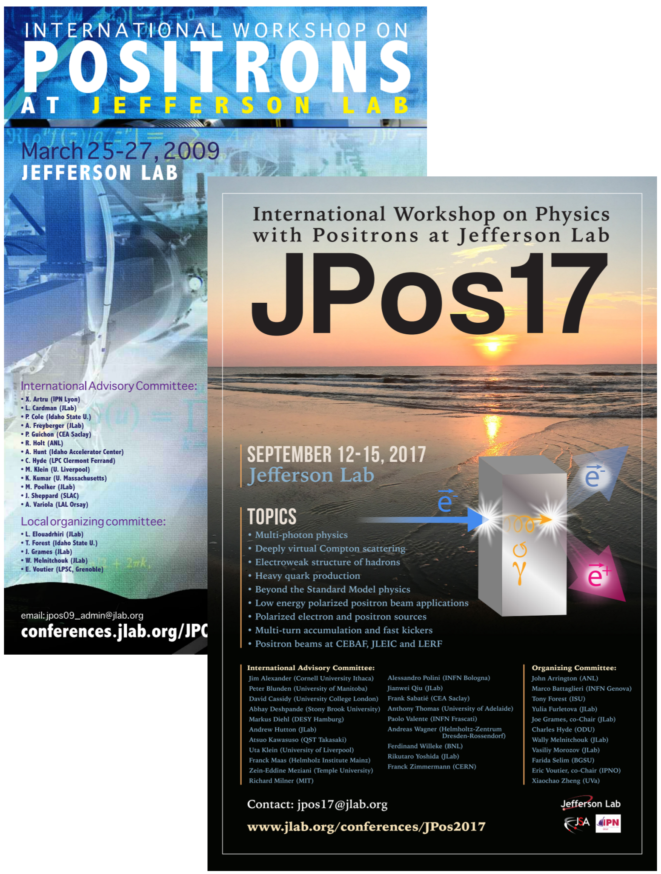
Neutral-Current Electroweak Physics with SoLID, a possible positron beam, and a possible energy upgrade of JLab

Xiaochao Zheng, Univ. of Virginia

March 29, 2022



Eur. Phys. J. A manuscript No.
(will be inserted by the editor)

https://arxiv.org/abs/2007.15081

An experimental program with high duty-cycle polarized and unpolarized positron beams at Jefferson Lab

Eur. Phys. J. A manuscript No.
(will be inserted by the editor)

https://arxiv.org/abs/2103.12555

Accessing weak neutral-current coupling g_{AA}^{eq} using positron and electron beams at Jefferson Lab

(A New Proposal to Jefferson Lab PAC-49)

Measurement of the Asymmetry $A_d^{e^+e^-}$ between $e^+-{}^2$H and $e^--{}^2$H Deep Inelastic Scattering Using SoLID and PEPPo at JLab

May 24, 2021

(pending approval from JLab Hall A and SoLID Collaborations)

The Landscape of Electroweak Physics Study

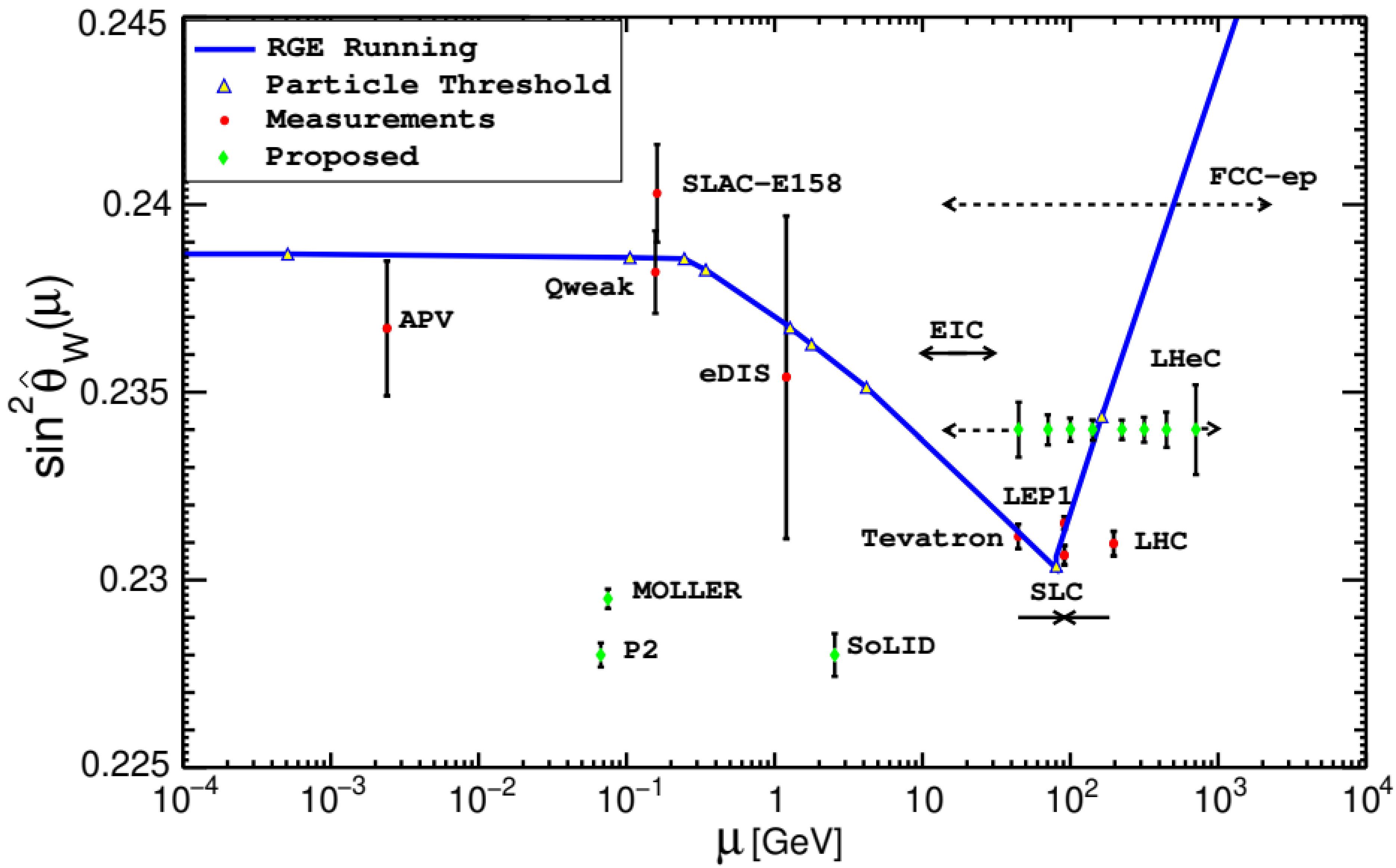
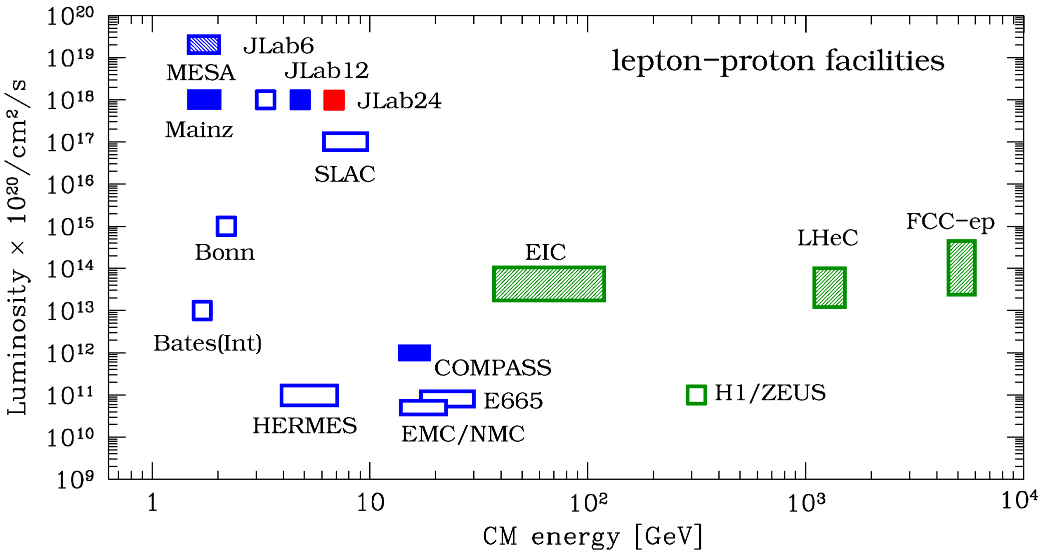
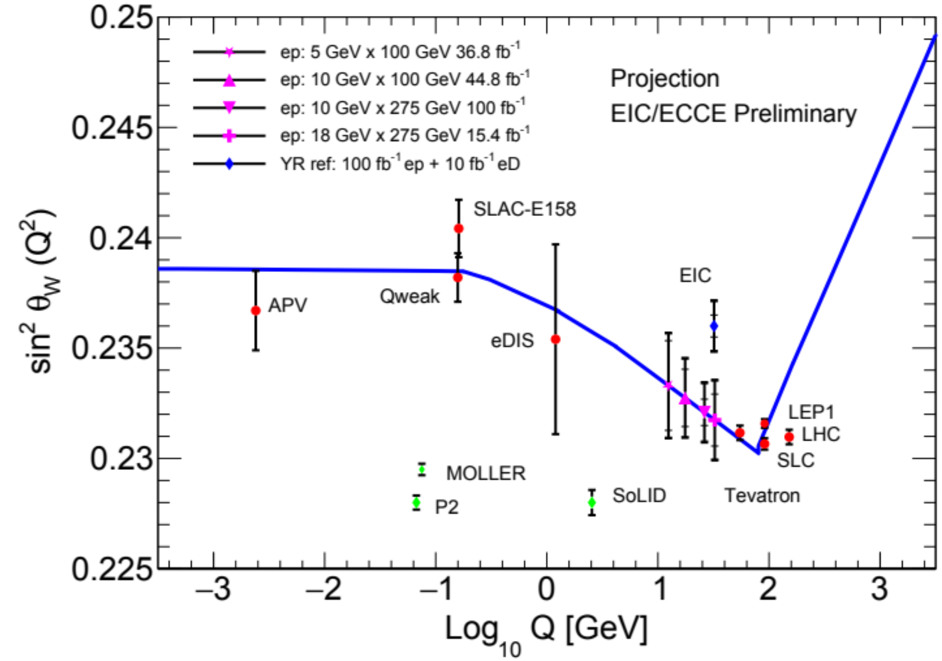


Figure updated from Erler, Ferro-Hernandez, JHEP03(2018) 196;
LHeC projection (60GeV x 7 TeV, ~1000fb^{-1}) from EPJC 80 (2020) 9, 831 arxiv.org/2007.11799;
(points with uncertainties comparable to or smaller than Qweak are shown, full range shown as arrows)

The Landscape of Electroweak Physics Study (next decade)



ECCE Detector Proposal and Analysis Notes

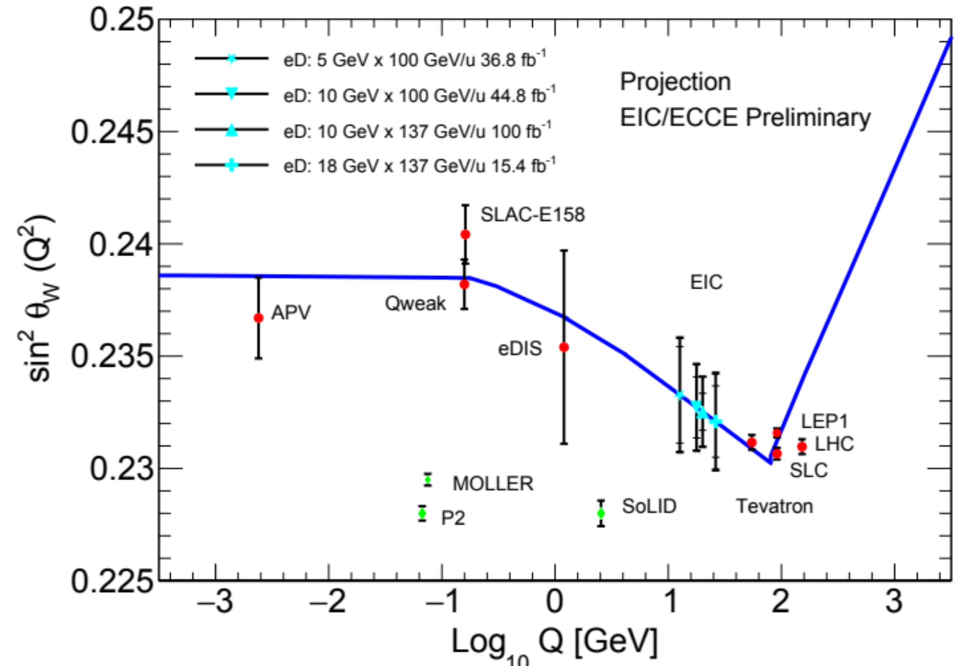


https://arxiv.org/abs/2203.13199

Snowmass 2021 White Paper:
Electron Ion Collider for High Energy Physics

to be posted first week of April (ECCE collaboration review starting today)

