Latest Ancillary Physics Results from Qweak

• Primary result reminder (just 1 slide)
• Final Results for $^{27}$Al PV Elastic $A_{PV}$ (Arex)
  – $R_n$, $R_n - R_p$, $F_{wk}$, $R_{wk}$, $R_{wk} - R_{ch}$
• Final Results for $^{12}$C & $^{27}$Al PC BNSSA (brief)

Greg Smith
Jefferson Lab
Hall C Winter Collaboration Meeting
Feb 17, 2022
Qweak Primary Results:

- \( Q_{\text{weak}} \) Expt. msrd ep \( A_{PV} = -226.5 \pm 9.3 \text{ ppb} @ Q^2 = 0.0248 \text{ GeV}^2 \)
  - Determined \( Q_W(p) = 0.0719 \pm 0.0045 \) for the 1\textsuperscript{st} time, < 0.2 \( \sigma \) from SM
  - \( \sin^2 \theta_W = 0.2383 \pm 0.0011 \) (MS-bar) (0.46%)
    - \( \text{Avg}(\text{APV, E158, Q}_{\text{weak}}) = 0.23861 \pm 0.00077 \) (0.32%)
  - Mass reach \( \Lambda = 26.6 \text{ TeV} \) (\( g^2=4\pi=\text{compositeness, 95}\% \text{ CL} \))
    - \( \Lambda = 2.3 \text{ TeV} \) (uud, \( g^2=4\pi\alpha=\text{leptoquarks, 95}\% \text{ CL} \))
    - \( \Lambda/g = 7.5 \text{ TeV} \) (proton, ie uud, 95\% CL)
    - \( \Lambda/g = 3.6 \text{ TeV} \) (\textit{flavor-independent}, 95\% CL)
  - Combined with APV: vector quark couplings \( C_{1u} \) & \( C_{1d} \)

- Publications:
  - Commissioning result: PRL 111, 141803 (2013)
  - Final \( Q^p_W \) result & SM test: Nature 557, 207 (2018)
  - \( Q^p_W \) cookbook & perspectives: ARNS 69, 191 (2019)
The Elastic PVES $^{27}$Al Experiment ("Arex")

- **Target:**
  - $^{27}$Al Alloy (7075), 4.2% $X_0$ (3.7 mm thick)
  - Located in $z$ at DS end of 35 cm long $Q_{\text{weak}}$ LH2 target

- **Beam:**
  - 1.16 GeV, 65 $\mu$A, $P=88.8\% \pm 0.6\%$

- **Spectrometer:**
  - $Q^2 = 0.02357 \pm 0.0001$ GeV$^2$, $<\theta_{\text{lab}}> = 7.61^\circ$, $5.8^\circ < \theta_{\text{lab}} < 11.6^\circ$

Elastic PVES on $^{27}\text{Al}$: Motivation

• Aluminum $A_{PV}$ msrd to correct for ($\sim 20\%$) tgt-window bkg in $Q_w^p$
• Al data now further analyzed to isolate the elastic $^{27}\text{Al} A_{PV}$ from non-elastic & other contributions:

$$A_{PV} = \frac{\sigma_+(\theta) - \sigma_-(\theta)}{\sigma_+(\theta) + \sigma_-(\theta)} \approx \frac{G_F Q^2 Q_W}{4\pi \alpha Z \sqrt{2}} \frac{F_W(Q^2)}{F_{EM}(Q^2)}$$

  – A>1 PVES: only $^{12}\text{C}$ (Bates), $^{4}\text{He}$ (Happex), $^{208}\text{Pb}$ (Prex) (soon $^{48}\text{Ca}$ too)
  – Comparing theory to $^{27}\text{Al} A_{PV}$ provides sanity check on $Q_w^p$ bkg corrections
  – Elastic $^{27}\text{Al} A_{PV}$ also provides $F_{wk}^{^{27}\text{Al}}, R_{n, wk}^{^{27}\text{Al}}$ & skin thicknesses

• We expect n-skin $\sim 0$ in a light $N\sim Z$ nucleus
  – serves as test case for the EW technique used to get $R_n$ in PREX & Crex
  – Important given tension noted in literature between Prex EW and non-EW results as well as recent LIGO/Virgo & NICER results
\textbf{EW vs non-EW Tension}

