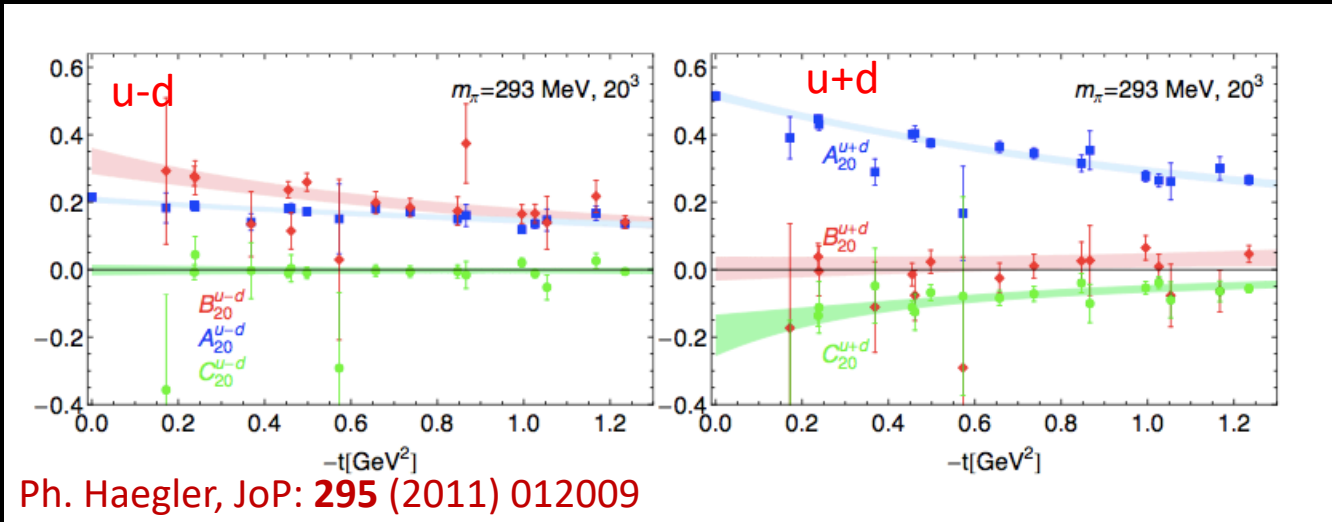
Operator \longleftrightarrow

$$T_q^{\mu\nu} + T_g^{\mu\nu} = \underbrace{\frac{1}{2} [\bar{\psi} \gamma^{(\mu} i \overrightarrow{D}^{\nu)} \psi + \bar{\psi} \gamma^{(\mu} i \overleftarrow{D}^{\nu)} \psi]}_{\text{quark}} + \underbrace{\frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu{}_\alpha}_{\text{gluon}}$$

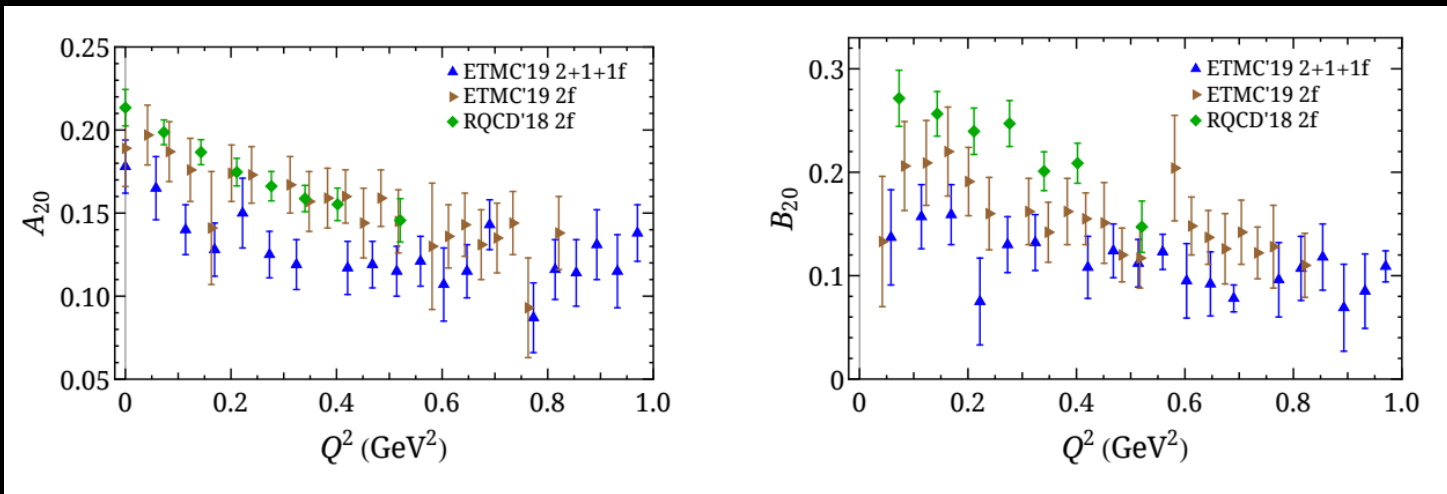
Matrix Elements \longleftrightarrow

	OPE	\longleftrightarrow	EMT
$\int dx x H(x, \xi, t)$	$A_{20}(t) + \xi^2 C_{20}(t)$	\equiv	$A(t) + \xi^2 C(t)$ ← D-term
$\int dx x E(x, \xi, t)$	$B_{20}(t) - \xi^2 C_{20}(t)$	\equiv	$B(t) - \xi^2 C(t)$

Calculable on lattice...

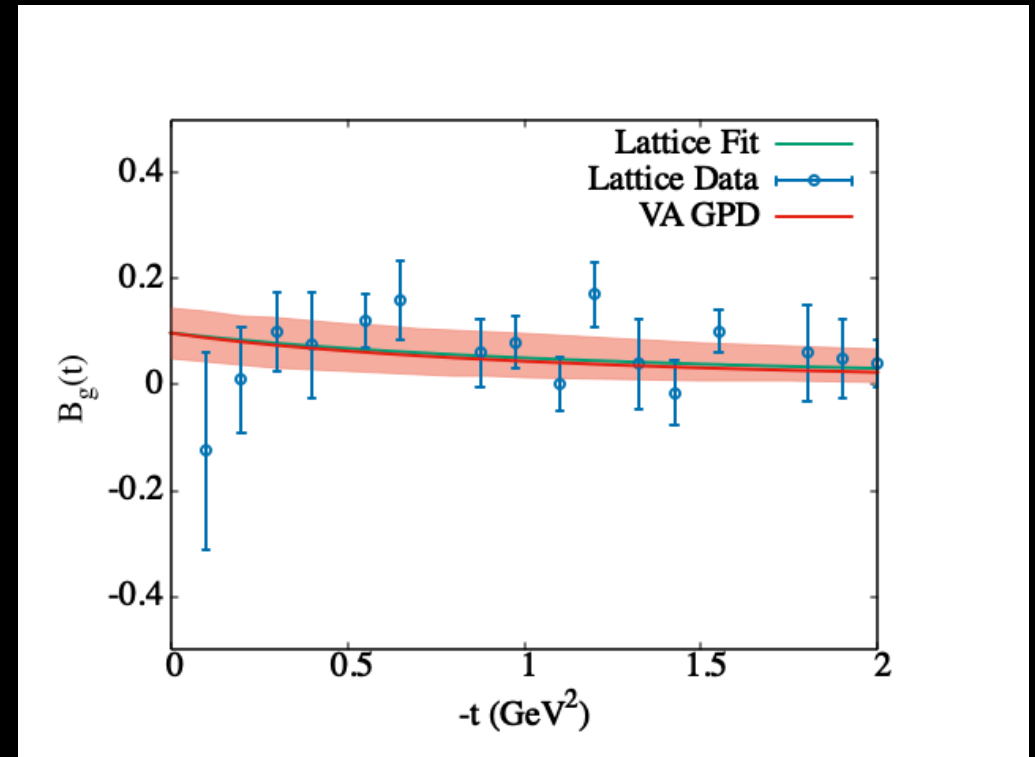
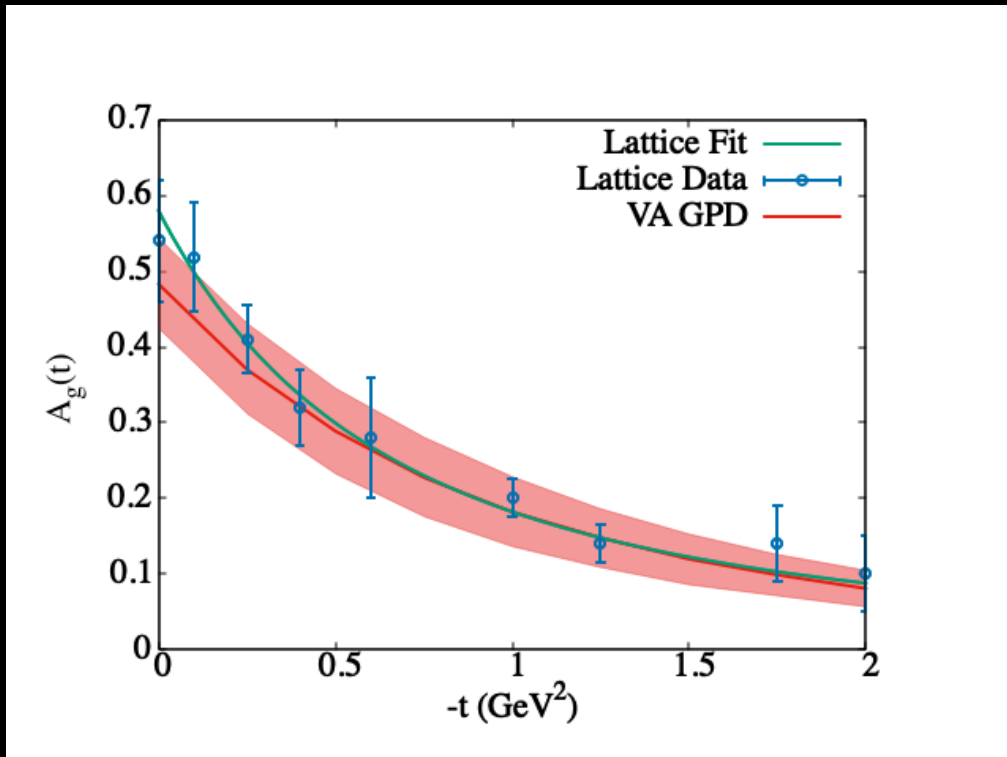