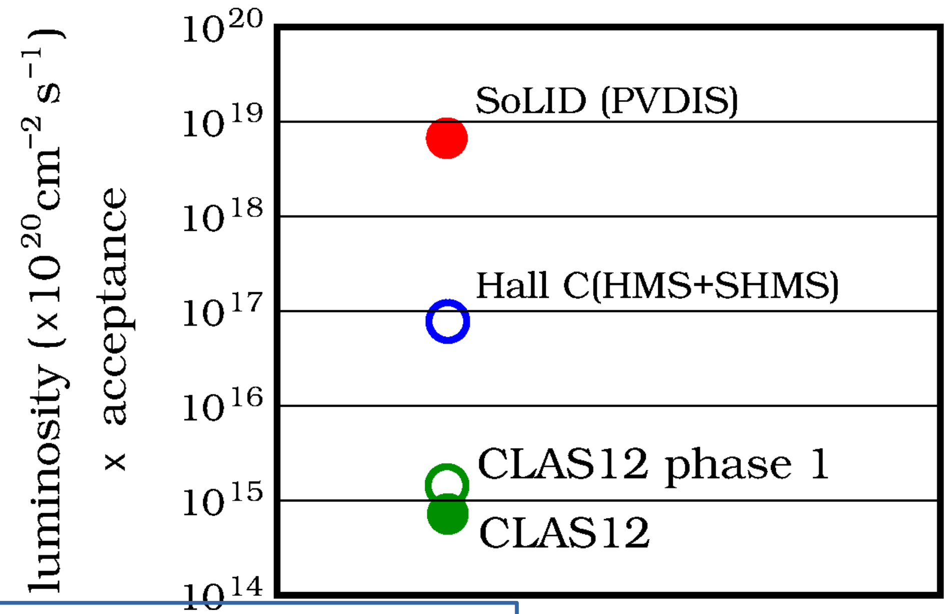
Neutral-Current Electroweak Physics and SMEFT Studies at the EIC

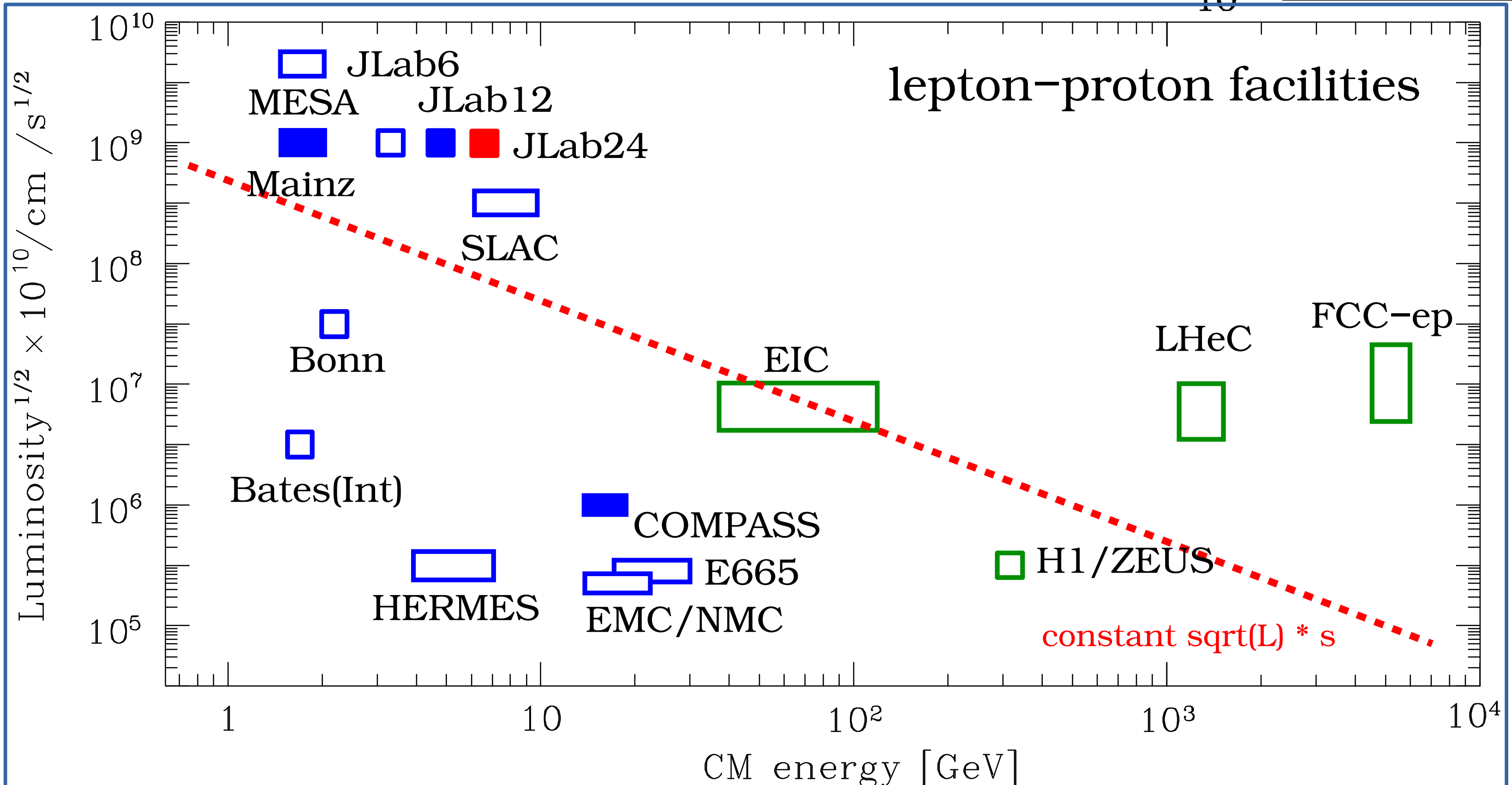


Keep JLab Competitive

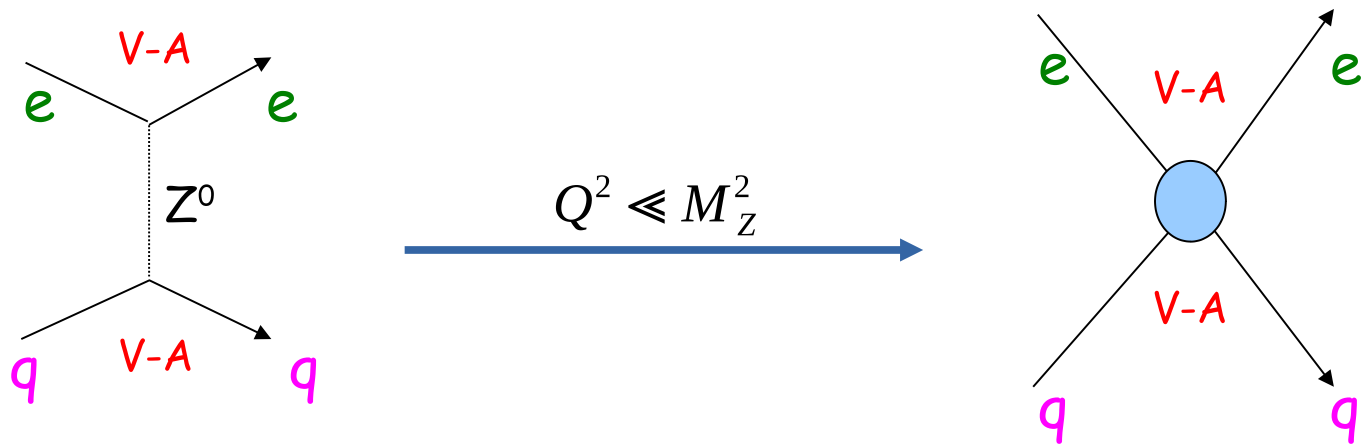
BSM Physics search FOM: $\sqrt{L}\,s$

Need large acceptance spectrometers





Neutral-Current Effective Couplings in (Low Energy) Electron Scattering



$$L_{NC}^{lq}=\frac{G_F}{\sqrt{2}}\sum_q\left[C_{0q}\,\bar{l}\,\gamma^{\mu}l\,\bar{q}\,\gamma_{\mu}q+C_{1q}\,\bar{e}\,\gamma^{\mu}\gamma_5 l\,\bar{q}\,\gamma_{\mu}q+C_{2q}\,\bar{e}\,\gamma^{\mu}e\,\bar{q}\,\gamma_{\mu}\gamma_5 q+C_{3q}\,\bar{l}\,\gamma^{\mu}\gamma_5 l\,\bar{q}\,\gamma_{\mu}\gamma_5 q\right]$$

VV (identical to γ) AV, VA (parity-violating) AA

$$C_{1u}=2g_A^e g_V^u=-\frac{1}{2}+\frac{4}{3}\sin^2(\theta_W) \qquad C_{2u}=2g_V^e g_A^u=-\frac{1}{2}+2\sin^2(\theta_W) \qquad C_{3u}=-2g_A^e g_A^u=\frac{1}{2}$$

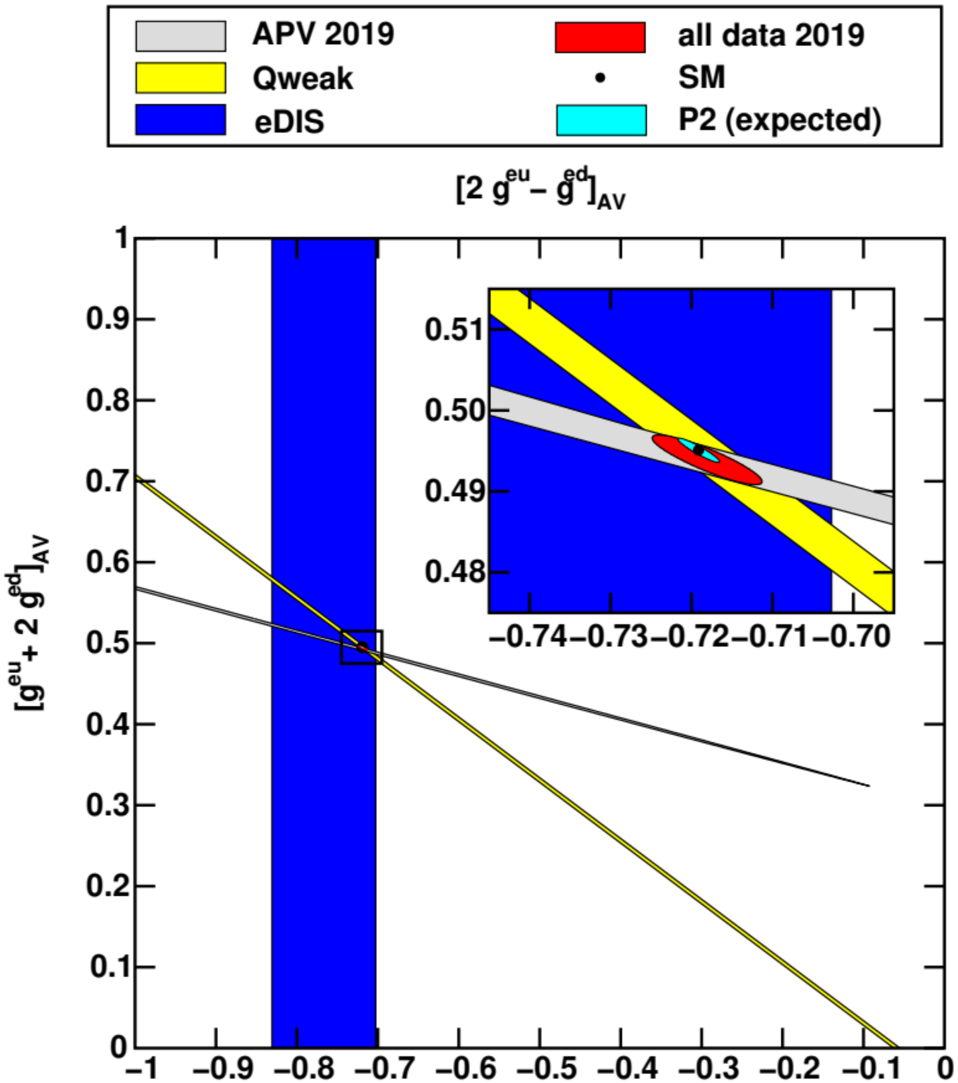
$$C_{1d}=2g_A^e g_V^d=\frac{1}{2}-\frac{2}{3}\sin^2(\theta_W) \qquad C_{2d}=2g_V^e g_A^d=\frac{1}{2}-2\sin^2(\theta_W) \qquad C_{3d}=-2g_A^e g_A^d=-\frac{1}{2}$$