- PVES (EW) $^{208}\text{Pb}$ Prex: $R_{n-p} = 0.283 \pm 0.071$ fm (PRL 126, 172502 (2021))
- Non-EW $^{208}\text{Pb}$ results: $R_{n-p} = 0.18 \pm 0.027$ fm (Tsang et al, PRC86, 015803 (2012))
  - 2012 $^{208}\text{Pb}$ world avg: Elastic $\vec{p}A$ on Ni&Pb isotopes, 26 $\vec{p}A$ decays, EDP (electric dipole polarizability) & PDR
    - EDP: electric dipole field excites the GDR, the collective motion of p's against n's, and vibrations of the N=Z symmetric core against the n-skin
    - Uncertainty floor of $\pm 0.05$ fm (2* EDP error) was imposed!!!
    - Error reduces considerably if post-2012 data included
  - \textit{Conclusions: The $R_{\text{skin}}^{208}$ extracted from PVES and the EDP are two of the cleanest experimental tools used to constrain the symmetry energy. However, the recent PVES value of $R_{\text{skin}}^{208}$ that suggests a fairly stiff symmetry energy stands in stark contrast to the conclusions derived from the EDP. At present, I offer no solution to this dilemma.}
Analysis Challenges

• Target is not pure $^{27}$Al
  – Must correct for 8 other elements, total dilution 5.4%

• Spectrometer was designed for $\text{H}_2$
  – has a large momentum acceptance 150 MeV wide
  – Accepts non-elastic processes which dilute msrd $A_{PV}$
  – Have to also correct for
    • Nuclear excited states
    • GDR
    • Inelastic ($N \rightarrow \Delta$)
    • Quasielastic events
**Excited State Asymmetries (W.u.-hoo!)**

- **Nominally, Born approx.:**
  \[ A_{PV} \approx \frac{G_F Q^2 Q_W}{4\pi\alpha\sqrt{2}} \left( Q_W^p + \frac{N}{Z} Q_W^n \right) \approx 2.5 \text{ ppm} \]

  - **Sign** of \( A_{PV} \) depends on whether state is isoscaler/collective (+) or isovector (+ or -)
  - Shell model: \(^{27}\text{Al} (J^\pi=5/2^+ \text{ g.s.}) = 1d5/2 \) p-hole coupled to the 0\(^+\), 2\(^+\), and 4\(^+\) states in \(^{28}\text{Si}\)
  - Weisskopf units (W.u.'s) (~ E2 transition strength):
    - \( \sim \) # of nucleons participating in transition
    - W.u. \( \lesssim 1 \) → single-particle (isovector) state
    - W.u. > 1 → isoscalar/collective state
  - Wu > 1 (or excited in \( \Delta T=0 \) (\( \alpha,\alpha \)) transitions →
    \( A_{PV} = +2.5 \text{ ppm} \pm 50\% \text{ error} \)
  - Wu \( \lesssim 1 \) (which could be p or n s.p. states) →
    \( A_{PV} = +2.5 \text{ ppm} \pm 200\% \text{ error} \)
    - to cover potential contributions to \( A_{PV} \) of the opposite sign

- **Results:**\[
\begin{align*}
  f_{\text{nucl}} & : 3.83 \pm 0.23 \% \\
  A_{\text{nucl}} & : 2.58 \pm 1.40 \text{ ppm}
\end{align*}
\]

**References:**
- M. Basunia, Nuclear Data Sheets 112, 1875 (2011)
PVES Elastic 27Al Asymmetry Result

\[ A_{PV} = 2.16 \pm 0.19 \text{ ppm} \]
\[ [\pm 0.11 \text{ (stat)} \pm 0.16 \text{ (syst)}] \]

\[ Q^2 = 0.02357 \pm 0.00010 \text{ GeV}^2 \]
\[ <\theta_{lab}> = 7.61^\circ \pm 0.02^\circ \]
\[ <E_{lab}> = 1.157 \text{ GeV} \]

Dominant Systematics (\(\Delta A/A \%)\):
- \(f_{QE}\) 5.0%
- \(A_{QE}\) 2.4%
- \(A_{nucl}\) 3.6%
- \(A_{inel}\) 2.6%
- \(A_{alloy}\) 2.1%

Total systematics: 7.6%
Statistics: 5.1%

Agreement of predictions with our \(A_{PV}\) result is good!
Many-models correlation plot from C. Horowitz, F. Fattoyev & Z. Lin

RMF models tuned to reproduce binding energies, charge radii, GDR strengths, etc. in different nuclei

Test case using EW method to get \( R_n \) on \(^{27}\)Al passes!
PVES Elastic $^{27}$Al Weak Form Factor

\[ A_{PV} = \frac{\sigma_+(\theta) - \sigma_-(\theta)}{\sigma_+(\theta) + \sigma_-(\theta)} \approx \frac{G_F Q^2 Q_W F_W(Q^2)}{4\pi\alpha Z \sqrt{2} F_{EM}(Q^2)} \]