u-d



Gluons

Detmold and Shanahan PRD 2018



- Recent surge in activities to understand the various components of the EMT, in particular mass (from energy density, T_{00}), transverse spin, pressure
- The response will not be found debating the fine details of the various possible definitions but it will be in our ability to find experimental validations of the theory

Going beyond EMT definitions

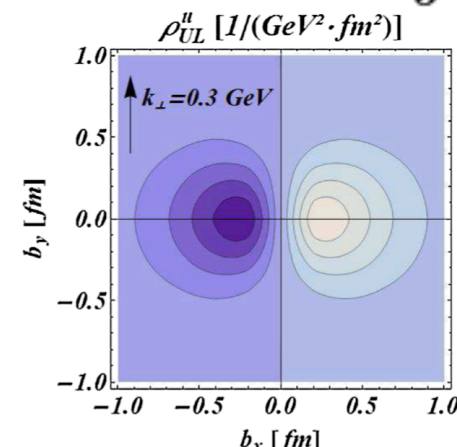
- can we understand, based on physical models, the various physical components of angular momentum, mass and internal forces/pressure?
- Can measurements shed light on the orbital and spin components for quarks and gluons?
- Can we distinguish the Higgs mass component from the dynamical mass originating from quark and gluon interactions?
- Role of the trace anomaly
- What are the observables for pressure, internal forces



Towards quantitative evaluations/validations

- Focus on the **quark** sector: a method to separate the **orbital** and **spin** components of J that gives results directly in terms of kinematic, (x, ξ, t, k_T) dependence of observables (Rajan, Engelhardt, SL)
- Based on generalized **Lorentz Invariance Relations** which establish a connection between k_T^2 **moments** of TMDs/GTMDs and **twist 3** PDFs/GPDs

$$\begin{aligned}
J_L &= L_L + S_L \\
\frac{1}{2} \int dx x(H + E) &= \int dx x(\tilde{E}_{2T} + H + E) + \frac{1}{2} \int dx \tilde{H} \\
&= - \int dx F_{14}^{(1)} + \frac{1}{2} \int dx \tilde{H}
\end{aligned}$$



Role of genuine twist three terms

We find that a LIR violating term gives the difference between the Jaffe Manohar and Ji decompositions described through staple link in Burkardt, PRD 2013

$$L^{JM}(x) - L^{Ji}(x) = \mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0} = - \int_x^1 dy \mathcal{A}_{F_{14}}(y).$$

Can we measure this?

- There are two types of genuine twist three terms that cannot be separated in one single experiments
- The staple link component can, however, be determined in lattice QCD

Transverse Angular Momentum Sum Rule

$$J_q = L_q + \frac{1}{2} \Delta \Sigma_q$$

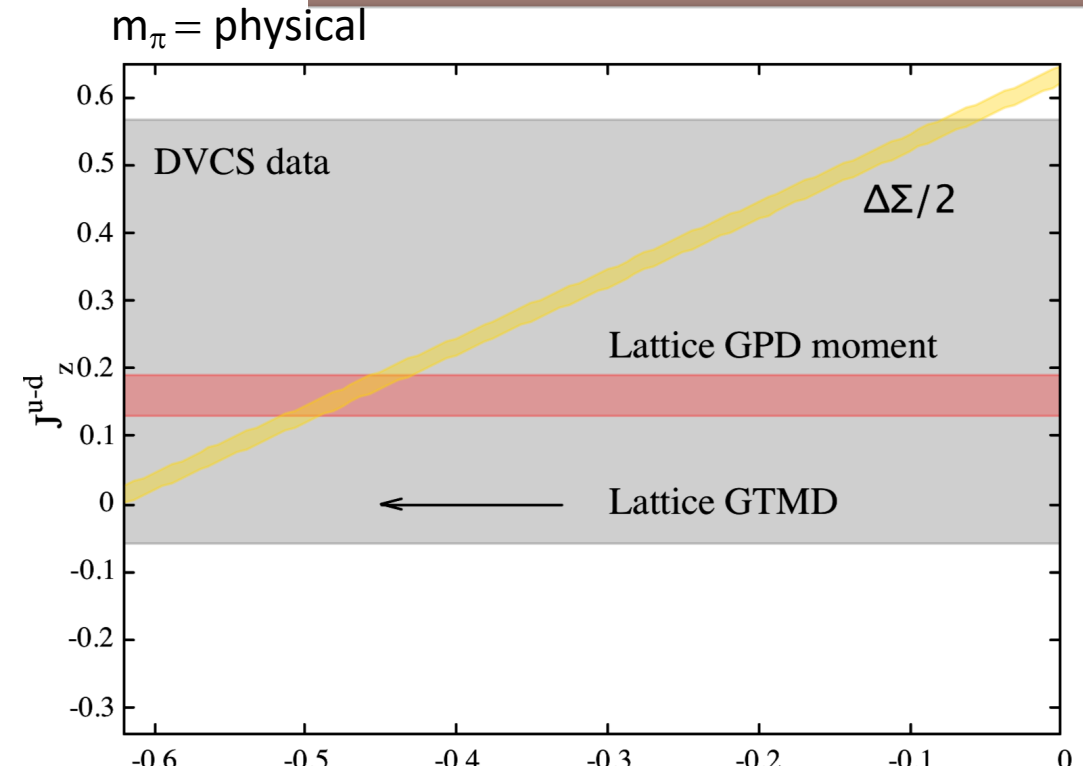
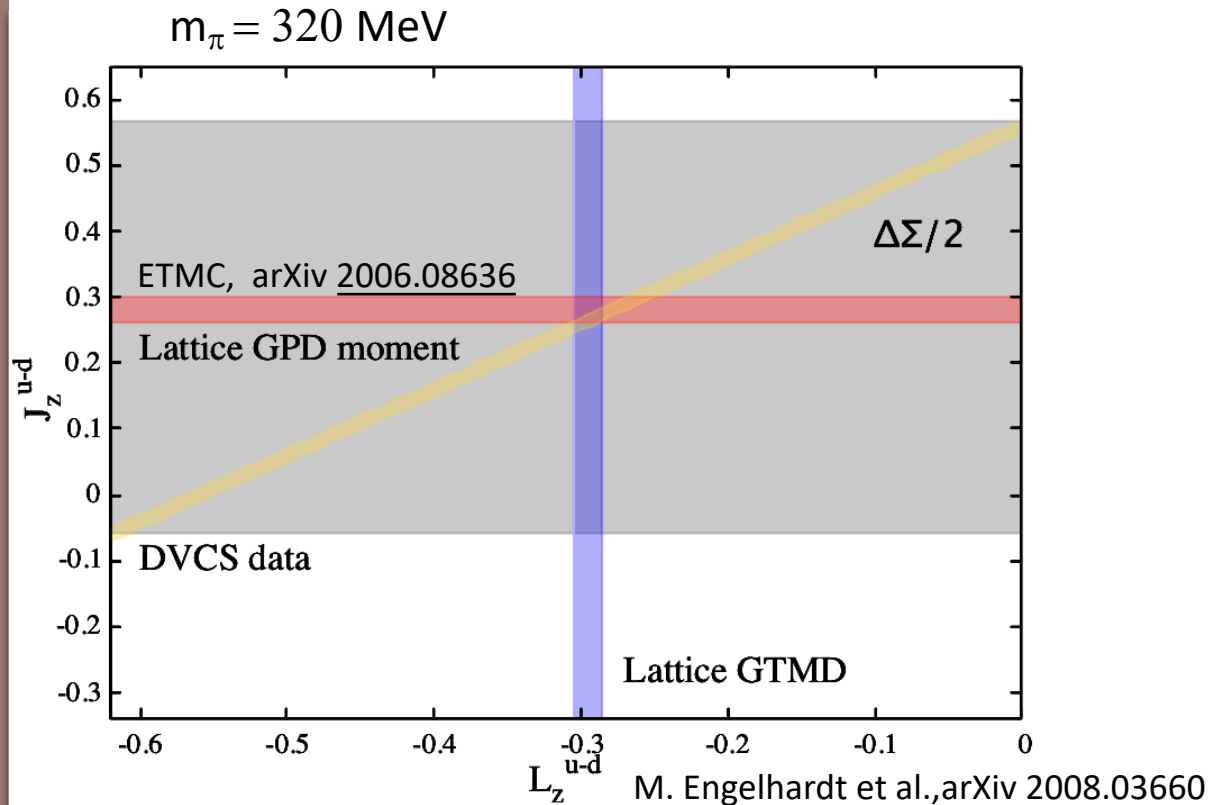
A. Rajan, O. Alkassasbeh, M. Engelhardt, SL, *to be submitted*

