

**MICHAEL PAOLONE**

NEW MEXICO STATE UNIVERSITY

FOR THE E05-110 COLLABORATION.

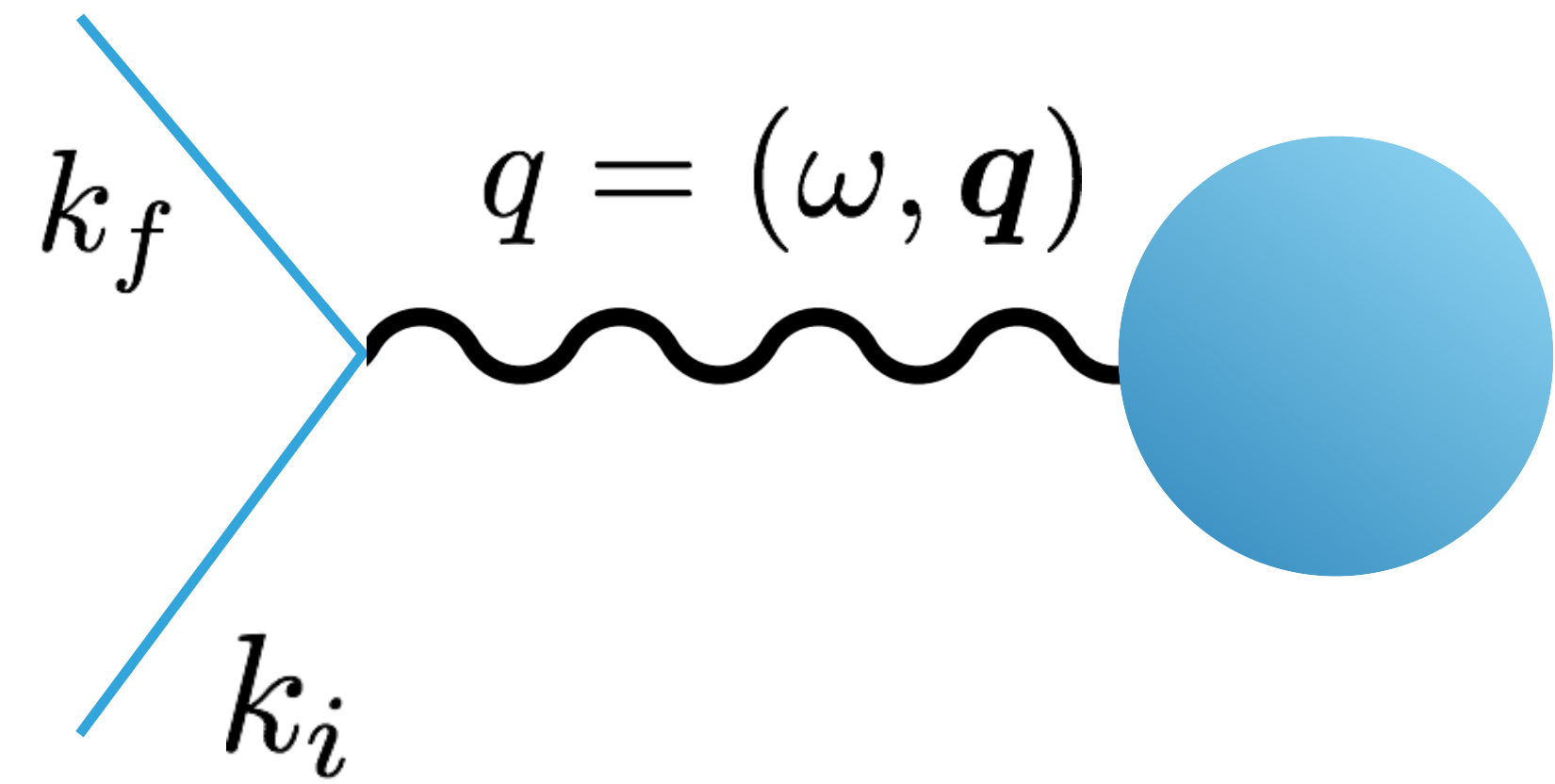
# THE COULOMB SUM RULE IN NUCLEI



# COULOMB SUM RULE

Inclusive electron scattering cross-section:

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[ \frac{q^4}{|\mathbf{q}|^4} R_L(\omega, |\mathbf{q}|) + \left( \frac{q^2}{2|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, |\mathbf{q}|) \right]$$



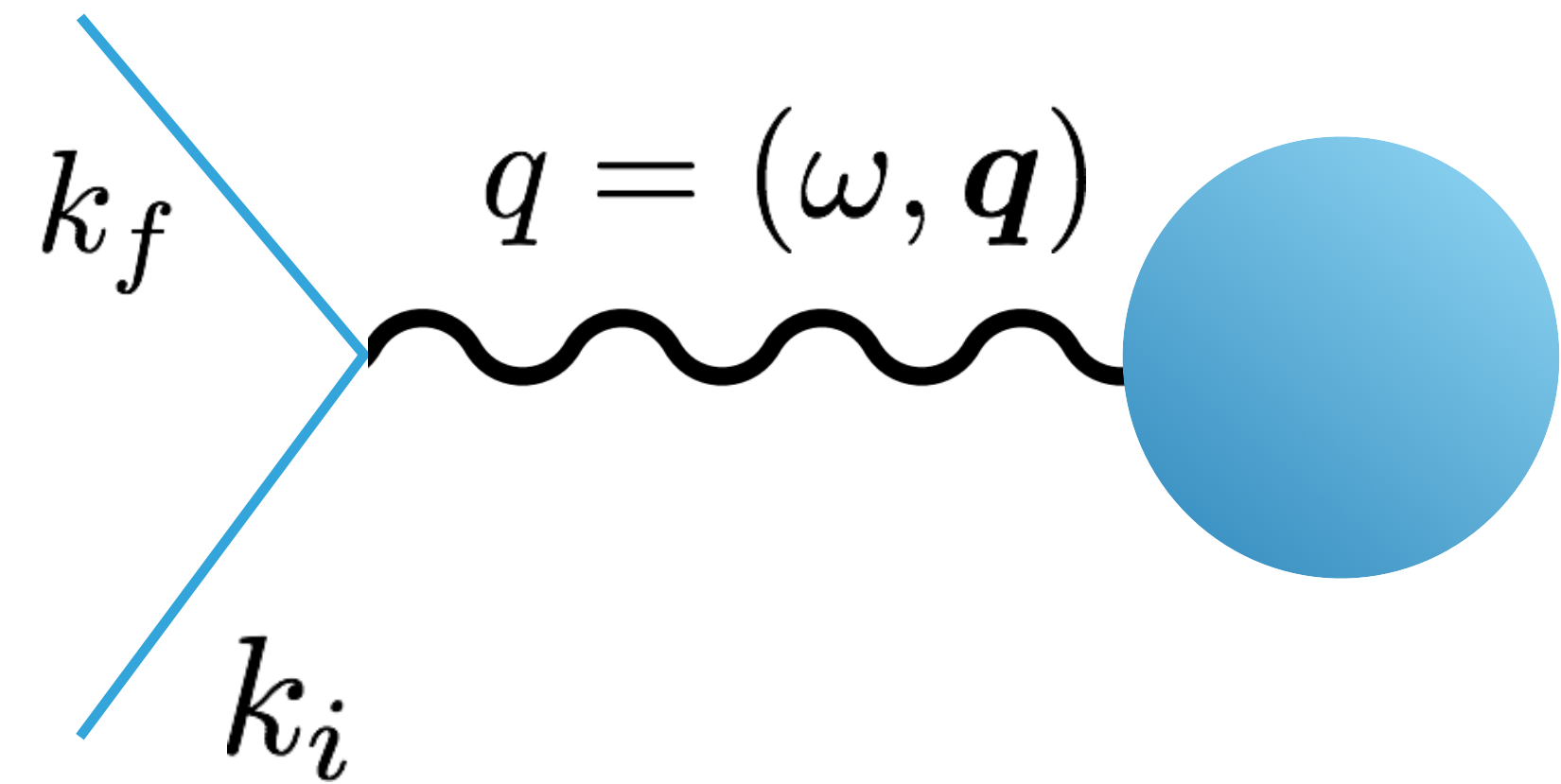
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due to **charge** properties

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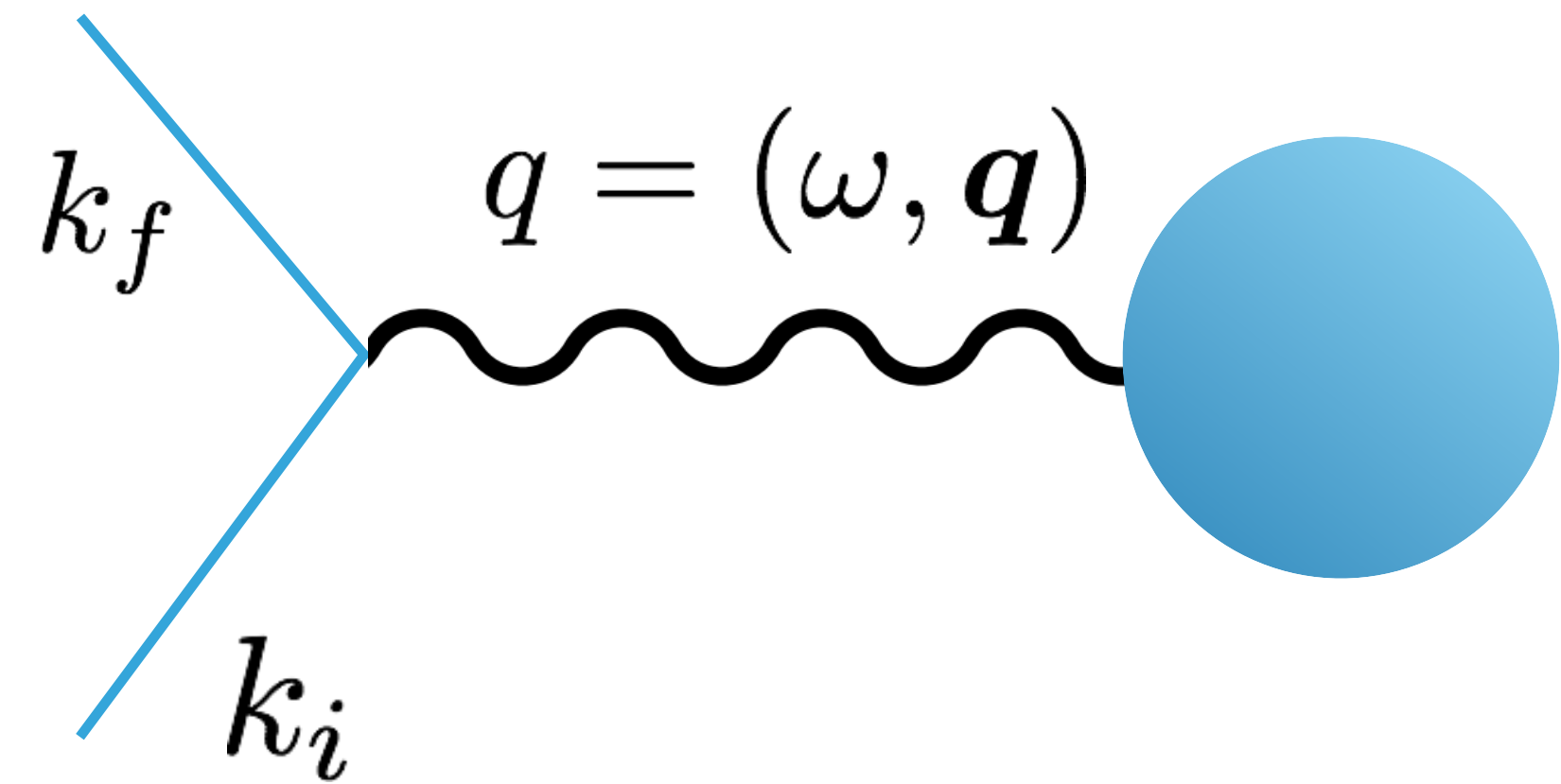
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Coulomb Sum Rule definition:

$$S_L(|\mathbf{q}|) = \int_{\omega^+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.





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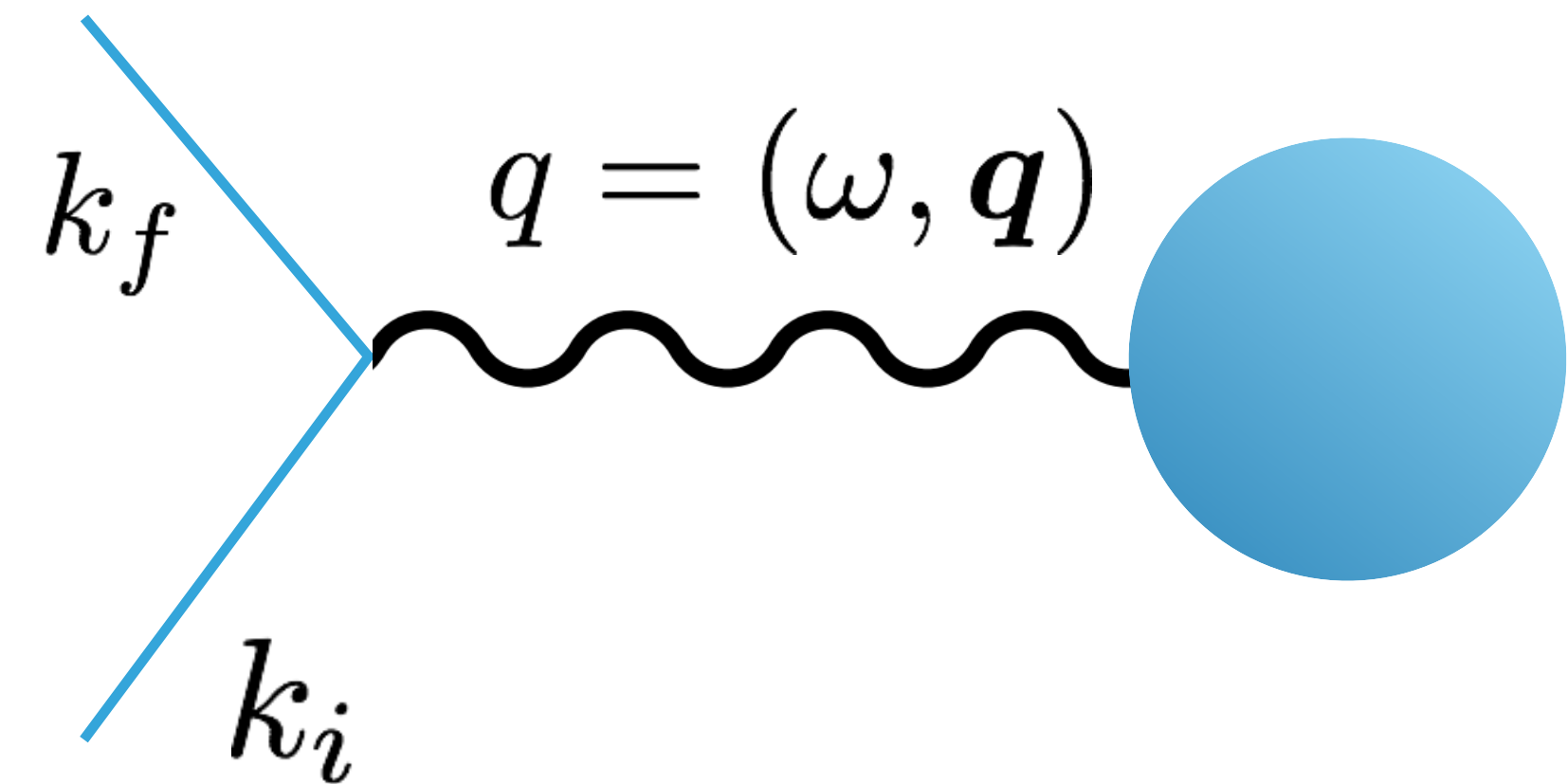
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due to long range nuclear effects, Pauli blocking.  
(directly calculable, well understood).



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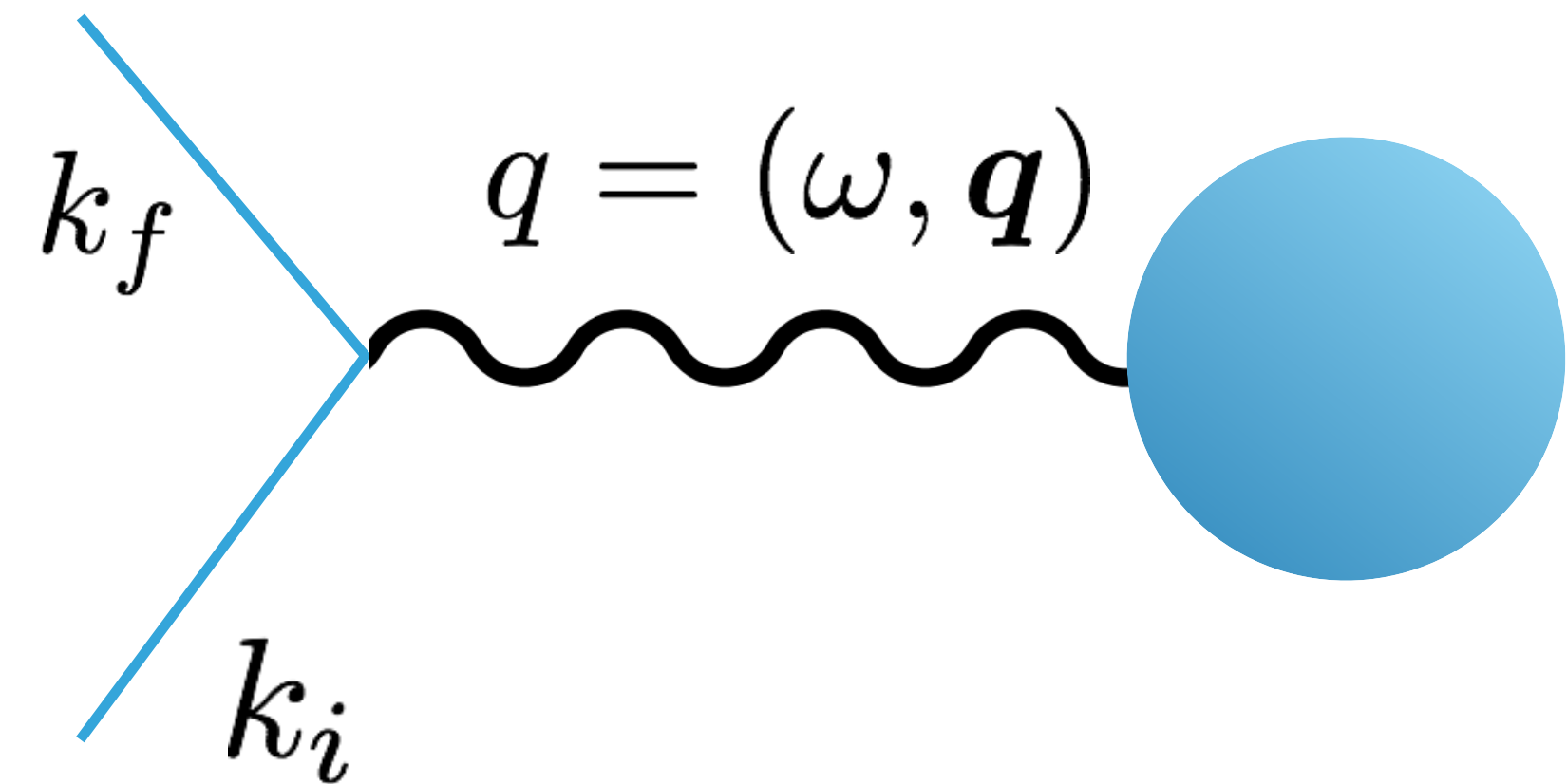
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**At large  $|\mathbf{q}| \gg 2k_f$ ,  $S_L$  should go to 1. Any significant\* deviation from this  
would be an indication of relativistic or medium effects distorting the nucleon form factor!**

\*Short range correlations will also quench  $S_L$ , but only by  $< 10\%$





## THE COULOMB SUM RULE IN NUCLEI

## COULOMB SUM RULE

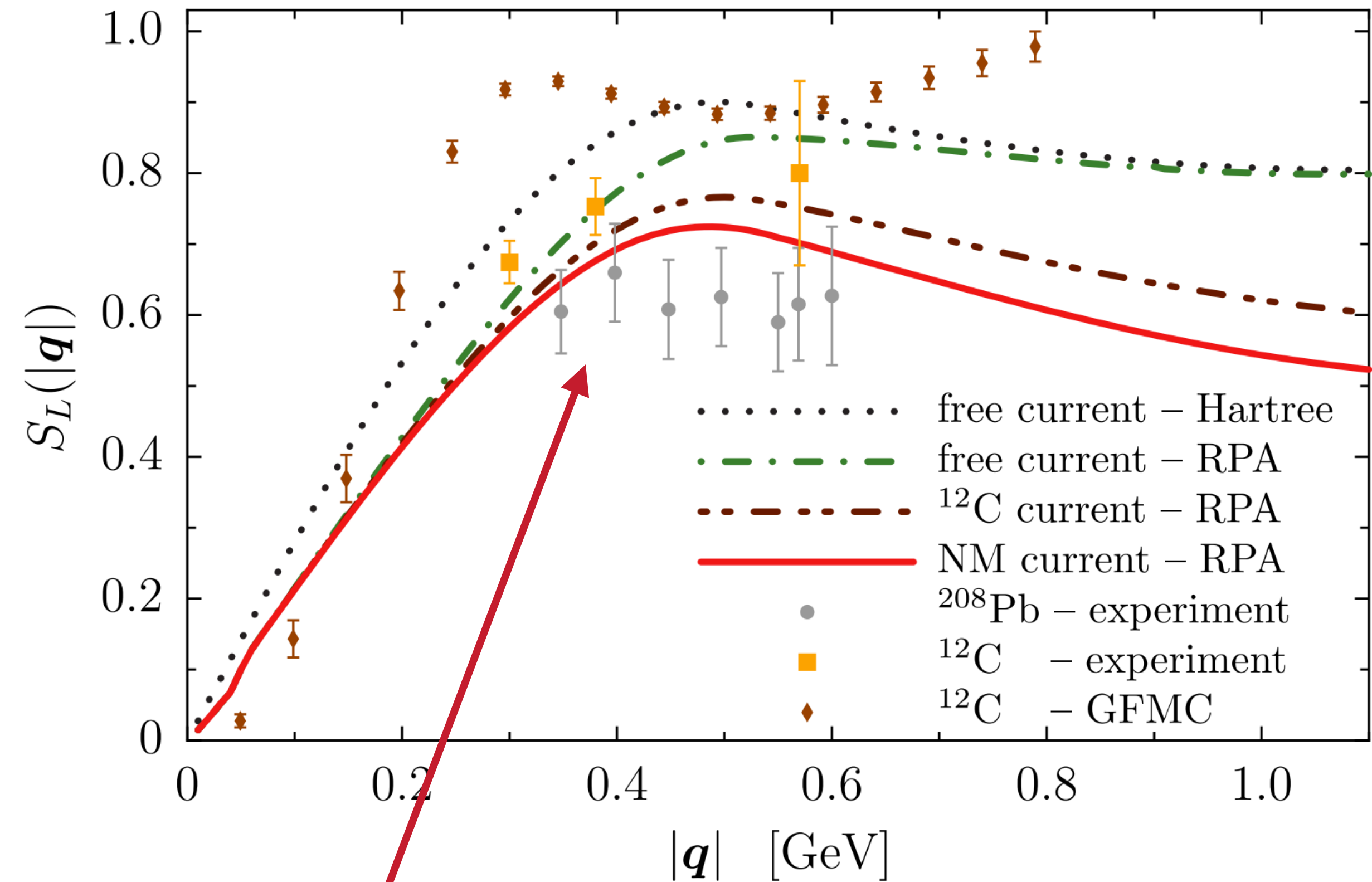
- ▶ Long standing issue with many years of theoretical interest.
- ▶ Even most state-of-the-art models cannot predict existing data.
- ▶ New precise data at larger  $|q|$  would provide crucial insight and constraints to modern calculations.