- Measure elastic $^{27}$Al $A_{PV} = 2.16 \pm 0.19$ ppm
- Calculate $Q_W^{(27\text{Al})} = -12.92 \pm 0.014$
  - using fully radiated formula in PDG EW Review
- Calculate $F_{EM}(Q^2=0.02357 \text{ GeV}^2) = 0.384 \pm 0.012$
  - Following K. Mesick (nee Meyers) thesis:
    - used method described by Stovall, Vinciguerra, & Bernheim, NPA 91, 513 (1967)
    - checked to 3% using xsec/FF data from Li, Yearian & Sick, PRC 9, 1861 (1974)
- Plug in to Born (tree-level) expression to get
  \[ F_W^{(27\text{Al}, Q^2=0.02357 \text{ GeV}^2)} = 0.393 \pm 0.038 \]
**Born (tree-level) for weak Observables**

- DWBA accurately predicts our msrd $A_{PV}$
- We have a 9.1% result for our $^{27}$Al $A_{PV}$
  - radiative effects typically $\sim$1%
  - Note: our determination of $Q_w(\text{}^{27}\text{Al}) = -12.92 \pm 0.01$ fully radiated
- $Z=13$ means less Coulomb distortion ($\propto Z$)
  - relative to $^{208}\text{Pb}$ ($Z=82$)
- Followed Koshchii et al, PRC 102, 022501 (2020), “Weak charge & weak radius of $^{12}\text{C}$”:

<table>
<thead>
<tr>
<th>TABLE III. Derived $^{27}\text{Al}$ Observables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observable</td>
</tr>
<tr>
<td>$R_n$</td>
</tr>
<tr>
<td>$R_n - R_p$</td>
</tr>
<tr>
<td>$F_{wk}(Q^2 = 0.0236 \text{ GeV}^2)$</td>
</tr>
<tr>
<td>$\Delta = ZA_{PV}/(A_0Q_w) - 1$</td>
</tr>
<tr>
<td>$R_{wskin} = -3\Delta/(Q^2R_{ch})$</td>
</tr>
<tr>
<td>$R_{wk} = R_{wskin} + R_{ch}$</td>
</tr>
<tr>
<td>$\lambda \equiv (R_{wk} - R_{ch})/R_{ch}$</td>
</tr>
</tbody>
</table>
Beam polarization orientation:

- **Longitudinal** → PV asymmetries $A_{PV} \to Q_w^p$
- **Transverse** (Vertical or Horizontal) → PC asymmetries $B_n$ or BNSSA

$B_n = 0$ in OPE

$B_n \neq 0 \to$ TPE (Im(TPE))

TPE is leading explanation for proton FF puzzle ($LT vs PT G_E^p \;/ G_M^p$)

Test predictions of Im(TPE) by comparing to $B_n$

$B_n$ manifests itself as the amplitude of an azimuthal variation of the asymmetry when beam is polarized transverse to its incident p
Effect of Coulomb Distortions on the Optical Model Calculations of Gorchtein et al.

Without Coulomb distortions

With Coulomb distortions

With Coulomb distortions

Without Coulomb distortions

12C

27Al

PRC104, 014606 (2021)

PRC103, 064316 (2021)

PREX 208Pb

PREX 12C

MAMI
**Scaling:** \( B_n = kQ A/Z \)

Scaling: PRL 109, 192501 (2012)

- **Red:** Qweak data on \(^1\text{H}, ^{12}\text{C}, ^{27}\text{Al}\) (\(Q=0.157\) GeV)
- **Symbols:**
  - Solid: \( \theta \lesssim 10^\circ \)
  - Open: \( \theta \gtrsim 10^\circ \)
- **Blue curve is** \(kA/Z\), with \(k = -30\) ppm/GeV (and \(Q=0.157\) GeV)
- **PREX** \(^{208}\text{Pb}\) datum an outlier with other data, scaling, & theory - not understood why:
  - Coulomb distortions? No. See Koshchii et al., PRC103, 064316 (2021)
  - “nuclear” region of the photoabsorption \(x\)-sec?
Scaling: $B_n = kQ A/Z$

$kQ = Z/A \ B_n$

- $Q > 0.35 \text{ GeV} \ ^1\text{H} \ \text{data require a quadratic term}$
- Mainz $\theta \gtrsim 10^\circ \ \text{data have twice the slope of other data with } \theta \lesssim 10^\circ$

$k = Z/(AQ) \ B_n$

- All the far-forward angle ($\theta < 10^\circ$) data from $^1\text{H}$ to $^{27}\text{Al}$ can be described by the same slope out to $Q \sim 0.35 \text{ GeV}$.
- PREX $^{208}\text{Pb}$ datum still an outlier
Summary