$$\underbrace{\frac{1}{2} \int dx x (H + E)}_{J_T} - \frac{1}{2} \int dx \mathcal{M}_T = \underbrace{\int dx x (\tilde{E}_{2T} + H + E)}_{L_T} + \underbrace{\frac{H_{2T}}{\xi}}_{S_T} + \frac{1}{2} \int dx g_T - \frac{1}{2} \int dx x \mathcal{A}_T$$

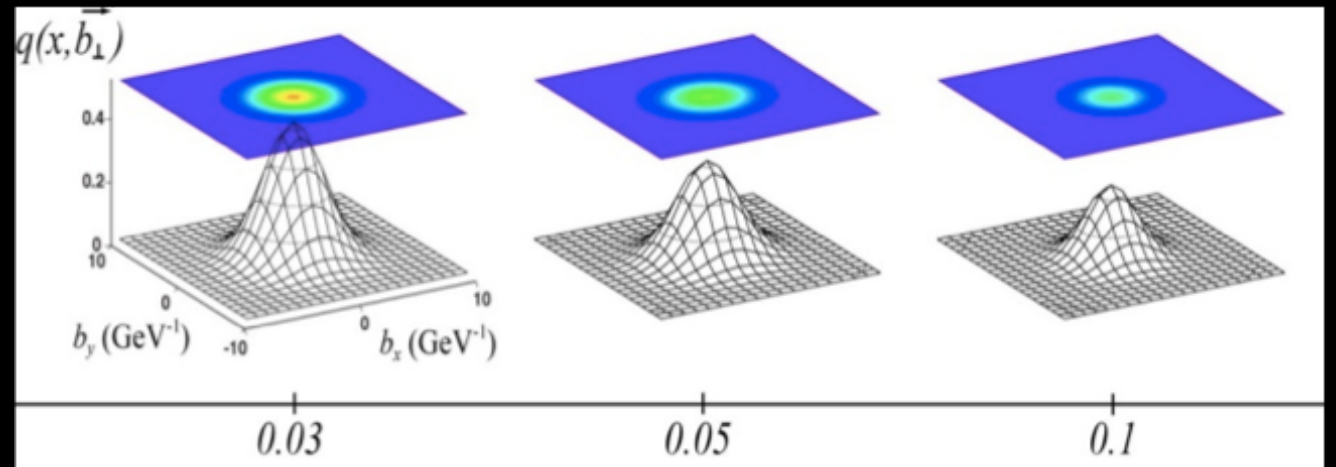
See also recent papers by Guo, Ji, Shiells [2101.05243](#)
Lorce' et al.

What a sum rule should predict

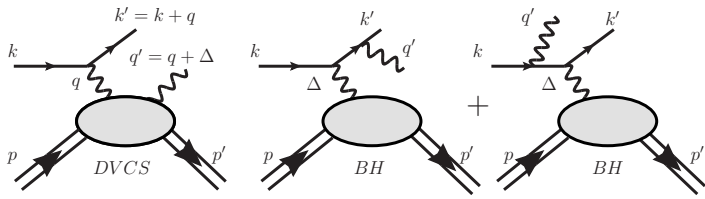
$$J_q = L_q + \frac{1}{2} \Delta\Sigma_q$$