$$S_L(|\mathbf{q}|) = \int_{\omega^+}^{\omega} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

## Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule

Ian C. Cloët,<sup>1</sup> Wolfgang Bentz,<sup>2</sup> and Anthony W. Thomas<sup>3</sup><sup>1</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA<sup>2</sup>Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan<sup>3</sup>CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, University of Adelaide, Adelaide South Australia 5005, Australia

(Received 23 June 2015; published 19 January 2016)



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# QUASI-ELASTIC SCATTERING

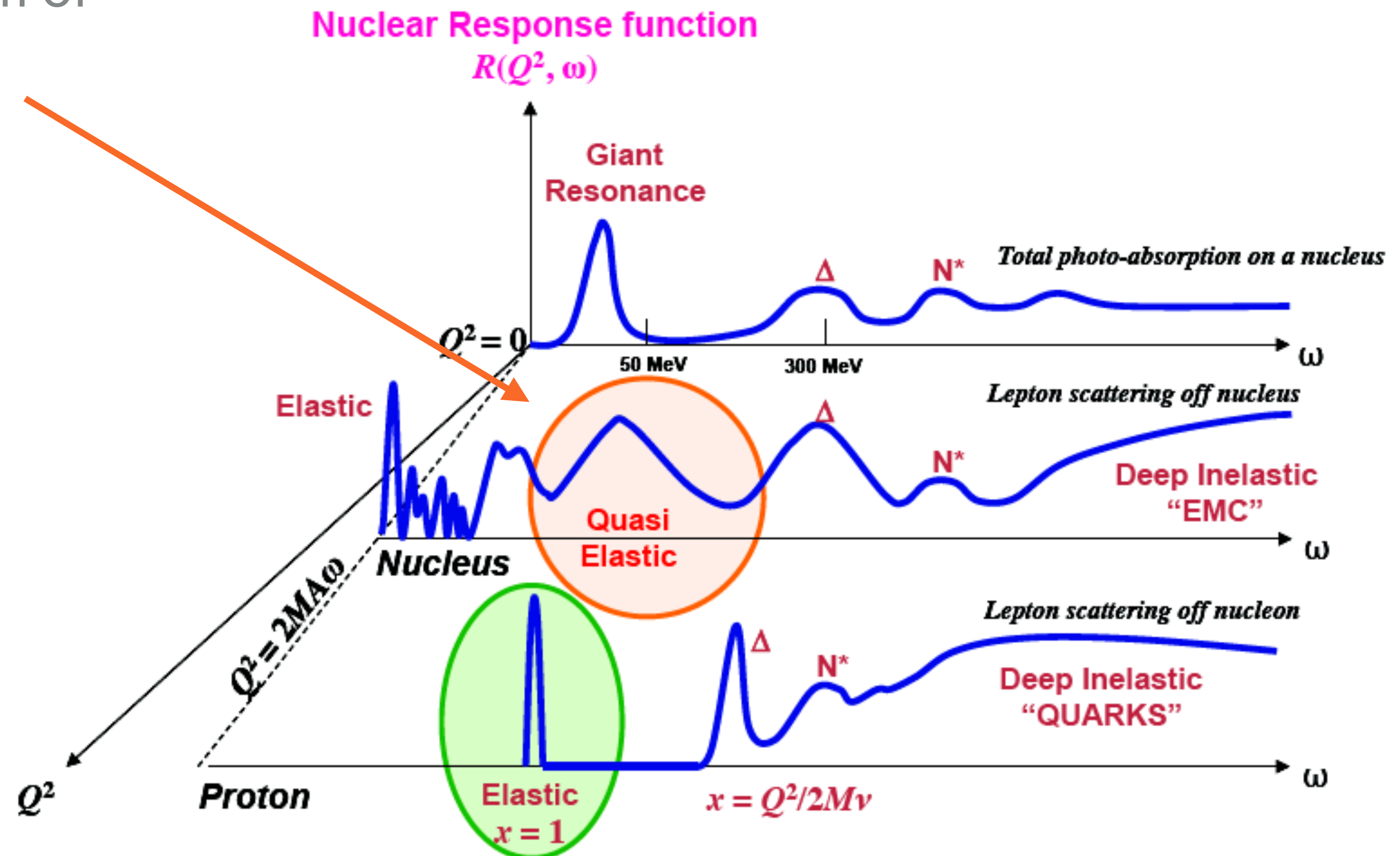
- ▶ Quasi-elastic scattering at intermediate  $Q^2$  is the region of interest for our experiment:

- ▶ Nuclei investigated:

- ▶  $^4\text{He}$
- ▶  $^{12}\text{C}$
- ▶  $^{56}\text{Fe}$
- ▶  $^{208}\text{Pb}$

$$S_L(|\mathbf{q}|) = \int_{\omega_+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

We want to integrate above the coherent elastic peak:  
Quasi-elastic is “elastic” scattering on constituent nucleons inside nucleus.

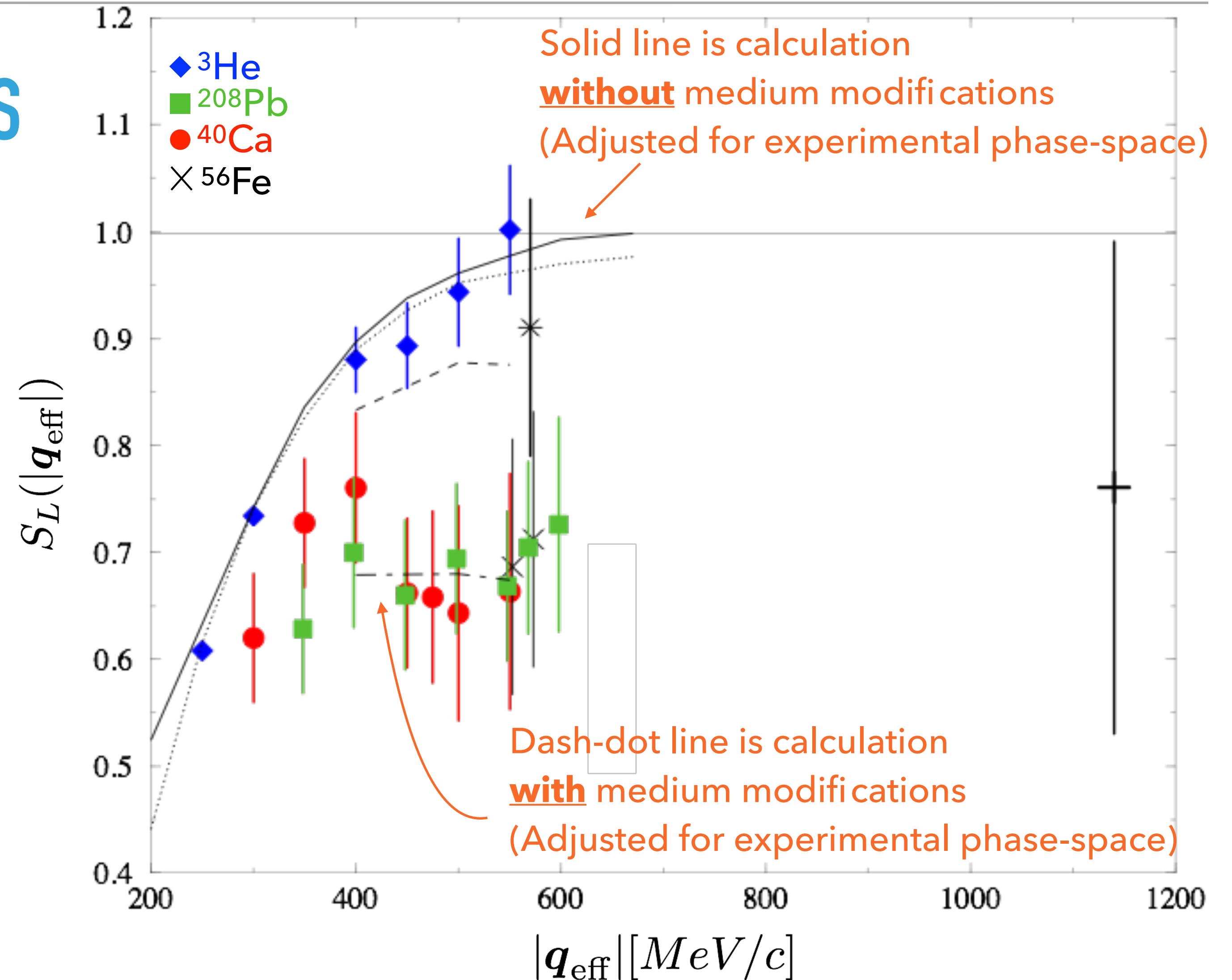




## PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of  $S_L$  consistent with medium modified form-factors.

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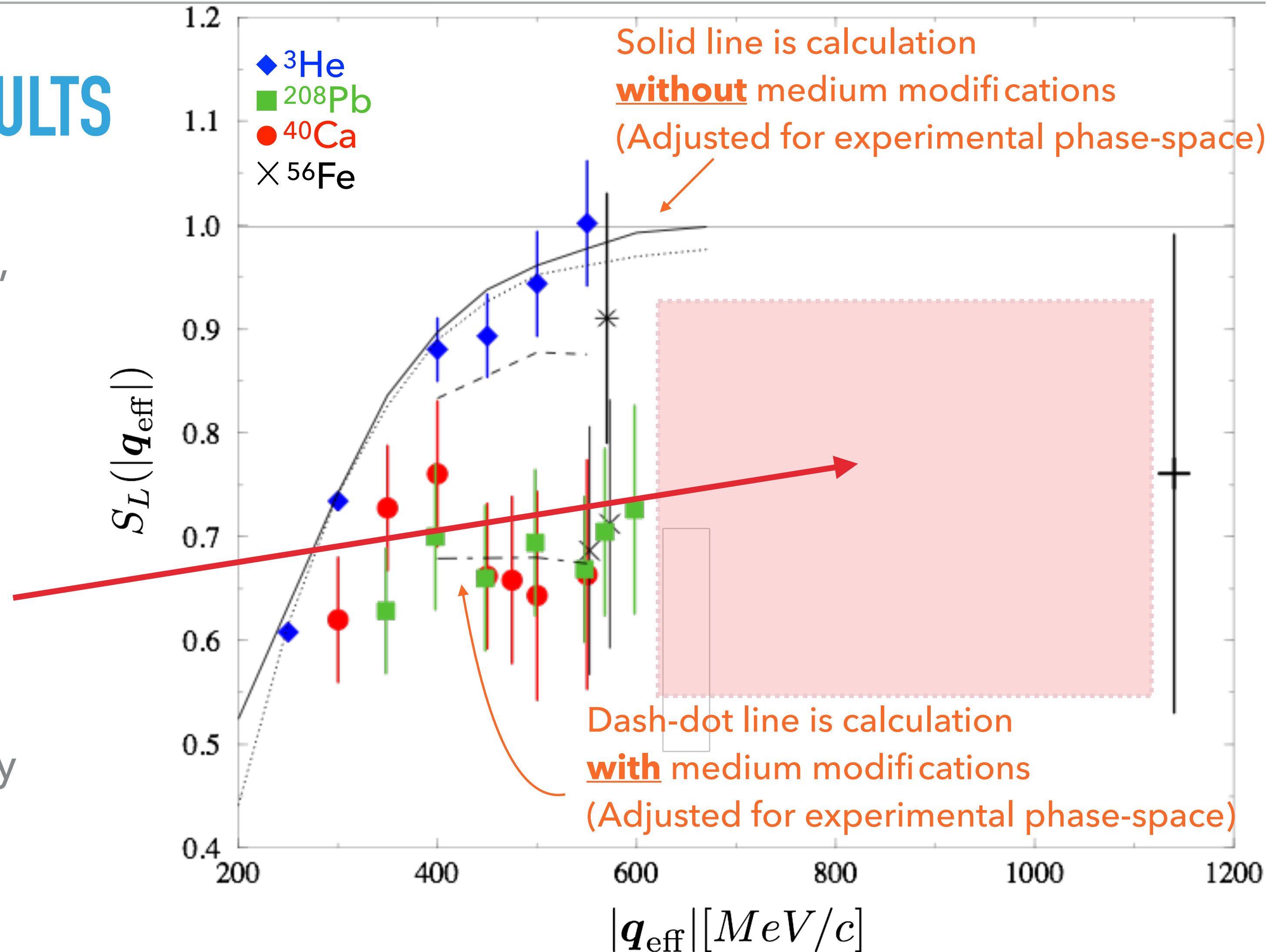


$|\mathbf{q}_{\text{eff}}|$  is  $|\mathbf{q}|$  corrected for a nuclei dependent mean coulomb potential.  
Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.



## PUBLISHED EXPERIMENTAL RESULTS

- ▶ First group of experiments from Saclay, Bates, and SLAC show a quenching of  $S_L$  consistent with medium modified form-factors.
- ▶ Very little data above  $|\mathbf{q}|$  of 600 MeV/c, where the cleanest signal of medium effects should exist!
- ▶ Saclay, Bates limited in beam energy reach up to 800 MeV.
- ▶ SLAC limited in kinematic coverage of scattered electron at  $|\mathbf{q}|$  below 1150 MeV/c.



$|\mathbf{q}_{\text{eff}}|$  is  $|\mathbf{q}|$  corrected for a nuclei dependent mean coulomb potential.  
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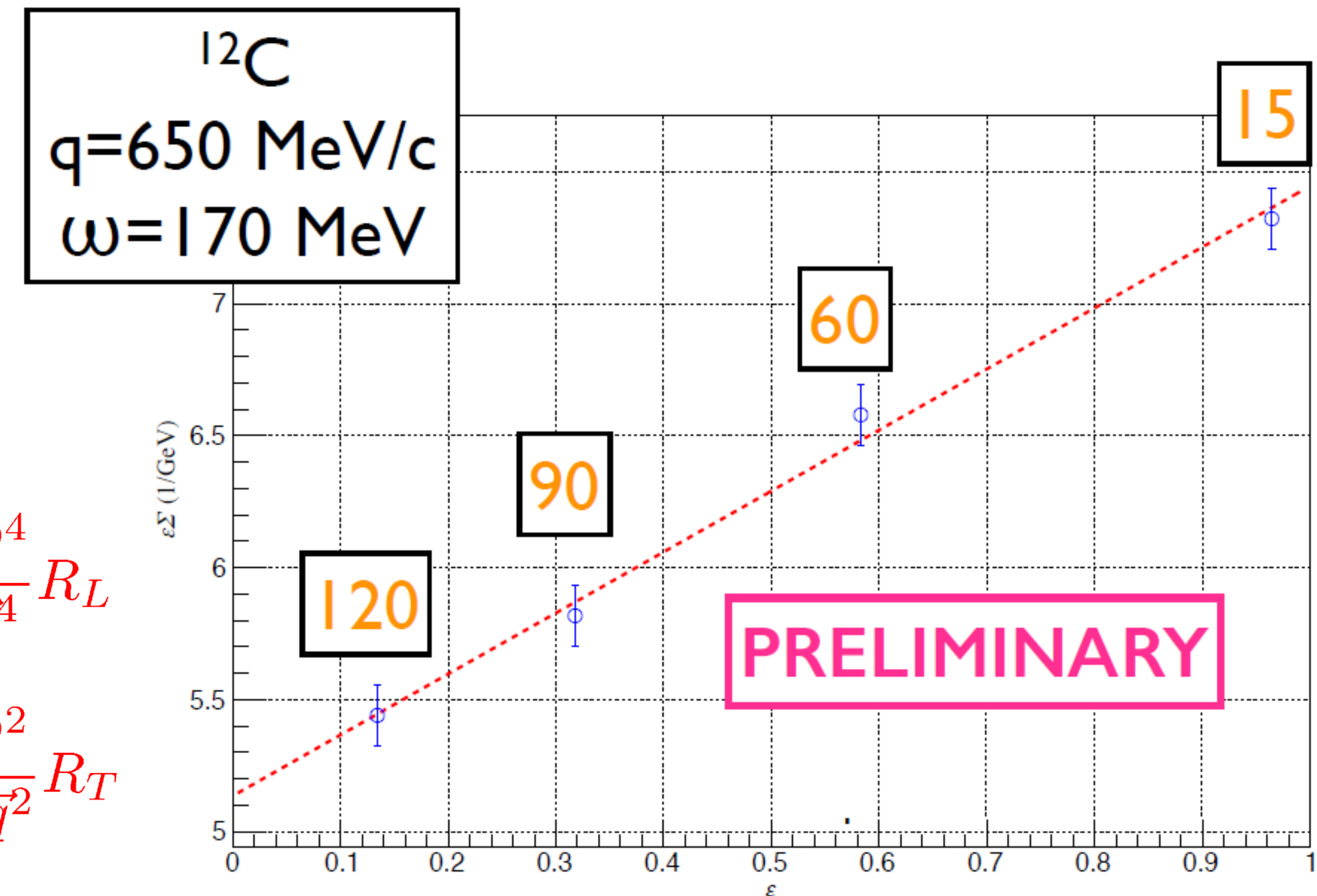
# EXPERIMENTAL DESIGN

- ▶ Need  $R_L$  → Use Rosenbluth separation!

$$S_L(|\mathbf{q}|) = \int_{\omega^+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

$$\text{Slope} = \frac{Q^4}{\vec{q}^4} R_L$$

$$\text{Intercept} = \frac{Q^2}{2\vec{q}^2} R_T$$



- ▶ Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of  $R_L$ !
- ▶ Need data for each angle at a constant  $|\mathbf{q}|$  over an  $\omega$  range starting above the elastic peak up to  $|\mathbf{q}|$ .
  - ▶ When running a single arm experiment with fixed beam energy and scattering angle,  $|\mathbf{q}|$  is NOT constant over your momentum acceptance.
  - ▶ Need to take data at varying beam energies, and “map-out”  $|\mathbf{q}|$  and  $\omega$  space.

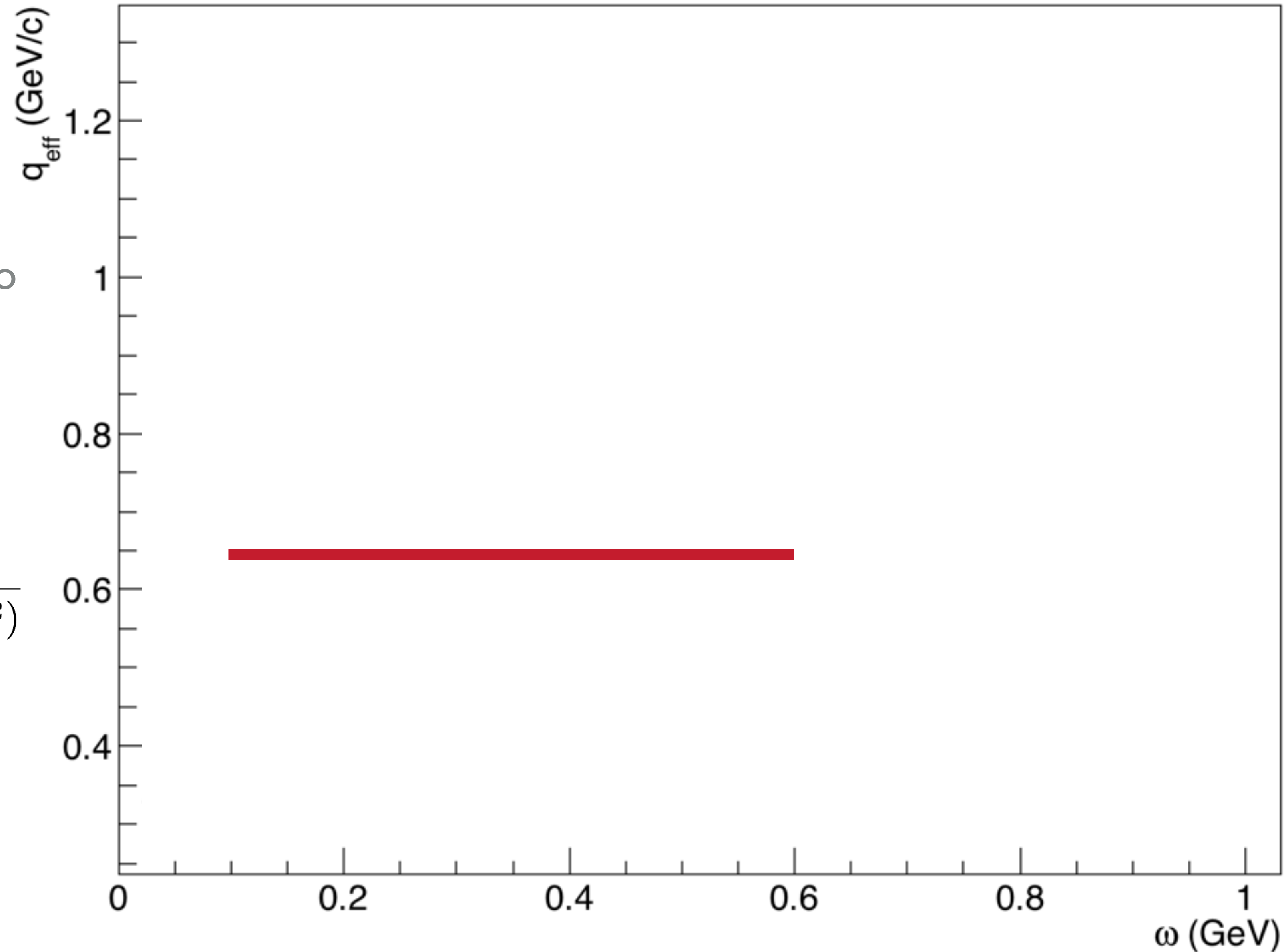


## EXPERIMENTAL DESIGN

- ▶ If one wants to measure from 100 to 600 MeV  $\omega$  at constant  $|\mathbf{q}| = 650$  MeV/c

CSR calculated at constant  $|\mathbf{q}|$  !!