• Elastic PVES $A_{PV}$ on $^{27}$Al: \texttt{arXiv:2112.15412} [nucl-ex], submitted to PRL
  – First msr of $^{27}$Al $A_{PV}$
  – DWBA predictions accurately predict msrd $A_{PV}$
  – n-skin $\sim 0$ as expected for N$\sim$Z $^{27}$Al
  – Successful benchmark of EW method used to get $R_n$
    • Despite tension noted in EW $R_n$ of $^{208}$Pb

• Elastic BNSSA on $^{12}$C & $^{27}$Al: Phys. Rev. C 104, 014606 (2021)
  – First $B_n$ on $^{27}$Al
  – Qweak ($^{1}$H, $^{12}$C, & $^{27}$Al) & all other $\theta \lesssim 10^\circ$ data consistent with Gorchtein TPE calculations:
    • Except $^{208}$Pb
    • Agreement with Mainz $\theta > 10^\circ$ $^{12}$C & $^{28}$Si results fails with Coulomb corrections
  – Empirically: Q-dependence of all $^{1}$H $B_n Z/A$ data, & all the $\theta < 10^\circ$, $A > 1$ data can be described by the same slope out to $Q = 0.35$ GeV
    • $Q > 0.35$ GeV data have a higher-order Q-dependence
    • Larger-angle ($\theta > 10^\circ$) data from Mainz $^{12}$C & $^{28}$Si consistent with a slope $\sim 2^*$ steeper
    • $^{208}$Pb an outlier
Thank you!

The Qweak Collaboration

101 collaborators 27 grad students
10 post docs 23 institutions

Institutions:
1 University of Zagreb
2 College of William and Mary
3 A. I. Alikhanyan National Science Laboratory
4 Massachusetts Institute of Technology
5 Thomas Jefferson National Accelerator Facility
6 Ohio University
7 Christopher Newport University
8 University of Manitoba
9 University of Virginia
10 TRIUMF
11 Hampton University
12 Mississippi State University
13 Virginia Polytechnic Institute & State Univ
14 Southern University at New Orleans
15 Idaho State University
16 Louisiana Tech University
17 University of Connecticut
18 University of Northern British Columbia
19 University of Winnipeg
20 George Washington University
21 University of New Hampshire
22 Hendrix College, Conway
23 University of Adelaide


Spokespersons: F.J. Fattoyev, and Z. Lin

C.J. Horowitz, F.J. Fattoyev, and Z. Lin
BACKUP SLIDES
• Commissioning result: PRL 111, 141803 (2013)
• Apparatus: NIM A781, 105 (2015)
• Final $Q_W^p$ result & SM test: Nature 557, 207 (2018)
• $Q_W^p$ cookbook & perspectives: ARNS 69, 191 (2019)
• Layman’s description: NPN 29, 15 (2019)
• 3-pass $A_{\text{inel}}$ in resonance region: PRC 101, 055503 (2020)
• $^1$H BNSSA: PRL 125, 112502 (2020):
• $^{12}$C & $^{27}$Al BNSSA: PRC 104, 014606 (2021)
• $^{27}$Al Longitudinal ($A_{P^V, Q_{W}^{27Al}, \delta R_{np}^{27Al}}$): submitted to PRL
• N $\rightarrow$ $\Delta$ BNSSA @ 1160 MeV (Nurruzaman) & 877 MeV (Anna Lee)
  – This & next 2 need elastic radiative tail simulations (new: Devi Adhikari)
• N $\rightarrow$ $\Delta$ Inel $A_{P^V}$ ($d_{\Delta}$) @ 877 MeV (Anna Lee) & 1160 (Leacock, Thamraa, Hend)
• Moller scattering BNSSA
• 27 students/theses, several instrumentation papers
# PVES Elastic 27Al Corrections & Systematic Error Contributions