Measuring All This

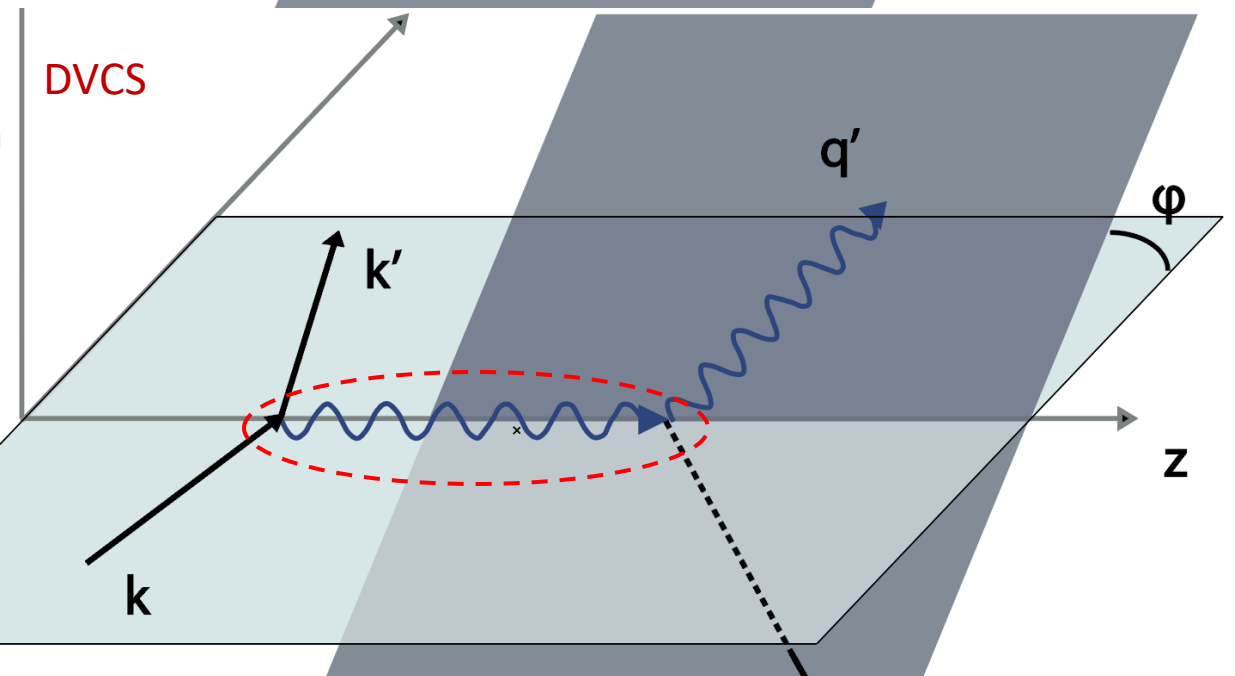
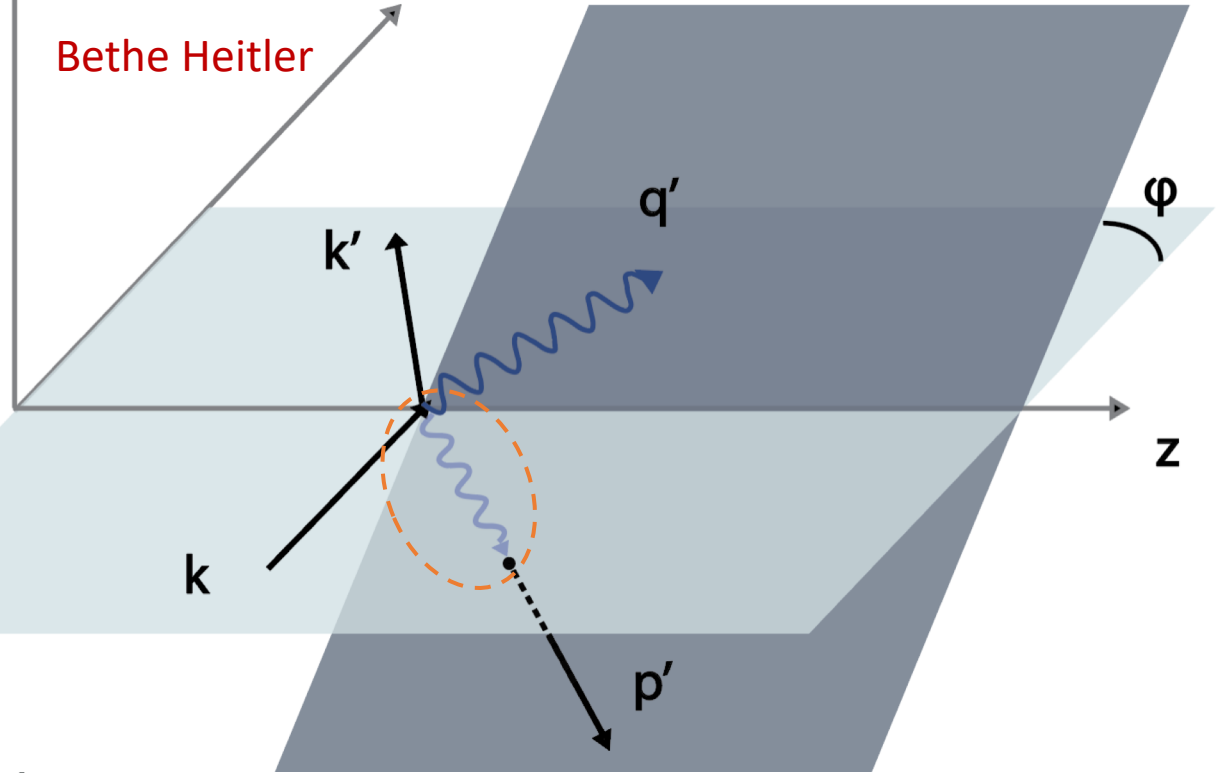


graph from M. Defurne



Demystification of harmonics formalism

- In DVCS the virtual photon is along the z axis: φ dependence from usual rotation of polarization vector in helicity amp
- In BH the virtual photon is along the direction of p'
- Mismatch complicates the BH-DVCS term



Parametrization of Quark and Gluon Generalized Parton Distributions in a Dynamical Framework

2101.01826

Brandon Kriesten,^{1,*} Philip Velie,^{1,†} Emma Yeats,^{1,‡} Fernanda Yopez Lopez,^{1,§} and Simonetta Liuti^{1,¶}

¹*Department of Physics, University of Virginia, Charlottesville, VA 22904, USA.*

PRD, accepted

2012.04801

Deep learning analysis of deeply virtual exclusive photoproduction

Jake Grigsby, Brandon Kriesten, Joshua Hoskins, Simonetta Liuti, Peter Alonzi, and Matthias Burkardt
Phys. Rev. D **104**, 016001 – Published 1 July 2021

2011.04484

Novel Rosenbluth Extraction Framework for Compton Form Factors from Deeply Virtual Exclusive Experiments

PLB, accepted

Brandon Kriesten,^{*} Simonetta Liuti,[†] and Andrew Meyer[‡]

2004.08890

Theory of deeply virtual Compton scattering off the unpolarized proton

Brandon Kriesten and Simonetta Liuti
Phys. Rev. D **105**, 016015 – Published 18 January 2022

1903.05742

Extraction of generalized parton distribution observables from deeply virtual electron proton scattering experiments

Brandon Kriesten, Andrew Meyer, Simonetta Liuti, Liliet Calero Diaz, Dustin Keller, Gary R. Goldstein, and J. Osvaldo Gonzalez-Hernandez
Phys. Rev. D **101**, 054021 – Published 16 March 2020

Using formalism of helicity
amplitudes

$$f_{\Lambda\Lambda'}^{\pm 1\pm 1} = \sum_{\lambda, \lambda'} g_{\lambda\lambda'}^{\pm 1\pm 1}(x, \xi, t; Q^2) \otimes A_{\Lambda'\lambda', \Lambda\lambda}(x, \xi, t),$$

$$A_{++,++} = \sqrt{1 - \xi^2} \left(\frac{H + \tilde{H}}{2} - \frac{\xi^2}{1 - \xi^2} \frac{E + \tilde{E}}{2} \right),$$

$$A_{-+,-+} = \sqrt{1 - \xi^2} \left(\frac{H - \tilde{H}}{2} - \frac{\xi^2}{1 - \xi^2} \frac{E - \tilde{E}}{2} \right),$$

$$A_{++, -+} = -e^{-i\varphi} \frac{\sqrt{t_0 - t}}{2m} \frac{E - \xi\tilde{E}}{2},$$

$$A_{-+, ++} = e^{i\varphi} \frac{\sqrt{t_0 - t}}{2m} \frac{E + \xi\tilde{E}}{2},$$

M. Diehl, Phys. Rep.388(2003)
B. Kriesten et al. PRD 101(2020)

BH

$$\frac{d^5 \sigma_{unpol}^{BH}}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} \equiv \frac{\Gamma}{t} F_{UU}^{BH} = \frac{\Gamma}{t} \left[A(y, x_{Bj}, t, Q^2, \phi) (F_1^2 + \tau F_2^2) + B(y, x_{Bj}, t, Q^2, \phi) \tau G_M^2(t) \right]$$

$$A = \frac{16 M^2}{t(k q')(k' q')} \left[4\tau \left((k P)^2 + (k' P)^2 \right) - (\tau + 1) \left((k \Delta)^2 + (k' \Delta)^2 \right) \right]$$
$$B = \frac{32 M^2}{t(k q')(k' q')} \left[(k \Delta)^2 + (k' \Delta)^2 \right],$$

Longitudinal to transverse photon polarization ratio

$$\epsilon_{BH} = \left(1 + \frac{B}{A} (1 + \tau) \right)^{-1}$$

Kriesten, SL, hep-ph/2004.08890

...compared
to ELASTIC
SCATTERING

10/21/21

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon(G_E^N)^2 + \tau(G_M^N)^2}{\epsilon(1 + \tau)},$$

where $N = p$ for a proton and $N = n$ for a neutron, (the recoil-corrected relativistic point-particle (Mott) and τ, ϵ are dimensionless kinematic variables:

$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}\right]^{-1},$$

J. Arrington, G. Cates, S. Riordan, Z. Ye, B. Wojsetowski, A. Puckett ...

...compared to BKM, NPB (2001)

$$|\mathcal{T}_{\text{BH}}|^2 = \frac{e^6}{x_{\text{B}}^2 y^2 (1 + \epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \times \left\{ c_0^{\text{BH}} + \sum_{n=1}^2 c_n^{\text{BH}} \cos(n\phi) + s_1^{\text{BH}} \sin(\phi) \right\},$$

$$\begin{aligned} c_{0,\text{unp}}^{\text{BH}} = & 8K^2 \left\{ (2 + 3\epsilon^2) \frac{Q^2}{\Delta^2} \left(F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) + 2x_{\text{B}}^2 (F_1 + F_2)^2 \right\} \\ & + (2 - y)^2 \left\{ (2 + \epsilon^2) \left[\frac{4x_{\text{B}}^2 M^2}{\Delta^2} \left(1 + \frac{\Delta^2}{Q^2} \right)^2 \right. \right. \\ & \quad \left. \left. + 4(1 - x_{\text{B}}) \left(1 + x_{\text{B}} \frac{\Delta^2}{Q^2} \right) \right] \left(F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) \right. \\ & \quad \left. + 4x_{\text{B}}^2 \left[x_{\text{B}} + \left(1 - x_{\text{B}} + \frac{\epsilon^2}{2} \right) \left(1 - \frac{\Delta^2}{Q^2} \right)^2 \right. \right. \\ & \quad \left. \left. - x_{\text{B}}(1 - 2x_{\text{B}}) \frac{\Delta^4}{Q^4} \right] (F_1 + F_2)^2 \right\} \\ & + 8(1 + \epsilon^2) \left(1 - y - \frac{\epsilon^2 y^2}{4} \right) \\ & \times \left\{ 2\epsilon^2 \left(1 - \frac{\Delta^2}{4M^2} \right) \left(F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) - x_{\text{B}}^2 \left(1 - \frac{\Delta^2}{Q^2} \right)^2 (F_1 + F_2)^2 \right\}. \end{aligned}$$

A.V. Belitsky et al. / Nuclear Physics B 629 (2002) 323–392

$$\begin{aligned} c_{1,\text{unp}}^{\text{BH}} = & 8K(2 - y) \left\{ \left(\frac{4x_{\text{B}}^2 M^2}{\Delta^2} - 2x_{\text{B}} - \epsilon^2 \right) \left(F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) \right. \\ & \left. + 2x_{\text{B}}^2 \left(1 - (1 - 2x_{\text{B}}) \frac{\Delta^2}{Q^2} \right) (F_1 + F_2)^2 \right\}, \\ c_{2,\text{unp}}^{\text{BH}} = & 8x_{\text{B}}^2 K^2 \left\{ \frac{4M^2}{\Delta^2} \left(F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) + 2(F_1 + F_2)^2 \right\}. \end{aligned}$$

BH-DVCS interference

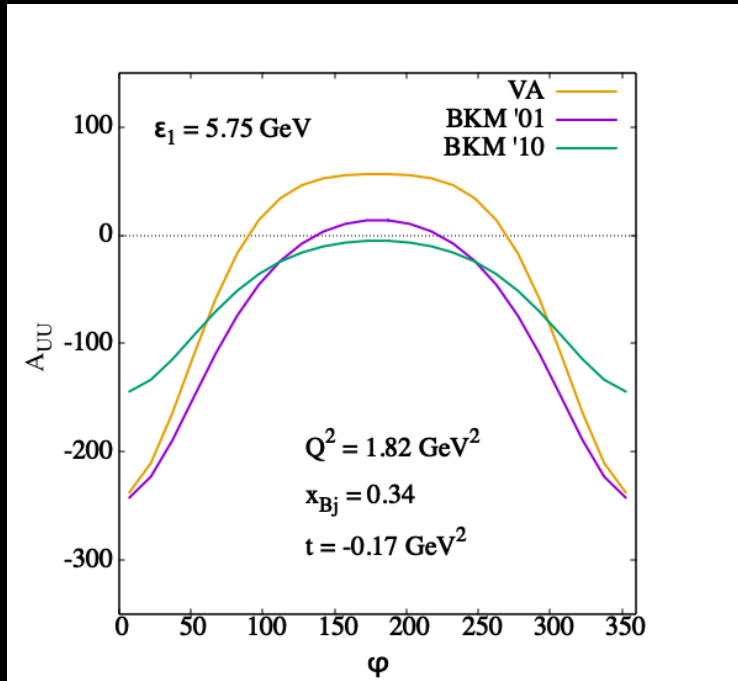
$$F_{UU}^{\mathcal{I}} = F_{UU}^{\mathcal{I},tw2} + \frac{K}{\sqrt{Q^2}} F_{UU}^{\mathcal{I},tw3}$$

$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re \left(F_1 \mathcal{H} + \tau F_2 \mathcal{E} \right) + B_{UU}^{\mathcal{I}} G_M \Re (\mathcal{H} + \mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re \tilde{\mathcal{H}}$$

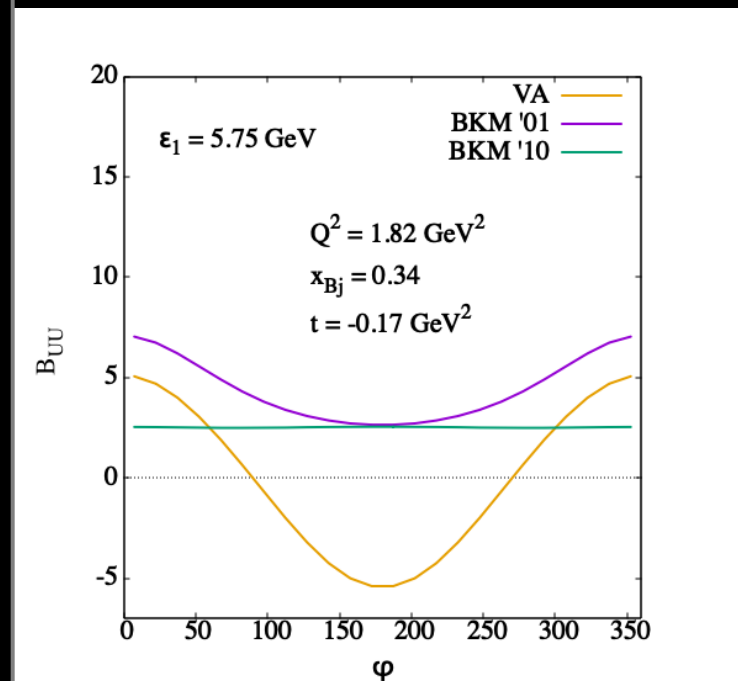
$A_{UU}^{\mathcal{I}}$ $B_{UU}^{\mathcal{I}}$ $C_{UU}^{\mathcal{I}}$