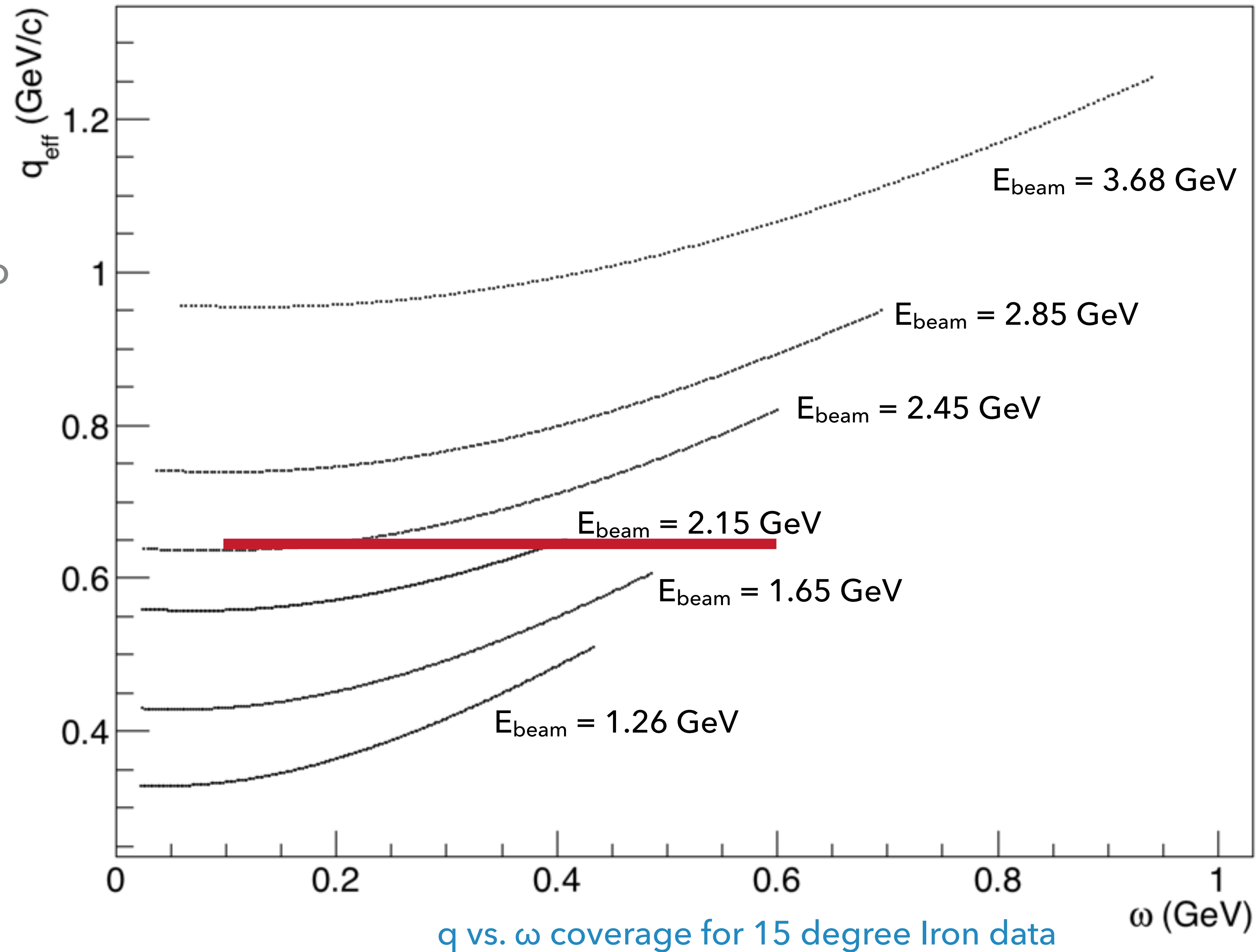
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## EXPERIMENTAL DESIGN

- ▶ If one wants to measure from 100 to 600 MeV  $\omega$  at constant  $|q| = 650$  MeV/c
- ▶ Take data at different beam energies, and interpolate to determine cross-section at constant  $|q|$ .

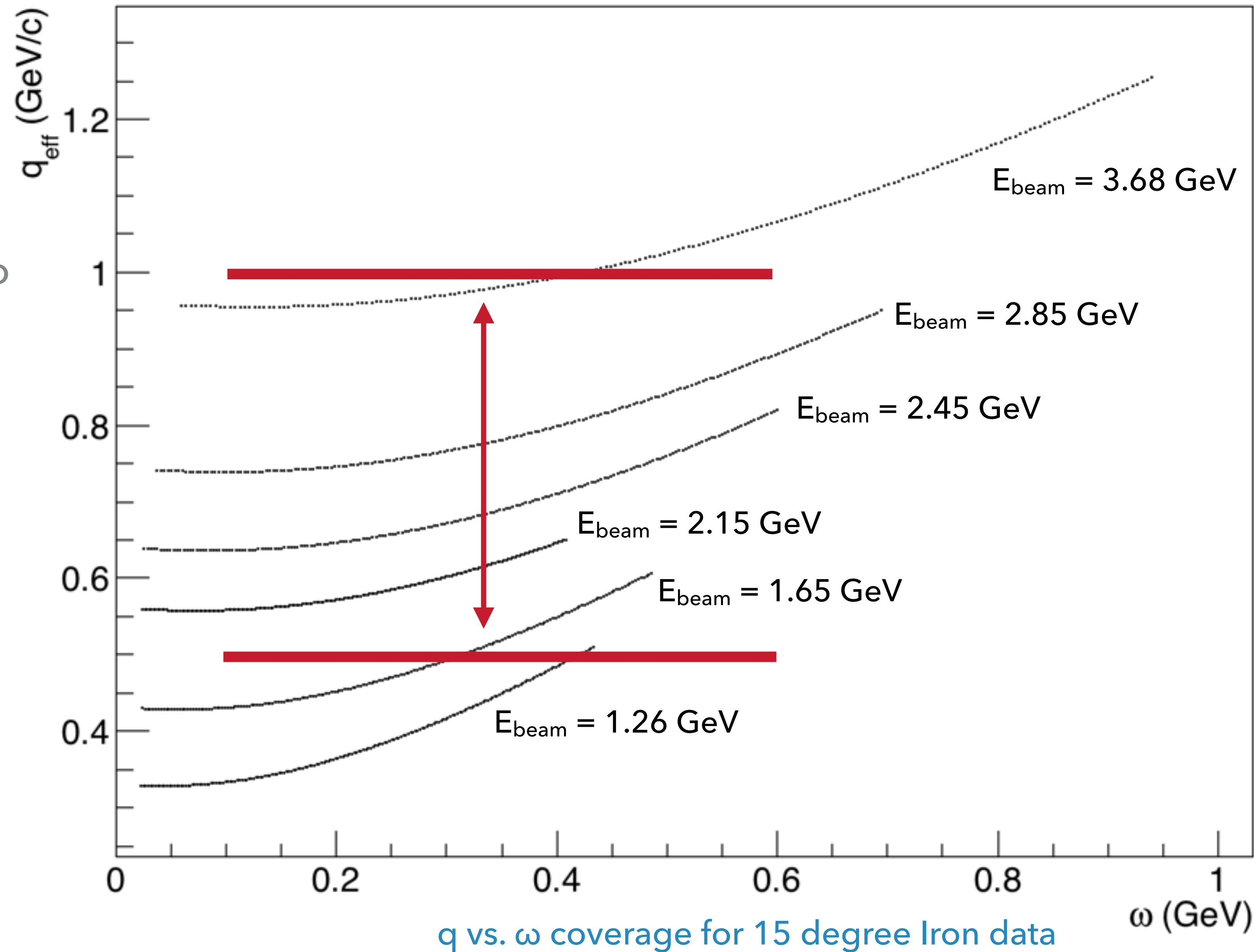




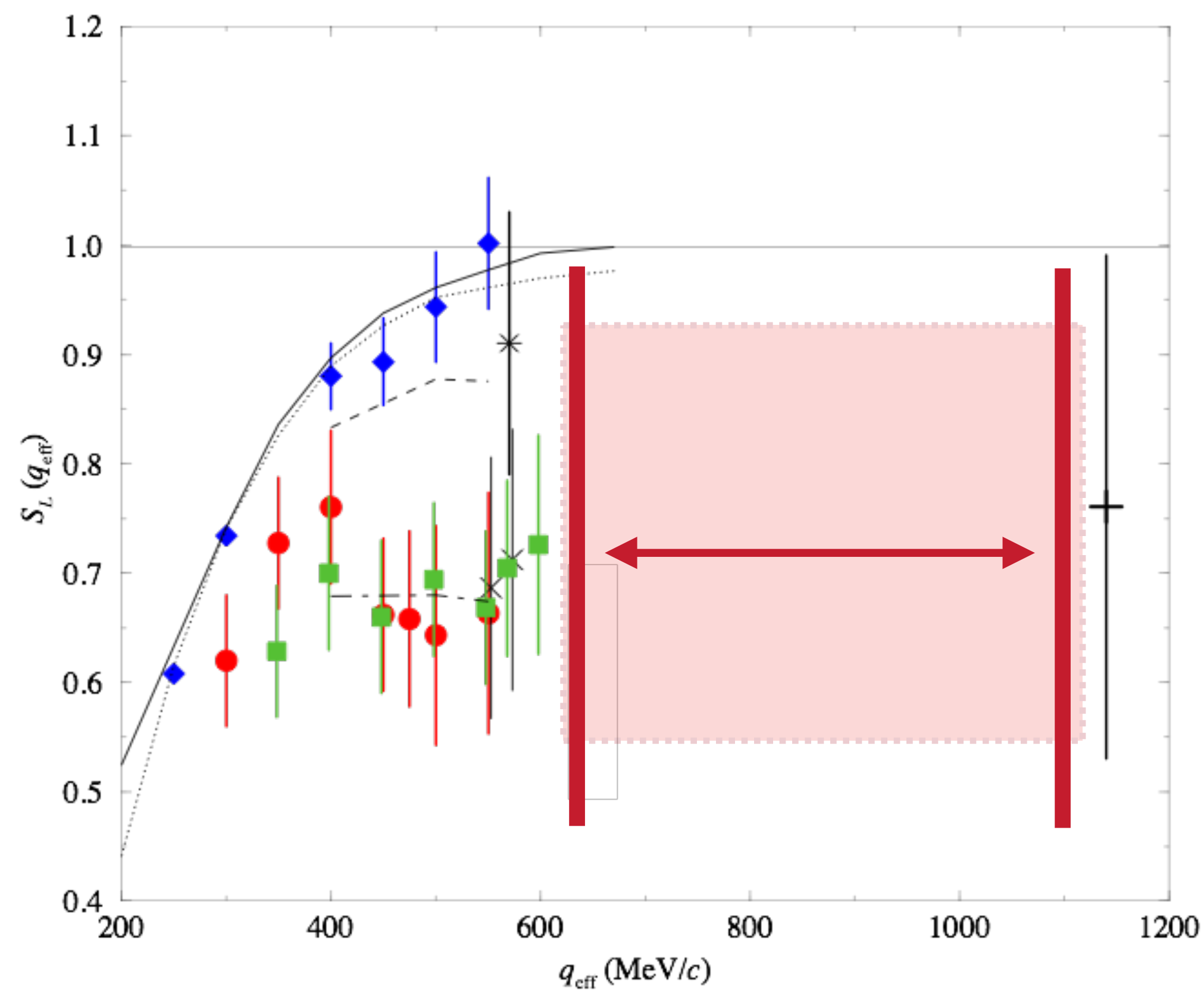
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- ▶ Take data at different beam energies, and interpolate to determine cross-section at constant  $|q|$ .
- ▶  $|q|$  can be selected between 550 and 1000 MeV/c

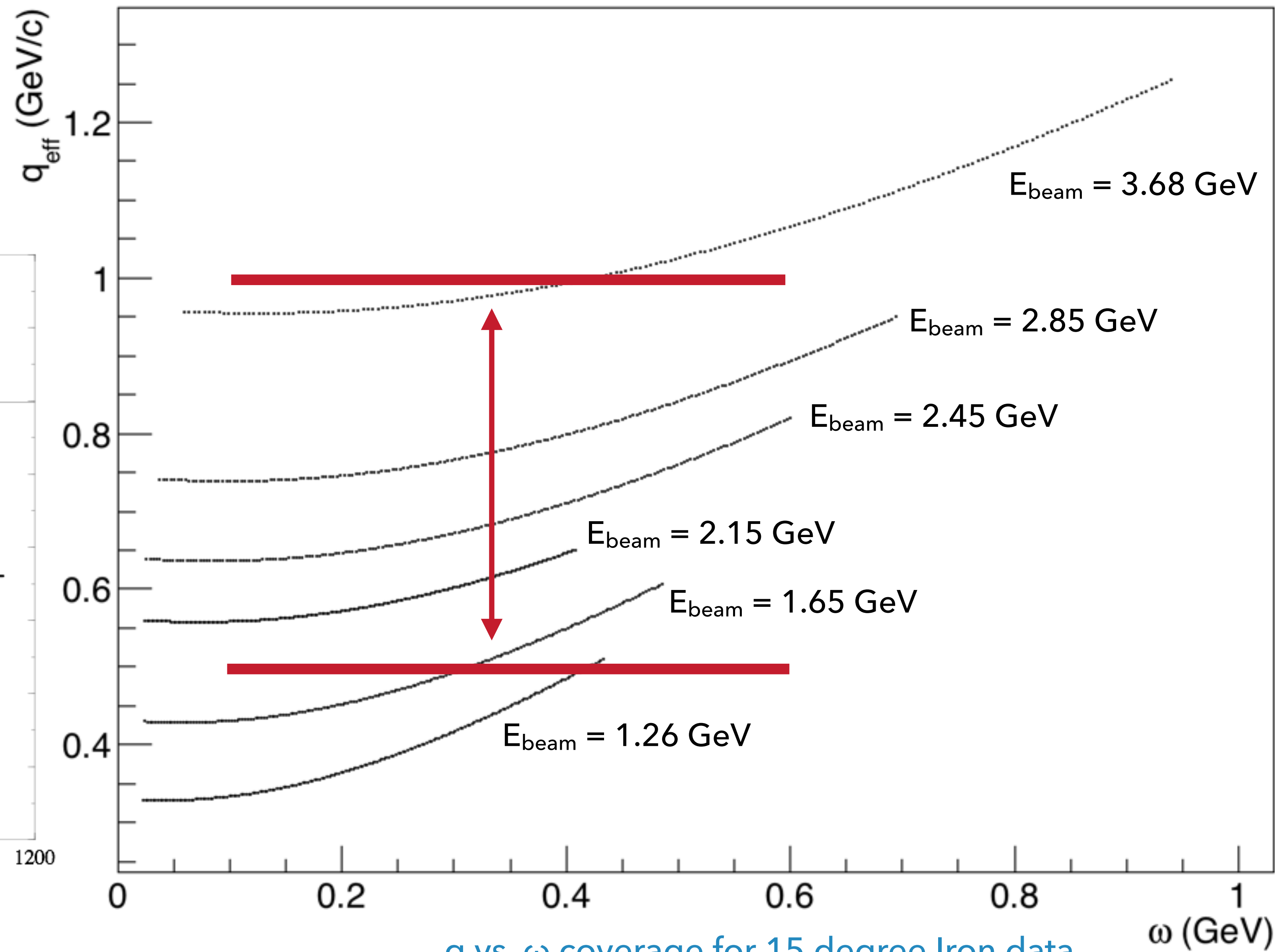
Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.



## EXPERIMENTAL DESIGN



Repeat this "mapping" for 60, 90, and 120 degree spectrometer central angles.



$q$  vs.  $\omega$  coverage for 15 degree Iron data

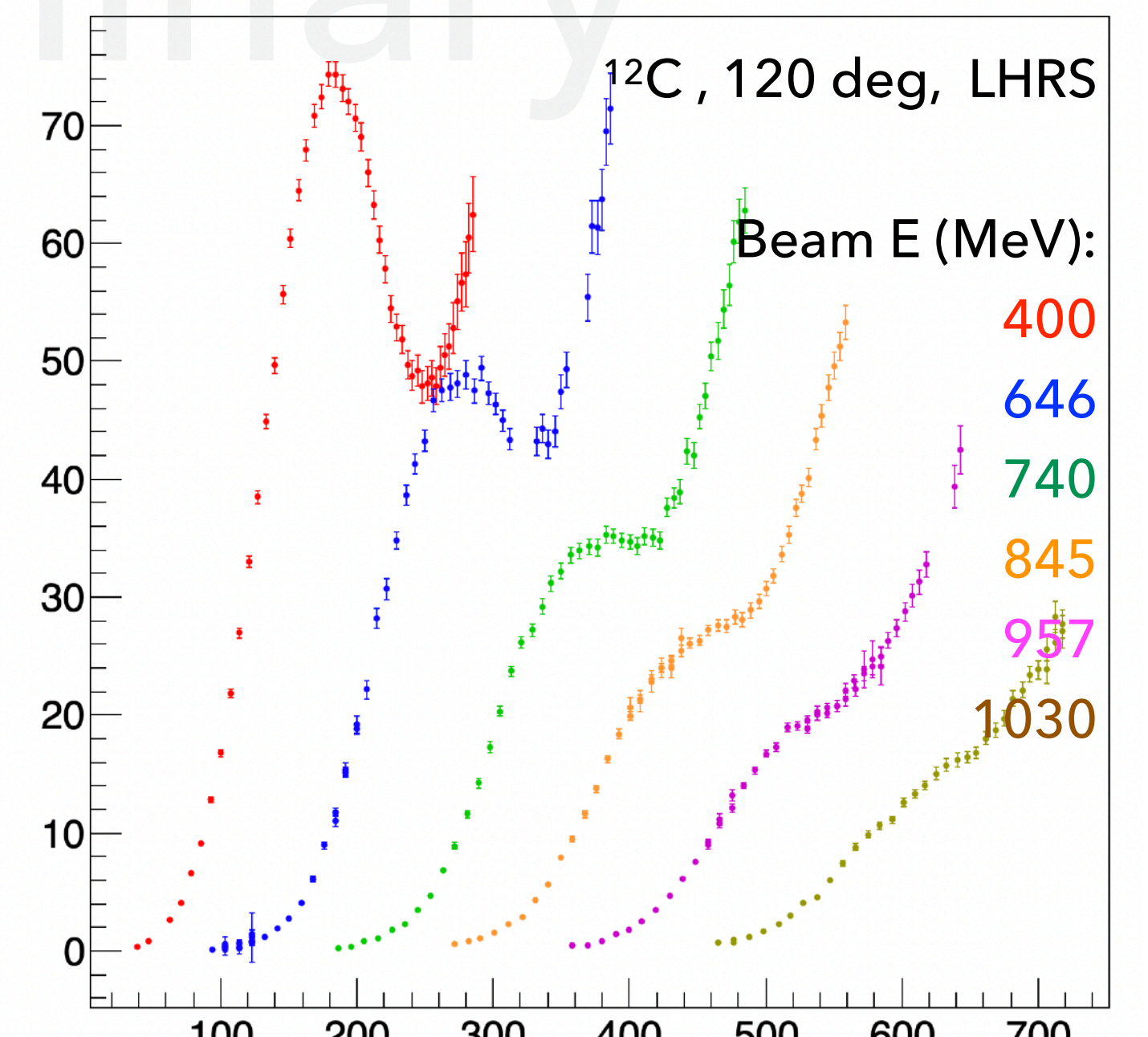
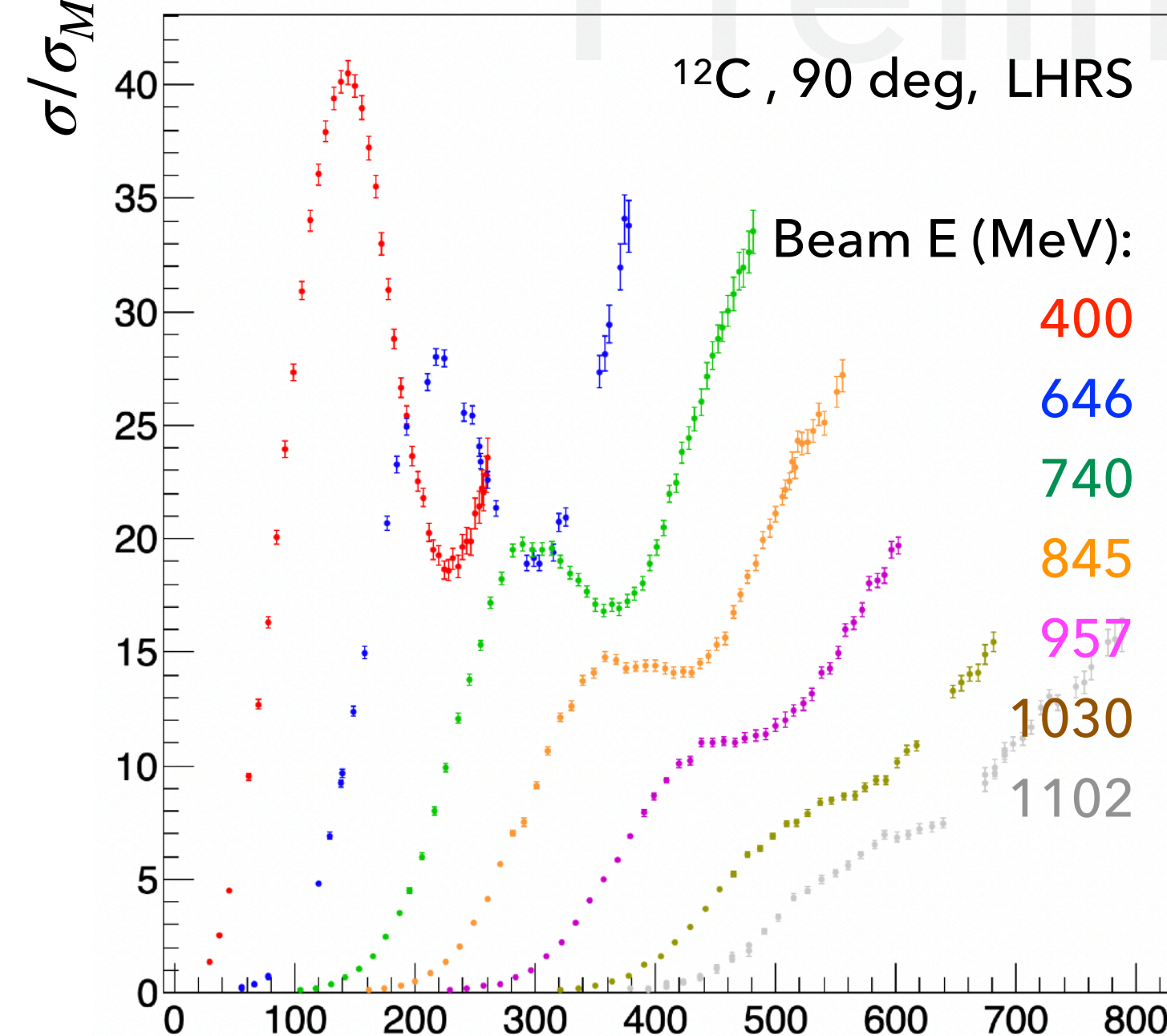
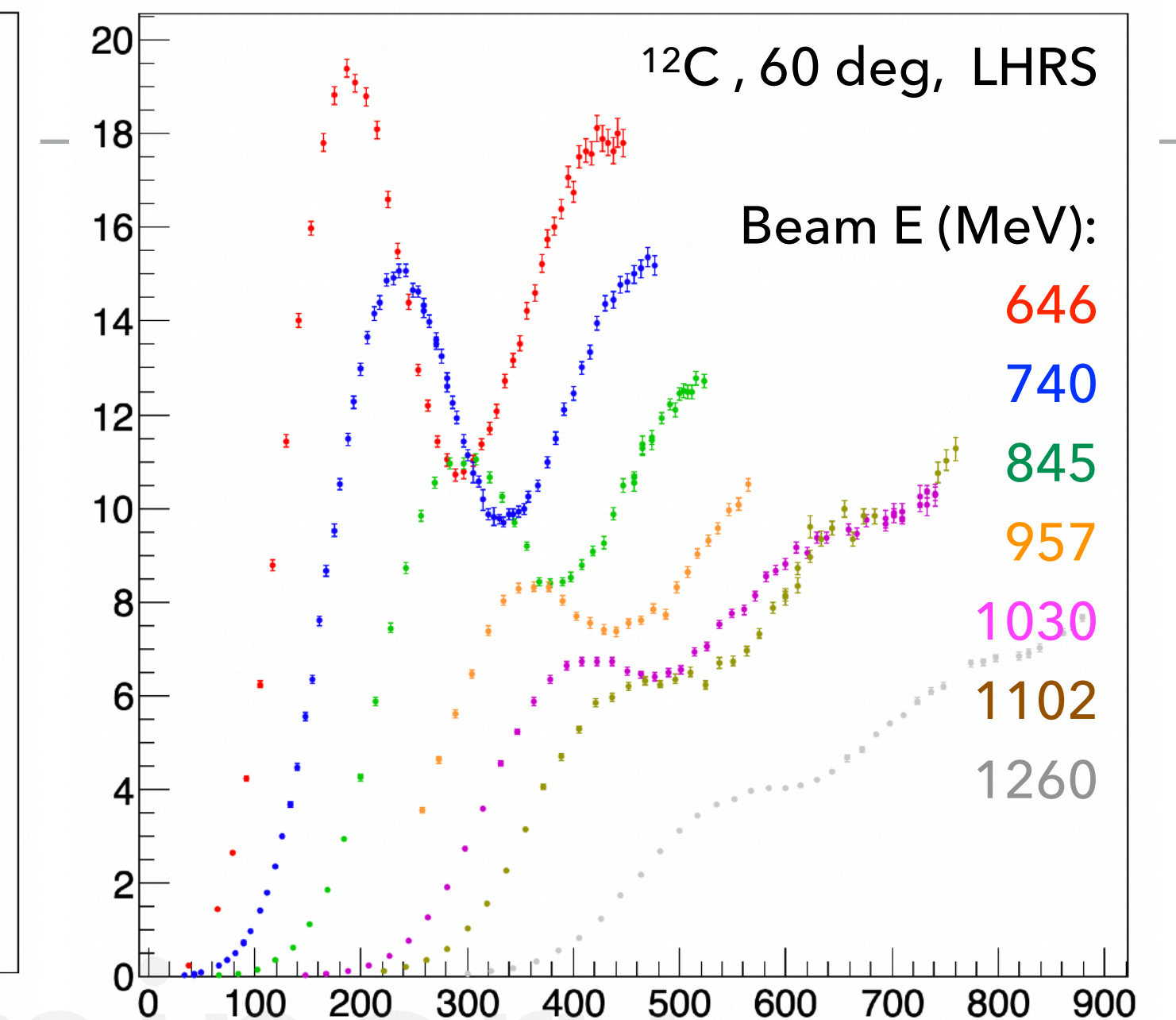
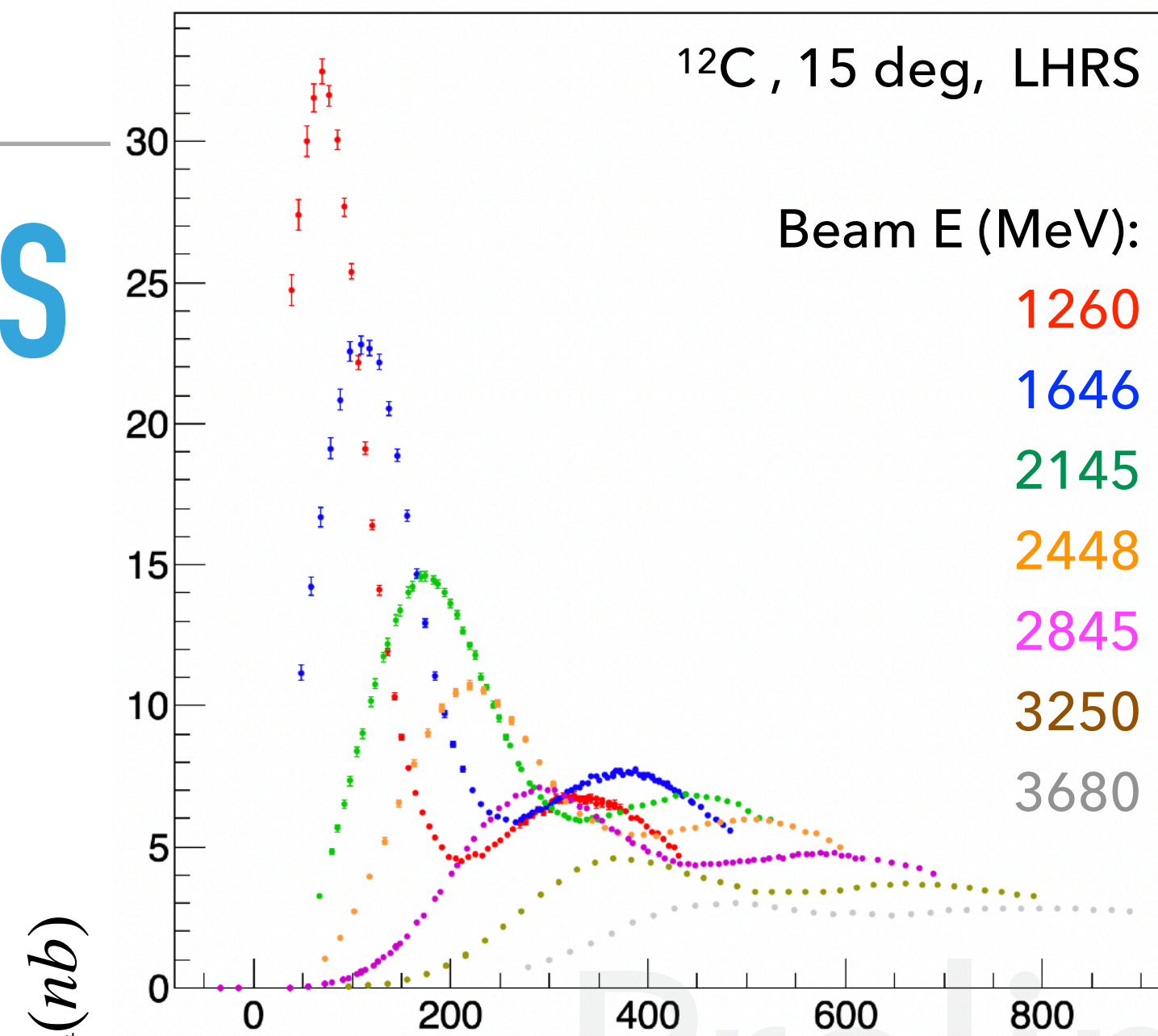