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>$\Delta A_{PV}/A_{PV}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{msr}$</td>
<td>$1.436 \pm 0.014$ ppm</td>
<td>1.0</td>
</tr>
<tr>
<td>$P$</td>
<td>$0.8880 \pm 0.0055$</td>
<td>0.7</td>
</tr>
<tr>
<td>$R_{tot}$</td>
<td>$0.9855 \pm 0.0087$</td>
<td>0.9</td>
</tr>
<tr>
<td>$f_{QE}$</td>
<td>$21.2 \pm 2.9$ %</td>
<td>5.0</td>
</tr>
<tr>
<td>$A_{QE}$</td>
<td>$-0.34 \pm 0.17$ ppm</td>
<td>2.4</td>
</tr>
<tr>
<td>$f_{nucl}$</td>
<td>$3.83 \pm 0.23$ %</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_{nucl}$</td>
<td>$2.58 \pm 1.40$ ppm</td>
<td>3.6</td>
</tr>
<tr>
<td>$f_{inel}$</td>
<td>$0.665 \pm 0.099$ %</td>
<td>0.2</td>
</tr>
<tr>
<td>$A_{inel}$</td>
<td>$-0.58 \pm 5.83$ ppm</td>
<td>2.6</td>
</tr>
<tr>
<td>$f_{alloy}$</td>
<td>$5.41 \pm 0.34$ %</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_{alloy}$</td>
<td>$1.90 \pm 0.58$ ppm</td>
<td>2.1</td>
</tr>
<tr>
<td>$f_{pions}$</td>
<td>$0.06 \pm 0.06$ %</td>
<td>0.1</td>
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<tr>
<td>$A_{pions}$</td>
<td>$0 \pm 20$ ppm</td>
<td>0.8</td>
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<tr>
<td>$f_{neutral}$</td>
<td>$0 \pm 0.45$ %</td>
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<tr>
<td>$A_{neutral}$</td>
<td>$1.7 \pm 0.2$ ppm</td>
<td>0.0</td>
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<tr>
<td>$f_{beamline}$</td>
<td>$0.69 \pm 0.06$ %</td>
<td>0.1</td>
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<tr>
<td>$f_{GDR}$</td>
<td>$0.045 \pm 0.023$ %</td>
<td>0.1</td>
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<tr>
<td>$A_{GDR}$</td>
<td>$-2.22 \pm 1.11$ ppm</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Total Systematic</strong></td>
<td><strong>7.6</strong></td>
<td></td>
</tr>
</tbody>
</table>
Astrophysical Tension

Reed, Fattoyev, Horowitz, Piekarewicz, PRL 126, 172503:

- Blue box is overlap of 1st NICER & Prex.
  - 2nd NICER result has less overlap, as indicated by lower purple horizontal datum
- Original LIGO tidal deformability $\Lambda_{1.4} < 800$ is consistent (PRL 119, 161101 (2017))
- New GW170817 tidal deformability result is “more challenging”:
  $\Lambda_{1.4} = 190^{+390}_{-120} \lesssim 580$ (red line, grey box)
  - Favors models with smaller $^{208}$Pb skin, more compatible with non-EW results than Prex
- However, given the existing uncertainties, there are no gross discrepancies between any of these results
27Al QE & Inel dilution Corrections

- QE & Inel $f_i$ : dilutions from simulations using a generator based on phenomenological fits from Bosted/Mamyan, later scaled to Christy’s fits
  - HUGE improvement over Bosted/Mamyan at (our) low $Q^2$!

- Inel $A_i$ scaled from brief msrmnt at low spect. B:

<table>
<thead>
<tr>
<th></th>
<th>$f_i$ (%)</th>
<th>$A_i$ (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QE</td>
<td>21.2 ± 2.9</td>
<td>-0.34 ± 0.17</td>
</tr>
<tr>
<td>Inel (N→∆)</td>
<td>0.665 ± 0.99</td>
<td>-0.58 ± 5.83</td>
</tr>
</tbody>
</table>

“MEC” strength is bogus in B-M model at low $Q^2$
Qweak Ancillary Results

From systematic studies made to support our primary ep $A_{PV}$ result

- PV ep $A_{inel}$ above the resonance region

- Elastic $^1$H BNSSA

- Elastic $^{12}$C & $^{27}$Al BNSSA
  - M.J. McHugh & K. Bartlett theses

- Inelastic ep→e′Δ BNSSA
  - Nuruzzaman thesis
  - Elastic $A_{PV}^{^{27}Al}$, $Q_{W}^{^{27}Al}$, $\delta R_{np}^{^{27}Al}$ submitted to PRL
    - K. Bartlett thesis

- Inelastic ep→e′Δ $A_{PV}$
  - A. Lee, H. Nuhait, T. AlShayeb theses
Beam Helicity slow reversals:
- Insertable half-wave plate in the polarized source every ~8h
- Reversals of the double Wien spin filter in injector every ~month
- 1 pass → 2 pass → 1 pass: g-2 flip

- Asymmetry well-behaved under 3 kinds of slow helicity reversal
- Corrections for HC beam properties small: 0.4 ± 1.4 ppb
Beam Normal Single Spin Asymmetry in $\Delta$ Resonance