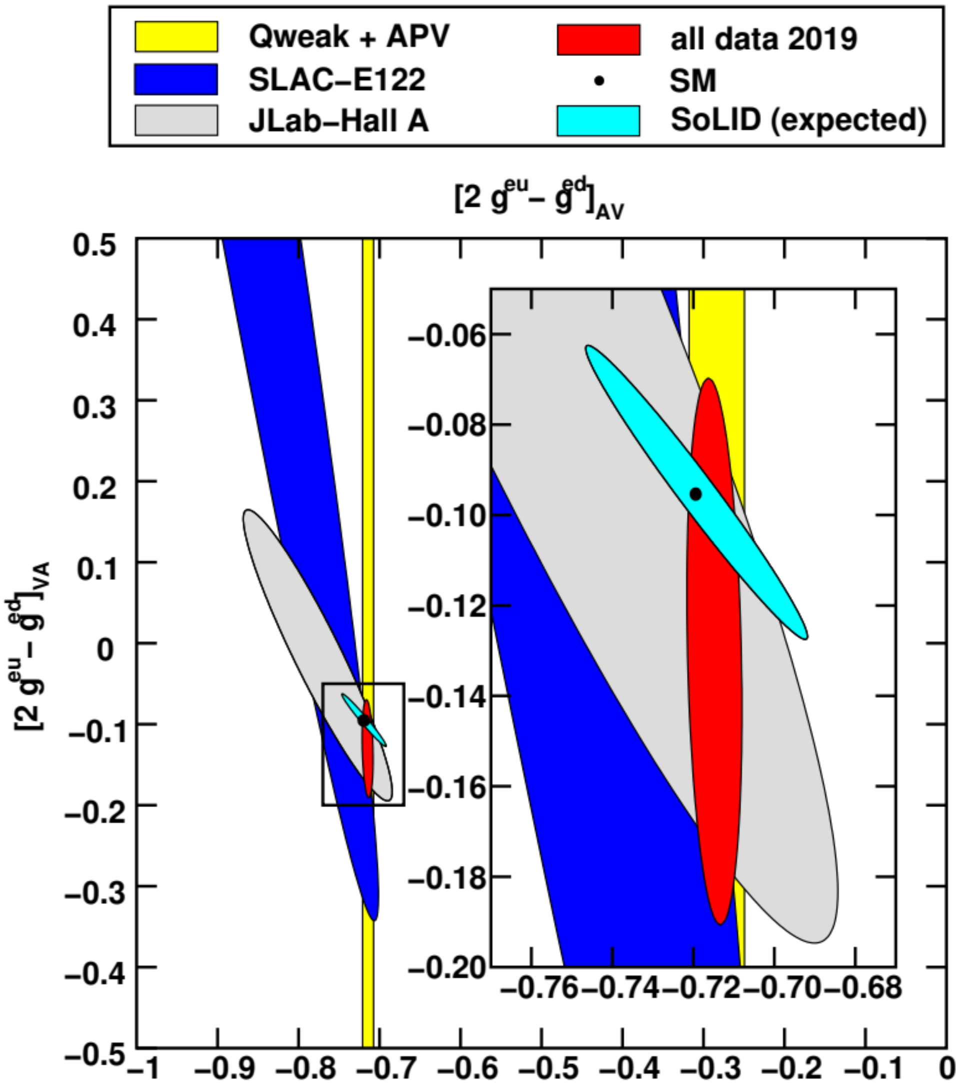
- A new set of notation $g_{AV,VA,AA}^{eq}$ introduced in 2013 – Erler&Su, Prog. Part. Nucl. Phys. 71, 119 (2013)
- Example: In PVES, we can measure $C_{1,2}$

Current Knowledge on $C_{1q,}C_{2q}$

all are 68% C.L. limit

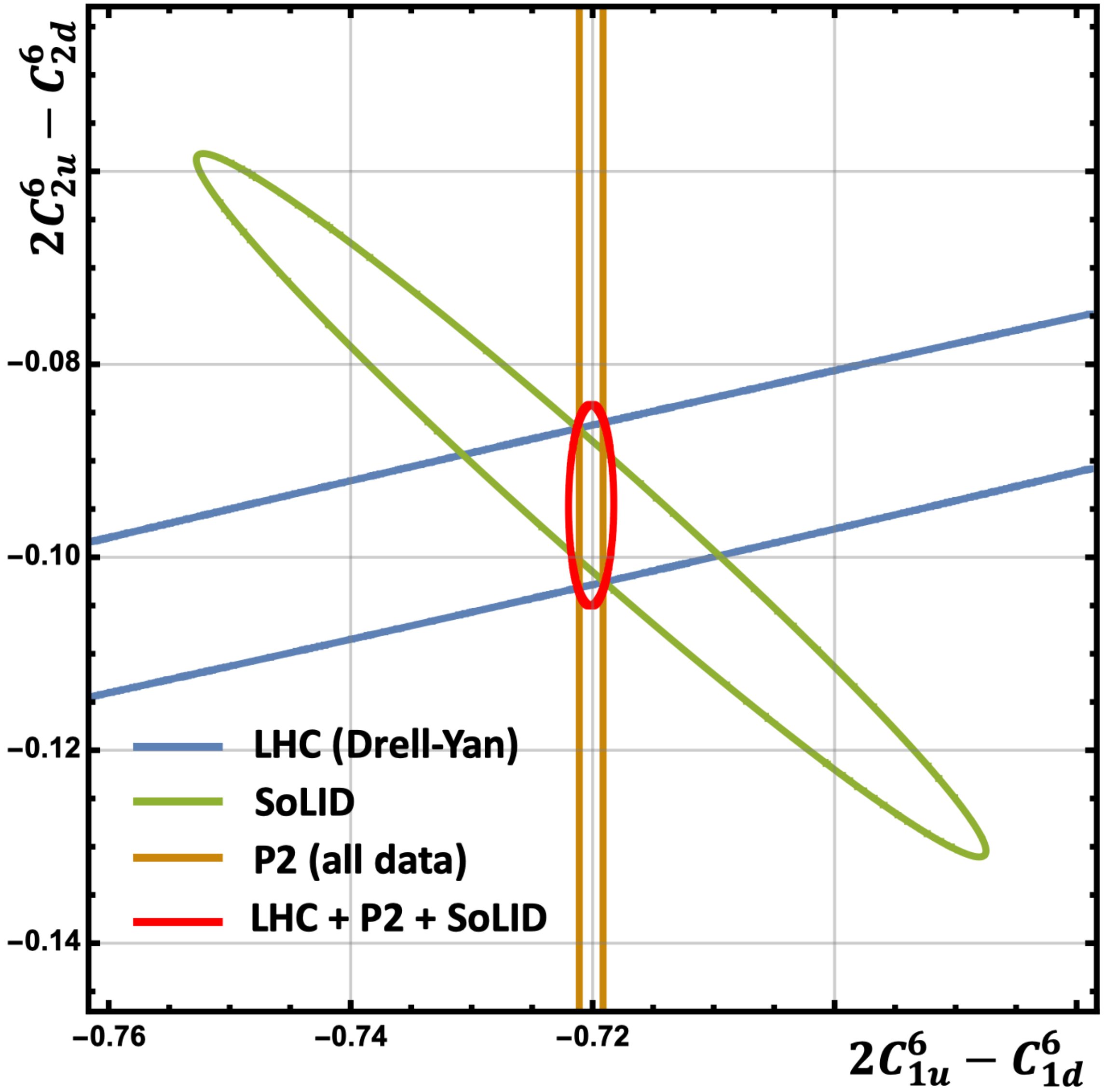
https://arxiv.org/abs/2103.12555





SMEFT with PVES

R. Boughezal, F. Petriello and D. Wiegand, Phys.Rev.D 104 (2021) 1, 016005; 2104.03979 [hep-ph]



Caveat: LHC is sensitive to a combination of Wilson coefficients, to put it on this graph, one has to set the other coefficients to zero.

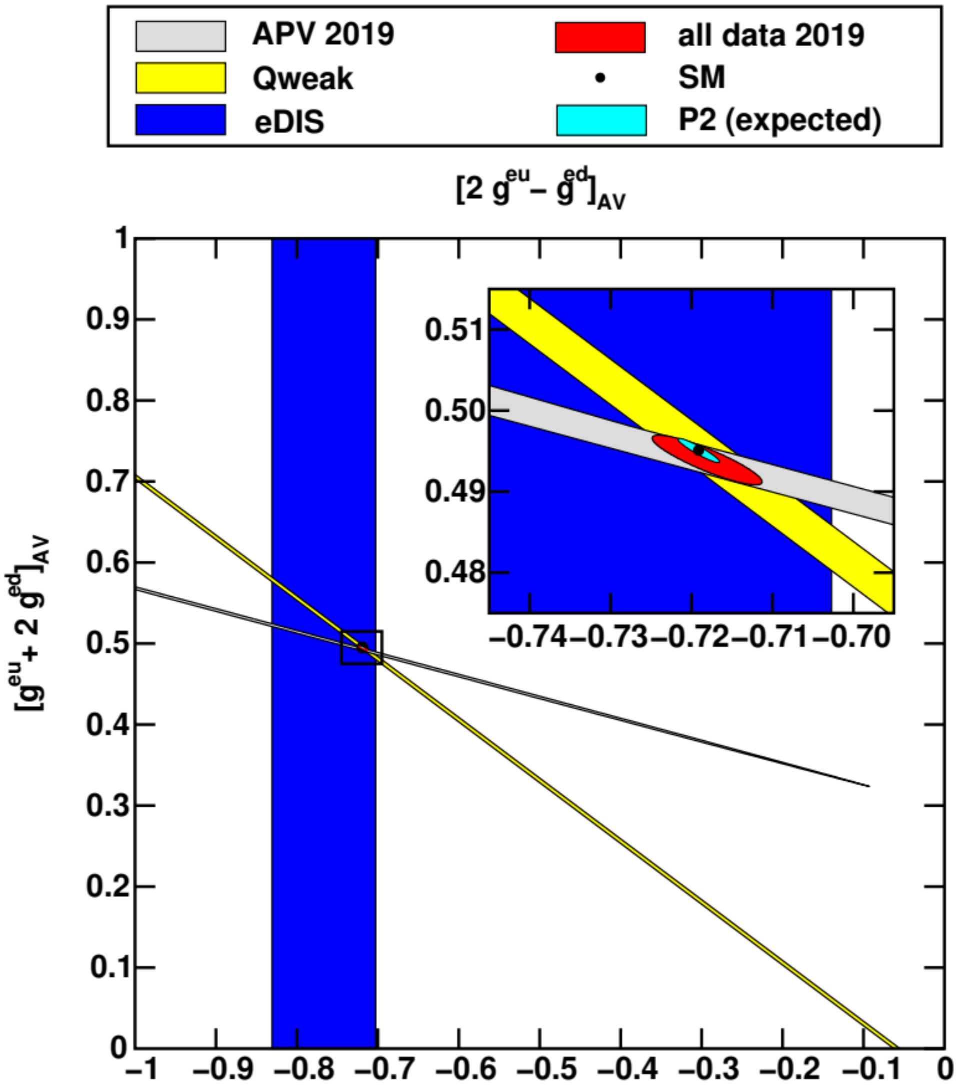
While SoLID and P2 determinations are without ambiguity.