# THE COULOMB SUM RULE IN NUCLEI

## EXPERIMENTAL SPECIFICS

### ► E05-110:

- Data taken from October 23rd 2007 to January 16th 2008
- 4 central angle settings: 15, 60, 90, 120 degs.
- Many beam energy settings: 0.4 to 4.0 GeV
- Many central momentum settings: 0.1 to 4.0 GeV
- LHRS and RHRS independent (redundant) measurements for most settings
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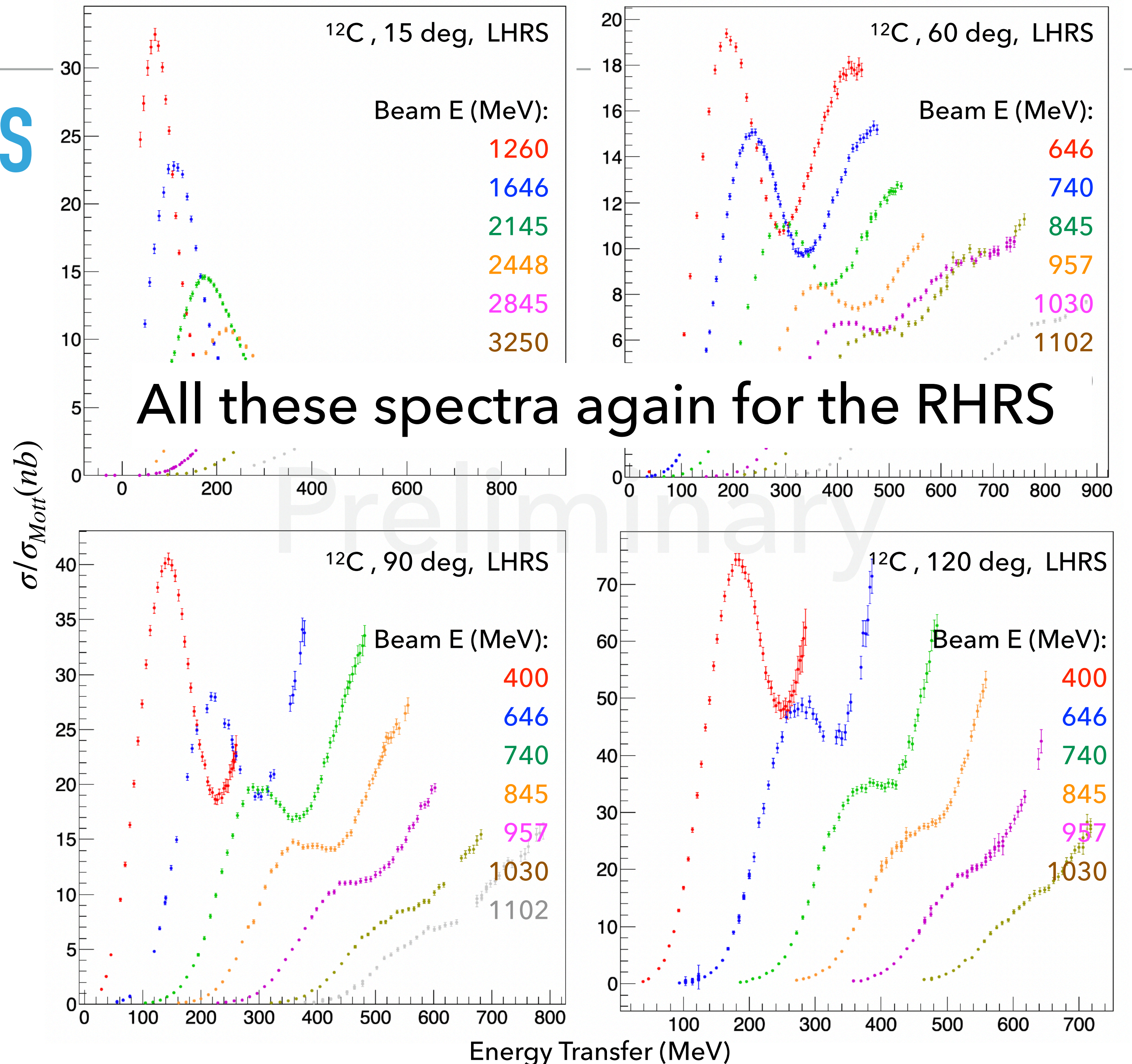
Energy Transfer (MeV)



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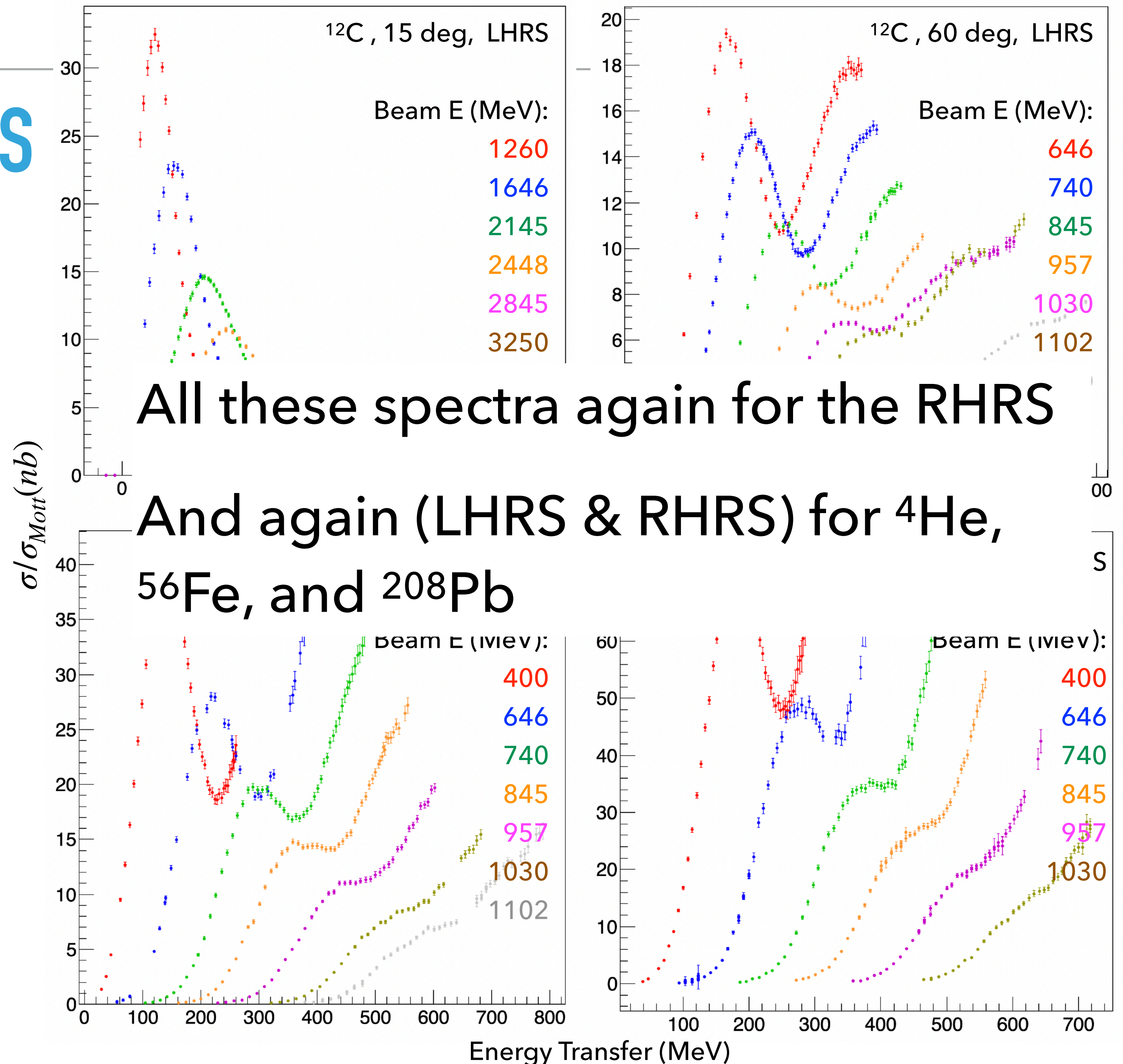




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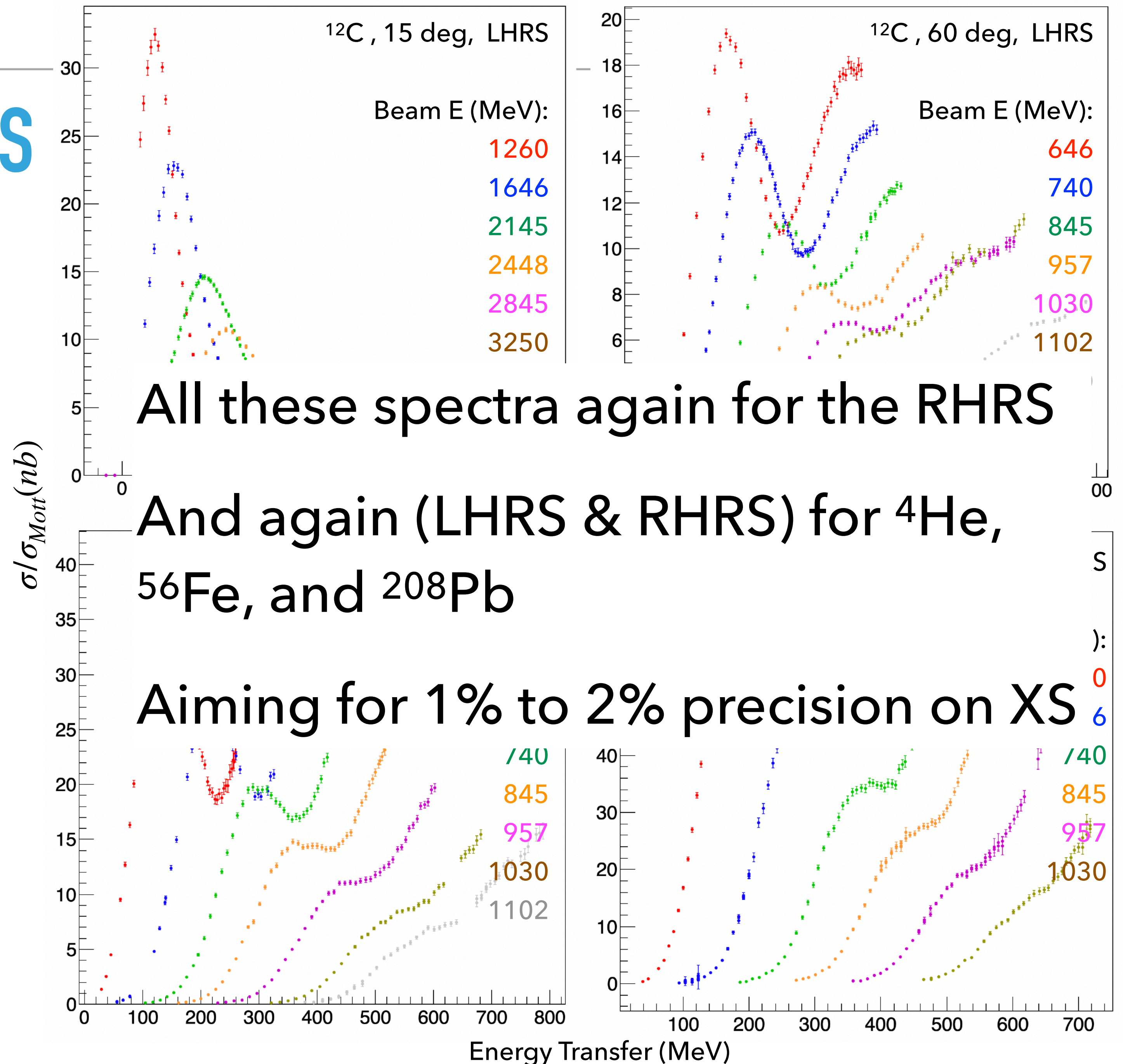
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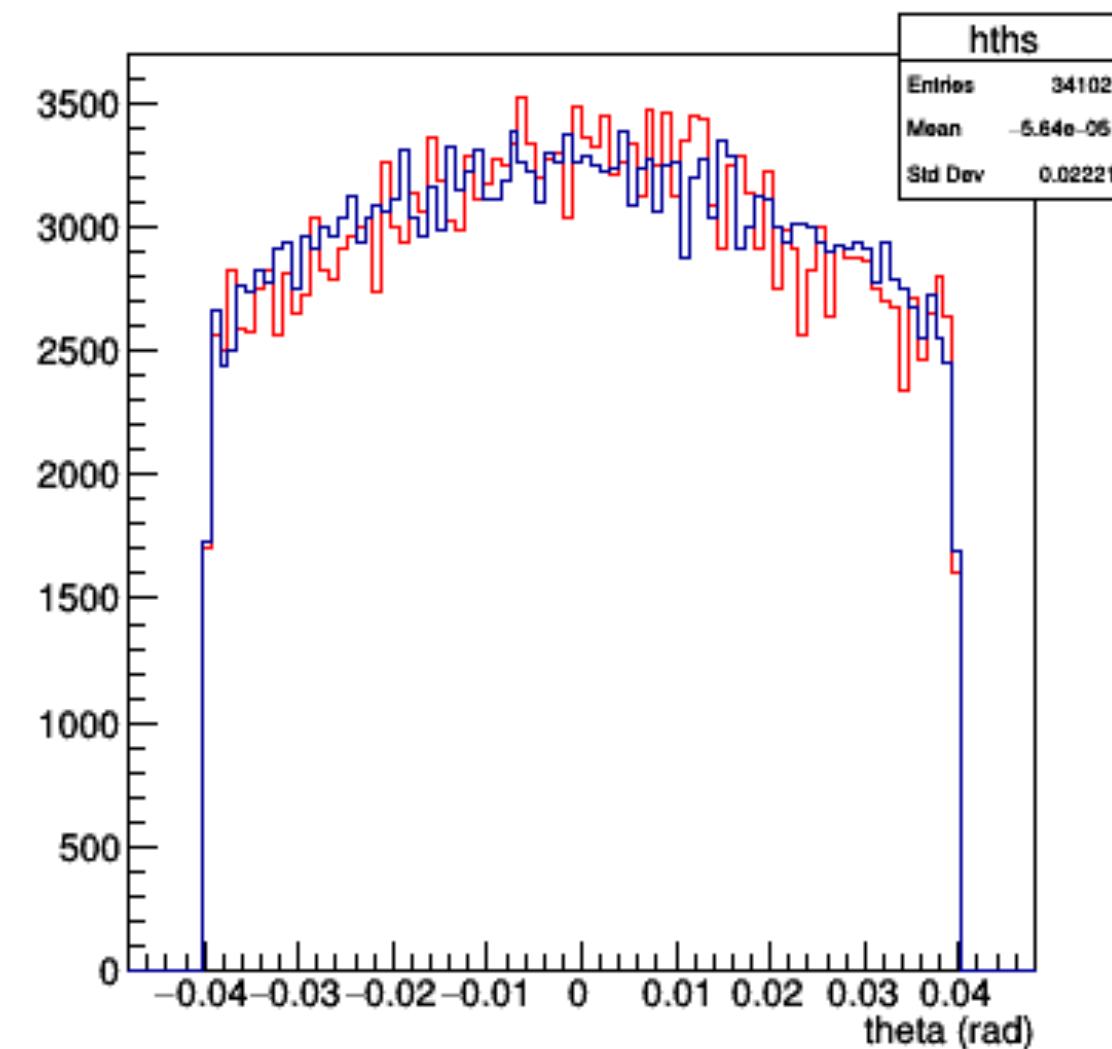
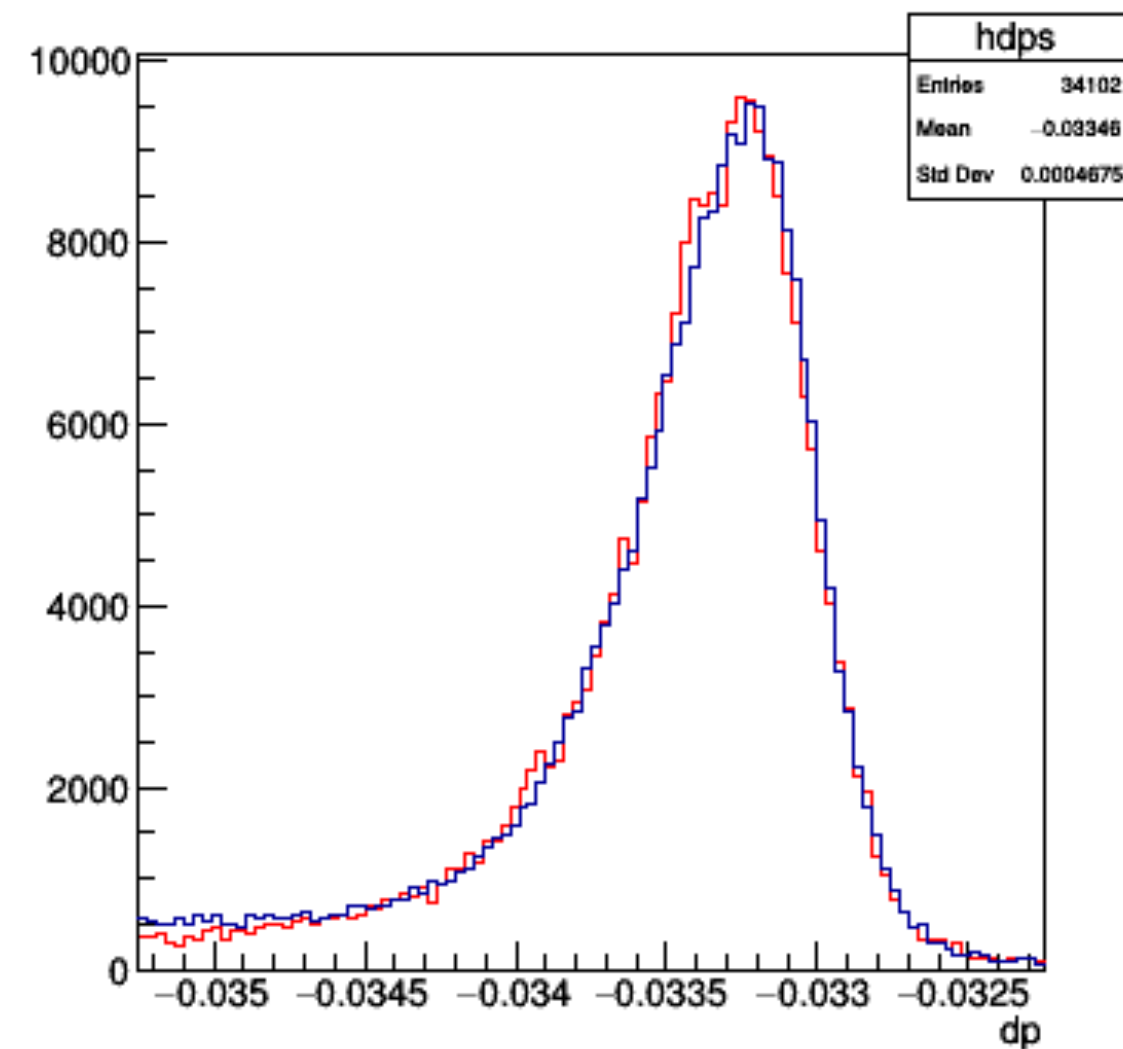
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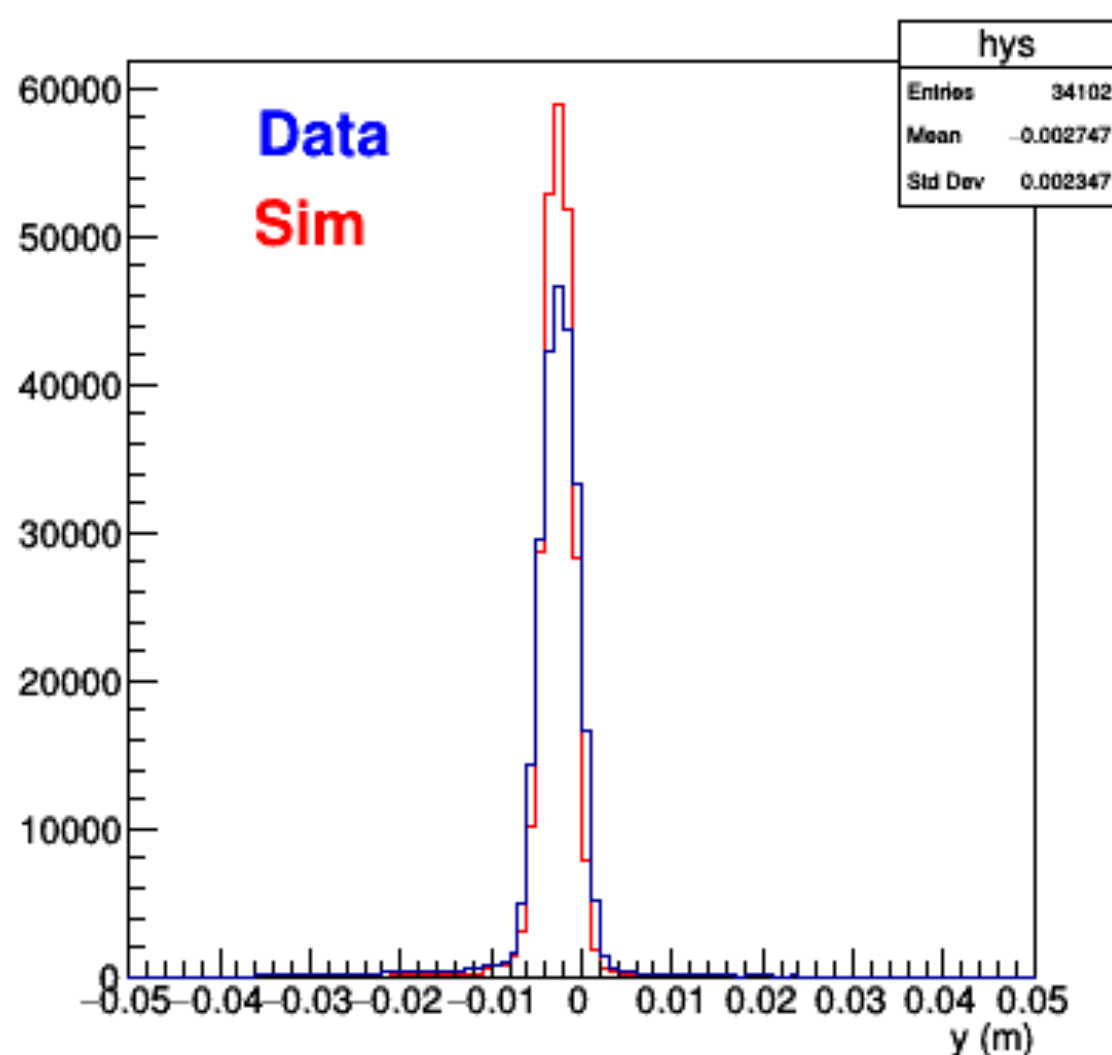
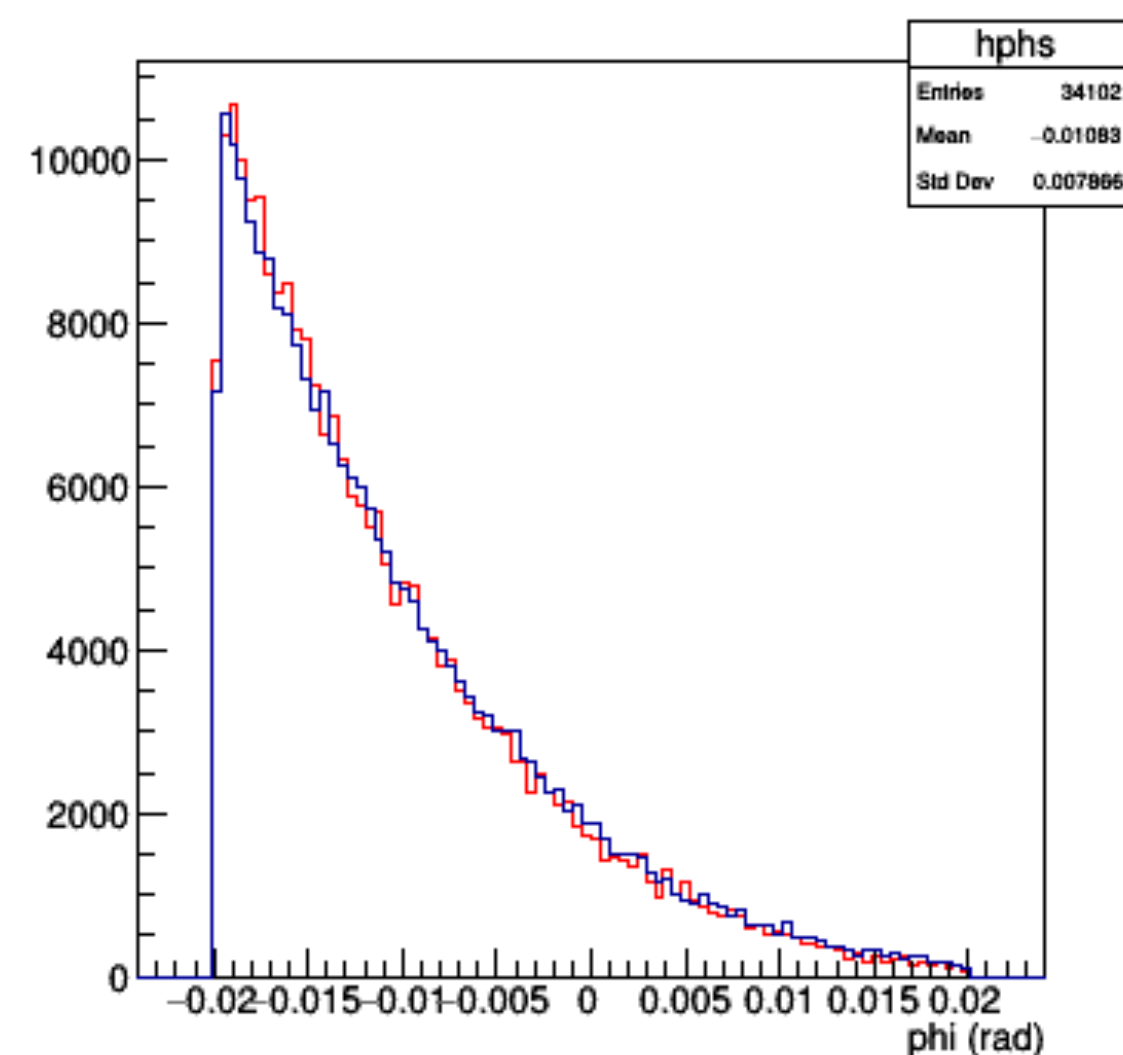




# ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS

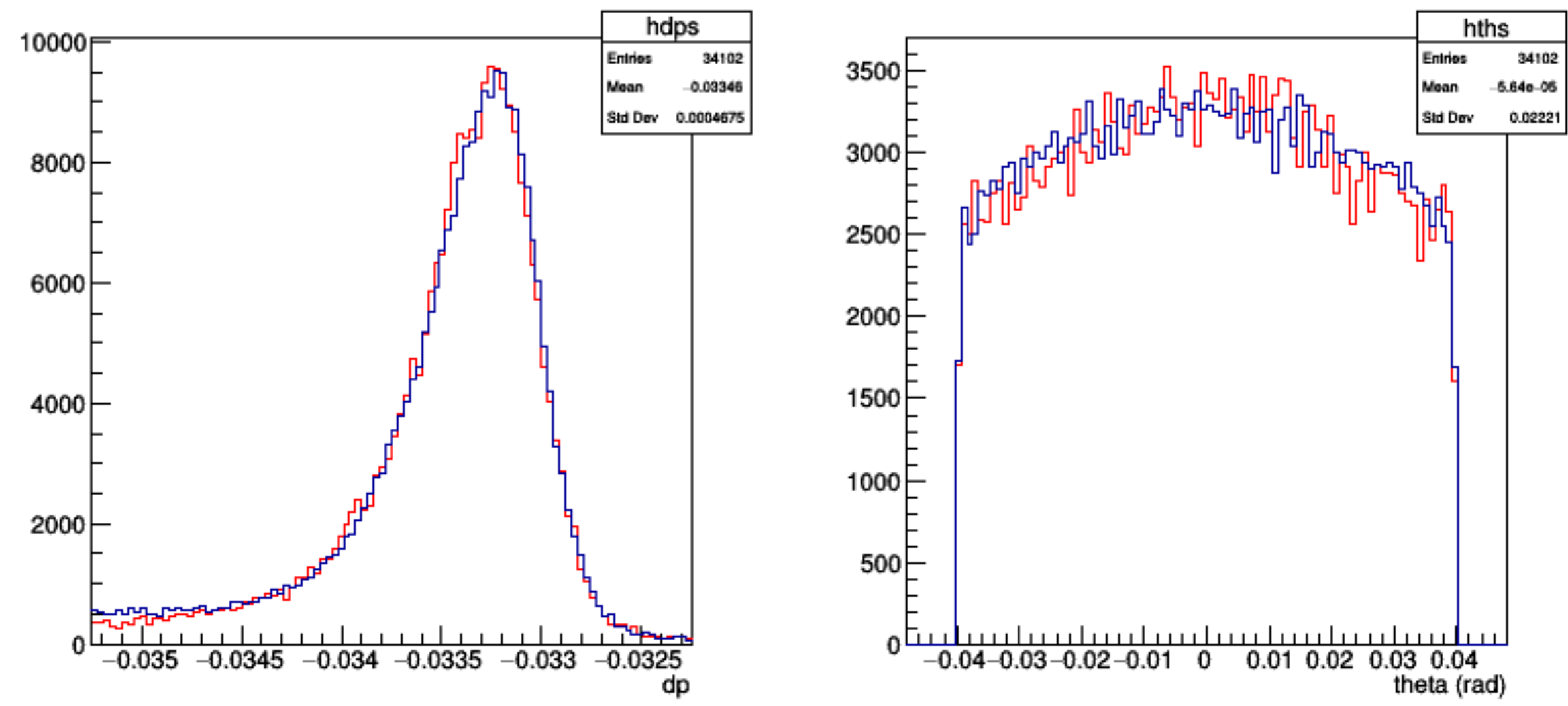


**$^{12}\text{C}$  elastic XS at 1260 MeV, 15 degrees**

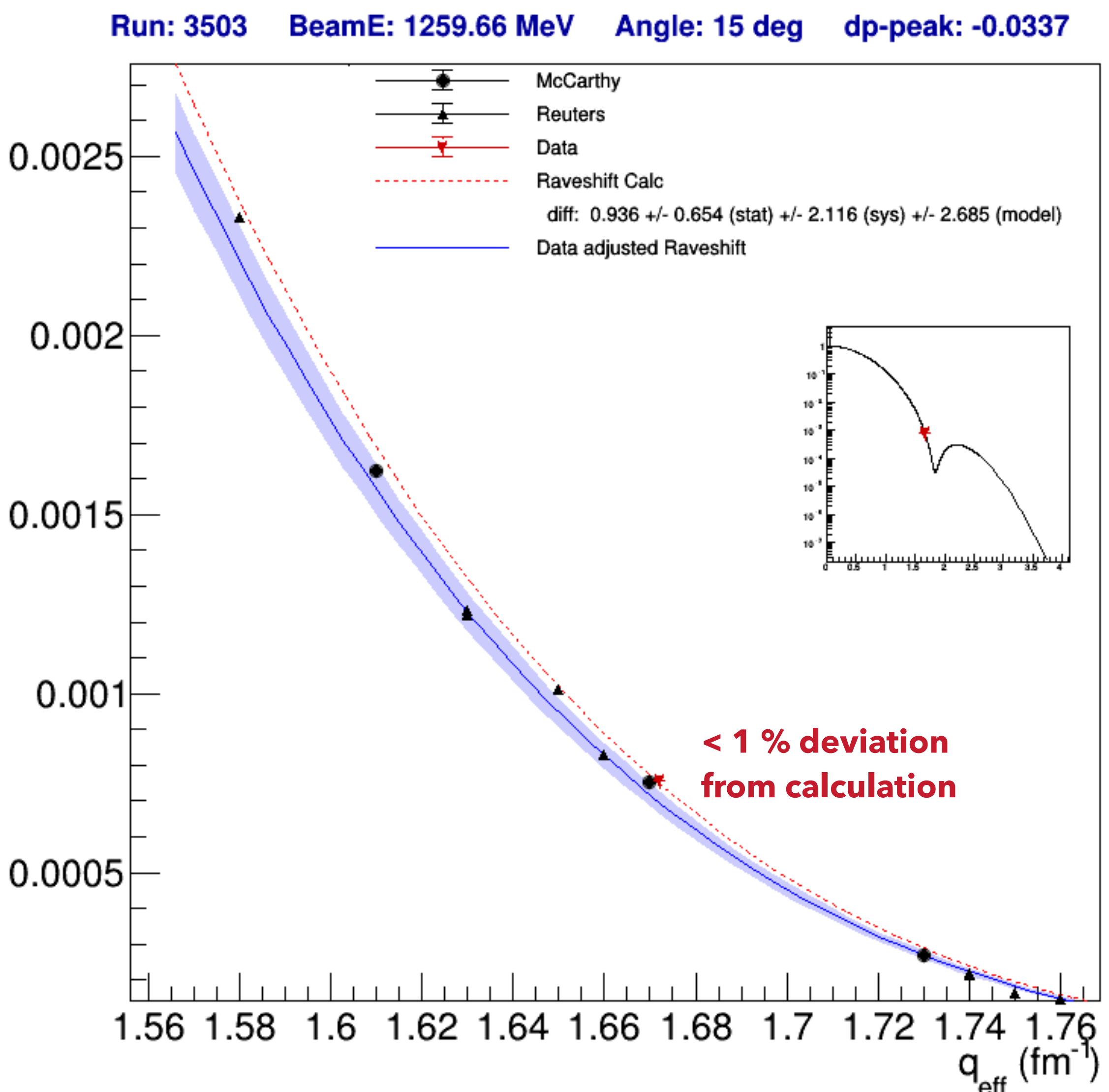
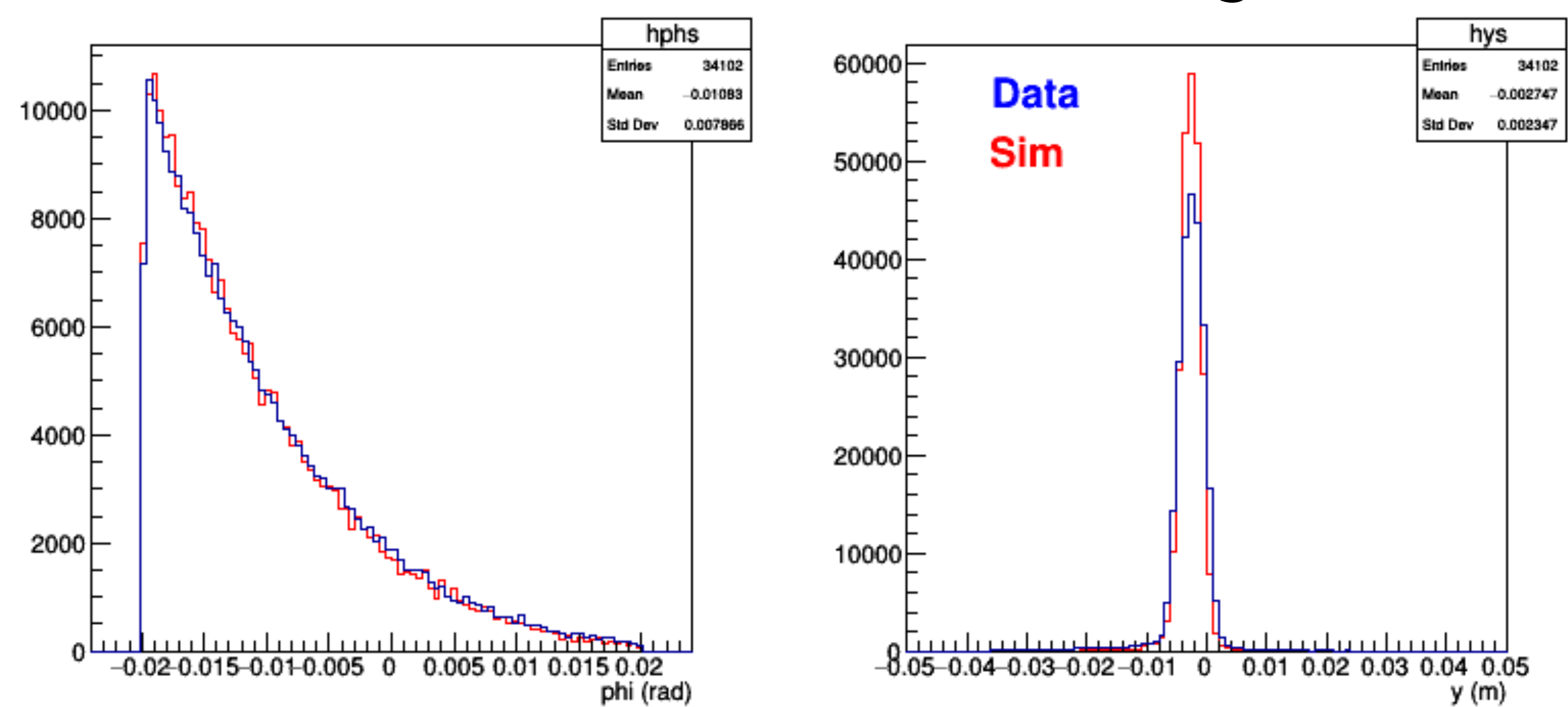


- ▶ Blue histograms are reconstructed data.
- ▶ Red histograms are monte-carlo:
  - ▶ Event sample generated from expected XS calculations (Fourier-Bessel fit to world data)
  - ▶ Radiative effects (internal, external, vertex) are handled, including exact bremsstrahlung distributions.
  - ▶ Resolution effects are applied by calculating the expected material effects of tracks passing through the VDC chamber materials.