Q-weak has measured Beam Normal Single Spin Asymmetry ($B_n$) in the N-to-$\Delta$ transition on H$_2$

$$B_n = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\downarrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\downarrow\downarrow}} = \frac{2\text{Im}(T_{1\gamma} \times T_{2\gamma})}{|T_{1\gamma}|^2}$$

After correcting for polarization and backgrounds

$$B_n = 43 \pm 16 \text{ ppm}$$

- $<E> = 1.16 \text{ GeV}$
- $<W> = 1.2 \text{ GeV}$
- $<\theta> = 8.3^\circ$
- $<Q^2> = 0.021 \text{ GeV}^2$

- Unique tool to study $\gamma^*\Delta\Delta$ form factors
- Q-weak along with world data has potential to constrain models and study charge radius and magnetic moment of $\Delta$
PV ep $A_{inel}$ above the resonance region

- Helps validate modeling of the $\gamma Z$ interference structure functions $F_1^{\gamma Z}$ & $F_2^{\gamma Z}$, used for determination of the two-boson exchange $\gamma Z$ box diagram contribution to PV elastic scattering measurements.
- A positive PV asymmetry for inclusive $\pi^-$ production was observed, as well as a positive BNSSA for scattered electrons, and a negative BNSSA for inclusive $\pi^-$ production.

**Kinematics:**
- $<E> = 3.35$ GeV  $<W> = 2.23$ GeV
- $<Q^2> = 0.082$ GeV$^2$
- $<P_{mixed}> = 0.870 \pm 0.006$, but mixed 94% (long) & 34% (hor)

**Special Corrections:**
- $e/\pi/\mu/\gamma/n$ fraction (higher $E \rightarrow$ more $\pi$’s)
- 3 GeV elastics punch-thru shieldwall designed for 1 GeV elastics