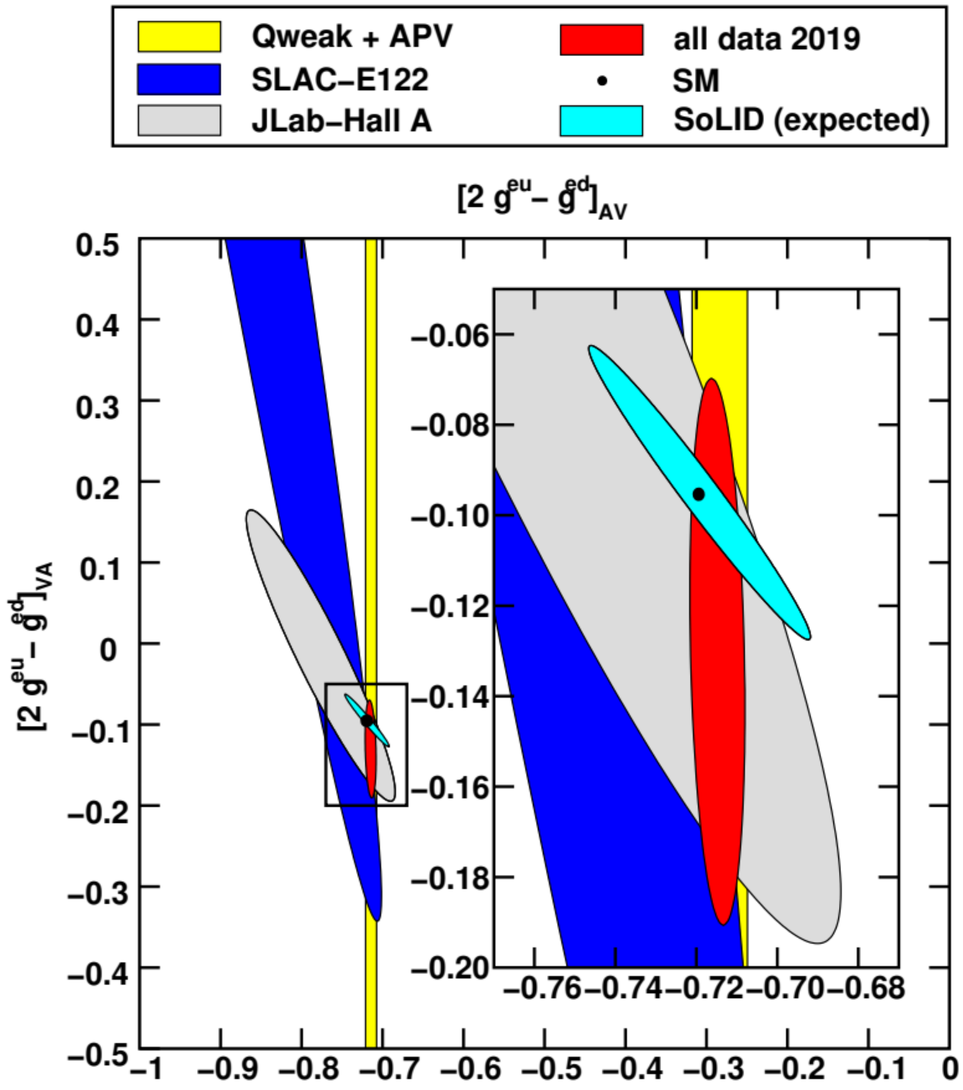
To do:
Find a way to visualize Wilson coefficients that emphasize SoLID

Current Knowledge on $C_{1q,} C_{2q}$

all are 68% C.L. limit

https://arxiv.org/abs/2103.12555





CERN for muon: $2C_{3u}^{\mu q} - C_{3d}^{\mu q} = 1.57 \pm 0.38$ Argento et al., PLB120B, 245 (1983)

In the Parton Model

Parity-Violating

$$A_{RL}^{e^\pm}=\frac{\sigma_R^{e^\pm}-\sigma_L^{e^\pm}}{\sigma_R^{e^\pm}+\sigma_L^{e^\pm}}$$

$$\left(A_{RL}^{e^\pm}=-A_{LR}^{e^\pm}\right)$$

$$A_d=|\lambda|(108\,ppm)Q^2[(2C_{1u}-C_{1d})+Y(y)(2C_{2u}-C_{2d})R_V(x)]$$

beam polarization

$$Y(y)=\frac{1-(1-y)^2}{1+(1-y)^2}$$

(indicates spin flip of quarks)

$$R_V(x)=\frac{u_V(x)+d_V(x)}{u(x)+\bar{u}(x)+d(x)+\bar{d}(x)}$$

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$$A_{RL}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_L^{e^-}}{\sigma_R^{e^+}+\sigma_L^{e^-}}$$

$$\left(A_{RL}^{e^+e^-}\neq-A_{LR}^{e^+e^-}\right)$$

Lepton-Charge

$$A_{RR}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_R^{e^-}}{\sigma_R^{e^+}+\sigma_R^{e^-}}$$

$$\left(A_{RR}^{e^+e^-}\neq-A_{LL}^{e^+e^-}\right)$$

$$A_{unpol}^{e^+e^-}=\frac{\sigma^{e^+}-\sigma^{e^-}}{\sigma^{e^+}+\sigma^{e^-}}$$

In the Parton Model

Parity-Violating

$$A_{RL}^{e^\pm}=\frac{\sigma_R^{e^\pm}-\sigma_L^{e^\pm}}{\sigma_R^{e^\pm}+\sigma_L^{e^\pm}} \qquad A_d = |\lambda|(108\,ppm)Q^2[(2C_{1u}-C_{1d})+Y(y)(2C_{2u}-C_{2d})R_V(x)]$$

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$$Y(y)=\frac{1-(1-y)^2}{1+(1-y)^2} \qquad R_V(x)=\frac{u_V(x)+d_V(x)}{u(x)+\bar{u}(x)+d(x)+\bar{d}(x)}$$

(indicates spin flip of quarks)

Lepton-Charge

$$A_{RL}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_L^{e^-}}{\sigma_R^{e^+}+\sigma_L^{e^-}} \qquad A_{RL,d}^{e^+e^-}=(108\,ppm)Q^2Y(y)R_V(x)[|\lambda|(2C_{2u}-C_{2d})-(2C_{3u}-C_{3d})]$$

$$(A_{RL}^{e^+e^-}\neq -A_{LR}^{e^+e^-})$$

(flip |λ| for LR)

$$A_{RR}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_R^{e^-}}{\sigma_R^{e^+}+\sigma_R^{e^-}} \qquad A_{RR,d}^{e^+e^-}=(108\,ppm)Q^2[|\lambda|(2C_{1u}-C_{1d})-Y(y)R_V(x)(2C_{3u}-C_{3d})]$$

(flip |λ| for LL)

$$(A_{RR}^{e^+e^-}\neq -A_{LL}^{e^+e^-})$$

$$A_{unpol}^{e^+e^-}=\frac{\sigma^{e^+}-\sigma^{e^-}}{\sigma^{e^+}+\sigma^{e^-}} \qquad A_d^{e^+e^-}=-(108\,ppm)Q^2Y(y)R_V(x)(2C_{3u}-C_{3d})$$

In the Parton Model

Parity-Violating

$$A_{RL}^{e^\pm}=\frac{\sigma_R^{e^\pm}-\sigma_L^{e^\pm}}{\sigma_R^{e^\pm}+\sigma_L^{e^\pm}}$$

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$$A_d=|\lambda|(108\,ppm)Q^2[(2C_{1u}-C_{1d})+Y(y)(2C_{2u}-C_{2d})R_V(x)]$$

beam polarization

$$Y(y)=\frac{1-(1-y)^2}{1+(1-y)^2}$$

(indicates spin flip of quarks)

$$R_V(x)=\frac{u_V(x)+d_V(x)}{u(x)+\bar{u}(x)+d(x)+\bar{d}(x)}$$

Lepton-Charge

$$A_{RL}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_L^{e^-}}{\sigma_R^{e^+}+\sigma_L^{e^-}}$$

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$$\boxed{A_{RL,d}^{e^+e^-}=(108\,ppm)Q^2Y(y)R_V(x)[|\lambda|(2C_{2u}-C_{2d})-(2C_{3u}-C_{3d})]}$$

(flip |λ| for LR)

“B” in CERN measurement

$$A_{RR}^{e^+e^-}=\frac{\sigma_R^{e^+}-\sigma_R^{e^-}}{\sigma_R^{e^+}+\sigma_R^{e^-}}$$

$$\left(A_{RR}^{e^+e^-}\neq -A_{LL}^{e^+e^-}\right)$$

$$A_{RR,d}^{e^+e^-}=(108\,ppm)Q^2[|\lambda|(2C_{1u}-C_{1d})-Y(y)R_V(x)(2C_{3u}-C_{3d})]$$

(flip |λ| for LL)

(no polarization needed!)

$$A_{unpol}^{e^+e^-}=\frac{\sigma^{e^+}-\sigma^{e^-}}{\sigma^{e^+}+\sigma^{e^-}}$$

$$A_d^{e^+e^-}=-(108\,ppm)Q^2Y(y)R_V(x)(2C_{3u}-C_{3d})$$

“direct” access to $2C_{3u}$-C_{3d}

e^+e^- for Structure Function Study

Approximately: $$A^{e^+e^-}_{\text{unpol}} = \frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{g_A^e}{2} Y(y) \frac{F_3^{\gamma Z}}{F_1^{\gamma}}$$

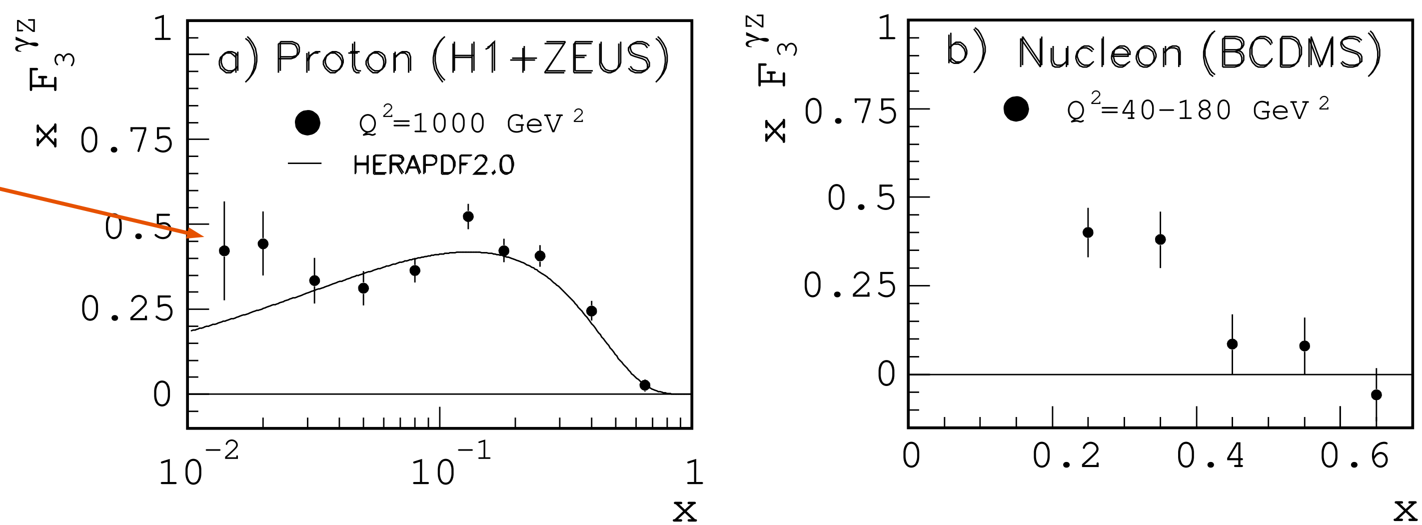
In the parton model: $$\boxed{F_1^{\gamma}(x,Q^2) = 1/2 \sum Q_q^2 [q+\bar{q}]}$$

$$\boxed{F_3^{\gamma Z}(x,Q^2) = 2\sum g_A^q [q-\bar{q}]}$$

Low x HERA data pose question on

$$q_{\text{sea}} = \bar{q}_{\text{sea}}$$

($\rightarrow$ LHeC)