are φ dependent coefficients

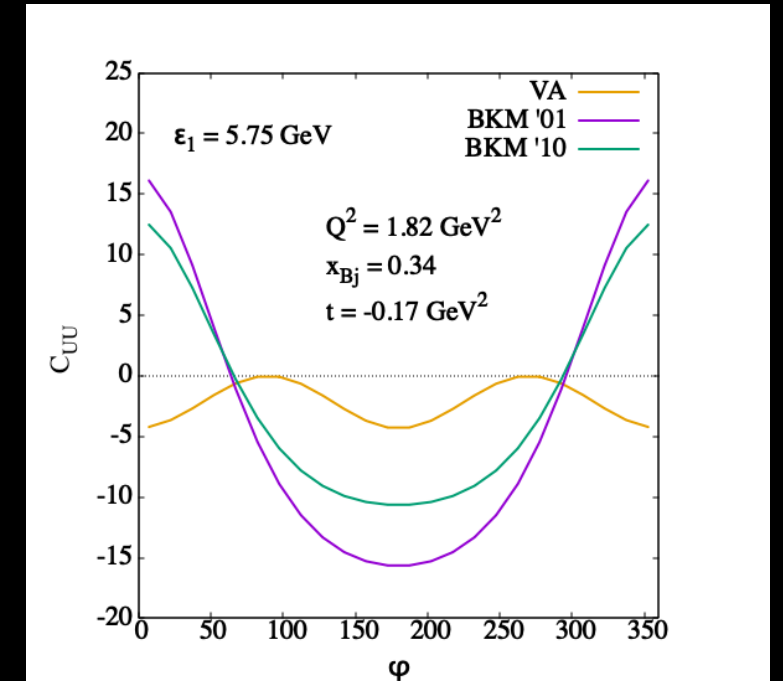
Electric



Magnetic

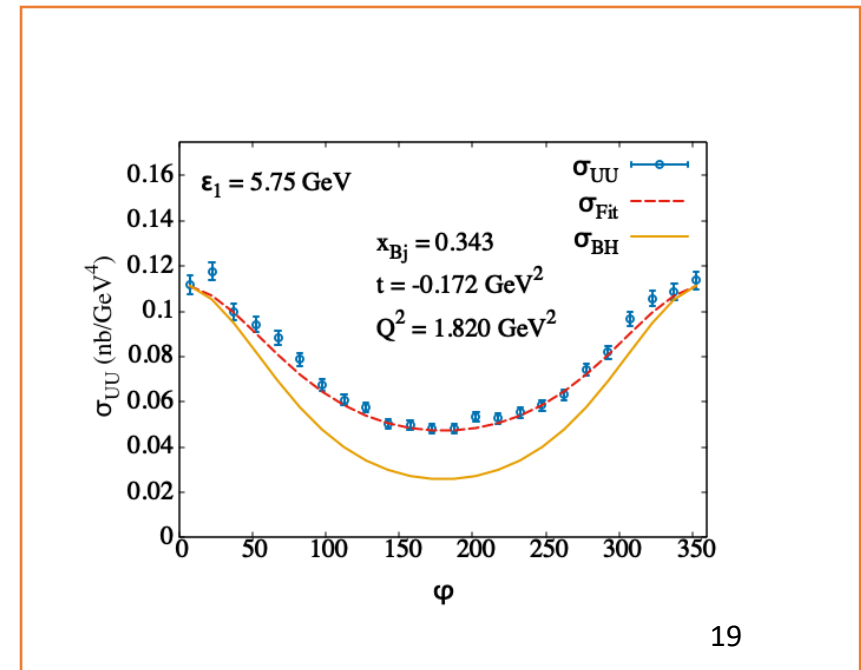
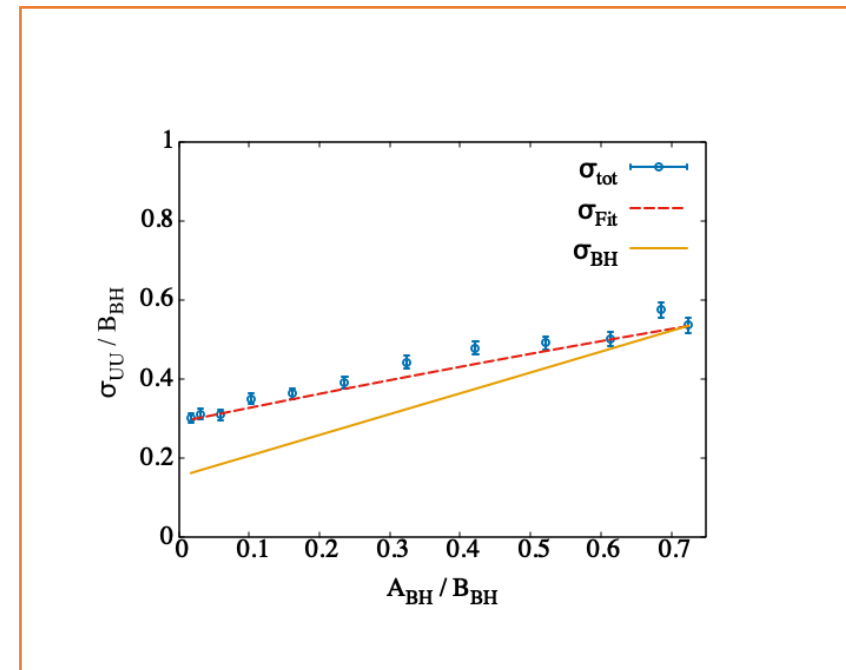


Axial



Linear Plots!!!

- Rosenbluth Separated BH-DVCS interference data



Twist 3 BH-DVCS interference

$$F_{UU}^{\mathcal{I}} = F_{UU}^{\mathcal{I},tw2} + \frac{K}{\sqrt{Q^2}} F_{UU}^{\mathcal{I},tw3}$$

$$F_{UU}^{\mathcal{I},tw3} = A_{UU}^{(3)\mathcal{I}} \left[F_1 \left(\Re(2\tilde{\mathcal{H}}_{2T} + \mathcal{E}_{2T}) - \Re(2\tilde{\mathcal{H}}'_{2T} + \mathcal{E}'_{2T}) \right) + F_2 \left(\Re(\mathcal{H}_{2T} + \tau\tilde{\mathcal{H}}_{2T}) - \Re(\mathcal{H}'_{2T} + \tau\tilde{\mathcal{H}}'_{2T}) \right) \right]$$

$$+ B_{UU}^{(3)\mathcal{I}} G_M (\Re\tilde{\mathcal{E}}_{2T} - \Re\tilde{\mathcal{E}}'_{2T}) \quad \text{Orbital Angular Momentum}$$

$$+ C_{UU}^{(3)\mathcal{I}} G_M \left[2\xi(\Re\mathcal{H}_{2T} - \Re\mathcal{H}'_{2T}) - \tau \left(\Re(\tilde{\mathcal{E}}_{2T} - \xi\mathcal{E}_{2T}) - \Re(\tilde{\mathcal{E}}'_{2T} - \xi\mathcal{E}'_{2T}) \right) \right]$$

Twist 3 GPDs Physical Interpretation

GPD	$P_q P_p$	TMD	Ref. 1
H^\perp	UU	f^\perp	$2\tilde{H}_{2T} + E_{2T}$
\tilde{H}_L^\perp	LL	g_L^\perp	$2\tilde{H}'_{2T} + E'_{2T}$
H_L^\perp	UL	$f_L^\perp^{(*)}$	$\tilde{E}_{2T} - \xi E_{2T}$
\tilde{H}^\perp	LU	$g^\perp^{(*)}$	$\tilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \tilde{H}_{2T}$
$\tilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \tilde{H}'_{2T}$



1/Q correction to H



1/Q correction to \tilde{H}

NEW!!

Orbital Angular Momentum \mathbf{L}

NEW!!