$A_{inel} = -13.5 \pm 4.4$ ppm
BNSSA Backup Slides
<table>
<thead>
<tr>
<th>Expt</th>
<th>A</th>
<th>( \theta(\text{lab}) ) (deg)</th>
<th>( E(\text{lab}) ) (GeV)</th>
<th>( Q ) (GeV)</th>
<th>( B_n ) (ppm)</th>
<th>( \Delta B_n ) (ppm)</th>
<th>Fitting Group</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>33.9</td>
<td>0.3151</td>
<td>0.179</td>
<td>-2.220</td>
<td>0.587</td>
<td>1,1a</td>
<td>[25]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>34.1</td>
<td>0.5102</td>
<td>0.286</td>
<td>-9.320</td>
<td>0.884</td>
<td>1,1a</td>
<td>[25]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>34.1</td>
<td>0.8552</td>
<td>0.467</td>
<td>-7.460</td>
<td>1.973</td>
<td>1</td>
<td>[25]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>34.3</td>
<td>0.4202</td>
<td>0.239</td>
<td>-6.880</td>
<td>0.676</td>
<td>1,1a</td>
<td>[25]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>34.1</td>
<td>1.5084</td>
<td>0.783</td>
<td>-0.060</td>
<td>3.459</td>
<td>1</td>
<td>[25]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>35.0</td>
<td>0.5693</td>
<td>0.326</td>
<td>-8.590</td>
<td>1.164</td>
<td>1,1a</td>
<td>[24]</td>
</tr>
<tr>
<td>A4</td>
<td>(^1\text{H})</td>
<td>35.3</td>
<td>0.8552</td>
<td>0.480</td>
<td>-8.520</td>
<td>2.468</td>
<td>1</td>
<td>[24]</td>
</tr>
<tr>
<td>G0</td>
<td>(^1\text{H})</td>
<td>7.5</td>
<td>3.0310</td>
<td>0.387</td>
<td>-4.060</td>
<td>1.173</td>
<td>1</td>
<td>[22]</td>
</tr>
<tr>
<td>G0</td>
<td>(^1\text{H})</td>
<td>9.6</td>
<td>3.0310</td>
<td>0.500</td>
<td>-4.820</td>
<td>2.111</td>
<td>1</td>
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</tr>
<tr>
<td>Qweak</td>
<td>(^1\text{H})</td>
<td>7.9</td>
<td>1.1490</td>
<td>0.157</td>
<td>-5.194</td>
<td>0.106</td>
<td>1,1a</td>
<td>[27]</td>
</tr>
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<td>HAPPEX</td>
<td>(^1\text{H})</td>
<td>6.0</td>
<td>3.0260</td>
<td>0.310</td>
<td>-6.800</td>
<td>1.540</td>
<td>1,1a</td>
<td>[33]</td>
</tr>
<tr>
<td>HAPPEX</td>
<td>(^{4}\text{He})</td>
<td>6.0</td>
<td>2.7500</td>
<td>0.280</td>
<td>-13.970</td>
<td>1.450</td>
<td>1,1a</td>
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</tr>
<tr>
<td>A1</td>
<td>(^{12}\text{C})</td>
<td>15.1</td>
<td>0.5700</td>
<td>0.152</td>
<td>-15.984</td>
<td>1.252</td>
<td>2</td>
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<tr>
<td>A1</td>
<td>(^{12}\text{C})</td>
<td>17.7</td>
<td>0.5700</td>
<td>0.173</td>
<td>-20.672</td>
<td>1.106</td>
<td>2</td>
<td>[34]</td>
</tr>
<tr>
<td>A1</td>
<td>(^{12}\text{C})</td>
<td>20.6</td>
<td>0.5700</td>
<td>0.202</td>
<td>-21.933</td>
<td>2.219</td>
<td>2</td>
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<tr>
<td>A1</td>
<td>(^{12}\text{C})</td>
<td>23.5</td>
<td>0.5700</td>
<td>0.197</td>
<td>-23.877</td>
<td>1.225</td>
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<tr>
<td>A1</td>
<td>(^{12}\text{C})</td>
<td>25.9</td>
<td>0.5700</td>
<td>0.221</td>
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<td>1.480</td>
<td>2</td>
<td>[34]</td>
</tr>
<tr>
<td>PREX</td>
<td>(^{12}\text{C})</td>
<td>5.0</td>
<td>1.0630</td>
<td>0.099</td>
<td>-6.490</td>
<td>0.380</td>
<td>1,1a</td>
<td>[33]</td>
</tr>
<tr>
<td>Qweak</td>
<td>(^{12}\text{C})</td>
<td>7.9</td>
<td>1.1580</td>
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<td>-10.680</td>
<td>1.065</td>
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<tr>
<td>Qweak</td>
<td>(^{27}\text{Al})</td>
<td>7.9</td>
<td>1.1580</td>
<td>0.154</td>
<td>-12.160</td>
<td>0.849</td>
<td>1,1a</td>
<td>-</td>
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<tr>
<td>A1</td>
<td>(^{28}\text{Si})</td>
<td>19.4</td>
<td>0.5700</td>
<td>0.190</td>
<td>-21.807</td>
<td>1.480</td>
<td>2</td>
<td>[35]</td>
</tr>
<tr>
<td>A1</td>
<td>(^{28}\text{Si})</td>
<td>23.5</td>
<td>0.5700</td>
<td>0.195</td>
<td>-23.302</td>
<td>1.470</td>
<td>2</td>
<td>[35]</td>
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<tr>
<td>A1</td>
<td>(^{90}\text{Zr})</td>
<td>20.7</td>
<td>0.5700</td>
<td>0.205</td>
<td>-16.787</td>
<td>5.688</td>
<td>2</td>
<td>[35]</td>
</tr>
<tr>
<td>A1</td>
<td>(^{90}\text{Zr})</td>
<td>23.5</td>
<td>0.5700</td>
<td>0.205</td>
<td>-17.033</td>
<td>3.848</td>
<td>2</td>
<td>[35]</td>
</tr>
<tr>
<td>PREX</td>
<td>(^{208}\text{Pb})</td>
<td>5.0</td>
<td>1.0630</td>
<td>0.094</td>
<td>0.280</td>
<td>0.250</td>
<td>-</td>
<td>[33]</td>
</tr>
</tbody>
</table>
## 12C & 27Al BNSSA Corrections