By measuring $A^{e+e-}_{p,d}$ we can access $F_3^{\gamma Z}(x,Q^2)$

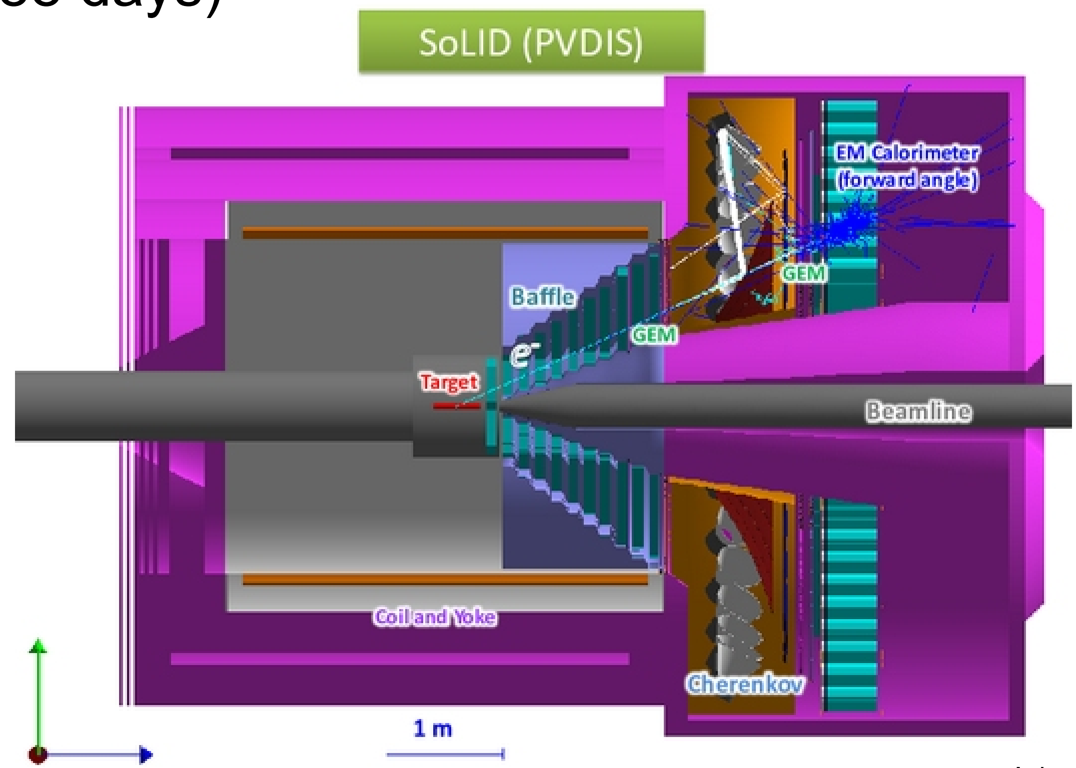
Designing the Experiment

Need high Q^2, high Y(y) → **SoLID PVDIS** configuration is ideal (40cm LD2)

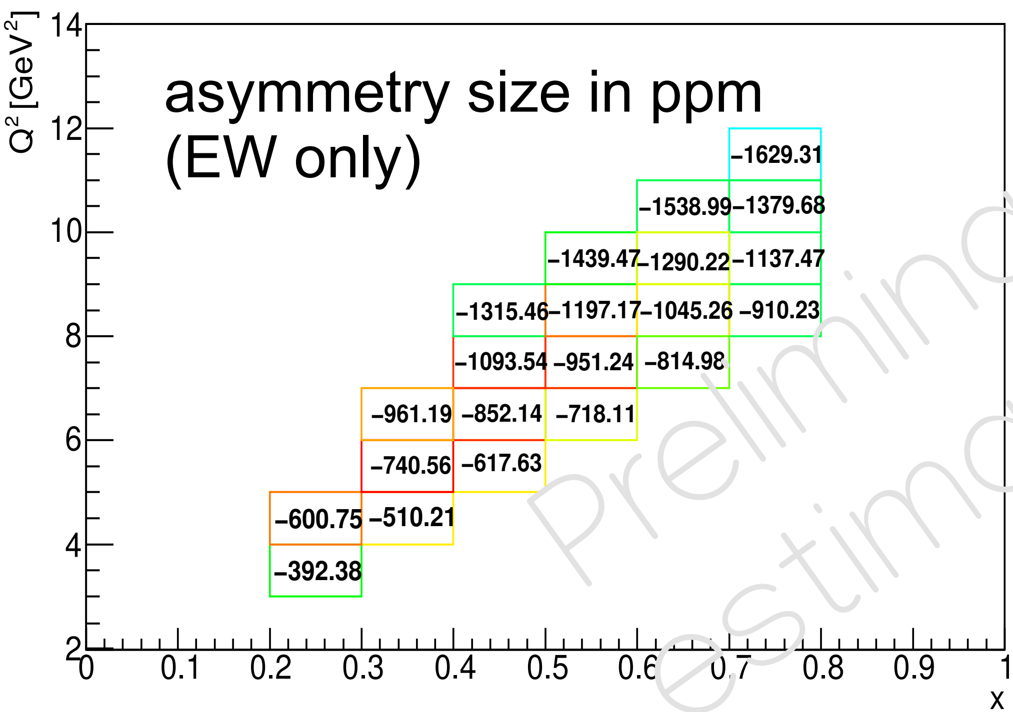
Need positron beam → **PEPPo**: up to 5uA for unpolarized. We ask for 3uA, 88 days at 11 GeV, 8 days at 6.6 GeV, each split between e+ and e- runs.

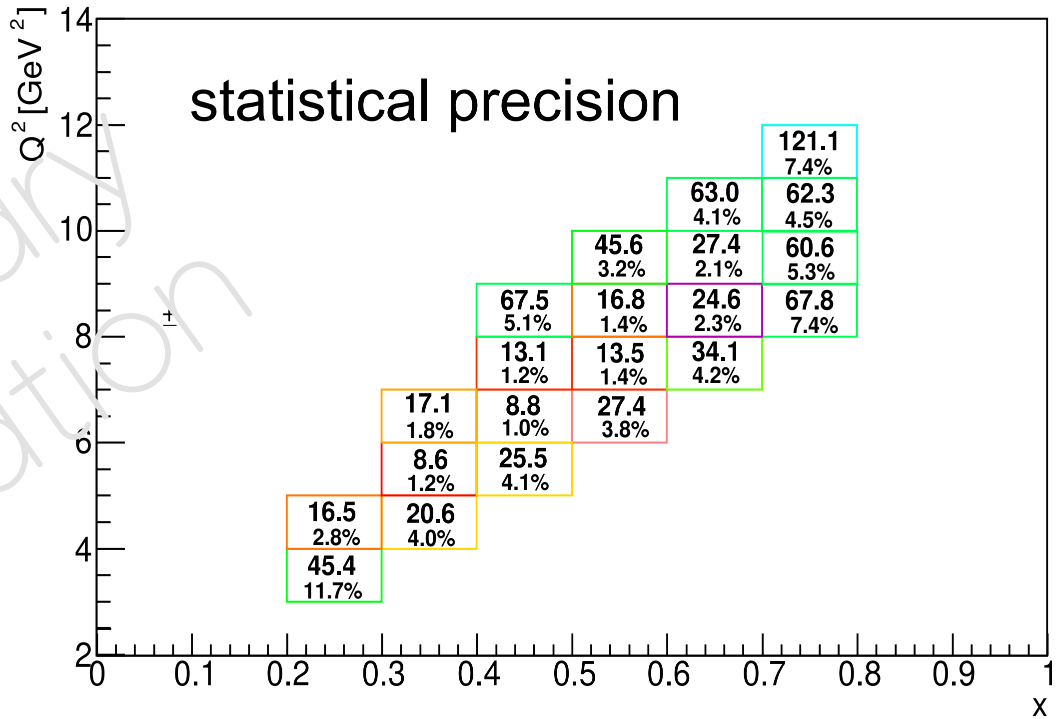
Need positron detection → **reverse magnet** polarity of SoLID, run magnets always at full saturation (field mapping needed to control field diff. < 10^{-5})

For each of e+ and e- run, also need **reverse polarity runs** to determine pair production background (8 of 88 days)



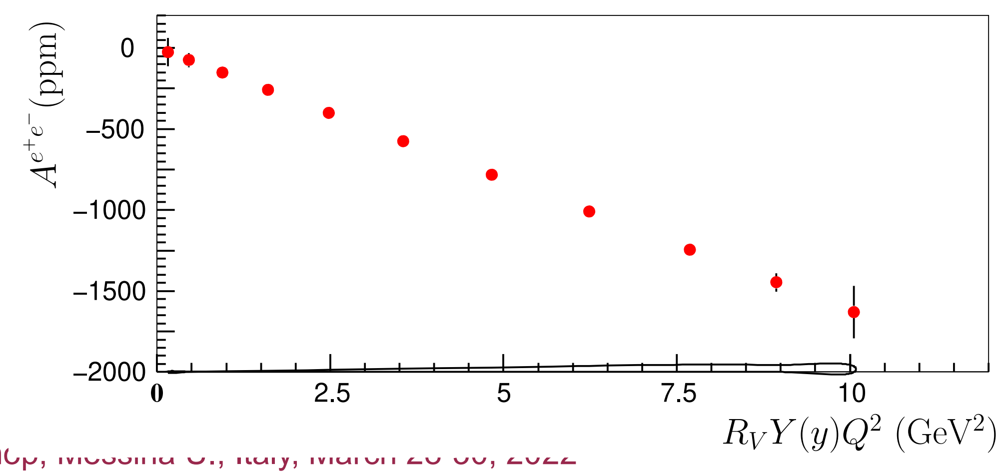
What can we do with 80 days of 3uA beam on a 40cm LD2 target? (in absence of all challenges):





if we consider only statistics and assume A=0 at Q²=0: $\boxed{1.5 \pm 0.007}$

$$A_d^{e^+e^-} = -(108\,ppm)\,Q^2\,Y\,R_V\,(2C_{3u} - C_{3d})$$



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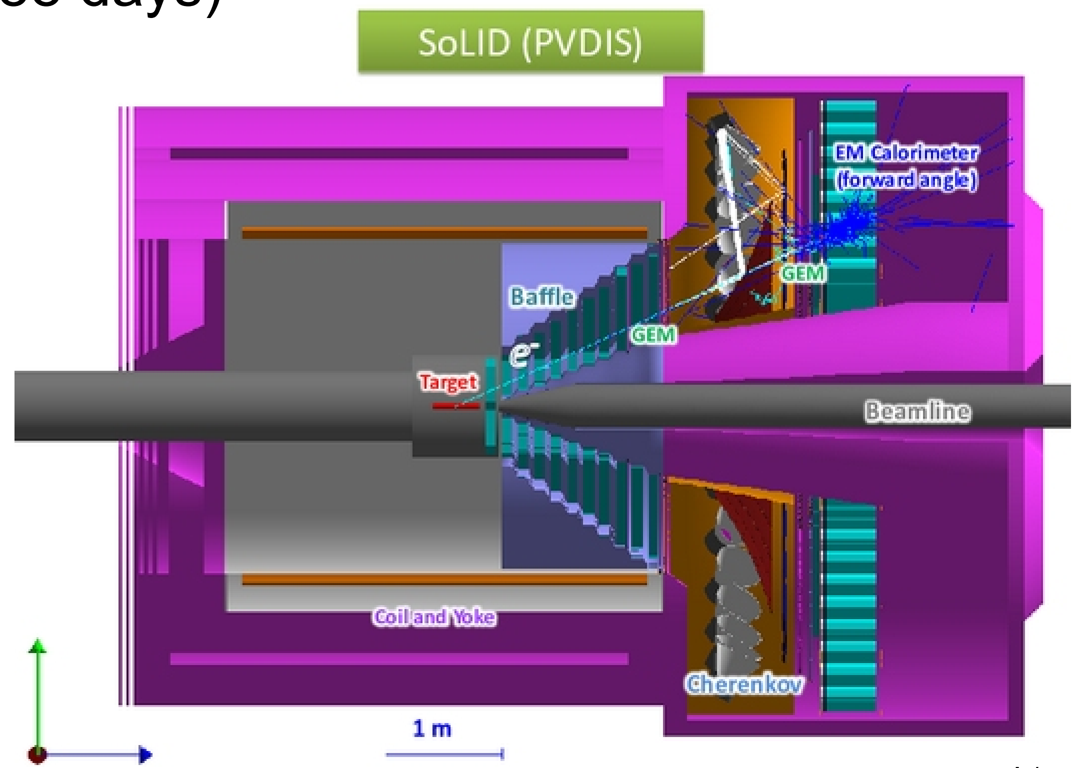