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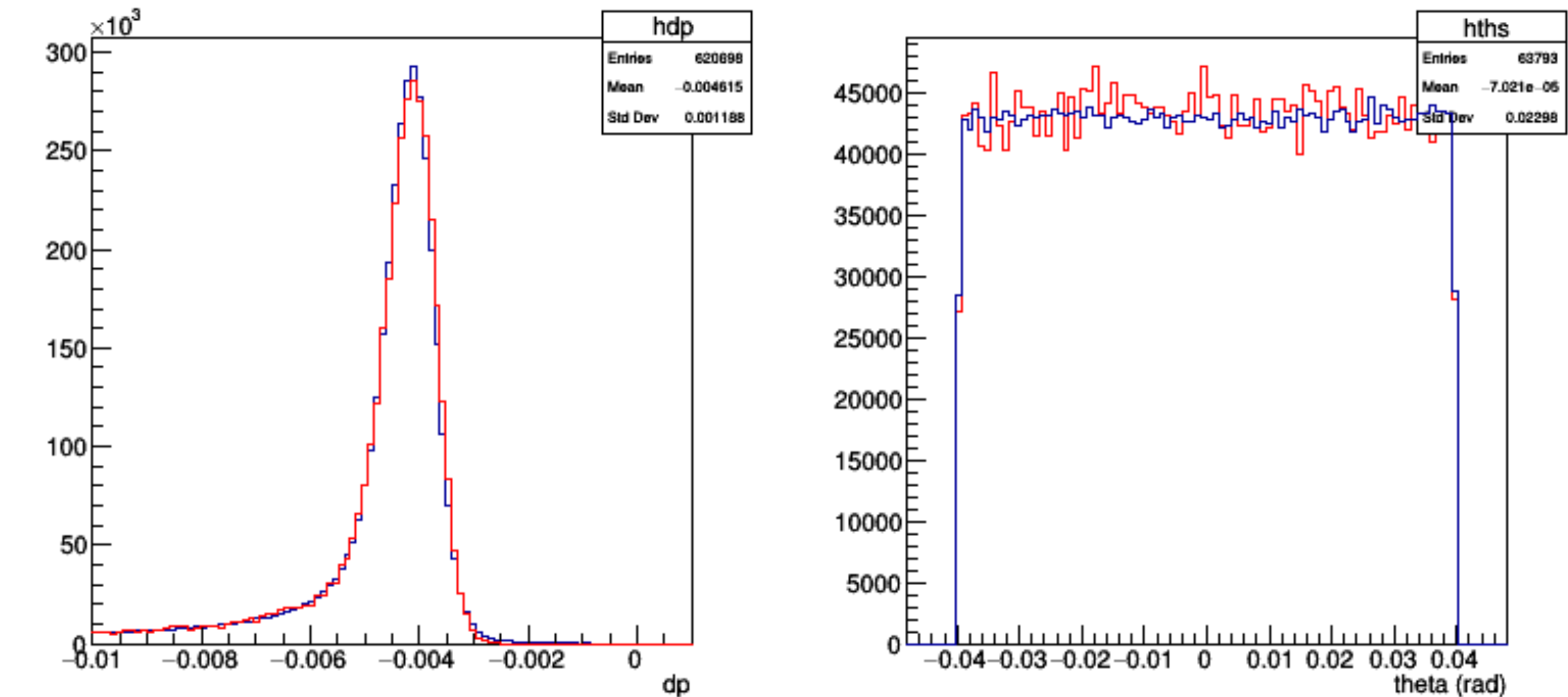


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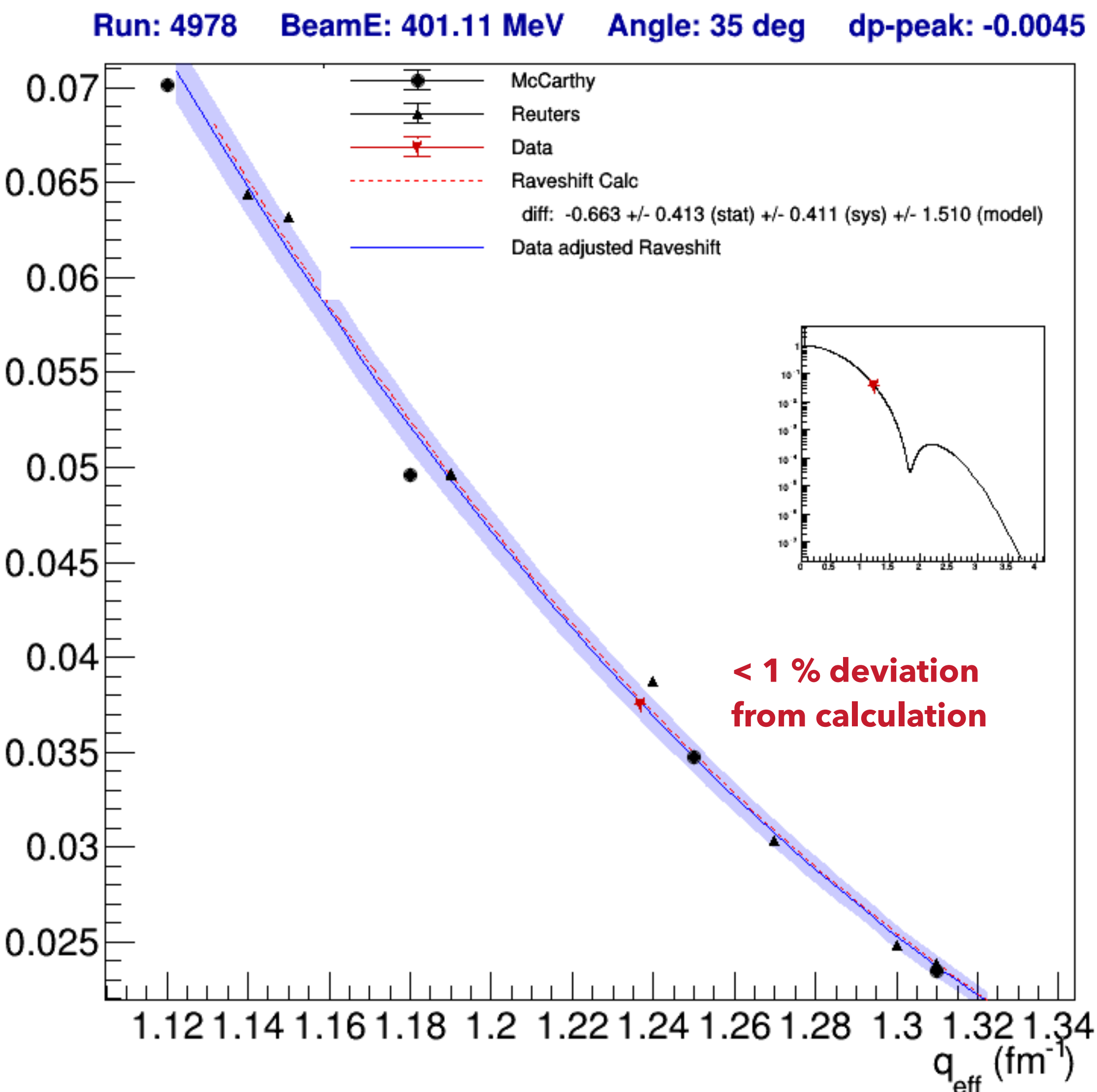
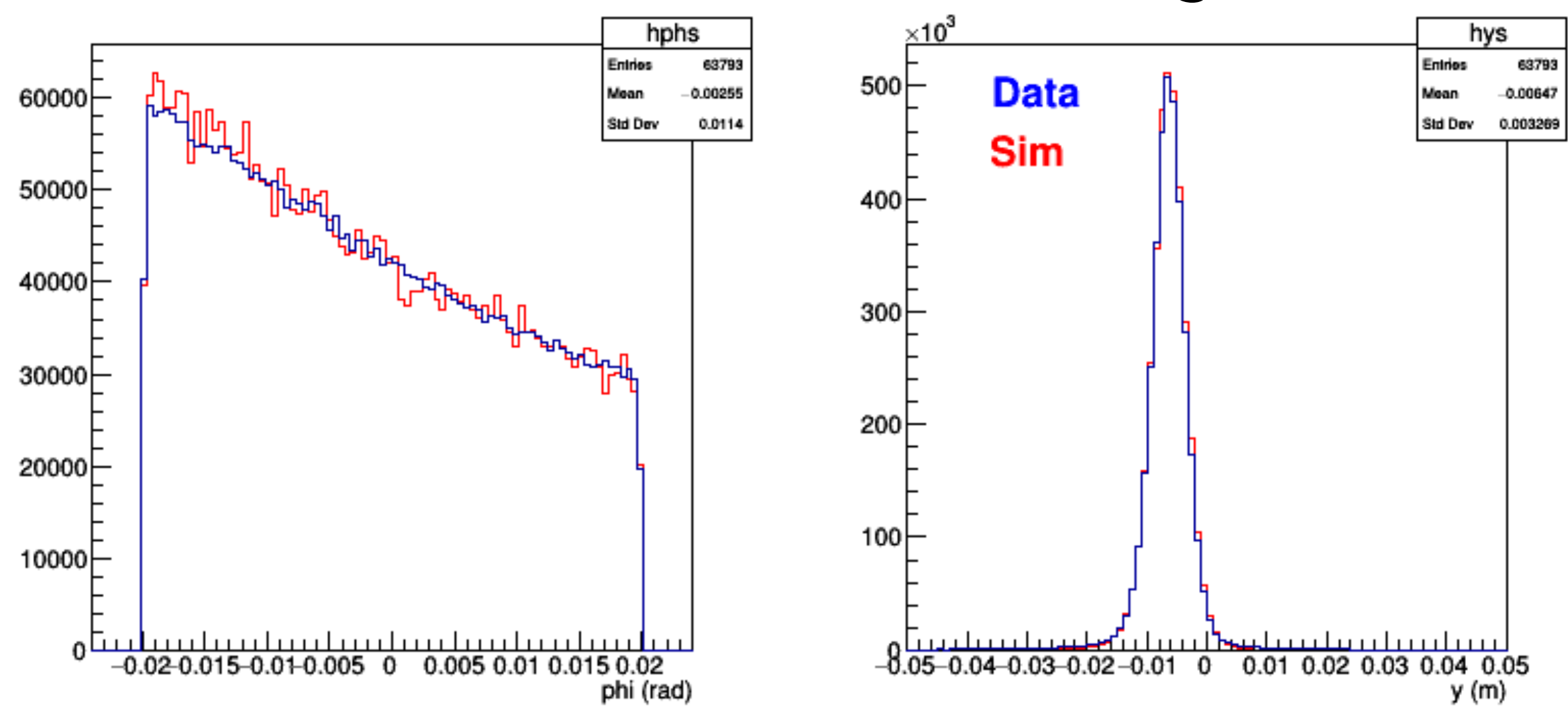




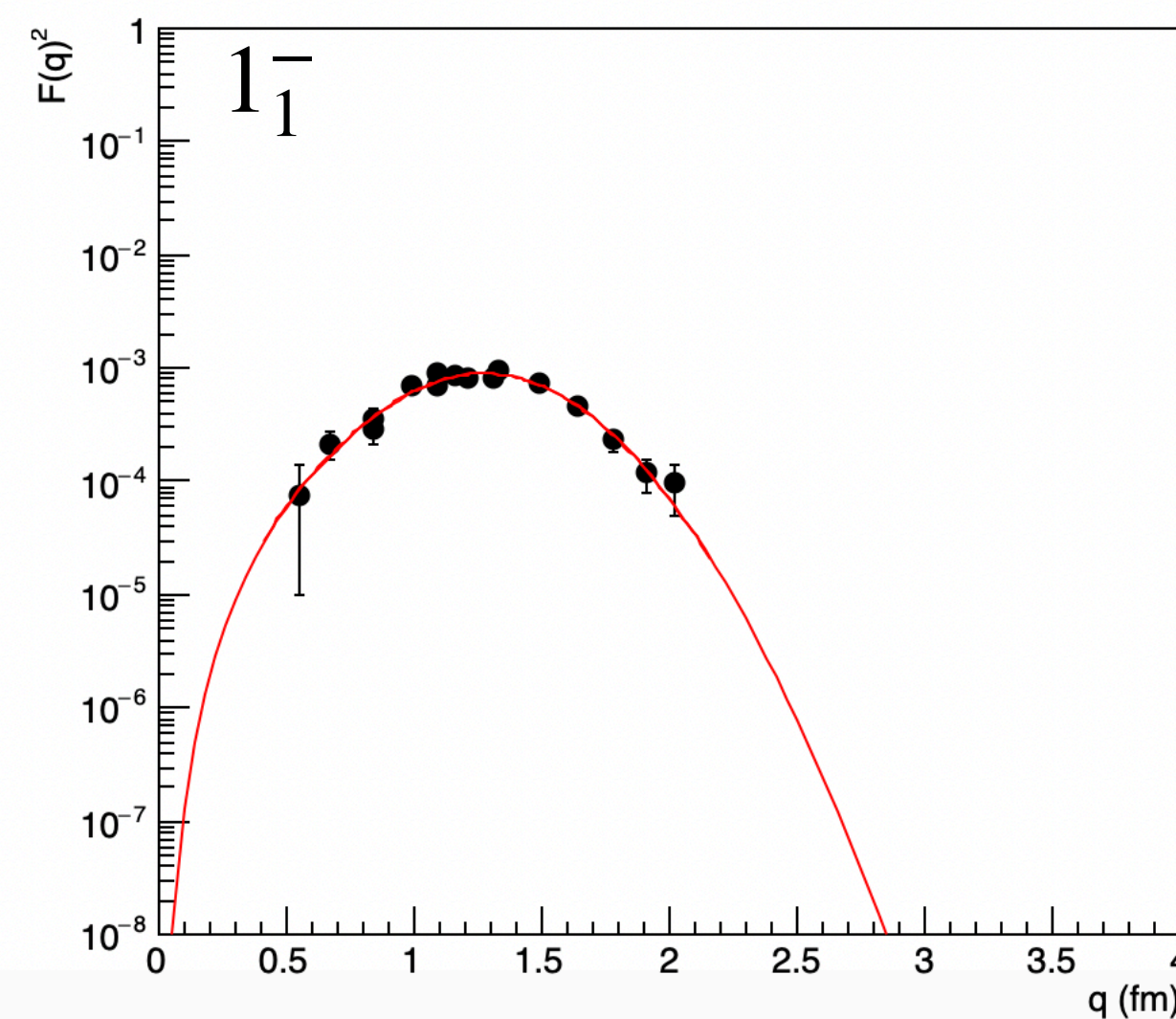
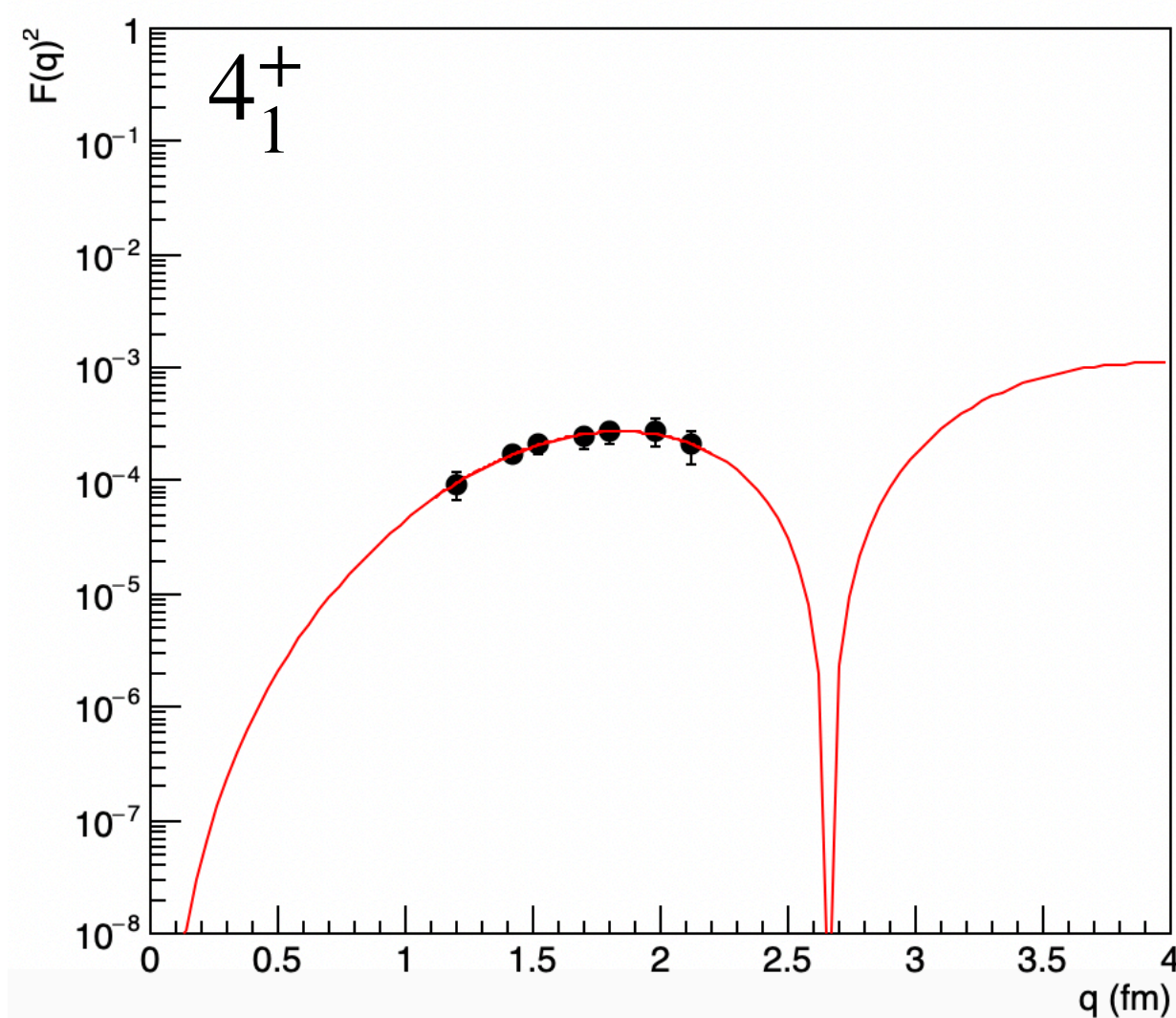
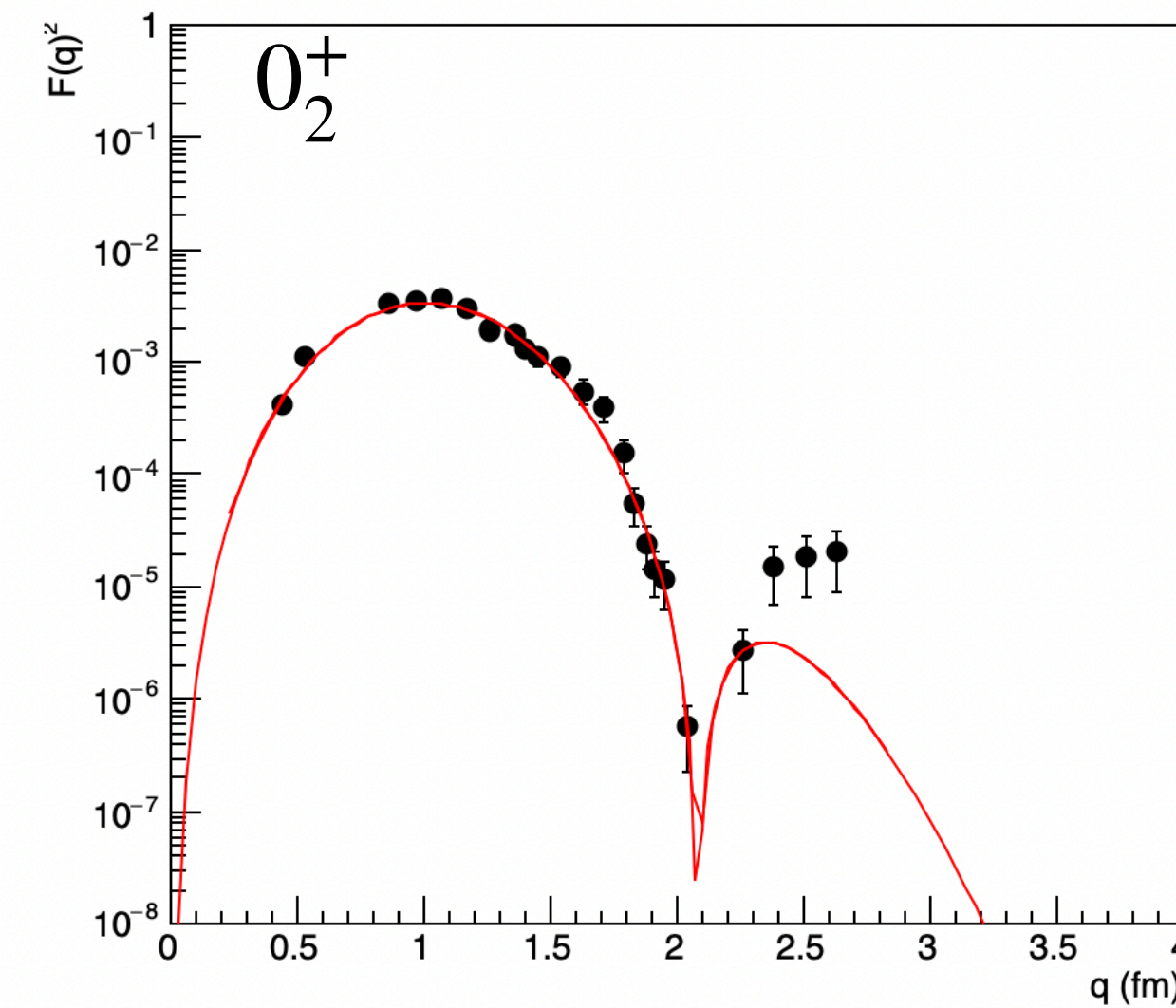
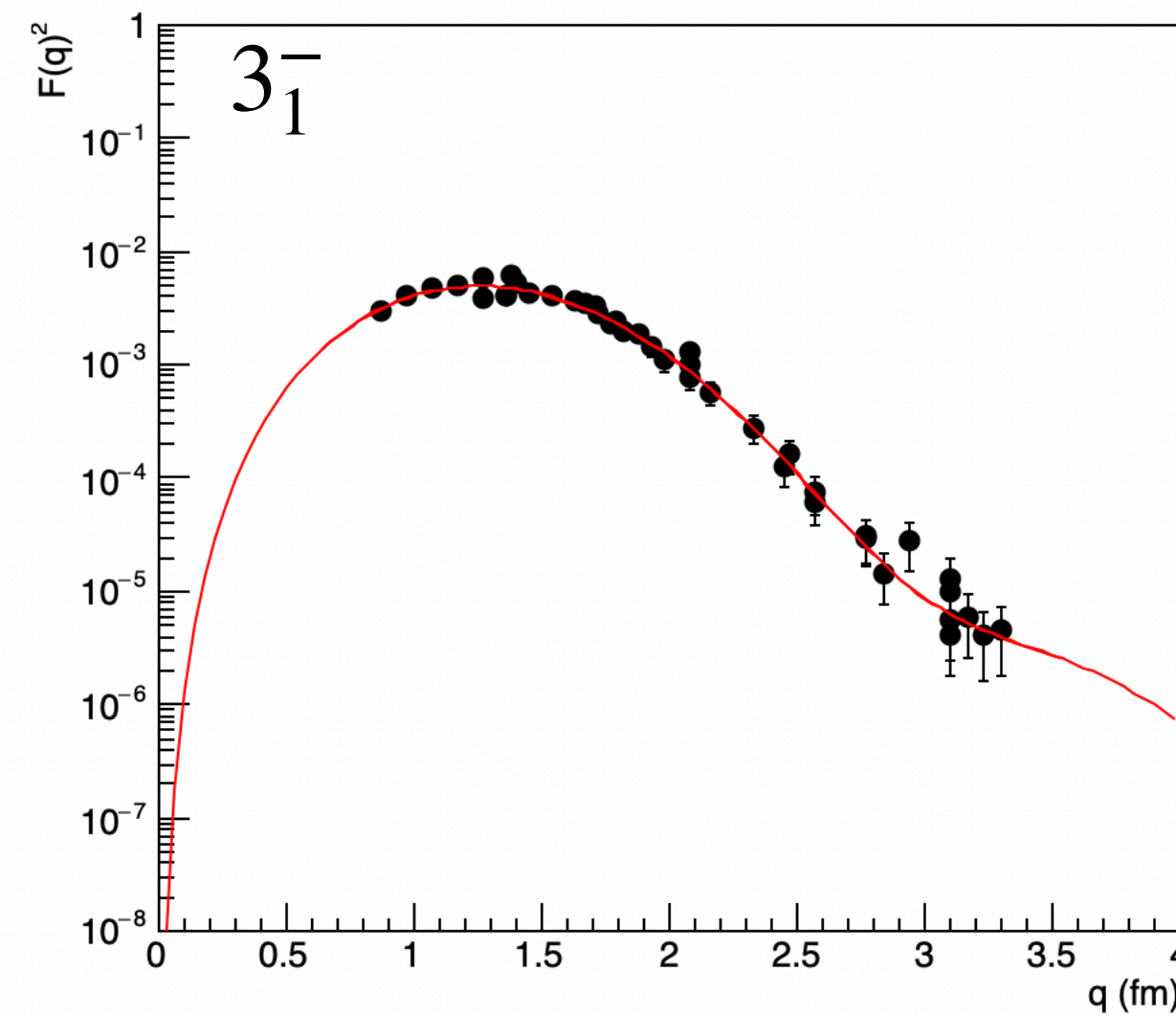
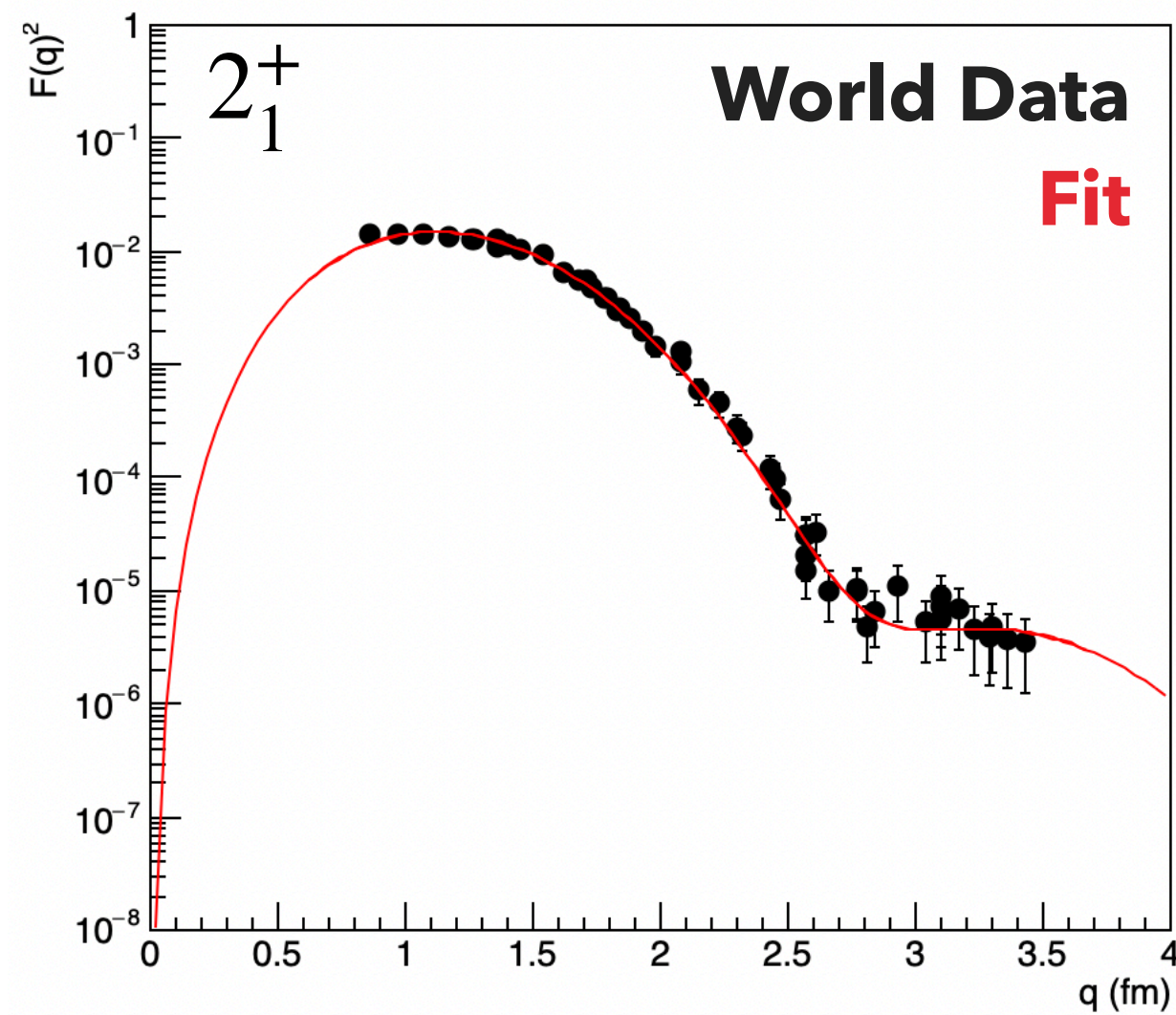
# ELASTIC XS CALCULATIONS, AND ELASTIC TAIL CORRECTIONS



**$^{12}\text{C}$  elastic XS at 400 MeV, 35 degrees**



# EXCITED ELASTIC STATES



Extractions of excited elastic states based on fit of transition form-factors to world data.