Spin Orbit correlation $\mathbf{L} \cdot \mathbf{S}$



1/Q correction to E

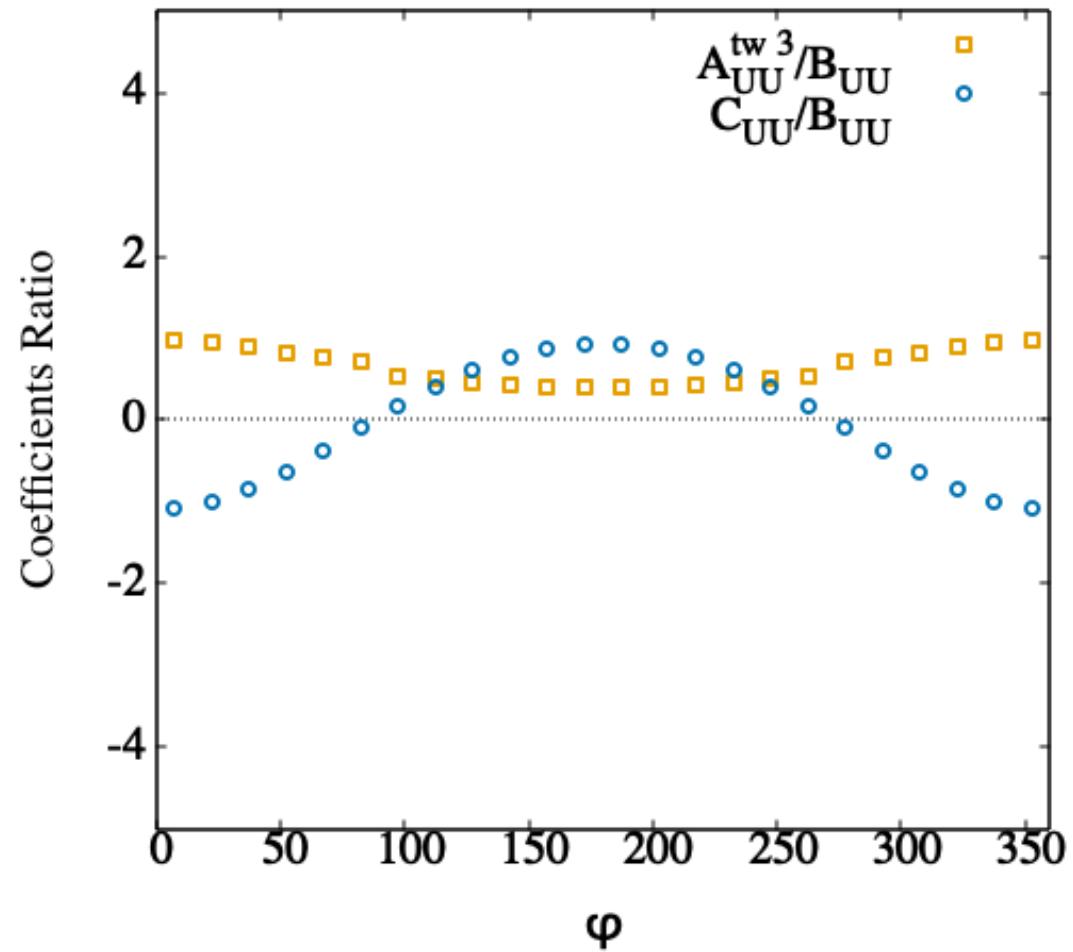


1/Q correction to \tilde{E}

(*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

Twist 3 seems small



but we can extract it by comparing DV/GS and TGS



Beam target helicity

Twist 2

GPD	Twist	$P_q P_p$	TMD	$P_{Beam} P_p$ (DVCS)	$P_{Beam} P_p$ (\mathcal{I})
$\mathbf{H} + \frac{\xi^2}{1-\xi} E$	2	UU	f_1	$UU, LL, UT^{\sin(\phi-\phi_s)}, LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi}, LU^{\sin\phi}$
$\tilde{\mathbf{H}} + \frac{\xi^2}{1-\xi} \tilde{E}$	2	LL	g_1	$UU, LL, UT^{\sin(\phi-\phi_s)}, LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\sin\phi}, UT^{\frac{\cos\phi}{\sin\phi}}, LT^{\cos\phi}$
\mathbf{E}	2	UT	$f_{1T}^{\perp(*)}$	$UT^{\sin(\phi-\phi_s)}, LT^{\cos(\phi-\phi_s)}$	$UU^{\cos\phi}, LU^{\sin\phi}, UT, LT, UT^{\cos\phi}, UT^{\sin\phi}$
$\tilde{\mathbf{E}}$	2	LT	g_{1T}	$UT^{\sin(\phi-\phi_s)}, LT^{\cos(\phi-\phi_s)}$	$UL^{\sin\phi}, LL^{\cos\phi}, UT^{\cos\phi}, UT^{\sin\phi}$
$\mathbf{H} + \mathbf{E}$	2	-	-	-	$UU^{\cos\phi}, LU^{\sin\phi}, UL^{\sin\phi}, LL^{\cos\phi}, UT^{\cos\phi}, UT^{\sin\phi}$

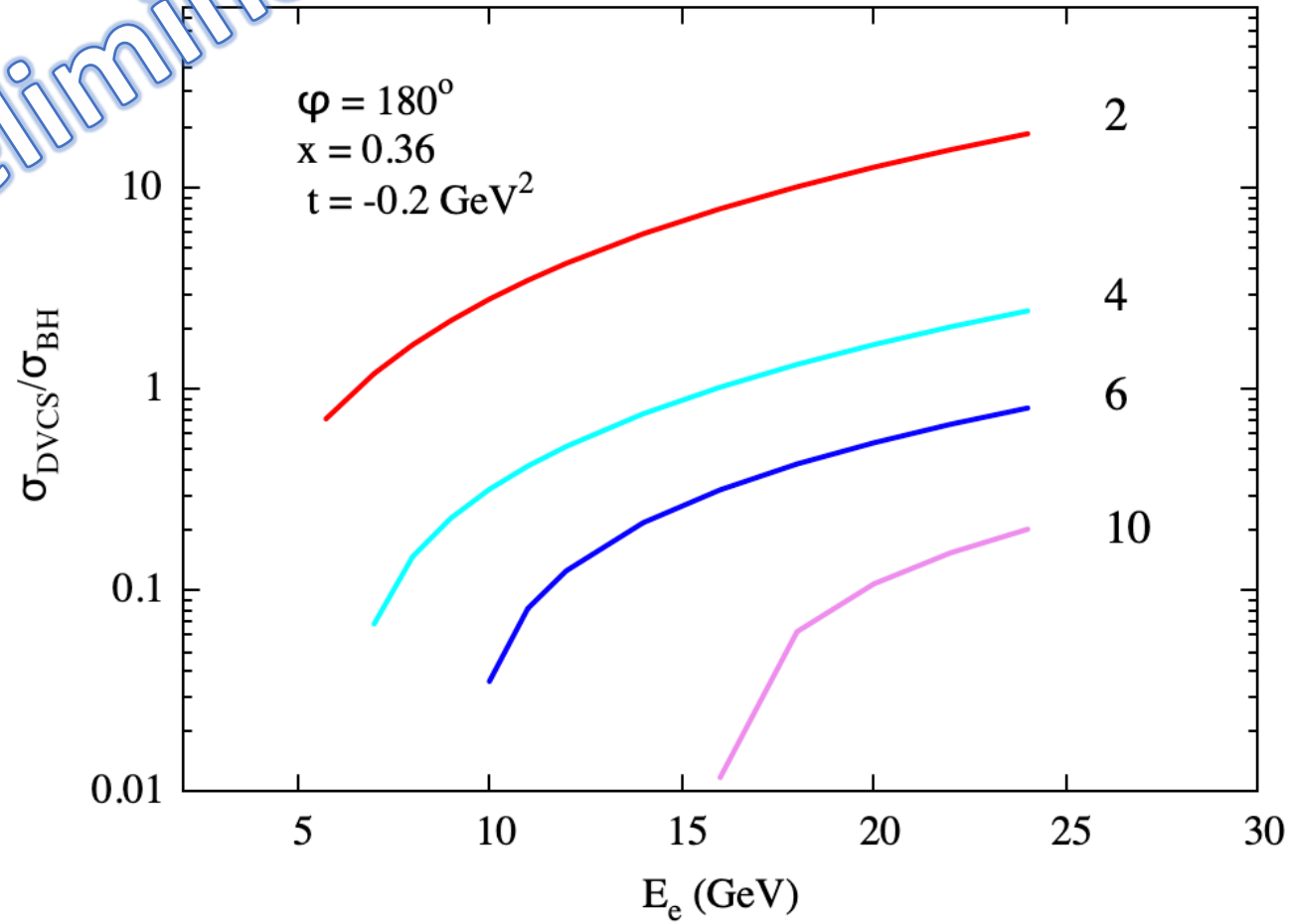
Twist 3

$2\tilde{\mathbf{H}}_{2T} + \mathbf{E}_{2T} - \xi\tilde{E}_{2T}$	3	UU	f^{\perp}	$UU^{\cos\phi}, LU^{\sin\phi}$	UU, LU
$2\tilde{\mathbf{H}}'_{2T} + \mathbf{E}'_{2T} - \xi\tilde{E}'_{2T}$	3	LL	g_L^{\perp}	$UU^{\cos\phi}, LU^{\sin\phi}$	UU, LU
$\mathbf{H}_{2T} + \frac{t_o - t}{4M^2} \tilde{\mathbf{H}}_{2T}$	3	UT	$f_T^{(*)}, f_T^{\perp(*)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU Transv. OAM
$\mathbf{H}'_{2T} + \frac{t_o - t}{4M^2} \tilde{\mathbf{H}}'_{2T}$	3	LT	g'_T, g_T^{\perp}	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU
$\tilde{\mathbf{E}}_{2T} - \xi E_{2T}$	3	UL	$f_L^{\perp(*)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU, UT OAM
$\tilde{\mathbf{E}}'_{2T} - \xi E'_{2T}$	3	LU	$g^{\perp(*)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU, UT Spin Orbit
$\tilde{\mathbf{H}}_{2T}$	3	UT _x	$f_T^{\perp(*)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU, UT
$\tilde{\mathbf{H}}'_{2T}$	3	LT _x	g_T^{\perp}	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU, UT