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value $^{12}$C</th>
<th>Value $^{27}$Al</th>
<th>$\Delta B_n/B_n$ (%) $^{12}$C</th>
<th>$\Delta B_n/B_n$ (%) $^{27}$Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$: Beam Polarization</td>
<td>0.8852 ± 0.0068</td>
<td>0.8872 ± 0.0070</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_{\text{tot}}$: Kinematics &amp; Radiative effects</td>
<td>1.0054 ± 0.0046</td>
<td>1.0054 ± 0.0046</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$R_{\text{av}}$: Acceptance averaging</td>
<td>0.9862 ± 0.0036</td>
<td>0.9862 ± 0.0036</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$R_{\ell}$: Electronic non-linearity</td>
<td>1.0014 ± 0.0050</td>
<td>1.0014 ± 0.0050</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$B_{\text{fit}}$: Fitting</td>
<td>0 ± 0.042 ppm</td>
<td>0 ± 0.050 ppm</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$B_{\text{reg}}$: Linear Regression</td>
<td>0 ± 0.002 ppm</td>
<td>0 ± 0.020 ppm</td>
<td>&lt; 0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$B_{\text{bias}}$: Rescattering Bias</td>
<td>0.125 ± 0.041 ppm</td>
<td>0.125 ± 0.041 ppm</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$f_{\text{neutral}}$:</td>
<td>0.69 ± 0.45 %</td>
<td>0.69 ± 0.45 %</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>$B_{\text{neutral}}$:</td>
<td>0 ± 10 ppm</td>
<td>0 ± 10 ppm</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>$f_{\text{alloy}}$:</td>
<td>—</td>
<td>5.41 ± 0.34 %</td>
<td>—</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>$B_{\text{alloy}}$:</td>
<td>—</td>
<td>−10.7 ± 2.0 ppm</td>
<td>—</td>
<td>1.3</td>
</tr>
<tr>
<td>$f_{\text{QE}}$:</td>
<td>15.9 ± 2.2 %</td>
<td>21.2 ± 2.9 %</td>
<td>1.5</td>
<td>2.4</td>
</tr>
<tr>
<td>$B_{\text{QE}}$:</td>
<td>−5.2 ± 1.0 ppm</td>
<td>−5.2 ± 1.0 ppm</td>
<td>2.0</td>
<td>2.6</td>
</tr>
<tr>
<td>$f_{\text{inel}}$:</td>
<td>0.40 ± 0.06 %</td>
<td>0.66 ± 0.10 %</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>$B_{\text{inel}}$:</td>
<td>43 ± 16 ppm</td>
<td>43 ± 16 ppm</td>
<td>0.8</td>
<td>1.3</td>
</tr>
<tr>
<td>$f_{\text{nucl}}$:</td>
<td>4.71 ± 0.31 %</td>
<td>3.88 ± 0.23 %</td>
<td>&lt; 0.1</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>$B_{\text{nucl}}$:</td>
<td>−10.5 ± 10.5 ppm</td>
<td>−12 ± 5.5 ppm</td>
<td>3.9</td>
<td>2.6</td>
</tr>
<tr>
<td>Total Systematic</td>
<td>5.3 %</td>
<td>5.2 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**BNSSA Empirical Fits**

### TABLE VII. Fit results.

<table>
<thead>
<tr>
<th>Group</th>
<th>Linear ($\vec{B}_n$) (ppm/GeV)</th>
<th>Quadratic ($\beta$) (ppm/GeV$^2$)</th>
<th># data</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-41.1 \pm 1.1$</td>
<td>56.0 $\pm 4.8$</td>
<td>15</td>
<td>4.4</td>
</tr>
<tr>
<td>1a</td>
<td>$-31.8 \pm 0.5$</td>
<td>$-$</td>
<td>10</td>
<td>6.4</td>
</tr>
<tr>
<td>2</td>
<td>$-58.3 \pm 1.4$</td>
<td>$-$</td>
<td>9</td>
<td>2.0</td>
</tr>
</tbody>
</table>
\[ B_n = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} = \frac{2 \text{Im} \left( M_{\gamma \gamma} M_{\gamma}^* \right)}{|M_{\gamma}|^2} \]

\[ A_{\text{exp}}(\phi) \approx B_n \vec{P} \cdot \hat{n} \]

\[ R_l R_{\text{av}} B_{\text{exp}} \sin(\phi_s - \phi_i + \phi_{\text{off}}) + C \]

\[ B_n = R_{\text{tot}} \left[ \frac{B_{\text{exp}}}{P} - \sum_i f_i B_i \right] + B_{\text{bias}} \]

\[ B_n = -5.19 \pm 0.07 \text{ (stat)} \pm 0.08 \text{ (syst) ppm} \]

Pasquini & Vanderhagen: model intermediate hadronic state (VVCS) with electro-absorption amplitudes. Limited to \( \pi N \) states only (bad), but should apply at all angles (good).

Afanasev & Merenkov, and Gorchtein: use the optical theorem to relate the VVCS amplitude to the total photo-absorption \( \sigma \). Includes all intermediate states (good), but only strictly valid in the forward-angle limit (bad).
Predictions (open squares) at different kinematics from each group are connected by solid (Gorchtein), dashed (Pasquini & Vanderhagen) & dash-dot (Afanasev & Merenkov) lines to guide the eye.

Agreement of predictions with the far-forward angle ($\theta<10^\circ$) data (solid symbols) is better than for the $\theta>10^\circ$ data (open symbols).
12C & 27Al BNSSA Data

27Al Horizontal IHWP Uncorrected

Final Bn Results after all corrections are ~ 10%