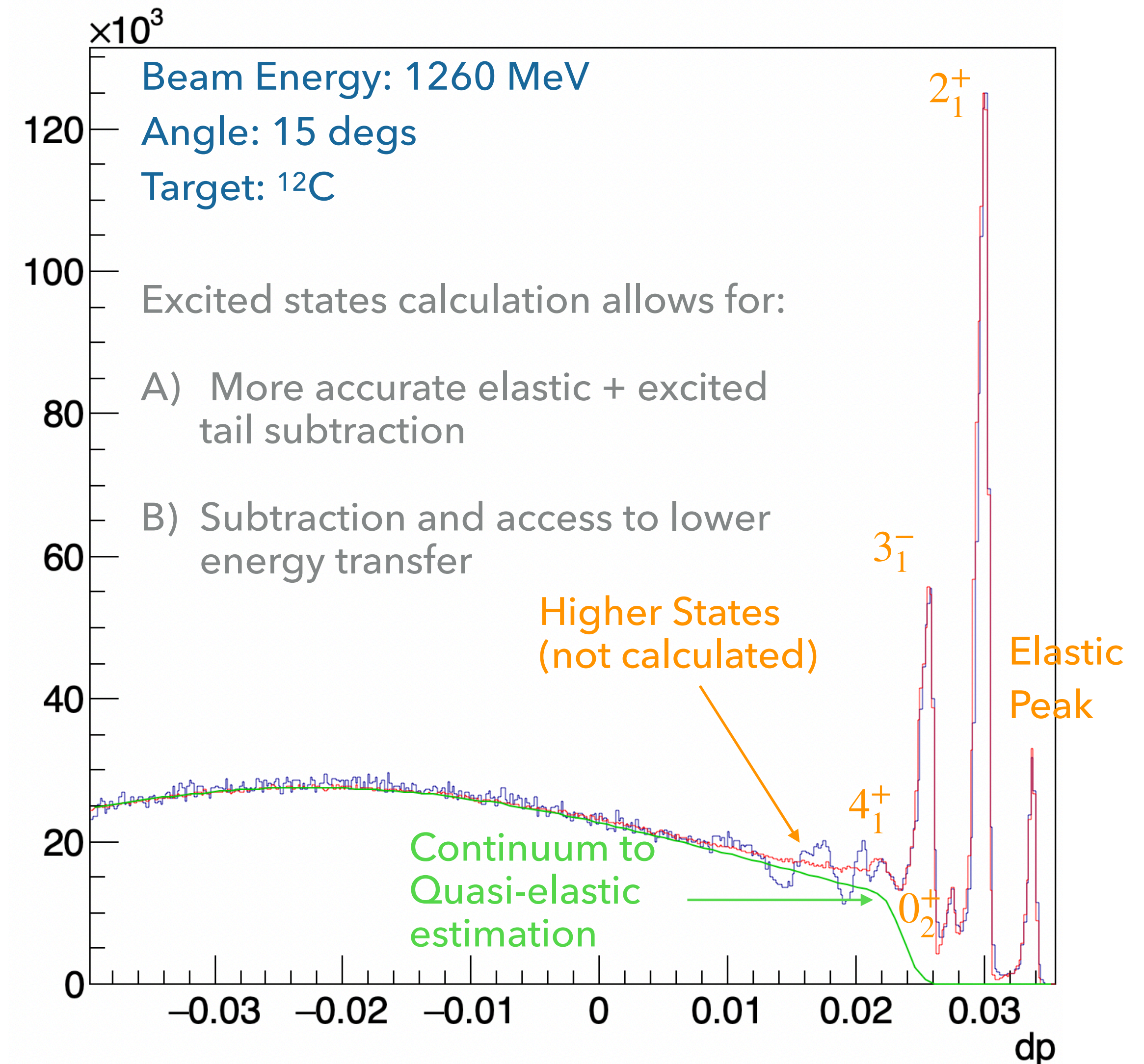
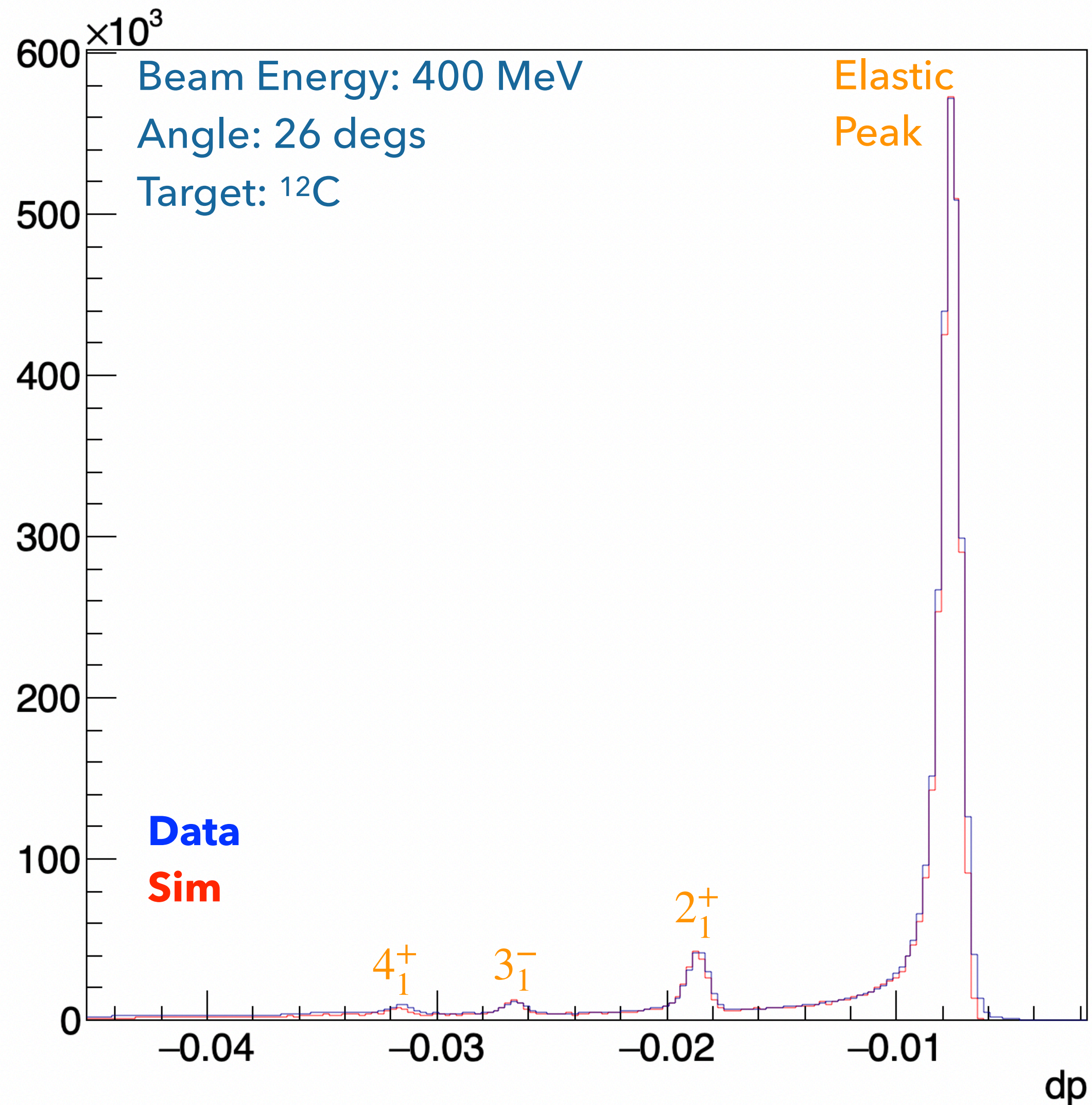
Functional form follows an analytic, global, and model-independent analysis introduced recently\* (mostly in the study of the  $0_2^+$  "Hoyle" state)

$$F(q) = \frac{1}{Z} e^{-\frac{1}{2}(bq)^2} \sum_{n=1}^{n_{\max}} c_n (bq)^{2n}$$

\* M. Chernykh, *et al.* Phys. Rev. Lett. 105



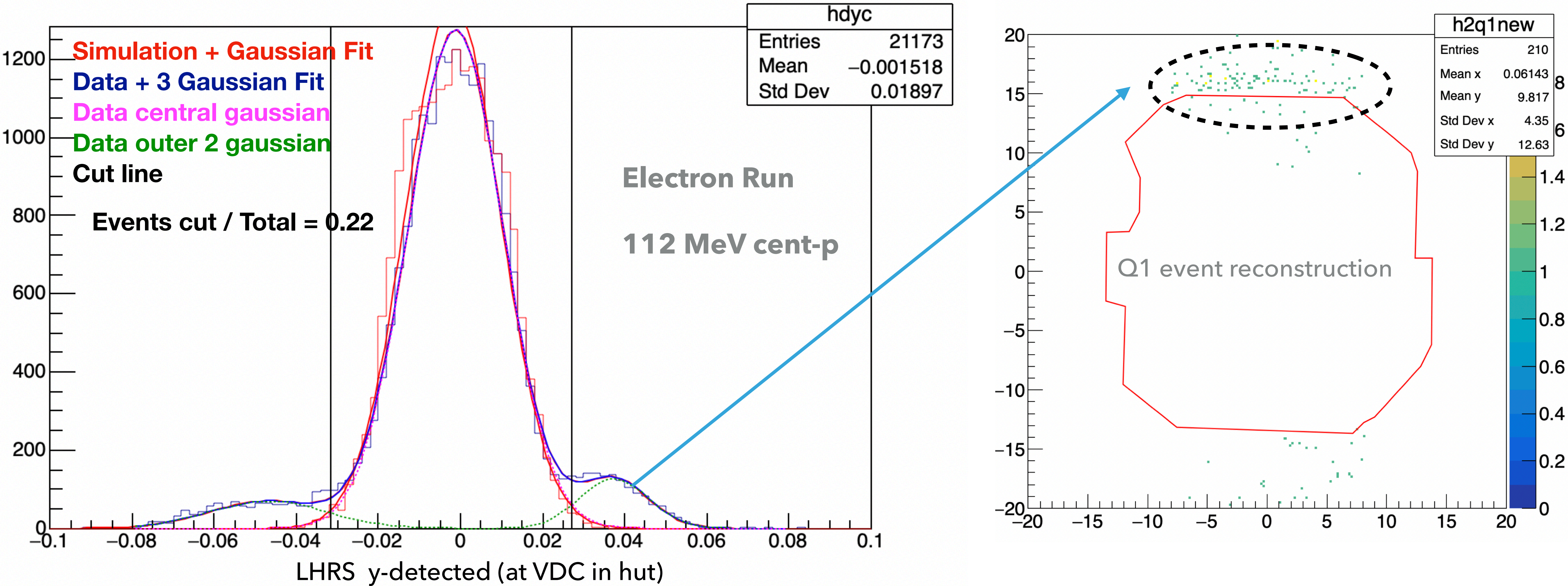
# EXCITED ELASTIC STATES





# MAGNET RESCATTERING

► At low central momentum, magnet-rescattered events may be present.



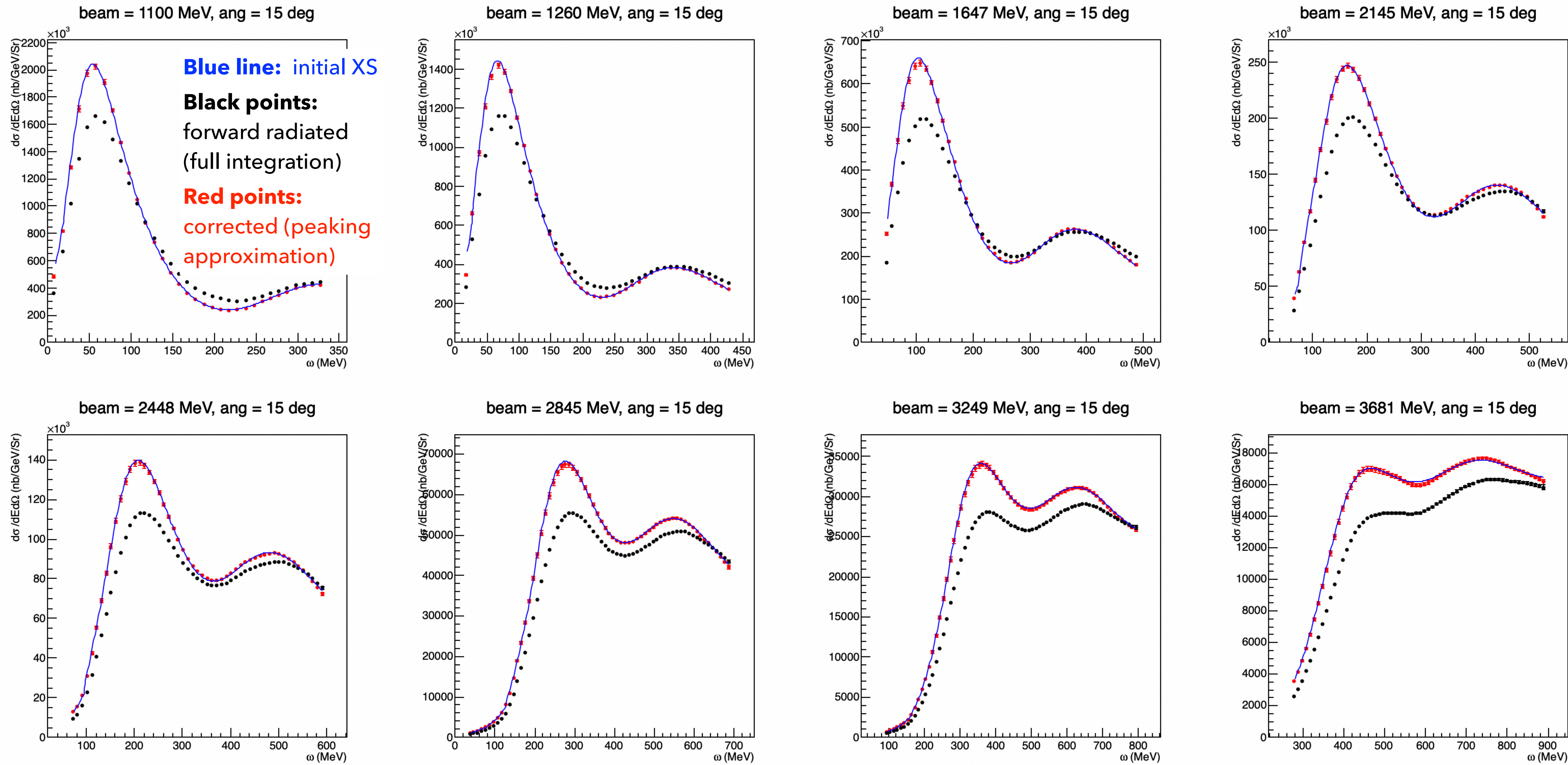


## RADIATIVE CORRECTIONS

- ▶ The radiative corrections are calculated via peaking approximation from Mo & Tsai formalism. Using multiple energy spectra, the corrections are unfolded.
- ▶ As a test to the radiative correction peaking approximation, a full forward-radiating Monte-Carlo was developed using the equivalent radiator method for internal radiation calculation.
- ▶ Test procedure: Use  $^{54}\text{Fe}$  XS predictions (F1F2 Bosted program) for test:
  - ▶ Radiate forward, with full integration over target length, radiated incident energy, and scattered electron energy. Vertex level corrections and ionization effects are also applied.
    - ▶ Use identical energies, angles, and measurement spacings as seen in data.
  - ▶ Use radcor fortran code to do peaking calculations and unfolding for radiative corrections.
    - ▶ Then compare input to output (ideally identical)