Experimental challenges:
- Ebeam, luminosity, charged pion and pair production background, magnet and detector stability

Theoretical challenges:
- higher-order QED corrections

Experimental Challenges

luminosity difference up to 1% (scaled by 1/10 in the plot) →

Δ Lumi

Eb difference up to 5×10^{-4}

$\Delta A_{E_b,max}$

E' difference up to 1×10^{-5}

$\Delta A_{E',max}$

Coulomb correction

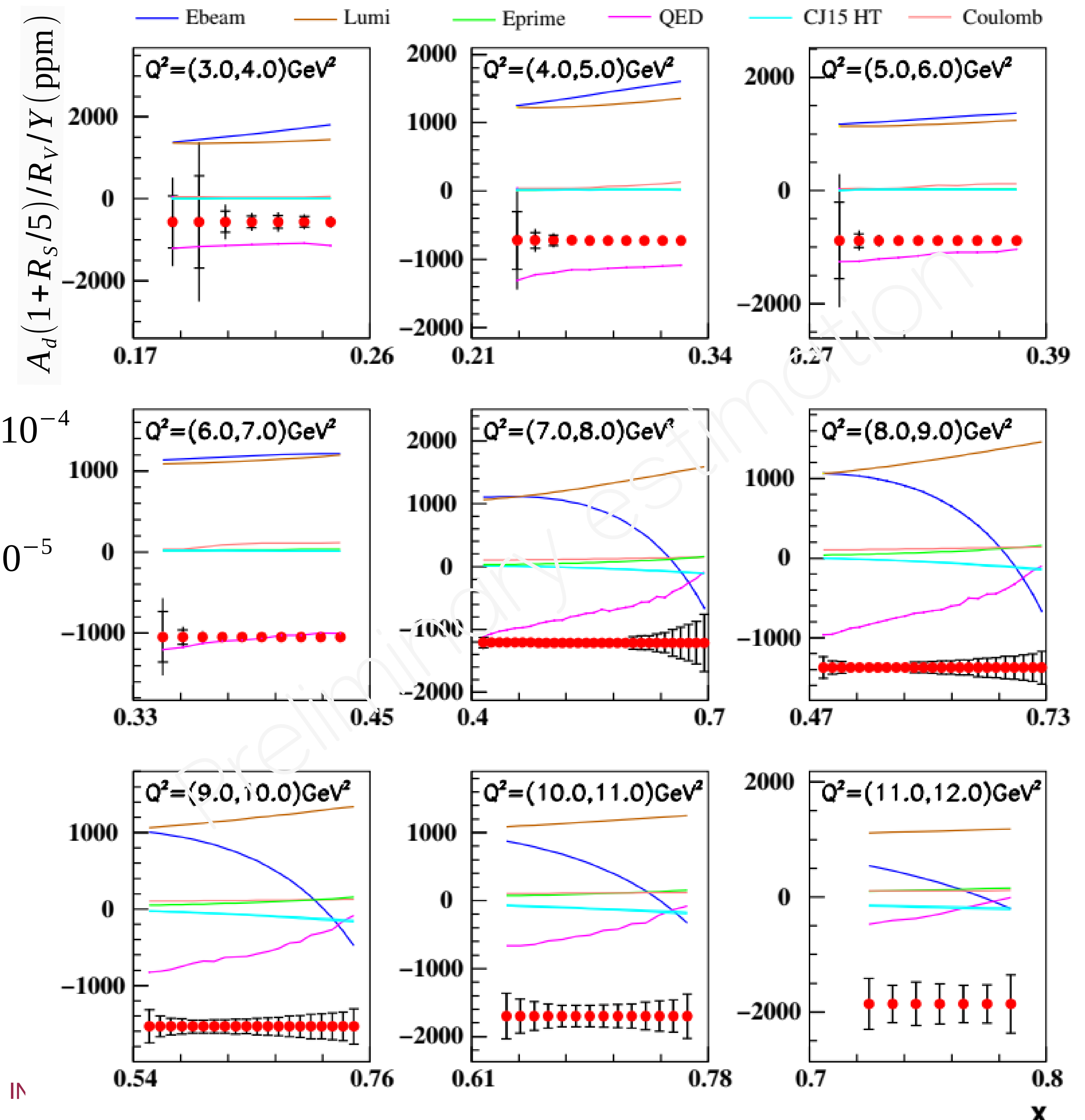
$\Delta A_{Coulomb}$

QED higher order (scaled by 1/5) →

ΔA_{QED}

CJ15 HT:

ΔA_{CJ15}



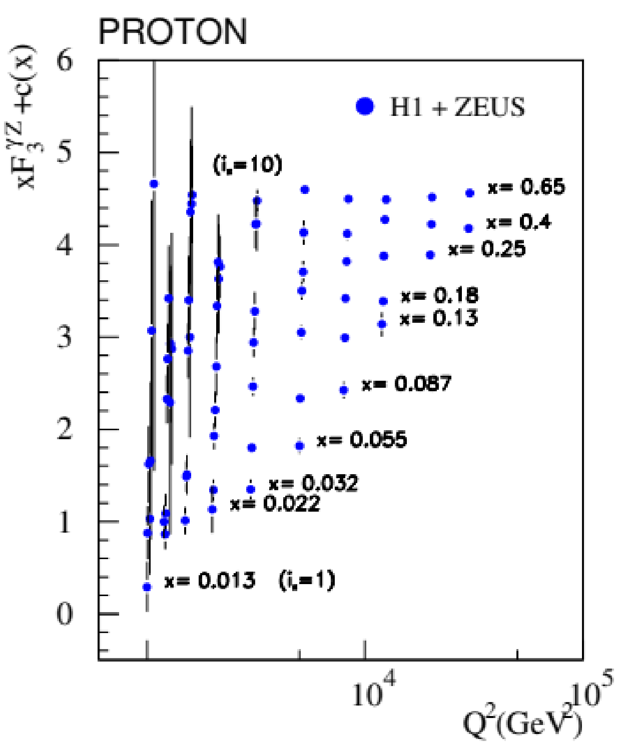
PR12-21-006 Lepton Charge Asymmetry

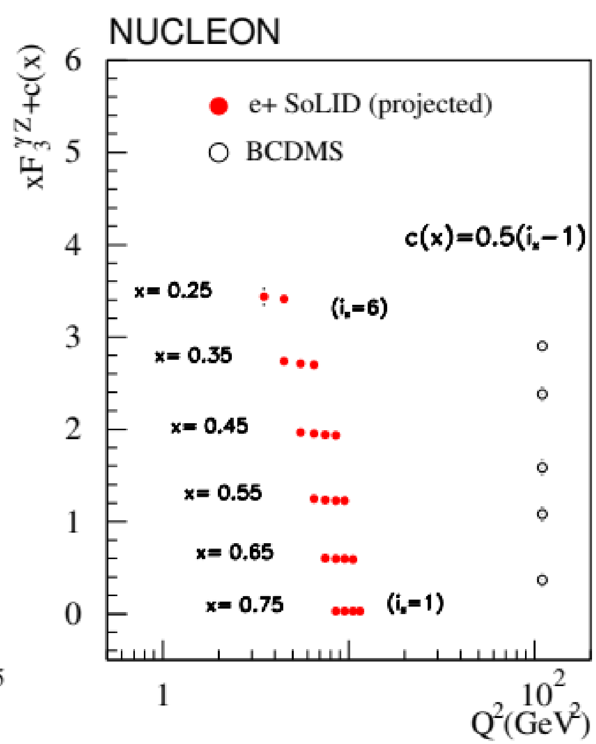
- 104 PAC days
- positron beam 3uA unpolarized
- beam control (1E-4 beam energy, ? beam position, “fast switch”)

PAC49 report:

Issues: The PAC is pleased to see such an interesting and far-reaching proposal. … ...At the same time, the requirements on the accelerator and theory are both daunting.

$$\Delta\left(2C_{3u} - C_{3d}\right)_{\text{total}} = \pm 0.053\left(\exp\right) \pm 0.009\left(1\%\ \text{QED}\right)$$
$$+0.000 - 0.035\left(\text{HT, CJ15}\right) \approx \pm 0.060$$





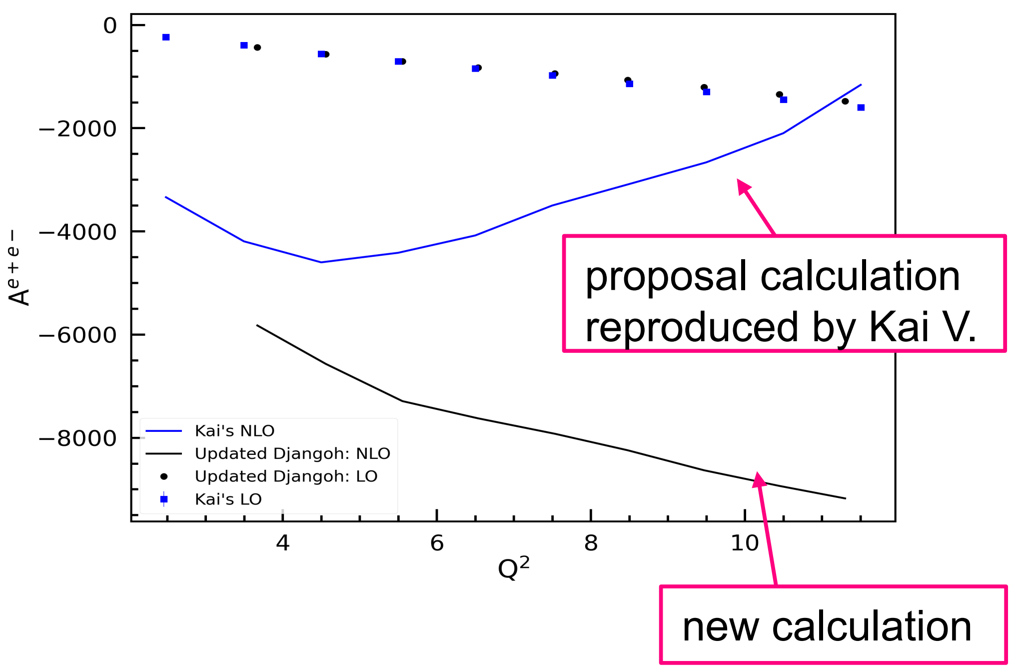
Summary: This proposal will require a tour-de-force effort, and the PAC encourages the group to proceed with development. To allow the community better usage of the results, the proposal should include estimates of asymmetry and cross section uncertainties. At this time, our concerns about the details of having the proper beam and the optimal theory extraction of the electron-quark couplings leads us to defer the proposal in its present form.