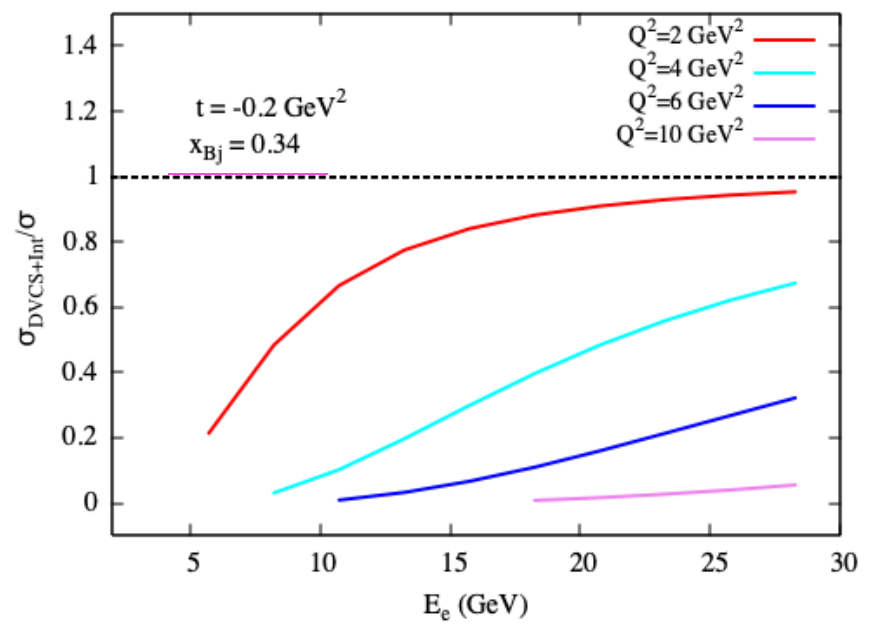
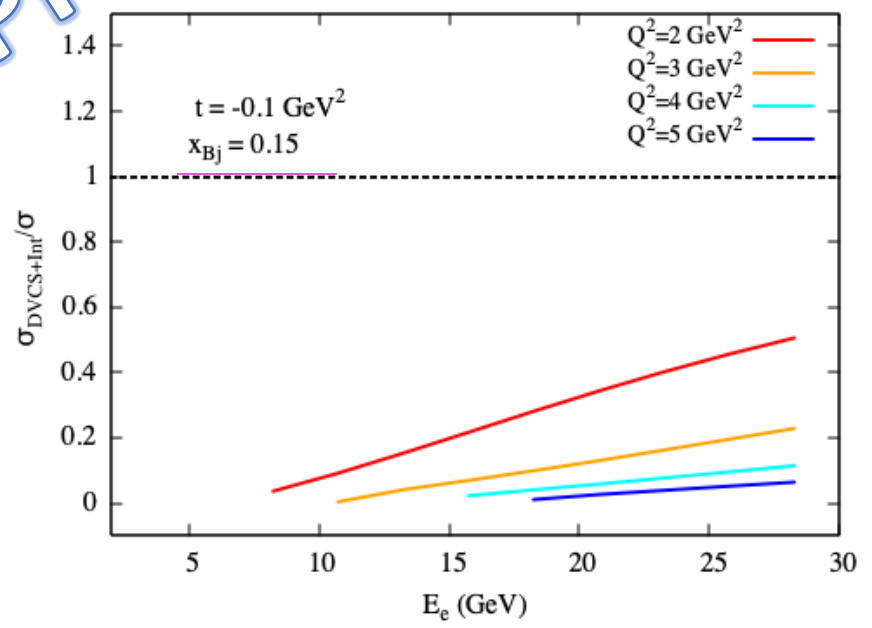


Kriesten et al., Phys.Rev.D 101 (2020)
 Kriesten and SL, 2004.08890

Preliminary



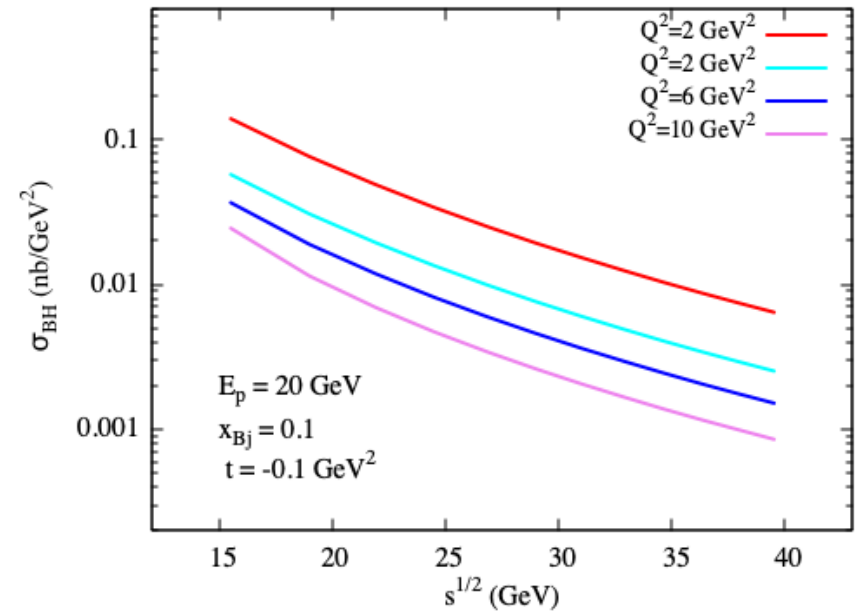
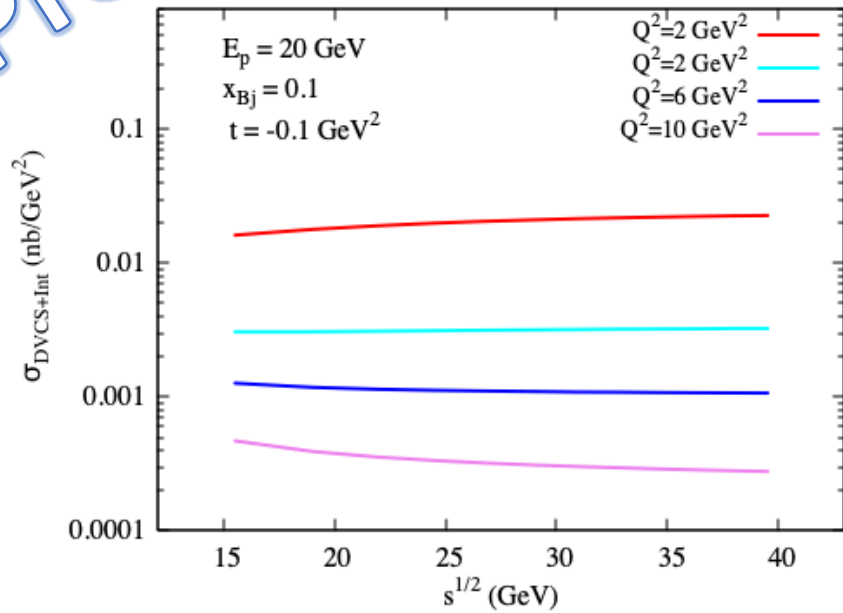
Preliminary



B. Kriesten, SL, in preparation

Preliminary

Comparison with EIC low energy scenario



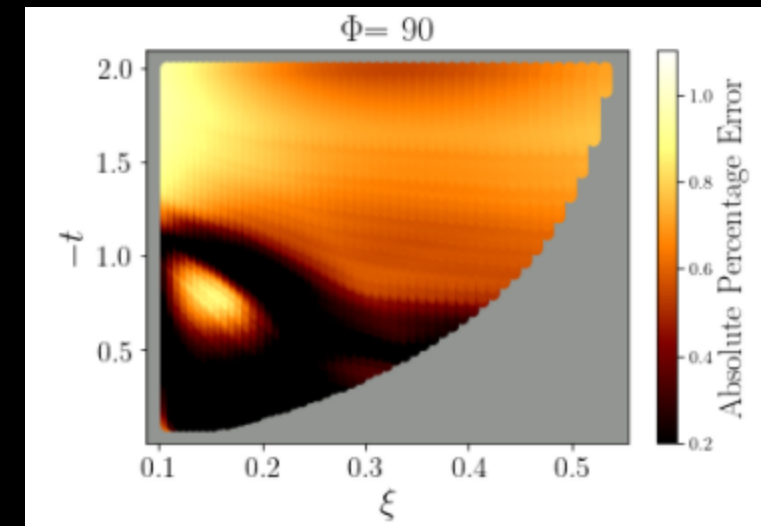
B. Kriesten, SL, in preparation

CONCLUSIONS

- We presented avenues to identify observables sensitive to both longitudinal and transverse OAM
- Jefferson Lab @24 GeV will make history as the we uncover the mechanical properties the of the proton and observe its spatial images!
- To observe, evaluate and interpret GPDs and Wigner distributions at the subatomic level requires stepping up data analyses. To accomplish this we need
 - Physics motivated cross section formulation
 - Develop methods to extract quantities from data including numerical/analytic/advanced ML and quantum computing method

This program is on its way! We would be happy to interact

UVA+ODU ML group





Finally, advertisements!!

QCD Evolution Workshop, May 9-13-22 at University of Virginia

<https://discovery.phys.virginia.edu/research/groups/qcd22/workshop.html>

Brandon Kriesten invited talk at APS April meeting (April 10)