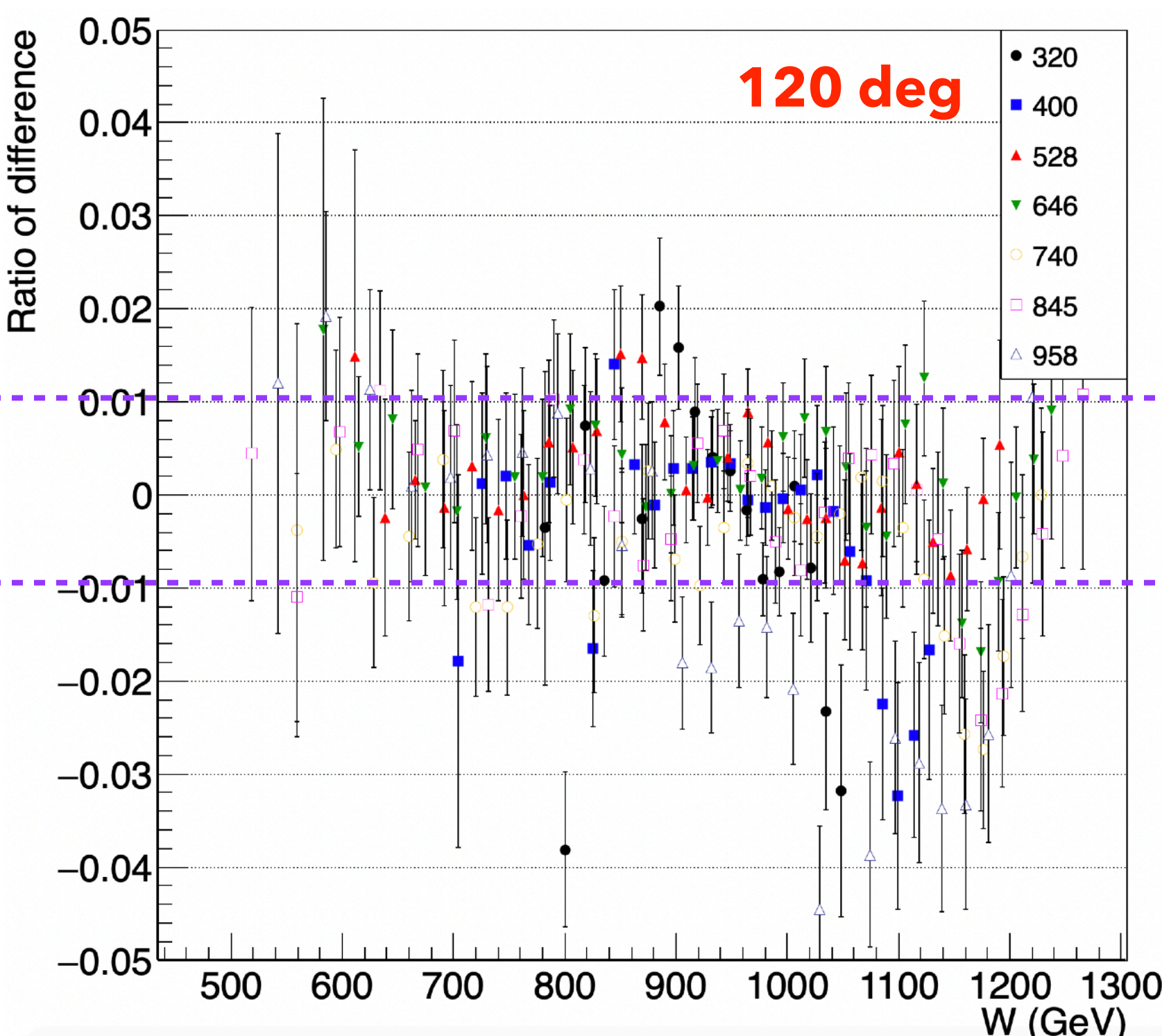
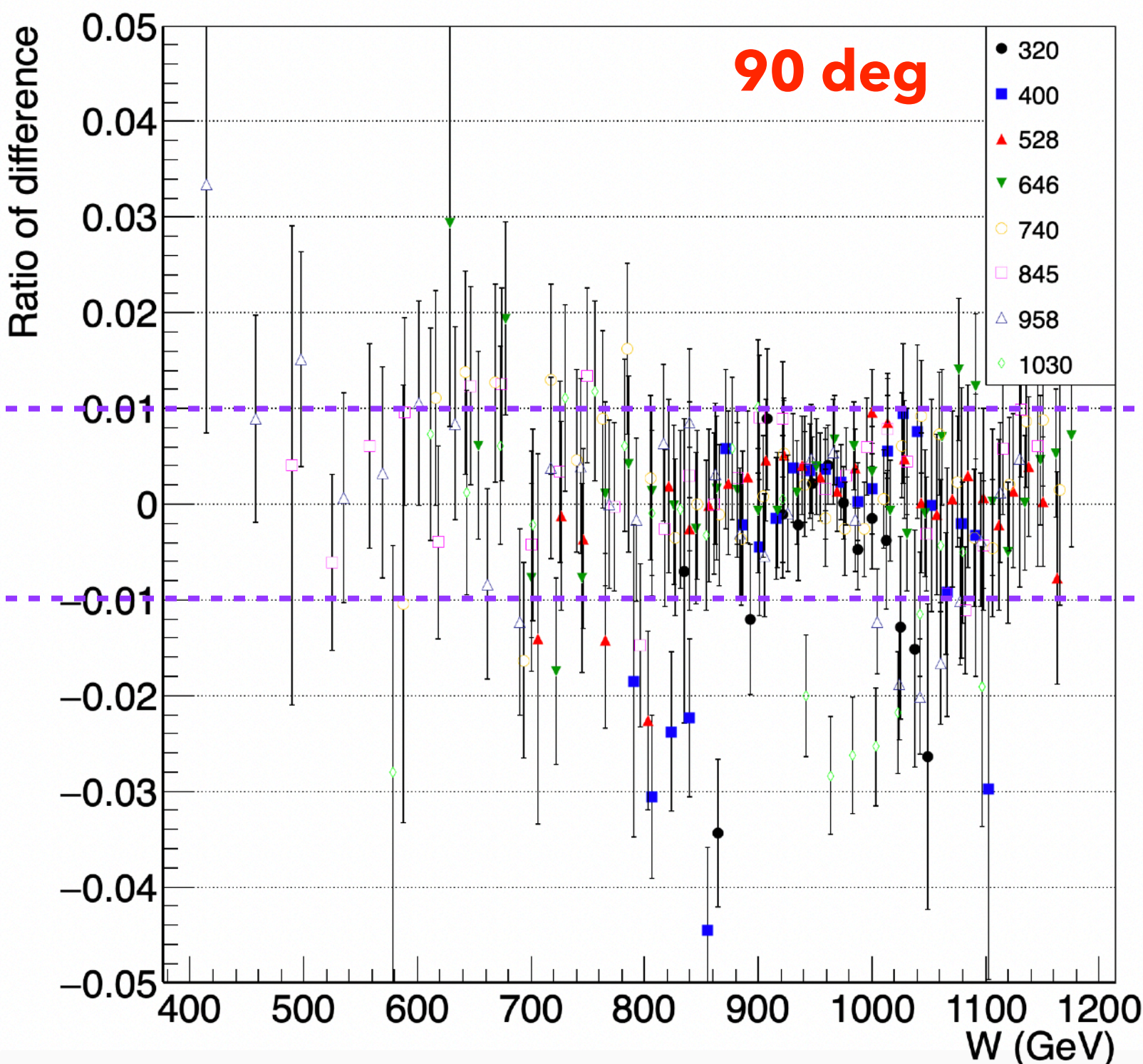
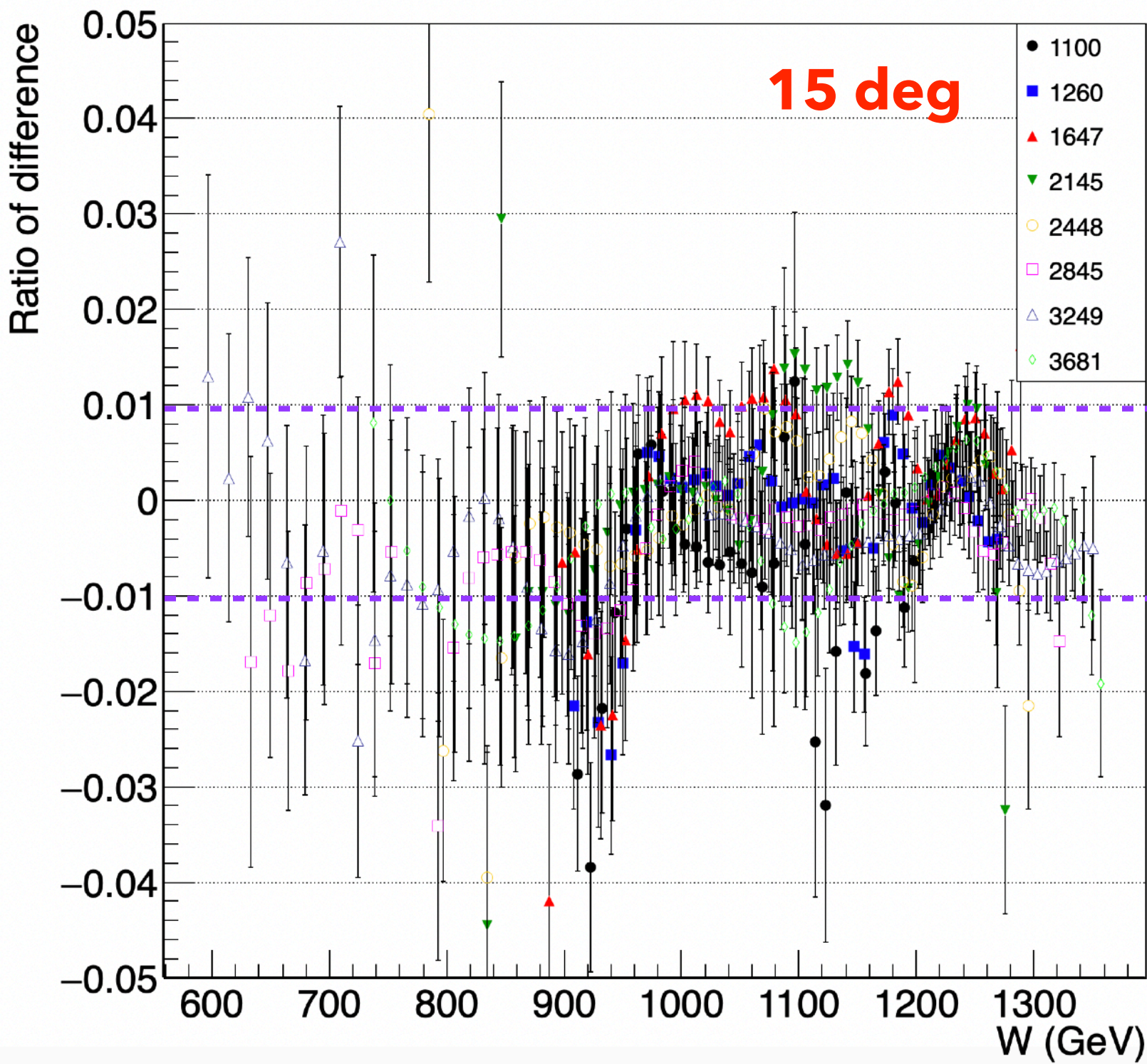
# RADIATIVE CORRECTIONS





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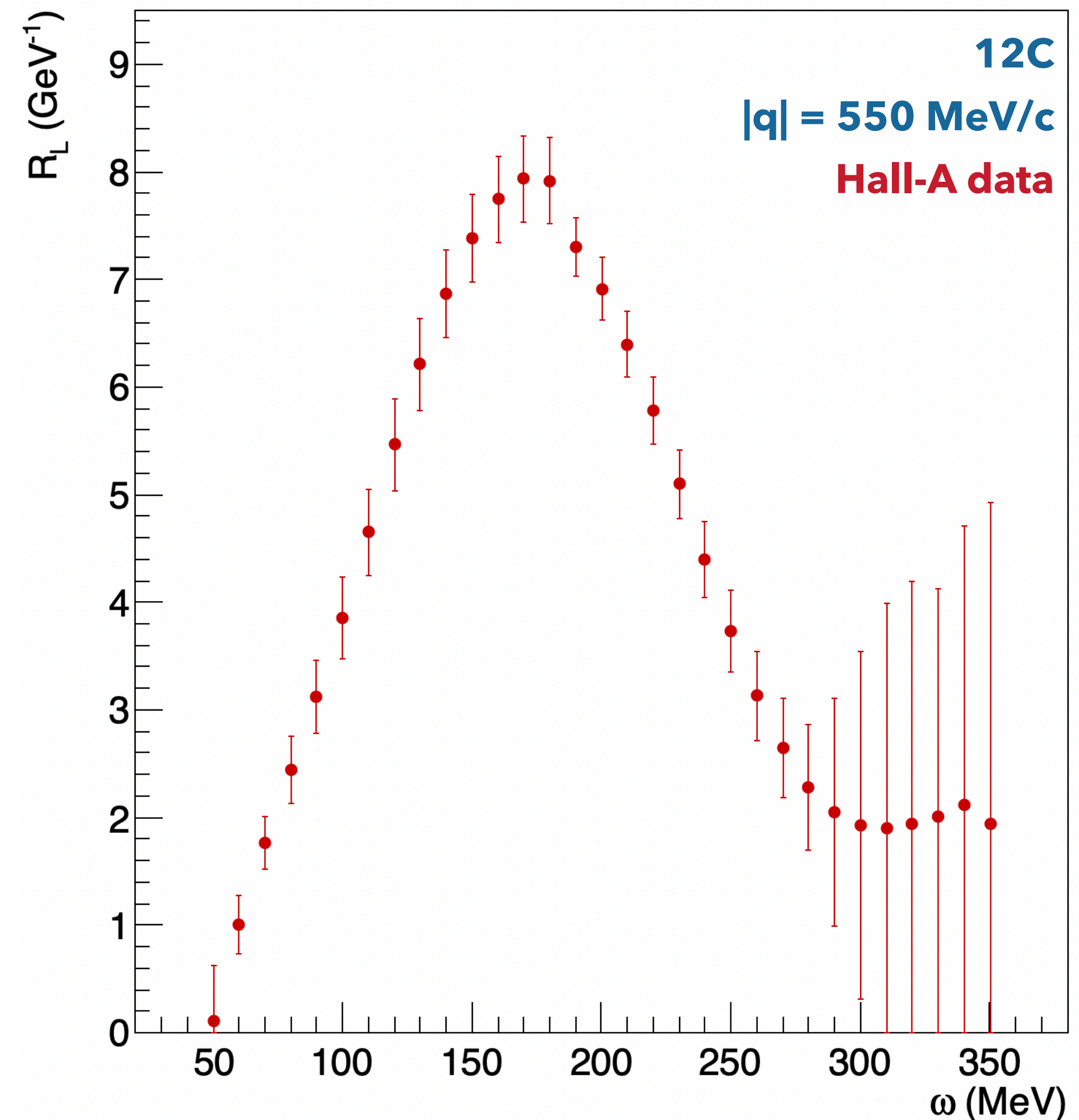
Difference mostly at the  $< 1\%$  level





## COMPARISONS TO EXISTING SUM-RULE CALCULATIONS

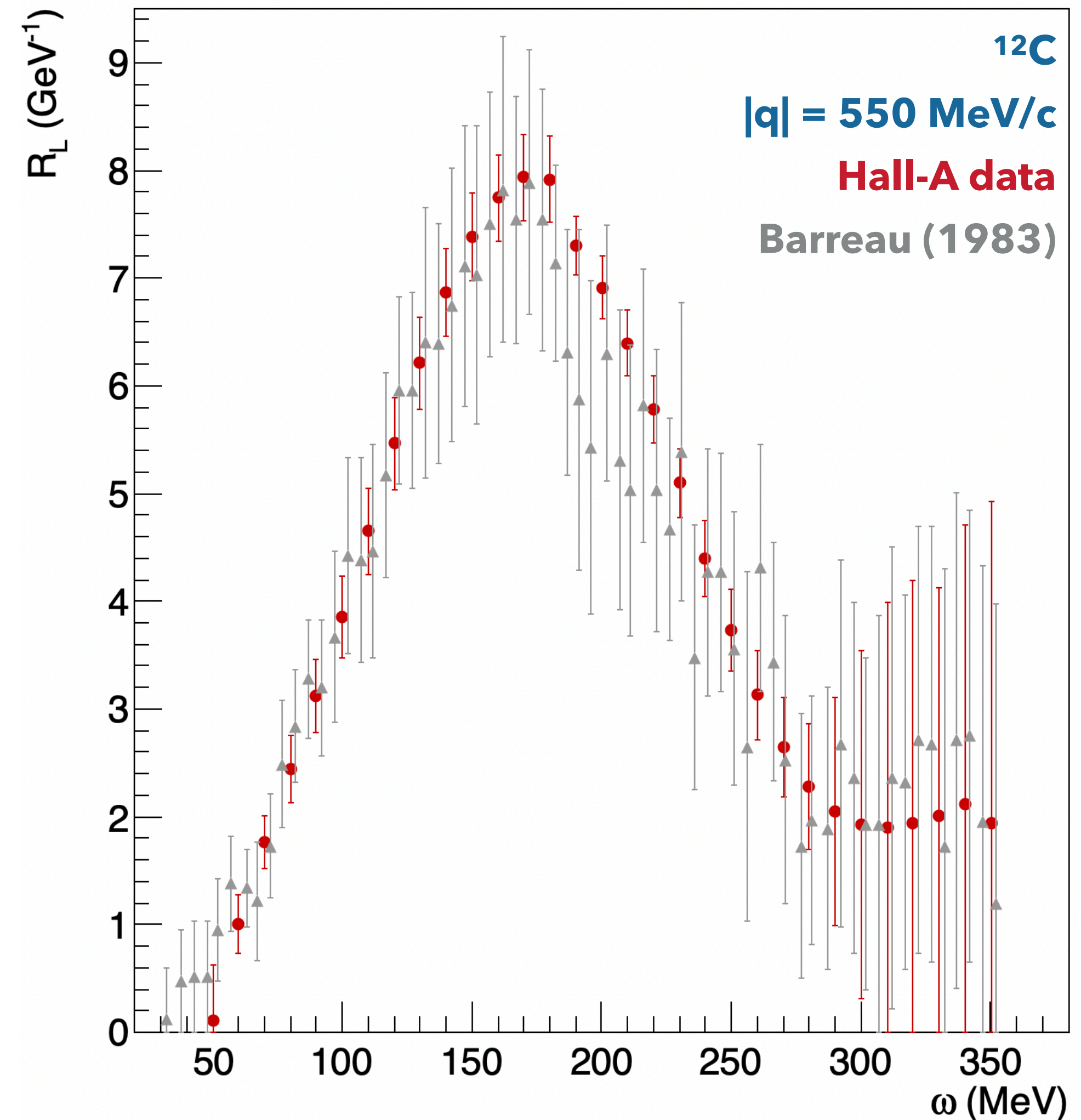
- ▶ We have some coverage at the lowest end of  $|q|$  to compare results at  $|q| = 550$  MeV/c with prior calculations.
  - ▶ Note: This is on the edge of our available phase space. Most slopes are calculated with only 2 or 3 angles.
- ▶ For Carbon:





## COMPARISONS TO EXISTING SUM-RULE CALCULATIONS

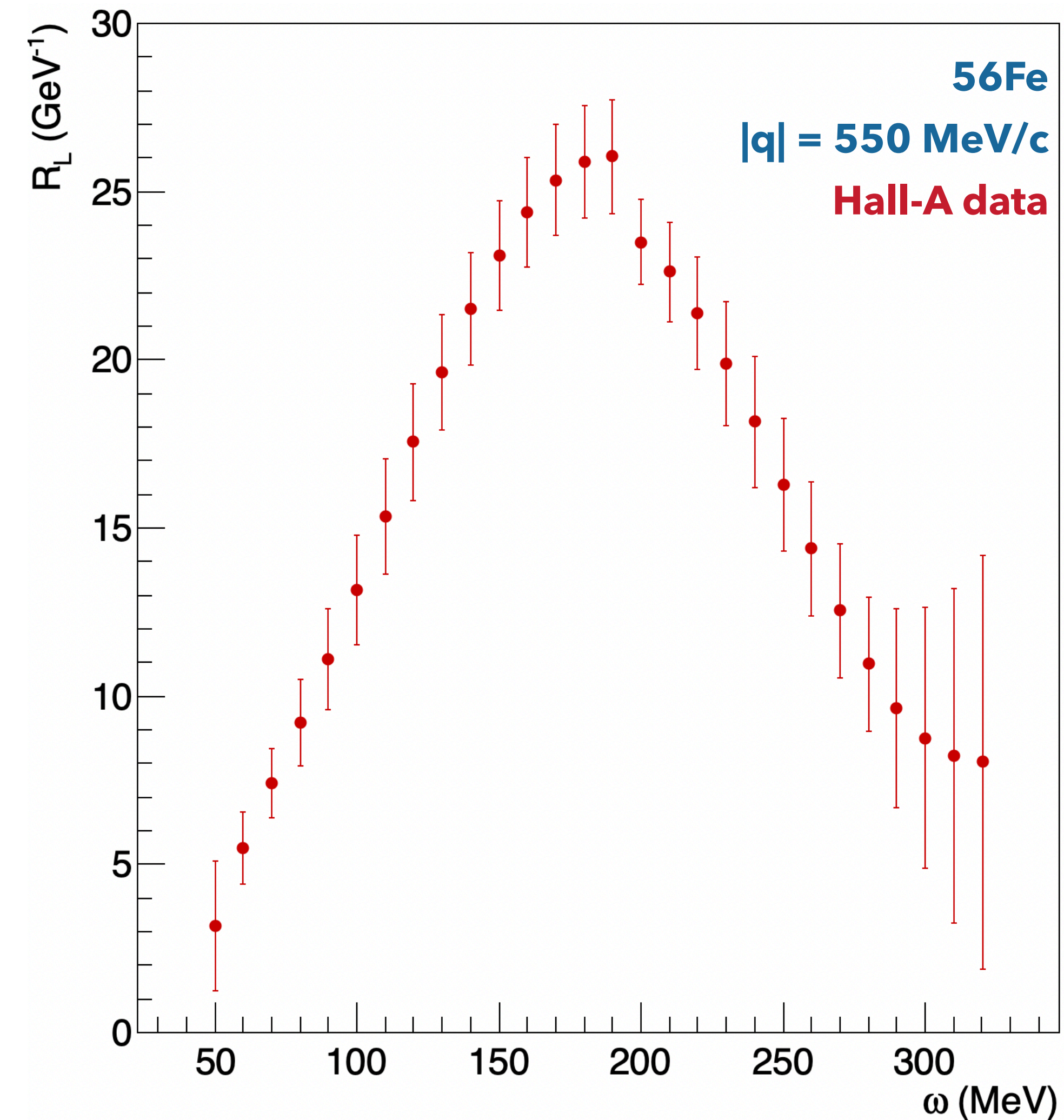
- ▶ We have some coverage at the lowest end of  $|q|$  to compare results at  $|q| = 550$  MeV/c with prior calculations.
  - ▶ Note: This is on the edge of our available phase space. Most slopes are calculated with only 2 or 3 angles.
- ▶ For Carbon: we have very nice agreement with Barreau, et. al.





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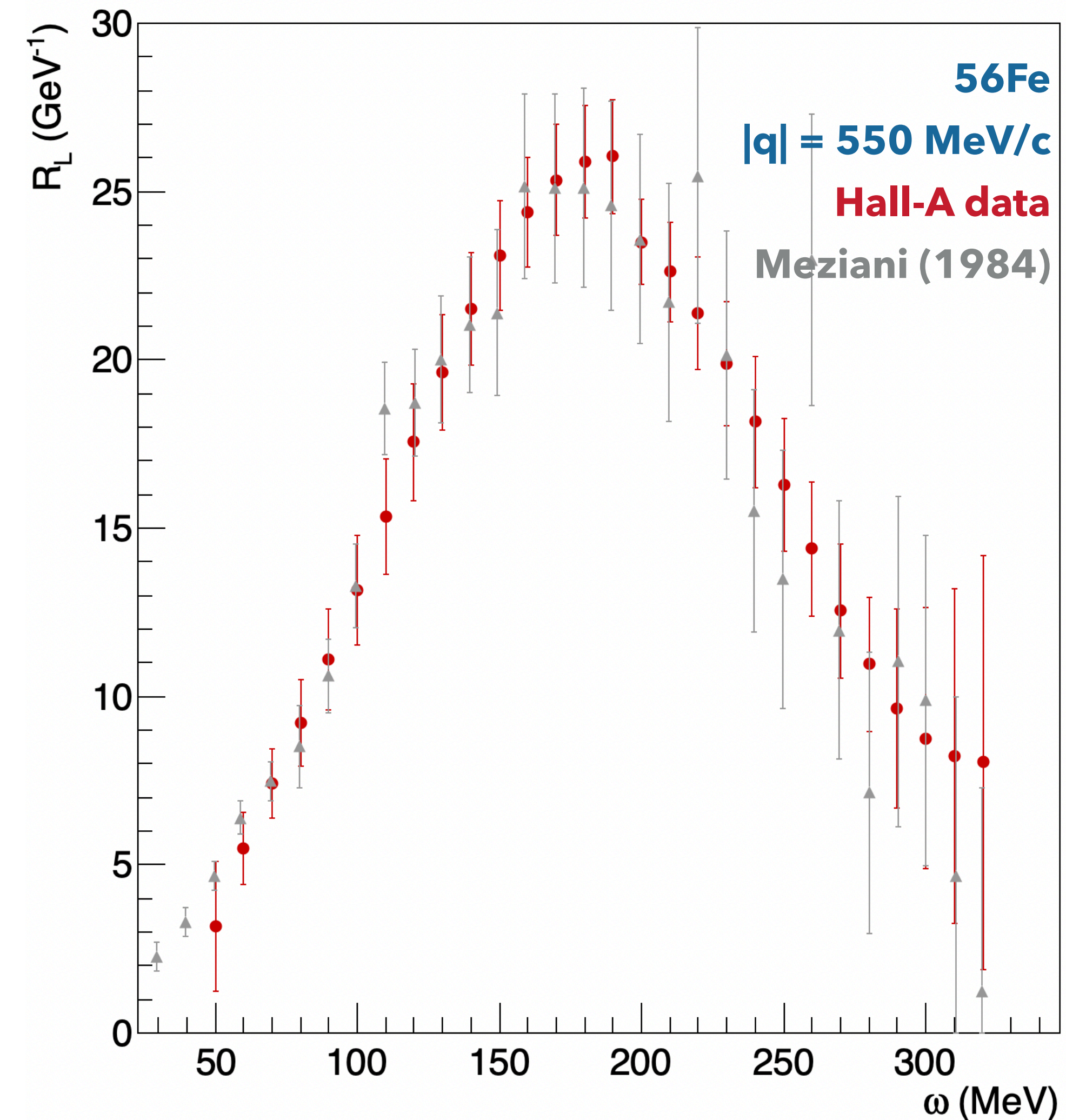
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- ▶ For Iron:





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- ▶ We have some coverage at the lowest end of  $|q|$  to compare results at  $|q| = 550 \text{ MeV}/c$  with prior calculations.
  - ▶ Note: This is on the edge of our available phase space. Most slopes are calculated with only 2 or 3 angles.
- ▶ For Carbon: we have very nice agreement with Barreau, et. al.
- ▶ For Iron: we also have nice agreement with Meziani, et. al.





## SUMMARY / LOOKING AHEAD

- ▶ Recent efforts:
  - ▶ Confirmation of radiative correction technique
  - ▶ Comparison to existing CSR data.
- ▶ Up next:
  - ▶ Systematic studies:
    - ▶ Comparison of Left to Right arm data.
  - ▶ Publish (a draft is in the works).



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and  
*Hall-A collaboration*

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