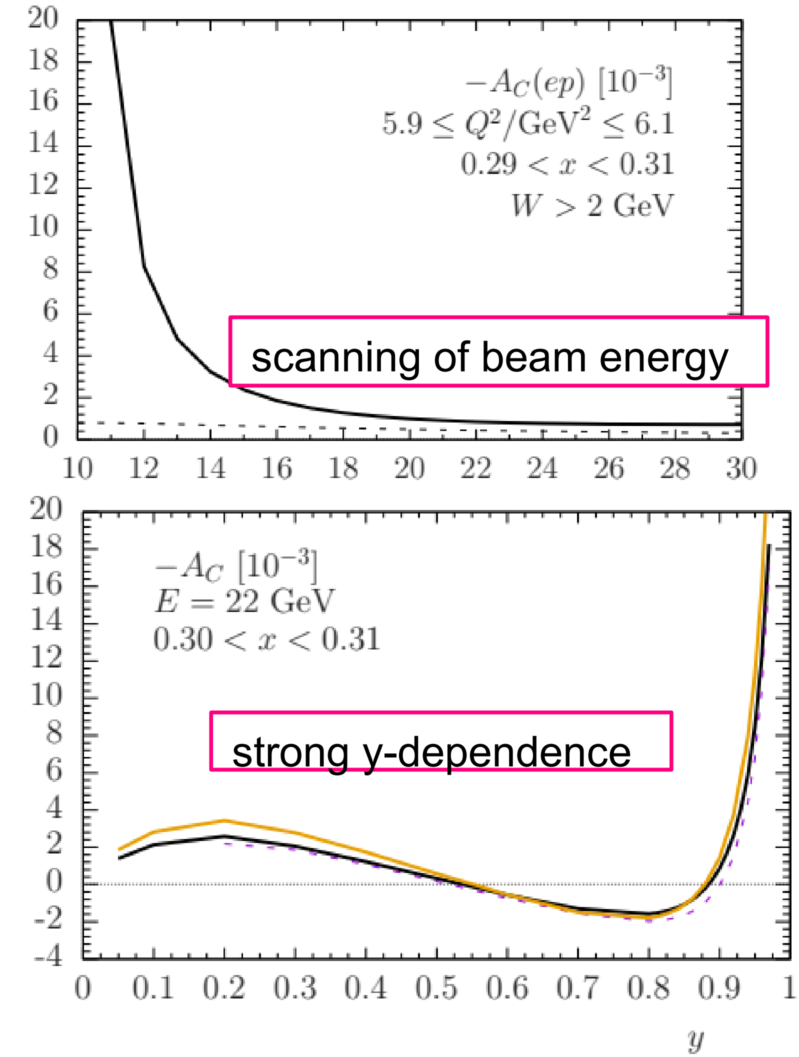
Idea: with positron beam, study TPE DIS (QED NLO) first

- TPE in DIS using positrons:
  - New calculation shows that NLO asymmetry is larger now for 11 GeV (than in the proposal), but at least 20 times much smaller at 22 GeV. Djangoh developer (Hubert S.) also suggested lower y settings;

H. Spiesberger, DJANGOH.4.6.19 16.

Djangoh 4.6.16 vs. 4.6.19 (lepton-charge for deuteron fixed target)





Idea: with positron beam, study TPE DIS (QED NLO) first

- TPE in DIS using positrons:
  - New calculation shows that NLO asymmetry is larger now for 11 GeV (than in the proposal), but at least 20 times much smaller at 22 GeV. Djangoh developer (Hubert S.) also suggested lower y settings;
  - We now have the tool for calculation, can do **FOM study** [target position/ scattering angle/ (x,Q2,y)]:
    - develop the physics case (TPE in DIS?); multi-stage approach?
    - calculation of A^{e+e-} LO and NLO over a wide range of (x,Q^2), optimize kinematics separately for:
      - TPE study (test NLO calculations, need NLO>>LO)
      - electroweak study (need NLO<<LO), measure C_{3q} ;
      - possibly study NLO at 11 GeV and C_{3q} at 22 GeV?
  - Proposal focusing on testing TPE DIS calculation possible (2024?), and e+@22 GeV in the (far) future.

Summary of Challenges and Why They Exist?

– With a positron beam, the best physics impact comes from comparison between e+ and e- scattering, rather than measuring the same observable (e.g. Apv) as electrons

– If positron vs. electron comparison is our goal, then all systematic effect related to the beam need to be controlled to high precision

– Frequent ("weekly") and fast switch between e+ and e- beams is required to control differences in beam and run conditions → impact on positron beam design. We need the systematic uncertainty small enough to match statistical uncertainties (from the high luminosity)

– Measurements where signal is tiny (EW physics) will be extremely difficult

– We have not even looked into particle background effects on the detector, trigger, and DAQ system.

– There is no well established calculation nor experimental test of TPE (QED NLO) in DIS. All previous (SLAC) data indicated zero but with poor precision;

– HERA data provided only slight constraint on QED NLO in DIS – "without the QED NLO term, the fit quality isn't very good";

– We could consider a "phased" approach: study DIS TPE with 11 GeV and see if it's realistic to study EW physics with 22 GeV (?)

Past Experiment – BCDMS

1983 CERN, using polarized μ+ vs. μ- beams:

$$2\,C_{3u}^{\mu q} - C_{3d}^{\mu q} = 1.57 \pm 0.38$$

Volume 120B, number 1,2,3 PHYSICS LETTERS 6 January 1983

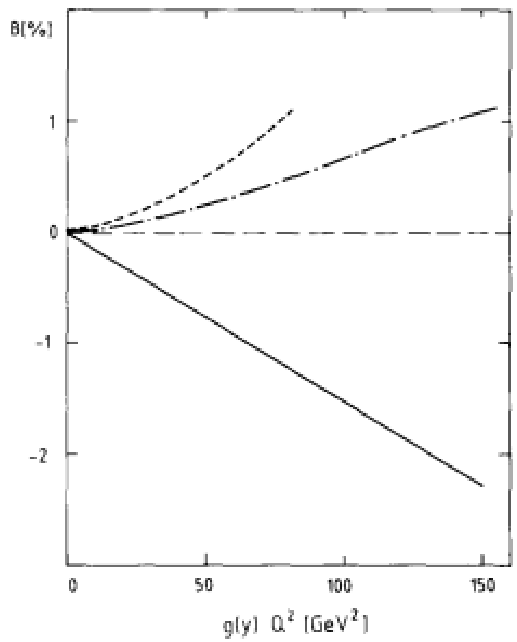


Fig. 2. The B asymmetry from $\gamma - Z^0$ interference to first order, calculated for a polarization $\lambda = 0.81$ and $\sin^2\theta_W = 0.23$ (solid line), and the asymmetry expected from higher order electromagnetic processes at beam energies of 120 GeV (dashed line) and 200 GeV (dashed–dotted line).

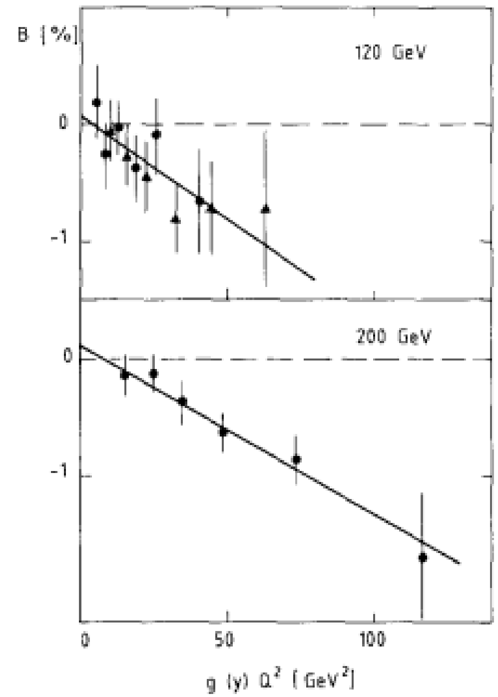


Fig. 3. The measured B asymmetry after radiative corrections at 120 GeV and 200 GeV beam energy versus $g(y)Q^2 = Q^2 \times [1-(1-y)^2]/[1+(1-y)^2]$ [eq. (3)]. For the 120 GeV data, circles represent data with $Q^2 > 15$ GeV. Solid lines are straight line fits to the data.

netic and weak-electromagnetic effects according to ref. [6]: the magnitude of these corrections is shown

a measurement for the electron is highly desired

Past Experiments – SLAC, HERMES, OLYMPUS (elastic), HERA

- D.L. Fancher et al, Phys.Rev.Lett.37, 1323 (1976)

13.5-GeV beams at **Stanford Linear Accelerator Center**, compared electron and positron inelastic scattering in $1.2 < Q^2 < 3.3\ (\text{GeV}/c)^2$, $2 < \nu < 9.5$ GeV. Found "e+/e- cross section ratio = 1.0027 ± 0.0035 (including stat and syst effects), with no significant dependence on Q^2 or v. This result has appreciably smaller errors to fine TPE effects in electron or muon scattering."

Note: Ae+e- ~ 1E-4, Coulomb ~ 1E-5 to 1E-4, QED NLO ~1E-4 for these kinematic settings.

- A. Airapetian et al., JHEP 05 (2011) 126 – **HERMES** inclusive paper; G. Schnell p.v.:

Overall normalization of DIS xsection was at 8% level.

- B.S. Henderson et al., Phys. Rev. Lett. 118 (2017) 092501 **OLYMPUS**

"The relative luminosity between the two beam species was monitored using tracking telescopes of interleaved gas electron multiplier and multiwire proportional chamber detectors at 12°, as well as symmetric Moller or Bhabha calorimeters at 1.29°. The uncertainty in the relative luminosity between beam species of 0.36% was achieved."

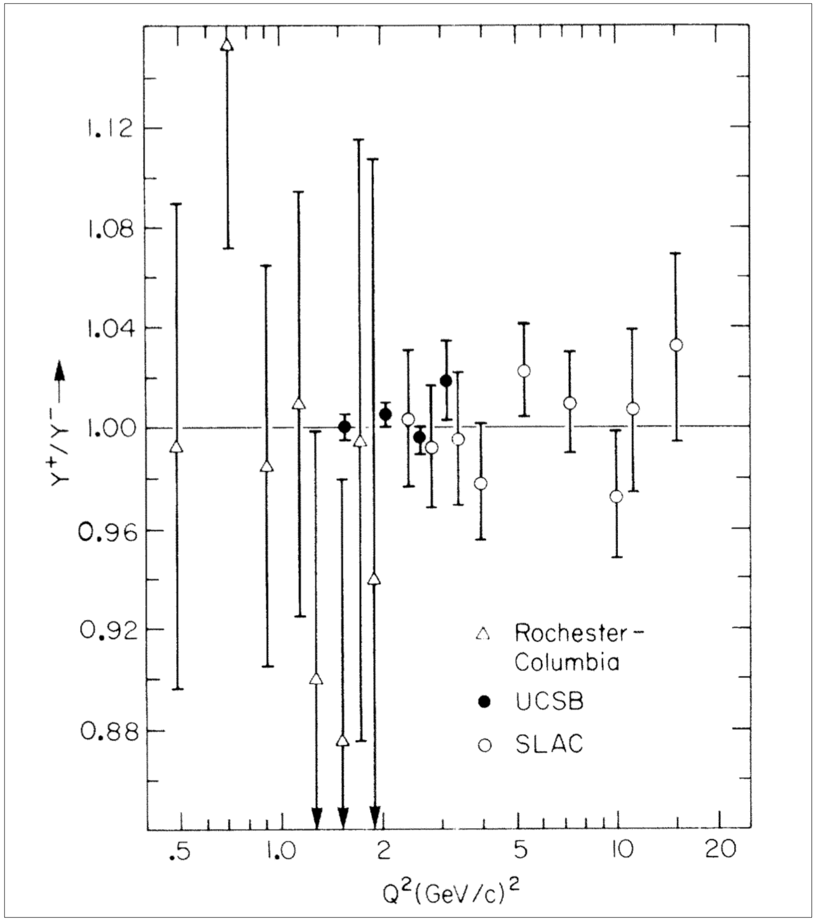
Note: 0.36% luminosity control is not going to help us

- V. Andreev et al. (**H1** Collaboration), Eur. Phys. J. C 78 (2018) 9, 777

luminosity ~ 2% with partial cancellations, measured e- and e+ DIS cross sections.

Note: At HERA energy, QED NLO is relatively small

SLAC 1976 Proton Inelastic Measurement



D.L. Fancher et al, Phys.Rev.Lett.37, 1323 (1976)

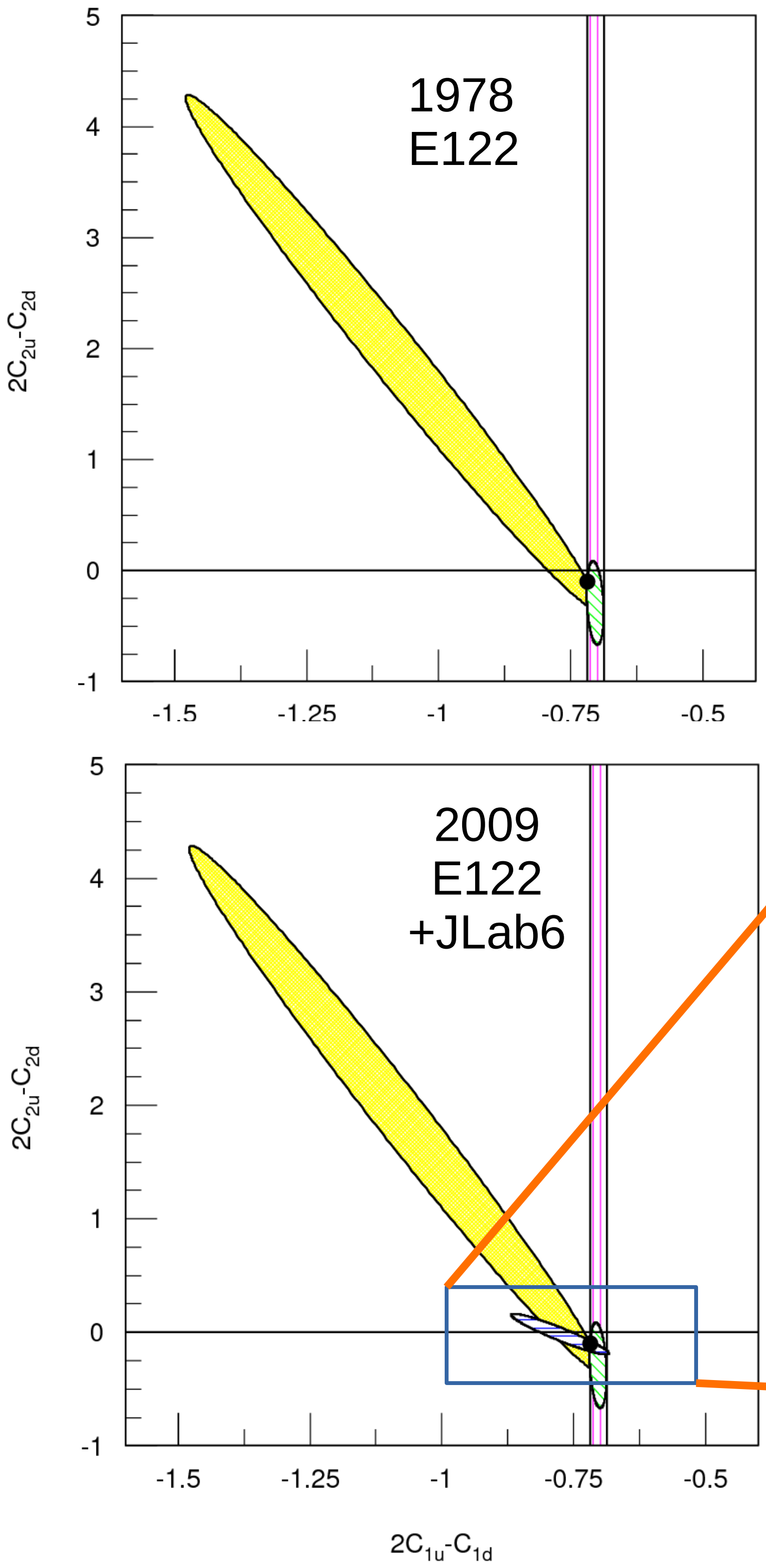
Q^2 $(GeV/c)^2$	Y_+	Y_-	Y_+/Y_-
1.3–1.8	227 054 ± 784	227 010 ± 729	1.0002 ± 0.0047
1.8–2.3	287 029 ± 804	285 228 ± 780	1.0063 ± 0.0039
2.3–2.8	167 359 ± 579	167 997 ± 583	0.9962 ± 0.0049
2.8–3.3	20 148 ± 210	19 766 ± 214	1.0191 ± 0.0150

E	Q^2	E’	ν	x
13.5	1.5	5.7	7.8	0.10
13.5	2.05	7.8	5.7	0.19
13.5	2.55	9.7	3.8	0.36
13.5	3.05	11.6	1.9	0.86

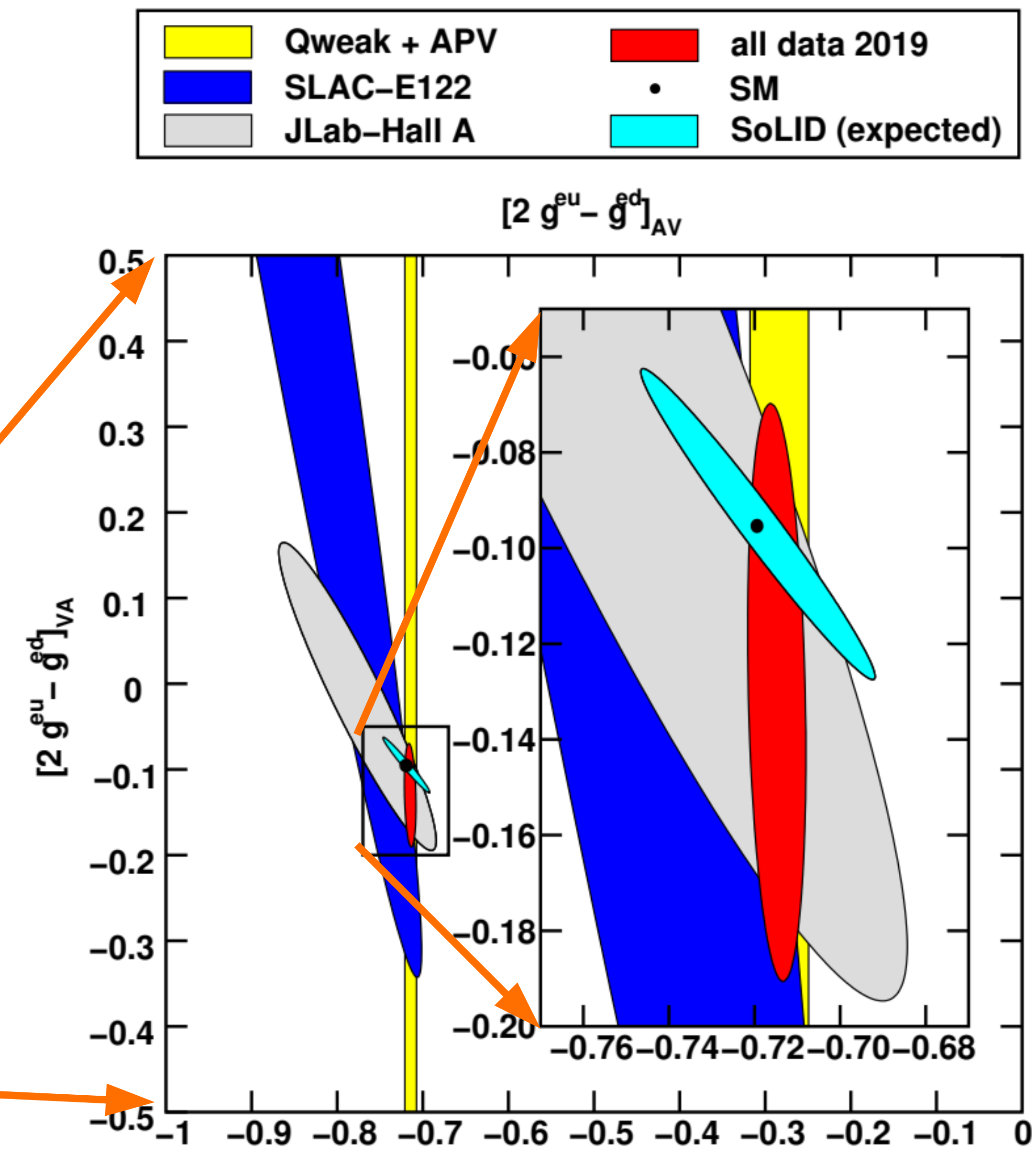
(Calculations done by M. Nycz, preliminary, consistent with data)

x_min	x_max	Q^2_min	Q^2_max	sig(e-)_LO	sig(e+)_LO	sig(e-)_NLO	sig(e+)_NLO	A_LO	A_NLO
0.08	0.14	1.3	1.8	7.679204	7.677651	7.948650	7.9462437	-0.000101	-0.0001514
0.14	0.26	1.8	2.3	5.269455	5.268194	5.205612	5.2043891	-0.000120	-0.0001174
0.26	0.52	2.3	2.8	2.853423	2.852809	2.526783	2.5263637	-0.000108	-0.0000830

PVDIS past, present, and future



2030(?) E122+JLab6+JLab12(SoLID)



The general case

(done)

based on
Anselmino et al. [arXiv:hep-ph/9401264]

For PVDIS:

$$A_{RL}^{e^-} = \frac{|\lambda|\,\eta_{\gamma Z}\left[g_A^e\, 2\,y\,F_1^{\gamma Z} + g_A^e\left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma Z} + g_V^e(2-y)F_3^{\gamma Z}\right]}{2\,y\,F_1^{\gamma} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma} - \eta_{\gamma Z}\left[g_V^e\, 2\,y\,F_1^{\gamma Z} + g_V^e\left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma Z} + g_A^e(2-y)F_3^{\gamma Z}\right]}$$

For $A^{e^+ e^-}$:

$$A_{RL}^{e^+ e^-} = \frac{\eta_{\gamma Z}\left(|\lambda|\,g_V^e + g_A^e\right)(2-y)F_3^{\gamma Z}}{2\,y\,F_1^{\gamma} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma} - \eta_{\gamma Z}\left(g_V^e + g_A^e\right)\left[2\,y\,F_1^{\gamma Z} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma Z}\right]}$$

$$A_{RR}^{e^-} = \frac{\eta_{\gamma Z}\, g_A^e\left[-|\lambda| 2\,y\,F_1^{\gamma Z} - |\lambda|\left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma Z} + (2-y)F_3^{\gamma Z}\right]}{2\,y\,F_1^{\gamma} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma} - \eta_{\gamma Z}\, g_V^e\left[2\,y\,F_1^{\gamma Z} + \left(\frac{2}{xy} - \frac{2}{x} - \frac{2M^2 xy}{Q^2}\right)F_2^{\gamma Z} + (2-y)F_3^{\gamma Z}\right]}$$

$$\eta_{\gamma Z} = \frac{G_F Q^2}{2\sqrt{2}\,\pi\alpha}\,\frac{M_Z^2}{M_Z^2 + Q^2}$$

$$\eta_Z = \eta_{\gamma Z}^2$$

Complete formula (also including Z terms):

– for numerator, replace $F_{1,2}^{\gamma Z} \rightarrow F_{1,2}^{\gamma Z} - 2\,\eta_{\gamma Z}\, g_V^e F_{1,2}^Z$ and $g_V^e F_3^{\gamma Z} \rightarrow g_V^e F_3^{\gamma Z} - \eta_{\gamma Z}(g_V^e g_V^e + g_A^e g_A^e) F_3^Z$

– for denominator, replace $g_V^e F_{1,2}^{\gamma Z} \rightarrow g_V^e F_{1,2}^{\gamma Z} - \eta_{\gamma Z}(g_V^e g_V^e + g_A^e g_A^e) F_{1,2}^Z$ $\quad \boxed{F_2^Z = 1/2\sum (g_V^q g_V^q + g_A^q g_A^q)[q+\bar{q}]}$

and $g_A^e F_3^{\gamma Z} \rightarrow g_A^e F_3^{\gamma Z} - 2\,\eta_{\gamma Z}(g_V^e g_A^e) F_3^Z$ $\quad \boxed{F_3^Z = 2\sum g_V^q g_A^q [q+\bar{q}]}$